STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **x**, Month 202x, pp 0–13.
Published online in International Academic Press (www.IAPress.org)

On Some Properties of SNM Semi-Open Sets in Supra Neutrosophic Multiset Topological Spaces

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Abstract This paper introduces the concept of supra neutrosophic multiset semi-open sets in supra neutrosophic multiset topological spaces. The proposed concept facilitates the study of uncertainty, indeterminacy, and repeated membership values within a unified topological structure. Fundamental concepts such as supra neutrosophic multiset interior and supra neutrosophic multiset closure are discussed. Also, we investigate the properties of supra neutrosophic multiset α-open, supra neutrosophic multiset regular-open, supra neutrosophic multiset preopen, and supra neutrosophic multiset β-open. Various characterizations and fundamental properties of supra neutrosophic open multiset are obtained and illustrated through examples.

Keywords Supra neutrosophic multiset topology, Supra neutrosophic multiset semi-open sets, Supra neutrosophic multiset interior, Supra neutrosophic multiset closure.

AMS 2010 subject classifications 00A05, 03E70, 54B10, 54G20

DOI: 10.19139/soic-2310-5070-4591

1. Introduction

In recent years, multisets and neutrosophic sets have attracted significant attention among researchers. Mathematicians often seek to simplify complex problems and identify feasible solutions. In 2016, Serkan Karatas and Cemil Kuru [14] established neutrosophic topology and investigated related properties such as neutrosophic closure, interior, exterior, boundary, and subspace. In 2021 Neutrosophic multi topological group was introduced by Basumatary et.al [2]. Earlier, R. Devi et al. [5] (2008) introduced the concept of supra α-open sets, while A.S. Mashhour et al. (1983) studied supra semi-continuous functions and their fundamental properties. These ideas were later generalised to neutrosophic settings, where Dhavaseelan, Ganster, Jafari, and Parimala [1] introduced neutrosophic α-supra open sets and neutrosophic α-supra continuous functions. subsequently Fathi [7] and S.A. El-Sheikh [11] contributed to the developement of the supra topological spaces, while M.E. El-Shafei [15] further investigated preopen sets in these spaces. A.A. Salama and S.A. Alblowi [13] proposed neutrosophic crisp sets and neutrosophic topological spaces. Later, G. Jayaparthasarathy et al. [8] defined neutrosophic supra topological spaces and applied single-valued neutrosophic score functions to medical diagnosis problems. Combining the concept of supra topology and neutrosophic multiset topology to construct a new topology named supra neutrosophic multiset topology was introduced by Priyalatha et al. [17]

In this paper we introduce the concept of supra neutrosophic multiset semi-open ($\mathcal{SNMSO}$) in ($\mathcal{SNMTS}$) . Fundamental definitions concerning supra neutrosophic multiset interior and closure operators are presented, and

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ISSN 2310-5070 (online) ISSN 2311-004X (print)

several characterizations and properties of supra neutrosophic multiset semi-open sets are obtained. Furthermore, the relationships between $\mathcal{SNM}$ semi-open and other sets such as $\mathcal{SNM}$ preopen, $\mathcal{SNM}$ α-open, $\mathcal{SNM}$ β-open, and $\mathcal{SNM}$ regular open in $\mathcal{SNMTS}$ are derived and the properties of unions, intersections, of $\mathcal{SNM}$ semi-open sets are investigated and illustrated with examples. A diagram connecting all implications proved in Section 3 is given in Figure 1.

1.1. Neutrosophic Multiset Topology and Supra Neutrosophic Multiset Topological Spaces

Several distinct research directions have combined multiset theory with neutrosophic set theory, and it is useful to distinguish their scope explicitly. Smarandache et al. [16] worked at the algebraic level, studying the algebraic structures of neutrosophic triplets, neutrosophic duplets, and neutrosophic multisets without imposing any topological structure. Rakal Das et al. [12] introduced neutrosophic multiset topological spaces (NMTS), in which the usual finite-intersection and arbitrary-union axioms are imposed directly on neutrosophic multisets. Basumatary *et al.* [2] extended this framework by equipping a neutrosophic multi topology with a compatible group structure, thereby introducing neutrosophic multi topological groups. Building on the topological framework of Das *et al.* [12], Priyalatha *et al.*[17] combined the supra topology with neutrosophic multiset topology to introduce supra neutrosophic multiset topological spaces (SNMTS), in which the finite-intersection axiom is replaced by the requirement of closure under arbitrary unions, thereby providing a more flexible framework for modeling vagueness, indeterminacy, and repeated observations.

Table 1. Evolution of neutrosophic multiset-related structures.

Model	**Basic form**	**Short description**
Multiset (MS) [4]	$M = \{(x, m(x)) : x \in X\}$, where $m(x) \in \mathbb{N}$.	Extends the classical set by allowing an element to occur multiple times through its multiplicity function.
Neutrosophic set (NS) [13]	$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$.	Assigns to each element three independent membership degrees representing truth, indeterminacy, and falsity.
Neutrosophic multiset (NM) [16]	$A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$, where an element x may occur repeatedly.	Combines the concepts of multisets and neutrosophic sets by allowing repeated occurrences of an element, each with its own truth, indeterminacy, and falsity degrees.
Neutrosophic multiset topological space (NMTS) [12]	$(W_X, \mathcal{T})$, where $W_X \neq \emptyset$ and $\mathcal{T}$ is closed under finite intersections and arbitrary unions.	Introduces a topology on neutrosophic multisets by requiring closure under finite intersections and arbitrary unions.
Supra neutrosophic multiset topological space (SNMTS) [17]	(X, τ_μ^*), where $\emptyset_{NM}, 1_{NM} \in \tau_\mu^*$ and τ_μ^* is closed under arbitrary unions.	In NMTS by replacing the finite-intersection axiom with closure under arbitrary unions only.

1.2. Our Contribution

This paper introduces the concept of supra neutrosophic multiset semi-open ($\mathcal{SNMSO}$) in supra neutrosophic multiset topological spaces ($\mathcal{SNMTS}$), together with their complementary class of supra neutrosophic multiset semi-closed ($\mathcal{SNMSC}$). Using the $\mathcal{SNM}$ interior as $\mathrm{SInt}_{\mathcal{NM}}$ and the $\mathcal{SNM}$ closure as $\mathrm{SCl}_{\mathcal{NM}}$, we establish two equivalent Several characterizations of $\mathcal{SNMSO}$ are presented. It is proved that arbitrary unions of $\mathcal{SNMSO}$ sets remain $\mathcal{SNMSO}$, while arbitrary intersections need not possess this property. Furthermore, we show that if $A \subseteq B \subseteq \mathrm{SCl}_{\mathcal{NM}}(A)$, then B is a $\mathcal{SNMSO}$ whenever A is a $\mathcal{SNMSO}$. We also establish that every supra neutrosophic multiset open ($\mathcal{SNMO}$) is $\mathcal{SNMSO}$, and that the union of a $\mathcal{SNMO}$ with a $\mathcal{SNMSO}$ is again a $\mathcal{SNMSO}$. The relationships between $\mathcal{SNMSO}$ and $\mathcal{SNM}\alpha\mathcal{O}$, $\mathcal{SNMRO}$, $\mathcal{SNMPO}$, and $\mathcal{SNM}\beta\mathcal{O}$ are also investigated. Finally, the proposed concepts and results are illustrated through examples based on student performance.

Table 2. Comparison of supra neutrosophic multiset open sets

Class	Defining condition	Short description
$\mathcal{SNMSO}$	$\exists \mathcal{U} \in \mathcal{SNMO}$ such that $\mathcal{U} \subseteq A \subseteq \mathrm{SCl}_{\mathcal{NM}}(\mathcal{U})$	A supra neutrosophic multiset lying between an $\mathcal{SNM}$ open multiset and its $\mathcal{SNM}$ closure.
$\mathcal{SNMPO}$	$A \subseteq \mathrm{SInt}_{\mathcal{NM}}(\mathrm{SCl}_{\mathcal{NM}}(A))$	A supra neutrosophic multiset contained in the $\mathcal{SNM}$ interior of its $\mathcal{SNM}$ closure.
$\mathcal{SNM}\alpha\mathcal{O}$	$A \subseteq \mathrm{SInt}_{\mathcal{NM}}(\mathrm{SCl}_{\mathcal{NM}}(\mathrm{SInt}_{\mathcal{NM}}(A)))$	A stronger form of openness obtained by applying an additional $\mathcal{SNM}$ interior operation.
$\mathcal{SNM}\beta\mathcal{O}$	$A \subseteq \mathrm{SCl}_{\mathcal{NM}}(\mathrm{SInt}_{\mathcal{NM}}(\mathrm{SCl}_{\mathcal{NM}}(A)))$	The weakest openness condition among the classes considered.
$\mathcal{SNMRO}$	$A = \mathrm{SInt}_{\mathcal{NM}}(\mathrm{SCl}_{\mathcal{NM}}(A))$	A supra neutrosophic multiset equal to the $\mathcal{SNM}$ interior of its own closure.

1.3. *Relation to $\mathcal{SNM}$ Preopen, $\mathcal{SNM}$ α-Open, $\mathcal{SNM}$ β-Open, and $\mathcal{SNM}$ Regular-Open Sets*

The relationships between $\mathcal{SNMSO}$ and other classes of open sets in a supra neutrosophic multiset topological space are also investigated. It is established that every $\mathcal{SNM}\alpha\mathcal{O}$ and every $\mathcal{SNMRO}$ is a $\mathcal{SNMSO}$, Furthermore, $\mathcal{SNMSO}$ and $\mathcal{SNMPO}$ are shown to be independent of each other, demonstrating that neither class is contained in the other. Hierarchical relationships among $\mathcal{SNM}\alpha\mathcal{O}$, $\mathcal{SNMRO}$, $\mathcal{SNMSO}$, $\mathcal{SNMPO}$, and $\mathcal{SNM}\beta\mathcal{O}$ sets are given Figure 1.

2. Preliminaries

Definition 2.1. [13] Let X be a nonempty fixed set. A neutrosophic set [briefly NS] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\mu_A(x), \sigma_A(x), \gamma_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of the indeterminacy (namely $\sigma_A(x)$) and the degree of the non membership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

Definition 2.2. [12] A neutrosophic multiset is a neutrosophic set where one or more elements are repeated with the same neutrosophic components, or with different neutrosophic components.

Definition 2.3. [12] The Empty neutrosophic multiset is denoted by N_Θ and defined by N_Θ =$\{\langle x_{(0,1,1)}\rangle : \forall_x \in X\}$ where x can be repeated.

Definition 2.4. [12] The Whole neutrosophic multiset is denoted by W_x and defined by $W_x = \{\langle x_{(1,0,0)}\rangle : \forall_x \in X\}$ where x can be repeated.
The power set of neutrosophic multiset is denoted by P(X).
The collection of all possible subsets of X is called the power set of the neutrosophic multiset.

Definition 2.5. [12] Let X be neutrosophic multiset and a non-empty family $\mathcal{T}$ subsets of W_X is said to be neutrosophic multiset topological space if the following axioms hold:

(1) $N_\emptyset, W_X \in \mathcal{T}$.
(2) $A \cap B \in \mathcal{T}$, for $A, B \in \mathcal{T}$.
(3) $\bigcup_{i\in\Lambda} A_i \in \mathcal{T}, \forall\{A_i : i \in \Lambda\} \subseteq \mathcal{T}$.

In this case, the pair $(W_X, \mathcal{T})$ is called a neutrosophic multiset topological space (NMTS in short) and any neutrosophic multiset in $\mathcal{T}$ is known as an open neutrosophic multiset (ONMS in short) in W_X. The elements of $\mathcal{T}^c$ are called closed neutrosophic multisets, otherwise, a neutrosophic set F is closed if and only if its complement F^c is an open neutrosophic multiset.

Definition 2.6. [12] Let (W_X, τ) be a neutrosophic multiset topological space with base β. The interior of the neutrosophic multiset A is the union of basis element of τ which is contained in A and it is denoted by NMint(A), i.e, NMint(A)= $\{\cup\beta_i : \beta_i \subseteq Aand\beta_i \in \beta\}$

Definition 2.7. [12] Let (W_X, τ) be a neutrosophic multiset topological. The closure of the neutrosophic multiset A is the intersection of all closed neutrosophic multiset containing the set A and it is denoted by NMcl(A), i.e, NMcl(A)= $\{\cap F_i : A \subseteq F_i$ and $F_i^c \in \tau\}$

Definition 2.8. [17] Let X be neutrosophic multiset and a non empty family τ_μ^* neutrosophic
submultiset of X is said to be supra neutrosophic multiset topological space if the following axioms hold.

(i) $0_{NM}, 1_{NM} \in \tau_\mu^*$
(ii) $\bigcup_{i\in\wedge} A_i \in \tau\mu$ for all $\{A_i : i \in \wedge\} \in \tau_\mu^*$.

The pair (X, τ_μ^*) is called a supra neutrosophic multiset topological space (SNMTS) and any supra neutrosophic multiset in τ_μ^* is known as supra neutrosophic open multiset (SNOM). The elements of τ_μ^{*c} are called supra neutrosophic closed multiset (SNCM).

Definition 2.9. [17] Let (X, τ_μ^*) be SNMTS and let $A = \{< x, \mu_{NM}, \sigma_{NM}, \delta_{NM} >: x \in X\}$ be a supra neutrosophic multiset, then the supra neutrosophic interior multiset of A is the union of all supra neutrosophic open multisets (SNOM) of X contains in A and is defined as
$Sint_{NM}(A) = \{< x, \cup_{(max)}\mu_{NM}, \cap_{(min)}S_{NM}, \cap_{(min)}\delta_{NM} >: x \in X\}$

Definition 2.10. [17] The empty neutrosophic multiset is denoted by N_θ and defined by $N_\phi = \{< x_{(0,1,1)} >: \forall x \in X\}$ where x can be repeated.

Proposition 2.11
[17] Let (X, τ_μ^*) be a supra neutrosophic multiset topological space. Let A, B be two SNMTS on X, then the following property hold:

(i) $Sint_{NM}(0_{NM}) = 0_{NM}, Sint_{NM}(1_{NM}) = 1_{NM}$
(ii) $Sint_{NM}A \subseteq A$.
(iii) $A \subseteq B \Rightarrow Sint_{NM}(A) \subseteq Sint_{NM}(B)$.
(iv) $Sint_{NM}(A \cap B) = Sint_{NM}(A) \cap Sint_{NM}(B)$.
(v) $Sint_{NM}(A) \cup Sint_{NM}(B) \subseteq Sint_{NM}(A \cup B)$.
(iv) $SCl_{NM}(0_{NM}) = 0_{NM}, SCl_{NM}(1_{NM}) = 1_{NM}$
(vii) $A \subseteq SCl_{NM}(A)$
(viii) A is SNCM if and only if $A \subseteq SCl_{NM}(P)$
(ix) $A \subseteq B \Rightarrow SCl_{NM}(A) \subseteq SCl_{NM}(B)$

The following definitions are some existing concepts in supra neutrosophic multiset topological spaces.

Definition 2.12. [17] Let (X, τ_μ^*) be a supra neutrosophic multiset topological space. Then a supra neutrosophic multiset A of X is said to be supra neutrosophic multiset regular open (SNMRO) if A = $SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(A))$ and supra neutrosophic multiset regular closed (SNMRC) if $SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) = A$.

3. On Supra Neutrosophic Multiset Semi-Open

This section introduce the concept of supra neutrosophic multiset semi-open Moreover, the concept of supra neutrosophic multiset semi-closed sets, defined as the complements of supra neutrosophic multiset semi-open sets, is introduced. Several characterizations of these sets are established and supported by illustrative examples..

Definition 3.1. [17] Let (X, τ_μ^*) be a $\mathcal{SNMTS}$ and let $A = \{< x, \mu_{NM}, \sigma_{NM}, \delta_{NM} >: x \in X\}$ be neutrosophic multiset, then the supra neutrosophic multiset interior and supra neutrosophic multiset closure of A is defined as
$Sint_{NM}(A)(or)SInt_{\mathcal{NM}}(A) = \{< x, \cup_{(max)}\mu_{NM}, \cap_{(min)}S_{NM}, \cap_{(min)}\delta_{NM} >: x \in X\}$
$Scl_{NM}(A)(or)SCl_{\mathcal{NM}}(A) = \{< x, \cap_{(max)}\mu_{NM}, \cup_{(min)}S_{NM}, \cup_{(min)}\delta_{NM} >: x \in X$
Throughout this paper, the supra neutrosophic multiset interior of A will be denoted by $SInt_{\mathcal{NM}}(A)$ and the supra neutrosophic multiset closure of A will be denoted by $SCl_{\mathcal{NM}}(A)$. It can also be shown that $SCl_{\mathcal{NM}}(A)$ is the supra neutrosophic closed multiset ($\mathcal{SNCM}$) and $SInt_{\mathcal{NM}}(A)$ is the supra neutrosophic open multiset ($\mathcal{SNOM}$). The following are the important characterization of supra neutrosophic multiset interior and supra neutrosophic multiset closure.

(i) A is a supra neutrosophic open multiset ($\mathcal{SNOM}$) if and only if $A = SInt_{\mathcal{NM}}(A)$.
(ii) A is a supra neutrosophic closed multiset ($\mathcal{SNCM}$) if and only if $A = SCl_{\mathcal{NM}}(A)$.

Definition 3.2. [17] Let (X, τ_μ^*) be a supra neutrosophic multiset topological space. A supra neutrosophic multiset A is said to be a supra neutrosophic multiset semi-open if there exists a supra neutrosophic open multiset U such that U $\subseteq A \subseteq SCl_{\mathcal{NM}}$(U).

Example 3.3. Consider $X = \{Anita\}$, where S represents a single student (Anita) considered for excellence-grade evaluation in Mathematics. Let $\tau_\mu^* = \{0_{NM}, A, B, C, A \cup B, 1_{NM}\}$ with $C(\tau) = \{1_{NM}, C(A), C(B), C(C), C(A \cup B), 0_{NM}\}$ Here,

- A denotes the evaluation assigned by three independent examiners.
- B denotes the evaluation provided by the class teacher.
- C denotes the evaluation provided by the coaching institute.
- $A \cup B$ denotes the combined evaluation obtained from A and B.

$A = \{\langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle,\ \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle,\ \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.6,\ f_\gamma\, 0.2\rangle\}$
$B = \{\langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.2\rangle,\ \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle,\ \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.7,\ f_\gamma\, 0.3\rangle\}$
$C = \{\langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle,\ \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle,\ \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.1\rangle\}$
$A \cup B = \{\langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle,\ \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle,\ \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.6,\ f_\gamma\, 0.2\rangle\}$
$C(A) = \{\langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.9,\ f_\gamma\, 0.8\rangle,\ \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.8,\ f_\gamma\, 0.7\rangle,\ \langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.4,\ f_\gamma\, 0.9\rangle\}$
$C(B) = \{\langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle,\ \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle,\ \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.3,\ f_\gamma\, 0.8\rangle\}$
$C(C) = \{\langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.9,\ f_\gamma\, 0.8\rangle,\ \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.8,\ f_\gamma\, 0.7\rangle,\ \langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.4,\ f_\gamma\, 0.9\rangle$
$C(A \cup B) = \{\langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.9,\ f_\gamma\, 0.8\rangle,\ \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.8,\ f_\gamma\, 0.7\rangle,\ \langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.4,\ f_\gamma\, 0.9\rangle\}$
Since $SCl_{\mathcal{NM}}(B) = \bigcap\{1_{NM}\} = 1_{NM}$, it follows that $B \subseteq A \subseteq SCl_{\mathcal{NM}}(B) = 1_{NM}$, with B being a supra neutrosophic multiset open set on (X, τ_μ^*). Therefore, by Definition 3.2, A qualifies as a supra neutrosophic multiset semi-open set.

Proposition 3.4
For a supra neutrosophic multiset A in (X, τ_μ^*) we have

(i) $SCl_{\mathcal{NM}}(C(A)) = C(SInt_{\mathcal{NM}}(A))$
(ii) $SInt_{\mathcal{NM}}(C(A)) = C(SCl_{\mathcal{NM}}(A))$

Proof

(i) Let $A = \{\langle x, \mu_{\mathcal{NM}_i}, \sigma_{\mathcal{NM}_i}, \delta_{\mathcal{NM}_i}\rangle : i \in J\}$. By the definition of the $\mathcal{SNM}$ interior, $SInt_{\mathcal{NM}}(A) = \{\langle x, \cup\mu_{\mathcal{NM}_i}, \cap\sigma_{\mathcal{NM}_i}, \cap\delta_{\mathcal{NM}_i}\rangle : i \in J\}$. By taking the complement, we obtain $C(SInt_{\mathcal{NM}}(A)) = \{\langle x, \cap\delta_{\mathcal{NM}_i}, \cup(1 - \sigma_{\mathcal{NM}_i}), \cup\mu_{\mathcal{NM}_i}\rangle : i \in J\}$. The complement of A is given by $C(A) = \{\langle x, \delta_{\mathcal{NM}_i}, 1 - \sigma_{\mathcal{NM}_i}, \mu_{\mathcal{NM}_i}\rangle : i \in J\}$.Applying the supra neutrosophic multiset closure operator yields $SCl_{\mathcal{NM}}(C(A)) = \{\langle x, \cap\delta_{\mathcal{NM}_i}, \cup(1 - \sigma_{\mathcal{NM}_i}), \cup\mu_{\mathcal{NM}_)}\rangle : i \in J\}$. Since both expressions are identical, it follows that $SCl_{\mathcal{NM}}(C(A)) = C(SInt_{\mathcal{NM}}(A))$.

(ii) Replacing A with $C(A)$ in part (i), we obtain $SCl_{\mathcal{NM}}(C(CA))) = C(SInt_{\mathcal{NM}}(C(A)))$. Using the identity $C(C(A)) = A$, this reduces to $SCl_{\mathcal{NM}}(A) = C(SInt_{\mathcal{NM}}(C(A)))$. Finally, taking complements on both sides gives $C(SCl_{\mathcal{NM}}(A)) = SInt_{\mathcal{NM}}(C(A))$, This concludes the proof. □

Proposition 3.5
For a supra neutrosophic multiset A in (X, τ_μ^*) we have

(i) $SInt_{\mathcal{NM}}(A)$ is the greatest $\mathcal{SNOM}$ contained in A
(ii) $SInt_{\mathcal{NM}}(A) = A$ iff A is a $\mathcal{SNOM}$
(iii) $SInt_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) = SInt_{\mathcal{NM}}(A)$

Proof

(i) By Definition 3.1, $SInt_{\mathcal{NM}}(A) = \{\langle x, \cup_{(max)}\mu_{NM}, \cap_{(min)}\sigma_{NM}, \cap_{(min)}\delta_{NM}\rangle : x \in X\}$, which is obtained as the union of all supra neutrosophic multiset open sets contained in A. Hence, $SInt_{\mathcal{NM}}(A)$ is itself a $\mathcal{SNOM}$. Moreover, by Proposition 2.15(ii), $SInt_{\mathcal{NM}}(A) \subseteq A$. Let $\mathcal{U} \in \tau_\mu^*$ be any $\mathcal{SNOM}$ satisfying $\mathcal{U} \subseteq A$. Since $\mathcal{U}$ is one of the members involved in the union defining $SInt_{\mathcal{NM}}(A)$, it follows that $\mathcal{U} \subseteq SInt_{\mathcal{NM}}(A)$ Therefore, $SInt_{\mathcal{NM}}(A)$ is the largest $\mathcal{SNOM}$ contained in A.
(ii) Assume that A is a $\mathcal{SNOM}$ of (X, τ_μ^*). By part (i), the largest $\mathcal{SNOM}$ contained in A is A itself. Hence, $SInt_{\mathcal{NM}}(A) = A$. Conversely, if $SInt_{\mathcal{NM}}(A) = A$, then, since $SInt_{\mathcal{NM}}(A)$ is always a $\mathcal{SNOM}$ by part (i), it follows that A is also a $\mathcal{SNOM}$.
(iii) By part (i), $SInt_{\mathcal{NM}}(A)$ is a $\mathcal{SNOM}$. Therefore, applying the result established in part (ii) to the set $SInt_{\mathcal{NM}}(A)$ yields $SInt_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) = SInt_{\mathcal{NM}}(A)$

□

Proposition 3.6
For a supra neutrosophic multiset A in (X, τ_μ^*) we have

(i) $SCl_{\mathcal{NM}}(A \bigcap B) \subseteq SCl_{\mathcal{NM}}(A) \bigcap SCl_{\mathcal{NM}}(B)$
(ii) $SCl_{\mathcal{NM}}(A \bigcup B) = SCl_{\mathcal{NM}}(A) \bigcup SCl_{\mathcal{NM}}(B)$
(iii) $SCl_{\mathcal{NM}}(\bigcup_{i\in I} A_i) \supseteq \bigcup_{i\in I} SCl_{\mathcal{NM}}(A_i)$
(iv) $SCl_{\mathcal{NM}}(SCl_{\mathcal{NM}}(A)) = SCl_{\mathcal{NM}}(A)$
(v) $SInt_{\mathcal{NM}}(\bigcap_{i\in I} A_i) \supseteq \bigcap_{i\in I} SInt_{\mathcal{NM}}(A_i)$

Proof
Parts (i)–(iv) follow directly from the corresponding closure properties of Proposition 3.5 together with the identity $SCl_{\mathcal{NM}}(A) = C(SInt_{\mathcal{NM}}(C(A)))$ For completeness, we present the proofs of parts (ii) and (v). For (ii), by Proposition 3.4(ii), $SCl_{\mathcal{NM}}(A \cup B) = C(SInt_{\mathcal{NM}}(C(A \cup B)))$ Since $(C(A \cup B) = C(A) \cap C(B)$, it follows that $SCl_{\mathcal{NM}}(A \cup B) = C(SInt_{\mathcal{NM}}(C(A) \cap C(B)))$ Applying Proposition 2.15(iv), we obtain $SInt_{\mathcal{NM}}(C(A) \cap C(B)) = SInt_{\mathcal{NM}}(C(A)) \cap SInt_{\mathcal{NM}}(C(B))$, and taking complements on both sides together with Proposition 3.4(ii) gives $SCl_{\mathcal{NM}}(A \cup B) = SCl_{\mathcal{NM}}(A) \cup SCl_{\mathcal{NM}}(B)$ For (v), Proposition 3.5(i) implies that $SInt_{\mathcal{NM}}(A_i) \subseteq A_i$ for every $i \in I$. Consequently, $\bigcap_{i\in I} SInt_{\mathcal{NM}}(A_i) \subseteq \bigcap_{i\in I} A_i$ Moreover, $\bigcap_{i\in I} SInt_{\mathcal{NM}}(A_i)$ is a supra neutrosophic open multiset by Definition 2.12(ii), and since it is contained in $\bigcap_{i\in I} A_i$, it is one of the supra neutrosophic open multiset contained in this intersection. Hence, by Proposition 3.5(i), $\bigcap_{i\in I} SInt_{\mathcal{NM}}(A_i) \subseteq SInt_{\mathcal{NM}}\left(\bigcap_{i\in I} A_i\right)$.

□

Definition 3.7. Let (X, τ_μ^*) be a supra neutrosophic multiset topological space. Then a supra neutrosophic multiset A of X is said to be supra neutrosophic multiset preopen ($\mathcal{SNMPO}$) if $A \subseteq SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(A))$ and supra neutrosophic multiset preclosed ($\mathcal{SNMPC}$) if $SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) \subseteq A$.

Example 3.8. Using the student certification setting of Example 3.3 (Excellence-in-Mathematics Certification), let A denote Anita's candidate certification profile and B the baseline teacher's assessment. Then $\mathrm{SCl}_{NM}(A) = 1_{NM}$, and consequently, $\mathrm{SInt}_{NM}\big(\mathrm{SCl}_{NM}(A)\big) = \mathrm{SInt}_{NM}(1_{NM}) = 1_{NM}$. Hence,

$$\begin{aligned} A &= \{\langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.6,\ f_\gamma\, 0.2\rangle\} \\ &\subseteq 1_{NM} = \mathrm{SInt}_{NM}\big(\mathrm{SCl}_{NM}(A)\big). \end{aligned}$$

Therefore, A satisfies the defining condition of a supra neutrosophic multiset preopen set. Hence, A is a supra neutrosophic multiset preopen set ($\mathcal{SNMPO}$).

Definition 3.9. Let (X, τ^*_μ) be a supra neutrosophic multiset topological space. Then a supra neutrosophic multiset A of X is said to be supra neutrosophic multiset α-open ($\mathcal{SNM}\alpha\mathcal{O}$) if $A \subseteq SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)))$ and supra neutrosophic multiset α-closed (SNMαC) if $SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(A))) \subseteq A$.

Example 3.10. Let $X = \{\text{Anita}, \text{Rahul}\}$, where two students undergo an extended exam stress-test. Three test cycles (an algebra test, a geometry test, and a timed calculus test) are bundled into one neutrosophic multiset rating per student. Let $\tau = \{0_{NM}, 1_{NM}, A\}$ be a supra neutrosophic multiset topological space, where

$$A = \{\langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.9,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.4,\ i_\sigma\, 0.8,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.9,\ f_\gamma\, 0.8\rangle,$$
$$\langle Rahul,\ t_\mu\, 0.4,\ i_\sigma\, 0.7,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.8,\ f_\gamma\, 0.4\rangle, \langle Rahul,\ t_\mu\, 0.3,\ i_\sigma\, 0.8,\ f_\gamma\, 0.7\rangle\}$$

Clearly $SInt_{NM}(A) = A,\ SCl_{NM}(A) = 1_{NM},\ SInt_{NM}(SCl_{NM}(SInt_{NM}(A))) = 1_{NM}$. Hence the α-open condition $A \subseteq SInt_{NM}(SCl_{NM}(SInt_{NM}(A)))$ holds. The assessment of Anita and Rahul satisfies the α-open property, indicating a stable classification under repeated applications of the interior and closure operators.

Definition 3.11. Let (X, τ^*_μ) be a supra neutrosophic multiset topological space. Then a supra neutrosophic multiset A of X is said to be supra neutrosophic multiset semi preopen ($\mathcal{SNMSPO}$) (or) supra neutrosophic multiset β-open ($\mathcal{SNM}\beta O$) if $A \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(A)))$.

Example 3.12. Continuing the exam stress-test above, Anita and Rahul now receive a weaker, borderline rating A from an intermediate examining panel, where

$$A = \{\langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.7,\ f_\gamma\, 0.6\rangle,$$
$$\langle Rahul,\ t_\mu\, 0.2,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle\}$$

Clearly $\text{SCl}_{NM}(A) = A^C$, $\text{SInt}_{NM}(\text{SCl}_{NM}(A)) = A$. Then
$\text{SCl}_{NM}(\text{SInt}_{NM}(\text{SCl}_{NM}(A))) =$

$$\{\langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.7,\ f_\gamma\, 0.6\rangle,$$
$$\langle Rahul,\ t_\mu\, 0.2,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle\}$$

Hence $A \subseteq \text{SCl}_{NM}(\text{SInt}_{NM}(\text{SCl}_{NM}(A)))$. Therefore A is supra neutrosophic multiset semi-preopen (or) β-open. The evaluations of Anita and Rahul satisfies the supra neutrosophic multiset β-open.

Theorem 3.13
A supra neutrosophic multiset A of a supra neutrosophic multiset topological space (X, τ^*_μ) is a supra neutrosophic multiset semi-open set if and only if $A \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$.

Proof
Let A be a supra neutrosophic multiset semi-open of (X, τ^*_μ). Then by Definition 3.2 $U \subseteq A \subseteq SCl_{\mathcal{NM}}(U)$ for some supra neutrosophic open multiset U. Thus $U = SInt_{\mathcal{NM}}(U) \subseteq SInt_{\mathcal{NM}}(A)$. That is, $U \subseteq SInt_{\mathcal{NM}}(A)$. This implies that $SCl_{\mathcal{NM}}(U) \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$. Hence $A \subseteq SCl_{\mathcal{NM}}(U) \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$. Conversely, let $A \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$ and $U = SInt_{\mathcal{NM}}(A)$. Then by Proposition 2.15(ii) $SInt_{\mathcal{NM}}(A) \subseteq A$ and $A \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$ implies that $U \subseteq A \subseteq SCl_{\mathcal{NM}}(U)$. Hence A is a supra neutrosophic multiset semi-open. □

Example 3.14. Using the same construction as in Example 3.3, and the supra neutrosophic multisets A and B are considered, we have $\text{SInt}_{\mathcal{NM}}(A) = \bigcup\{0_{NM}, A, B\} = A$, and $\text{SCl}_{\mathcal{NM}}(\text{SInt}_{\mathcal{NM}}(A)) = \text{SCl}_{\mathcal{NM}}(A) = \bigcap\{1_{NM}\} = 1_{NM}$. Hence, $A = \{\langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle,\ \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle,\ \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.6,\ f_\gamma\, 0.2\rangle\} \subseteq$

$1_{NM} = \mathrm{SCl}_{\mathcal{NM}}(\mathrm{SInt}_{\mathcal{NM}}(A))$. Therefore, A satisfies the defining condition of a supra neutrosophic multiset semi-open set. Hence, A is a $\mathcal{SNMSO}$. From Example 3.14 $SInt_{\mathcal{NM}}(A) = \bigcup\{0_{NM}, A, B\} = A$ and $SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) = SNM \sim cl(A) = \bigcap\{1_{NM}\} = 1_{NM}$. Therefore $A = \left\{ \langle_{snm}a, {}_{t_\mu}0.8, {}_{i_\sigma}0.1, {}_{f_\gamma}0.2\rangle, \langle_{snm}a, {}_{t_\mu}0.7, {}_{i_\sigma}0.2, {}_{f_\gamma}0.3\rangle, \langle_{snm}a, {}_{t_\mu}0.9, {}_{i_\sigma}0.6, {}_{f_\gamma}0.2\rangle \right\} \subseteq 1_{NM} = SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$ Thus A is a supra neutrosophic multiset semi-open.

Theorem 3.15
A supra neutrosophic multiset A is a supra neutrosophic multiset semi-open if and only if $SCl_{\mathcal{NM}}(A) = SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$.

Proof
Let A be a supra neutrosophic multiset semi-open of (X, τ_μ^*). Then by Theorem 3.13 $A \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$. Thus $SCl_{\mathcal{NM}}(A) \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$. Since $SInt_{\mathcal{NM}}(A) \subseteq A, SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) \subseteq SCl_{\mathcal{NM}}(A)$. Hence $SCl_{\mathcal{NM}}(A) = SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$.

Conversely, let $SCl_{\mathcal{NM}}(A) = SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$. Take $U = SInt_{\mathcal{NM}}(A)$. Then U is a supra neutrosophic open multiset such that $U \subseteq A \subseteq SCl_{\mathcal{NM}}(A) = SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) = SCl_{\mathcal{NM}}(U)$. Hence by Definition 3.2, A is a supra neutrosophic multiset semi-open. □

Theorem 3.16
Let (X, τ_μ^*) be a supra neutrosophic multiset topological space. Then the union of two supra neutrosophic multiset semi-open is a supra neutrosophic multiset semi-open of (X, τ_μ^*).

Proof
Let A and B be two supra neutrosophic multiset semi-open of a supra neutrosophic multiset topological space (X, τ_μ^*). Then by Theorem 3.13, $A \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$ and $B \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(B))$. Now $A \bigcup B \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) \bigcup SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(B))$. By Proposition 3.6(ii), $A \bigcup B \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A) \bigcup SNM \sim int(B))$. Again by Proposition 2.15 (v) $A \bigcup B \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A \bigcup B))$. Hence by Theorem 3.13, $A \bigcup B$ is a supra neutrosophic multiset semi-open of a supra neutrosophic multiset topological space (X, τ_μ^*). □

Theorem 3.17
Let (X, τ_μ^*) be a supra neutrosophic multiset topological space. If $\{A_\alpha\}_{\alpha \in I}$ is a collection of a supra neutrosophic multiset semi-open of a supra neutrosophic multiset topological space (X, τ_μ^*), then $\bigcup_{\alpha \in I} A_\alpha$ is a supra neutrosophic multiset semi-open set of X.

Proof
For each $\alpha \in I$, there exist a supra neutrosophic open multiset B_α such that $B_\alpha \subseteq A_\alpha \subseteq SCl_{\mathcal{NM}}(B_\alpha)$. Then by Proposition 3.6(iii) $\bigcup_{\alpha \in I} B_\alpha \subseteq \bigcup_{\alpha \in I} A_\alpha \subseteq \bigcup_{\alpha \in I} SCl_{\mathcal{NM}}(B_\alpha) \subseteq SCl_{\mathcal{NM}}(\bigcup_{\alpha \in I} B_\alpha)$. Let $B = \bigcup_{\alpha \in I} B_\alpha$. Then $B \subseteq \bigcup_{\alpha \in I} A_\alpha \subseteq SCl_{\mathcal{NM}}(B)$. By Definition 3.2 $\bigcup_{\alpha \in I} A_\alpha$ is a supra neutrosophic multiset semi-open of (X, τ_μ^*) □

Remark 3.18. In a supra neutrosophic multiset topological space (X, τ_μ^*), the intersection of two supra neutrrosophic multiset semi-open may not be a supra neutrosophic multiset semi-open as shown by the following example.

Example 3.19. Let $X = \{\text{Anita}, \text{Rahul}\}$, two students under certification, with $\tau^*_\mu = \{0_{NM}, A, B, A \cup B, 1_{NM}\}$ and $C(\tau^*_\mu) = \{1_{NM}, C(A), C(B), C(A \cup B), 0_{NM}\}$.

$$A = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.1,\ i_\sigma\, 0.8,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.8,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.4,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle \end{matrix} \right\},$$

$$B = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.8,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.6,\ f_\gamma\, 0.9\rangle, \\ \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.1,\ i_\sigma\, 0.6,\ f_\gamma\, 0.4\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.8,\ f_\gamma\, 0.7\rangle \end{matrix} \right\},$$

$$A \cup B = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.6,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.4,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle \end{matrix} \right\},$$

$$C(A) = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.4,\ i_\sigma\, 0.2,\ f_\gamma\, 0.1\rangle, \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.8,\ f_\gamma\, 0.7\rangle, \\ \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.2,\ f_\gamma\, 0.8\rangle, \langle Rahul,\ t_\mu\, 0.2,\ i_\sigma\, 0.6,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.9\rangle \end{matrix} \right\},$$

$$C(B) = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.4,\ f_\gamma\, 0.9\rangle, \\ \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.4,\ i_\sigma\, 0.4,\ f_\gamma\, 0.1\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.7\rangle \end{matrix} \right\},$$

$$C(A \cup B) = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.8,\ f_\gamma\, 0.9\rangle, \\ \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.4,\ f_\gamma\, 0.8\rangle, \langle Rahul,\ t_\mu\, 0.2,\ i_\sigma\, 0.6,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.9\rangle \end{matrix} \right\}.$$

Let two further independent evaluation ratings for the same class be:

$$A_1 = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.4,\ f_\gamma\, 0.2\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle, \\ \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.6,\ f_\gamma\, 0.4\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.6\rangle \end{matrix} \right\} \quad \text{and}$$

$$A_2 = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.8,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.1\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.5,\ f_\gamma\, 0.2\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.6,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.5\rangle \end{matrix} \right\}$$

Then $\text{SInt}_{NM}(A_1 \cap A_2) = \bigcup\{0_{NM}, B\} = B$, $\text{SCl}_{NM}(\text{SInt}_{NM}(A_1 \cap A_2)) = \bigcap\{1_{NM}, C(B)\} = C(B)$. Thus $A_1 \cap A_2 \not\subseteq \text{SCl}_{NM}(\text{SInt}_{NM}(A_1 \cap A_2)) = C(B)$. Hence $A_1 \cap A_2$ is not a supra neutrosophic multiset semi-open set. Since the truth-membership value of $A_1 \cap A_2$ is greater than that of $C(B)$, we have$A_1 \cap A_2 \not\subseteq C(B)$. Since the truth-membership value of $A_1 \cap A_2$ is greater than that of $C(B)$, we have $A_1 \cap A_2 \not\subseteq C(B)$. Hence, the intersection of two supra neutrosophic multiset semi-open sets need not be a supra neutrosophic multiset semi-open set.Hence, the intersection of two supra neutrosophic multiset semi-open sets need not be a supra neutrosophic multiset semi-open set.

Theorem 3.20
Let A be a supra neutrosophic multiset semi open of a supra neutrosophic multiset topological space (X, τ^*_μ) and $A \subseteq B \subseteq SCl_{\mathcal{NM}}(A)$. Then B is a supra neutrosophic multiset semi-open of (X, τ^*_μ).

Proof
Let A be a supra neutrosophic multiset semi-open of (X, τ^*_μ). Then by Definition 3.2, there exist a supra neutrosophic open multiset U such that $U \subseteq A \subseteq SCl_{\mathcal{NM}}(U)$. Since $U \subseteq A$ and $A \subseteq B$. Again since $A \subseteq SCl_{\mathcal{NM}}(U), SCl_{\mathcal{NM}}(A) \subseteq SCl_{\mathcal{NM}}(SCl_{\mathcal{NM}}(U))$. Thus by Proposition 2.4 (iv), $SCl_{\mathcal{NM}}(A) \subseteq SCl_{\mathcal{NM}}(U)$.
By hypothesis, $B \subseteq SCl_{\mathcal{NM}}(A)$. Therefore $B \subseteq SCl_{\mathcal{NM}}(U)$. Thus we have $U \subseteq B \subseteq SCl_{\mathcal{NM}}(U)$, where U is a supra neutrosophic open multiset of (X, τ^*_μ) and hence B is a supra neutrosophic multiset semi-open of (X, τ^*_μ). □

Proposition 3.21
Every supra neutrosophic open multiset is supra neutrosophic multiset semi open.

Proof
Let A be a supra neutrosophic open multiset in $(\mathfrak{X}, \tau^*_\mu)$. Then by Definition 3.1 (i)
$A = SInt_{\mathcal{NM}}(A)$, Also by Proposition 2.5 (vii) $A \subseteq SCl_{\mathcal{NM}}(A)$. Then $A = SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$ Hence by Theorem 3.13, A is a supra neutrosophic multiset semi-open. □

Note 3.22. The converse of the above Proposition need not be true as sown by the following example.

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Example 3.23. Let $X = \{\text{Anita}, \text{Rahul}\}$, two students under a class certification programme, with $\tau^*_\mu = \{0_{NM}, A, B, 1_{NM}\}$. Several independent evaluation panels produce the following candidate ratings; some of these qualify as supra neutrosophic multiset semi-open sets:

$$A = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.6,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \\ \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle \end{matrix} \right\},$$

$$B = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.4\rangle \end{matrix} \right\},$$

$$C = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.2\rangle, \\ \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.4\rangle \end{matrix} \right\},$$

$$D = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.1,\ f_\gamma\, 0.3\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.3\rangle \end{matrix} \right\},$$

$$E = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.4,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.4,\ f_\gamma\, 0.2\rangle \end{matrix} \right\},$$

$$F = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle, \\ \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle \end{matrix} \right\},$$

$$G = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.1\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.1\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.3\rangle \end{matrix} \right\},$$

$$H = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.1\rangle \end{matrix} \right\},$$

$$I = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.2\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle \end{matrix} \right\},$$

$$J = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.3\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.2\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.2\rangle \end{matrix} \right\}.$$

Hence C, D, E, F, G, H, I, J are supra neutrosophic multiset semi-open sets but are not supra neutrosophic open multisets. Ratings C through J represent different certification assessments for Anita and Rahul. Each rating satisfies the semi-open property, although none is a supra neutrosophic multiset open set.

Theorem 3.24
If A is a supra neutrosophic open multiset and B is a supra neutrosophic multiset semi-open set of X, then $A \bigcup B$ is a supra neutrosophic multiset semi-open set of X.

Proof
Let A be a supra neutrosophic open multiset and B is a supra neutrosophic multiset semi-open set of X. Then $SInt_{\mathcal{NM}}(A) = A$ and $B \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(B)$. By Propositon 3.6 (ii) and Proposition 2.15 (v) $SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A \bigcup B)) \supseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A) \bigcup SInt_{\mathcal{NM}}(B)) = SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A)) \bigcup SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(B)) \supseteq SCl_{\mathcal{NM}}(A) \bigcup B \supseteq A \bigcup B$. Therefore $SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A \bigcup B)) \subseteq A \bigcup B$.
Hence by Theorem 3.13, $A \bigcup B$ is a supra neutrosophic multiset semi-open. □

Example 3.25. Let $X = \{\text{Anita}, \text{Rahul}, \text{Priya}\}$, three students in a class under certification, with $\tau^*_\mu = \{0_{NM}, A, B, 1_{NM}\}$ and $C(\tau^*_\mu) = \{0_{NM}, C, D, 1_{NM}\}$, where

$$A = \left\{\begin{matrix} \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.8,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.7,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.9,\ f_\gamma\, 0.9\rangle, \\ \langle Rahul,\ t_\mu\, 0.4,\ i_\sigma\, 0.9,\ f_\gamma\, 0.8\rangle, \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.8,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.8,\ f_\gamma\, 0.8\rangle, \\ \langle Priya,\ t_\mu\, 0.3,\ i_\sigma\, 0.7,\ f_\gamma\, 0.5\rangle, \langle Priya,\ t_\mu\, 0.2,\ i_\sigma\, 0.8,\ f_\gamma\, 0.9\rangle, \langle Priya,\ t_\mu\, 0.8,\ i_\sigma\, 0.8,\ f_\gamma\, 0.9\rangle \end{matrix}\right\},$$

$$B = \left\{\begin{matrix} \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.8,\ f_\gamma\, 0.9\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.9,\ f_\gamma\, 0.9\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.8,\ f_\gamma\, 0.8\rangle, \\ \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.9,\ f_\gamma\, 0.8\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.6\rangle, \\ \langle Priya,\ t_\mu\, 0.6,\ i_\sigma\, 0.8,\ f_\gamma\, 0.7\rangle, \langle Priya,\ t_\mu\, 0.5,\ i_\sigma\, 0.9,\ f_\gamma\, 0.6\rangle, \langle Priya,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle \end{matrix}\right\},$$

$$A \cup B = \left\{\begin{matrix} \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.8,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.7,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.8,\ f_\gamma\, 0.8\rangle, \\ \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.9,\ f_\gamma\, 0.8\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.6,\ f_\gamma\, 0.6\rangle, \\ \langle Priya,\ t_\mu\, 0.6,\ i_\sigma\, 0.7,\ f_\gamma\, 0.5\rangle, \langle Priya,\ t_\mu\, 0.5,\ i_\sigma\, 0.8,\ f_\gamma\, 0.6\rangle, \langle Priya,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle \end{matrix}\right\},$$

$$C(A) = \left\{\begin{matrix} \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.2,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.1,\ f_\gamma\, 0.8\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.4\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.6\rangle, \\ \langle Priya,\ t_\mu\, 0.5,\ i_\sigma\, 0.3,\ f_\gamma\, 0.3\rangle, \langle Priya,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.2\rangle, \langle Priya,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.8\rangle \end{matrix}\right\},$$

$$C(B) = \left\{\begin{matrix} \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.2,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.1,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.7\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \\ \langle Priya,\ t_\mu\, 0.7,\ i_\sigma\, 0.2,\ f_\gamma\, 0.6\rangle, \langle Priya,\ t_\mu\, 0.6,\ i_\sigma\, 0.1,\ f_\gamma\, 0.5\rangle, \langle Priya,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle \end{matrix}\right\},$$

$$C(A \cup B) = \left\{\begin{matrix} \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.2,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.8\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.1,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.4,\ f_\gamma\, 0.6\rangle, \\ \langle Priya,\ t_\mu\, 0.5,\ i_\sigma\, 0.3,\ f_\gamma\, 0.6\rangle, \langle Priya,\ t_\mu\, 0.6,\ i_\sigma\, 0.2,\ f_\gamma\, 0.5\rangle, \langle Priya,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle \end{matrix}\right\}.$$

Now, consider the rating provided by another evaluation panel for the same class.

$$E = \left\{\begin{matrix} \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.7,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle, \\ \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.7,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.8,\ f_\gamma\, 0.6\rangle, \\ \langle Priya,\ t_\mu\, 0.4,\ i_\sigma\, 0.6,\ f_\gamma\, 0.4\rangle, \langle Priya,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Priya,\ t_\mu\, 0.8,\ i_\sigma\, 0.7,\ f_\gamma\, 0.8\rangle \end{matrix}\right\}.$$

Then, $\mathrm{SInt}_{\mathcal{NM}}(E) = A$ and $\mathrm{SCl}_{\mathcal{NM}}(\mathrm{SInt}_{\mathcal{NM}}(E)) = C(A)$. Thus, $E \subseteq C(A)$, and hence E is a supra neutrosophic multiset semi-open set. Since $A \cup E = E$, it follows that $A \cup E$ is also a supra neutrosophic multiset semi-open set. Panel E's assessment satisfies the semi-open condition with respect to A, and therefore E is a supra neutrosophic multiset semi-open set.

Remark 3.26. Supra neutrosophic multiset semi-open and supra neutrosophic multiset preopen are independent with each other. It was shown by the following example.

Example 3.27.

(i) Let $X = \{\text{Anita}\}$, a single student under certification, with $\tau^*_\mu = \{0_{NM}, A, B, A \cup B, 1_{NM}\}$ and $C(\tau^*_\mu) = \{0_{NM}, C(A), C(B), C(A \cup B), 1_{NM}\}$ where

$A = \{\langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.7,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.9\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.8,\ f_\gamma\, 0.8\rangle\}$

$B = \{\langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.3,\ f_\gamma\, 0.8\rangle\}$

$A \cup B = \{\langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.3,\ f_\gamma\, 0.8\rangle\}$

$C(A) = \{\langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.2,\ f_\gamma\, 0.7\rangle\}$

$C(B) = \{\langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.7,\ f_\gamma\, 0.5\rangle\}$

$C(A \cup B) = \{\langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.7,\ f_\gamma\, 0.7\rangle\}$

Here, $SCl_{NM}(SInt_{NM}(C(A))) = C(A)$ is a supra neutrosophic multiset semi-open set. But $SInt_{NM}(SCl_{NM}(C(A))) = A \cup B$. Thus $C(A) \nsubseteq A \cup B$, which is not a supra neutrosophic multiset preopen set. The rejected-certification profile C(A) for Anita is semi-open but not preopen. Hence, it satisfies the semi-open condition but cannot be generated from the open sets A and B via the preopen operation.

(ii) Let $X = \{\text{Anita}\}$ with $\tau^*_\mu = \{0_{NM}, A, B, A \cup B, 1_{NM}\}$ and $C(\tau^*_\mu) = \{0_{NM}, C(A), C(B), C(A \cup B), 1_{NM}\}$ where $A = \{\langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle\}$
$B = \{\langle Anita,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.3,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle\}$
$A \cup B = \{\langle Anita,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.3,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle\}$
$C(A) = \{\langle Anita,\ t_\mu\, 0.4,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle\}$
$C(B) = \{\langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.7,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle\}$
$C(A \cup B) = \{\langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.4\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.7,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle\}$
Here, $SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(C(A)) = 1_{NM}$. Hence $C(A)$ is preopen, but $SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(C(A))) = 0_{NM}$. Thus $C(A) \nsubseteq 0_{NM}$ which is not semi-open. Hence, by the definition of supra neutrosophic multiset preopen and semi-open sets are independent. Here, the rejected-certification profile C(A) for Anita is preopen but not semi-open. Hence, parts (i) and (ii) show that supra neutrosophic multiset preopen and supra neutrosophic multiset semi-open sets are independent.

Proposition 3.28
Every supra neutrosophic multiset α-open is a supra neutrosophic multiset semi-open.

Proof
Let A is a supra neutrosophic mutliset α-open of (X, τ^*_μ). Then by Definition 3.7
$A \subseteq SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))) \subseteq SCl_{\mathcal{NM}}(SInt_{\mathcal{NM}}(A))$. By Theorem 3.13, A is a supra neutrosophic multiset semi-open. □

Note 3.29. The converse of the above proposition need not be true as shown by the following example

Example 3.30. Scenario. Let $X = \{\text{Anita}\}$, a single student under certification, with $\tau^*_\mu = \{0_{NM}, A, B, A \cup B, 1_{NM}\}$ and $C(\tau^*_\mu) = \{0_{NM}, C(A), C(B), C(A \cup B), 1_{NM}\}$ where
$A = \{\langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.7\rangle\}$
$B = \{\langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.7,\ f_\gamma\, 0.9\rangle\}$
$A \cup B = \{\langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.7\rangle\}$
$C(A) = \{\langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle\}$
$C(B) = \{\langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.6\rangle\}$
$C(A \cup B) = \{\langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle\}$
Here, $\mathrm{SInt}_{\mathcal{NM}}(B) = B$. Therefore, $\mathrm{SCl}_{\mathcal{NM}}\,(\mathrm{SInt}_{\mathcal{NM}}(B)) = C(A)$, which implies that $B \subseteq \mathrm{SCl}_{\mathcal{NM}}\,(\mathrm{SInt}_{\mathcal{NM}}(B))$ Hence, B satisfies the defining condition of a supra neutrosophic multiset semi-open set. Furthermore, $\mathrm{SInt}_{\mathcal{NM}}\,(\mathrm{SCl}_{\mathcal{NM}}\,(\mathrm{SInt}_{\mathcal{NM}}(B))) = 0_{\mathcal{NM}}$. Since $B \nsubseteq 0_{\mathcal{NM}}$, B does not satisfy the defining condition of a supra neutrosophic multiset α-open set. Consequently, B is a supra neutrosophic multiset semi-open set but not a supra neutrosophic multiset α-open set. The class teacher's rating B for Anita is semi-open but not α-open. Although B satisfies the semi-open condition, it fails to satisfy the defining condition of an α-open set.

Proposition 3.31
Every supra neutrosophic multiset regular open is supra neutrosophic multiset semi-open.

Proof
Let A be a supra neutrosophic multiset regular open of X. Then by Definition 3.11,
$A = SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(A))$. Since supra neutrosophic multiset interior of any supra neutrosophic multiset is a supra neutrosophic open multiset, $SInt_{\mathcal{NM}}(SCl_{\mathcal{NM}}(A)) = A$ is a supra neutrosophic open multiset. Hence by Proposition 3.21, A is a supra neutrosophic multiset semi-open. □

Note 3.32. The converse of the above proposition need not be true as shown by the following example.

Example 3.33. Let $X = \{\text{Anita}, \text{Rahul}\}$ be the set of two students under certification. Consider $\tau_\mu^* = \{0_{NM}, A, B, A \cup B, 1_{NM}\}$ and $C(\tau_\mu^*) = \{0_{NM}, C(A), C(B), C(A \cup B), 1_{NM}\}$, where

$$A = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.4,\ f_\gamma\, 0.2\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \\ \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.5,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.3,\ f_\gamma\, 0.8\rangle \end{matrix} \right\},$$

$$B = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.7,\ f_\gamma\, 0.8\rangle, \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \\ \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.3,\ f_\gamma\, 0.8\rangle, \langle Rahul,\ t_\mu\, 0.5,\ i_\sigma\, 0.4,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle \end{matrix} \right\},$$

$$A \cup B = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.4,\ f_\gamma\, 0.2\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.6,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \\ \langle Rahul,\ t_\mu\, 0.7,\ i_\sigma\, 0.3,\ f_\gamma\, 0.3\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.4,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.9,\ i_\sigma\, 0.3,\ f_\gamma\, 0.8\rangle \end{matrix} \right\},$$

$$C(A) = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.6,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle, \\ \langle Rahul,\ t_\mu\, 0.3,\ i_\sigma\, 0.5,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.7,\ f_\gamma\, 0.7\rangle \end{matrix} \right\},$$

$$C(B) = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.5,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \langle Anita,\ t_\mu\, 0.8,\ i_\sigma\, 0.3,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.5\rangle, \\ \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.7,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.6,\ f_\gamma\, 0.5\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.5,\ f_\gamma\, 0.9\rangle \end{matrix} \right\},$$

$$C(A \cup B) = \left\{ \begin{matrix} \langle Anita,\ t_\mu\, 0.2,\ i_\sigma\, 0.6,\ f_\gamma\, 0.6\rangle, \langle Anita,\ t_\mu\, 0.7,\ i_\sigma\, 0.4,\ f_\gamma\, 0.7\rangle, \langle Anita,\ t_\mu\, 0.6,\ i_\sigma\, 0.5,\ f_\gamma\, 0.8\rangle, \\ \langle Rahul,\ t_\mu\, 0.3,\ i_\sigma\, 0.7,\ f_\gamma\, 0.7\rangle, \langle Rahul,\ t_\mu\, 0.6,\ i_\sigma\, 0.6,\ f_\gamma\, 0.6\rangle, \langle Rahul,\ t_\mu\, 0.8,\ i_\sigma\, 0.7,\ f_\gamma\, 0.9\rangle \end{matrix} \right\}.$$

The class teacher's reference rating represented by B for Anita and Rahul satisfies the condition of a supra neutrosophic multiset semi-open set. However, $\mathrm{SInt}_{\mathcal{NM}}(\mathrm{SCl}_{\mathcal{NM}}(B)) = 1_{\mathcal{NM}}$. Hence, B does not satisfy the defining condition of a supra neutrosophic multiset regular open set.

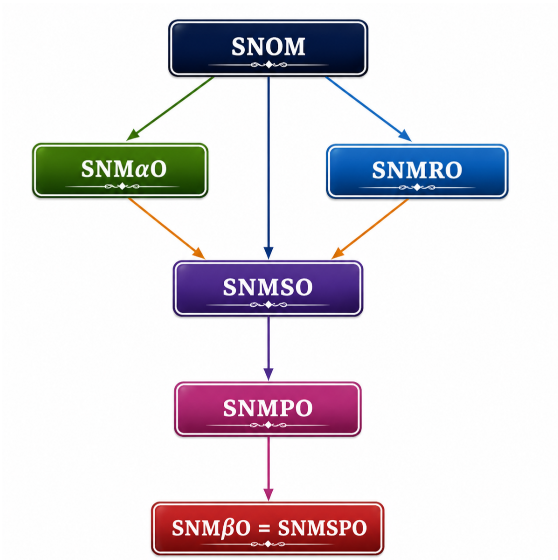

Figure 1. Relationships among the $\mathcal{SNOM}$ sets in $\mathcal{SNMTS}$

4. Conclusion

In this paper we have introduced the class of supra neutrosophic multiset semi-open sets ($\mathcal{SNMSO}$) in a supra neutrosophic multiset topological space ($\mathcal{SNMTS}$) and obtained two characterizations of this class in terms of $\mathcal{SNM}$ interior and $\mathcal{SNM}$ closure. We established that the arbitrary union of $\mathcal{SNMSO}$ is again a $\mathcal{SNMSO}$, whereas the intersection of two $\mathcal{SNMSO}$ need not be $\mathcal{SNMSO}$. Furthermore, the relationship between $\mathcal{SNMSO}$ and the classes of $\mathcal{SNMPO}$, $\mathcal{SNM}\alpha\mathcal{O}$, $\mathcal{SNM}\beta\mathcal{O}$, and $\mathcal{SNMRO}$ was investigated. In particular, every $\mathcal{SNM}\alpha\mathcal{O}$ set and every $\mathcal{SNMRO}$ is a $\mathcal{SNMSO}$, while the classes of $\mathcal{SNMSO}$ and $\mathcal{SNMPO}$ are independent of one another. The relationships among the sets are illustrated in the diagram shown in Figure 1.The examples discussed throughout, based on student performance, illustrate that candidate evaluations remain meaningful even when they are not fully open sets. Further work may extend these results to supra neutrosophic multiset continuous functions and to separation axioms defined by means of $\mathcal{SNMSO}$.

Acknowledgement

This work was completed solely done by the authors without any external assistance or specific support.

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