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Nonlinear dynamics and stability of a three-dimensional bank balance-sheet model

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Abstract This paper analyzes the nonlinear dynamics of a three-dimensional bank balance-sheet model describing the interactions among deposits (D), loans (L), and equity (Q) under a reserve requirement mechanism. Under the parameter regime considered, the system admits the zero-activity equilibrium E_0, the deposit–equity equilibrium E_1, and a positive interior equilibrium E^*. The local stability analysis shows that E_0 is unstable when the net deposit attraction rate (γ) is greater than the deposit withdrawal rate (w), whereas E_1 and E^* are locally asymptotically stable under their corresponding parameter conditions. Lyapunov functions and LaSalle's Invariance Principle are employed to derive sufficient conditions for the global asymptotic stability of E_1 and E^* within the corresponding positively invariant regions. Numerical simulations using the fourth-order Runge–Kutta method illustrate the convergence behaviour under two parameter configurations and are consistent with the analytical stability results. The study provides a unified theoretical framework for examining the equilibrium structure and long-term dynamics of a nonlinear deposit–loan–equity system.

Keywords Bank Balance-Sheet Model, Dynamical Systems, Deposit-Loan-Equity System, Global Stability Analysis

AMS 2010 subject classifications 34D20, 34D23, 37C60, 37N40

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1. Introduction

Financial stability is a fundamental pillar of modern economies, in which banks contribute to economic growth primarily through their role as financial intermediaries [1, 2]. Banks collect funds in the form of deposits and allocate part of these funds to lending activities [3, 4]. The effective performance of this function requires careful balance-sheet management, since a bank's financial position is reflected in its assets, liabilities, and equity [5]. In a simplified balance-sheet structure, loans and reserves constitute major asset components, customer deposits represent a principal liability [6, 7], and equity serves as a buffer against unexpected losses [8, 9]. Understanding the evolution of deposits, loans, and equity is therefore important for examining the financial condition and long-term stability of banking institutions [10, 11, 12, 13].

This study develops a theoretical framework for the mathematical modelling of banking dynamics. Instead of reproducing the full accounting, institutional, and regulatory structure of a real-world commercial bank, the proposed framework isolates the fundamental interactions among deposits, loans, and equity. These interactions are formulated through a system of nonlinear differential equations, providing a basis for the analysis of the qualitative behaviour and dynamic characteristics of the resulting banking system. This formulation allows the equilibrium structure, stability conditions, and long-term behaviour of the model to be examined analytically.

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A deterministic continuous-time formulation is adopted to isolate the structural feedback mechanisms among the balance-sheet components and to derive transparent analytical stability conditions before introducing stochastic shocks or time-varying policy effects.

The operational environment of banking institutions is strongly influenced by regulatory policies that constrain liquidity allocation [14, 15]. One important policy instrument is the Reserve Requirement (RR), known in Indonesia as the Giro Wajib Minimum (GWM), which requires banks to retain a specified fraction of customer deposits as reserves at the central bank [16, 17]. Although reserve requirements support liquidity management and financial stability, they also reduce the proportion of deposited funds available for lending [18, 19]. Consequently, changes in reserve requirement policy may affect deposit accumulation, lending capacity, and the development of bank equity [20].

Banking balance sheets are also influenced by fluctuations in market confidence, interest-rate adjustments [21], deposit withdrawals, credit demand, and changes in non-performing loans [22, 23]. These factors may alter the availability of funds and disturb the relationship among deposits, loans, reserves, and equity [24, 25]. Since changes in one component may propagate through the balance sheet, the resulting feedback mechanisms can produce nonlinear adjustment patterns [6, 26, 27]. A nonlinear dynamical-system approach is therefore useful for examining whether the modelled banking system converges to a stable operating state or moves away from equilibrium under different structural and regulatory conditions.

Mathematical models formulated through nonlinear differential equations provide an analytically tractable framework for studying these temporal interactions [28]. Related studies have investigated global stability in fractional infection models [29], numerical solutions of nonlinear fractional differential equations [30], stability in delayed predator–prey dynamics [31], and local stability in compartmental corruption models [32]. Within this framework, equilibrium points characterize the possible steady-state configurations of the modelled banking system. Their stability indicates whether small deviations from a steady state diminish over time or cause the system to move away from that configuration. An equilibrium with positive deposits, loans, and equity may be interpreted as a fully active banking state, whereas equilibria with one or more zero components represent boundary states in which certain banking activities are absent. These interpretations are intended to provide insight into the mathematical behaviour of the system and do not seek to capture the full range of institutional conditions present in actual banking practice.

Several studies have examined banking dynamics through deterministic and nonlinear differential equation models. Sumarti et al. [33] developed loan–deposit interaction models based on the Lotka–Volterra framework, including a formulation involving reserve requirements. Damayanti et al. [34] extended the loan–deposit framework by incorporating deposit withdrawals and non-performing loans, while a subsequent study [35] introduced a reserve requirement mechanism into this class of models. In a different direction, Ansori et al. [28] formulated a balance-sheet model involving deposits, loans, and equity. Sumarti et al. [36] further developed the balance-sheet framework through a profit formulation combining loan revenues, returns from other assets, deposit costs, equity costs, and operational expenses. More recent studies have incorporated macroprudential reserve requirements [37] and further modifications of deterministic banking balance-sheet models [38]. These studies provide the main mathematical components used in the present work, but they consider them within different model structures.

Relative to earlier two-dimensional loan–deposit formulations [33, 34, 35], the present model introduces equity as an endogenous state variable, which allows the interaction between deposit–loan dynamics and capital accumulation to be examined within the same system. Compared with the stochastic balance-sheet formulation of Ansori et al. [28], the present study adopts a deterministic continuous-time framework and explicitly couples reserve-adjusted deposit–loan conversion with profit-driven equity dynamics. Macroprudential formulations such as [37] emphasize broader regulatory mechanisms, whereas the present model places the reserve requirement directly in the nonlinear loan-conversion term. Consequently, the current formulation yields boundary and interior equilibria whose admissibility and stability depend jointly on lending, reserve, and equity-related parameters.

In the present study, these mechanisms are integrated into a single three-dimensional system describing deposits, loans, and equity. The model combines logistic deposit growth, a saturating deposit-loan interaction under a reserve requirement, withdrawal and non-performing-loan effects, and profit-driven equity dynamics. Thus, the novelty

does not lie in introducing these mechanisms individually, but in their specific integration within a coupled deposit–loan–equity framework and in the qualitative analysis that follows from this formulation.

The mathematical contribution of this study can be identified across several levels. First, the equilibrium structure of the resulting three-dimensional system is characterized by identifying its boundary and economically admissible interior equilibria. Second, sufficient parameter conditions for local asymptotic stability are derived from the linearized system. Third, Lyapunov functions together with LaSalle's Invariance Principle are used to obtain sufficient conditions for global asymptotic convergence within the corresponding positively invariant regions. Numerical simulations are then used to illustrate the analytical results and the adjustment paths of deposits, loans, and equity under different parameter configurations. The contribution is therefore primarily mathematical, providing a unified framework for examining equilibrium configurations and sufficient local and global stability conditions in a nonlinear banking system.

The remainder of this paper is organized as follows. Section 2 introduces the model formulation and the interpretation of its components. Section 3 presents the equilibrium and stability analyses. Section 4 provides numerical simulations illustrating the theoretical results, and Section 5 concludes the study and outlines possible directions for future model development.

2. Model Formulation

To describe the main mechanisms of bank balance-sheet dynamics, we formulate a theoretical nonlinear system involving deposits, loans, and equity. Deposit accumulation is represented by a logistic growth term, following the modelling approach in [33, 34, 35], which accounts for deposit attraction subject to a finite carrying capacity. The conversion of available deposits into loans is represented by a saturating nonlinear interaction, where the factor $(1-m)D$ denotes the proportion of deposits available for lending after the reserve requirement, while b represents friction in the deposit-loan conversion process.

The equity dynamics are linked to bank profit and losses associated with non-performing loans. Following [36], the profit formulation incorporates loan-interest revenue, returns from other assets, deposit-interest payments, equity costs, and operational costs. These mechanisms are integrated into a three-dimensional nonlinear system designed for qualitative analysis of equilibrium configurations and stability.

Based on these assumptions, the proposed balance-sheet dynamics are formulated as follows:

$$\begin{aligned}
\frac{dD}{dt} &= \boldsymbol{\gamma} D\left(1-\frac{D}{k}\right) - \frac{p(1-m)DL}{b(1-m)D+1} - wD,\\
\frac{dL}{dt} &= \frac{p(1-m)DL}{b(1-m)D+1} - (\sigma+n)L - c(1-\mu)L - hL^2,\\
\frac{dQ}{dt} &= \alpha P - nL.
\end{aligned} \tag{1}$$

where

$$L + A = D + Q$$

is the balance-sheet identity, A denotes the volume of other bank assets, $R = mD$ denotes the reserve volume required by the reserve requirement policy, and P represents bank profit, defined as

$$P = \sigma_L(1-\mu)L + r_A(A-R) - \gamma_D(1-w_D)D - r_E Q - (C_D D + C_L L).$$

Since $R(t) = mD(t)$, system (1) can be rewritten as follows.

$$\begin{aligned}
\frac{dD}{dt} &= \gamma D\left(1-\frac{D}{k}\right) - \frac{p(1-m)DL}{b(1-m)D+1} - wD,\\
\frac{dL}{dt} &= \frac{p(1-m)DL}{b(1-m)D+1} - HL - hL^2,\\
\frac{dQ}{dt} &= \alpha\left(\kappa_1 L + \kappa_2 D + \kappa_3 Q\right) - nL.
\end{aligned} \tag{2}$$

where

$$\begin{aligned} H &= \sigma + n + c(1-\mu), \\ \kappa_1 &= \sigma_L(1-\mu) - r_A - C_L, \\ \kappa_2 &= r_A(1-m) - \gamma_D(1-w_D) - C_D, \\ \kappa_3 &= r_A - r_E. \end{aligned}$$

The model variables and parameters are summarized in Table 1.

Table 1. Description of model variables and parameters.

Symbol	Description
A	Volume of other bank assets.
D	Deposit volume.
L	Loan volume.
Q	Equity volume.
R	Reserve volume, with $R = mD$.
P	Bank profit.
k	Deposit carrying capacity.
b	Friction parameter in converting available deposits into loans.
c	Loan repayment rate.
h	Loan competition rate.
n	Non-performing-loan loss rate affecting loan and equity dynamics.
p	Maximum loan issuance rate.
γ	Net deposit attraction rate.
μ	Proportion of non-performing loans affecting repayment and loan-interest revenue.
σ	Loan interest rate.
m	Reserve requirement proportion.
α	Equity growth rate from profit.
σ_L	Proportion of loan interest revenue.
r_A	Return proportion from other assets.
γ_D	Proportion of deposit interest payments.
w	Deposit withdrawal rate in the deposit dynamics.
w_D	Withdrawal proportion used in the deposit-interest payment term.
r_E	Equity cost proportion.
C_D	Operational cost proportion for deposits.
C_L	Operational cost proportion for loans.

For dimensional consistency, let M and T denote the monetary-volume and time units, respectively. Then

$$[D] = [L] = [Q] = [A] = [R] = [k] = [P] = M.$$

The parameters γ, w, σ, n, c, and α have units of T^{-1}, while

$$[b] = M^{-1}, \qquad [p] = [h] = M^{-1}T^{-1}.$$

The quantities m, μ, w_D, σ_L, r_A, γ_D, r_E, C_D, and C_L are dimensionless proportions. Consequently, κ_1, κ_2, and κ_3 are dimensionless, and every right-hand side of system (2) has dimension MT^{-1}.

The well-posedness of system (2) is investigated by establishing the existence and uniqueness of solutions and verifying their nonnegativity and boundedness in the following subsections.

2.1. Existence and Uniqueness of Solutions

Define

$$\Omega_2 = \left\{(D, L, Q) \in \mathbb{R}^2_+ \times \mathbb{R} : \max\{|D|, |L|, |Q|\} \leq M_2\right\}.$$

Let $M_2 > 0$ be arbitrary. The set Ω_2 is therefore a bounded subset of the state space on which the Lipschitz estimate is established. The nonnegativity of $D(t)$ and $L(t)$ and the a priori bounds required for global continuation are established in the subsequent subsections. Let

$$X_2 = (D, L, Q), \qquad \bar{X}_2 = (\bar{D}, \bar{L}, \bar{Q}),$$

and define the mapping

$$G(X_2) = (G_1(X_2), G_2(X_2), G_3(X_2)),$$

where

$$\begin{aligned}
G_1(X_2) &= \gamma D\left(1 - \frac{D}{k}\right) - \frac{p(1-m)DL}{b(1-m)D+1} - wD, \\
G_2(X_2) &= \frac{p(1-m)DL}{b(1-m)D+1} - HL - hL^2, \\
G_3(X_2) &= \alpha\left(\kappa_1 L + \kappa_2 D + \kappa_3 Q\right) - nL.
\end{aligned}$$

Then, for any $X_2, \bar{X}_2 \in \Omega_2$, we obtain

$$\begin{aligned}
\|G(X_2) - G(\bar{X}_2)\| &= |G_1(X_2) - G_1(\bar{X}_2)| + |G_2(X_2) - G_2(\bar{X}_2)| + |G_3(X_2) - G_3(\bar{X}_2)| \\
&\leq \gamma|D - \bar{D}| + \frac{2\gamma M_2}{k}|D - \bar{D}| + w|D - \bar{D}| \\
&\quad + \frac{2p(1-m)M_2}{(b(1-m)D+1)(b(1-m)\bar{D}+1)}|D - \bar{D}| \\
&\quad + \frac{2p(1-m)M_2}{b(1-m)\bar{D}+1}|L - \bar{L}| + H|L - \bar{L}| + 2hM_2|L - \bar{L}| \\
&\quad + \alpha|\kappa_1||L - \bar{L}| + \alpha|\kappa_2||D - \bar{D}| + \alpha|\kappa_3||Q - \bar{Q}| + n|L - \bar{L}|.
\end{aligned}$$

Since $D \geq 0$, $b \geq 0$, and $0 \leq m < 1$, and $b(1-m)D + 1 \geq 1$, we obtain

$$\begin{aligned}
\|G(X_2) - G(\bar{X}_2)\| &\leq \left(\gamma + \frac{2\gamma M_2}{k} + w + 2p(1-m)M_2 + \alpha|\kappa_2|\right)|D - \bar{D}| \\
&\quad + \left(2p(1-m)M_2 + H + 2hM_2 + \alpha|\kappa_1| + n\right)|L - \bar{L}| \\
&\quad + \alpha|\kappa_3||Q - \bar{Q}| \\
&= \mathcal{K}_1|D - \bar{D}| + \mathcal{K}_2|L - \bar{L}| + \alpha|\kappa_3||Q - \bar{Q}|.
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{K}_1 &= \gamma + \frac{2\gamma M_2}{k} + w + 2p(1-m)M_2 + \alpha|\kappa_2|, \\
\mathcal{K}_2 &= 2p(1-m)M_2 + H + 2hM_2 + \alpha|\kappa_1| + n, \\
\mathcal{K} &= \max\left\{\mathcal{K}_1, \mathcal{K}_2, \alpha|\kappa_3|\right\}.
\end{aligned}$$

Therefore,

$$\|G(X_2) - G(\bar{X}_2)\| \leq \mathcal{K}\left(|D - \bar{D}| + |L - \bar{L}| + |Q - \bar{Q}|\right) = \mathcal{K}\|X_2 - \bar{X}_2\|.$$

The function $G(X_2)$ therefore satisfies the Lipschitz condition on Ω_2 [39]. Hence, for a given initial condition in Ω_2, system (2) admits a unique local solution. The a priori bounds established in Section 2.3 ensure that, under the stated boundedness condition, the solution can be continued for all $t \geq 0$. Thus, for a given initial condition and fixed parameter values, the model generates a unique trajectory for the evolution of deposits, loans, and equity.

2.2. *Nonnegativity of Deposits and Loans and the Behaviour of Equity*

Suppose, that for some $t > 0$, the function $D(t)$ becomes negative. This assumption implies the existence of a time $t_1 > 0$ such that $D(t) > 0$ for $0 < t < t_1$, $D(t_1) = 0$, and $D(t) < 0$ for $t > t_1$. From system (2), we obtain

$$\left.\frac{dD}{dt}\right|_{t=t_1} = 0,$$

which implies that $D(t) = 0$ for $t \in (t_1, t_1 + \epsilon)$, where $\epsilon > 0$ is an arbitrarily small constant. This contradicts the assumption that $D(t) < 0$ for $t > t_1$. Therefore, the assumption is false, and we conclude that $D(t) \geq 0$ for all $t > 0$. By an analogous argument, it follows that $L(t) \geq 0$ for all $t > 0$. Unlike deposits and loans, the equity variable $Q(t)$ is not required to remain nonnegative in the present model. Economically, $Q = 0$ represents a state in which the bank's assets equal its liabilities, marking the boundary between positive and negative equity. From the third equation of system (2), at $Q = 0$, we obtain

$$\left.\frac{dQ}{dt}\right|_{Q=0} = (\alpha\kappa_1 - n)L + \alpha\kappa_2 D.$$

Since this expression may be negative, the nonnegativity of $Q(t)$ cannot be guaranteed without additional parameter restrictions. Negative equity may indicate financial distress or insolvency.

The model guarantees the nonnegativity of the deposit and loan variables, consistent with the fact that these balance-sheet components cannot be negative. In contrast, the equity variable is allowed to take either positive or negative values depending on the parameter values and the resulting system dynamics. A positive equity value represents a solvent financial position, whereas a negative equity value may be interpreted as a state of financial distress or insolvency.

2.3. *Boundedness of the Solutions*

We now show that the solutions $D(t)$ and $L(t)$ are bounded for all $t \geq 0$, for initial conditions

$$(D(0), L(0)) \in \mathbb{R}^2_+ \cup \{\vec{0}\}.$$

Let

$$V(t) = D(t) + L(t).$$

Then, using system (2), we obtain

$$\begin{aligned}
\frac{dV}{dt} &= \gamma D\left(1 - \frac{D}{k}\right) - \frac{p(1-m)DL}{b(1-m)D+1} - wD + \frac{p(1-m)DL}{b(1-m)D+1} - HL - hL^2 \\
&= (\gamma - w)D - \frac{\gamma}{k}D^2 + (c\mu - c - \sigma - n)L - hL^2 \\
&\leq (\gamma - w)D - \frac{\gamma}{k}D^2 + (c\mu - c - \sigma - n)L. \qquad (3)
\end{aligned}$$

For any constant $\xi > 0$, we obtain

$$\frac{dV}{dt} + \xi V \leq (\gamma + \xi - w)D - \frac{\gamma}{k}D^2 + (\xi + c\mu - c - \sigma - n)L.$$

Choosing $0 < \xi < c(1-\mu) + \sigma + n$ gives

$$\frac{dV}{dt} + \xi V \leq (\gamma + \xi - w)D - \frac{\gamma}{k}D^2.$$

Completing the square yields

$$\frac{dV}{dt} + \xi V \leq -\frac{\gamma}{k}\left(D - \frac{k(\gamma + \xi - w)}{2\gamma}\right)^2 + \frac{k(\gamma + \xi - w)^2}{4\gamma}.$$

Therefore,

$$\frac{dV}{dt} \leq \frac{k(\gamma+\xi-w)^2}{4\gamma} - \xi V.$$

Solving this differential inequality gives

$$V(t) \leq \frac{k(\gamma+\xi-w)^2}{4\xi\gamma} + \left(V(0) - \frac{k(\gamma+\xi-w)^2}{4\xi\gamma}\right) e^{-\xi t}.$$

Taking the upper limit as $t \to \infty$,

$$\limsup_{t\to\infty} V(t) \leq \frac{k(\gamma+\xi-w)^2}{4\xi\gamma}.$$

Hence, the solutions $D(t)$ and $L(t)$ remain bounded and eventually lie in

$$\Omega_{DL} = \left\{(D,L) \in \mathbb{R}^2_+ : D + L \leq \frac{k(\gamma+\xi-w)^2}{4\xi\gamma}\right\}.$$

For the equity variable, from the third equation of system (2),

$$\frac{dQ}{dt} = \alpha\kappa_3 Q + (\alpha\kappa_1 - n)L + \alpha\kappa_2 D.$$

Since $D(t)$ and $L(t)$ are bounded, there exists a constant $C_Q > 0$ such that

$$|(\alpha\kappa_1 - n)L + \alpha\kappa_2 D| \leq C_Q.$$

Using the variation of constants formula,

$$Q(t) = e^{\alpha\kappa_3 t}Q(0) + \int_0^t e^{\alpha\kappa_3(t-s)}\left[(\alpha\kappa_1 - n)L(s) + \alpha\kappa_2 D(s)\right] ds.$$

If $\kappa_3 < 0$, then

$$|Q(t)| \leq |Q(0)|e^{\alpha\kappa_3 t} + \frac{C_Q}{-\alpha\kappa_3}\left(1 - e^{\alpha\kappa_3 t}\right),$$

which shows that $Q(t)$ is bounded. For $\kappa_3 \geq 0$, uniform boundedness of $Q(t)$ is not guaranteed in general; therefore, the subsequent boundedness and global-stability results are stated under the condition $\kappa_3 < 0$. Therefore, under the condition $\kappa_3 < 0$, all state variables of system (2) remain bounded. The boundedness of $D(t)$ and $L(t)$ reflects the limiting mechanisms incorporated in the deposit and loan dynamics. For the equity variable, the condition $\kappa_3 < 0$, or equivalently $r_A < r_E$, provides a stabilizing effect on the equity dynamics and, together with the boundedness of deposits and loans, prevents unbounded growth of $Q(t)$. Thus, under this parameter condition, the model trajectories remain within finite levels.

3. Dynamical Analysis

This section examines the qualitative behavior of the proposed banking model by analyzing its long-term dynamics. We begin by identifying the equilibrium points that represent steady-state configurations of deposits, loans, and equity. We then analyze the local stability of these equilibria to assess the system's response to small perturbations around each equilibrium. Finally, we investigate the global stability behaviour and derive sufficient conditions under which the system trajectories converge to an equilibrium within the corresponding positively invariant regions.

3.1. Equilibrium Points

The equilibrium points of system (2) are obtained by solving

$$\frac{dD}{dt} = \frac{dL}{dt} = \frac{dQ}{dt} = 0.$$

The equilibrium points of the system are described as follows.

1. Zero-activity equilibrium
   The zero equilibrium is given by
   $$E_0 = (0, 0, 0).$$
   This equilibrium represents a zero-activity state of the model in which deposits, loans, and equity are all zero. It is interpreted here as a mathematical boundary state without assigning a specific institutional or regulatory meaning to it.
2. Deposit–equity equilibrium

   For $L = 0$ and $D > 0$, the first equilibrium equation becomes
   $$\gamma D\left(1 - \frac{D}{k}\right) - wD = 0.$$
   Hence,
   $$D_{(1)} = \frac{k}{\gamma}(\gamma - w),$$
   which is positive whenever $\gamma > w$. Using the third equilibrium equation, for $\kappa_3 \neq 0$, we obtain
   $$Q_{(1)} = -\frac{\kappa_2 D_{(1)}}{\kappa_3} = -\frac{\kappa_2 k(\gamma - w)}{\kappa_3 \gamma}.$$
   Therefore, the deposit–equity equilibrium is $E_1 = \left(D_{(1)}, 0, Q_{(1)}\right)$. The equity component is positive whenever $Q_{(1)} > 0$, or equivalently, $\kappa_2\kappa_3 < 0$. Thus, positive equity occurs when either $\kappa_2 < 0$ and $\kappa_3 > 0$, or $\kappa_2 > 0$ and $\kappa_3 < 0$. The equilibrium E_1 represents a boundary state of the model in which deposits and equity may remain positive while the loan volume vanishes.
3. Full banking equilibrium
   $$E^* = (D^*,\, L^*,\, Q^*),$$
   where
   $$L^* = \frac{1}{h}\left(\frac{p(1-m)D^*}{1 + b(1-m)D^*} - H\right),$$
   which is positive provided that
   $$\frac{p(1-m)D^*}{1 + b(1-m)D^*} > H.$$
   For $\kappa_3 \neq 0$, the equilibrium equity is given by
   $$Q^* = \frac{L^*(n - \alpha\kappa_1) - \alpha\kappa_2 D^*}{\alpha\kappa_3},$$
   which is in surplus whenever
   $$s > 0, \qquad \kappa_3 > 0,$$
   or
   $$s < 0, \qquad \kappa_3 < 0,$$

where

$$s = L^*(n - \alpha\kappa_1) - \alpha\kappa_2 D^*.$$

The equilibrium value D^* is determined by a positive real root of the cubic polynomial

$$(D^*)^3 + B_1(D^*)^2 + B_2 D^* + B_3 = 0, \tag{4}$$

where

$$\begin{aligned} B_1 &= -\frac{(\gamma - w)khb^2(1-m)^2 - 2hb(1-m)\gamma}{\gamma hb^2(1-m)^2}, \\ B_2 &= -\frac{2khb(\gamma - w)(1-m) + pkb(1-m)^2 H - \gamma h - p^2 k(1-m)^2}{\gamma hb^2(1-m)^2}, \\ B_3 &= -\frac{(\gamma - w)kh + pk(1-m)H}{\gamma hb^2(1-m)^2}. \end{aligned}$$

The real roots of equation (4) can be determined using Cardano's method as described in [40, 34]. Among these roots, any positive real root D^* satisfying $L^* > 0$ and $Q^* > 0$ determines an admissible interior equilibrium. The existence and convergence toward the corresponding equilibrium are further illustrated in the numerical simulations. This equilibrium may represent a fully functioning banking operation, in which deposit inflows are transformed into loan assets, while the resulting financial returns contribute to maintaining a positive equity position.

3.2. Local stability analysis

The local stability analysis is performed by linearizing the system through the Jacobian matrix as follows.

$$J(D, L, Q) = \begin{pmatrix} J_1 & -H_2 & 0 \\ H_1 & J_2 & 0 \\ \alpha\kappa_2 & \alpha\kappa_1 - n & \alpha\kappa_3 \end{pmatrix}.$$

where

$$\begin{aligned} J_1 &= \gamma\left(1 - \frac{2D}{k}\right) - H_1 - w, \\ J_2 &= H_2 - H - 2hL, \\ H_1 &= \frac{p(1-m)L}{(b(1-m)D + 1)^2}, \\ H_2 &= \frac{p(1-m)D}{b(1-m)D + 1}, \\ H &= \sigma + n + c(1-\mu). \end{aligned}$$

Throughout the following local stability analysis, we assume that $\gamma > w$ which also guarantees the positivity of the deposit component $D_{(1)}$ of the equilibrium E_1.

Theorem 1. The local stability of the equilibrium points is characterized as follows:

1. Under the assumption $\gamma > w$, the equilibrium point E_0 is unstable.
2. If $\kappa_3 < 0$ and $H_{2_{(1)}} < H$, then the deposit–equity equilibrium point E_1 is locally asymptotically stable. If $\kappa_3 > 0$ or $H_{2_{(1)}} > H$, then E_1 is unstable. For $\kappa_3 = 0$ or $H_{2_{(1)}} = H$, the linearization is inconclusive.
3. The full banking equilibrium point E^* is locally asymptotically stable if $\kappa_3 < 0$, $\mathrm{tr}(J^*) < 0$, and $\det(J^*) > 0$, where J^* is specified in the proof.

Proof.

1. The Jacobian matrix at E_0 is given by

$$J(0,0,0) = \begin{pmatrix} \gamma - w & 0 & 0 \\ 0 & -H & 0 \\ \alpha\kappa_2 & \alpha\kappa_1 - n & \alpha\kappa_3 \end{pmatrix},$$

from which the eigenvalues are obtained as follows:

$$\begin{aligned} \lambda_1 &= \gamma - w > 0, \\ \lambda_2 &= -H < 0, \\ \lambda_3 &= \alpha\kappa_3. \end{aligned}$$

Under the assumption $\gamma > w$, we have $\lambda_1 = \gamma - w > 0$. Therefore, E_0 has at least one positive eigenvalue and is unstable.

2. The Jacobian matrix at E_1 is given by

$$J(D_{(1)}, 0, Q_{(1)}) = \begin{pmatrix} -(\gamma - w) & -H_{2_{(1)}} & 0 \\ 0 & H_{2_{(1)}} - H & 0 \\ \alpha\kappa_2 & \alpha\kappa_1 - n & \alpha\kappa_3 \end{pmatrix},$$

where

$$H_{2_{(1)}} = \frac{p(1-m)D_{(1)}}{b(1-m)D_{(1)} + 1}.$$

The characteristic equation is obtained as

$$(\alpha\kappa_3 - \lambda) \det \begin{pmatrix} -(\gamma - w) - \lambda & -H_{2_{(1)}} \\ 0 & H_{2_{(1)}} - H - \lambda \end{pmatrix} = 0,$$

which yields the eigenvalues

$$\begin{aligned} \lambda_1 &= \alpha\kappa_3, \\ \lambda_2 &= -(\gamma - w) < 0, \\ \lambda_3 &= H_{2_{(1)}} - H. \end{aligned}$$

Since $\alpha > 0$ and $\gamma > w$, all eigenvalues have negative real parts if $\kappa_3 < 0$ and $H_{2_{(1)}} < H$. Therefore, E_1 is locally asymptotically stable under these conditions. If $\kappa_3 > 0$ or $H_{2_{(1)}} > H$, at least one eigenvalue is positive, and hence E_1 is unstable. In the boundary cases $\kappa_3 = 0$ or $H_{2_{(1)}} = H$, the Jacobian has a zero eigenvalue, so the linearization is inconclusive.

3. For E^*, the Jacobian matrix is given by

$$J(D^*, L^*, Q^*) = \begin{pmatrix} J_1^* & -H_2^* & 0 \\ H_1^* & J_2^* & 0 \\ \alpha\kappa_2 & \alpha\kappa_1 - n & \alpha\kappa_3 \end{pmatrix},$$

where

$$\begin{aligned} J_1^* &= \gamma\left(1 - \frac{2D^*}{k}\right) - H_1^* - w, \\ J_2^* &= H_2^* - H - 2hL^*, \\ H_1^* &= \frac{p(1-m)L^*}{(b(1-m)D^* + 1)^2}, \\ H_2^* &= \frac{p(1-m)D^*}{b(1-m)D^* + 1}, \\ H &= \sigma + n + c(1-\mu). \end{aligned}$$

The characteristic equation is obtained as

$$(\alpha\kappa_3 - \lambda)\det(J^* - \lambda I) = 0,$$

where

$$J^* = \begin{pmatrix} J_1^* & -H_2^* \\ H_1^* & J_2^* \end{pmatrix}.$$

Equivalently,

$$(\alpha\kappa_3 - \lambda)\left[\lambda^2 - \mathrm{tr}(J^*)\lambda + \det(J^*)\right] = 0.$$

Hence,

$$\lambda_1 = \alpha\kappa_3, \qquad \lambda_{2,3} = \frac{\mathrm{tr}(J^*) \pm \sqrt{\mathrm{tr}(J^*)^2 - 4\det(J^*)}}{2}.$$

Clearly, the first eigenvalue $\lambda_1 = \alpha\kappa_3$ is negative if $\kappa_3 < 0$. The remaining two eigenvalues have negative real parts if

$$\mathrm{tr}(J^*) < 0 \quad \text{and} \quad \det(J^*) > 0.$$

Therefore, E^* is locally asymptotically stable if

$$\kappa_3 < 0, \qquad \mathrm{tr}(J^*) < 0, \qquad \det(J^*) > 0.$$

3.3. Global stability analysis

In this subsection, sufficient conditions for the global asymptotic stability of the equilibrium points are established using appropriate Lyapunov functions and LaSalle's Invariance Principle [41].

For the global stability analysis, define

$$\Omega_1 = \{(D, L, Q) \in \mathbb{R}^3 : D > 0,\ L \geq 0\},$$

and

$$\Omega_* = \{(D, L, Q) \in \mathbb{R}^3 : D > 0,\ L > 0\}.$$

The nonnegativity and uniqueness results established in Section 2.2 ensure that these conditions on D and L are preserved along the corresponding solutions.

The results of the global stability analysis are formally stated in **Theorem 2.**

Theorem 2. The global stability of the equilibrium points is characterized as follows:

1. If $p(1-m)D_{(1)} \leq H$ and $\kappa_3 < 0$, then E_1 is globally asymptotically stable in Ω_1.
2. If $\frac{\gamma}{k} > pb(1-m)^2L^*$ and $\kappa_3 < 0$, then the interior equilibrium E^* is globally asymptotically stable in Ω_*.

The condition $p(1-m)D_{(1)} \leq H$ indicates that the loan-generation effect is not sufficient to offset the aggregate loan-reduction term at E_1. Meanwhile, $\gamma/k > pb(1-m)^2L^*$ is a sufficient condition obtained from the Lyapunov analysis for the global stability of E^*. These conditions are sufficient, but not necessary, for stability.

Proof.

1. Suppose the Lyapunov function is defined as follows

$$\mathcal{L}_1 = D - D_{(1)} - D_{(1)}\ln\frac{D}{D_{(1)}} + L.$$

At the equilibrium point E_1, we have

$$\gamma \quad = \quad \frac{\gamma D_{(1)}}{k} + w.$$

The first derivative of the Lyapunov function $\mathcal{L}_1$ with respect to t is given by

$$\begin{aligned}
\frac{d\mathcal{L}_1}{dt} &= \left(1-\frac{D_{(1)}}{D}\right)\frac{dD}{dt}+\frac{dL}{dt}\\
&= (D-D_{(1)})\left(\gamma\left(1-\frac{D}{k}\right)-\frac{p(1-m)L}{b(1-m)D+1}-w\right)+\frac{p(1-m)DL}{b(1-m)D+1}-HL-hL^2\\
&= (D-D_{(1)})\left(\frac{\gamma D_{(1)}}{k}+w-\frac{\gamma D}{k}-\frac{p(1-m)L}{b(1-m)D+1}-w\right)+\frac{p(1-m)DL}{b(1-m)D+1}-HL-hL^2\\
&= (D-D_{(1)})\left(\frac{-\gamma}{k}(D-D_{(1)})\right)-\frac{p(1-m)L(D-D_{(1)})}{b(1-m)D+1}+\frac{p(1-m)DL}{b(1-m)D+1}-HL-hL^2\\
&= -\frac{\gamma}{k}(D-D_{(1)})^2+\left(\frac{p(1-m)D_{(1)}}{b(1-m)D+1}-H\right)L-hL^2\\
&\leq -\frac{\gamma}{k}(D-D_{(1)})^2+\left(p(1-m)D_{(1)}-H\right)L-hL^2.
\end{aligned}$$

If $p(1-m)D_{(1)}\leq H$, then $\frac{d\mathcal{L}_1}{dt}\leq 0$, with equality only at $(D,L)=(D_{(1)},0)$. By LaSalle's invariance principle [41],

$$(D,L)\longrightarrow(D_{(1)},0)\qquad\text{as}\qquad t\longrightarrow\infty.$$

From the third equation of system (2),

$$\frac{dQ}{dt}=\alpha\kappa_3 Q+\alpha\kappa_2 D+(\alpha\kappa_1-n)L.$$

Since $D(t)\to D_{(1)}$ and $L(t)\to 0$, and $\kappa_3<0$, the variation-of-constants formula yields

$$Q(t)\longrightarrow-\frac{\kappa_2 D_{(1)}}{\kappa_3}=Q_{(1)}\qquad\text{as}\qquad t\longrightarrow\infty.$$

Hence, the equilibrium point E_1 is globally asymptotically stable if $p(1-m)D_{(1)}\leq H$ and $\kappa_3<0$.

2. Suppose the Lyapunov function is defined as

$$\mathcal{L}_2=D-D^*-D^*\ln\frac{D}{D^*}+v\left(L-L^*-L^*\ln\frac{L}{L^*}\right),$$

where $v=b(1-m)D^*+1$. At the equilibrium point E^*, we obtain

$$\begin{aligned}
\gamma-w &= \frac{\gamma D^*}{k}+\frac{p(1-m)L^*}{b(1-m)D^*+1},\\
\sigma+n+c(1-\mu) &= \frac{p(1-m)D^*}{b(1-m)D^*+1}-hL^*.
\end{aligned}$$

The first derivative of $\mathcal{L}_2$ with respect to t becomes

$$\begin{aligned}
\frac{d\mathcal{L}_2}{dt} &= \left(1-\frac{D^*}{D}\right)\frac{dD}{dt}+v\left(1-\frac{L^*}{L}\right)\frac{dL}{dt}\\
&\leq -\frac{\gamma}{k}(D-D^*)^2-vh(L-L^*)^2+pb(1-m)^2L^*(D-D^*)^2.
\end{aligned}$$

Since $v=b(1-m)D^*+1$, we obtain

$$\frac{d\mathcal{L}_2}{dt}\leq-\frac{\gamma}{k}(D-D^*)^2-vh(L-L^*)^2+pb(1-m)^2L^*(D-D^*)^2.$$

If $\frac{\gamma}{k}>pb(1-m)^2L^*$, then $\frac{d\mathcal{L}_2}{dt}\leq 0$ for $D>0$ and $L>0$, with equality only at $(D,L)=(D^*,L^*)$. By LaSalle's invariance principle [41],

$$(D,L)\longrightarrow(D^*,L^*)\qquad\text{as}\qquad t\longrightarrow\infty.$$

From the third equation of system (2),

$$\frac{dQ}{dt} = \alpha\kappa_3 Q + (\alpha\kappa_1 - n)L + \alpha\kappa_2 D.$$

Since $D(t) \to D^*$ and $L(t) \to L^*$, and $\kappa_3 < 0$, the variation-of-constants formula yields

$$Q(t) \longrightarrow -\frac{(\alpha\kappa_1 - n)L^* + \alpha\kappa_2 D^*}{\alpha\kappa_3} = Q^* \qquad \text{as} \qquad t \longrightarrow \infty.$$

Therefore, the equilibrium point E^* is globally asymptotically stable if $\frac{\gamma}{k} > pb(1-m)^2 L^*$ and $\kappa_3 < 0$.

4. Numerical Simulations

Numerical simulations are conducted to observe the dynamical behavior of the banking model by generating solution curves under different parameter configurations. In this study, two numerical experiments are performed using the parameter sets provided in Table 2, each with a specified initial condition. These simulations are used to illustrate how parameter variations influence the evolution of deposits, loans, and equity over time and to examine the consistency of the numerical trajectories with the analytical stability results. The numerical solutions of the differential equation system are computed using the fourth-order Runge–Kutta method, as employed in previous studies such as [42, 43]. This method is chosen because of its accuracy and reliability in solving nonlinear dynamical systems. The numerical computations are implemented in MATLAB with a time step of $\Delta t = 0.1$ over the interval $0 \leq t \leq 800$. The initial conditions are selected as positive off-equilibrium states to illustrate the convergence behaviour of the system. The simulation horizon is chosen sufficiently long to capture the transient dynamics and the subsequent approach toward the corresponding equilibrium. These simulations are intended to illustrate the analytical results rather than to provide empirical validation of the model.

Table 2. Parameter values.

Parameter	**Simulation 1**	**Simulation 2**
γ	0.12	0.12
w	0.08	0.08
α	0.15	0.15
k	3	3
p	0.2	0.4
m	0.02	0.02
b	0.02	0.0025
σ	0.12	0.08
n	0.03	0.02
c	0.25	0.25
μ	0.15	0.1
h	0.1	0.1
σ_L	0.15	0.1
γ_D	0.02	0.02
w_D	0.08	0.08
r_A	0.07	0.07
r_E	0.13	0.13
C_D	0.02	0.02
C_L	0.03	0.03

The parameter values used in the numerical simulations are summarized in Table 2. Several parameter values were selected based on empirical ranges or official banking information. In particular, the non-performing loan

(NPL) rate (n) is chosen within the range commonly observed in the banking sector, where recent statistics from the Financial Services Authority report that the gross NPL ratio of Indonesian banks is approximately 2-3%, which remains below the regulatory threshold of 5% [44]. The reserve requirement proportion (m) is taken from the reserve requirement policy reported by Bank Indonesia [45]. The loan interest rate (σ) and the deposit interest proportion (γ_D) are selected based on banking interest rate statistics reported in the World Bank's World Development Indicators database [46]. The remaining parameters represent behavioural or structural coefficients of the model. Since direct empirical estimates for these coefficients are not available in the present study, their values are specified hypothetically for the purpose of illustrating the model dynamics. Therefore, the parameter values used in the simulations should be interpreted as partially informed by empirical data rather than as a fully calibrated parameter set. Using the initial condition $(D(0), L(0), Q(0)) = (1.5,\ 1,\ 0.5)$ and the parameter values listed in the

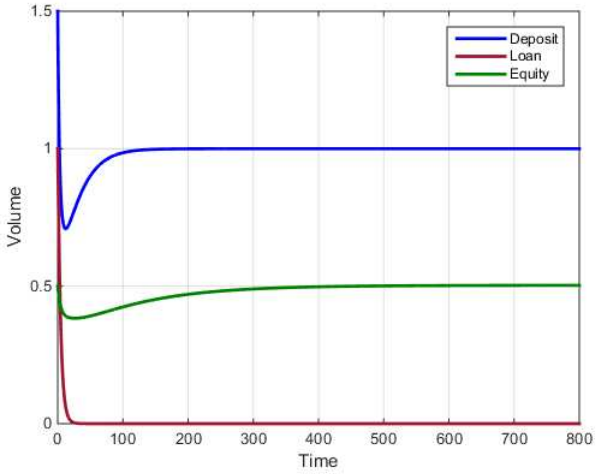


Figure 1. Solution curves toward E_1.

Simulation 1 column of Table 2, the numerical trajectory evolves toward the deposit–equity equilibrium E_1. For this parameterisation,

$$\kappa_3 = -0.06 < 0 \qquad \text{and} \qquad \kappa_2 = 0.0302 > 0.$$

Since $\kappa_3 = r_A - r_E$, the condition $\kappa_3 < 0$ implies $r_A < r_E$. Together with $\kappa_2 > 0$, these parameter values yield a positive equilibrium equity value at E_1. The computed equilibrium is

$$E_1 = (1, 0, 0.5033).$$

These equilibrium values represent model-scale balance-sheet levels and should not be interpreted as predictions for a particular banking institution. The corresponding parameter values satisfy the local and global stability conditions derived in **Theorems 1** and **2**.

$$H_{2_{(1)}} = \frac{p(1-m)D_{(1)}}{b(1-m)D_{(1)}+1} = 0.1922 < p(1-m)D_{(1)} = 0.196 < 0.3625 = H.$$

Figure 1 illustrates the convergence of $D(t)$, $L(t)$, and $Q(t)$ toward E_1, consistent with the analytical stability results. The decline of the loan variable can be understood from the loan equation. When

$$\frac{p(1-m)D}{1+b(1-m)D} < H,$$

the effective loan-generation term is insufficient to offset the aggregate loan-reduction term represented by H and the nonlinear term hL. Consequently, $\dot{L} < 0$. Under the parameter configuration of Simulation 1, the loan variable

decreases and approaches zero. For $\kappa_3 < 0$, which corresponds to $r_A < r_E$, the equilibrium equity value $Q_{(1)}$ is positive when $\kappa_2 > 0$. Since

$$\kappa_2 = r_A(1-m) - \gamma_D(1-w_D) - C_D,$$

the condition $\kappa_2 > 0$ indicates that, within the model, the return contribution from non-loan assets after the reserve requirement exceeds the deposit-related and operational cost terms. Thus, the model admits a positive equity equilibrium even when the loan volume approaches zero. Under these conditions, the long-run model state is characterized by vanishing lending while deposits and equity remain positive. The temporary decline and subsequent recovery of $D(t)$ and $Q(t)$ in Figure 1 represent transient adjustments toward E_1. As $L(t)$ decreases, the deposit–loan interaction term becomes weaker, while the deposit and equity variables gradually approach their corresponding equilibrium values.

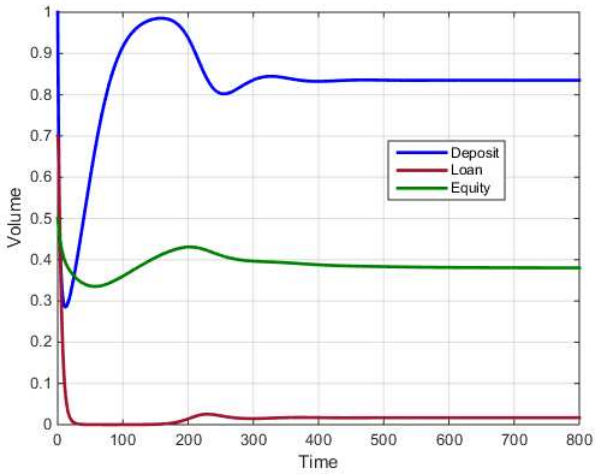


Figure 2. Solution curves toward E^*.

Using the parameter values in the Simulation 2 column of Table 2 together with the initial condition

$$(D(0), L(0), Q(0)) = (1, 0.7, 0.5),$$

the numerical trajectory approaches the interior equilibrium

$$E^* = (0.8351, 0.0169, 0.38).$$

These values likewise represent equilibrium levels within the theoretical model rather than institution-specific predictions. For this parameter set,

$$\kappa_3 = -0.06 < 0 \qquad \text{and} \qquad s = -0.0034 < 0,$$

which yield $Q^* > 0$. The global stability condition is satisfied since

$$\frac{\gamma}{k} = 0.04 > 1.6194 \times 10^{-5} = pb(1-m)^2 L^*.$$

The local stability conditions are also satisfied, since

$$\mathrm{tr}(J^*) = -0.0351 < 0, \qquad \det(J^*) = 0.0022 > 0.$$

Figure 2 illustrates the convergence of the solution trajectories toward E^*, consistent with the analytical results. Within the model, the condition

$$\frac{\gamma}{k} > pb(1-m)^2 L^*$$

is a sufficient condition used in the global stability analysis of the interior equilibrium. Under Simulation 2, the higher value of p together with the lower values of b, σ, n, and μ produces a parameter configuration in which a positive loan level can be maintained. Consequently, the trajectories approach E^*, where deposits, loans, and equity coexist at positive equilibrium levels.

As a numerical convergence check, the maximum componentwise absolute deviation from the corresponding equilibrium at $t = 800$ is approximately 1.49×10^{-4} for Simulation 1 and 2.09×10^{-4} for Simulation 2. These small deviations are consistent with the convergence shown in the numerical trajectories.

In summary, the two numerical simulations demonstrate distinct long-term dynamical behaviours of the model. In Simulation 1, the loan variable approaches zero and the trajectories converge to the deposit–equity equilibrium E_1. In Simulation 2, the trajectories converge to the positive interior equilibrium E^*, where deposits, loans, and equity remain positive. These numerical results are consistent with the corresponding analytical stability conditions and illustrate how different parameter configurations may lead to different asymptotic states of the model.

5. Conclusions

This study investigated the nonlinear dynamics of a three-dimensional bank balance-sheet model describing the interactions among deposits (D), loans (L), and equity (Q). Under the parameter regime considered, the system admits three equilibrium configurations: the zero-activity equilibrium E_0, the deposit–equity equilibrium E_1, and the positive interior equilibrium E^*. The local stability analysis shows that E_0 is unstable under $\gamma > w$, whereas E_1 and E^* are locally asymptotically stable under their corresponding parameter conditions.

Global stability was investigated using suitable Lyapunov functions and LaSalle's Invariance Principle. Sufficient conditions were established for the global asymptotic stability of E_1 and E^* within the corresponding positively invariant regions. These results characterize the long-term behaviour of the model under the specified parameter conditions.

Numerical simulations illustrate the adjustment paths of deposits, loans, and equity toward the corresponding equilibria and are consistent with the analytical stability results. Overall, the study contributes to mathematical banking modelling by providing a unified nonlinear framework for analysing equilibrium configurations and local and global stability under a reserve requirement mechanism.

Within the present theoretical framework, the reserve requirement parameter m directly determines the fraction $(1 - m)D$ of deposits available for lending. Consequently, changes in the reserve requirement can modify the effective loan-generation mechanism and the parameter conditions associated with long-run equilibrium behaviour. These implications are qualitative and model-based and should not be interpreted as direct policy prescriptions without empirical calibration.

The model, however, has several limitations. Although the non-performing loan rate, reserve requirement proportion, loan interest rate, and deposit interest proportion were selected from empirical ranges and official banking information, several behavioural and structural coefficients were specified hypothetically because empirical estimates were unavailable. Therefore, the simulations should be interpreted as illustrations of the model dynamics rather than predictions for a particular bank. Practical implementation of the model requires calibrating these parameters using relevant institution-specific or sector-specific data. In the present formulation, all parameters are assumed to remain constant over time. Allowing the reserve requirement, interest rates, or non-performing-loan rates to vary with time or the system state may alter the equilibrium structure and stability conditions. Future research may incorporate empirical parameter estimation, stochastic effects, bank-run-type withdrawal shocks, interbank interactions, liquidity and capital requirements, and time-varying regulatory policies.

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