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Employing hybrid meta-heuristic algorithm for nonlinear regression model estimation

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Abstract Modeling of stochastic phenomenon is widely done using nonlinear regression models and the estimation of parameters problem plays a central role in the inference in nonlinear regression models Nonlinear regression models are applicable in solving optimization problems.As the traditional techniques would fail to produce the satisfactory results in parameters estimation problem.The methods that have to be taken into consideration are Newton-Raphson Method (NRM), Gauss-Newton Method (GNM), The Levenberg-Marquardt Method(LMM), Quasi-Newton methods (QNM), Davidon Fletcher Powell Method (DFP), Broyden Fletcher Goldfarb Shanno Method(BFGSM) and Nonlinear Conjugate Gradient Method (NCG), In this paper we propose A new hybrid algorithm Harris hawks optimization (HHO)that can enjoy the strengths of both the descent algorithms and the natural optimisations algorithms. Via computer simulation, it has been demonstrated how the (HHO-NRM), (HHO-GNM), (HHO-LMM), (HHO- QNM), (HHO-DFPM), (HHO -BFGSM) and (HHO-NCGM) hybrid algorithm- based formula that is powered by the conventional algorithms greatly outperform the conventional algorithms by squared error rates. This was done by use of nonlinear functions. The computational findings indicate that the summed up HHO is favorable over to the numerical techniques (NRM), (GNM), (LMM), (QNM), (BFGSM) and (NCGM) in majority of the problem formulations used in this work.

Keywords Nonlinear Regression, Numerical Parameter Estimation, Harris Hawks Optimization

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1. Introduction

The nonlinear regression models have found quite popular use in domains such as statistics, chemistry, physics, psychology, health science, and biology. There exist a linear correlation between some of the parameters [9, 23, 24, 25].

In both a theory and practice of control engineering, parameter estimation (non-linear estimation in particular) is a highly important issue. In a nutshell the parameter estimation is a multi dimensional numerical optimization challenge, which has several local optimum hence making it extremely difficult to optimally solve, particularly in some class of parameters. As an example, it is demonstrated that time-delay, a special system parameter varying in a different way (not correlated) to other parameters and model order, is extremely hard to estimate it accurately [12, 27, 28].

further and especially interesting area of study in applied statistical sciences is now-a-days nonlinear model building. In Mathematics or even in any other scientific field, formulation of a nonlinear model is definitely one

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of the problems quite facing a research worker. Many nonlinear models have been structure and used effectively in the studies dealing with various situations in real life concerning a number of research concerns in the different aspects of applied mathematics. Nevertheless, the huge number of problems that had not been nonlinearly modelled exist due to either the complexity of situation, or intractability of situations mathematically or statistic-wise [1, 30, 31, 32, 33].

Engineers and computer scientists have already successfully used swarm-based and other ature-inspired metaheuristic algorithms to solve any type of optimization problems. All their optimization problems appear much more complicated and may involve the mixture of continuous and discrete variables. The variables to optimise can easily be in hundreds. I find their ideas of algorithm so relaxing, thrilling, and motivating [15, 34, 35].

The solution of algebraic equations and transcendental equations is one of the objectives practiced in most cases in applied mathematics and it requires arithmetic operations that are very tedious and what is implied under the non-linear equation is any equation that consists of variable powers of (x) or trigonometric or exponential or logarithmic functions or what is known as old fashioned (transcendental function) and we will find that there is no theoretical means of solution or an explicit law can be adopted to seek the roots of such equations so methods of obtaining approximate solutions are used in the case of non-linear [15, 22, 36, 37, 38].

The estimation of parameters of the models of Non-Linear Regression has been tackled through different articles. The nonlinearity theory, in its turn, renders the estimation method and statistical analysis of parameter estimates more, however [**?**, 11, 39, 40] mixed up and difficult. The limitations of these (traditional) methods of estimating parameters that are nonlinear are also mentioned. As well, it is cumbersome to the practitioner to control and so many auxiliary data are needed to operate effectively. Such difficulties are formed due to the great number of parameters used in the goal function and the multimodality of the goal functions. It is very difficult to minimize the mean squared deviations function $S(x)$ by typical optimization methods [**?**]. Consequently, use of advanced meta-heuristic procedures, which possess a number of benefits such as simplicity of implementation, is credible, dependable, and assists in mitigating these predicaments. In order to solve the Non-Linear Multiple regression, methods Harris hawks optimization (HHO) were applied in the paper By hybridising with numerical methods with a view of enhancing the estimation and minimise the error.

2. Methods numerical Estimation Non-linear

The least squares principle is very common in the analysis of non linear and linear models. LS principle is simply a basic notion that requires the minimization of the sums of perversion squared between the observed reaction variable Y and the accommodated value of the model Y [22]. Suppose that the letter ϵ should be employed to refer to the error term, the objective function will be achieved in the following way:

$$S(\beta) = \sum_{i=1}^{n} \left(Y_i - f\left(X_i, \hat{\beta} \right) \right)^2 = \sum_{i=1}^{n} r_i^2 \tag{1}$$

Where; Y: a variable of response, $f(X, \beta)$ with regression model, play of thumb: r_i residual.

Sure that the objective function $S(\beta)$, is known, one performs the following operation; by taking the partial derivative of the objective function $S(\beta)$, with respect to the unknown parameters $(\beta_i \, 's)$ one by one; a normal equation $g(\beta_i) = \frac{\partial S(\beta)}{\partial \beta_i} = 0$ is found. The se normal equations are then solved to get the corresponding estimate results.

Since most nonlineal models lack analytical solutions, iteration method have to be carried out. The most used iteration methods that have been developed and employed to obtain nonlinear least sqaures (NLS) estimates of parameters will be highlighted [17].

2.1. Newton Raphson method (NRM)

Named after Isaac Newton and Raphson, Newton (or Newton Raphson) is a variant of the general technique that is used to find successively better approximations to the solutions (or zeroes) of a real-valued functional equation.

$$f(x) = 0 \tag{2}$$

Newton Raphson algorithm (one variable) is in the following form. Offered a function f defined on the reals x, and it possessing a derivative f, we suppose a first trial x_0 in a root of the function f. In case the function meets all of the assumptions during the derivation of the formula, an improved approximation x_1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}. \tag{3}$$

This is repeated by following the process once again.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \tag{4}$$

Till an adequate accurate value is achieved [3].

2.2. Gauss-Newton method (GNM)

An optimization problem arises when an objective is either minimized or maximized regarding a collection of restraints. In our piece of work the nonlinear least squares problem is treated as a non constrained optimization problem and the SSE is set as the objective. In the course of the computation process the SSE is minimized, but the unknown parameters in the proposed model are estimated in the optimal sense [13]. The unconstraint optimization problem, nonlinear least squares problem, can be characterized here as follows:

$$\min_{x \in \Re^n} f(x) = \frac{1}{2} r(x)^T r(x) = \frac{1}{2} \sum_{i=1}^{m} \left(r_i(x)\right)^2 \tag{5}$$

with objective function $f(x) : \Re^n \to \Re$ which is also known as residual function $r_i(x)$, which is characterized by

$$r_i(x) = \phi(t_i, x) - y_i, i = 1, 2, \ldots, m, \tag{6}$$

Where $\phi(t_i, x)$ is suggested to be the function and y_i i is the observation information. The gradient of $f(x)$ is equal to

$$g(x) = J(x)^T r(x) \tag{7}$$

that is, where $J(x)$ denotes the Jacobian of the residual function $r(x)$. The Hessian matrix is designated by

$$G(x) = \sum_{i=1}^{m} \left(S_i(x) + \nabla r_i(x)^T \nabla r_i(x)\right), \tag{8}$$

where

$$S(x) = \nabla^2 r(x)\, r(x), \tag{9}$$

2.3. The Levenberg-Marquardt method (LMN)

The LevenbergMarquardt algorithm adjustively mixes the coefficients of updating both to the gradient descent update and that of GaussNewton as per given need.

$$\left[J^\top W J + \lambda I\right] h_{lm} = J^\top W (y - \hat{y}), \tag{10}$$

in which very small values of the damping coefficient lambda lead to a Gauss Newton update and big values of lambda corresponding to a gradient decline update. To have small initial updates as first steps in the steepest-descent direction, the damping coefficient λ is initially large. When chance occurs that one of the iterations lead to a worse approximation (when $\left(\chi^2\left(a+h_{lm}\right)>\chi^2(a)\right)$, then lambda is increased. Otherwise, the better the solution the smaller the 1, the Levenberg-Marquardt is close to (GNM) and locally the solution generally speeds to the local minimum [7, 41, 42].

In the update relationship by Marquardt [13], a learning rate (sometimes called damping coefficient λ) is multiplied by the diagonal of the Hessian $J^\top W J$ per coefficient.

$$\left[J^\top W J+\lambda diag\left(J^\top W J\right)\right] h_{lm}=J^\top W(y-\hat{y}) \tag{11}$$

2.4. Quasi-Newton methods(QNM)

A family of numerical optimization algorithms (which find local optima or optima of functions, or zeros of functions) based on iterative processes; they involve estimating the Hessian matrix of second derivatives needed at each iteration, without necessarily computing it. In contrast to Newton methods they do not necessarily require that the Hessian matrix (or Jacobian in the case of systems) be explicitly computed; rather, they involve the incremental construction of an approximation of the Hessian (or inverse Hessian) using only gradient data, thus being more convenient and efficient computationally both in large-scale settings and in those situations where the Hessian is more costly to calculate exactly [17].

$$S\left(\beta^k+\delta^k\right)=S\left(\beta^k\right)+\nabla S\left(\beta^k\right)^T\delta^k+\frac{1}{2}\delta^{k^T}H_k\delta^k \tag{12}$$

where; beta k: current inference; delta k: step size in the k^{th} iteration; $\nabla S\left(\beta^k\right)$ = -first derivative at the actual estimate; H_k: Hessian matrix. General iterative GNM algorithms is

$$\beta^{k+1}=\beta^k-\alpha_k H_k^{-1}g_k \tag{13}$$

g_k: gradient and α_k: step size selected using line search.

2.5. Davidon-Fletcher-Powell method(DFPM)

DavidonFletcherPowell (DFP) is a classical form of quasiNewton optimization algorithm applied to finding the optimum of nonlinear, unconstrained minimization problems. It was the earliest generalisation of the secant method to the multiple dimensions of which updates an approximation to the matrix inverse Hessian of the objective function iteratively and used it to direct the search direction in which to minimize the objective function [2]. The approximation of the Hessian matrix is carried out in the Eq. (14).

$$H_{k+1}=\left(I-\frac{\varphi_k\delta^{k^T}}{\varphi_k{}^T\delta^k}\right)H_k\left(I-\frac{\delta^k\varphi_k{}^T}{\varphi_k{}^T\delta^k}\right)+\frac{\varphi_k\varphi_k{}^T}{\varphi_k{}^T\delta^k} \tag{14}$$

where

$$\varphi_k=\nabla S\left(\beta^k+\delta^k\right)-\nabla S\left(\beta^k\right)$$

and

$$\gamma_k=\frac{1}{\varphi_k{}^T\delta^k}$$

As described, the other consequence of the approach is the formula to guide approximation to the inverse Hessian matrix in the same way as the Eq. (15).

$$B_{k+1}=B_k+\frac{\delta^k\delta^{k^T}}{\delta^{k^T}\varphi_k}-\frac{B_k\varphi_k\varphi_k{}^TB_k}{\varphi_kB_k\varphi_k^T} \tag{15}$$

2.6. The Broyden Fletcher-Goldfarb-Shanno (BFGSM) method

Algorithm is a common iterative scheme to unconstrained nonlinear optimization problems. It is one of the quasi-Newton methods, where the Hessian matrix (second derivative) of the objective function is approximated only without evaluating the direct calculation of Hessian, which is complex and computationally time-consuming.

As the Eq. (16) displays, the approximation formula of Hessian appears as follows [8, 17].

$$H_{k+1} = H_k + \frac{\varphi_k \varphi_k^T}{\varphi_k^T \delta^k} - \frac{H_k \delta^k \delta^{k^T} H_k^T}{\delta^{k^T} H_k \delta^k} \tag{16}$$

Another value pointed by the method is to a explicit approximation to inverse Hessian as in the Eq. (17).

$$B_{k+1} = \left(I - \frac{\delta^k \varphi_k^T}{\varphi_k^T \delta^k}\right) B_k \left(I - \frac{\varphi_k \delta^{k^T}}{\varphi_k^T \delta^k}\right) + \frac{\delta^{k^T} \delta^k}{\varphi_k^T \delta^k} \tag{17}$$

where

$$\varphi_k = \nabla S\left(\beta^k + \delta^k\right) - \nabla S\left(\beta^k\right) \tag{18}$$

2.7. Nonlinear Conjugate gradient method (NCG)

(NCG) Method Extended to the solution of nonlinear optimization, i.e. unconstrained minimization of nonlinear functions. As opposed to the linear conjugate gradient method, used to solve the large linear system whose coefficient matrix is positive definite, the nonlinear form is a means of optimizing on the local minima to nonlinear functions with only the gradient information available. The algorithm (NCG)

Algorithm 1 NCG

Set 1: $p_{-1} = 0, \beta_0 = 0,$
Set 2: For $i = 0, 1, \ldots$
Set 3: If $\|\nabla f(xi)\| < \epsilon$, stop.
Set 4: If $i > 0$, set $\beta i = \nabla f\left(x_i\right)^T \nabla f\left(x_i\right) / \nabla f\left(x_{i-1}\right)^T \nabla f\left(x_i - 1\right)$
Set 5: Set $p_i = -\nabla f\left(x_i\right) + \beta_i p_{i-1}$.
Set 6: Use $x_{i+1} = x_i + \alpha_i p_i$.

The algorithm can be used to unconstrained general problems: minimize $f(x)$.[14]

3. Nonlinear regression models

Some of the nonlinear regression models are important as given below [19, 14, 16, 15].

1. Militky Model 2(Militkty2)
$$\exp\left(\beta_1 x\right) + \exp\left(\beta_2 x\right) \tag{19}$$
2. Ratkowsky Model 2(Ratkowsky2)
$$\frac{\beta_1}{1 + \exp\left(\beta_2 - \beta_3 x\right)} \tag{20}$$
3. Eckerle Model 4(Eckerle4)
$$\frac{\beta_1}{\beta_2} \exp\left(\frac{-\left(x - \beta_3\right)^2}{2\beta_2^2}\right) \tag{21}$$
4. Ratkowsky Model 3(Ratkowsky3)
$$\frac{\beta_1}{\left(1 + \exp\left(\beta_2 - \beta_3 x\right)\right)^{\frac{1}{\beta_4}}} \tag{22}$$

5. MGH09 Rational Function Model (MGH09)

$$\frac{\beta_1\left(x^2+x\beta_2\right)}{x^2+x\beta_3+\beta_4} \tag{23}$$

6. Roszman Model 1(Roszman1)

$$\beta_1-\beta_2 x-\frac{\arctan\frac{\beta_3}{x-\beta_4}}{\pi} \tag{24}$$

7. Misrald Saturation Growth Model (Misrald)

$$\frac{\beta_1\beta_2 x}{1+\beta_2 x} \tag{25}$$

8. Misralc Square Root Model (Misralc)

$$\beta_1\left(1-\frac{1}{\sqrt{1+2\beta_2 x}}\right) \tag{26}$$

9. Nelson Exponential Interaction Model (Nelson)

$$\exp\left(\beta_1-\beta_2 x_1\exp\left(-\beta_3 x_2\right)\right) \tag{27}$$

10. Misralb Rational Saturation Model (Misralb)

$$\beta_1\left(1-\frac{1}{\left(1+\frac{\beta_2 x}{2}\right)^2}\right) \tag{28}$$

11. DanWood Power Law Model (DanWood)

$$\beta_1 x^{\beta_2} \tag{29}$$

4. Harris Hawks optimization (HHO)

In the given part we represent the exploration, exploitation stages of the suggested HHO based on the exploration of fur, surprise attack, and attacking patterns of HHO. HHO is a population-based strategy for optimization that does not rely on gradients, making it adaptable to any problem as long as the problem is clearly defined. The complete sequence of operations for HHO is outlined in Fig. 1, and the next subsections will detail each of these steps.

4.1. Exploration phase

In this paragraph, the HHO exploration mechanism is submitted. When we put into consideration the type of hawks that were possessed by Harris, they can locate and feel the quarry by their sharp sighted eyes and sometimes the quarry may be too hard to recognize. The hawks consequently wait, watch and scan the desert region to quarry on a victim possibly after hours. According to HHO, the Harris hawks will be the solutions and the fittest in each step will accumulate as the desired quarry named as the almost the optimum solution. After conducting the two strategies, the hawks of Harris in HHO randomly sit on portions of some site and await the moment when it detects a quarry. When q, the probability of striking, is equal with all perching strategies they perch on the position of other members of the family (to get close to them) and the bunny itself [10], which is represented in Eq. perch on random tall trees (random sites within the home range of the group), (1) when in the state of $q<0.5$, or (2) when in the state of $q<0.5$, and these two cases are modeled in Eq. (30). when $q>=0.5$.

$$X\left(t+1\right)=\begin{cases}X_{rand}\ \left(t\right)-r_1\left|X_{rand}\ \left(t\right)-2r_2X\left(t\right)\right| & q\geq 0.5\\ \left(X_{bunny}\ \left(t\right)-X_m\left(t\right)\right)-r_3\left(LB+r_4\left(UB-LB\right)\right) & q<0.5\end{cases} \tag{30}$$

in which, $X(t+1)$ is the position of hawks at the next iteration t, $X_{bunny}\ (t)$ is position of the bunny, X(t) is current position of hawks, r_1, r_2, r_3, r_4, and q are random numerals between $(0, 1)$, which are modernized according to iteration, LB , UB are the upper and more down limits of the variable, $X_{rand}\ (t)$ is a haphazardly chosen hawk of a current inhabitance of hawks, and X m is average position of current inhabitance choosing of h.

To randomly create sites within the home range of the group (LB, UB), we suggested a fictitious model. First rule comes up with solution depending on random site and other hawks. The variation between the position of beter so far and the average function of the set and a ingredient with random scaling founded on the range of the variables is present in second base of Eq. (1), r_3 is another scaling coefficient that makes the random nature of base even more random because when r_4 accepts near values to 1 and follows close distribution patterns then they may appear. What we do in this base is add any randomly scaled length of movement to the LB. Next, we enabled random scaling coefficient of the ingredient to get access to additional diversification patterns and see some other areas in the feature space. Different updating rules can be built, although we used the most basic one that could replicate behaviors of hawks. By means of Eq. (31), the mean site of hawks can be gained:

$$X_m\,(t) = \frac{1}{N}\sum_{i=1}^{N} X_i\,(t) \tag{31}$$

In which $X_i(t)$ represents the position of each of the hawks on loop t and N the number of hawks. The average site can be derived in various manners but we used the rule of thumb.

4.2. Shifting to exploitation transition

HHO algorithm is able to switch between exploration and utilization and then switch between various profiteerer actions depending on the flee vigor of the quarry. Through the escaping behavior the vigor of a quarry is reduced significantly. In order to model this truth, vigor of a quarry is assumed to behave as follows:

$$E = 2E_0\left(1 - \frac{t}{T}\right) \tag{32}$$

where E is a rate of escaping vigor of the quarry, T is a maximum number of iterations and E_0 is the initial condition of its vigor. At every iteration, E_0 randomly varies within the range $(-1, 1)$ in HHO. Physically when the value of E_0 changes to negative one, the bunny is physically flagging and when the value of E_0 changes to positive one then the meaning is the bunny is truth. The kinetic vigor E used to escape has the mode of decreasing trend in the iterations. As the escaping vigor $|E|$ is increasing $|E| \geq 1$, the hawks explore other areas to seek a bunny site, and therefore, HHO undertakes the exploration steps, and on the other side, when $|E| < 1$ then the algorithm tries exploring the neighborhood of the solutions in the exploitation stages. Concisely, exploration is when $|E| \geq 1$ but exploitation occurs in subsequent steps when $|E| < 1$. Time dependent behavior of E is also illustrated.

4.3. Exploitation phase

At this stage, the Harris hawks take the surprise pounce (seven kills as referred as [5] in that they take on the target quarry identified during the preceding phase. But in most cases, quarrys usually escape when they are in a do-or-die situation. Therefore, in the real life, various chasing styles are present. The four possible strategies thus suggested in the HHO model the attacking stage based on the escaping patterns of the quarry and the chasing patterns of the Harris hawks.

The quarrys are always willing to flee life threatening situations. Assume that r is the probability of a quarry who either will manage to survive $((r < 0.5))$ or will not manage to survive $(r \geq 0.5)$ before surprise pounce. The hawks will, whatever the quarry does, besiege it hard or soft, in order to get it. It implies that they will approach the quarry at various directions either making soft or hard attacks according to the amount of stored vigor of the quarry. In the real life, the hawks must come closer and nearer to the target to have a better run killing the bunny cooperatively by executing the surprise pounce. After some few minutes, the fleeing animal will become weaker and weaker; at this time the hawks will increase the siege action so that they easily nab the dying animal. In order

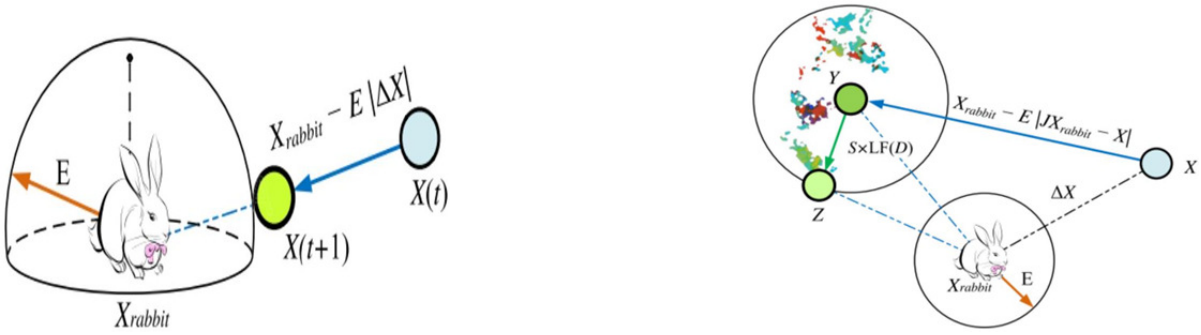
to simulate this plan and allow the HHO to change the soft and the hard besiege processes, E parameter is used. In this aspect, the soft besiege takes place when $|E| \geq 0.5$ and the hard one takes place when $|E| < 0.5$.

4.3.1. Soft besiege In case r is not negative and not less than 0.5 and $|E|$ the same then to start with its bunny has not yet run out of vigor and it will make some misleading attempts to jump away in some haphazard and random directions but it is unable to. In these efforts the Harris hawks surround it gently and so render the bunny more fatigued and then execute the surprise leap. Such behavior is exemplified by the following rules:

$$X\left(t+1\right) = \Delta X\left(t\right) - E|X_{bunny}\left(t\right) - X\left(t\right)| \tag{33}$$

$$\Delta X\left(t\right) = X_{bunny}\left(t\right) - X\left(t\right) \tag{34}$$

in which $\Delta X(t)$ refers to the difference between the position vector of the bunny and the current position in iteration t, r_5 a random number within $(0, 1)$ and $J = 2\left(1 - r_5\right)$ the random magnitude of the jump of the bunny used in the escaping process. The J value will be random at each iteration to contain the nature of bunny movements.



(a) (b)

Figure 1. Step for one hawk, (a) General vectors example in case of tough blockade; (b) Example of general vectors in the case of soft blockade with progressive quick dips.

4.3.2. Hard besiege In case $r \geq 0.5$ and $|E| < 0.5$, the quarry is victims to such an extent, and it has little escaping vigor. Moreover, the hawks of Harris do not spend a lot of time surrounding a possible victim before making that unexpected attack. Here, the existing (old) positions are changed to the latest ones according to the Eq.(35):

$$X\left(t+1\right) = X_{bunny}\ \left(t\right) - E\left|\Delta X\left(t\right)\right| \tag{35}$$

It is conveniently illustrated by a simple example of one hawk in Fig. 1.

4.3.3. Low besiege with gradual rapid plunge Even when $|E|$ is as low as 0.5 but $r < 0.5$ there is plenty of vigor left to evade and a gentle besiege is built before the ambush. This process is smarter as opposed to the former example.

The HHO algorithm utilizes the levy flight (LF) concept to model escaping characteristics of the quarry in mathematical terms, as well as leapfrog behavior (which is so called in [5]). The LF is used to give the simulation of the real zigzag fake movement of the quarrys (especially bunny s) during their escaping phase and intermittent and sudden and rapid escaping by the hawks about the escaping quarry. Actually, the hawks perform various team rapid flights around the bunny and strive to get more and more coordinated in their sites and movements to the deceptive quarrys. The other realistic observations in other competitive cases in nature also mark the support of the mechanism. It has been established that among the foragers/predators, LF-based activities are the best searching strategies under non-destructive foraging conditions. Moreover, amongst others, it has been observed that the LF-based patterns are observed in animals such as monkeys during chasing behaviors and the shark. Therefore, the motions of LF were applied in this stage of HHO technique [10, 20].

Based on the observation of actual actions of hawks, we assumed that hawks have the ability to gradually determine the most adequate dive to the quarry when they have the plan of capturing the quarry during the competitive instances. Hence, in order to achieve a soft besiege, we assumed that, in response to the given Eq.(36) the hawks are able to assess (decide) their next step relying on the given rule:

$$Y = X_{bunny}(t) - E\left|JX_{bunny}(t) - X(t)\right| \tag{36}$$

Then, they compare the potential outcome of such a moving with the previous dive to identify that will it be good dive or not. Not being reasonable (when they notice that the quarry are carrying out more deceptive movements), they also begin to make irregular, sudden, quick dives at the bunny. Using the next rule we assumed that they will plunge depending on the LF-based patterns as shown:

$$Z = Y + S \times LF(D) \tag{37}$$

where D is size of problem and S is a random vector of size $1 \times D$ and LF is the levy flight calculating executed with the help of Eq. (38):

$$F(x) = 0.01 \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}}, \sigma = \left(\frac{\Gamma(1+\beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{\left(\frac{\beta-1}{2}\right)}}\right)^{\frac{1}{\beta}} \tag{38}$$

the values of u, v are random numbers within $(0, 1)$, β, cannot change 0, 1, and the code 1.5 is a constant default. Therefore, updating the hawks in soft besiege strategy may be completed with the help of a final strategy, which can be carried out with the use of Eq. (38):

$$X(t+1) = \begin{cases} Y & if\ F(Y) < F(X(t)) \\ Z & if\ F(Z) < F(X(t)) \end{cases} \tag{39}$$

where Y and Z are found with the help of Eqs. It is Eq.(7) and Eq. (8). It is just a matter of illustrating this step in the case of one hawk as illustrated in Fig. 1. It is important to note here that the history of the positions of LF-based leapfrog movement patterns across some iterations are also stored and presented, as well, by this illustration. The footprint locations of LF-based patterns (commands) are the colored dots in the one experiment and then, the HHO goes to the position Z. At every step, we will only pick the superior place among Y or Z and chose it as the next place. This approach is incorporated on all search agents [10, 20, 21, 22].

4.3.4. Hard besiege with gradual fencing dives In case $|E| < 0.5$ and $r < 0.5$, the bunny does not possess sufficient vigor to overcome it and a rigorous siege is built before the ambush jump to put and kill the quarry. This case in the quarry side is similar to the case of the soft besiege, except that in this one, the hawks attempt to narrow the average sites between themselves and the quarry who is escaping. Hence, the ensuing rule is executed during hard besiege condition:

$$X(t+1) = \begin{cases} Y & if\ F(Y) < F(X(t)) \\ Z & if\ F(Z) < F(X(t)) \end{cases} \tag{40}$$

where Y and Z are calculated with new rules in Eqs. In (41) and (42).

$$Y = X_{bunny}(t) - E|X_{bunny}(t) - X_m(t)| \tag{41}$$

$$Z = Y + S \times LF(D) \tag{42}$$

with $X_m(t)$ calculated with the help of the Eq. (31).

The pseudocode for the suggested Harris Hawks Optimizer algorithm can be found in publication [10, 21].

5. Experimental results and discussions

To demonstrate the enhancement on the nonlinear egression parameter estimation of HHO algorithm, we have considered famous 11 nonlinear regression models In the models, the number of parameters lies in the range of two to four. The precision of the given method is the main issue of this essay. The proposed approach (HHO) is implemented in R and, depending on the discussed subject, Simulation was done on the basis of Mean Squares Error (MSE) to estimate the parameters of 11 models of nonlinear regression ((Militkty2), (Ratkowsky2) (Eckerle4), (Ratkowsky3), (MGH09), (Roszman1), (Misrald), (Misralc), (Nelson), (Misralb) and (DanWood)). The value of response variable yi was dependent on xi, which were generated in accordance with the exponential distribution ei, whereas the random variable was generated in accordance with the $N(0, \sigma 2)$. Besides, there were 1000 replications of each simulation condition. Different sample sizes are tried 25,50, 100,150 and 200.

Seven numerical methods were used in the research Newton Raphson Method (NRM), Gauss Newton Method (GNM), The Levenberg Marquardt Method(LMM), Quasi Newton methods (QNM), Davidon Fletcher Powell Method (DFPM), Broyden Fletcher Goldfarb Shanno Method(BFGSM) and Nonlinear Conjugate Gradient Method (NCGM), and the seven numerical methods were hybridized with the HHO algorithm by conducting simulations (HHO-NRM), (HHO-GNM),(HHO-LMM), (HHO- QNM), (HHO-DFPM), (HHO -BFGSM) and (HHO-NCGM) hybrid algorithm. The Tables 1-5 show the results before and after the hybridization processes for the purpose of improving the estimation process.

Table 1. Comparison of simulation results for the 10 models and all methods using the mean square error MSE criterion and a sample size of 25

Method	Number Model										
	1	2	3	4	5	6	7	8	9	10	11
NRM	4.325	2.716	3.484	3.023	2.345	4.746	4.004	3.008	5.345	3.746	4.004
GNM	3.873	2.364	3.198	3.204	2.610	2.422	3.095	3.487	5.345	3.386	4.101
LMM	5.001	3.478	6.185	4.008	3.329	4.278	4.184	4.937	4.853	4.886	5.114
QNM	2.694	4.282	3.697	5.019	3.395	1.17	1.784	3.722	2.119	1.565	1.795
DFPM	3.173	2.328	2.289	3.71	3.145	3.892	3.467	3.505	1.264	5.271	3.939
BFGSM	4.89	3.941	4.477	4.835	4.878	2.716	3.04	3.706	5.193	1.997	4.121
NCGM	1.281	1.99	1.208	4.136	4.365	4.261	2.037	1.691	2.976	4.487	5.149
HHO-NRM	0.862	0.085	0.596	0.588	0.18	0.78	0.782	0.257	0.852	0.604	0.756
HHO-GNM	0.182	0.371	0.25	0.815	0.494	0.098	0.384	0.354	0.517	0.137	0.502
HHO-LMM	0.286	0.426	0.467	0.045	0.118	0.893	0.316	0.245	0.118	0.861	0.482
HHO- QNM	0.025	0.716	0.613	0.692	0.544	0.39	0.09	0.254	0.5	0.064	0.243
HHO-DFPM	0.585	0.849	0.514	0.81	0.739	0.239	0.484	0.321	0.814	0.825	0.473
HHO -BFGSM	0.83	0.132	0.458	0.739	0.342	0.658	0.707	0.25	0.192	0.493	0.785
HHO-NCGM	0.338	0.145	0.77	0.397	0.375	0.738	0.416	0.837	0.399	0.588	0.765

Table 2. Comparison of simulation results for the 10 models and all methods using the mean square error MSE criterion and a sample size of 50

Method	Number Model										
	1	2	3	4	5	6	7	8	9	10	11
NRM	2.181	2.584	2.967	2.81	1.824	1.265	2.756	2.318	2.314	2.514	2.712
GNM	2.786	1.085	2.289	2.033	2.15	1.227	2.862	1.229	2.485	3.207	1.944
LMM	2.015	2.866	2.533	1.836	3.099	1.645	1.5	2.841	1.442	1.14	2.149
QNM	1.792	3.158	3.176	2.715	3.033	1.097	2.198	2.611	2.274	2.74	3.26
DFPM	1.582	2.71	2.189	1.519	1.028	2.472	2.643	2.439	3.257	2.988	1.073
BFGSM	1.095	2.942	1.578	2.671	3.303	1.854	2.356	1.989	1.349	1.111	1.367
NCGM	1.125	2.294	2.754	2.151	2.901	1.262	2.586	2.606	1.456	1.58	3.186
HHO-NRM	0.148	0.451	0.306	0.298	0.491	0.218	0.342	0.288	0.548	0.164	0.17
HHO-GNM	0.096	0.192	0.018	0.113	0.014	0.368	0.341	0.037	0.31	0.032	0.408
HHO-LMM	0.207	0.155	0.262	0.477	0.455	0.223	0.102	0.166	0.058	0.253	0.209
HHO- QNM	0.471	0.529	0.519	0.282	0.577	0.23	0.026	0.496	0.117	0.021	0.096
HHO-DFPM	0.035	0.048	0.376	0.02	0.061	0.034	0.532	0.055	0.055	0.499	0.392
HHO -BFGSM	0.041	0.363	0.516	0.577	0.358	0.548	0.076	0.214	0.258	0.48	0.402
HHO-NCGM	0.53	0.324	0.53	0.429	0.394	0.594	0.419	0.019	0.607	0.284	0.546

Table 3. Comparison of simulation results for the 10 models and all methods using the mean square error criterion and a sample size of 100

Method	Number Model										
	1	2	3	4	5	6	7	8	9	10	11
NRM	2.78	2.33	2.163	1.301	1.464	1.082	2.794	1.925	2.839	2.445	1.157
GNM	2.446	1.144	2.881	2.913	1.021	2.767	2.073	2.566	2.188	2.218	2.27
LMM	2.705	2.747	2.78	1.165	1.415	1.708	1.74	2.014	1.717	2.327	1.557
QNM	1.931	1.269	1.261	1.331	1.484	2.413	1.87	1.439	2.529	1.54	1.132
DFPM	1.644	2.119	1.601	2.139	2.548	1.845	2.755	2.988	1.361	1.718	1.13
BFGSM	1.958	1.251	1.494	2.368	1.55	1.168	2.046	1.895	2.105	2.92	2.728
NCGM	1.523	1.173	1.804	2.654	2.935	2.203	2.109	1.925	2.424	1.524	3
HHO-NRM	0.432	0.402	0.014	0.382	0.054	0.439	0.221	0.024	0.485	0.467	0.201
HHO-GNM	0.324	0.263	0.482	0.067	0.431	0.412	0.37	0.251	0.496	0.172	0.16
HHO-LMM	0.267	0.433	0.428	0.402	0.252	0.244	0.424	0.508	0.095	0.311	0.243
HHO- QNM	0.146	0.224	0.387	0.468	0.164	0.05	0.232	0.315	0.047	0.493	0.281
HHO-DFPM	0.267	0.049	0.207	0.146	0.06	0.156	0.094	0.359	0.446	0.225	0.146
HHO -BFGSM	0.094	0.063	0.316	0.054	0.423	0.106	0.432	0.119	0.352	0.386	0.439
HHO-NCGM	0.272	0.496	0.342	0.26	0.291	0.271	0.167	0.204	0.208	0.189	0.028

Table 4. Comparison of simulation results for the 10 models and all methods using the mean square error MSE criterion and a sample size of 150

Method	Number Model										
	1	2	3	4	5	6	7	8	9	10	11
NRM	1.552	2.914	2.43	2.682	1.389	1.842	2.219	2.03	1.576	2.348	2.522
GNM	2.561	2.713	1.285	2.41	1.396	1.25	2.596	2.723	1.236	2.835	2.599
LMM	2.357	2.234	1.24	2.121	2.979	2.704	2.108	1.943	2.299	2.745	1.124
QNM	1.538	2.488	2.075	2.296	1.666	2.86	2.456	0.998	2.296	2.337	2.704
DFPM	1.932	1.961	2.094	1.147	0.992	2.14	1.062	1.853	2.234	2.302	1.579
BFGSM	1.543	2.683	1.289	1.158	2.384	2.725	2.501	2.838	1.975	1.871	1.888
NCGM	2.742	2.783	1.77	2.879	1.116	2.201	2.563	2.649	1.559	1.518	1.933
HHO-NRM	0.164	0.295	0.084	0.205	0.303	0.096	0.079	0.255	0.115	0.21	0.308
HHO-GNM	0.082	0.156	0.106	0.018	0.117	0.103	0.186	0.126	0.305	0.155	0.258
HHO-LMM	0.129	0.242	0.16	0.148	0.29	0.135	0.224	0.256	0.071	0.153	0.127
HHO- QNM	0.121	0.058	0.307	0.02	0.166	0.03	0.096	0.254	0.046	0.046	0.109
HHO-DFPM	0.024	0.19	0.231	0.01	0.12	0.007	0.139	0.273	0.285	0.069	0.01
HHO -BFGSM	0.021	0.043	0.198	0.052	0.149	0.279	0.233	0.225	0.248	0.18	0.057
HHO-NCGM	0.191	0.078	0.287	0.039	0.204	0.243	0.05	0.081	0.008	0.162	0.153

Table 5. Comparison of simulation results for the 10 models and all methods using the mean square error MSE criterion and a sample size of 200

Method	Number Model										
	1	2	3	4	5	6	7	8	9	10	11
NRM	2.502	1.162	2.004	1.263	1.364	1.955	1.091	2.328	2.279	2.071	2.408
GNM	1.392	2.297	1.061	1.351	1.67	1.59	1.818	2.539	1.624	1.031	1.628
LMM	2.39	1.182	1.292	2.374	1.362	1.703	2.172	1.974	2.166	2.391	2.439
QNM	2.273	2.102	1.449	1.723	1.801	1.845	1.974	2.09	1.313	2.322	2.346
DFPM	1.27	2.075	1.519	2.356	1.151	2.512	1.375	1.011	1.284	1.082	1.298
BFGSM	2.27	2.232	1.674	1.051	2.124	1.916	1.521	2.229	1.251	2.367	2.478
NCGM	1.215	1.12	2.029	1.166	1.287	2.146	1.445	1.008	2.444	2.048	1.153
HHO-NRM	0.008	0.192	0.172	0.128	0.123	0.114	0.198	0.229	0.034	0.213	0.215
HHO-GNM	0.074	0.224	0.129	0.251	0.176	0.134	0.012	0.2	0.019	0.19	0.023
HHO-LMM	0.216	0.167	0.129	0.182	0.175	0.179	0.115	0.011	0.25	0.155	0.03
HHO- QNM	0.061	0.021	0.006	0.236	0.11	0.096	0.156	0.235	0.249	0.016	0.217
HHO-DFPM	0.076	0.126	0.184	0.006	0.223	0.195	0.213	0.113	0.033	0.151	0.004
HHO -BFGSM	0.101	0.125	0.151	0.219	0.047	0.2	0.088	0.012	0.201	0.162	0.214
HHO-NCGM	0.048	0.114	0.011	0.191	0.248	0.145	0.01	0.214	0.214	0.173	0.043

The other condition that is the subject interest of this comparative study is sample size. In this study, it falls in the small and large sample size. The sample size is taken as 200 and the small size is taken as 25. The distinction between them cannot be underestimated as we are interested in observing how some algorithms will behave in this

kind of situation. The intention here is to note which of them (one or more) happens to do this situation better in comparison with its competitive counterparts when the other conditions are kept constant, i.e., when errors are normally distributed and good initial values. The results of the simulation are as shown in Tables 1-4 and Table 5 respectively, in the case of a small sample and large sample simulation. The findings are indicated in Tables 1-4, and 5, respectively. Obviously, the estimates will be with good statistical characteristics when the errors are normal distributed. The findings are sensible under the assumption that the errors are non-normal.

Tables 1-4, till 5 measures the performance of HHO-NRM Hybrid algorithm and numerical algorithm. The findings revealed that, HHO Hybrid performs better than the numerical in most experiments concerning minimum sum of squares. They emphasized the fact HHO Hybrid has advantages in computer resources as opposed to numerical.

The eleven models were applied to real data using a single hybrid method, which is the HHO-NRM, through the application to Iraqi economic data representing a study of the impact of economic and social factors on the unemployment rate in Iraq from the website of the Central Statistical Organization and the Iraqi Ministry of Planning for 18 governorates, where the dependent variable represented Y the unemployment rate and the independent variables were the first X1 average income, the second X2 the percentage of university education, the third X3 the number of investment projects, and the fourth X4 the number of socially active population. The results are shown in the Table 6.

Table 6. Shows the estimation of parameters for the eleven models using the HHO-NRM method through application to Iraqi economic data

Number Model	Model parameters			
	β_1	β_2	β_3	β_4
1	2.345	3.432		
2	3.455	3.567	1.345	
3	1.345	0.345	0.134	
4	1.456	0.934	0.934	
5	10.234	6.467	12.567	9.456
6	8.953	5.936	11.947	5.336
7	25.345	11.367		
8	6.374	0.376		
9	0.267	0.954	2.536	
10	30.456	9.584		
11	0.788	2.456		

6. Conclusion

In the study, Harris hawks optimization (HHO) was applied as an alternative way of estimating parameters of nonlinear regression models The eleventh. A simulation study was undertaken to validate the algorithm Proposed hybrid . The results revealed that the algorithm has a general advantage of estimating parameters with; both the nonlinear Newton-Raphson Method (NRM), Gauss-Newton Method (GNM), The Levenberg-Marquardt Method(LMM), Quasi-Newton (QNM), Davidon-Fletcher-Powell (DFP) and Broyden Fletcher-Goldfarb-Shanno (BFGS) methods as well as Nonlinear Conjugate-Gradient Method (NCG) methods since the hybrid algorithm performed better than all others across sample sizes. The outputs also revealed that the parameters of every model in the algorithm were improved when compared to any of the fluctuations and each of the algorithm hybrid namely; (HHO-NRM), (HHO-GNM), (HHO-LMM), (HHO- QNM), (HHO-DFP), (HHO -BFGS) and (HHO-NCG) had the lowest MSE of all models. It was observed that the error reduced as the sample size became larger and we got the accuracy in detecting the parameters. We suggest in using other hybrid.

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