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On Binary Generalized Pre and Binary Generalized Semi Pre Continuous Functions in Binary Topological Spaces

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Abstract The aim of this paper is to introduce and study the binary generalized pre continuous and binary generalized semi pre continuous functions (briefly b-gpCF(b-gspCF)) in binary topological spaces. Basic properties of b-gpCF(b-gspCF) are analyzed and also, we study the relationship between b-gpCF(b-gspCF) and other existing binary continuous functions. Further we have given an appropriate example to understand the abstract concepts clearly. Also, we discuss some forms b-gpCF(b-gspCF) in binary topological spaces.

Keywords b-gp-closed, b-gsp-closed, b-gpCF and b-gspCF

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1. Introduction

Continuity between topological spaces plays a dominant role in analysis. The concept of generalized pre closed functions introduced by Noiri et. al [4] in the year 1998. The notion of generalized semi pre closed funtions introduced by Dontchev [5] in the year 1995.In the year 2011, Nithyanantha Jothi and Thangavelu [1] introduced binary topological spaces. Later in 2012, Nithyanantha Jothi and Thangavelu [6] introduced the concept of coninuous functions in binary topological spaces. Further, the concept of generalized binary continuity introduced by Nazir Ahmad et. al [10] in the year 2018. In 2021, Carlos Granados [2] introduced binary pre-open sets and binary pre-ω open sets in binary topological spaces. The binary generalized pre closed sets and binary generalized semi pre closed sets in binary topological spaces were introduced by Sathishmohan et. al[11] in the year 2024.

The concept of continuity plays a central role in topology, and its generalizations have been widely studied. Different forms of continuity, such as semi-continuity, pre-continuity, α-continuity, and β-continuity, have been introduced to capture finer variations in the behavior of functions. These notions are closely related to corresponding types of advanced open sets and have been investigated extensively in classical and advanced topological settings. Extending these ideas to advanced binary topological spaces leads to the study of binary continuous mappings and their various forms.This binary continuous functions are used to analyze mappings between binary topological spaces. Real-world data modeling such as in Rough Set Theory, digital image segmentation, and asymmetric neural networks frequently encounters incomplete, fragmented, or noisy information. Traditional continuous mappings fail in these granular environments due to sharp discontinuities and structural gaps. This motivates to introduce b-gp continuous and b-gsp continuous functions within binary

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topological spaces. By utilizing generalized pre-open and semi-pre-open sets, these mappings can smoothly bridge localized data irregularities and fragmented boundaries where standard interiors fail to exist. This theoretical framework provides a robust topological foundation for designing error-tolerant approximation operators and multi-attribute decision-making algorithms capable of operating over highly uncertain datasets. The binary topological structures (including neutrosophic micro binary topology) are used to analyze complex data sets and identify critical factors in games, and key reasons for failure.

The structure of this paper is as follows: In section 2, study the basic concepts of binary topological spaces. In section 3, we introduce and study the notion of b-gpCF and b-gspCF in binary topological spaces and in section 4, study some other types of b-gpCF and b-gspCF in binary topological spaces.

Throughout this paper, $\mathscr{P}(P)$ and $\mathscr{P}(Q)$ are the power sets of P and Q. We use the notations b-gpCF, b-gspCF, CL, O, CLO, BTS and TS to represent binary generalized pre continuous functions, binary generalized semi pre continuous functions, closed, open, clopen, binary topological spaces and topological spaces respectively.
The basic definitions used in this paper is given below.

2. Preliminaries

Definition 2.1
[1] Let P and Q be any two non-empty sets and C $\subseteq$ P and D $\subseteq$ Q. A BT from P to Q is binary structure $\mathscr{M} \subseteq \mathscr{P}(P)\times\mathscr{P}(Q)$ that satisfies the following axioms:

- ($\emptyset$,$\emptyset$) and (P,Q) $\in \mathscr{M}$.
- $(C_1 \cap C_2, D_1 \cap D_2) \in \mathscr{M}$, for any $(C_1, D_1) \in \mathscr{M}$ and $(C_2, D_2) \in \mathscr{M}$.
- If $\{(C_\alpha, D_\alpha) : \alpha \in \Delta\}$ is a family of members of $\mathscr{M}$, then $(\bigcup_{\delta\in\Delta} C_\delta, \bigcup_{\delta\in\Delta} D_\delta) \in \mathscr{M}$.

Definition 2.2
[1] If $\mathscr{M}$ is a BT from P to Q, then the triplet (P,Q,$\mathscr{M}$) is said to be the BTS.

Definition 2.3
[1] Let (P,Q,$\mathscr{M}$) be a BTS and let (p,q) $\in$ (P,Q) . The b-O set (A,B) is said to be a binary neighbourhood of (p,q) if p $\in$ A and q $\in$ B.

Proposition 2.4
[1] Let (C,D) $\subseteq$ (P,Q) and (P,Q,$\mathscr{M}$) be a BTS. Then the following statements hold:

- int(C,D) $\subseteq$ (C,D)
- If (C,D) is b-O, then int(C,D)=(C,D)
- int((C,D) $\subseteq$ int(E,F)
- int(int(C,D)) = int (C,D)
- (C,D) $\subseteq$ cl(C,D)
- If (C,D) is b-CL, then cl(C,D) = (C,D)
- cl(C,D) $\subseteq$ cl(E,F)
- cl(cl(C,D)) = cl(C,D)

Definition 2.5
A subset (C,D) of a BTS (P,Q,$\mathscr{M}$) is called

- b-sCL [9], if b-int(b-cl(C,D)) $\subseteq$ (C,D)
- b-pCL[3], if b-cl(b-int(C,D)) $\subseteq$ (C,D)
- b-αCL [3], if b-int(b-cl(b-int(C,D))) $\subseteq$ (C,D)
- b-spCL[3], if b-cl(b-int(b-cl(C,D))) $\subseteq$ (C,D)
- b-rCL [8], if (C,D) = b-cl(b-int(C,D))

Definition 2.6
Let (P,Q,$\mathscr{M}$) be a BTS. Let (C,D) $\subseteq$ (P,Q) .Then (C,D) is called

- gb-CL[7], if b-cl(C,D) $\subseteq$ (U,V) whenever (C,D) $\subseteq$ (U,V) and (U,V) is b-O.
- b-gpCL(b-gp-closed) [11], if b-pcl(C,D) $\subseteq$ (U,V) whenever (C,D) $\subseteq$ (U,V) and (U,V) is b-O.
- b-gspCL(b-gsp-closed) [11], if b-spcl(C,D) $\subseteq$ (U,V) whenever (C,D) $\subseteq$ (U,V) and (U,V) is b-O.

Definition 2.7
[1] Let C $\subseteq$ P and D $\subseteq$ Q and Ψ: R $\rightarrow$ P$\times$ Q be a function. Then Ψ^{-1}(C,D) = $\{$r $\in$ R : Ψ(r) = (x,y) $\in$ (C,D)$\}$.

Lemma 2.8
[1] Let C $\subseteq$ P and D $\subseteq$ Q and Ψ : R $\rightarrow$ P $\times$ Q be a funcion. Then R - Ψ^{-1}(C,D) = Ψ^{-1}(C, Q-D) $\cup$ Ψ^{-1}(P-C, D) $\cup$ Ψ^{-1} (P-C, Q-D).

Definition 2.9
Let (P,Q,$\mathscr{M}$) be a BTS and (R,τ) be a TS. Then Ψ: (R,τ) $\rightarrow$ P $\times$ Q is said to be

- b-CF [6], if Ψ^{-1}(C,D) is open in R for every b-O set (C,D) in (P,Q,$\mathscr{M}$) .
- b-sCF [9], if Ψ^{-1}(C,D) is semi-O in R for every b-O set (C,D)in (P,Q,$\mathscr{M}$).
- gb-CF[10], if Ψ^{-1}(C,D) is g-O in R for every b-O set (C,D) in (P,Q,$\mathscr{M}$).
- Totally b-CF[9], if Ψ^{-1}(C,D) is CLO in R for every b-O (C,D) in (P,Q,$\mathscr{M}$).
- Totally b-sCF [9], if Ψ^{-1}(C,D) is semi-CLO in R for every b-O (C,D) in (P,Q,$\mathscr{M}$).
- Strongly b-CF [9], if Ψ^{-1}(C,D) is CLO in R for every binary set (C,D) in (P,Q,$\mathscr{M}$).
- Strongly b-sCF [9], if Ψ^{-1}(C,D) is semi-CLO in R for every binary set (C,D) in (P,Q,$\mathscr{M}$).

3. b-gp(b-gsp)-Continuous Functions

Here we introduce and study a new class of continuous functions in BTS called b-gpCF and b-gspCF and investigate the relationship between these functions with other existing functions. Further we discuss their basic properties.

Definition 3.1
The function Ψ:(R,τ) $\rightarrow$ P $\times$ Q from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) is said to be b-gpCF if Ψ^{-1}(C,D) is gp-O in (R,τ) for every b-O set (C,D) in (P,Q,$\mathscr{M}$).

Example 3.2
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{2,3\},R\}$ and $\mathscr{M}$= $\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\nu\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\rightarrow$P $\times$Q by Ψ(1)= $(\{\mu\},\{\mu,\nu\})$, $\Psi(2)$=$(\{\nu\},\{\nu\})$=$\Psi(3)$. Now $\Psi^{-1}(\emptyset,\emptyset)=\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\nu\})$=$\{1\}$.This shows that pre image of every b-O set in (P,Q,$\mathscr{M}$) is gp-O set in R. Hence Ψ is b-gpCF.

Theorem 3.3
Let (R,τ) be a TS and let (P,Q,$\mathscr{M}$) be a BTS. Let Ψ: (R,τ) $\rightarrow$ P $\times$ Q be a function such that R$\setminus$ Ψ^{-1}(C,D) = Ψ^{-1}(P$\setminus$C,Q$\setminus$D) for all C $\subseteq$ P and D $\subseteq$ Q . Then Ψ is b-gpCF iff Ψ^{-1}(C,D) is gp-CL set in (R,τ) for every b-CL sets (C,D) in (P,Q,$\mathscr{M}$).
Proof: Assume that μ is b-gpCF. Let (C,D) be a b-CL set in (P,Q,$\mathscr{M}$). $\therefore$ (P$\setminus$C,Q$\setminus$D) is a b-O set. That is, (P$\setminus$C,Q$\setminus$D) $\in$ $\mathscr{M}$. $\because$ Ψ is b-gpCF, we have Ψ^{-1}(P$\setminus$C,Q$\setminus$D) is gp-O set in (R,τ) . $\therefore$ R$\setminus$ Ψ^{-1}(C,D) is gp-O set in (R,τ). Hence, Ψ^{-1}(C,D) is gp-CL set in (R,τ).

Conversely, let Ψ^{-1}(C,D) is gp-CL set in (R,τ) for every b-CL sets (C,D) in (P,Q,$\mathscr{M}$). Let (C,D) be a b-O set in (P,Q,$\mathscr{M}$) . $\therefore$ by assumption Ψ^{-1}(P$\setminus$C,Q$\setminus$D) is gp-CL set in (R,τ). Thus, R$\setminus$ Ψ^{-1}(C,D) is gp-CL set in (R,τ) . This shows that Ψ^{-1}(C,D) is gp-O set in (R,τ).This implies that Ψ is b-gpCF.

Theorem 3.4
Let (P,Q,$\mathscr{M}$) be a BTS, such that (C,Q\D) and (P\C,D) are b-O in (P,Q,$\mathscr{M}$), whenever (C,D) is b-O. Then Ψ: (R,τ) $\to$ P $\times$ Q is b-gpCF iff Ψ^{-1}(C,D) is gp-CL set in (R,τ) for every b-CL set in (C,D) in (P,Q,$\mathscr{M}$).
Proof: Assume that Ψ : (R,τ) $\to$ P $\times$ Q is b-gpCF. Let (C,D) be a b-CL set in (P,Q,$\mathscr{M}$). $\therefore$ (P\C,Q\D) is a b-O set. That is,(P\C,Q\D) $\in\mathscr{M}$. $\because$ Ψ is b-gpCF, we have Ψ^{-1}(P\C,Q\D) is gp-O set in (R,τ) . $\because$ (C,Q\D) and (P\C,D) are b-O in (P,Q,$\mathscr{M}$), we have R\ Ψ^{-1}(C,D) is gp-O set in (R,τ). Hence,Ψ^{-1}(C,D) is gp-CL set in (R,τ) . Conversely, assume that Ψ^{-1}(C,D) is gp-CL set in (R,τ) for every b-CL sets (C,D) in (P,Q,$\mathscr{M}$). Let (C,D) be a b-O set. $\because$ (C,D) $\in\mathscr{M}$, we have (P\C,Q\D) is b-CL set in (P,Q,$\mathscr{M}$) . $\therefore$ by our assumption Ψ^{-1}(P\C,Q\D) is gp-CL set in (R,τ) .

Theorem 3.5
Let a function Ψ : (R,τ) $\to$ P $\times$ Q from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) is b-gpCF iff for every r $\in$R and for every b-O set (C,D) with Ψ(r) $\in$(C,D) there exist a gp-O set U $\subseteq$R such that Ψ(U) $\subseteq$(C,D).
Proof: Let Ψ:(R,τ) $\to$ P $\times$ Q is b-gpCF. Let (C,D) be a b-O set with Ψ(r)=(x,y) $\in$(C,D). Then r $\in$ Ψ^{-1}(C,D) . Take U=Ψ^{-1}(C,D) . Then U is gp-O set in R with r $\in$U . Also Ψ(U)=$\{\Psi$(u): u$\in$U $\subseteq$(C,D)$\}$.

Conversely, for all r $\in$R and for every b-O set (C,D), with Ψ(r) $\in$(C,D) there exist gp-O set U in R with r $\in$U, Ψ(U) $\subseteq$(C,D). Let (C,D) be a b-O set. To prove that Ψ^{-1}(C,D) is gp-O set in (R,τ) . Let u $\in$ Ψ^{-1}(C,D) . Then Ψ(u) $\in$(C,D) . By assumption there exist gp-O set U with Ψ(U) $\subseteq$(C,D). $\therefore$ Ψ^{-1}(Ψ(U)) $\subseteq\Psi^{-1}$(C,D). That is, U $\subseteq\Psi^{-1}$(C,D). This proves that for each u $\in$ Ψ^{-1}(C,D) there is gp-O set U containing u such that U $\subseteq\Psi^{-1}$(C,D) that implies Ψ^{-1}(C,D) is a union of gp-O set. This proves that Ψ^{-1}(C,D) is gp-O set in R, hence Ψ is b-gpCF.

Theorem 3.6
For a function Ψ : (R,τ) $\to$ P $\times$ Q , the following statements are equivalent:
i)Ψ is b-gpCF
ii)For each (p,q) $\in$ (P,Q) and each b-O set (V,U) in P $\times$ Q containing (Ψ(p),Ψ(q)) there exists an gp-O set (N,M) in (R,τ) containing (p,q) such that Ψ(N,M) $\subset$(V,U).
Proof: (i) $\Longrightarrow$ (ii) Let (p,q) $\in$(P,Q) and (V,U) be any b-O set in P $\times$ Q , such that (Ψ(p),Ψ(q)) $\in$ (V,U), then (p,q) $\in$ Ψ^{-1}(V,U) and since Ψ is b-gpCF, Ψ^{-1}(V,U) is gp-O set in (R,τ). If (N,M)= Ψ^{-1}(V,U), then (N,M) is gp-O set in (R,τ) containing (p,q) such that Ψ(N,M)=Ψ(Ψ^{-1}(V,U)) $\subset$(V,U).
(ii) $\Longrightarrow$ (i) Let (V,U) be any b-O set in P $\times$ Q and (p,q) $\in$ Ψ^{-1} (V,U), then (Ψ(p),Ψ(q)) $\in$(V,U) and by (ii) there exists an open set $(N,M)_p$ in (R,τ) such that (x,y) $\in$ $(N,M)_{(x,y)}$ and $\Psi((N,M)_{(x,y)})$ $\subset$(V,U). Thus (p,q) $\in$ $(N,M)_{(p,q)}\subset$ Ψ^{-1} ($\Psi((N,M)_{(p,q)})$) $\subset$ Ψ^{-1} (V,U). Then, we have Ψ^{-1} (V,U) is gp-O set in (R,τ) and so Ψ is b-gpCF.

Theorem 3.7
Let Ψ : (R,τ) $\to$ P $\times$Q is a function from TS (R,τ) to BTS (P,Q,$\mathscr{M}$), then
(i)Every b-C function is b-gpCF
(ii) Every b-sC function is b-gpCF
(iii)Every b-αC function is b-gpCF
(iv) Every b-rC function is b-gpCF
(v)Every b-gC function is b-gpCF
Proof: Let (C,D) be a b-O in (P,Q,$\mathscr{M}$) . $\because$ Ψ is b-CF, we have Ψ^{-1}(C,D) is open in (R,τ) and every open set is gp-O set in (R,τ) , then Ψ^{-1}(C,D) is gp-O in (R,τ). Hence Ψ is b-gpCF.
(ii) to (v) Similar to proof (i).

Remark 3.8
The following example gives the converse of the above theorem.

Example 3.9
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{2,3\},R\}$ and $\mathscr{M}$= $\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\nu\}),(P,Q)\}$ BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(1)$= $(\{\mu\},\{\mu,\nu\})$=$\Psi(3)$, $\Psi(2)$=$(\{\nu\},\{\nu\})$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\nu\})$=$\{1,3\}$ which gp-O set in R but not open, semi open, α-O, r-O and g-O set in R. Clearly, Ψ is b-gpCF but not b-CF, b-sCF, b-αCF, b-rCF and b-gCF.

Theorem 3.10
Ψ : (R,τ) $\to$ P $\times$ Q is a b-gpCF iff for every X $\subseteq$ P and Y $\subseteq$ Q , Ψ^{-1}(b-gp-int(X,Y)) $\subseteq$ gp-int(Ψ^{-1}(X,Y)).
Proof: Suppose μ : (R,τ) $\to$ P $\times$ Q is a b-gpCF. Let X $\subseteq$P and Y $\subseteq$Q. $\because$ b-gp-int(X,Y) is b-gpO in (P,Q,$\mathscr{M}$) and contained in (X,Y) . $\therefore$ Ψ^{-1}(b-gp-int(X,Y)) is gp-O set in R. Now, b-gp-int(X,Y) $\subseteq$ (X,Y). This implies that, Ψ^{-1}(b-gp-int(X,Y)) $\subseteq$ Ψ^{-1}(X,Y) $\implies$ gp-int (Ψ^{-1}(b-gp-int(X,Y))) $\subseteq$ gp-int (Ψ^{-1}(X,Y)). This follows that, Ψ^{-1}(b-gp-int(X,Y)) $\subseteq$ gp-int(Ψ^{-1}(b-gp-int(X,Y)).

Conversely, let Ψ^{-1}(b-gp-int(X,Y)) $\subseteq$ gp-int(Ψ^{-1}(X,Y)) for every C $\subseteq$ P and D $\subseteq$ Q . Let (X,Y) is a b-gpO set. Then b-gp-int(X,Y) = (X,Y). $\therefore$ Ψ^{-1}(X,Y) = Ψ^{-1}(b-gp-int(X,Y)) $\subseteq$ gp-int(Ψ^{-1}(X,Y)). $\therefore$ gp-int(Ψ^{-1}(X,Y)) is gp-O set in R . Hence Ψ is a b-gpCF.

Definition 3.11
The function Ψ:(R,τ) $\to$ P $\times$ Q from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) is said to be b-gspCF if Ψ^{-1}(C,D) is gsp-O in (R,τ) for every b-O set (C,D) in (P,Q,$\mathscr{M}$).

Example 3.12
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu.\nu,\omega\}$. Let τ=$\{\emptyset,\{3\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\omega\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(2)$ = $(\{\mu\},\{\mu,\omega\})$ and $\Psi(1)$=$(\{\nu\},\{\nu\})$=$\Psi(3)$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\omega\})$=$\{2\}$. This shows that the pre image of every b-O set in (P,Q,$\mathscr{M}$) is gsp-O set in R. $\therefore$ Ψ is b-gspCF.

Theorem 3.13
Let Ψ:(R,τ) $\to$ P $\times$ Q is a function from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) , then (i) Every b-CF is b-gspCF
(ii) Every b-sCF is b-gspCF
(iii) Every b-αCF is b-gspCF
(iv) Every b-rCF is b-gspCF
(v) Every b-gCF is b-gspCF
Proof: (i) Let (C,D) be a b-O in (P,Q,$\mathscr{M}$) . $\because$ Ψ is b-CF, we have Ψ^{-1}(C,D) is open in (R,τ). $\because$ every open set is gsp-O set in (R,τ), then Ψ^{-1}(C,D) is gsp-O in (R,τ). Hence Ψ is b-gspCF.

Proof of (ii) to (v) is similar to proof (i).

Remark 3.14
The following examples gives the converse of theorem 3.13.

Example 3.15
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu.\nu,\omega\}$. Let τ=$\{\emptyset,\{3\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\omega\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(2)$=$(\{\mu\},\{\mu,\omega\})$ and $\Psi(1)$=$(\{\nu\},\{\nu\})$=$\Psi(3)$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q) = $\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\omega\})$=$\{2\}$ which is gsp-open set but not open, semi open, α-open and r-open. Clearly, Ψ is b-gspCF but not b-CF, b-sCF, b-αCF and b-rCF.

Example 3.16
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu.\nu,\omega\}$. Let τ=$\{\emptyset,\{1\},\{1,2\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\omega\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(1)$ = $(\{\mu\},\{\mu,\omega\})$=$\Psi(3)$ and $\Psi(2)$=$(\{\nu\},\{\nu\})$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$,$\Psi^{-1}$(P,Q)= $\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\omega\})$=$\{1,3\}$, which is gsp-O set but not g-O. Clearly, Ψ is b-gspCF but not g-CF.

Theorem 3.17
If Ψ:(R,τ)$\to$P $\times$ Q is b-sgCF from TS (R,τ) to BTS (P,Q,$\mathscr{M}$), then it is b-gspCF.
Proof: Let (C,D) be a b-O in (P,Q,$\mathscr{M}$) . $\because$ Ψ is b-sgCF, we have Ψ^{-1}(C,D) is sg-O in (R,τ) . $\because$ every sg-O set is gsp-O set in (R,τ) , then Ψ^{-1}(C,D) is gsp-O in (R,τ) . Hence Ψ is b-gspCF.

Example 3.18
From example 3.15, Define Ψ:(R,τ)$\rightarrow$P $\times$Q by $\Psi(2)=(\{\mu\},\{\mu,\omega\})$ and $\Psi(1)=(\{\nu\},\{\nu\})=\Psi(3)$. Now $\Psi^{-1}(\emptyset,\emptyset)=\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\omega\})=\{2\}$ which is gsp-O set but not sg-O set. Clearly, Ψ is b-gspCF but not b-sgCF.

Theorem 3.19
Let (P,Q,$\mathscr{M}$) be a BTS and (R,τ) be a TS. If Ψ:(R,τ)$\rightarrow$P $\times$ Q is b-gpCF, then it is b-gspCF.
Proof: Let (C,D) be a b-O set in (P,Q,$\mathscr{M}$). $\because$ Ψ is b-gpCF, we have Ψ^{-1}(C,D) is gp-O in (P,Q,$\mathscr{M}$) . $\because$ every gp-O set is gsp-O set in (R,τ), then Ψ^{-1}(C,D) is gsp-O in (R,τ). Hence Ψ is b-gspCF.

Example 3.20
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let $\tau=\{\emptyset,\{1\},\{1,2\},R\}$ and $\mathscr{M}=\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\omega\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\rightarrow$P $\times$Q by $\Psi(1)=(\{\mu\},\{\mu,\omega\})=\Psi(3)$ and $\Psi(2)=(\{\nu\},\{\nu\})$. Now $\Psi^{-1}(\emptyset,\emptyset)=\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\omega\})=\{1,3\}$ which is gsp-O set but not gp-O set. Clearly, Ψ is b-gspCF but not b-gpCF.

Theorem 3.21
Let (R,τ) be a TS and let (P,Q,$\mathscr{M}$) be a BTS. Let Ψ:(R,τ)$\rightarrow$P $\times$ Q is a function such that R $\setminus\Psi^{-1}$(C,D) = Ψ^{-1}(P$\setminus$C,Q$\setminus$D) for all C $\subseteq$P and D $\subseteq$Q . Then Ψ is b-gspCF iff Ψ^{-1}(C,D) is gsp-CL set in (R,τ) for every b-CL sets (C,D) in (P,Q,$\mathscr{M}$).
Proof: Follows the theorem 3.3.

Theorem 3.22
Let (P,Q,$\mathscr{M}$) be a BTS, such that (C,Q$\setminus$D) and (P$\setminus$C,D) are b-O sets in (P,Q,$\mathscr{M}$) , whenever (C,D) is b-O. Then Ψ:(R,τ)$\rightarrow$P $\times$ Q is b-gspCF iff Ψ^{-1}(C,D) is gsp-CL set in (R,τ) for every b-CL set (C,D) in (P,Q,$\mathscr{M}$) .
Proof: Follows the theorem 3.4.

Theorem 3.23
Let (P,Q,$\mathscr{M}$) be a BTS and (R,τ) be a TS then Ψ: (R,τ)$\rightarrow$P $\times$ Q is b-gspCF iff for every r $\in$ R and for every b-O set (C,D) with Ψ(r) $\in$ (C,D) there is an gsp-O set U $\subseteq$ R such that Ψ(U) $\subseteq$ (C,D).
Proof: Follows the theorem 3.5.

Remark 3.24
The following table gives the relationship of b-gpCF(b-gspCF) with other existing functions. The symbol "1" in a cell means that a function implies the other function and the symbol "0" means that a function does not imply the other function.

functions	b-CF	b-sCF	b-αCF	b-rCF	b-gCF	b-sgCF	b-gpCF	b-gspCF
b-CF	1	1	1	1	1	1	1	1
b-sCF	0	1	0	0	0	1	1	1
b-αCF	0	0	1	0	0	0	1	1
b-rCF	0	0	0	1	0	0	1	1
b-gCF	0	0	0	0	1	1	1	1
b-sgCF	0	0	0	0	0	1	1	1
b-gpCF	0	0	0	0	0	0	1	1
b-gspCF	0	0	0	0	0	0	0	1

- Every b-CF, b-sCf, b-αCF, b-rCF, b-gCF and b-sgCF is b-gpCF.
- Every b-CF, b-sCf, b-αCF, b-rCF, b-gCF and b-sgCF is b-gspCF.
- Every b-gpCF is b-gspCF.

4. Some forms of b-gp(b-gsp)-Continuous functions

In this section, we introduce and study some forms of b-gpCF(b-gspCF) in BTS called Totally b-gpCF(b-gspCF), Strongly b-gpCF(b-gspCF) and b-gp(b-gsp)-irresolute functions. Also discuss some of their basic properties.

Definition 4.1
A function Ψ:(R,τ)$\to$P $\times$ Q from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) is said to be
i)Totally b-gpCF if Ψ^{-1}(C,D) is gp-CLO in (R,τ) for every b-O set (C,D) in (P,Q,$\mathscr{M}$).
ii)Totally b-gspCF if Ψ^{-1}(C,D) is gsp-CLO in (R,τ) for every b-O set (C,D) in (P,Q,$\mathscr{M}$).

Example 4.2
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{3\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\omega\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(2)$=$(\{\mu\},\{\mu,\omega\})$ and $\Psi(1)$=$(\{\nu\},\{\nu\})$=$\Psi(3)$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$ = R and $\Psi^{-1}(\{\mu\},\{\mu,\omega\})$=$\{2\}$.This shows that pre image of every b-O set in (P,Q,$\mathscr{M}$) is gp-CLO set in R. $\therefore$ Ψ is b-gpCF.

Example 4.3
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{1\},\{1,2\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\omega\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(1)$=$(\{\mu\},\{\mu,\omega\})$=$\Psi(3)$ and $\Psi(2)$=$(\{\nu\},\{\nu\})$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$ = R and $\Psi^{-1}(\{\mu\},\{\mu,\omega\})$=$\{1,3\}$.This shows that pre image of every b-O set in (P,Q,$\mathscr{M}$) is gsp-CLO set in R. $\therefore$ Ψ is b-gspCF.

Theorem 4.4
If Ψ:(R,τ)$\to$P $\times$ Q is a totally b-CF from TS (R,τ) to BTS (P,Q,$\mathscr{M}$), then it is totally b-gpCF (b-gspCF).
Proof: Let (C,D) be a b-O in (P,Q,$\mathscr{M}$). $\because$ Ψ is totally b-CF, we have Ψ^{-1}(C,D) is both open and closed in R. $\because$ every open set is gp-O(gsp-O) set and every closed set is gp-CL(gsp-CL) set in R. $\therefore$ Ψ^{-1}(C,D) is gp-CLO(gsp-CLO) set in R. Hence Ψ is totally b-gpCF(b-gspCF).

Remark 4.5
From example 4.2, Ψ is totally b-gpCF but not totally b-CF and from example 4.3, Ψ is totally b-gspCF but not totally b-CF.

Theorem 4.6
A function Ψ:(R,τ)$\to$P $\times$ Q is totally b-gpCF iff pre image of every b-CL subset of (P,Q,$\mathscr{M}$) is gp-CLO in R.
Proof: Suppose Ψ:(R,τ)$\to$P $\times$ Q is totally b-gpCF. Let (C,D) be a b-CL in (P,Q,$\mathscr{M}$). Then (P\C,Q\D) is b-O. $\because$ Ψ is totally b-gpCF, Ψ^{-1} (P\C,Q\D) is gp-CLO in R. This implies Ψ^{-1} (C,D) is gp-CLO in R.
Conversely, suppose pre image of every b-CL subset of (P,Q,$\mathscr{M}$) is gp-CLO in R. Let (C,D) be a b-O. Then (P\C,Q\D) is b-CL by hypothesis, Ψ^{-1}(P\C,Q\D) = R\(Ψ^{-1} (C,D)) is gp-CLO in R, that is Ψ^{-1} (C,D) is gp-CLO in R. Hence Ψ is totally b-gpCF.

Theorem 4.7
A functionΨ:(R,τ)$\to$P $\times$Q is totally b-gspC iff pre image of every b-CL subset of (P,Q,$\mathscr{M}$) is gsp-CLO in R.
Proof: Follows the theorem 4.6.

Definition 4.8
A function Ψ:(R,τ)$\to$P $\times$ Q from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) is said to be
i) Strongly b-gpCF if Ψ^{-1}(C,D) is gp-CLO in (R,τ) for every binary set (C,D) in (P,Q,$\mathscr{M}$).
ii) Strongly b-gspCF if Ψ^{-1}(C,D) is gsp-CLO in (R,τ) for every binary set (C,D) in (P,Q,$\mathscr{M}$).

Theorem 4.9
If Ψ:(R,τ)$\to$P $\times$ Q is a strongly b-CF from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) , then it is strongly b-gpCF.
Proof: Let (C,D) be a binary set in (P,Q,$\mathscr{M}$). $\because$ Ψ is strongly b-CF, we have Ψ^{-1}(C,D) is both open and closed in R. $\because$ every open set is gp-O set and every closed set is gp-CL set in R. $\therefore$ Ψ^{-1}(C,D) is gp-CLO set in R. $\therefore$ Ψ is strongly b-gpCF.

Example 4.10
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{2,3\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\nu\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(1)$=$(\{\mu\},\{\mu,\nu\})$, $\Psi(2)$=$(\{\nu\},\{\nu\})$= $\Psi(3)$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\nu\})$=$\{1\}$, $\Psi^{-1}(\emptyset,\{\mu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\nu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\omega\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\mu,\nu\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\nu,\omega\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\mu\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\nu\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\omega\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\mu,\nu\})$=$\{1\}$, $\Psi^{-1}(\{\mu\},\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\nu,\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\nu\})$ = $\{2,3\}$, $\Psi^{-1}(\{\nu\},\{\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu,\nu\})$ =$\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\nu,\omega\})$=$\emptyset$, $\Psi^{-1}(\emptyset,Q)$=$\emptyset$, $\Psi^{-1}(P,\emptyset)$=$\emptyset$, $\Psi^{-1}(P,\{\mu\})$=$\{1\}$, $\Psi^{-1}(P,\{\nu\})$=$\{2,3\}$, $\Psi^{-1}(P,\{\omega\})$=$\emptyset$, $\Psi^{-1}(P,\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(P,\{\mu,\nu\})$=$\{1\}$, $\Psi^{-1}(P,\{\nu,\omega\})$=$\{2,3\}$. This gives pre image of every binary sets in (P,Q,$\mathscr{M}$) is gp-CLO in R. Hence Ψ is strongly b-gpCF but not strongly b-CF.

Theorem 4.11
If Ψ:(R,τ)$\to$P $\times$ Q is a strongly b-CF from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) , then it is strongly b-gspCF.
Proof: Follows the theorem 4.9.

Example 4.12
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{1\},\{1,2\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(\{\mu\},\{\mu,\omega\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(1)$=$(\{\mu\},\{\mu,\omega\})$=$\Psi(3)$ and $\Psi(2)$=$(\{\nu\},\{\nu\})$ Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\omega\})$=$\{1,3\}$,$\Psi^{-1}(\emptyset,\{\mu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\nu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\omega\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\mu,\nu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\nu,\omega\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\mu\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\nu\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\omega\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\mu,\nu\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\mu,\omega\})$=$\{1,3\}$, $\Psi^{-1}(\{\mu\},\{\nu,\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\nu\})$=$\{2\}$, $\Psi^{-1}(\{\nu\},\{\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu,\nu\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\nu,\omega\})$=$\emptyset$, $\Psi^{-1}(\emptyset,Q\})$=$\emptyset$, $\Psi^{-1}(P,\emptyset\})$=$\emptyset$, $\Psi^{-1}(P,\{\mu\})$=$\{2\}$, $\Psi^{-1}P,\{\omega\})$ = $\emptyset$, $\Psi^{-1}(P,\{\mu,\omega\})$=$\{1,3\}$, $\Psi^{-1}(P,\{\mu,\nu\})$=$\{2\}$, $\Psi^{-1}(P,\{\nu,\omega\})$ = $\{2\}$, This gives pre image of every binary sets in (P,Q,$\mathscr{M}$) is gsp-CLO in R. Hence Ψ is strongly b-gspCF but not strongly b-CF.

Definition 4.13
A function Ψ:(R,τ)$\to$P $\times$ Q from TS (R,τ) to BTS (P,Q,$\mathscr{M}$) is said to be
i) b-gp-irresolute if Ψ^{-1}(C,D) is gp-O in (R,τ) for every b-gpO set (C,D) in (P,Q,$\mathscr{M}$).
ii) b-gsp-irresolute if Ψ^{-1}(C,D) is gsp-O in (R,τ) for every b-gspO set (C,D) in (P,Q,$\mathscr{M}$).

Example 4.14
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{1\},\{1,2\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(P,\{\mu,\nu\}),(\{\mu\},\{\mu\}),(\{\nu\},\{\nu\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$ P $\times$Q by $\Psi(1)$=$(\{\mu\},\{\mu\})$=$\Psi(3)$, $\Psi(2)$=$(\{\nu\},\{\nu\})$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$ = R and $\Psi^{-1}(\{\mu\},\{\mu,\nu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\mu\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\nu\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\omega\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\mu,\nu\})$ = $\emptyset$,$\Psi^{-1}(\emptyset,\{\mu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\nu,\omega\})$=$\emptyset$,$\Psi^{-1}(\{\nu\},\{\nu\})$=$\{2\}$, $\Psi^{-1}(\{\mu\},\{\nu\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\omega\})$ = $\emptyset$, $\Psi^{-1}(\{\mu\},\{\nu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\{\mu\},\{\mu\})$ = $\{1,3\}$, $\Psi^{-1}(\{\nu\},\{\omega\})$ = $\emptyset$,$\Psi^{-1}(\{\nu\},\{\mu,\nu\})$ = $\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\{\nu\},\{\nu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,Q\})$ = $\emptyset$, $\Psi^{-1}(\{\mu\},Q\})$ = $\{1,3\}$, $\Psi^{-1}(\{\nu\},Q\})$ = $\{2\}$, $\Psi^{-1}(P,\emptyset\})$ = $\emptyset$, $\Psi^{-1}(P,\{\mu\})$ = $\{1,3\}$, $\Psi^{-1}(P,\{\nu\})$ = $\{2\}$, $\Psi^{-1}(P,\{\omega\})$ = $\emptyset$, $\Psi^{-1}(P,\{\mu,\omega\})$ = $\emptyset$, $\Psi^{-1}(P,\{\mu,\nu\})$ = $\emptyset$, $\Psi^{-1}(P,\{\nu,\omega\})$ = $\emptyset$. For this function Ψ pre image of every b-gspO sets in (P,Q,$\mathscr{M}$) is gsp-O in R. $\therefore$ Ψ is b-gsp-irresolute function.

Theorem 4.15
Let Ψ:(R,τ)$\to$P $\times$ Q be a function from TS (R,τ) to BTS (P,Q,$\mathscr{M}$)
(i) If Ψ is binary irresolute, then it is b-gp(b-gsp)-irresolute
(ii) If Ψ is b-gp-irresolute, then it is b-gsp-irresolute.
Proof:(i) Let (C,D) be b-O in (P,Q,$\mathscr{M}$). Since Ψ is binary irresolute, we have Ψ^{-1}(C,D) is open in R. Since every b-O set is b-gpO(b-gspO) set and every open set is gp-O(gsp-O) set. This implies that (C,D) be a b-gpO(b-gspO) in (P,Q,$\mathscr{M}$) and Ψ^{-1}(C,D) is gp-O(gsp-O) in R. Hence Ψ is b-gp(b-gsp)-irresolute.
Proof of (ii) is similar to (i).

Example 4.16
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{3\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(P,\{\mu\}),(\{\mu\},\{\mu,\nu\},(P,\{\mu,\nu\}),(\{\mu\},\{\mu\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$P $\times$Q by $\Psi(3)$=$(\{\mu\},\{\mu,\nu\})$=$\Psi(2)$, $\Psi(3)$=$(\{\mu\},\{\mu\})$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$=R and $\Psi^{-1}(\{\mu\},\{\mu,\nu\})$=$\{2,3\}$, $\Psi^{-1}(\emptyset,\{\mu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\nu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\omega\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\mu,\nu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\nu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\{\nu\},\{\nu\})$ =$\emptyset$, $\Psi^{-1}(\{\mu\},\{\nu\})$ = $\emptyset$, $\Psi^{-1}(\{\mu\},\{\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\nu\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},\{\omega\})$=$\emptyset$,$\Psi^{-1}(\{\nu\},\{\nu\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu,\nu\})$ =$\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(\{\nu\},\{\nu,\omega\})$=$\emptyset$, $\Psi^{-1}(\emptyset,Q\})$=$\emptyset$, $\Psi^{-1}(\{\mu\},Q)$=$\{2,3\}$, $\Psi^{-1}(P,\emptyset)$=$\emptyset$, $\Psi^{-1}(P,\{\mu\})$=$\{2\}$, $\Psi^{-1}(P,\{\nu\})$=$\emptyset$, $\Psi^{-1}(P,\{\omega\})$= $\emptyset$, $\Psi^{-1}(P,\{\mu,\omega\})$=$\emptyset$, $\Psi^{-1}(P,\{\mu,\nu\})$=$\{2,3\}$, $\Psi^{-1}(P,\{\nu,\omega\})$=$\emptyset$. This gives pre image of every b-gpO(b-gspO) sets in (P,Q,$\mathscr{M}$) is gp-O(gsp-O) in R. Hence Ψ is b-gp(b-gsp)-irresolute but not binary irresolute function.

Example 4.17
Let R=$\{1,2,3\}$, P=$\{\mu,\nu\}$ and Q=$\{\mu,\nu,\omega\}$. Let τ=$\{\emptyset,\{1\},\{1,2\},R\}$ and $\mathscr{M}$=$\{(\emptyset,\emptyset),(P,\{\mu,\nu\}),(\{\mu\},\{\mu\}),(\{\nu\},\{\nu\}),(P,Q)\}$ is BT from P to Q. Define Ψ:(R,τ)$\to$ P $\times$Q by $\Psi(1)$=$(\{\mu\},\{\mu\})$=$\Psi(3)$, $\Psi(2)$=$(\{\nu\},\{\nu\})$. Now $\Psi^{-1}(\emptyset,\emptyset)$=$\emptyset$, Ψ^{-1}(P,Q)=$\{1,2,3\}$ = R and $\Psi^{-1}(\{\mu\},\{\mu,\nu\})$=$\emptyset$, $\Psi^{-1}(\emptyset,\{\mu\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\nu\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\omega\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\mu,\nu\})$ = $\emptyset$,$\Psi^{-1}(\emptyset,\{\mu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,\{\nu,\omega\})$=$\emptyset$,$\Psi^{-1}(\{\nu\},\{\nu\})$=$\{2\}$, $\Psi^{-1}(\{\mu\},\{\nu\})$ = $\emptyset$, $\Psi^{-1}(\{\mu\},\{\omega\})$ = $\emptyset$, $\Psi^{-1}(\{\mu\},\{\nu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\{\mu\},\{\mu\})$ = $\{1,3\}$, $\Psi^{-1}(\{\nu\},\{\omega\})$ = $\emptyset$,$\Psi^{-1}(\{\nu\},\{\mu,\nu\})$ = $\emptyset$, $\Psi^{-1}(\{\nu\},\{\mu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\{\nu\},\{\nu,\omega\})$ = $\emptyset$, $\Psi^{-1}(\emptyset,Q\})$ = $\emptyset$, $\Psi^{-1}(\{\mu\},Q\})$ = $\{1,3\}$, $\Psi^{-1}(\{\nu\},Q\})$ = $\{2\}$, $\Psi^{-1}(P,\emptyset\})$ = $\emptyset$, $\Psi^{-1}(P,\{\mu\})$ = $\{1,3\}$, $\Psi^{-1}(P,\{\nu\})$ = $\{2\}$, $\Psi^{-1}(P,\{\omega\})$ = $\emptyset$, $\Psi^{-1}(P,\{\mu,\omega\})$ = $\emptyset$, $\Psi^{-1}(P,\{\mu,\nu\})$ = $\emptyset$, $\Psi^{-1}(P,\{\nu,\omega\})$ = $\emptyset$. This gives pre image of every b-gspO sets in (P,Q,$\mathscr{M}$) is gsp-O in R. $\therefore$ Ψ is b-gsp-irresolute but not b-gp irresolute function.

5. Conclusion

In this paper, we introduced the notions binary generalized pre continuous and binary generalized semi pre continuous functions (briefly b-gpCF(b-gspCF)), Totally b-gpCF(b-gspCF), Strongly b-gpCF(b-gspCF) and b-gp(b-gsp)-irresolute functions in BTS. Also we discussed some of the properties and characterizations. Further the relationship between the other existing continuous functions were investigated in BTS. In future we will map these continuity concepts onto Rough Set Theory and Nano Topologies to better handle uncertainty, vague boundaries, and incomplete information in modern machine learning datasets. Our future work is to explore a separation axioms in binary topological spaces to examine the characteristics of these axioms within the framework of classical topological spaces and highly likely to find the applications in the theory of b-gp(b-gsp) closed sets.

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