STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **x**, Month 202x, pp 0–35.
Published online in International Academic Press (www.IAPress.org)

Comparing Different Estimation Methods for Structural Equation Modeling Parameters

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Abstract
Structural equation modeling (SEM) is widely used to examine relationships among observed and latent variables. However, selecting an appropriate estimation method remains challenging when analyzing ordinal data, particularly under different sample sizes, numbers of response categories, and distributional conditions. This study employed a Monte Carlo simulation to compare the performance of five estimation methods: Maximum Likelihood (ML), Robust Maximum Likelihood (MLR), Weighted Least Squares (WLS), Diagonally Weighted Least Squares (DWLS), and Unweighted Least Squares (ULS). Two SEM models with different levels of complexity were examined across five sample sizes (200, 350, 750, 900, and 1500), three numbers of response categories (3, 5, and 8), and symmetric and asymmetric distributions, with 500 replications per condition. Estimator performance was evaluated using goodness-of-fit indices (CFI, NFI, and RMSEA), mean squared error (MSE), and convergence rates. The results indicated that DWLS generally produced the most favorable goodness-of-fit values, followed closely by ULS. ML and MLR demonstrated similar fit performance, whereas WLS tended to yield relatively higher RMSEA values, particularly in the more complex model. Regarding parameter estimation accuracy, DWLS, ULS, and WLS generally achieved lower MSE values than ML and MLR. Distributional asymmetry was associated with higher MSE values for ML and MLR, particularly when fewer than eight response categories were used. Increasing sample size generally improved model fit and estimation accuracy, while increasing the number of categories had a less consistent effect on goodness-of-fit indices. Model complexity had a relatively limited influence on MSE but affected NFI, particularly at smaller sample sizes. Overall, the findings suggest that DWLS and ULS are promising estimation methods for ordinal SEM under the examined conditions, highlighting the importance of considering both model fit and parameter estimation accuracy when selecting an estimator.

Keywords Structural equation modeling, Estimation methods, Monte Carlo simulation, Categorical data

AMS 2010 subject classifications 62H12

DOI: 10.19139/soic-2310-5070-4604

1. Introduction

Categorical data are widely encountered in many fields such as social sciences, education, medicine, and economics [5, 1, 6]. These data represent variables that describe qualitative characteristics rather than numerical quantities, for example, gender, marital status, disease diagnosis, or responses measured on Likert scales. Because of their prevalence in empirical research, the analysis of categorical data has become an important topic in multivariate statistical analysis. Appropriate analytical techniques are required to correctly model the relationships between categorical variables and to draw reliable conclusions from such data [3, 4].

Several statistical methods have been developed for analyzing categorical data, including contingency table analysis, logistic regression, log–linear models, and generalized linear models. Although these techniques are

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ISSN 2310-5070 (online) ISSN 2311-004X (print)
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useful in many situations, they are often limited when research involves complex relationships among multiple variables. Many traditional methods focus primarily on analyzing a single dependent variable or examining relationships between variables in isolation. Consequently, they may fail to capture the underlying structure of relationships among variables or reveal indirect and hidden effects that occur through intermediate variables.

Similarly, conventional multivariate techniques, such as multiple regression and analysis of variance, also have limitations when modeling complex relationships. These approaches typically depend on simple model structures and focus on observable variables, which makes the estimation of indirect effects and interactions among several variables within a single framework more challenging.

To address these limitations, structural equation modeling (SEM) has become popular as a statistical technique to examine complex relationships between observed and latent variables [14]. SEM combines regression analysis, path analysis, and factor analysis within a unified framework. This combination enables researchers to analyze several dependent variables in a simulation, examine theoretical relationships, and assess both direct and indirect effects.

In addition, SEM is effective for investigating latent constructs that are not directly observable, such as perceptions, attitudes, or psychological traits. These constructs are often measured through questionnaire items with categorical or ordinal response scales (e.g., Likert scales), making SEM particularly suitable for analyzing multivariate categorical data widely encountered in social and behavioral research. By treating observed responses as indicators of latent variables, SEM offers a more complete and accurate representation of the theoretical relationships among the constructs.

However, the use of categorical or ordinal indicators in SEM raises important methodological issues, especially when selecting an appropriate estimation method. The accuracy of the SEM results depends on whether the estimation technique is appropriate for the characteristics of the data. Accurate SEM estimation relies on several key assumptions related to the type of data (continuous or ordered categorical) and their distribution (normal or non-normal). Traditional estimation methods, such as maximum likelihood, assume that the observed variables are continuous and follow a multivariate normal distribution. However, empirical data frequently violate these assumptions, especially when variables are measured using ordinal response scales. Such violations may lead to biased parameter estimates, underestimated standard errors, and misleading model fit statistics [13].

Alternative methods, such as weighted least squares and robust estimators, have been developed to accommodate categorical and non-normal data. However, these methods also have their own requirements, such as large sample sizes, to ensure reliable results. When these assumptions are violated, the model fit and parameter estimates can be severely affected [11].

Recent research has emphasized that estimation methods should be chosen according to the characteristics of the data used in SEM. Beauducel and Herzberg [7] found that mean- and variance-adjusted weighted least squares estimation yielded more accurate chi-square rejection rates and more precise factor loadings than maximum likelihood when variables had only two or three categories.

Rhemtulla et al. [25] compared robust maximum likelihood and categorical least squares (cat-LS) for ordinal data in confirmatory factor analysis. They found that cat-LS performed better with fewer than five categories, whereas robust maximum likelihood is adequate and performs similarly when variables have five or more categories and symmetric thresholds. Xia and Yang [29] showed that unweighted least squares and diagonally weighted least squares tended to produce smaller RMSEA values and larger CFI and TLI values than maximum likelihood for ordered categorical data, which may cause conventional fit cutoffs to miss model misfit. Together, these studies show that SEM estimation should be chosen carefully based on whether the indicators are continuous or categorical, the number of categories, and the distributional properties of the data.

Oldenburg [31] investigated a residual-based least squares approach for estimating nonlinear structural equation models and found that appropriate weighting strategies could improve parameter estimation compared with uncorrected least squares methods. The study also emphasized that no single estimation method consistently performed best across all model structures. In a subsequent study, Oldenburg [32] proposed geometric weighted least squares (GWLS) and its controlled version (GWLSc). The simulation results showed that GWLSc generally provided more accurate parameter estimates, particularly at smaller sample sizes, although this improvement came at a substantially higher computational cost. The study also demonstrated that estimator performance could be

sensitive to distributional assumptions. These findings highlight the importance of sample size, and distributional characteristics in estimation performance.

Adedia et al. [2] compared several SEM estimators using a real health data set and showed that the ridge maximum likelihood estimators produced the best overall fit of the model, while the unweighted least squares and the asymptotic distribution-free estimation often led to convergence problems or unreliable results. In addition, , Zulkifli et al. [30] demonstrated that regularized unweighted least squares improved fit indices such as CFI, TLI, RMSEA, and SRMR, particularly under non-normal data and stronger factor loadings.

Despite the growing body of research on estimation methods in SEM, existing studies have often focused on either parameter accuracy or overall model fit. Limited attention has been given to comprehensive evaluations that integrate multiple performance criteria and examine estimation efficiency and consistency across varying sample sizes. Moreover, the performance of different estimators under diverse data conditions, such as varying numbers of ordinal categories and non-normal distributions, remains underexplored.

Therefore, the present study provides a systematic comparison of maximum likelihood, robust maximum likelihood, weighted least squares, diagonally weighted least squares, and unweighted least squares estimators within the SEM framework. Using a Monte Carlo simulation, this study evaluates estimator performance from multiple perspectives, including estimation efficiency, consistency, and model fit, under a range of realistic data conditions. The findings are expected to offer practical guidance for researchers in selecting appropriate estimation methods when analyzing categorical and non-normal data in SEM.

2. Structural Equation Modeling

SEM is a statistical approach designed to examine complex relationships between observed and latent variables. It integrates two main components: the measurement model and the structural model that represent the relationships between variables. The basic elements of the SEM model include observed variables, latent variables, and paths that illustrate the relationships between variables. The measurement model links the latent variables with the indicators.

Observed variables (indicators), also referred to as manifest variables or items, are variables that are directly measured or observed in a study. A latent variable is an underlying construct or concept that cannot be directly observed or measured but is inferred from observed indicators or measurements. Latent variables that function exclusively as independent variables are known as exogenous latent variables. In contrast, latent variables that act as dependent variables or serve as both independent and dependent variables are referred to as endogenous latent variables (see Figure 1) [27]. This formula can be expressed by the following:

$$\eta = \alpha + B\eta + \Gamma\xi + \zeta \tag{1}$$

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_r \end{bmatrix}; \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_r \end{bmatrix}; \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_s \end{bmatrix}; \zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_r \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & b_{12} & \cdots & b_{1r} \\ b_{21} & 0 & \cdots & b_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ b_{r1} & b_{r2} & \cdots & 0 \end{bmatrix}; \Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1s} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{r1} & \gamma_{r2} & \cdots & \gamma_{rs} \end{bmatrix}$$

Equation 1 represents the structural function model in the SEM framework [24].
In this equation,

(a) α is an $(r \times 1)$ vector of intercepts associated with endogenous latent variables.
(b) B is an $(r \times r)$ matrix containing the structural coefficients that describe the relationships among the endogenous latent variables. The diagonal elements of this matrix are assumed to be zero.

(c) Γ is an $(r \times s)$ matrix of regression coefficients representing the effects of exogenous latent variables on endogenous latent variables.
(d) η is an $(r \times 1)$ vector of endogenous latent variables, while ξ is an $(s \times 1)$ vector of exogenous latent variables.
(e) ζ is an $(r \times 1)$ vector of disturbance terms representing unexplained variation in endogenous latent variables. It is assumed to follow a multivariate normal distribution with $N(0, \Psi_\zeta)$ and to be uncorrelated with exogenous variables.

$$y = \alpha_y + \Lambda_y \eta + \epsilon \tag{2}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}; \alpha_y = \begin{bmatrix} \alpha_{y1} \\ \alpha_{y2} \\ \vdots \\ \alpha_{yp} \end{bmatrix}; \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_p \end{bmatrix}$$

$$\Lambda_y = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{p1} & \lambda_{p2} & \cdots & \lambda_{pn} \end{bmatrix}$$

$$x = \alpha_x + \Lambda_x \xi + \delta \tag{3}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_q \end{bmatrix}; \alpha_x = \begin{bmatrix} \alpha_{x1} \\ \alpha_{x2} \\ \vdots \\ \alpha_{xq} \end{bmatrix}; \delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_q \end{bmatrix}$$

$$\Lambda_x = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1k} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{q1} & \lambda_{q2} & \cdots & \lambda_{qk} \end{bmatrix}$$

Equation 2 and 3 represent the measurement models [24].

(a) α_y and α_x are $(p \times 1)$ and $(q \times 1)$ vectors of intercepts, respectively.
(b) Λ_y is a $(p \times n)$ loading matrix relating the observed variables y to the endogenous latent variables η, while Λ_x is a $(q \times k)$ loading matrix relating the observed variables x to the exogenous latent variables ξ.
(c) ξ and η denote vectors of latent exogenous and endogenous variables, respectively, and are assumed to follow a multivariate normal distribution $N(0, \Phi)$.
(d) ϵ is a $(p \times 1)$ vector of measurement errors associated with y, assumed to follow a multivariate normal distribution with $N(0, \Psi_\epsilon)$.
(e) δ is a $(q \times 1)$ vector of measurement errors associated with x, assumed to follow a multivariate normal distribution with $N(0, \Psi_\delta)$.

Figure 1 illustrates three key components: the structural model, the measurement model of exogenous latent variables, and the measurement model of endogenous latent variables. The part of the structural model contains exogenous latent variables (ξ_1 and ξ_2), and endogenous latent variables (η_1 and η_2) that connect to the error terms (ζ_1 and ζ_2). (γ_1 and γ_2) are the coefficients of the exogenous latent variables to endogenous latent variables. β is the coefficient between the endogenous latent variables.

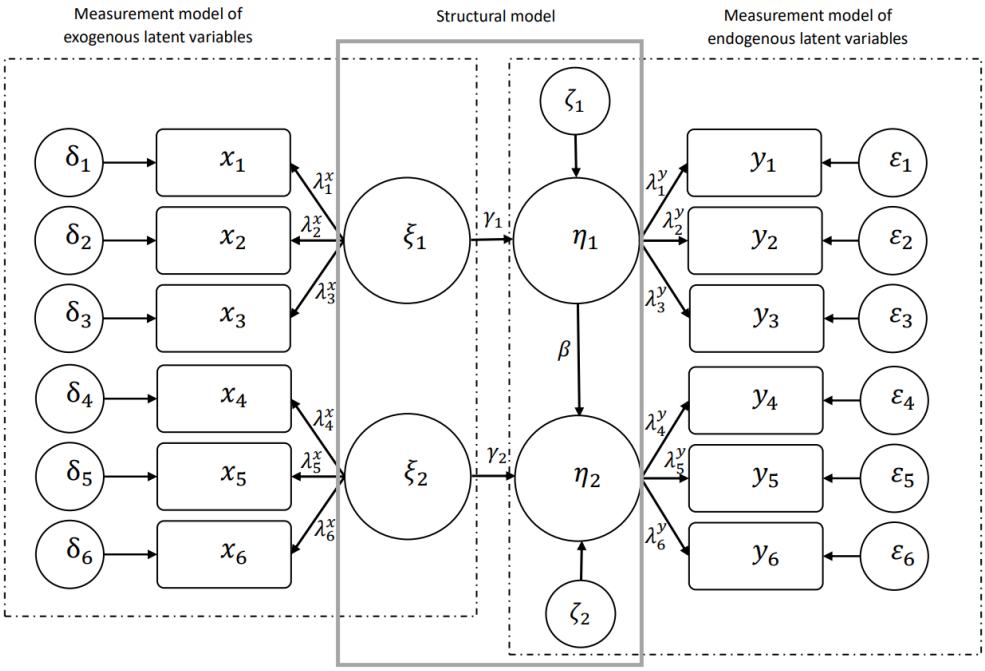


Figure 1. Path diagram of the four-factor model.

Second part is the measurement model of exogenous latent variables, where it represents the connection between the exogenous latent variables (ξ_1 and ξ_2) with x indicators (x_1 to x_6) by the x's loadings (λ_1^x to λ_6^x). Each indicator has error term (δ_1 to δ_6).

Last part is the measurement model of endogenous latent variables, where it represents the connection between the endogenous latent variables (η_1 and η_2) with y indicators (y_1 to y_6) by the y's loadings (λ_1^y to λ_6^y). Each indicator has error term (ϵ_1 to ϵ_6).

3. Estimation Methods

In this study, five estimation methods were employed within the SEM framework, maximum likelihood, robust maximum likelihood, weighted least squares, diagonally weighted least squares, and unweighted least squares, to examine their performance under different data conditions.

3.1. Maximum Likelihood

Maximum Likelihood (ML) is one of the most commonly applied estimation methods in SEM. It assumes that the observed indicators follow a multivariate normal distribution. The ML discrepancy function can be expressed as:

$$F_{ML} = \ln|\Sigma(\boldsymbol{\psi})| + \operatorname{tr}\left(\mathbf{S}\Sigma^{-1}(\boldsymbol{\psi})\right) - \ln|\mathbf{S}| - q, \tag{4}$$

where $\boldsymbol{\psi}$ denotes the vector of model parameters, $\Sigma(\boldsymbol{\psi})$ represents the model-implied covariance matrix, $\mathbf{S}$ is the sample covariance matrix, tr denotes the trace operator (sum of the diagonal elements, and q is the number of observed variables in the model [19].

3.2. Robust Maximum Likelihood

Robust Maximum Likelihood (MLR) is an extension of the conventional ML estimator designed to reduce the sensitivity of statistical inference to violations of the multivariate normality assumption. MLR applies robust corrections to standard errors. Specifically, standard errors are calculated using a sandwich-type estimator to account for nonnormality in the observed data. These corrections improve the robustness of statistical inference when the data exhibit skewness, kurtosis, or both [19].

3.3. Weighted Least Squares

Weighted Least Squares (WLS) is an estimation method that minimizes the weighted differences between observed and model-implied statistics. When ordinal indicators are specified as ordered categorical variables, WLS uses polychoric correlations and thresholds. The WLS fitting function is defined as:

$$F_{WLS} = [\mathbf{v} - \boldsymbol{\sigma}(\boldsymbol{\psi})]' \mathbf{W}^{-1} [\mathbf{v} - \boldsymbol{\sigma}(\boldsymbol{\psi})]. \tag{5}$$

where $\boldsymbol{\psi}$ is the vector of unknown model parameters, $\mathbf{v}$ represents the vector of sample statistics, including polychoric correlations and thresholds, $\boldsymbol{\sigma}(\boldsymbol{\psi})$ denotes the vector of model-implied statistics, and $\mathbf{W}$ is the full asymptotic covariance matrix of the sample statistics [19].

3.4. Diagonally Weighted Least Squares

Diagonally Weighted Least Squares (DWLS) is an estimation method suitable for ordinal data. When the observed variables are specified as ordered categorical variables, DWLS uses polychoric correlations and thresholds. The DWLS fitting function is defined as:

$$F_{DWLS} = [\mathbf{v} - \boldsymbol{\sigma}(\boldsymbol{\psi})]' \mathbf{W}_D^{-1} [\mathbf{v} - \boldsymbol{\sigma}(\boldsymbol{\psi})]. \tag{6}$$

where $\boldsymbol{\psi}$ is the vector of unknown model parameters, $\mathbf{v}$ represents the vector of sample statistics, including polychoric correlations and thresholds, $\boldsymbol{\sigma}(\boldsymbol{\psi})$ denotes the vector of model-implied statistics, and $\mathbf{W}_D$ is a diagonal matrix containing the diagonal elements of the asymptotic covariance matrix of the sample statistics [20].

3.5. Unweighted Least Squares

Unweighted Least Squares (ULS) is an estimation method that minimizes the unweighted differences between observed and model-implied statistics. When ordinal indicators are specified as ordered categorical variables, ULS uses polychoric correlations and thresholds. Unlike WLS and DWLS, ULS employs an identity weight matrix, assigning equal weights to the discrepancies between observed and model-implied statistics. The ULS fitting function is defined as:

$$F_{ULS} = [\mathbf{v} - \boldsymbol{\sigma}(\boldsymbol{\psi})]' \mathbf{I} [\mathbf{v} - \boldsymbol{\sigma}(\boldsymbol{\psi})]. \tag{7}$$

where $\boldsymbol{\psi}$ is the vector of unknown model parameters, $\mathbf{v}$ represents the vector of sample statistics, including polychoric correlations and thresholds, $\boldsymbol{\sigma}(\boldsymbol{\psi})$ denotes the vector of model-implied statistics, and I is the identity matrix.

4. Assessment of Overall Model Fit

In this study, three goodness-of-fit indices were used to evaluate model fit: the Comparative Fit Index, the Normed Fit Index, and the Root Mean Square Error Approximation. Their definitions and formulations are presented below.

4.1. Comparative Fit Index

The comparative fit index (CFI) ranges from 0 to 1, with values closer to 1 indicating a better model fit. A CFI value of 0.95 or higher is considered to indicate an excellent model fit, whereas values between 0.90 and 0.95 are regarded as acceptable. In contrast, values below 0.90 suggest an unacceptable model fit. The formula for CFI can be expressed as:

$$CFI = 1 - \frac{Max((\chi^2_{model} - df_{model}), 0)}{Max((\chi^2_{null} - df_{null}), 0)}, \tag{7}$$

χ^2_{model} and df_{model} are Chi-square and its degrees of freedom of the model, respectively. Here, Max indicates the maximum of the values given in brackets. χ^2_{null} and df_{null} are the chi-square and degrees of freedom of the null model, respectively. The null model assumes that there are no relationships among the observed variables [9].

4.2. Normed Fit Index

The normed fit index (NFI) is bounded between 0 and 1, with larger values indicating a better fit model. An NFI of at least 0.95 is generally interpreted as excellent model fit, while values from 0.90 to 0.95 indicate an acceptable fit. Scores below 0.90 suggest an unacceptable model fit. The index is obtained by comparing the proposed model with the baseline model, as shown below:

$$NFI = \frac{\chi^2_b - \chi^2_m}{\chi^2_b} \tag{8}$$

where χ^2_b denotes the chi-square statistics of the baseline model, whereas χ^2_m represents the chi-square statistic for the proposed model [2].

4.3. Root Mean Square Error Approximation

The root mean square error approximation (RMSEA) range value is between 0 to 1, with values closer to 0 indicating a better model fit. An RMSEA value below 0.05 indicates an excellent model fit. Values between 0.05 and 0.08 indicate an acceptable model fit. Conversely, values greater than 0.08 indicate an unacceptable model fit. It formed as below:

$$RMSEA = \left[\frac{(\chi^2 - df)}{df(n-1)}\right]^{0.5}, \tag{9}$$

where $\chi^2 = (n-1)F_{min}$, F_{min} is the minimum value of the fit function of the estimating method, and df is the degrees of freedom. RMSEA would work well with large sample size and less number of variables in the model [9].

5. Assessment of Parameter Estimation Accuracy

The present study evaluated parameter estimation accuracy using the mean squared error (MSE). This measure combines both estimation bias and variability, making it a comprehensive indicator of estimation accuracy. The MSE is defined as:

$$\text{MSE}(\hat{\phi}) = \frac{1}{R}\sum_{r=1}^{R}\left(\hat{\phi}_r - \phi\right)^2, \tag{10}$$

where $\hat{\phi}_r$ represents the parameter estimate obtained from the rth replication, and ϕ denotes the corresponding population parameter. The symbol R refers to the total number of replications. The MSE is calculated as the average squared deviation between the estimated and true parameter values and, for an unbiased estimator, is equivalent to the sampling variance [22].

6. Simulation Framework

This study employed a simulation-based methodology to evaluate the performance of SEM estimation methods for categorical data. Five estimation methods ML, MLR, WLS, DWLS, and ULS, were used to estimate the model parameters and compare their performance.

Performance assessment was carried out under a range of simulation conditions, including different sample sizes, model complexity, number of response categories, and underlying data distributions. This simulation design provides a comprehensive evaluation of the robustness and efficiency of each estimation method under conditions representative of practical research settings.

Model fit was assessed using three goodness-of-fit indices (CFI, NFI, and RMSEA), while estimation accuracy was evaluated using MSE. The details of each simulation condition are presented below.

6.1. SEM Model Size

Two model sizes were specified to examine the effect of model complexity on estimation performance. Model A consisted of a two-factor (latent variable) structure with six observed indicators, with three indicators loading on each factor. Model B represented a more complex structure, comprising three latent factors and nine observed indicators, with three indicators per factor. Model A had 8 degrees of freedom; Model B had 24 degrees of freedom.

Figures 2 and 3 present the path diagrams for the two specified models, respectively. In these diagrams, latent variables are denoted by F_k, where k indicates the latent factor number, taking values (k = 1, 2) for Model A and (k = 1, 2, 3) for Model B. The observed indicators are represented by X_p, where p indexes the observed variable number, ranging from (p = 1, . . . , 6) in Model A and (p = 1, . . . , 9) in Model B. Correspondingly, the factor loadings λ_p represent the relationships between each observed variable and its associated latent factor.

The parameters γ_v denote the structural paths between the latent factors, where v indexes the path number. The number of such parameters depends on the model structure, with one path in Model A and multiple paths in Model B. All structural path parameters γ_v were fixed at 0.5, while the factor loadings λ_p were set to 0.7.

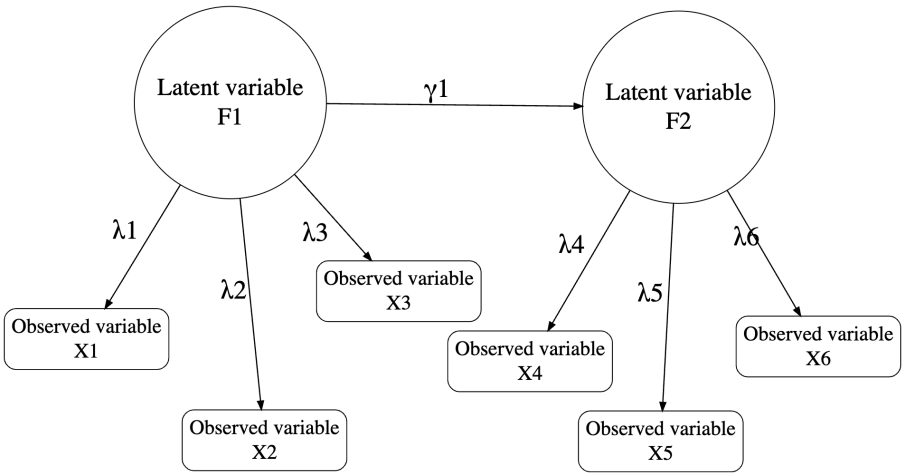


Figure 2. Model A: Path diagram of the two-factor model.

6.2. Number of Categories

The simulation generated categorical data based on specific distributions (symmetric and asymmetric) with three levels: 3, 5, and 8 categories.

The selection of these category levels was guided by previous simulation and empirical studies on Likert-type scales. Prior research has shown that odd-numbered response categories, particularly those including a middle

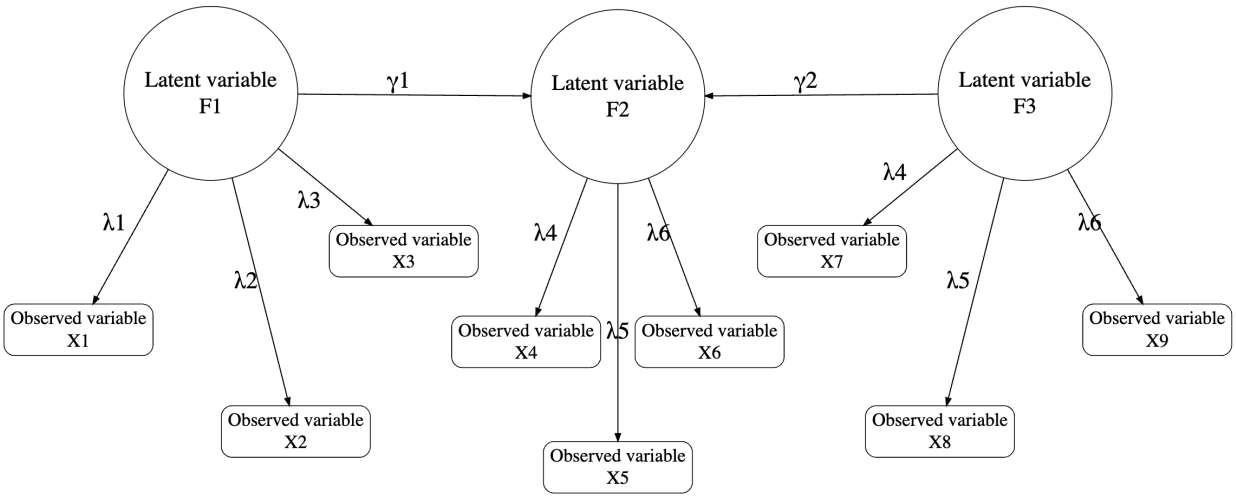


Figure 3. Model B: Path diagram of the three-factor model.

option, are more commonly used in practice [20]. Among various psychometric instruments, five-category scales are the most frequently employed. The eight-category condition was included to examine whether using a larger number of response categories improves estimator performance and reduces estimation error. This condition also allowed the evaluation of whether the commonly suggested recommendation of using more than five response categories provides additional advantages in structural equation modeling.

6.3. Underlying Distribution

Two distributions were utilized: a symmetric normal condition with skewness and excess kurtosis equal to zero, and a non-normal distribution with asymmetric data. According to Cain et al. [8], skewness and kurtosis are commonly used measures of normality, where values equal to zero indicate a normal distribution, and deviations from zero indicate asymmetry or skewness and departures from normality in terms of tail behavior and peakedness. Figure 4 presents a visualization of the categorical data and its distribution.

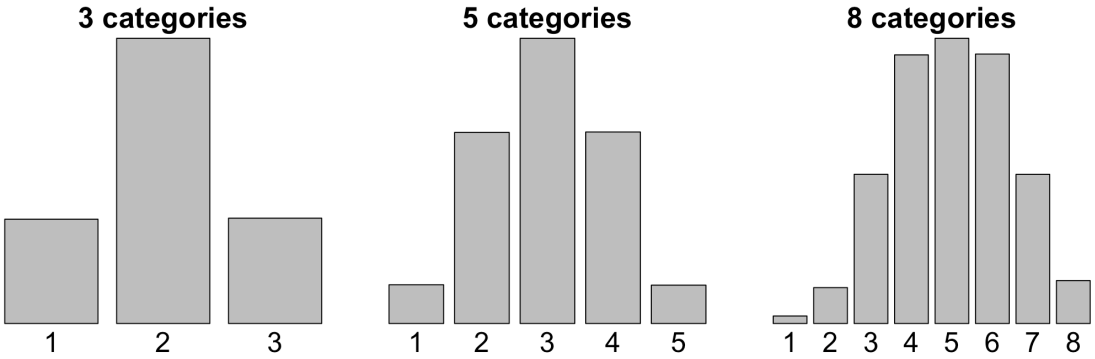
6.4. Sample Size

Sample size is considered one of the most important factors affecting parameter estimation accuracy and model stability in SEM, particularly in Monte Carlo simulation studies. Small sample sizes may lead to inaccurate parameter estimates, unstable standard errors, non-convergence problems, and improper solutions [20]. In addition, estimator performance and goodness-of-fit indices are generally expected to improve as sample size increases.

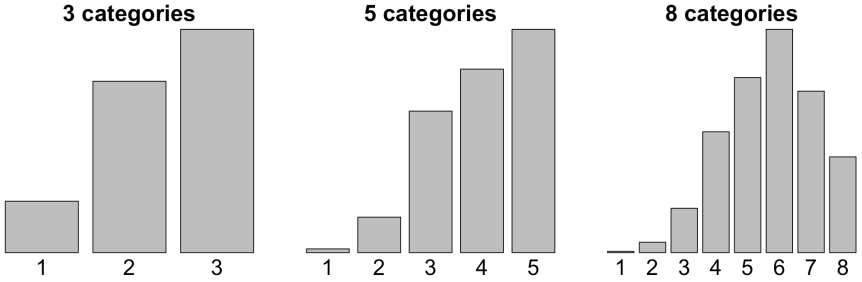
Previous SEM and CFA studies have frequently employed a wide range of sample sizes to evaluate estimator behavior under different analytical conditions. Previous reviews by DiStefano and Hess (2005) and Jackson et al. (2009) reported that approximately 19% and 20% of CFA studies, respectively, used samples smaller than 200. However, small samples may lead to inaccurate parameter estimates, unstable standard errors, nonconvergence, and inadmissible solutions (Li, 2021).

Accordingly, this study employed five sample sizes: $N = 200$, 350, 750, 900, and 1500. These values were selected to represent a broad range of realistic empirical conditions, ranging from small to relatively large sample sizes.

In the preliminary stage of this study, smaller sample sizes were initially examined. However, the results indicated unstable MSE values, inaccurate parameter estimates, and, in some cases, non-convergence and improper solutions. Consequently, $N = 200$ was adopted as the smallest sample size in the final simulation design, as it



(a) Symmetric distribution



(b) Asymmetric distribution

Figure 4. Categorical data with different types of distribution.

produced more stable parameter estimates while remaining representative of sample sizes commonly encountered in empirical studies.

In addition, larger sample sizes ($N = 750, 900, and 1500$) were incorporated to examine changes in estimator performance under more stable conditions. These sample sizes facilitate the assessment of improvements in parameter estimation accuracy, MSE, and goodness-of-fit indices across the different estimation methods.

6.5. Data Generation and Analysis

Monte Carlo simulation was conducted the `R` statistical computing environment and implemented through `RStudio` [23]. Several R packages were utilized for specific tasks. The `lavaan` package [26] was employed for model estimation and calculation of goodness-of-fit indices, while functions from the `MASS` package [28] were used to generate multivariate normal latent variables.

A total of 60 experimental conditions were generated based on the full factorial combination of model size (2 levels), ordinal data distribution (2 levels), number of response categories (3 levels), and sample size (5 levels). For each condition, 500 datasets were simulated, resulting in a total of 30,000 generated datasets.

The selection of 500 replications per condition was intended to reduce sampling variability, ensure adequate statistical power, and maintain computational feasibility [19]. For each simulated dataset, model parameters, MSE, and goodness-of-fit indices (CFI, NFI, and RMSEA) were estimated.

All analyses were conducted using five estimation methods, ML, MLR, WLS, DWLS, and ULS, implemented in RStudio version 2026 [23].

Two categorical observed distributions were manipulated to represent ordinal conditions: (1) a symmetric distribution and (2) an asymmetric distribution.

For the symmetric distribution, the threshold values for the three-category observed variables were specified as $[-0.8, 0.8]$, corresponding approximately to response proportions of 21.19%, 57.62%, and 21.19% across

the categories, respectively. The threshold values for the five-category observed variables were specified as $[-1.633, -0.5, 0.5, 1.633]$, corresponding approximately to response proportions of 5.13%, 25.72%, 38.29%, 25.72%, and 5.13%. For the eight-category observed variables, the threshold values were specified as $[-2.5, -1.8, -1.0, -0.3, 0.3, 1.0, 1.8]$, corresponding approximately to response proportions of 0.62%, 2.97%, 12.27%, 22.34%, 23.58%, 22.34%, 12.27%, and 3.59%, respectively.

For the asymmetric distribution, the threshold values for the three-category observed variables were specified as $[-1.2, 0]$, corresponding approximately to response proportions of 11.51%, 38.49%, and 50.00%, respectively. The threshold values for the five-category observed variables were specified as $[-2.5, -1.5, -0.5, 0.3]$, corresponding approximately to response proportions of 0.62%, 6.06%, 24.17%, 31.94%, and 37.21%, respectively. For the eight-category observed variables, the threshold values were specified as $[-3, -2.2, -1.5, -0.8, -0.2, 0.5, 1.2]$, corresponding approximately to response proportions of 0.13%, 1.26%, 5.29%, 14.50%, 20.89%, 27.07%, 19.35%, and 11.51%, respectively. Response proportions of categorical observed variables used in this study are displayed in Figure 4.

6.6. Algorithm

Using Monte Carlo simulation, the study followed these steps:

1. Generate latent variables from a multivariate normal distribution.
2. Generate continuous observed variables using the population factor loadings ($\lambda = 0.70$), structural path ($\gamma = 0.50$), and random error terms.
3. Convert the continuous variables into ordinal categorical variables according to predefined threshold values for each response category condition (3, 5, and 8 categories), under symmetric and asymmetric distributions, as illustrated in Section 6.5.
4. Specify the SEM models for both Model A and Model B.
5. Estimate the model parameters (factor loadings and structural paths) using using ML, MLR, WLS, DWLS, and ULS through the **sem()** function in the lavaan package in R. For WLS, DWLS, and ULS, the ordinal indicators are specified as ordered categorical variables. The default lavaan convergence criteria are used. Since the simulated datasets contain no missing values, no missing-data handling procedure is required.
6. Compute the bias and squared bias for the factor loadings and structural paths.
7. Calculate the model fit using the fit indices CFI, NFI, and RMSEA.
8. Repeat the simulation process 500 times for each experimental condition and sample size.
9. Calculate the MSE, the average parameter estimates, and the average values of fit indices CFI, NFI, and RMSEA.

7. Results

7.1. Convergence Rates

Convergence refers to the estimation algorithm reaching a solution that satisfies the specified numerical stopping criteria. Nonconvergence occurs when these criteria are not met within the maximum number of iterations. Convergence rates were calculated as the proportion of converged replications within each simulation condition [18].

ML, MLR, DWLS, and ULS achieved 100% convergence across all conditions in both models. Convergence difficulties were confined to WLS, although it achieved complete convergence under all three-category conditions.

For Model A, WLS convergence rates under symmetric distributions were 88.0% and 97.2% in the eight-category conditions at N = 200 and N = 350, respectively, and 100% in all remaining symmetric conditions. Under asymmetric distributions with five categories, convergence rates were 89.0% at N = 200 and 96.6% at N = 350, while reaching 100% at larger sample sizes. With eight asymmetric categories, the rates were 88.0%, 92.0%, 94.0%, 92.8%, and 98.8% at N = 200, 350, 750, 900, and 1,500, respectively.

For Model B, WLS convergence rates under symmetric distributions were 81.8% and 94.0% in the eight-category conditions at N = 200 and N = 350, respectively, and 100% in all remaining symmetric conditions. Under asymmetric distributions with five categories, convergence rates were 85.6%, 94.4%, and 99.6% at N = 200, 350, and 750, respectively, reaching 100% at N = 900 and N = 1,500. With eight asymmetric categories, the rates were 75.4%, 87.0%, 91.6%, 90.6%, and 97.0% at N = 200, 350, 750, 900, and 1,500, respectively.

Overall, WLS convergence difficulties were more pronounced under asymmetric distributions, particularly with eight response categories, and were greater in Model B than in Model A. Increasing sample size generally improved convergence.

7.2. Estimated Parameter Values

This study assesses the performance of parameter estimation under varying conditions, including sample sizes, category levels, model sizes, and data distributions. The objective is to identify the most consistent and reliable techniques for estimating standardized parameters. The parameter estimates were expected to approach their true population values as the sample size and the number of response categories increased.

Table 1 to Table 6 represent the estimated parameters and corresponding MSE for Model A, and Table 7 to Table 12 for Model B. The columns ($\hat{\lambda}$) represent the estimated factor loadings that link each observed indicator to its latent factor in the measurement model. The rows labeled ($\hat{\gamma}$) in Model A, and ($\hat{\gamma}_1$ and $\hat{\gamma}_2$) in Model B, represent the estimated structural paths between latent factors. The parameter estimates were obtained using ML, MLR, WLS, DWLS, and ULS.

The results show that estimator performance depended on the number of response categories, sample size, and distribution. The estimated parameters generally became closer to the population values (≈ 0.70 for loadings and ≈ 0.50 for structural paths), as the sample size and number of response categories increased in both models.

For factor loadings, ML and MLR produced similar estimates and consistently underestimated the population value of 0.70. This underestimation was most pronounced with three response categories, where estimates remained approximately between 0.62 and 0.63, even at the largest sample size. Increasing the number of categories reduced this discrepancy: estimates were approximately 0.67 under symmetric conditions and 0.65 under asymmetric conditions with five categories, and approximately 0.68 with eight categories. Thus, increasing the number of response categories brought the ML and MLR loading estimates closer to the population value, whereas increasing sample size alone did not eliminate their underestimation.

DWLS and ULS generally produced loading estimates close to 0.70 across sample sizes and distributional conditions. WLS tended to overestimate factor loadings at smaller sample sizes, particularly in Model B. As sample size increased, WLS estimates moved closer to 0.70, reducing the differences between WLS, DWLS, and ULS. In contrast, the loading estimates obtained using ML and MLR remained below those obtained using these three methods, especially with three response categories.

The structural path estimates were generally close to the population value of 0.50, with smaller differences between estimators than those observed for factor loadings. Under symmetric conditions, most estimates were close to 0.50, although ML yielded slightly lower values in Model A with three categories. Under asymmetric conditions with three or five categories, both ML and MLR generally yielded structural path estimates below 0.50, including at larger sample sizes. WLS generally produced values above 0.50 in smaller samples, whereas DWLS and ULS usually produced estimates close to 0.50. With eight response categories, differences between the five estimators were generally small.

The main patterns were broadly similar in Models A and B. However, at smaller sample sizes, WLS loading estimates were further above the population value of 0.70 in Model B than in Model A. The effect of distributional asymmetry also depended on the number of response categories. With five categories, ML and MLR loading estimates were further below 0.70 under asymmetric conditions than under symmetric conditions, whereas the corresponding differences were smaller with three and eight categories. DWLS and ULS loading estimates remained close to 0.70 under both distributions.

Overall, DWLS and ULS demonstrated the highest accuracy in estimating parameter, consistently producing estimates closest to the true value under both symmetric and asymmetric distributions. WLS also provided relatively accurate estimates, whereas ML and MLR showed larger deviations from the true value, particularly as the number

Table 1. Parameter estimates and MSE for the symmetric distribution in Model A with 3 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

			Model A - 3 category - Symmetric									
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6248	0.0113	0.6249	0.0106	0.7071	0.0069	0.7044	0.0059	0.7032	0.0062
		x2	0.6278	0.011	0.6212	0.0117	0.7104	0.007	0.7046	0.007	0.7038	0.0073
		x3	0.6241	0.0111	0.6195	0.0114	0.719	0.0069	0.6983	0.0066	0.697	0.0068
	f2	x4	0.6266	0.0111	0.6222	0.0108	0.718	0.0068	0.6996	0.0056	0.6983	0.006
		x5	0.622	0.012	0.6217	0.0114	0.7112	0.0063	0.7022	0.0058	0.7	0.0062
		x6	0.6243	0.011	0.6253	0.0111	0.7106	0.0064	0.7031	0.0055	0.7021	0.0057
	$\hat{\gamma}$		0.4801	0.0109	0.4998	0.0093	0.5121	0.0102	0.5048	0.0082	0.5028	0.008
350	f1	x1	0.6217	0.0095	0.6198	0.0092	0.7097	0.0035	0.7045	0.003	0.704	0.0032
		x2	0.6213	0.009	0.624	0.0087	0.7102	0.0038	0.6996	0.0031	0.6983	0.0033
		x3	0.6273	0.0086	0.6266	0.0082	0.7072	0.0032	0.7027	0.0033	0.7026	0.0035
	f2	x4	0.626	0.0089	0.6235	0.0088	0.7121	0.0033	0.7026	0.0033	0.7026	0.0036
		x5	0.6169	0.0097	0.6209	0.009	0.7029	0.0034	0.6976	0.0035	0.6961	0.0038
		x6	0.6249	0.0088	0.6225	0.0089	0.7057	0.0035	0.7004	0.0033	0.6999	0.0035
	$\hat{\gamma}$		0.4778	0.0063	0.494	0.0055	0.5136	0.0053	0.4997	0.0049	0.4987	0.0049
750	f1	x1	0.6231	0.0073	0.6253	0.0067	0.7049	0.0014	0.7033	0.0017	0.7043	0.0016
		x2	0.623	0.0073	0.6264	0.0066	0.7035	0.0014	0.7018	0.0017	0.7007	0.0014
		x3	0.6237	0.0072	0.6259	0.0068	0.7035	0.0015	0.7017	0.0016	0.6976	0.0016
	f2	x4	0.6231	0.0074	0.6269	0.0067	0.7022	0.0017	0.7014	0.0015	0.7005	0.0017
		x5	0.6241	0.0071	0.6226	0.0074	0.7052	0.0017	0.6991	0.0017	0.701	0.0018
		x6	0.621	0.0076	0.6223	0.0073	0.7046	0.0014	0.6996	0.0015	0.6984	0.0018
	$\hat{\gamma}$		0.4895	0.003	0.4971	0.0023	0.5031	0.0027	0.503	0.0024	0.5006	0.0025
900	f1	x1	0.6245	0.0069	0.6258	0.0066	0.7033	0.0012	0.6971	0.0014	0.6981	0.0015
		x2	0.6226	0.0071	0.6239	0.0069	0.7041	0.0014	0.7016	0.0013	0.7003	0.0013
		x3	0.6207	0.007	0.6243	0.0067	0.7061	0.0014	0.7001	0.0013	0.6983	0.0015
	f2	x4	0.6236	0.0071	0.6242	0.0069	0.7032	0.0014	0.6994	0.0013	0.6993	0.0013
		x5	0.6237	0.007	0.6266	0.0065	0.7027	0.0012	0.7007	0.0013	0.6984	0.0015
		x6	0.622	0.0073	0.6267	0.0063	0.7027	0.0013	0.6985	0.0013	0.7014	0.0014
	$\hat{\gamma}$		0.4841	0.0023	0.4971	0.002	0.5039	0.0021	0.4991	0.0019	0.5002	0.002
1500	f1	x1	0.6227	0.0068	0.625	0.0061	0.7009	0.0008	0.6994	0.0008	0.6992	0.0008
		x2	0.6231	0.0066	0.6247	0.0063	0.7042	0.0008	0.7015	0.0007	0.7013	0.0008
		x3	0.6214	0.0069	0.6241	0.0065	0.7015	0.0007	0.7001	0.0008	0.6999	0.0008
	f2	x4	0.6223	0.0067	0.6257	0.0062	0.7028	0.0008	0.7006	0.0007	0.7006	0.0008
		x5	0.6234	0.0065	0.6243	0.0063	0.7029	0.0007	0.6989	0.0008	0.699	0.0008
		x6	0.6234	0.0066	0.6227	0.006	0.7027	0.0008	0.6979	0.0007	0.6974	0.0008
	$\hat{\gamma}$		0.4847	0.0016	0.501	0.0011	0.501	0.0013	0.5011	0.0011	0.5007	0.0011

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}$ = structural path between latent variables ($f1 \Rightarrow f2$).

of response categories decreased. These results indicate that larger samples alone were insufficient to remove the loading underestimation observed for ML and MLR within the examined conditions.

7.3. Mean Squared Error

This study assessed parameter estimation accuracy using MSE. From Table 1 to Table 12, the MSE values generally decreased as the sample sizes and the number of response categories increased, in both models. Lower MSE values

Table 2. Parameter estimates and MSE for the symmetric distribution in Model A with 5 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model A- 5 category - Symmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6625	0.0052	0.6614	0.0057	0.7111	0.0046	0.6987	0.0047	0.7036	0.0043
		x2	0.6673	0.0046	0.6703	0.0049	0.7117	0.004	0.7011	0.0043	0.7019	0.0048
		x3	0.6695	0.0047	0.6612	0.0052	0.7088	0.0046	0.702	0.0043	0.6985	0.0048
	f2	x4	0.6625	0.0054	0.6626	0.0054	0.7105	0.0048	0.6972	0.0047	0.6989	0.0041
		x5	0.664	0.005	0.666	0.0052	0.7096	0.0043	0.6974	0.0039	0.6984	0.0048
		x6	0.6633	0.0054	0.6663	0.0045	0.7096	0.0044	0.7034	0.0047	0.7003	0.0042
	$\hat{\gamma}$		0.496	0.0076	0.4981	0.0074	0.5189	0.008	0.505	0.0071	0.5034	0.0071
350	f1	x1	0.662	0.0035	0.6646	0.0033	0.7052	0.0021	0.7068	0.0022	0.7065	0.0024
		x2	0.667	0.0035	0.6701	0.003	0.707	0.0025	0.6979	0.0023	0.6971	0.0025
		x3	0.6657	0.0033	0.664	0.0037	0.7071	0.0027	0.6986	0.0026	0.6984	0.0029
	f2	x4	0.6628	0.0035	0.6661	0.0031	0.7083	0.0025	0.7008	0.0023	0.7004	0.0025
		x5	0.6655	0.0032	0.6636	0.0032	0.7065	0.0024	0.6978	0.0024	0.6971	0.0026
		x6	0.6666	0.0033	0.6619	0.0036	0.7087	0.0025	0.7007	0.0023	0.7002	0.0025
	$\hat{\gamma}$		0.4966	0.0042	0.4969	0.0041	0.5107	0.0048	0.4978	0.004	0.4968	0.004
750	f1	x1	0.6676	0.0021	0.6667	0.002	0.7042	0.0011	0.7021	0.001	0.7018	0.0011
		x2	0.6668	0.002	0.6652	0.0022	0.7036	0.0011	0.6987	0.001	0.6985	0.0011
		x3	0.6659	0.0021	0.6679	0.002	0.7051	0.001	0.6993	0.0012	0.699	0.0013
	f2	x4	0.6659	0.0021	0.6628	0.0024	0.7055	0.0011	0.7015	0.0011	0.7012	0.0012
		x5	0.6678	0.002	0.6688	0.0019	0.7011	0.0011	0.7008	0.0011	0.7004	0.0012
		x6	0.6661	0.0022	0.6669	0.0021	0.7044	0.001	0.6996	0.0011	0.6995	0.0012
	$\hat{\gamma}$		0.4961	0.002	0.5017	0.0018	0.5043	0.0022	0.5027	0.002	0.5022	0.002
900	f1	x1	0.6685	0.0018	0.6676	0.0019	0.7012	0.0009	0.699	0.001	0.6986	0.001
		x2	0.6636	0.002	0.6684	0.0017	0.706	0.001	0.698	0.0009	0.7019	0.0011
		x3	0.6653	0.002	0.6696	0.0017	0.7034	0.0009	0.7002	0.001	0.6973	0.001
	f2	x4	0.667	0.0019	0.6648	0.0021	0.7039	0.0009	0.6999	0.0009	0.7014	0.001
		x5	0.6681	0.0018	0.6668	0.0019	0.7015	0.0009	0.6995	0.0009	0.7008	0.001
		x6	0.6663	0.0019	0.6662	0.002	0.7026	0.001	0.7002	0.001	0.6985	0.0011
	$\hat{\gamma}$		0.4967	0.0017	0.4997	0.0017	0.5053	0.0015	0.5016	0.0017	0.4981	0.0016
1500	f1	x1	0.665	0.0017	0.6662	0.0017	0.7023	0.0005	0.6993	0.0006	0.7024	0.0006
		x2	0.6661	0.0016	0.6651	0.0017	0.701	0.0005	0.7012	0.0006	0.6996	0.0006
		x3	0.6656	0.0017	0.6659	0.0017	0.7012	0.0005	0.6997	0.0005	0.7009	0.0005
	f2	x4	0.6657	0.0017	0.6651	0.0017	0.7014	0.0005	0.7004	0.0005	0.6979	0.0007
		x5	0.6669	0.0015	0.6649	0.0017	0.7014	0.0006	0.7004	0.0005	0.6988	0.0007
		x6	0.6673	0.0016	0.6675	0.0015	0.7042	0.0006	0.6983	0.0006	0.7004	0.0006
	$\hat{\gamma}$		0.4972	0.0011	0.4971	0.0011	0.5012	0.001	0.5009	0.001	0.5007	0.0009

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}$ = structural path between latent variables ($f1 \Rightarrow f2$).

suggest that the estimated parameters were closer to their true population values, reflecting greater parameter estimation accuracy.

The differences in MSE between ML and MLR were generally small. However, the both methods yielded higher MSE values than WLS, DWLS, and ULS across the examined sample sizes and distribution types. For example, under the symmetric three-category condition in Model A at $N = 1500$, MSE values ranged from 0.006 to 0.0069 for ML and MLR, compared with approximately from 0.0007 to 0.0008 for WLS, DWLS, and ULS.

Table 3. Parameter estimates and MSE for the symmetric distribution Model A with 8 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model A - 8 category - Symmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6857	0.0036	0.6832	0.0037	0.7067	0.0035	0.7045	0.0034	0.7036	0.004
		x2	0.6846	0.0039	0.685	0.0036	0.7136	0.0038	0.7	0.0034	0.7008	0.0043
		x3	0.6824	0.0035	0.6823	0.0035	0.707	0.0032	0.7025	0.0036	0.6982	0.0041
	f2	x4	0.6863	0.004	0.6788	0.0037	0.7115	0.0035	0.6968	0.0037	0.7004	0.0038
		x5	0.6828	0.0042	0.6835	0.0037	0.7104	0.0037	0.6989	0.0039	0.6981	0.004
		x6	0.6868	0.0034	0.6846	0.0039	0.7103	0.0041	0.704	0.0036	0.6985	0.0037
	$\hat{\gamma}$		0.4963	0.0065	0.5019	0.0063	0.5114	0.0067	0.5125	0.0075	0.5021	0.0064
350	f1	x1	0.6821	0.002	0.6845	0.0021	0.7048	0.0022	0.7048	0.0021	0.7043	0.0022
		x2	0.6859	0.0021	0.6835	0.0021	0.703	0.0019	0.6966	0.0021	0.6959	0.0023
		x3	0.6805	0.0024	0.682	0.0022	0.7064	0.0019	0.702	0.0021	0.702	0.0023
	f2	x4	0.684	0.0021	0.683	0.0021	0.7037	0.0021	0.6997	0.0021	0.6994	0.0023
		x5	0.6834	0.0022	0.6862	0.0023	0.7061	0.002	0.6978	0.0021	0.6971	0.0022
		x6	0.6851	0.0021	0.6841	0.0022	0.7063	0.002	0.702	0.0019	0.7017	0.0021
	$\hat{\gamma}$		0.4994	0.0042	0.4965	0.0039	0.5052	0.0042	0.4975	0.0036	0.4963	0.0036
750	f1	x1	0.6841	0.0011	0.6843	0.0011	0.7026	0.0009	0.7016	0.0009	0.7016	0.001
		x2	0.6855	0.001	0.6854	0.001	0.7011	0.0009	0.6984	0.0009	0.6982	0.001
		x3	0.6849	0.0011	0.6844	0.0011	0.7024	0.001	0.6997	0.0009	0.6992	0.0011
	f2	x4	0.6861	0.0012	0.6836	0.0012	0.7027	0.001	0.7003	0.0009	0.7003	0.001
		x5	0.6844	0.0011	0.683	0.0012	0.7045	0.0009	0.7007	0.001	0.7003	0.0011
		x6	0.6832	0.0012	0.6863	0.001	0.7023	0.001	0.6996	0.001	0.6994	0.0011
	$\hat{\gamma}$		0.4991	0.0018	0.5014	0.0015	0.5035	0.0017	0.5022	0.0019	0.5017	0.0018
900	f1	x1	0.6853	0.001	0.6846	0.001	0.7037	0.0008	0.7034	0.0008	0.6995	0.0009
		x2	0.6815	0.001	0.6821	0.001	0.7005	0.0008	0.6983	0.0008	0.7026	0.0009
		x3	0.6838	0.001	0.6851	0.001	0.6998	0.0008	0.7002	0.0008	0.6972	0.0009
	f2	x4	0.6841	0.0011	0.6846	0.001	0.7013	0.0008	0.7001	0.0008	0.7023	0.001
		x5	0.6835	0.0011	0.6844	0.0009	0.6997	0.0008	0.6987	0.0007	0.6991	0.0009
		x6	0.6854	0.001	0.6833	0.001	0.7011	0.0008	0.7002	0.0007	0.6978	0.0009
	$\hat{\gamma}$		0.497	0.0016	0.4991	0.0017	0.5019	0.0014	0.5033	0.0013	0.498	0.0014
1500	f1	x1	0.6847	0.0007	0.6851	0.0006	0.6999	0.0004	0.6989	0.0005	0.7007	0.0005
		x2	0.6841	0.0007	0.6842	0.0007	0.7004	0.0004	0.7007	0.0005	0.699	0.0005
		x3	0.6847	0.0006	0.6842	0.0007	0.7016	0.0004	0.6995	0.0005	0.7018	0.0004
	f2	x4	0.6847	0.0007	0.6847	0.0006	0.7012	0.0005	0.6994	0.0005	0.6981	0.0006
		x5	0.6836	0.0007	0.6853	0.0007	0.7007	0.0004	0.6998	0.0004	0.699	0.0006
		x6	0.6858	0.0007	0.6855	0.0006	0.7011	0.0004	0.6984	0.0004	0.6996	0.0006
	$\hat{\gamma}$		0.5005	0.0009	0.499	0.0009	0.5024	0.0009	0.4996	0.0009	0.5016	0.0009

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}$ = structural path between latent variables ($f1 \Rightarrow f2$).

Thus, although increasing sample size reduced MSE for all methods, relative differences remained between the estimation methods.

Although the MSE values for ML and MLR generally decrease as the number of response categories increases, they tend to be relatively higher under asymmetric distributions, particularly when the number of categories is fewer than eight. For example, in Model A with five response categories, the MSE values for ML and MLR range from 0.0015 to 0.0017 under the symmetric distribution, compared with 0.0025 to 0.0029 under asymmetric distributions. In contrast, WLS, DWLS, and ULS maintain relatively low MSE values even under asymmetric

Table 4. Parameter estimates and MSE for the asymmetric distribution in Model A with 3 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model A - 3 category - Asymmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6279	0.0106	0.6199	0.012	0.7127	0.0062	0.7019	0.0064	0.7	0.0068
		x2	0.6234	0.0122	0.6279	0.0107	0.7119	0.0065	0.7064	0.0065	0.7051	0.0069
		x3	0.6209	0.0118	0.6192	0.0119	0.7052	0.0074	0.7025	0.0061	0.7014	0.0066
	f2	x4	0.6222	0.0117	0.6285	0.0108	0.7197	0.0073	0.7041	0.0064	0.7029	0.0068
		x5	0.6293	0.0111	0.6183	0.0125	0.7111	0.0072	0.6991	0.0066	0.6974	0.0071
		x6	0.619	0.0133	0.6226	0.0116	0.7066	0.0072	0.7029	0.0062	0.702	0.0066
	$\hat{\gamma}$		0.4872	0.0097	0.483	0.0112	0.5258	0.0115	0.5029	0.0098	0.5001	0.0095
350	f1	x1	0.6216	0.0095	0.6191	0.0097	0.7092	0.0041	0.6996	0.0039	0.6991	0.0041
		x2	0.6185	0.0098	0.6259	0.0088	0.7131	0.0035	0.698	0.0033	0.6969	0.0036
		x3	0.6225	0.0096	0.6239	0.0086	0.7114	0.0042	0.7051	0.004	0.7044	0.0043
	f2	x4	0.6264	0.0088	0.6225	0.0096	0.7124	0.0037	0.7034	0.0039	0.7029	0.0042
		x5	0.6225	0.0089	0.6218	0.0095	0.7065	0.0039	0.6944	0.0032	0.6935	0.0034
		x6	0.6224	0.0093	0.6236	0.0089	0.7101	0.0036	0.702	0.0035	0.7012	0.0037
	$\hat{\gamma}$		0.4837	0.0061	0.4837	0.0062	0.5135	0.0059	0.4966	0.0057	0.495	0.0057
750	f1	x1	0.6208	0.0078	0.6233	0.0073	0.7027	0.0018	0.7006	0.0016	0.7005	0.0017
		x2	0.6231	0.0073	0.6201	0.0079	0.7026	0.0016	0.7003	0.0016	0.7001	0.0017
		x3	0.6245	0.0071	0.6268	0.0068	0.7045	0.0017	0.7002	0.0017	0.6993	0.0018
	f2	x4	0.6218	0.0078	0.6216	0.0077	0.7051	0.0014	0.7005	0.0018	0.7001	0.0019
		x5	0.6238	0.0073	0.6253	0.0072	0.7053	0.0016	0.7009	0.0015	0.7001	0.0017
		x6	0.6217	0.0075	0.6211	0.0077	0.7025	0.0017	0.6982	0.0016	0.6982	0.0017
	$\hat{\gamma}$		0.486	0.0028	0.4909	0.0027	0.5102	0.003	0.5053	0.0028	0.5045	0.0028
900	f1	x1	0.6234	0.007	0.6243	0.0069	0.7041	0.0014	0.7018	0.0014	0.7013	0.0015
		x2	0.6216	0.007	0.6249	0.0068	0.7024	0.0013	0.7	0.0013	0.6996	0.0014
		x3	0.6257	0.0067	0.624	0.007	0.705	0.0016	0.6981	0.0014	0.6979	0.0015
	f2	x4	0.6236	0.007	0.6203	0.0075	0.7037	0.0013	0.7009	0.0015	0.7005	0.0016
		x5	0.6255	0.0068	0.6236	0.007	0.7009	0.0015	0.7	0.0014	0.6991	0.0015
		x6	0.6216	0.0074	0.6243	0.0071	0.7051	0.0015	0.6998	0.0013	0.7001	0.0014
	$\hat{\gamma}$		0.4869	0.0026	0.4838	0.0023	0.5027	0.0023	0.5003	0.0021	0.4997	0.0021
1500	f1	x1	0.6242	0.0065	0.622	0.0068	0.7014	0.0008	0.6997	0.0009	0.6997	0.001
		x2	0.6222	0.0068	0.6228	0.0067	0.7037	0.0008	0.6993	0.0008	0.6988	0.0009
		x3	0.6223	0.007	0.6245	0.0064	0.7002	0.0007	0.6992	0.0008	0.6992	0.0009
	f2	x4	0.6231	0.0066	0.6244	0.0064	0.7039	0.0009	0.6989	0.0008	0.6984	0.0009
		x5	0.6209	0.007	0.623	0.0066	0.7008	0.0009	0.701	0.0007	0.7012	0.0008
		x6	0.6252	0.0063	0.6224	0.0067	0.703	0.0008	0.7005	0.0008	0.7003	0.0009
	$\hat{\gamma}$		0.486	0.0014	0.4884	0.0014	0.5031	0.0014	0.4997	0.0014	0.4993	0.0014

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}$ = structural path between latent variables ($f1 \Rightarrow f2$).

distributions. Specifically, in Model A with five response categories, their MSE values range from 0.0005 to 0.0007 under the symmetric distribution and from 0.0005 to 0.0008 under asymmetric distributions. Furthermore, the MSE values remain relatively similar between Models A and B, with only minor differences observed across the two models, suggesting that model complexity may have a limited impact on parameter estimation accuracy. Overall, these findings indicate that ML and MLR are more sensitive to distributional asymmetry, whereas WLS, DWLS, and ULS demonstrate greater stability and generally provide more accurate parameter estimates, as reflected by their consistently lower MSE values across different distributional conditions and model complexities.

Table 5. Parameter estimates and MSE for the asymmetric distribution in Model A with 5 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model A - 5 category - Asymmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6533	0.0065	0.6535	0.0071	0.7069	0.0051	0.7031	0.0042	0.7059	0.0052
		x2	0.6512	0.0069	0.6527	0.0068	0.711	0.0048	0.7038	0.005	0.7019	0.0059
		x3	0.6578	0.006	0.6527	0.0068	0.7082	0.005	0.703	0.0045	0.699	0.0056
	f2	x4	0.6477	0.0077	0.6543	0.0066	0.7142	0.005	0.6985	0.005	0.699	0.0051
		x5	0.6516	0.007	0.6502	0.0074	0.7133	0.005	0.6993	0.0049	0.6982	0.0055
		x6	0.6584	0.006	0.6512	0.0076	0.7098	0.0056	0.7043	0.0049	0.6998	0.0049
	$\hat{\gamma}$		0.489	0.0098	0.4863	0.0087	0.5093	0.0076	0.515	0.009	0.5	0.0081
350	f1	x1	0.655	0.0044	0.6516	0.0048	0.7123	0.003	0.7043	0.0029	0.7036	0.0031
		x2	0.6547	0.0048	0.6549	0.0046	0.7044	0.0028	0.6977	0.0025	0.6968	0.0027
		x3	0.6553	0.0047	0.6552	0.0044	0.7062	0.0027	0.7012	0.003	0.7011	0.0033
	f2	x4	0.6576	0.0044	0.6513	0.0051	0.7063	0.0027	0.7004	0.0028	0.7001	0.0031
		x5	0.6561	0.0046	0.6525	0.0049	0.7057	0.0026	0.6988	0.0026	0.6981	0.0027
		x6	0.6512	0.0049	0.6534	0.0047	0.7073	0.0028	0.7011	0.0025	0.7006	0.0027
	$\hat{\gamma}$		0.4915	0.005	0.4957	0.0046	0.5124	0.0052	0.4981	0.0046	0.4966	0.0045
750	f1	x1	0.6541	0.0033	0.6556	0.0032	0.7016	0.0012	0.7019	0.0012	0.7018	0.0013
		x2	0.6531	0.0034	0.6558	0.0033	0.7045	0.0014	0.6993	0.0012	0.6991	0.0013
		x3	0.6537	0.0034	0.6552	0.0031	0.7068	0.0014	0.6988	0.0012	0.6984	0.0013
	f2	x4	0.6534	0.0032	0.6526	0.0035	0.705	0.0012	0.7005	0.0012	0.7001	0.0013
		x5	0.6552	0.0031	0.6573	0.003	0.7033	0.0011	0.7005	0.0013	0.6999	0.0014
		x6	0.6539	0.0034	0.6516	0.0035	0.7044	0.0013	0.6985	0.0012	0.6986	0.0013
	$\hat{\gamma}$		0.4889	0.0025	0.4913	0.0023	0.5055	0.002	0.5033	0.0022	0.5027	0.0022
900	f1	x1	0.6534	0.0032	0.6518	0.0033	0.7037	0.0011	0.6994	0.0011	0.7005	0.0011
		x2	0.6546	0.0029	0.6526	0.0033	0.7036	0.0011	0.6987	0.001	0.7029	0.0012
		x3	0.6535	0.0033	0.6543	0.003	0.7024	0.0009	0.6986	0.0012	0.6943	0.0012
	f2	x4	0.6527	0.0032	0.654	0.0031	0.6992	0.001	0.6996	0.0011	0.7018	0.0013
		x5	0.6524	0.003	0.6533	0.0032	0.7023	0.001	0.7003	0.0011	0.6999	0.0012
		x6	0.6504	0.003	0.6505	0.0034	0.705	0.0011	0.6998	0.001	0.6975	0.0012
	$\hat{\gamma}$		0.4927	0.0018	0.4921	0.0021	0.5016	0.0017	0.5015	0.0018	0.4971	0.0018
1500	f1	x1	0.6531	0.0028	0.6519	0.0029	0.7019	0.0006	0.6993	0.0007	0.7024	0.0006
		x2	0.6531	0.0028	0.6554	0.0025	0.7021	0.0005	0.6996	0.0007	0.6998	0.0006
		x3	0.6513	0.0029	0.6529	0.0029	0.7005	0.0006	0.6994	0.0006	0.7019	0.0006
	f2	x4	0.6527	0.0028	0.6512	0.0029	0.7027	0.0006	0.7003	0.0006	0.6984	0.0008
		x5	0.6514	0.0029	0.6536	0.0028	0.7008	0.0007	0.7005	0.0006	0.6996	0.0008
		x6	0.6538	0.0027	0.6547	0.0026	0.7006	0.0007	0.699	0.0006	0.7002	0.0007
	$\hat{\gamma}$		0.4892	0.0013	0.4907	0.0012	0.5043	0.0011	0.5001	0.0011	0.5005	0.0011

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}$ = structural path between latent variables ($f1 \Rightarrow f2$).

For the structural path coefficients, MSE also decreased substantially with increasing sample size. Differences between estimators were generally less pronounced than those observed for factor loadings, particularly in larger samples. For example, with eight response categories at $N = 1500$, structural path MSE values were approximately 0.0008–0.0010 across the two models and distributional conditions. In smaller samples, the estimator with the lowest MSE varied across conditions, and WLS often yielded higher structural path MSE values than the other methods in Model B.

Table 6. Parameter estimates and MSE for the asymmetric distribution in Model A with 8 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model A - 8 category - Asymmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6806	0.0043	0.6816	0.0039	0.7103	0.0038	0.7047	0.0034	0.7039	0.0037
		x2	0.6793	0.0037	0.6832	0.0039	0.7059	0.0046	0.7022	0.004	0.702	0.0043
		x3	0.6815	0.0042	0.6797	0.0035	0.7076	0.0038	0.698	0.0037	0.6967	0.004
	f2	x4	0.6784	0.0038	0.6821	0.0039	0.7117	0.0038	0.7004	0.0034	0.6998	0.0037
		x5	0.68	0.004	0.6845	0.0033	0.7144	0.0037	0.7005	0.0038	0.6992	0.0042
		x6	0.6794	0.0039	0.6871	0.0036	0.7093	0.0037	0.6999	0.0035	0.6986	0.0039
	$\hat{\gamma}$		0.4918	0.0072	0.5007	0.0068	0.5091	0.0072	0.5012	0.0066	0.4997	0.0066
350	f1	x1	0.6798	0.0023	0.6815	0.0021	0.7028	0.0021	0.7047	0.0021	0.7044	0.0023
		x2	0.6815	0.0024	0.6822	0.0024	0.7037	0.0019	0.6983	0.002	0.6975	0.0022
		x3	0.6801	0.0025	0.6844	0.0022	0.7084	0.0022	0.6991	0.0023	0.6991	0.0026
	f2	x4	0.6771	0.0025	0.6795	0.0022	0.7055	0.002	0.7011	0.0021	0.7009	0.0023
		x5	0.6832	0.0022	0.6786	0.0026	0.7058	0.0021	0.698	0.0019	0.6971	0.0021
		x6	0.6788	0.0023	0.6826	0.0023	0.7072	0.0021	0.7022	0.002	0.7019	0.0022
	$\hat{\gamma}$		0.4994	0.004	0.4942	0.0041	0.5117	0.0038	0.4971	0.0038	0.496	0.0038
750	f1	x1	0.6843	0.0012	0.6833	0.0011	0.7033	0.001	0.7019	0.001	0.7017	0.001
		x2	0.6833	0.0012	0.6815	0.0013	0.7028	0.0009	0.6984	0.0009	0.6985	0.001
		x3	0.6793	0.0013	0.6811	0.0013	0.7011	0.0009	0.6983	0.001	0.6978	0.0011
	f2	x4	0.682	0.0012	0.6822	0.0013	0.7047	0.0009	0.7011	0.001	0.7008	0.0011
		x5	0.6821	0.0012	0.6795	0.0014	0.7024	0.001	0.7004	0.001	0.7002	0.0011
		x6	0.6816	0.0012	0.6787	0.0014	0.7024	0.0009	0.6995	0.001	0.6993	0.001
	$\hat{\gamma}$		0.4995	0.0016	0.4975	0.0017	0.507	0.0018	0.5014	0.0019	0.5009	0.0018
900	f1	x1	0.6812	0.0011	0.6828	0.001	0.7014	0.0008	0.6989	0.0009	0.6987	0.0009
		x2	0.6799	0.0012	0.6814	0.0012	0.7035	0.0009	0.6982	0.0008	0.702	0.0009
		x3	0.6824	0.0011	0.6822	0.0011	0.7015	0.0008	0.6995	0.0009	0.6971	0.0009
	f2	x4	0.683	0.001	0.6826	0.001	0.7044	0.0008	0.699	0.0008	0.7006	0.001
		x5	0.6795	0.0012	0.6819	0.0011	0.7022	0.0008	0.6993	0.0008	0.6997	0.0009
		x6	0.6787	0.001	0.6814	0.0011	0.7014	0.0009	0.701	0.0008	0.6978	0.001
	$\hat{\gamma}$		0.4985	0.0016	0.4962	0.0016	0.5029	0.0015	0.5014	0.0016	0.4981	0.0015
1500	f1	x1	0.6806	0.0009	0.681	0.0008	0.7035	0.0005	0.6994	0.0005	0.701	0.0005
		x2	0.6825	0.0008	0.682	0.0007	0.7025	0.0005	0.7001	0.0005	0.6997	0.0005
		x3	0.6829	0.0008	0.6821	0.0008	0.7006	0.0005	0.6997	0.0005	0.7016	0.0005
	f2	x4	0.6813	0.0008	0.6822	0.0007	0.7006	0.0005	0.6994	0.0005	0.6977	0.0006
		x5	0.6822	0.0008	0.6823	0.0007	0.7018	0.0005	0.7002	0.0005	0.6986	0.0006
		x6	0.6821	0.0007	0.6812	0.0008	0.701	0.0005	0.6986	0.0004	0.7	0.0006
	$\hat{\gamma}$		0.4971	0.001	0.4983	0.0009	0.5013	0.0009	0.4998	0.0009	0.5011	0.0009

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}$ = structural path between latent variables ($f1 \Rightarrow f2$).

7.4. Goodness of Fit Results

A major objective of this study was to examine how the estimation methods performed with respect to overall model fit under different sample sizes, response category conditions, and data distributions. Model fit was evaluated using three goodness-of-fit indices: CFI, NFI, and RMSEA.

Tables 13 and 14 summarize the goodness-of-fit results for Model A under symmetric and asymmetric distributions, respectively, while Tables 15 and 16 present the corresponding results for Model B.

Table 7. Parameter estimates and MSE for the symmetric distribution in Model B with 3 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model B - 3 category - Symmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6185	0.0118	0.6203	0.0113	0.7187	0.0074	0.7065	0.006	0.705	0.0063
		x2	0.6326	0.0094	0.6264	0.0104	0.7318	0.0076	0.6969	0.0059	0.6963	0.0064
		x3	0.6238	0.0105	0.6243	0.0106	0.7214	0.0067	0.7012	0.0061	0.6996	0.0065
	f2	x4	0.6189	0.0112	0.6239	0.0104	0.733	0.0067	0.7029	0.0044	0.7011	0.0046
		x5	0.6273	0.01	0.6251	0.0098	0.7274	0.0065	0.7025	0.0048	0.7017	0.005
		x6	0.6304	0.0085	0.6238	0.0101	0.7277	0.007	0.6984	0.0051	0.6966	0.0053
	f3	x7	0.6261	0.0104	0.6253	0.0109	0.722	0.0081	0.7006	0.0059	0.6988	0.0062
		x8	0.6278	0.01	0.6263	0.0101	0.7206	0.0083	0.6976	0.0064	0.6961	0.0067
		x9	0.6208	0.0114	0.6244	0.0109	0.7226	0.0074	0.7003	0.0062	0.6998	0.0067
	$\hat{\gamma}_1$		0.4978	0.009	0.5078	0.0089	0.508	0.0104	0.5114	0.0089	0.5081	0.0086
	$\hat{\gamma}_2$		0.5012	0.0089	0.4982	0.0092	0.5088	0.0108	0.511	0.0097	0.5073	0.0094
350	f1	x1	0.6278	0.0079	0.6266	0.0083	0.7121	0.0037	0.7008	0.0031	0.6997	0.0033
		x2	0.6282	0.0079	0.6215	0.0089	0.7176	0.0038	0.7039	0.0037	0.7039	0.0039
		x3	0.6245	0.0085	0.6231	0.0087	0.7129	0.0041	0.7059	0.0035	0.7052	0.0037
	f2	x4	0.625	0.0077	0.6256	0.008	0.7169	0.0033	0.697	0.0031	0.6959	0.0032
		x5	0.6255	0.0078	0.6224	0.0082	0.7178	0.0031	0.7045	0.0026	0.7033	0.0027
		x6	0.6275	0.0073	0.6222	0.0082	0.7186	0.0034	0.7004	0.0026	0.6997	0.0027
	f3	x7	0.6243	0.0086	0.626	0.0082	0.7181	0.0039	0.7037	0.0035	0.7031	0.0037
		x8	0.624	0.0082	0.6244	0.0084	0.716	0.0039	0.6936	0.0036	0.6924	0.0038
		x9	0.6233	0.0087	0.6236	0.0085	0.7125	0.0036	0.7061	0.0036	0.7056	0.0039
	$\hat{\gamma}_1$		0.4907	0.0047	0.4992	0.005	0.5038	0.0054	0.5089	0.0049	0.5066	0.0048
	$\hat{\gamma}_2$		0.4985	0.0051	0.5015	0.0052	0.5085	0.0055	0.5099	0.0049	0.5078	0.0049
750	f1	x1	0.6255	0.0068	0.6242	0.0071	0.7094	0.0016	0.7026	0.0016	0.702	0.0017
		x2	0.6256	0.0069	0.6249	0.0069	0.7034	0.0017	0.6972	0.0016	0.6971	0.0018
		x3	0.6286	0.0063	0.6257	0.007	0.7041	0.0017	0.6992	0.0016	0.6988	0.0017
	f2	x4	0.6275	0.0064	0.6242	0.0068	0.7058	0.0014	0.6995	0.0014	0.6992	0.0015
		x5	0.6255	0.0066	0.6259	0.0065	0.7079	0.0014	0.701	0.0014	0.7005	0.0014
		x6	0.6247	0.0067	0.6248	0.007	0.7082	0.0014	0.6989	0.0014	0.6985	0.0014
	f3	x7	0.6281	0.007	0.6259	0.0067	0.7062	0.0015	0.7001	0.0016	0.6998	0.0017
		x8	0.6237	0.0071	0.622	0.0073	0.7069	0.0017	0.7016	0.0016	0.7013	0.0017
		x9	0.6257	0.0068	0.6236	0.0071	0.7051	0.0014	0.6995	0.0016	0.6992	0.0017
	$\hat{\gamma}_1$		0.4965	0.0021	0.5014	0.0021	0.5011	0.0023	0.5024	0.0025	0.5013	0.0024
	$\hat{\gamma}_2$		0.4997	0.0022	0.4938	0.0025	0.5006	0.0026	0.4992	0.0023	0.4982	0.0023

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

Using the adopted fit criteria, all reported CFI values exceeded 0.95, and all RMSEA values were below 0.05 for both models under symmetric and asymmetric distributions. NFI values also exceeded 0.95 across all conditions for Model A. For Model B, however, several NFI values fell within the acceptable range of 0.90 to below 0.95 at (N = 200). These occurred for ML and MLR across all three response-category conditions and for WLS with three and five categories under both distributions. Under symmetric distributions, these NFI values ranged from 0.9259 to 0.9481, whereas under asymmetric distributions, they ranged from 0.9209 to 0.9454. DWLS and ULS yielded NFI values above 0.95 throughout. At sample sizes of 350 or greater, all estimators yielded NFI values above 0.95 for both models.

Table 7. Parameter estimates and MSE for the symmetric distribution in Model B with 3 response categories (continued).

Model B - 3 category - Symmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
900	f1	x1	0.6255	0.0066	0.6244	0.0067	0.7026	0.0013	0.7005	0.0013	0.7019	0.0014
		x2	0.6242	0.0069	0.6277	0.006	0.7097	0.0012	0.7028	0.0013	0.7008	0.0014
		x3	0.6256	0.006	0.6226	0.007	0.7061	0.0013	0.6984	0.0013	0.7014	0.0014
	f2	x4	0.624	0.006	0.623	0.0068	0.7073	0.0011	0.7018	0.0011	0.7002	0.0012
		x5	0.6246	0.0066	0.6255	0.0065	0.7085	0.0012	0.7022	0.001	0.7016	0.0011
		x6	0.6256	0.0064	0.6235	0.007	0.7095	0.0013	0.7033	0.001	0.6983	0.0012
	f3	x7	0.6232	0.007	0.6253	0.0066	0.7037	0.0011	0.7004	0.0012	0.7016	0.0016
		x8	0.6259	0.0065	0.6215	0.0072	0.7069	0.0014	0.7014	0.0012	0.702	0.0015
		x9	0.6239	0.0067	0.6262	0.0065	0.7061	0.0012	0.7	0.0013	0.6989	0.0012
	$\hat{\gamma}_1$		0.4972	0.0021	0.5003	0.0021	0.5014	0.002	0.4993	0.0017	0.4995	0.0019
	$\hat{\gamma}_2$		0.4993	0.0017	0.5015	0.0019	0.5007	0.0019	0.5018	0.0019	0.5007	0.0019
1500	f1	x1	0.624	0.0064	0.6235	0.0065	0.7017	0.0007	0.6983	0.0007	0.6984	0.0008
		x2	0.6252	0.0062	0.6238	0.006	0.7044	0.0008	0.7015	0.0009	0.7029	0.0008
		x3	0.6267	0.006	0.6264	0.0061	0.702	0.0007	0.6987	0.0007	0.6995	0.0009
	f2	x4	0.6239	0.006	0.6253	0.0061	0.7057	0.0007	0.7019	0.0006	0.6991	0.0007
		x5	0.6244	0.0062	0.6252	0.0061	0.7053	0.0006	0.7016	0.0006	0.6997	0.0007
		x6	0.6246	0.0062	0.6244	0.0063	0.703	0.0007	0.6995	0.0007	0.7019	0.0007
	f3	x7	0.6234	0.0065	0.6236	0.0064	0.7022	0.0007	0.6987	0.0008	0.7001	0.0008
		x8	0.6235	0.0065	0.6257	0.0061	0.7028	0.0007	0.7	0.0007	0.7025	0.0009
		x9	0.624	0.0064	0.6241	0.0063	0.7043	0.0008	0.7012	0.0008	0.6987	0.0009
	$\hat{\gamma}_1$		0.4999	0.0012	0.4979	0.0012	0.5003	0.0013	0.4996	0.0013	0.5021	0.0011
	$\hat{\gamma}_2$		0.5018	0.0012	0.4969	0.0012	0.5019	0.0012	0.5013	0.0011	0.5011	0.0012

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

Among the five estimators, DWLS and ULS generally produced the most favorable goodness-of-fit (GOF) values across both models under symmetric and asymmetric distributions. DWLS consistently achieved CFI values close to 1, ranging from 0.9988 to 1.0000 in Model A and from 0.9992 to 1.0000 in Model B. Similarly, ULS produced high CFI values, ranging from 0.9963 to 1.0000 in Model A and from 0.9935 to 1.0000 in Model B. Regarding NFI, DWLS generally outperformed the other estimators, with values ranging from 0.9705 to 0.9990 across the examined conditions, compared with 0.9620 to 0.9987 for ULS. In terms of RMSEA, DWLS generally yielded the lowest values, ranging from 0.0003 to 0.0054, whereas ULS values ranged from 0.0003 to 0.0195. However, ULS occasionally achieved lower RMSEA values than DWLS, particularly with eight response categories. For example, under the asymmetric distribution in Model B with (N = 200) and eight categories, ULS yielded an RMSEA of 0.0009 compared with 0.0027 for DWLS. Overall, DWLS demonstrated the most favorable GOF values, followed closely by ULS.

ML and MLR demonstrated relatively similar GOF values across both models and distributional conditions. The CFI values for ML ranged from 0.9868 to 0.9993, while those for MLR ranged from 0.9866 to 0.9993. Similarly, NFI values ranged from 0.9209 to 0.9962 for ML and from 0.9212 to 0.9962 for MLR. Regarding RMSEA, ML produced values ranging from 0.0056 to 0.0237, compared with 0.0055 to 0.0231 for MLR. Although MLR occasionally yielded slightly more favorable RMSEA values, the differences between the two estimators were generally small, with neither estimator consistently outperforming the other across all conditions. Both estimators demonstrated satisfactory model fit, although their GOF values were generally less favorable than those obtained using DWLS and ULS, particularly at smaller sample sizes.

Table 8. Parameter estimates and MSE for the symmetric distribution in Model B with 5 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model B - 5 category - Symmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6714	0.0045	0.6637	0.0053	0.718	0.0053	0.6985	0.005	0.7019	0.0046
		x2	0.6628	0.0054	0.6704	0.0045	0.7204	0.0053	0.6932	0.0045	0.6993	0.0048
		x3	0.6617	0.005	0.6654	0.0048	0.7166	0.0054	0.7024	0.0042	0.6956	0.0048
	f2	x4	0.6669	0.0046	0.6676	0.0045	0.7189	0.005	0.6976	0.0037	0.7016	0.0032
		x5	0.6661	0.0049	0.6615	0.0047	0.7277	0.0044	0.6993	0.0037	0.6995	0.0036
		x6	0.666	0.0043	0.6662	0.0043	0.7239	0.0045	0.7015	0.003	0.7003	0.0038
	f3	x7	0.6655	0.0046	0.6628	0.005	0.7179	0.0061	0.6992	0.0043	0.6966	0.0043
		x8	0.6634	0.0051	0.6677	0.0044	0.714	0.005	0.6946	0.004	0.6992	0.0047
		x9	0.6652	0.0054	0.669	0.0048	0.7191	0.0052	0.7036	0.0039	0.7045	0.0044
		$\hat{\gamma}_1$	0.5087	0.0071	0.4954	0.0078	0.5086	0.0086	0.5053	0.0077	0.5068	0.0071
		$\hat{\gamma}_2$	0.494	0.0078	0.5016	0.0068	0.5038	0.0092	0.5108	0.0065	0.5073	0.0077
350	f1	x1	0.6661	0.0034	0.6625	0.0035	0.7079	0.0023	0.7003	0.0024	0.7007	0.0023
		x2	0.6685	0.0031	0.6668	0.0032	0.7133	0.0027	0.7009	0.0021	0.7011	0.0027
		x3	0.6651	0.0032	0.665	0.0035	0.7113	0.0026	0.7012	0.0023	0.7027	0.0026
	f2	x4	0.6648	0.0031	0.6684	0.0026	0.7133	0.0022	0.7017	0.002	0.6993	0.0022
		x5	0.6706	0.0028	0.6638	0.0034	0.7146	0.0026	0.6986	0.002	0.6998	0.0022
		x6	0.6652	0.003	0.667	0.0029	0.7113	0.0024	0.7013	0.0021	0.6988	0.0019
	f3	x7	0.6651	0.0036	0.6662	0.003	0.7129	0.0025	0.7064	0.0023	0.7029	0.0027
		x8	0.6673	0.0033	0.6671	0.0034	0.7065	0.0024	0.7011	0.0024	0.6981	0.0026
		x9	0.6666	0.0033	0.6664	0.0031	0.7107	0.0025	0.7006	0.0026	0.7025	0.0025
		$\hat{\gamma}_1$	0.5013	0.0041	0.4973	0.0038	0.5076	0.0049	0.5041	0.004	0.5054	0.0037
		$\hat{\gamma}_2$	0.5031	0.0038	0.4983	0.0035	0.506	0.0046	0.502	0.0042	0.5061	0.0042
750	f1	x1	0.668	0.002	0.6672	0.0021	0.7062	0.0011	0.7013	0.0011	0.7008	0.0011
		x2	0.6666	0.002	0.6662	0.0022	0.7058	0.0011	0.7017	0.0012	0.6967	0.0013
		x3	0.667	0.0021	0.6683	0.0019	0.7047	0.0012	0.6993	0.001	0.6998	0.0011
	f2	x4	0.6661	0.0021	0.6667	0.0019	0.7044	0.001	0.7028	0.001	0.6985	0.001
		x5	0.6646	0.0021	0.6678	0.0018	0.7049	0.001	0.7003	0.0009	0.7013	0.0011
		x6	0.6689	0.002	0.6686	0.002	0.7076	0.001	0.7008	0.0009	0.6988	0.001
	f3	x7	0.667	0.0021	0.6646	0.0022	0.7035	0.001	0.704	0.001	0.7021	0.0012
		x8	0.6667	0.002	0.6643	0.0022	0.7053	0.0011	0.6995	0.0011	0.7003	0.0012
		x9	0.6678	0.0019	0.6683	0.002	0.7041	0.0011	0.6993	0.001	0.6997	0.0012
		$\hat{\gamma}_1$	0.4996	0.0016	0.4991	0.0017	0.4986	0.0019	0.5003	0.0016	0.4987	0.0019
		$\hat{\gamma}_2$	0.5009	0.0017	0.4974	0.0019	0.5033	0.0021	0.5001	0.0018	0.5004	0.0019

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

WLS generally produced less favorable GOF values than DWLS and ULS, particularly in terms of RMSEA. Its CFI values ranged from 0.9830 to 0.9992, while NFI values ranged from 0.9233 to 0.9958, indicating relatively high model fit across the examined conditions. However, RMSEA values ranged from 0.0058 to 0.0304, which were generally higher than those obtained using the other estimators, particularly in Model B. For example, under the asymmetric distribution in Model B with (N = 200) and three response categories, WLS produced an RMSEA of 0.0304, compared with 0.0029 for DWLS and 0.0195 for ULS. Nevertheless, WLS still achieved satisfactory GOF values across the examined conditions, despite its relatively less favorable performance compared with DWLS and ULS.

Table 8. Parameter estimates and MSE for the symmetric distribution in Model B with 5 response categories (continued).

Model B - 5 category - Symmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
900	f1	x1	0.6661	0.0019	0.6658	0.002	0.7064	0.0009	0.6994	0.001	0.7027	0.0011
		x2	0.6645	0.002	0.6652	0.002	0.703	0.0009	0.6999	0.001	0.6993	0.0009
		x3	0.6652	0.002	0.6677	0.0019	0.7024	0.0009	0.7001	0.0009	0.7004	0.0009
	f2	x4	0.666	0.0018	0.6668	0.0018	0.7055	0.0009	0.701	0.0008	0.6996	0.0008
		x5	0.6656	0.0019	0.6665	0.0018	0.7053	0.0008	0.7023	0.0007	0.7011	0.0008
		x6	0.6659	0.002	0.6658	0.002	0.7057	0.0009	0.7016	0.0009	0.6968	0.0009
	f3	x7	0.6657	0.002	0.666	0.002	0.7043	0.0008	0.6981	0.0008	0.7007	0.0009
		x8	0.6649	0.002	0.6667	0.002	0.7059	0.0009	0.7006	0.0009	0.6999	0.0011
		x9	0.6686	0.0018	0.6663	0.002	0.7062	0.0009	0.7001	0.001	0.7	0.0009
	$\hat{\gamma}_1$		0.5044	0.0014	0.4998	0.0014	0.4999	0.0016	0.4991	0.0013	0.4981	0.0016
	$\hat{\gamma}_2$		0.4973	0.0017	0.5012	0.0014	0.5012	0.0015	0.5007	0.0014	0.5023	0.0016
1500	f1	x1	0.6668	0.0015	0.6659	0.0016	0.7017	0.0006	0.6987	0.0006	0.7003	0.0006
		x2	0.6667	0.0015	0.6658	0.0016	0.7037	0.0006	0.7017	0.0006	0.7017	0.0005
		x3	0.6688	0.0015	0.6674	0.0015	0.7023	0.0005	0.6994	0.0005	0.701	0.0006
	f2	x4	0.6664	0.0015	0.6659	0.0016	0.7042	0.0005	0.7012	0.0004	0.6989	0.0005
		x5	0.6648	0.0017	0.6656	0.0016	0.7057	0.0005	0.7022	0.0005	0.699	0.0005
		x6	0.6649	0.0017	0.6662	0.0016	0.7026	0.0005	0.6998	0.0005	0.7024	0.0005
	f3	x7	0.6662	0.0017	0.6664	0.0016	0.7012	0.0005	0.6983	0.0005	0.7007	0.0006
		x8	0.6678	0.0015	0.666	0.0017	0.7025	0.0005	0.7005	0.0005	0.7008	0.0006
		x9	0.6666	0.0017	0.6648	0.0017	0.7026	0.0005	0.6999	0.0005	0.6982	0.0007
	$\hat{\gamma}_1$		0.4984	0.0009	0.5013	0.001	0.5003	0.001	0.4994	0.001	0.4998	0.0009
	$\hat{\gamma}_2$		0.5004	0.0009	0.5003	0.001	0.502	0.001	0.5017	0.001	0.5014	0.0009

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

In general, increasing the sample size from 200 to 1500 improved the GOF values across all five estimators, with CFI and NFI values approaching 1 and RMSEA values generally decreasing toward 0. This improvement was particularly noticeable for ML, MLR, and WLS, whereas DWLS and ULS maintained favorable GOF values even at smaller sample sizes. For example, under the symmetric distribution in Model A with three response categories, the NFI value for ML increased from 0.9558 at (N = 200) to 0.9938 at (N = 1500), while its RMSEA decreased from 0.0237 to 0.0079. A similar improvement was observed for WLS in Model B under the asymmetric distribution with three categories, where NFI increased from 0.9233 to 0.9864 and RMSEA decreased from 0.0304 to 0.0064 as the sample size increased from 200 to 1500.

The effect of increasing the number of response categories from three to eight was less consistent than that of sample size. CFI and NFI generally increased with the number of categories, although some fluctuations were observed across estimators and conditions. For example, under the symmetric distribution in Model A at (N = 200), the NFI values for ML increased from 0.9558 to 0.9716 as the number of categories increased from three to eight. However, RMSEA did not consistently decrease with increasing categories, particularly for ML, MLR, and WLS. In contrast, ULS frequently exhibited lower RMSEA values with eight categories than with three categories. Overall, sample size showed a clearer and more consistent association with improved GOF values than the number of response categories.

The comparison between Models A and B depended on the fit index and estimation method. Model A consistently yielded higher NFI values than Model B. For example, under symmetric distributions with three categories and (N = 200), ML yielded NFI values of 0.9558 and 0.9267 for Models A and B, respectively. The corresponding values were 0.9581 and 0.9259 for MLR and 0.9627 and 0.9344 for WLS. The difference between

Table 9. Parameter estimates and MSE for the symmetric distribution in Model B with 8 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model B - 8 category - Symmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6888	0.0035	0.6848	0.0034	0.709	0.0046	0.697	0.0062	0.7004	0.0044
		x2	0.684	0.0033	0.683	0.0037	0.7151	0.0045	0.6986	0.0065	0.6987	0.0042
		x3	0.683	0.0036	0.6843	0.0036	0.714	0.0038	0.7058	0.006	0.6997	0.0042
	f2	x4	0.6833	0.0034	0.6824	0.0033	0.7205	0.0037	0.6999	0.0054	0.7016	0.0032
		x5	0.6858	0.0031	0.6829	0.0032	0.7163	0.0037	0.7015	0.0047	0.7004	0.0032
		x6	0.6813	0.0035	0.6846	0.003	0.7205	0.0041	0.7034	0.0055	0.6996	0.0034
	f3	x7	0.6874	0.0035	0.6844	0.0035	0.7111	0.0039	0.703	0.006	0.6958	0.0038
		x8	0.682	0.0036	0.6794	0.0039	0.7149	0.0048	0.6973	0.0064	0.6994	0.004
		x9	0.6848	0.004	0.6884	0.0037	0.715	0.0036	0.7014	0.0068	0.7025	0.0037
	$\hat{\gamma}_1$		0.5037	0.0066	0.5013	0.0066	0.5108	0.0077	0.5068	0.0088	0.5024	0.0065
	$\hat{\gamma}_2$		0.4943	0.0061	0.498	0.0077	0.5107	0.0076	0.5043	0.0086	0.5053	0.0072
350	f1	x1	0.684	0.0025	0.684	0.0021	0.7082	0.0022	0.6973	0.0032	0.6992	0.0022
		x2	0.6805	0.002	0.6843	0.0021	0.7117	0.0021	0.7011	0.0033	0.699	0.0025
		x3	0.6842	0.0022	0.6845	0.0021	0.7071	0.002	0.7025	0.0031	0.7033	0.0024
	f2	x4	0.6832	0.002	0.6828	0.0021	0.7124	0.0021	0.6988	0.0029	0.6983	0.002
		x5	0.6824	0.0019	0.6871	0.002	0.7116	0.0019	0.703	0.0029	0.6994	0.0018
		x6	0.6835	0.0021	0.6856	0.0019	0.7109	0.002	0.7014	0.0029	0.7007	0.0017
	f3	x7	0.6828	0.0023	0.6844	0.0019	0.7058	0.0022	0.702	0.0034	0.7005	0.0023
		x8	0.683	0.0022	0.6822	0.002	0.7102	0.0023	0.702	0.0034	0.6976	0.0023
		x9	0.6833	0.0022	0.6822	0.002	0.7105	0.0022	0.6967	0.004	0.7025	0.0021
	$\hat{\gamma}_1$		0.5007	0.004	0.4959	0.0034	0.5027	0.0042	0.506	0.0047	0.504	0.0035
	$\hat{\gamma}_2$		0.4984	0.0036	0.5018	0.0038	0.5067	0.0038	0.4997	0.0053	0.5052	0.0038
750	f1	x1	0.6836	0.0012	0.6851	0.0011	0.7052	0.0011	0.6989	0.0017	0.7004	0.001
		x2	0.685	0.001	0.6819	0.0013	0.7044	0.0009	0.6994	0.0016	0.6965	0.0012
		x3	0.6838	0.0011	0.6841	0.0011	0.7031	0.0009	0.6997	0.0015	0.6984	0.001
	f2	x4	0.6843	0.0012	0.6826	0.001	0.705	0.0008	0.6978	0.0013	0.6983	0.0009
		x5	0.6825	0.001	0.6828	0.001	0.7047	0.0008	0.7015	0.0012	0.7006	0.001
		x6	0.6848	0.001	0.6825	0.0011	0.7096	0.0009	0.7025	0.0014	0.6989	0.0009
	f3	x7	0.6856	0.0011	0.6805	0.0013	0.7065	0.0009	0.701	0.0015	0.7018	0.001
		x8	0.6826	0.0011	0.6793	0.0012	0.7039	0.001	0.7022	0.0018	0.6997	0.001
		x9	0.6825	0.0012	0.686	0.0011	0.7077	0.0009	0.7007	0.0014	0.6997	0.0011
	$\hat{\gamma}_1$		0.5026	0.0017	0.4966	0.0016	0.5019	0.0018	0.5041	0.0023	0.5	0.0017
	$\hat{\gamma}_2$		0.5016	0.0015	0.5002	0.0019	0.503	0.0018	0.5047	0.0025	0.498	0.0018

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

the models narrowed at larger sample sizes; for ML under the same distributional and category conditions, it decreased from 0.0291 at (N = 200) to 0.0047 at (N = 1500). However, the other indices did not consistently favor Model A. In the same small-sample condition, ML yielded a higher CFI and lower RMSEA for Model B, whereas WLS yielded a higher CFI and lower RMSEA for Model A. These findings suggest that model complexity had a more noticeable effect on NFI, particularly at smaller sample sizes, whereas its effect on CFI and RMSEA was less consistent.

The effect of distributional shape on GOF values varied across estimators and fit indices. In general, DWLS showed relatively stable GOF values under both symmetric and asymmetric distributions. ML and MLR exhibited

Table 9. Parameter estimates and MSE for the symmetric distribution in Model B with 8 response categories (continued).

			Model B - 8 category - Symmetric									
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
900	f1	x1	0.6843	0.0009	0.6837	0.0011	0.7023	0.0008	0.6999	0.0009	0.7013	0.0009
		x2	0.6841	0.0009	0.6824	0.0011	0.7018	0.0009	0.6999	0.0008	0.6978	0.0008
		x3	0.6845	0.0009	0.6844	0.001	0.7055	0.0008	0.6999	0.0008	0.7006	0.0009
	f2	x4	0.6854	0.0008	0.6844	0.0009	0.7046	0.0007	0.7003	0.0007	0.699	0.0008
		x5	0.6841	0.0009	0.6854	0.0009	0.704	0.0008	0.7018	0.0006	0.7007	0.0007
		x6	0.685	0.0009	0.6838	0.0009	0.705	0.0007	0.7015	0.0007	0.6985	0.0007
	f3	x7	0.6847	0.0009	0.6843	0.0009	0.7057	0.0007	0.6988	0.0007	0.7009	0.0008
		x8	0.6857	0.0009	0.6832	0.001	0.7064	0.0008	0.7011	0.0008	0.7002	0.0009
		x9	0.6847	0.0009	0.6836	0.001	0.7029	0.0008	0.7007	0.0008	0.7002	0.0008
	$\hat{\gamma}_1$		0.5008	0.0012	0.4987	0.0014	0.4989	0.0015	0.501	0.0012	0.4976	0.0015
	$\hat{\gamma}_2$		0.4997	0.0014	0.5014	0.0014	0.5046	0.0014	0.5001	0.0013	0.5025	0.0015
1500	f1	x1	0.6836	0.0006	0.6841	0.0007	0.7026	0.0005	0.6992	0.0005	0.7001	0.0005
		x2	0.6842	0.0007	0.6833	0.0007	0.7028	0.0005	0.7009	0.0005	0.7017	0.0004
		x3	0.6844	0.0007	0.6841	0.0006	0.7014	0.0005	0.6994	0.0004	0.6995	0.0006
	f2	x4	0.6843	0.0006	0.6827	0.0007	0.7004	0.0004	0.7018	0.0004	0.6986	0.0004
		x5	0.6828	0.0007	0.6835	0.0007	0.7024	0.0004	0.7019	0.0004	0.6991	0.0004
		x6	0.6837	0.0006	0.6851	0.0006	0.7041	0.0004	0.699	0.0004	0.7028	0.0004
	f3	x7	0.6831	0.0007	0.684	0.0007	0.7026	0.0005	0.6987	0.0004	0.7009	0.0005
		x8	0.6838	0.0008	0.6834	0.0007	0.7034	0.0004	0.6998	0.0004	0.7001	0.0005
		x9	0.6855	0.0006	0.6852	0.0006	0.7027	0.0005	0.7005	0.0005	0.6995	0.0006
	$\hat{\gamma}_1$		0.5009	0.0009	0.4999	0.0008	0.5014	0.0008	0.5006	0.0009	0.4997	0.0009
	$\hat{\gamma}_2$		0.498	0.0009	0.498	0.0009	0.5009	0.0008	0.5008	0.0009	0.5008	0.0008

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

similar sensitivity to distributional asymmetry, whereas WLS showed more noticeable changes in NFI under certain conditions. For example, in Model B with N = 200 and three response categories, the NFI value for WLS decreased from 0.9344 under the symmetric distribution to 0.9233 under the asymmetric distribution, compared with a smaller decrease from 0.9728 to 0.9705 for DWLS. In contrast, ULS showed a more noticeable change in RMSEA under the same conditions, increasing from 0.0141 to 0.0195, whereas DWLS increased only from 0.0027 to 0.0029. These findings suggest that the sensitivity of GOF values to distributional asymmetry depends on both the estimator and the fit index, with DWLS exhibiting relatively stable performance in the illustrated condition.

Figures 5, 6, and 7 illustrate the patterns of CFI, NFI, and RMSEA across different sample sizes, response categories, distributional shapes, and models. In each figure, the rows correspond to three, five, and eight response categories, while the columns correspond to symmetric and asymmetric distributions. Solid and dashed lines distinguish Models A and B, respectively. The estimators are differentiated by color and marker shape: circles for ML, crosses for MLR, triangles for WLS, squares for DWLS, and vertical bars for ULS.

The Figures show that, increasing sample size generally improved GOF values, with more pronounced improvements for ML, MLR, and WLS, whereas DWLS and ULS maintained favorable values across all conditions. DWLS consistently produced the highest or nearly highest CFI and NFI values and generally the lowest RMSEA values, followed closely by ULS. ML and MLR exhibited very similar performance across all three indices. WLS produced CFI and NFI values comparable to those of ML and MLR, occasionally outperforming them, but generally yielded higher RMSEA values, particularly in Model B. Increasing the number of response categories showed a less consistent effect on GOF values, while differences between symmetric and asymmetric

Table 10. Parameter estimates and MSE for the asymmetric distribution in Model B with 3 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model B - 3 category - Asymmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6193	0.0119	0.6193	0.0123	0.7346	0.0085	0.6957	0.0047	0.703	0.0083
		x2	0.6258	0.0116	0.6178	0.013	0.733	0.0088	0.706	0.0047	0.695	0.0078
		x3	0.624	0.0116	0.6276	0.0106	0.7318	0.009	0.6979	0.0046	0.7007	0.0075
	f2	x4	0.6216	0.011	0.626	0.0099	0.7296	0.0071	0.7015	0.0034	0.7	0.0058
		x5	0.6207	0.0113	0.6271	0.0099	0.7318	0.0072	0.7032	0.0042	0.7013	0.0059
		x6	0.6243	0.0106	0.6178	0.0118	0.7264	0.007	0.6993	0.0034	0.6981	0.0058
	f3	x7	0.6293	0.01	0.6278	0.0104	0.7312	0.0079	0.6981	0.0046	0.6986	0.0072
		x8	0.6249	0.0114	0.6223	0.0117	0.7277	0.0072	0.7075	0.0045	0.6982	0.0077
		x9	0.6231	0.0111	0.6244	0.0116	0.7242	0.0081	0.6986	0.0043	0.6989	0.0068
	$\hat{\gamma}_1$		0.4901	0.0099	0.4898	0.0104	0.5045	0.0125	0.4994	0.007	0.5001	0.0108
	$\hat{\gamma}_2$		0.4819	0.0105	0.4922	0.0103	0.5152	0.0117	0.5032	0.0073	0.5035	0.0102
350	f1	x1	0.6264	0.0085	0.6244	0.0086	0.7147	0.0042	0.7004	0.0028	0.6998	0.0038
		x2	0.622	0.0092	0.6256	0.0083	0.7155	0.004	0.7023	0.003	0.7016	0.0039
		x3	0.6204	0.0093	0.6223	0.0089	0.7135	0.004	0.6966	0.0028	0.7022	0.0044
	f2	x4	0.6281	0.0078	0.6232	0.0083	0.7218	0.0041	0.7029	0.002	0.6986	0.0031
		x5	0.6218	0.0084	0.6233	0.0083	0.7192	0.0035	0.701	0.0025	0.6995	0.0028
		x6	0.6227	0.0083	0.6223	0.0088	0.7159	0.0039	0.6999	0.0023	0.7023	0.0031
	f3	x7	0.6248	0.009	0.6209	0.0092	0.7178	0.0043	0.7007	0.0027	0.6997	0.004
		x8	0.6239	0.0089	0.6211	0.0096	0.7148	0.0043	0.7004	0.0028	0.6988	0.0039
		x9	0.6202	0.0096	0.6256	0.0087	0.7169	0.0041	0.6998	0.003	0.6996	0.0035
	$\hat{\gamma}_1$		0.4862	0.0059	0.4882	0.0052	0.5046	0.0063	0.5081	0.0044	0.503	0.0054
	$\hat{\gamma}_2$		0.488	0.0055	0.4869	0.0061	0.51	0.006	0.501	0.0045	0.5062	0.0058
750	f1	x1	0.6235	0.0073	0.6252	0.007	0.7054	0.0018	0.7007	0.0016	0.7003	0.0018
		x2	0.6216	0.0075	0.6216	0.0076	0.7097	0.0016	0.699	0.0018	0.6988	0.0019
		x3	0.6244	0.007	0.6251	0.0071	0.7114	0.0018	0.7003	0.0017	0.6997	0.0018
	f2	x4	0.6218	0.0074	0.6232	0.0072	0.709	0.0015	0.6984	0.0015	0.6983	0.0015
		x5	0.6255	0.007	0.6227	0.0073	0.7101	0.0015	0.7006	0.0015	0.6996	0.0015
		x6	0.6255	0.007	0.6208	0.0075	0.7094	0.0016	0.6973	0.0014	0.6969	0.0015
	f3	x7	0.6247	0.007	0.6213	0.0076	0.7065	0.0017	0.7026	0.0016	0.702	0.0017
		x8	0.6239	0.0073	0.6235	0.007	0.7078	0.0018	0.7009	0.0016	0.7002	0.0017
		x9	0.6251	0.007	0.6256	0.007	0.7071	0.0018	0.699	0.0016	0.6989	0.0018
	$\hat{\gamma}_1$		0.4898	0.0024	0.488	0.0025	0.5031	0.0023	0.503	0.0026	0.502	0.0026
	$\hat{\gamma}_2$		0.4866	0.0028	0.4881	0.0029	0.5009	0.0029	0.4992	0.0026	0.4983	0.0026

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

distributions were generally small. Model A consistently yielded higher NFI values than Model B, particularly at smaller sample sizes, whereas differences in CFI and RMSEA between the models were less consistent.

Table 10. Parameter estimates and MSE for asymmetric distribution in Model B with 3 response categories (continued).

Model B - 3 category - Asymmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
900	f1	x1	0.623	0.0071	0.6243	0.007	0.706	0.0015	0.6995	0.0015	0.6999	0.0015
		x2	0.6242	0.0069	0.6218	0.0073	0.7065	0.0013	0.6996	0.0015	0.6992	0.0014
		x3	0.6241	0.0068	0.6249	0.0067	0.7068	0.0013	0.6991	0.0013	0.7017	0.0016
	f2	x4	0.6249	0.0068	0.6244	0.0067	0.7064	0.0013	0.7009	0.0013	0.6988	0.0013
		x5	0.6212	0.007	0.6265	0.0064	0.7064	0.0012	0.7024	0.0011	0.7023	0.0013
		x6	0.6231	0.007	0.6245	0.0067	0.7065	0.0012	0.6996	0.001	0.6963	0.0014
	f3	x7	0.6216	0.007	0.6217	0.0073	0.7035	0.0016	0.7	0.0014	0.7006	0.0014
		x8	0.6235	0.007	0.6211	0.007	0.7057	0.0014	0.7021	0.0015	0.6998	0.0015
		x9	0.6221	0.007	0.6231	0.007	0.7059	0.0015	0.6997	0.0014	0.7012	0.0015
	$\hat{\gamma}_1$		0.4892	0.0023	0.4839	0.0023	0.5019	0.002	0.5011	0.0018	0.4998	0.0021
	$\hat{\gamma}_2$		0.4833	0.0023	0.4839	0.0023	0.5032	0.0022	0.5012	0.0019	0.5035	0.0023
1500	f1	x1	0.6237	0.0065	0.6231	0.0066	0.7015	0.0008	0.6997	0.0008	0.6991	0.0009
		x2	0.6218	0.0068	0.6238	0.0066	0.7032	0.0008	0.7007	0.0009	0.7018	0.0009
		x3	0.623	0.0066	0.6234	0.0065	0.7021	0.0007	0.7008	0.0009	0.6987	0.0009
	f2	x4	0.6226	0.0067	0.624	0.0063	0.7042	0.0008	0.7022	0.0007	0.6993	0.0007
		x5	0.6233	0.0065	0.6234	0.0064	0.7033	0.0007	0.7013	0.0007	0.6995	0.0007
		x6	0.6247	0.0063	0.623	0.0065	0.7052	0.0007	0.7002	0.0007	0.7016	0.0007
	f3	x7	0.6241	0.0065	0.6236	0.0066	0.7046	0.0008	0.6994	0.0008	0.7011	0.0009
		x8	0.6242	0.0064	0.6223	0.0068	0.7031	0.0008	0.6997	0.0008	0.7009	0.0009
		x9	0.6259	0.0062	0.6221	0.0068	0.7044	0.0008	0.701	0.0008	0.698	0.001
	$\hat{\gamma}_1$		0.4843	0.0014	0.4853	0.0014	0.499	0.0013	0.4997	0.0013	0.4983	0.0014
	$\hat{\gamma}_2$		0.4866	0.0014	0.4845	0.0015	0.502	0.0012	0.5025	0.0014	0.5024	0.0014

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

Table 11. Parameter estimates and MSE for asymmetric distribution in Model B with 5 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model B - 5 category - Asymmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6553	0.0061	0.6566	0.0066	0.7252	0.0056	0.7012	0.0035	0.7027	0.0059
		x2	0.6494	0.0074	0.6509	0.0069	0.7209	0.0061	0.7075	0.0037	0.6955	0.0059
		x3	0.6551	0.0065	0.6538	0.0066	0.7234	0.007	0.6969	0.0038	0.7009	0.0058
	f2	x4	0.6525	0.0061	0.652	0.0065	0.7262	0.0054	0.7005	0.003	0.7019	0.004
		x5	0.6567	0.0057	0.6567	0.0062	0.7242	0.0055	0.6968	0.003	0.6994	0.0043
		x6	0.655	0.0064	0.6577	0.0053	0.7206	0.0048	0.6943	0.0032	0.7002	0.0047
	f3	x7	0.6523	0.0068	0.6547	0.0067	0.7218	0.0058	0.6958	0.0037	0.6965	0.0049
		x8	0.6472	0.0075	0.6543	0.0067	0.7273	0.0059	0.6982	0.0039	0.697	0.0057
		x9	0.653	0.0066	0.6528	0.0067	0.7228	0.0056	0.7019	0.004	0.7034	0.0051
	$\hat{\gamma}_1$		0.4962	0.0082	0.4896	0.0086	0.5117	0.0093	0.5045	0.0069	0.5028	0.0087
	$\hat{\gamma}_2$		0.4908	0.0081	0.4872	0.0085	0.5082	0.0092	0.5068	0.0068	0.5088	0.0088
350	f1	x1	0.6542	0.0047	0.6533	0.0048	0.7156	0.0032	0.7009	0.0028	0.7001	0.003
		x2	0.6541	0.0048	0.655	0.0042	0.7199	0.0036	0.6997	0.0031	0.699	0.0034
		x3	0.6539	0.0044	0.6561	0.0041	0.7114	0.0032	0.7023	0.003	0.7021	0.0033
	f2	x4	0.6554	0.0041	0.652	0.0046	0.7145	0.0028	0.7006	0.0023	0.6993	0.0025
		x5	0.6563	0.0042	0.652	0.0048	0.7127	0.0026	0.7011	0.0023	0.7007	0.0024
		x6	0.6575	0.0037	0.6552	0.0042	0.7123	0.0026	0.6991	0.0021	0.6982	0.0022
	f3	x7	0.6452	0.0055	0.6555	0.0043	0.7104	0.0029	0.7019	0.0029	0.7012	0.0031
		x8	0.6522	0.0051	0.6525	0.0044	0.7188	0.003	0.6998	0.0029	0.6987	0.0031
		x9	0.652	0.0051	0.6539	0.0045	0.7129	0.0029	0.7011	0.0029	0.701	0.0031
	$\hat{\gamma}_1$		0.4922	0.0044	0.4945	0.0046	0.5063	0.005	0.5081	0.0047	0.5059	0.0046
	$\hat{\gamma}_2$		0.4924	0.0049	0.4876	0.0046	0.5031	0.0048	0.508	0.0046	0.5062	0.0045
750	f1	x1	0.6529	0.0034	0.6529	0.0034	0.7077	0.0013	0.7001	0.0012	0.6997	0.0013
		x2	0.6533	0.0033	0.6522	0.0035	0.7077	0.0013	0.6975	0.0013	0.6975	0.0014
		x3	0.6528	0.0033	0.6529	0.0034	0.7058	0.0013	0.7001	0.0013	0.6996	0.0014
	f2	x4	0.6579	0.003	0.6541	0.0032	0.7061	0.0012	0.699	0.0011	0.6987	0.0011
		x5	0.6557	0.003	0.6544	0.0031	0.7057	0.0012	0.7007	0.0012	0.7003	0.0013
		x6	0.6516	0.0033	0.6525	0.0033	0.709	0.0012	0.6994	0.001	0.699	0.0011
	f3	x7	0.6529	0.0033	0.6542	0.0033	0.7083	0.0013	0.7013	0.0013	0.7011	0.0014
		x8	0.6525	0.0035	0.6564	0.003	0.7045	0.0012	0.6999	0.0012	0.6992	0.0013
		x9	0.6546	0.0032	0.654	0.0034	0.7054	0.0012	0.7003	0.0013	0.7	0.0014
	$\hat{\gamma}_1$		0.4905	0.0022	0.4913	0.002	0.5039	0.0021	0.5003	0.0022	0.4995	0.0022
	$\hat{\gamma}_2$		0.4913	0.0021	0.4889	0.0022	0.5046	0.0022	0.5012	0.0023	0.5006	0.0022

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

Table 11. Parameter estimates and MSE for asymmetric distribution in Model B with 5 response categories (continued).

			Model B - 5 category - Asymmetric									
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
900	f1	x1	0.6526	0.0033	0.6539	0.0032	0.7066	0.0011	0.6991	0.0012	0.7018	0.0012
		x2	0.6519	0.0033	0.6534	0.0032	0.7049	0.0011	0.7001	0.0012	0.6984	0.0011
		x3	0.6526	0.0033	0.6516	0.0033	0.7047	0.0009	0.6994	0.001	0.7005	0.0011
	f2	x4	0.6565	0.003	0.6525	0.003	0.7049	0.001	0.7003	0.0009	0.6991	0.001
		x5	0.6532	0.003	0.6542	0.003	0.707	0.0009	0.7036	0.0008	0.7018	0.0009
		x6	0.6521	0.0031	0.6512	0.0033	0.7057	0.001	0.6997	0.0009	0.6977	0.0009
	f3	x7	0.653	0.0032	0.6549	0.003	0.7025	0.001	0.6982	0.0011	0.7021	0.0011
		x8	0.6509	0.0034	0.6562	0.0028	0.7059	0.0011	0.7004	0.0012	0.7	0.0011
		x9	0.6525	0.0032	0.6528	0.0031	0.7045	0.001	0.7005	0.0012	0.7013	0.0011
	$\hat{\gamma}_1$		0.4902	0.0017	0.4922	0.0019	0.5028	0.0017	0.5008	0.0015	0.4995	0.0017
	$\hat{\gamma}_2$		0.4921	0.0017	0.4939	0.0019	0.5011	0.0017	0.5022	0.0016	0.5022	0.0018
1500	f1	x1	0.6525	0.0028	0.6535	0.0028	0.7041	0.0006	0.6993	0.0007	0.6998	0.0007
		x2	0.6539	0.0026	0.6528	0.0028	0.7036	0.0006	0.7016	0.0007	0.702	0.0006
		x3	0.6538	0.0027	0.6542	0.0026	0.7045	0.0006	0.6996	0.0006	0.7004	0.0007
	f2	x4	0.6531	0.0027	0.6531	0.0027	0.7041	0.0005	0.7021	0.0005	0.6989	0.0006
		x5	0.6528	0.0027	0.6509	0.0029	0.7019	0.0006	0.7019	0.0005	0.6996	0.0006
		x6	0.6528	0.0027	0.6528	0.0028	0.7027	0.0006	0.6975	0.0005	0.7022	0.0006
	f3	x7	0.6528	0.0028	0.6511	0.003	0.701	0.0007	0.6992	0.0006	0.7011	0.0008
		x8	0.653	0.0028	0.6522	0.0028	0.7017	0.0006	0.6995	0.0006	0.7013	0.0007
		x9	0.6526	0.0029	0.654	0.0027	0.7034	0.0006	0.7004	0.0006	0.699	0.0009
	$\hat{\gamma}_1$		0.4888	0.0012	0.4924	0.001	0.5044	0.0011	0.501	0.0011	0.4986	0.0011
	$\hat{\gamma}_2$		0.4918	0.0011	0.4904	0.001	0.4992	0.001	0.5015	0.0011	0.5009	0.001

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

Table 12. Parameter estimates and MSE for asymmetric distribution in Model B with 8 response categories (set $\lambda = 0.70$ and $\gamma = 0.50$).

Model B - 8 category - Asymmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
200	f1	x1	0.6822	0.0036	0.6883	0.0035	0.7168	0.004	0.7019	0.0039	0.7008	0.0042
		x2	0.6804	0.0044	0.6819	0.0035	0.7198	0.0042	0.6978	0.0038	0.6975	0.0043
		x3	0.678	0.004	0.68	0.0036	0.7144	0.0042	0.7001	0.0039	0.699	0.0041
	f2	x4	0.6811	0.0036	0.6845	0.0031	0.7192	0.004	0.7021	0.0028	0.7014	0.003
		x5	0.6771	0.0041	0.6808	0.0032	0.7086	0.0038	0.7005	0.003	0.6997	0.0031
		x6	0.6794	0.0034	0.683	0.0034	0.7115	0.0039	0.6988	0.0033	0.6976	0.0035
	f3	x7	0.6798	0.0037	0.6841	0.0039	0.7116	0.0044	0.6978	0.0038	0.6968	0.004
		x8	0.6821	0.0034	0.6845	0.0032	0.7132	0.0041	0.6999	0.0038	0.6989	0.0041
		x9	0.6873	0.0034	0.6826	0.0036	0.7163	0.0048	0.7038	0.0037	0.7031	0.004
	$\hat{\gamma}_1$		0.4974	0.0061	0.4977	0.0069	0.5065	0.0082	0.5082	0.0065	0.5054	0.0063
	$\hat{\gamma}_2$		0.5006	0.0064	0.4946	0.0073	0.5116	0.0084	0.5074	0.0071	0.5047	0.007
350	f1	x1	0.6837	0.0023	0.6828	0.0022	0.7067	0.0021	0.6986	0.0021	0.6987	0.002
		x2	0.6847	0.0021	0.6839	0.0023	0.7096	0.0023	0.7028	0.0021	0.701	0.0025
		x3	0.6812	0.0022	0.6826	0.0022	0.7093	0.0023	0.7013	0.0023	0.7036	0.0025
	f2	x4	0.6822	0.002	0.6824	0.002	0.7139	0.0023	0.7	0.0019	0.6994	0.0021
		x5	0.6795	0.0021	0.6844	0.002	0.7149	0.0021	0.6972	0.0018	0.6998	0.0019
		x6	0.6827	0.0019	0.685	0.002	0.7108	0.002	0.6983	0.0016	0.7005	0.0017
	f3	x7	0.6816	0.0023	0.6826	0.0022	0.7105	0.0023	0.7026	0.0019	0.7006	0.0024
		x8	0.6784	0.0024	0.6791	0.0025	0.7019	0.0025	0.6975	0.0023	0.6962	0.0022
		x9	0.6809	0.0023	0.6842	0.002	0.7096	0.0021	0.7007	0.0023	0.7041	0.0023
	$\hat{\gamma}_1$		0.4941	0.0039	0.5025	0.0037	0.5061	0.004	0.5001	0.0042	0.5034	0.0036
	$\hat{\gamma}_2$		0.4996	0.004	0.5008	0.0036	0.5019	0.004	0.506	0.0036	0.5058	0.0038
750	f1	x1	0.681	0.0012	0.6812	0.0013	0.7039	0.001	0.7013	0.0009	0.701	0.001
		x2	0.6809	0.0013	0.6815	0.0012	0.7062	0.001	0.6979	0.0011	0.6978	0.0012
		x3	0.6808	0.0012	0.6828	0.0012	0.7054	0.0009	0.6995	0.0009	0.6991	0.001
	f2	x4	0.6828	0.001	0.6814	0.0011	0.7066	0.0008	0.6995	0.0009	0.6993	0.0009
		x5	0.6824	0.0011	0.6812	0.0012	0.7073	0.001	0.701	0.0009	0.7006	0.001
		x6	0.6808	0.0012	0.6787	0.0012	0.7045	0.0009	0.6979	0.0008	0.6977	0.0009
	f3	x7	0.6816	0.0013	0.6821	0.0012	0.7044	0.001	0.702	0.001	0.7017	0.0011
		x8	0.682	0.0012	0.6831	0.0011	0.7027	0.0011	0.7016	0.0009	0.7012	0.001
		x9	0.6818	0.0013	0.6839	0.0012	0.7045	0.0009	0.6994	0.001	0.6993	0.0011
	$\hat{\gamma}_1$		0.4974	0.0017	0.5004	0.0018	0.5002	0.0016	0.5005	0.0018	0.4997	0.0018
	$\hat{\gamma}_2$		0.5007	0.0018	0.5016	0.0017	0.5033	0.002	0.5005	0.0018	0.4999	0.0018

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

Table 12. Parameter estimates and MSE for asymmetric distribution in Model B with 8 response categories (continued).

Model B - 8 category - Asymmetric												
			ML		MLR		WLS		DWLS		ULS	
N	L.V	O.V	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE	$\hat{\lambda}$	MSE
900	f1	x1	0.6832	0.001	0.6821	0.0011	0.7044	0.0008	0.6994	0.0009	0.7017	0.0009
		x2	0.6807	0.0011	0.6805	0.0011	0.7024	0.0008	0.7005	0.0008	0.6979	0.0008
		x3	0.6809	0.0011	0.6822	0.001	0.7025	0.0008	0.6993	0.0008	0.7011	0.0009
	f2	x4	0.6854	0.0009	0.6813	0.001	0.7034	0.0007	0.7	0.0007	0.6988	0.0008
		x5	0.6807	0.001	0.6802	0.0011	0.7055	0.0007	0.7017	0.0006	0.7016	0.0008
		x6	0.6818	0.001	0.6813	0.001	0.7029	0.0008	0.7014	0.0007	0.6973	0.0007
	f3	x7	0.6807	0.0012	0.6815	0.0011	0.7032	0.0009	0.6991	0.0008	0.7005	0.0008
		x8	0.6807	0.0011	0.682	0.0011	0.7029	0.0008	0.7	0.0008	0.7004	0.0009
		x9	0.6797	0.0012	0.6833	0.0011	0.7023	0.0009	0.7001	0.0009	0.7002	0.0007
	$\hat{\gamma}_1$		0.4967	0.0014	0.4995	0.0013	0.4987	0.0015	0.5005	0.0012	0.499	0.0014
	$\hat{\gamma}_2$		0.4977	0.0014	0.4992	0.0015	0.5037	0.0016	0.4993	0.0013	0.5017	0.0015
1500	f1	x1	0.6808	0.0008	0.6816	0.0007	0.7032	0.0005	0.699	0.0005	0.6995	0.0005
		x2	0.6809	0.0008	0.6827	0.0007	0.7031	0.0005	0.7009	0.0005	0.7022	0.0005
		x3	0.6828	0.0007	0.6809	0.0008	0.7045	0.0005	0.6999	0.0005	0.7001	0.0006
	f2	x4	0.6811	0.0008	0.681	0.0008	0.7018	0.0004	0.7016	0.0004	0.699	0.0004
		x5	0.6821	0.0007	0.6825	0.0007	0.7035	0.0005	0.7016	0.0004	0.6996	0.0004
		x6	0.6805	0.0008	0.6807	0.0008	0.7018	0.0005	0.6992	0.0004	0.7021	0.0004
	f3	x7	0.6821	0.0007	0.6812	0.0008	0.703	0.0005	0.6986	0.0005	0.7016	0.0006
		x8	0.6812	0.0008	0.6806	0.0008	0.702	0.0004	0.701	0.0004	0.7014	0.0005
		x9	0.6827	0.0007	0.6823	0.0007	0.7027	0.0005	0.7	0.0005	0.6982	0.0006
	$\hat{\gamma}_1$		0.4985	0.0009	0.4982	0.0008	0.5015	0.0009	0.4998	0.0009	0.5	0.0009
	$\hat{\gamma}_2$		0.4989	0.0008	0.4987	0.0008	0.5029	0.0009	0.5013	0.0009	0.5013	0.0009

Note. N = sample size; L.V. = latent variable; O.V. = observed variable; MSE = mean squared error; $\hat{\lambda}$ = loading estimate; $\hat{\gamma}_1$ = structural path between latent variables ($f1 \Rightarrow f2$); $\hat{\gamma}_2$ = structural path between latent variables ($f3 \Rightarrow f2$).

Table 13. CFI, NFI, and RMSEA values for Model A under symmetric distribution.

Model A - Symmetric																
		ML			MLR			WLS			DWLS			ULS		
N	Category	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA
200	3	0.9884	0.9558	0.0237	0.9905	0.9581	0.0208	0.9904	0.9627	0.0228	0.9991	0.9861	0.0049	0.9973	0.983	0.0131
	5	0.993	0.9673	0.0201	0.9936	0.9685	0.0186	0.9936	0.9718	0.0226	0.9993	0.9902	0.0048	0.9993	0.9878	0.0041
	8	0.994	0.9716	0.019	0.9948	0.9716	0.018	0.9941	0.9751	0.0221	0.9995	0.9923	0.0044	0.9997	0.9898	0.0019
350	3	0.9941	0.9745	0.0162	0.9948	0.9756	0.0147	0.9942	0.9758	0.0167	0.9995	0.9922	0.0033	0.9987	0.9905	0.0082
	5	0.996	0.9812	0.0148	0.996	0.9811	0.0145	0.996	0.9819	0.0159	0.9998	0.9947	0.0025	0.9998	0.9934	0.0019
	8	0.9969	0.9838	0.0131	0.9967	0.9835	0.0136	0.9965	0.9843	0.0159	0.9998	0.9955	0.0025	0.9999	0.9944	0.0008
750	3	0.9972	0.988	0.0113	0.9975	0.9886	0.0104	0.9974	0.9879	0.0107	0.9998	0.9962	0.0021	0.9994	0.9955	0.0062
	5	0.9984	0.9914	0.0094	0.9982	0.9911	0.0103	0.9981	0.9908	0.0104	0.9999	0.9975	0.0021	0.9998	0.9969	0.0018
	8	0.9984	0.9922	0.0094	0.9985	0.9922	0.0095	0.9984	0.9921	0.0097	0.9999	0.9978	0.0017	0.9999	0.9973	0.0008
900	3	0.9975	0.9897	0.0107	0.9979	0.9905	0.0094	0.9978	0.9897	0.0098	0.9998	0.9967	0.0022	0.9995	0.9963	0.005
	5	0.9984	0.9925	0.0094	0.9986	0.9928	0.0088	0.9984	0.9923	0.0093	0.9999	0.9978	0.0016	0.9999	0.9974	0.0012
	8	0.9987	0.9935	0.009	0.9986	0.9934	0.0086	0.9986	0.993	0.0092	0.9999	0.9982	0.0015	1	0.9978	0.0007
1500	3	0.9987	0.9938	0.0079	0.9989	0.9944	0.0068	0.9987	0.9938	0.0073	0.9999	0.9981	0.0015	0.9997	0.9978	0.004
	5	0.999	0.9954	0.0077	0.9992	0.9956	0.0065	0.999	0.9952	0.007	1	0.9988	0.0009	1	0.9986	0.0007
	8	0.9993	0.9962	0.0062	0.9993	0.9962	0.0064	0.9992	0.9958	0.0066	1	0.999	0.0009	1	0.9987	0.0003

Note. N = sample size.

Table 14. CFI, NFI, and RMSEA values for Model A under asymmetric distribution.

Model A - Asymmetric																
		ML			MLR			WLS			DWLS			ULS		
N	Category	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA
200	3	0.9894	0.9568	0.0227	0.9889	0.9559	0.0231	0.9904	0.9599	0.0227	0.9988	0.9851	0.0054	0.9963	0.9814	0.0156
	5	0.991	0.9633	0.0224	0.9915	0.9634	0.0224	0.9929	0.9688	0.0222	0.9991	0.989	0.0051	0.9986	0.986	0.0069
	8	0.994	0.9704	0.0187	0.9933	0.9701	0.0212	0.9943	0.9747	0.0218	0.9994	0.9917	0.0049	0.9996	0.9897	0.0021
350	3	0.994	0.9747	0.0166	0.9946	0.975	0.0158	0.9947	0.9747	0.0162	0.9994	0.9914	0.0036	0.9982	0.9895	0.0107
	5	0.9953	0.9793	0.0164	0.9957	0.9797	0.0146	0.9955	0.9801	0.0162	0.9997	0.9937	0.0028	0.9995	0.9922	0.0036
	8	0.9962	0.9825	0.0157	0.9969	0.9837	0.0133	0.9964	0.9836	0.0155	0.9998	0.9954	0.0025	0.9999	0.9944	0.0009
750	3	0.9974	0.988	0.0108	0.9968	0.9874	0.0127	0.9973	0.987	0.0101	0.9998	0.996	0.002	0.9992	0.9952	0.0072
	5	0.998	0.9903	0.0104	0.9979	0.9902	0.0108	0.9979	0.9898	0.0101	0.9998	0.997	0.0019	0.9997	0.9964	0.0029
	8	0.9983	0.9919	0.0104	0.9982	0.9917	0.0107	0.9983	0.9917	0.0098	0.9999	0.9978	0.0017	0.9999	0.9973	0.0008
900	3	0.9976	0.9898	0.0104	0.9977	0.9898	0.0106	0.9978	0.9892	0.0091	0.9998	0.9966	0.0022	0.9993	0.9959	0.0069
	5	0.9983	0.9919	0.0094	0.9981	0.9916	0.0102	0.9982	0.9913	0.0089	0.9999	0.9974	0.0018	0.9998	0.9969	0.0027
	8	0.9986	0.9933	0.0091	0.9986	0.9932	0.0091	0.9986	0.993	0.0088	0.9999	0.9981	0.0017	1	0.9978	0.0005
1500	3	0.9984	0.9937	0.0086	0.9987	0.994	0.0076	0.9985	0.9931	0.0076	0.9999	0.998	0.0014	0.9996	0.9976	0.005
	5	0.9989	0.9949	0.0078	0.9988	0.9949	0.0076	0.9988	0.9945	0.0072	0.9999	0.9985	0.0014	0.9999	0.9982	0.0019
	8	0.9992	0.9959	0.0071	0.9992	0.996	0.0068	0.9992	0.9957	0.0066	1	0.9989	0.0011	1	0.9987	0.0005

Note. N = sample size.

Table 15. CFI, NFI, and RMSEA values for Model B under symmetric distribution.

Model B - Symmetric																
		ML			MLR			WLS			DWLS			ULS		
N	Category	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA
200	3	0.9904	**0.9267**	0.0158	0.9896	**0.9259**	0.0166	0.9836	**0.9344**	0.0302	0.9992	0.9728	0.0027	0.9956	0.9656	0.0141
	5	0.9922	**0.9413**	0.0158	0.9919	**0.9407**	0.0162	0.9873	**0.9481**	0.0302	0.9995	0.9812	0.0025	0.9995	0.9759	0.002
	8	0.9933	**0.9465**	0.0155	0.9934	**0.9464**	0.0156	0.9881	0.9535	0.0301	0.9996	0.984	0.0023	0.9998	0.9791	0.0007
350	3	0.9946	0.9561	0.0116	0.9937	0.9552	0.0126	0.9916	0.9562	0.0178	0.9995	0.9838	0.0023	0.9974	0.9801	0.0108
	5	0.9959	0.9655	0.0115	0.9956	0.9651	0.0119	0.9934	0.9651	0.0178	0.9998	0.9893	0.0015	0.9996	0.9857	0.0023
	8	0.9962	0.9689	0.0109	0.9963	0.9689	0.0117	0.9944	0.9697	0.0176	0.9998	0.9907	0.0015	0.9999	0.9878	0.0007
750	3	0.9976	0.9791	0.0078	0.9973	0.9787	0.0082	0.9965	0.9766	0.0097	0.9999	0.9924	0.001	0.9991	0.9907	0.0064
	5	0.998	0.9836	0.0078	0.9981	0.9836	0.0074	0.9973	0.9818	0.0097	0.9999	0.9949	0.0009	0.9998	0.9934	0.0011
	8	0.9983	0.9851	0.0078	0.9983	0.9851	0.0074	0.9977	0.984	0.0093	0.9999	0.9957	0.0007	1	0.9944	0.0004
900	3	0.9977	0.9822	0.0077	0.9977	0.982	0.0077	0.9971	0.9983	0.0086	0.9999	0.9939	0.001	0.9991	0.992	0.0067
	5	0.9982	0.9859	0.0077	0.9983	0.986	0.0076	0.9977	0.9989	0.0086	0.9999	0.9958	0.0007	0.9999	0.9943	0.0013
	8	0.9985	0.9874	0.0074	0.9987	0.9877	0.0066	0.9981	0.9992	0.0082	0.9999	0.9964	0.0007	1	0.9952	0.0003
1500	3	0.9986	0.9891	0.0061	0.9986	0.9892	0.0059	0.9984	0.9877	0.0061	0.9999	0.9962	0.0008	0.9995	0.9953	0.005
	5	0.999	0.9916	0.0058	0.9989	0.9915	0.0058	0.9988	0.9905	0.0058	1	0.9975	0.0004	0.9999	0.9967	0.001
	8	0.9991	0.9923	0.0058	0.9991	0.9924	0.0055	0.9989	0.9915	0.0058	1	0.9979	0.0003	1	0.9972	0.0003

Note. N = sample size; **Bold values** = Acceptable fit.

Table 16. CFI, NFI, and RMSEA values for Model B under asymmetric distribution.

Model B - Asymmetric																
		ML			MLR			WLS			DWLS			ULS		
N	Category	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA	CFI	NFI	RMSEA
200	3	0.9868	**0.9209**	0.0197	0.9866	**0.9212**	0.0198	0.983	**0.9323**	0.0304	0.9992	0.9705	0.0029	0.9935	0.962	0.0195
	5	0.9899	**0.9343**	0.0181	0.9897	**0.9341**	0.0186	0.9862	**0.9435**	0.0303	0.9994	0.9781	0.0028	0.9983	0.9709	0.0064
	8	0.9926	**0.9452**	0.0168	0.9926	**0.9454**	0.0166	0.9881	0.9519	0.0301	0.9996	0.9835	0.0027	0.9998	0.9782	0.0009
350	3	0.9922	0.9525	0.015	0.9926	0.9527	0.0146	0.991	0.9529	0.018	0.9996	0.9834	0.0016	0.9965	0.9784	0.0143
	5	0.9942	0.9608	0.0137	0.994	0.9607	0.0144	0.9926	0.9616	0.0178	0.9998	0.9875	0.0016	0.999	0.9833	0.005
	8	0.9963	0.9687	0.011	0.9962	0.9687	0.011	0.994	0.9681	0.0176	0.9998	0.9904	0.0016	0.9998	0.9875	0.0009
750	3	0.9967	0.9777	0.0094	0.9965	0.9774	0.01	0.9958	0.9743	0.0105	0.9999	0.992	0.001	0.9985	0.9898	0.0089
	5	0.9977	0.9817	0.0085	0.9975	0.9815	0.009	0.9968	0.9794	0.0101	0.9999	0.994	0.0009	0.9996	0.9922	0.0029
	8	0.9982	0.9848	0.0081	0.9984	0.9851	0.007	0.9976	0.9834	0.0096	0.9999	0.9956	0.0007	0.9999	0.9942	0.0005
900	3	0.9975	0.9814	0.008	0.9971	0.981	0.0089	0.9967	0.9783	0.009	0.9999	0.9933	0.0007	0.9986	0.9913	0.009
	5	0.9979	0.9844	0.008	0.9976	0.9841	0.0087	0.9974	0.9825	0.0088	0.9999	0.9951	0.0007	0.9996	0.9934	0.0031
	8	0.9985	0.9872	0.0071	0.9982	0.9869	0.0083	0.9979	0.9858	0.0085	1	0.9963	0.0006	0.9999	0.9951	0.0005
1500	3	0.9984	0.9885	0.0065	0.9984	0.9886	0.0064	0.9981	0.9864	0.0064	1	0.996	0.0005	0.9992	0.9948	0.0068
	5	0.9987	0.9907	0.0063	0.9988	0.9906	0.0063	0.9985	0.989	0.0064	1	0.997	0.0004	0.9998	0.9961	0.002
	8	0.9991	0.9923	0.0056	0.9991	0.9923	0.0057	0.9988	0.9912	0.0063	1	0.9978	0.0003	1	0.9971	0.0004

Note. N = sample size; **Bold values** = Acceptable fit.

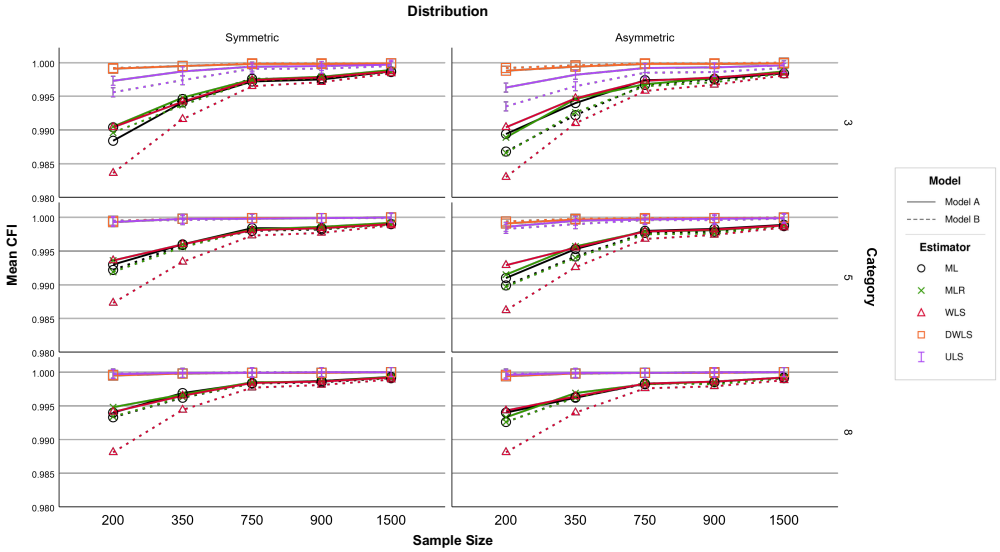


Figure 5. Mean CFI values.

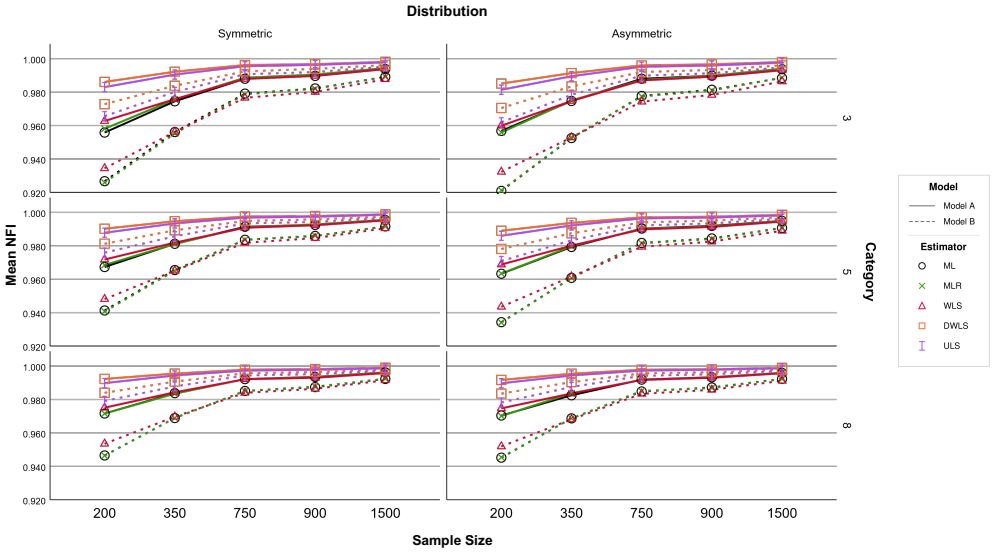


Figure 6. Mean NFI values.

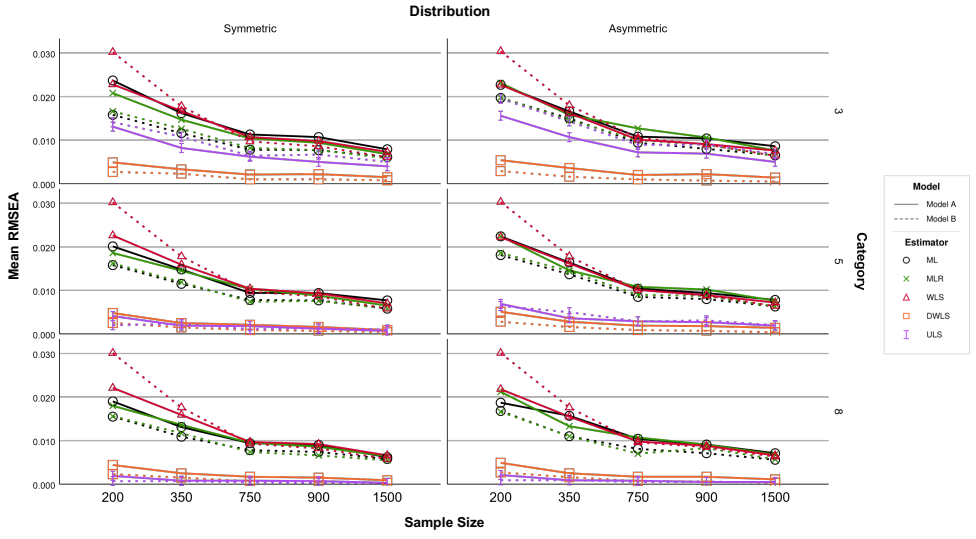


Figure 7. Mean RMSEA values.

8. Conclusions

ML estimation is one of the most commonly recommended methods in SEM due to its desirable statistical properties and broad applicability. However, ML performance may be influenced by several data characteristics, particularly sample size, response category conditions, and distributional assumptions. In applied research, researchers frequently encounter ordinal and non-normal data matrices that threaten the accuracy of parameter estimation and model fit evaluation [10]. Therefore, evaluating the performance of alternative estimation methods under such conditions remains an important issue in SEM research. Consequently, the present study executed a comprehensive Monte Carlo simulation that evaluated the performance of ML, MLR, WLS, DWLS, and ULS estimators under varying sample sizes, response category conditions, and distributional shapes within two distinct structural model configurations. The evaluation criteria were based on goodness-of-fit indices (CFI, NFI, and RMSEA), along with parameter estimation accuracy as assessed by MSE.

Overall, the findings of the present study are consistent with previous SEM literature suggesting that estimator performance depends on multiple interacting factors, including sample size, model complexity, response category conditions, and distributional characteristics [17, 12]. Among these factors, sample size had the greatest impact on estimation performance across both model structures and distributional conditions [21]. As the sample size increased, goodness-of-fit indices improved consistently and MSE values decreased, reflecting greater estimation stability and more accurate parameter recovery. Increasing the number of response categories also contributed to improved performance, particularly under smaller sample sizes and asymmetric distribution. However, the extent of this improvement differed across estimation methods.

The results revealed that estimator performance varied across the examined conditions. In terms of GOF, DWLS generally demonstrated the most favorable results, consistently producing CFI and NFI values close to 1 and the lowest RMSEA values in most conditions. ULS also showed competitive performance, particularly with larger numbers of response categories. ML and MLR exhibited very similar GOF values, whereas WLS tended to produce relatively higher RMSEA values, especially in the more complex model. Nevertheless, all five estimators generally achieved satisfactory model fit according to the adopted criteria.

Regarding parameter estimation accuracy, DWLS, ULS, and WLS generally produced lower MSE values than ML and MLR, indicating more accurate parameter estimates under the examined conditions. Although the MSE values of ML and MLR decreased as the number of response categories increased, they were more sensitive to distributional asymmetry, particularly when fewer than eight categories were used. In contrast, DWLS, ULS, and WLS maintained relatively low MSE values under both symmetric and asymmetric distributions. Furthermore, MSE values remained relatively similar between the two models, suggesting that model complexity had a limited influence on parameter estimation accuracy within the examined conditions.

Increasing sample size generally improved GOF values and reduced estimation errors across the five estimators, while increasing the number of response categories also tended to improve performance, although its effect on GOF was less consistent. Distributional asymmetry had a relatively limited and inconsistent influence on GOF values but was associated with higher estimation errors for ML and MLR. Model complexity affected NFI more noticeably than CFI and RMSEA, particularly at smaller sample sizes. These findings highlight the importance of considering multiple performance measures when evaluating estimation methods for ordinal data.

Overall, DWLS and ULS appear to be particularly suitable choices for estimating structural equation models with ordinal data, as they generally combined favorable GOF values with high parameter estimation accuracy across the examined conditions. WLS also demonstrated relatively accurate parameter estimation despite its less favorable RMSEA values, while ML and MLR showed greater sensitivity to distributional asymmetry in terms of MSE. These findings provide practical guidance for selecting estimation methods based on sample size, number of response categories, distributional characteristics, and model complexity. However, the conclusions should be interpreted within the investigated simulation settings, and further studies incorporating additional model structures, distributional conditions, and parameter configurations are recommended.

Limitations and Future Research

Despite the contributions of this study, several limitations should be acknowledged. First, the simulation design considered symmetric and asymmetric distributions; however, other forms of non-normality, including different degrees and directions of skewness and kurtosis, were not examined. Second, factor loadings and structural path coefficients were fixed at specific values, which may limit the generalizability of the findings to models with different parameter magnitudes. Third, Furthermore, Type I error rates and confidence interval coverage were not assessed. These aspects were beyond the scope of the present study, which primarily focused on comparing estimator performance in terms of estimation accuracy, goodness-of-fit indices, standard errors, and convergence rates. Future research could extend the simulation design to include a wider range of distributional characteristics, parameter values, and inferential performance measures to provide a more comprehensive evaluation of SEM estimators.

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