STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **x**, Month 202x, pp 0–24.
Published online in International Academic Press (www.IAPress.org)

A New Complex Integral Technique for solving The Differential Equations and Their Applications

Gharib M. Gharib[1], Maha Alsaoudi[2], Emad A. Kuffi[3], Mohamed A. Labeeb[4,*]

[1] *Department of Mathematics, Zarqa University, Zarqa , Jordan*
[2] *Department of Science, Applied Science Private University, Jordan*
[3] *College of Basic Education , Mustansiriyah University, Baghdad, Iraq*
[4] *Faculty of Artificial Intelligence-Egyptian Russian University, Cairo-Suez road, Badr City, Cairo Governorate 11829, Egypt*

Abstract This paper introduces a new multi-parametric complex integral technique designed to generalize and extend the traditional Laplace operator. By incorporating four independent complex-valued parameter functions and accommodating arbitrary positive bases, the proposed technique provides a versatile mathematical framework that unifies several well-known single-parameter techniques, including the Laplace, Aboodh, and Elzaki transforms as special boundary cases. We establish the basic operational calculus of the transform, rigorously deriving differential theorems for the ordinary and the partial derivatives for an arbitrary order. Furthermore, we calculate the utility and algebraic flexibility of this approach by obtaining exact analytical solutions for key higher-order ordinary differential equations (ODEs) and partial differential equations (PDEs), such as wave, telegraph, and Klein–Gordon boundary value problems in mathematical physics.

Keywords New complex integral technique, Inverse complex integral technique, Ordinary differential equations, Partial differential equations.

AMS 2010 subject classifications 35-XX, 35A08, 35F15, 35F16, 45-XX.

DOI: 10.19139/soic-2310-5070-4487

1. Introduction

The Laplace transform is the building block of this EM approach. Subsequent to the ease of EMG algebra and arithmetic manipulation and elementary properties, the new integral transform (EMG) approach was developed by Emad A. Kuffi, Mohamed A. Labeeb, and Gharib M. Gharib. for its application in some real problems in hard solution differential operator equations by direct methods.

Fourier, Laplace, Elzaki ,Aboodh , Kamal, AL-Zughair, mayan transform and Mahgoub transforms in general are important and effective tools for solving differential equations [1, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

A new function transforms for high request we think for functions in the set defined through [18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37].

Formulation of General Derivative Theorems: We derive exact theorems for applying the EMG operator to ordinary and partial derivatives of arbitrary orders, establishing the transform's operational calculus.

A new technique is established for function of exponential order.

We take function in the set B given by:

$$B = \{x(\tau) : \text{ there exist } \ H, l_1, l_2 >, \ \ \left| x(\tau) < He^{l_i|\tau|} \right| \text{ if } \ \tau \in (-1)^i \times R_+, \ i = 1, 2\}. \tag{1}$$

*Correspondence to: Mohamed A. Labeeb (Email: m.labeeb85@yahoo.com). Faculty of Artificial Intelligence-Egyptian Russian University, Cairo-Suez road, Badr City, Cairo Governorate 11829, Egypt.

ISSN 2310-5070 (online) ISSN 2311-004X (print)

For a specific function in the family, the constant H is a finite number, and n_1, n_2 may be finite or infinite. EM transform given by the operator $E(.)$ in terms of the integral equation:

$$\begin{aligned} E[x(\tau)] &= X(\alpha) \\ &= \beta(\alpha) \int_{\tau=0}^{\infty} (g(\alpha))^{-iq(\alpha)\tau} f[\eta(\alpha)\tau]\, d\tau \\ &= \frac{\beta(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} e^{-\dfrac{iq(\alpha)\ln g(\alpha)}{\eta(\alpha)}}\, d\tau, \end{aligned}$$

where, $p(\alpha), g(\alpha), q(\alpha), \eta(\alpha)$ are functions of complex parameter s and $\alpha = \alpha_1 + i\alpha_2$ and $\alpha \in C, \beta(\alpha)$ and $\eta(\alpha) \neq 0, g(\alpha) > 0, g(\alpha) \neq 1$, and $\Im\left(\dfrac{q(\alpha)}{\eta(\alpha)}\right) < 0$. Also, $\tau \geq 0, n_1 \leq \alpha \leq n_2$ and $i^2 = -1$.

Equivalently:

$$\begin{aligned} E[x(\tau)] &= F(\alpha) \\ &= \frac{\beta(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} [g(\alpha)]^{-\dfrac{iq(\alpha)}{\eta(\alpha)}} x(\tau)\ d\tau, \end{aligned} \tag{2}$$

where, the kernel of a new integral technique (EMG) is $\dfrac{\beta(\alpha)}{\eta(\alpha)}[g(\alpha)]^{-\dfrac{iq(\alpha)}{\eta(\alpha)}}$.

$F(\alpha)$ is its transform,

The invers EMG transform is

$$x(\tau) = \frac{1}{2\pi i} \int_{\tau=0}^{\infty} \frac{\eta(\alpha)}{\beta(\alpha)} F(\alpha)[g(\alpha)]^{\dfrac{iq(\alpha)}{\eta(\alpha)}\tau}\ d\tau$$

$$\begin{aligned} E[x(\tau)] &= F(\alpha) \\ &= \frac{\beta(\alpha)}{\eta(\alpha)} \sum_{t_k}^{\infty} x(\tau_k)[g(\alpha)]^{-\dfrac{iq(\alpha)}{\eta(\alpha)}\tau_k} \delta\tau, \qquad k = 0, 1, 2, \cdots. \end{aligned}$$

Thae standard Laplace transform for

$$x(\tau) = \int_{\tau=0}^{\infty} e^{-s\tau} x(\tau)\ d\tau$$

and the

The new integral transform (EMG) is a highly generalized, multi-parametric complex integral transform. This a new transform that introduces four independent, complex-valued parameter function $\beta(\alpha)$, $\eta(\alpha)$, $q(\alpha)$ and $g(\alpha)$ be a function, and α be a complex.

1.1. The comparison between the transformations as Aboodh, Elzaki and Kamal transforms and others [20, 25, 15].

The table shows some integral transform types and their definition.

Table 1. Integral Transforms and their definition

Name of Integral Transform	Definition of Integral Transform
Laplace transform	$L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$
Fourier transform	$F\{f(\omega)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty f(t)e^{-i\omega t}\,dt$
Sumudu transform	$S\{f(t)\} = \int_0^\infty f(st)e^{-t}\,dt = \frac{1}{s}\int_0^\infty f(t)e^{-t/s}\,dt = F(s)$
ZZ transform	$H\{f(t)\} = s\int_0^\infty f(vt)e^{-st}\,dt = Z(v,s)$
Shehu transform	$S\{f(t)\} = \int_0^\infty f(t)e^{-st/v}\,dt = F(s,v), \quad s,v>0$
Jafari transform	$T\{f(t)\} = P(s)\int_0^\infty f(t)e^{-q(s)t}\,dt = T(s),\ P(s)\neq 0,\ q(s)>0$
ARA transform	$G_m\{f(t)\} = s\int_0^\infty t^{m-1}e^{-st}f(t)\,dt = G(m,s),\ s>0$
ZMA transform	$A\{f(T)\} = \frac{1}{s}\int_0^\infty f(vt)e^{-t/s}\,dz = z_{MA}(s,v)$
Emad-Fatih transform	$EF\{f(t)\} = \frac{1}{s}\int_0^\infty f(t)e^{-s^2t}\,dt = T(s)$
Khalouta transform	$KH\{f(t)\} = s\int_0^\infty f(\gamma\mu t)e^{-st}\,dt = K(s,\gamma,\mu),\ (s,\gamma,\mu)>0$
KAJ transform	$S_m\{f(t)\} = \frac{1}{s^m}\int_0^\infty f(t/s)e^{-t}\,dt = K(s),\ m\in\mathbb{Z}$
Rishi transform	$R\{f(t)\} = \frac{s}{v}\int_0^\infty e^{-(v/s)t}\,dt = T(v,s),\ v,s>0$
SEA transform	$H^c\{f(t)\} = \frac{s}{v}\int_0^\infty f(t)e^{-i(s/v)}\,dt = Z(s,v),\ \Im(s/v)<0$
Mayan transform	$MA\{f(t)\} = \frac{1}{v^{i\beta}}\int_0^\infty e^{-v^{i\alpha}t}f(t)\,dt,\ \alpha,\beta\in\mathbb{Z},\ v\in\mathbb{C}$
General Polynomial transform	$I_s\{f(t)\} = \int_1^\infty t^{-(p(s)+1)}f(t)\,dt = F(p(s))$
NO transform	$NO\{f(t)\} = \left(\frac{v}{s}\right)^\beta\int_0^\infty e^{-ut}f(st)\,dt,\ \beta\in\mathbb{Z},\ u,v,s>0$
MAHA transform	$\mathcal{M}[f(t)] = (uv)^\beta\int_0^\infty f(t/u)e^{-t/u}\,dt,\ u,v>0,\ \beta\in\mathbb{Z}$
Elzaki transform	$E\{h(t)\} = s\int_0^\infty h(t)e^{-t/s}\,dt$
Natural transform	$n\{h(t)\} = R(s,u) = s\int_0^\infty h(ut)e^{-st}\,dt$
α-Laplace transform	$L_\alpha\{h(t)\} = \int_0^\infty h(t)e^{-t/\alpha}\,dt,\ \alpha\in\mathbb{R}_0^+$
Aboodh transform	$A\{h(t)\} = \frac{1}{s}\int_0^\infty h(t)e^{-st}\,dt$
Mohand transform	$m\{h(t)\} = R(s) = s^2\int_0^\infty h(t)e^{-st}\,dt$
Pourreza transform	$HJ\{h(t)\} = s\int_0^\infty h(t)e^{-st}\,dt$
Kamal transform	$K\{h(t)\} = G(s) = \int_0^\infty h(t)e^{-t/s}\,dt$
Sawi transform	$Sa\{h(t)\} = \frac{1}{s^2}\int_0^\infty h(t)e^{-t/s}\,dt$

1.2. Relation to Existing Transforms for some integral transformations

By choices of four functions $(\beta(\alpha), \eta(\alpha), q(\alpha), g(\alpha))$ that reduce the new Complex Integral transform (EMG) to the Laplace, Elzaki, Aboodh and Kamal transforms as table above.:

1. Reduction to the standard Laplace transform from the general definition of the new integral transform (EMG):

$$X(\alpha) = \frac{\beta(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} [g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$

Since

$$[g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} = e^{-\frac{iq(\alpha) ln g(\alpha)}{\eta(\alpha)}\tau}$$

So,

$$X(\alpha) = \frac{\beta(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} e^{-\frac{iq(\alpha) ln g(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$

If we put the parameter variable $\alpha = s$ and choose the four parameter functions as

$$\beta(\mathrm{s}) = \eta(\mathrm{s}) \ \ sothat \frac{\beta(\mathrm{s})}{\eta(\mathrm{s})} = 1, \eta(s) = 1, g(s) = e, q(s) = -is.$$

Substitute in the EMG transform to get the Laplace transform

$$X(s) = \int_{\tau=0}^{\infty} e^{-s\tau} x(\tau)\, d\tau$$

2. Reduction to the Elzaki transform from the general definition of the new integral transform (EMG):

$$X(\alpha) = \frac{\beta(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} [g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$

Since

$$[g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} = e^{-\frac{iq(\alpha) ln g(\alpha)}{\eta(\alpha)}\tau}$$

So,

$$X(\alpha) = \frac{\beta(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} e^{-\frac{iq(\alpha) ln g(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$

If we put the parameter variable $\alpha = v$ and choose the four parameter functions as

$$\beta(v) = v, \eta(v) = 1, g(v) = e, q(v) = \frac{i}{v}.$$

Substitute in the EMG transform to get the Elzaki transform

$$X(v) = v \int_{\tau=0}^{\infty} e^{-\frac{\tau}{v}} x(\tau)\, d\tau$$

3. Reduction to the Aboodh transform from the general definition of the new integral transform (EMG):

$$X(\alpha) = \frac{\beta(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} [g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$

Since

$$[g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} = e^{-\frac{iq(\alpha) ln g(\alpha)}{\eta(\alpha)}\tau}$$

So,

$$X(\alpha) = \frac{\beta(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} e^{-\frac{iq(\alpha) ln g(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$

If we put the parameter variable $\alpha = s$ and choose the four parameter functions as

$$\beta(s) = \frac{1}{s}, \eta(s) = 1, g(s) = e, q(s) = -is.$$

Substitute in the EMG transform to get the Aboodh transform

$$X(s) = \frac{1}{s}\int_{\tau=0}^{\infty} e^{-\ s\tau} x(\tau)\, d\tau$$

4. Reduction to the Kamal transform from the general definition of the new integral transform (EMG):

$$X(\alpha) = \frac{\beta(\alpha)}{\eta(\alpha)}\int_{\tau=0}^{\infty} [g(\alpha)]^{-\ \frac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$

Since

$$[g(\alpha)]^{-\ \frac{iq(\alpha)}{\eta(\alpha)}\tau} = e^{-\ \frac{iq(\alpha)lng(\alpha)}{\eta(\alpha)}\tau}$$

So,

$$X(\alpha) = \frac{\beta(\alpha)}{\eta(\alpha)}\int_{\tau=0}^{\infty} e^{-\ \frac{iq(\alpha)lng(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$

If we put the parameter variable $\alpha = v$ and choose the four parameter functions as

$$\beta(v) = 1, \eta(v) = 1, g(v) = e, q(v) = iv.$$

Substitute in the EMG transform to get the Kamal transform

$$X(v) = \int_{\tau=0}^{\infty} e^{e\ \frac{\tau}{v}} x(\tau)\, d\tau$$

Theorem 1.1. *Existence and Convergence of a new complex integral technique (EMG)*

Assume that $\mathrm{x}(\tau) \in B$ *is a piecewise condition function of an exponential order, and for a specific function in the family, the constant* H *is greater than zero and a finite number if*

$$\lceil \mathrm{x}(\tau) \rceil \leq He^{o|\tau|} forall \tau > 0.$$

Then $E[x(\tau)] = F(\alpha)$ *exists for all complex values of* α *that satisfy the relation* $Re[\frac{iq(\alpha)}{\eta(\alpha)}] > d$, *where* $\beta(\alpha), g(\alpha), q(\alpha), \eta(\alpha)$ *are functions of complex parameters and* $\alpha = \alpha_1 + i\alpha_2$ *where* $\alpha \in C, \alpha_1,\ \alpha_2 \in R,\ \mathrm{n}_1 \leq \alpha \leq \mathrm{n}_2$, *where the limits* $\mathrm{n}_1,\ \mathrm{n}_2$ *are finite real numbers or infinite.*

$$\beta(\alpha)\, and\, \eta(\alpha) \neq 0, g(\alpha) > 0, g(\alpha) \neq 1, and Im(\frac{q(\alpha)}{\eta(\alpha)}) \leq 0$$

. Also, $\tau \geq 0$.

Proof

To analyze convergence for the existence theorem of the EMG technique, we can determine the definition for the new integral transform (EMG)in the form:

$$\frac{\beta(\alpha)}{\eta(\alpha)}\int_{\tau=0}^{\infty} [g(\alpha)]^{-\ \dfrac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau$$
$$= \frac{\beta(\alpha)}{\eta(\alpha)}\int_{0}^{\mathrm{b}} [g(\alpha)]^{-\ \dfrac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau + \frac{\beta(\alpha)}{\eta(\alpha)}\int_{b}^{\infty} [g(\alpha)]^{-\ \dfrac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau) d\tau, \mathrm{b} > 0.$$

From the property that the function is piecewise continuous, then $\frac{\beta(\alpha)}{\eta(\alpha)} \int_0^b [g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)d\tau < \infty$. and

$$\left\lfloor \frac{\beta(\alpha)}{\eta(\alpha)} \int_b^\infty [g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau \right\rfloor \leq \frac{\beta(\alpha)}{\eta(\alpha)} \int_b^\infty [g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} \lfloor x(\tau) \rfloor\, d\tau$$

$$\leq \frac{\beta(\alpha)}{\eta(\alpha)} \int_b^\infty [g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} Hg(\alpha)^{d\tau} d\tau$$

$$\leq \frac{\beta(\alpha)\ H}{\eta(\alpha)} \int_b^\infty [g(\alpha)]^{-\left[\frac{iq(\alpha)}{\eta(\alpha)} - d\right]\tau} d\tau$$

After the simple simplification using the improper integral, we get

$$\left\lfloor \frac{\beta(\alpha)}{\eta(\alpha)} \int_b^\infty [g(\alpha)]^{-\frac{iq(\alpha)}{\eta(\alpha)}\tau} x(\tau)\, d\tau \right\rfloor = \frac{\beta(\alpha)\ H}{\eta(\alpha)} [\frac{[g(\alpha)]^{-\left[\frac{iq(\alpha)}{\eta(\alpha)} - d\right] b}}{lng(\alpha)\ \left[\frac{iq(\alpha)}{\eta(\alpha)} - d\right]}$$

The denominator must be greater than zero, then $\frac{iq(\alpha)}{\eta(\alpha)} - d > 0$, the condition satisfied.

□

2. Methodology

2.1. EMG Technique of Elementary Functions

Let $x(\tau)$ be an arbitrary function, we presuppose that the base case (2) occurs. The sufficiency sufficient cases for EMG procedure are that $x(\tau)$ for t greater than or equals zero be piecewise continuous and of extraordinary request, at least EMG transform could possibly occur explaining the step-by-step procedure for applying the EMG transform to ODEs and PDEs using the new integral transform and the improper integral for getting the exact solution and used the partial fractions, we show in the following sections.

Here is a comprehensive test suite designed to validate the implementation and accuracy of the EMG transform across various classes of ordinary and partial differential equations. Each test case includes the problem definition, initial and boundary conditions, the target algebraic equation in the EMG domain, and the verified analytical solution for direct validation.

In the subsequent section we evaluate EMG technique of elementary functions

(1) Let $(x\tau) = k$, where k be a constant, so via the definition we get:

$$
\begin{aligned}
E\,[\mathrm{k}] &= X\,(\alpha) \\
&= \frac{\beta\,(\alpha)}{\eta(\alpha)} \int_{t=0}^{\infty} k\ \left[g\,(\alpha)\right]^{-\dfrac{iq\,(\alpha)}{\eta\,(\alpha)}\tau} dt \\
&= k\frac{\beta(\alpha)}{\eta(\alpha)} \int_{t=0}^{\infty} [g(\alpha)]^{-\dfrac{iq\,(\alpha)}{\eta\,(\alpha)}\tau} d\tau\ , \\
&= k\frac{\beta\,(\alpha)}{\eta(\alpha)} \left[\frac{[g(\alpha)]^{-\dfrac{iq\,(\alpha)}{\eta\,(\alpha)}\tau}}{-\dfrac{iq\,(\alpha)}{\eta\,(\alpha)} lng(\alpha)}\right]_0^{\infty} \\
&= \frac{-ik\beta\,(\alpha)}{q(\alpha)lng\,(\alpha)}\ .
\end{aligned}
$$

(2) If $(x\tau) = \tau$, then:

$$
\mathrm{x}\,[\tau] = \mathrm{X}\,(\alpha) = \frac{\beta\,(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} \tau\ \left[g\,(\alpha)\right]^{-\dfrac{iq\,(\alpha)}{\eta\,(\alpha)}\tau} d\tau\ \ .
$$

Integration by parts, we get $\mathrm{E}\,[\tau] = -\dfrac{\dfrac{p(\alpha)}{\eta(\alpha)}}{[\dfrac{q\,(\alpha)}{\eta\,(\alpha)}lng\,(\alpha)\,]^{2}}$.

Also:

(i) $\mathrm{E}\left[\tau^2\right] = -\dfrac{2i\,\dfrac{\beta\,(\alpha)}{\eta(\alpha)}}{[\dfrac{q\,(\alpha)}{\eta\,(\alpha)}lng\,(\alpha)\,]^{3}}$.

(ii) $\mathrm{E}\left[\tau^3\right] = -\dfrac{6i\,\dfrac{\beta\,(\alpha)}{\eta(\alpha)}}{[\dfrac{q\,(\alpha)}{\eta\,(\alpha)}lng\,(\alpha)\,]^{4}}$.

(iii) Largely, let n be a positive constant, so

$$
\mathrm{E}\,[\tau^n] = \frac{-\,(n!)\,(i)\dfrac{\beta\,(\alpha)}{\eta(\alpha)}}{[\dfrac{q\,(\alpha)}{\eta\,(\alpha)}lng\,(\alpha)\,]^{n+1}}.
$$

(3) If $x\,(\tau) = e^{a\tau}$, where a is a constant, so

$$
\begin{aligned}
E\,[e^{a\tau}] = X\,(\alpha) &= \frac{\beta\,(\alpha)}{\eta(\alpha)} \int_{t=0}^{\infty} e^{a\tau}\ \left[g\,(\alpha)\right]^{-\dfrac{iq\,(\alpha)}{\eta\,(\alpha)}\tau} d\tau \\
&= \frac{\beta\,(\alpha)}{\eta(\alpha)} \int_{\tau=0}^{\infty} \left[\frac{[g(\alpha)]^{\dfrac{iq\,(\alpha)}{\eta\,(\alpha)}}}{e^{a}}\right]^{-\tau} d\tau\ .
\end{aligned}
$$

Integration by parts, we get

$$E\left[e^{a\tau}\right]=\frac{\dfrac{\beta\left(\alpha\right)}{\eta(\alpha)}}{[i\ \dfrac{q(s)}{\eta(s)}lng\left(\alpha\right)-a]}$$
$$=\frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\ \dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)+a\ ]}{(\dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)\ )^{2}\ +a^{2}}.$$

Similarly,

$$E\left[e^{a\tau}\right]=\frac{\dfrac{\beta\left(\alpha\right)}{\eta(\alpha)}}{[i\ \dfrac{q(s)}{\eta(s)}lng\left(\alpha\right)+a]}$$
$$=\frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\ \dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)-a\ ]}{(\dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)\ )^{2}\ +a^{2}}.$$

Similarly,

$$E\left[e^{ia\tau}\right]=\frac{\dfrac{\beta\left(\alpha\right)}{\eta(\alpha)}}{[i\ \dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)-ia]}$$
$$=\frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\ \dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)+ia\ ]}{(\dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)\ )^{2}\ -a^{2}}.$$

Similarly,

$$E\left[e^{-ia\tau}\right]=\frac{\dfrac{\beta\left(\alpha\right)}{\eta(\alpha)}}{[i\ \dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)+ia]}$$
$$=\frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\ \dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)-ia\ ]}{(\dfrac{q(\alpha)}{\eta(\alpha)}lng\left(\alpha\right)\ )^{2}\ -a^{2}}\ .$$

(4) If $x(\tau)=\sin(a\tau)$, where a is a constant, so

$$\begin{aligned}\mathrm{E}\left[\sin\left(a\tau\right)\right]=\mathrm{F}\left(\alpha\right)&=\frac{1}{2i}\left\{e^{ia\tau}-e^{-ia\tau}\right\}\\&=\frac{1}{2i}\left\{\frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\ \dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+ia-i\ \dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+ia]}{\left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^{2}-a^{2}}\right\}.\end{aligned}$$

And after simple simplifications, we obtain:

$$\mathrm{E}\left[sin\left(a\tau\right)\right]=\frac{-a\ \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^{2}-a^{2}}.$$

(5) Let $x(\tau)=\cos(a\tau)$, if a is positive integer, so

$$\begin{aligned}\mathrm{E}\left[cos\left(a\tau\right)\right]&=\mathrm{F}\left(\alpha\right)\\&=\frac{1}{2}\left\{e^{ia\tau}+e^{-ia\tau}\right\}\\&=\frac{1}{2}\left\{\frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\ \dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+ia+i\ \dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)-ia]}{\left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^{2}-a^{2}}\right\}.\end{aligned}$$

And after simple simplifications, we obtain:

$$\mathrm{E}\left[cos\left(a\tau\right)\right]=\frac{-\ \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\dfrac{q\left(\alpha\right)}{\eta\left(s\right)}lng\left(\alpha\right)]}{\left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^{2}-a^{2}}.$$

(6) If $x(\tau)=\sinh(a\tau)$, where a is a constant, so

$$\begin{aligned}\mathrm{E}\left[sinh\left(a\tau\right)\right]&=\mathrm{F}\left(\alpha\right)\\&=\frac{1}{2}\left\{e^{a\tau}-e^{-a\tau}\right\}\\&=\frac{1}{2}\left\{\frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\ \dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+a-i\ \dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+a]}{\left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^{2}+a^{2}}\right\}.\end{aligned}$$

And after simple simplifications, we obtain:

$$\mathrm{E}\left[sinh\left(at\right)\right]=\frac{-\ \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[a]}{\left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^{2}+a^{2}}.$$

(7) If $x(\tau) = \cosh(a\tau)$, where a is any constant, so

$$\begin{aligned}
\mathrm{E}\left[cosh\left(a\tau\right)\right] &= \mathrm{F}\left(\alpha\right) \\
&= \frac{1}{2}\left\{e^{a\tau} + e^{-a\tau}\right\} \\
&= \frac{1}{2}\left\{\frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\ \dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right) + a + i\ \dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right) - a]}{\left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^2 + a^2}\right\}.
\end{aligned}$$

And after simple simplifications, we obtain:

$$\mathrm{E}\left[sinh\left(a\tau\right)\right] = \frac{-\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)]}{\left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^2 + a^2}.$$

Theorem 2.1. *Let $X(s)$ is the EMG technique of $E[x(t)] = X(s)$ so:*

(i) $E\left[x'\left(\tau\right)\right] = -\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}x\left(0\right) + i\dfrac{q\left(s\right)}{\eta\left(s\right)}lng\left(s\right)\ X\left(s\right).$

(ii) $E\left[x''\left(\tau\right)\right] = -\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}x'\left(0\right) - i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)x\left(0\right)\ - \left(\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^2 X\left(s\right).$

(iii) $\mathrm{E}\left[x^{'''}\left(\tau\right)\right] = -\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}x^{''}\left(0\right) - \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)]\ x'\left(0\right) - \dfrac{p\left(\alpha\right)}{\eta\left(\alpha\right)}\left(i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^2 x\left(0\right) + \left(i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^3 x\left(s\right).$

(iv) $\mathrm{E}\left[f^{(4)}\left(\tau\right)\right] = -\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}x^{'''}\left(0\right) - \dfrac{p\left(\alpha\right)}{\eta\left(\alpha\right)}[i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)]x^{''}\left(0\right)\ - \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\left(i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^2 x^{'(0)} - \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\left(i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^3 x\left(0\right) + \left(i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^4 \mathrm{X}\left(\alpha\right).$

(v) $\mathrm{E}\left[x^{(\mathrm{m})}\left(\tau\right)\right] = \left(i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^m \mathrm{X}\left(\alpha\right) - \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\left(i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^{m-1} x\left(0\right) - \dfrac{\beta\left(\alpha\right)}{\eta\left(s\right)}\left(i\dfrac{q\left(s\right)}{\eta\left(s\right)}lng\left(s\right)\right)^{m-2} x^{'(0)} - \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\left(i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right)^{m-3} x^{''}\left(0\right) - \dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}x^{(\mathrm{m}-1)}\left(0\right).$

where, $m = 1, 2, 3, \ldots$.

Proof

(i) From the definition, we obtain:

$$E\left[x'\left(\tau\right)\right] = \frac{\beta\left(\alpha\right)}{\eta(\alpha)}\int_{t=0}^{\infty} x'\left(\tau\right)\ \left[g\left(\alpha\right)\right]^{-\dfrac{iq\left(\alpha\right)}{\eta\left(\alpha\right)}\tau} d\tau,$$

Using the integrating by parts,

$$u = \frac{\beta\left(\alpha\right)}{\eta(\alpha)}\ \left[g\left(\alpha\right)\right]^{-\dfrac{iq\left(\alpha\right)}{\eta\left(s\right)}\tau} \qquad du = -iq\left(s\right)\frac{\beta\left(\alpha\right)}{\eta(\alpha)}\ \left[g\left(\alpha\right)\right]^{-\dfrac{iq\left(\alpha\right)}{\eta\left(s\right)}\tau}$$

$$v = x(\tau) \qquad\qquad dv = x'(\tau)\, d\tau$$

we get:

$$E\left[x''(\tau)\right] = \frac{\beta(\alpha)}{\eta(\alpha)} \int_{t=0}^{\infty} x''(\tau)\ [g(\alpha)]^{-\dfrac{iq(s)}{\eta(s)}\tau}\, d\tau\ .$$

(ii) Also, via the definition we obtain:

$$E\left[x''(\tau)\right] = \frac{\beta(\alpha)}{\eta(\alpha)} \int_{t=0}^{\infty} x''(\tau)\ [g(\alpha)]^{-\dfrac{iq(s)}{\eta(s)}\tau}\, d\tau\ .$$

And so on, similarly (i), we get:

$$\mathrm{E}\left[x''(\tau)\right] = -\frac{\beta(\alpha)}{\eta(\alpha)} x'(0) - i\frac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\, x(0)\ - \left(\frac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\right)^2 X(s)$$

Similarly, the proof of (iii) and (iv).

(v) Proved via mathematical indication.

□

3. The Inverse of EMG Transform theory

Clarification of the Inverse EMG Transform Path the inverse EMG transform is fundamentally a complex line integral (contour integral) evaluated along c specific path in the complex α - plane.

Specification of the Integration Path (The Generalized Bromwich Contour)

Thus, the inverse EMG transform is formulated as a complex contour integral in the α plane

$$x(\tau) = E^{-1}\left[X(\alpha)\right] = \frac{1}{2\pi i} \int_{c} \frac{\eta(\alpha)}{\beta(\alpha)} X(\alpha)\ [g(\alpha)]^{\frac{iq(\alpha)}{\eta(\alpha)}\tau}\, d\alpha$$

where c is the generalized Bromwich contour in the complex α –plane.

If $E[x(\tau)] = X(\alpha)$ is the EM technique, so $x(\tau) = E^{-1}[X(\alpha)]$ is said to be an inverse of EMG technique.

In the next section, we introduce the inverse of EMG transform of elementary functions and we proved in section 2.1 and it rely entirely on partial fraction decomposition, integration by parts and taking the inverse:

(1) $E^{-1}\left[\dfrac{-i\beta(\alpha)}{q(\alpha)\, lng(\alpha)}\right] = 1.$

(2) $E^{-1}\left[-\dfrac{\dfrac{\beta(\alpha)}{\eta(s)}}{[\dfrac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\,]^2}\right] = \tau.$

(3) $E^{-1}\left[\dfrac{-\dfrac{\beta(\alpha)}{\eta(s)}(m!)\, i}{[\dfrac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\,]^{m+1}}\right] = \tau^m$, where $m > 0$ integer number.

(4) $E^{-1}\left[-\dfrac{\dfrac{\beta(\alpha)}{\eta(\alpha)}[i\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ -a]}{[\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,]^{2}-a^{2}}\right]=e^{-a\tau}$, where a is a constant number.

(5) $E^{-1}\left[-\dfrac{\dfrac{\beta(\alpha)}{\eta(\alpha)}[a]}{[\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,]^{2}-a^{2}}\right]=sin(a\tau)$

(6) $E^{-1}\left[-\dfrac{\dfrac{\beta(\alpha)}{\eta(\alpha)}[i\dfrac{q(s)}{\eta(s)}lng(\alpha)\,]}{[\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,]^{2}-a^{2}}\right]=cos(a\tau)$

(7) $E^{-1}\left[-\dfrac{\dfrac{\beta(\alpha)}{\eta(\alpha)}[a]}{[\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,]^{2}+a^{2}}\right]=sinh(a\tau)$

(8) $E^{-1}\left[-\dfrac{\dfrac{\beta(\alpha)}{\eta(\alpha)}[i\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,]}{[\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,]^{2}+a^{2}}\right]=cosh(a\tau)$

4. Problems of EMG technique of ordinary differential operator equations

EMG transform can be used as a successful technique, as stated in this article's presentation.

For examining, under introductory conditions, the fundamental characteristics of a direct framework represented by the operator differential equations.

The accompanying models represent the utilization of the EMG technique in taking care of certain underlying worth issue depicted via operator equations.

Assume that the ordinary differential equation

$$\frac{\mathrm{dx}}{\mathrm{dt}}+\mathrm{bx}=\mathrm{f}(\tau) \tag{3}$$

Where initial condition

$$x(0)=k \tag{4}$$

With b, k are any numbers and $f(t)$ is an outside input work with the aim that its EMG technique occur.

Take EMG technique for both sides' equation (3), we obtain:

$$E\left[\frac{dx}{d\tau}\right]+E[bx]=E[f(\tau)], \qquad \tau>0$$

So,

$$-\frac{\beta(\alpha)}{\eta(s)}x(0)+i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ X(\alpha)+bX(\alpha)=\mathrm{F}(\alpha)$$

$$X(\alpha)\left[i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ +b\right]=k\frac{p(\alpha)}{\eta(\alpha)}+F(\alpha)$$

$$X\left(s\right)=\frac{k\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\ +b}+\frac{F(\alpha)}{i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\ +b}$$

The inverse EMG technique prompts the arrangement. The second request ordinary differential equation has the form:

$$\frac{d^2x}{d t\tau^2}+2p\frac{\mathrm{dx}}{\mathrm{d}\tau}+\mathrm{qx}=\mathrm{f}\left(\tau\right),t>0 \tag{5}$$

The initial cases are

$$\mathrm{x}\left(0\right)=\mathrm{a},\frac{\mathrm{dx}}{\mathrm{d}\tau}\left(0\right)=k \tag{6}$$

With β, q, a, k are any numbers.
Using EMG transform to equation (5) gives:

$$\begin{aligned}&-\left[\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^2X\left(\alpha\right)-i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}x\left(0\right)-\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}x^{'}\left(0\right)\\&+2\beta\left[-\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}x\left(0\right)+i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)X\left(\alpha\right)\right]+qX\left(\alpha\right)=F(\alpha)\end{aligned}$$

$$\begin{aligned}&X\left(\alpha\right)\{-\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^2+2\beta\left[i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]+q\}\\&=F\left(\alpha\right)+2a\beta\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}+\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}k+ai\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\end{aligned}$$

After computations, we obtain:

$$\begin{aligned}X\left(s\right)=&\frac{-ia\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^2-2\beta\left[i\dfrac{q\left(s\right)}{\eta\left(s\right)}lng\left(s\right)\right]-q}\\&-\frac{\dfrac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\left(k+2ap\right)}{\left[\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^2-2\beta\left[i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]-q}\\&-\frac{F(\alpha)}{\left[\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^2-2\beta\left[i\dfrac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]-q}.\end{aligned}$$

Take the inverse of EMG technique to above equation gets the arrangement.

Example 4.1. *solve using EMG transform*

$$y^{'}(\tau)+2y(\tau)=\tau,\qquad\qquad y\left(0\right)=1$$

Solution
take EMG technique to above equation gets:

$$-\frac{\beta(\alpha)}{\eta(\alpha)}y(0)+i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,Y(\alpha)+2Y(\alpha)=-\frac{\frac{\beta(\alpha)}{\eta(\alpha)}}{[q(\alpha)\,lng(\alpha)]^2}.$$

$$-\frac{\beta(\alpha)}{\eta(\alpha)}+i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,Y(\alpha)+2Y(\alpha)=-\frac{\frac{\beta(\alpha)}{\eta(\alpha)}}{[q(\alpha)\,lng(\alpha)]^2}.$$

$$Y(\alpha)=\frac{-\frac{\beta(\alpha)}{\eta(\alpha)}}{[q(\alpha)\,lng(\alpha)]^2[i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\;+2]}+\frac{\frac{\beta(\alpha)}{\eta(\alpha)}}{i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\;+2}.$$

Using partial fraction for the first term $[\frac{-\frac{\beta(\alpha)}{\eta(\alpha)}}{[q(\alpha)\,lng(\alpha)]^2\left[i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\;+2\right]}]$, we find:

$$\frac{-1}{[q(\alpha)\,lng(\alpha)]^2[i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\;+2]}=[\frac{D}{[q(s)\,lng(\alpha)]^2}+\frac{E}{q(s)lng(s)}+\frac{F}{i\frac{q(\alpha)}{\eta(s)}lng(\alpha)\;+2}].$$

Then,

$$-1=D[i\frac{q(s)}{\eta(s)}lng(\alpha)]\;+E(q(s)lng(\alpha)\,)(i\frac{q(s)}{\eta(s)}lng(\alpha)\;+2)+F[q(\alpha)\,lng(\alpha)]^2].$$

After computations, we have:
$D=-\frac{1}{2},\;\; E=\frac{1}{4}i\eta(\alpha)\,,\; and\;\; F=\frac{1}{4}.$
So,

$$\frac{-\frac{\beta(\alpha)}{\eta(\alpha)}}{[q(\alpha)\,lng(\alpha)]^2[i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\;+2]}=\frac{-\frac{\beta(\alpha)}{2\eta(\alpha)}}{[q(\alpha)\,lng(\alpha)]^2}-\frac{-i\frac{\beta(\alpha)}{4\eta(\alpha)}}{\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)}+\frac{\frac{\beta(\alpha)}{4\eta(\alpha)}}{i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\;+2}.$$

So,

$$Y(\alpha)=\frac{-\frac{\beta(\alpha)}{2\eta(\alpha)}}{[q(\alpha)\,lng(\alpha)]^2}-\frac{-i\frac{\beta(\alpha)}{4\eta(\alpha)}}{\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)}+\frac{\frac{\beta(\alpha)}{4\eta(\alpha)}}{i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\;+2}+\frac{\frac{\beta(\alpha)}{\eta(\alpha)}}{i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\;+2},$$

Take the inverse of EMG, we have:

$$y(\tau)=\frac{1}{2}\tau-\frac{1}{4}+\frac{5}{4}e^{-2\tau}$$

ii) solve using Laplace transform

$$y^{'}(\tau)+2y(\tau)=\tau,\; y(0)=1$$

Solution: take Laplace technique to above equation gets:

$$sY(s)-1+2Y(s)=\frac{1}{s^2}.$$

$$Y\left(s\right)\left[s+2\right]=1+\frac{1}{s^{2}}=\frac{s^{2}+1}{s^{2}}.$$

$$Y\left(s\right)=\frac{s^{2}+1}{s^{2}(s+1)}$$

Using the partial fraction, and the simple simplifications, we get

$$Y\left(s\right)=\frac{1}{2s^{2}}-\frac{1}{4s}+\frac{5}{4(s+2)}$$

Taking the inverse LaPlace transform, we get

$$y\left(\tau\right)=\frac{1}{2}\tau-\frac{1}{4}+\frac{5}{4}e^{-2\tau}$$

Example 4.2. *solve:*

$$z^{'}+9z=cos(2\tau),\qquad z(0)=1,\qquad z(\frac{\pi}{2})=-1.$$

Solution
By taking EMG transform for the sides to this equation gives:

$$-\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}Z\left(\alpha\right)-i\frac{q\left(\alpha\right)}{\eta\left(s\right)}lng\left(\alpha\right)\frac{p\left(s\right)}{\eta\left(s\right)}z\left(0\right)-\frac{p\left(\alpha\right)}{\eta\left(\alpha\right)}z^{\prime}\left(0\right)+9Z\left(s\right)=-\frac{p\left(\alpha\right)}{\eta\left(\alpha\right)}\left[\frac{i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-4]}\right]$$

$$Z\left(\alpha\right)\left[-\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}+9\right]=i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}+a\frac{p\left(\alpha\right)}{\eta\left(\alpha\right)}-\frac{p\left(\alpha\right)}{\eta\left(\alpha\right)}\left[\frac{i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-4]}\right].$$

So,

$$Z\left(s\right)=\frac{-a\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-9}-\frac{-i\frac{q\left(s\right)}{\eta\left(s\right)}lng\left(s\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-9}+\frac{i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-4]\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-9]}.,$$

Using partial fraction for the third term $\left[\frac{i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-4]\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-9]}\right].$

We get:

$$\frac{\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)}{\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-4]\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-9]}=\frac{D\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+E}{\left[\frac{q\left(s\right)}{\eta\left(s\right)}lng\left(s\right)\right]^{2}-4]}-\frac{F\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+G}{\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^{2}-9]},$$

After computations, we obtain:
$D = -\frac{1}{5},\ \ E = 0,\ \ F = \frac{1}{5}\ and\ G = 0$. Then,

$$\frac{i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ \frac{\beta(\alpha)}{\eta(\alpha)}}{\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-4]\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-9]} = \frac{-i\frac{q(\alpha)}{5\,\eta(\alpha)}lng(\alpha)\ \frac{\beta(\alpha)}{\eta(\alpha)}}{\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-4]} - \frac{-i\frac{q(\alpha)}{5\,\eta(\alpha)}lng(s)\ \frac{\beta(\alpha)}{\eta(\alpha)}}{\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-9]},$$

S0,

$$Z(\alpha) = \frac{-a\frac{\beta(\alpha)}{\eta(\alpha)}}{\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-9} - \frac{-i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ \frac{\beta(\alpha)}{\eta(\alpha)}}{\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-9} + \frac{-i\frac{q(\alpha)}{5\,\eta(\alpha)}lng(\alpha)\ \frac{\beta(\alpha)}{\eta(\alpha)}}{\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-4]} - \frac{(-i)\frac{q(\alpha)}{5\,\eta(\alpha)}lng(\alpha)\ \frac{\beta(\alpha)}{\eta(\alpha)}}{\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-9]}.$$

Take the inverse of EMG, we have:

$$z(\tau) = \frac{a}{3}\sin(3\tau)\ + \frac{4}{5}\cos(3\tau)\ + \frac{1}{5}\cos(2\tau),$$

When $z\left(\frac{\pi}{2}\right), = -1.$
After computations, we obtain:

$$z(\tau) = \frac{4}{5}\sin(3\tau)\ + \frac{4}{5}\cos(3\tau)\ + \frac{1}{5}\cos(2\tau).$$

Example 4.3. *solve using EMG-transform*

$$\phi^{'} + 8\phi^{'} + 25\phi = 150, \qquad \phi(0) = \ \phi^{'}=0.$$

Solution
take EMG technique to above equation gets:

$$-\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2\phi(\alpha) - i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ \frac{\beta(\alpha)}{\eta(\alpha)}\phi(0) - \frac{\beta(\alpha)}{\eta(\alpha)}\phi'(0) + 8\left[-\frac{\beta(\alpha)}{\eta(\alpha)}\phi(0)\right]$$
$$+ 8i\frac{q(\alpha)}{\eta(\alpha)}lng(s)\ F(\alpha) + 25\phi = -\frac{150i\beta(\alpha)}{q(\alpha)ln(\alpha)}\ ,$$

$$H(\alpha) = \frac{-150ip(s)}{q(\alpha)\,lng(\alpha)\,[-\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2 8i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ +25]}.$$

Using partial fraction, we get:

$$\frac{-150i\beta(\alpha)}{q(\alpha)\,lng(\alpha)\,[-\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2 8i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ +25]} =$$
$$= \frac{D}{q(\alpha)\,lng(\alpha)} + \frac{Eq(\alpha)lng(\alpha)+F}{[-\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2 8i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ +25]},$$

After computations, we obtain:
$D=-6,\ \ E=\dfrac{-6}{[\eta(\alpha)]^2}\ and\ F=\dfrac{48i}{\eta(\alpha)}.$

Then,

$$H\left(\alpha\right)=\frac{-6i\beta\left(\alpha\right)}{q\left(\alpha\right)lng\left(\alpha\right)}+\frac{-6i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+4\right]^2+9]}+\frac{-24\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}}{\left[i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+4\right]^2+9]}$$

Take the inverse of EMG, we have:

$$h\left(\tau\right)=-6-6e^{-4\tau}\cos\left(3\tau\right)\ -8e^{-4\tau}\sin\left(3\tau\right).$$

Example 4.4. *A new Integral Transform (EMG) in a physical problem (Radioactive decay).*
Consider the ordinary differential equation (ODE) of the homogeneous first order [18].

$$\frac{dH}{d\tau}=-kH\left(\tau\right).$$

This equation forms the essential formula that defines the decay process. So, the decay constant k represents a steady rate of decay, which the function $H(\tau)$ represents the number of remaining atoms that have not decayed in a time interval τ.

By the EMG integral transform denoted as $E\{.\}$, one may derive as follows:

$$\mathrm{E}\{H'\left(\tau\right)\}+\mathrm{k}\{\mathrm{H}\left(\tau\right)\}=0, There,-\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}H\left(0\right)+i\,\frac{\mathrm{q}\left(\alpha\right)}{\eta\left(\alpha\right)}\ln g\left(\alpha\right)H(\alpha)+\mathrm{k}H\left(\alpha\right)=0$$

With initial condition $H\left(0\right)=H_0$.

So,

$$H\left(\alpha\right)[i\,\frac{\mathrm{q}\left(\alpha\right)}{\eta\left(\alpha\right)}\ln g\left(\alpha\right)+\mathrm{k}]=\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}H\left(0\right)$$

Then:

$$H\left(\alpha\right)=\frac{H_0\beta\left(\alpha\right)}{\eta\left(\alpha\right)[i\,\frac{\mathrm{q}(\alpha)}{\eta(\alpha)}\ln g\left(\alpha\right)+\mathrm{k}]},$$

Now, applying inverse A new integral transform EMG) on both sides, we have:

$$\mathrm{H}\left(\tau\right)=H_0e^{-k\tau}$$

The exact classical law of exponential nuclear decay.

5. Definition and Derivations EMG Technique of Partial Derivatives

To obtain the partial derivatives of EMG technique, we follow the following:

$$E\left[\frac{\partial f}{\partial\tau}(x,\tau)\right]=F\left(x,\alpha\right)$$

$$=\int_{t=0}^{\infty}\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\left[g\left(\alpha\right)\right]^{-\frac{iq\left(\alpha\right)}{\eta\left(\alpha\right)}\tau}\frac{\partial f}{\partial t}(x,\tau)d\tau\ ,$$

$$=\lim_{A\to\infty}\left[\left[\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}[g\left(\alpha\right)]^{-\frac{iq\left(\alpha\right)}{\eta\left(\alpha\right)}\tau}f\left(x,\tau\right)\right]_0^A+\mathrm{i}\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\int_{\tau=0}^{\infty}[g\left(\alpha\right)]^{-\frac{iq\left(s\right)}{\eta\left(s\right)}\tau}f\left(x,\tau\right)d\tau\right],$$

Using the property the improper integral in the first partial derivative, we get

$$E\left[\frac{\partial f}{\partial \tau}\right]=-\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}f\left(x,0\right)+\mathrm{i}\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)F\left(x,\alpha\right). \tag{7}$$

by assuming that f is a piecewise continues and is of exponential order.

So, $E\left[\frac{\partial f}{\partial x}(x,\tau)\right]=\int_{\tau=0}^{\infty}\frac{\beta\left(\alpha\right)}{\eta(\alpha)}[g\left(\alpha\right)]^{-\frac{iq\left(\alpha\right)}{\eta\left(\alpha\right)}\tau}\frac{\partial f}{\partial x}(x,\tau)d\tau=\frac{\partial}{\partial x}\int_{\tau=0}^{\infty}\frac{\beta\left(\alpha\right)}{\eta(\alpha)}\alpha f(x,\tau)d\tau=\frac{\partial}{\partial x}F(x,\alpha)$

$$E\left[\frac{\partial f}{\partial x}(x,\tau)\right]=\frac{\partial}{\partial x}F(x,\alpha) \tag{8}$$

Also, we can prove

$$E\left[\frac{\partial^2 f}{\partial x^2}(x,\tau)\right]=\frac{d^2}{dx^2}F(x,\alpha) \tag{9}$$

To find $E\left[\frac{\partial^2 f}{\partial \tau^2}(x,\tau)\right]$,

Let $\frac{\partial f}{\partial \tau}=r$, then

By applying equation (2), we get:

$$E\left[\frac{\partial^2 f}{\partial \tau^2}(x,\tau)\right]=E\left[\frac{\partial r}{\partial \tau}(x,\tau)\right]=-\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}r\left(x,0\right)+\mathrm{i}\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)E\left\{r\left(x,\tau\right)\right\},$$

$$E\left[\frac{\partial^2 f}{\partial \tau^2}(x,\tau)\right]=-\left[\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\right]^2\mathrm{F}\left(\mathrm{x},\alpha\right)-\mathrm{i}\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}\mathrm{f}\left(\mathrm{x},0\right)-\frac{p\left(\alpha\right)}{\eta\left(\alpha\right)}\frac{\partial f}{\partial \tau}(x,0) \tag{10}$$

we can easily extend this result to the $n\ th$ incomplete subsidiary.

5.1. Results and discussions

Here, in this section, we will describe a few significant uses of applications as example 1 in the first order partial differential equations and examples 1-3 are the second-order partial differential equations, such as the wave, the telegraph, the Klein-Gordon, because they are of utmost significance in physical, electrical, and engineering reality.

Example 5.1. *The solution of the first order PDE:*

$$z_x=2z_\tau+z, \qquad z\left(x,0\right)=6e^{-3x} \tag{11}$$

and z is limited by $x>0,\ \tau>0$.

By taking EMG transform for the sides of equation (11), we get:

$$\frac{dZ\left(x,\alpha\right)}{dx}-2[\frac{p\left(\alpha\right)}{\eta\left(\alpha\right)}+i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)]\,Z\left(\alpha\right)=Z\left(x,\alpha\right),$$

$$\frac{dZ\left(x,\alpha\right)}{dx}-2[6e^{-3x}(\frac{p\left(\alpha\right)}{\eta\left(\alpha\right)})+i\frac{q\left(s\right)}{\eta\left(s\right)}lng\left(\alpha\right)Z\left(\alpha\right)]\ -Z\left(x,\alpha\right)=0,$$

$$\frac{dZ\left(x,\alpha\right)}{dx}-[2i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+1]\ Z\left(x,\alpha\right)=-12\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}e^{-3x},$$

This is " LODE":

$p\left(x\right)=-[2i\frac{q\left(\alpha\right)}{\eta\left(\alpha\right)}lng\left(\alpha\right)+1]$ *and* $Q\left(x\right)=-12\frac{\beta\left(\alpha\right)}{\eta\left(\alpha\right)}e^{-3x}$.

So,

$$p(x) = e^{\int p(x)dx} = e^{\int -[2i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)+1]\ dx} = e^{-[2i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)+1]\ x}.$$

Therefore

$$Z(x,\alpha) = \frac{1}{p(x)}.\int p(x)\,Q(x)\,dx,$$

$$Z(x,\alpha) = e^{[2i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)+1]\ x}.\int e^{-[2i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)+1]\ x}(-12)\frac{\beta(\alpha)}{\eta(\alpha)}e^{-3x}dx$$

$$Z(x,\alpha) = e^{[2i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)+1]\ x}.\left[\frac{(-12)\frac{\beta(\alpha)}{\eta(\alpha)}}{-2[i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)+2]}e^{[-2i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)-4]\ x}+C\right],$$

$$Z(x,\alpha) = \left[\frac{6(\frac{\beta(\alpha)}{\eta(\alpha)}}{[i\frac{q(s)}{\eta(s)}lng(s)+2]}e^{-3x}+Ce^{[2i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)+1]\ x}\right]$$

Since Z is limited, C must be zero. Taking the inverse EMG technique, we get:

$$z(x,\tau) = 6e^{-3x}e^{-2\tau} = 6e^{-2\tau-3x}.$$

Example 5.2. *Solve the Homogeneous Wave Equation:*

$$v_{zz} = 4v_{xx},\quad v(x,0) = sin(\pi x)\,,\quad v_z(x,0) = 0,\quad z>0\,,x>0 \tag{12}$$

Using the EMG technique for the sides of the above equation (12) and utilizing occur, we obtain:

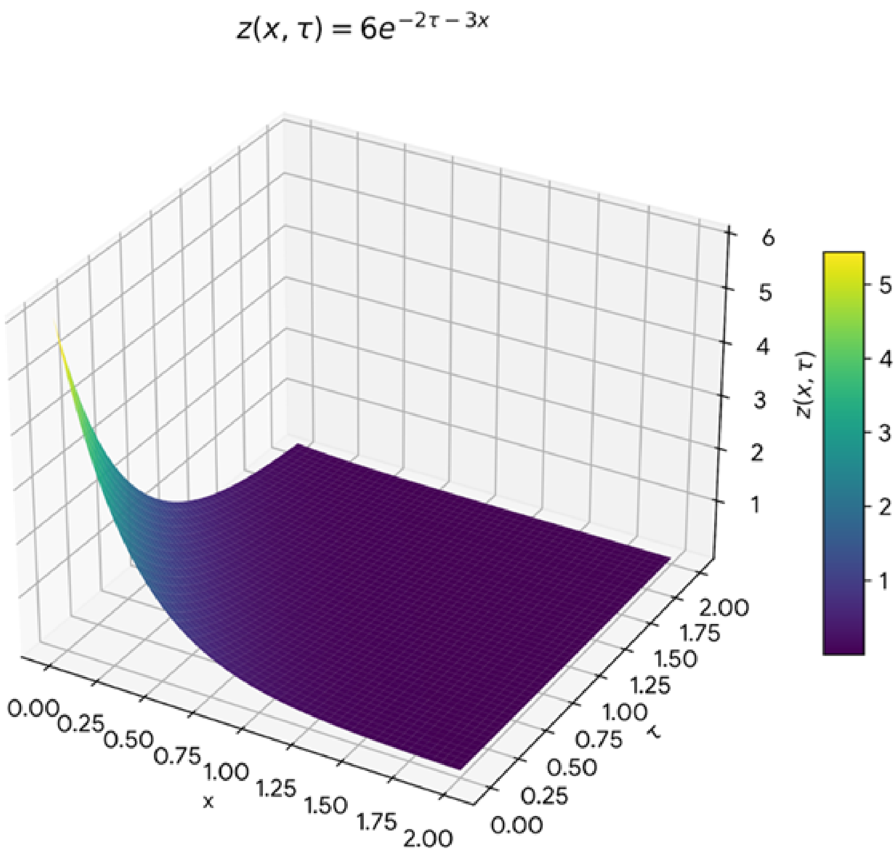
$$-\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2 V(x,\alpha) - i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)}v(x,0) - \frac{\beta(\alpha)}{\eta(\alpha)}v_t(x,0) = 4V''(x,s)$$

$$4V''(x,\alpha) + \left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2 v(x,s) = -i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)}\sin(\pi x),$$

$$D^2V(x,\alpha) + \left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2 v(x,s) = -i\frac{q(\alpha)}{4\,\eta(\alpha)}lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)}\sin(\pi x),$$

So,

$$\begin{aligned}V(x,\alpha) &= \frac{-i\frac{q(\alpha)}{4\,\eta(\alpha)}lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)}}{D^2+\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2}\sin(\pi x)\\ &= \frac{-i\frac{q(\alpha)}{4\,\eta(\alpha)}lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)}}{-\pi^2+\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2}\sin(\pi x)\\ &= \frac{-i\frac{q(\alpha)}{4\,\eta(\alpha)}lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)}}{-4\pi^2+\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2}\sin(\pi x)\,,\end{aligned}$$



$$V(x,\alpha)=\frac{-i\dfrac{q(\alpha)}{4\,\eta(\alpha)}lng(\alpha)\,\dfrac{\beta(\alpha)}{\eta(\alpha)}}{-4\pi^2+\left[\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2}\sin(\pi x)$$

Presently, we take EMG technique to track down the specific arrangement of case (6) in the form:

$$v(x,z)=cos(2\pi z)\,.sin(\pi x)\,.$$

Example 5.3. *Solve the Telegraph Equation:*

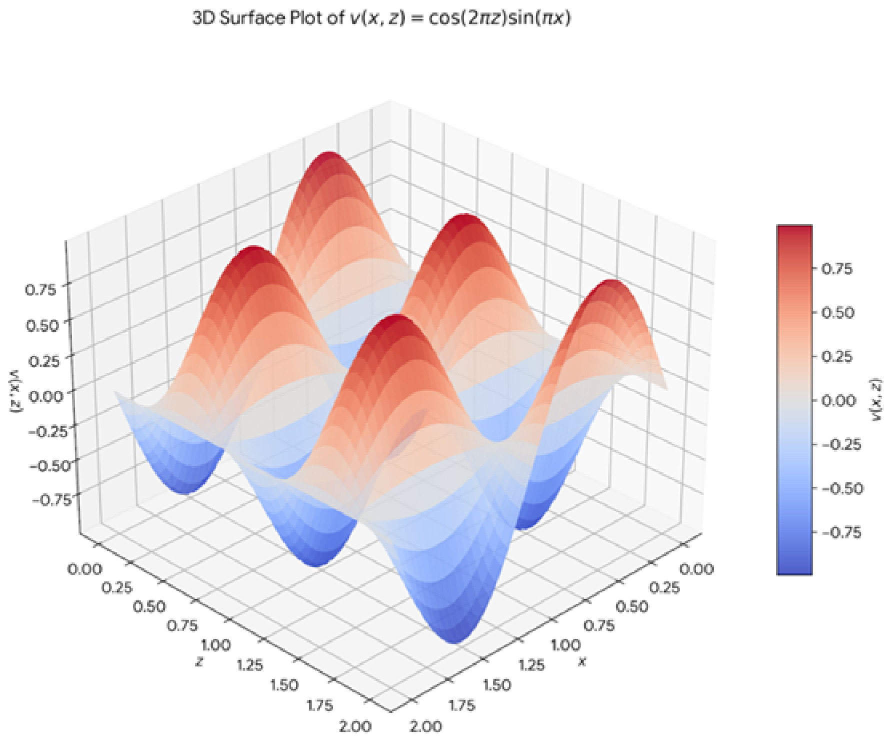
$$\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}+u$$

Subject to initial cases

$$u(z,0)=e^z$$

$$u_t(z.0)=-2e^z$$

Solution: by Applying EMG transform we have:



$$U''(z,\alpha) - \left[\frac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\right]^2 U(z,\alpha) - i\frac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)} u(z,0) - \frac{\beta(\alpha)}{\eta(\alpha)} u_t(z,0)$$
$$+ 2\left[-\frac{p(\alpha)}{\eta(\alpha)} u(z,0) + i\frac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\right] U(z,\alpha) + U(x,s)$$

$$U''(z,\alpha) + \left[\frac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\right]^2 U(z,\alpha) - 2i\frac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)} u(z,0)$$
$$+ i\frac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)} e^z + \frac{\beta(\alpha)}{\eta(\alpha)}(-2e^z) + 2\frac{\beta(\alpha)}{\eta(\alpha)}(e^z) - U(z,\alpha)$$

$$U(z,\alpha) = \frac{-i\dfrac{q(\alpha)}{\eta(\alpha)} lng(s)\,\dfrac{\beta(\alpha)}{\eta(\alpha)}}{D^2 + \left[\dfrac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\right]^2 - 2i\dfrac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\ - 1} e^z$$

$$U(z,\alpha) = \frac{-i\dfrac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\,\dfrac{p(\alpha)}{\eta(\alpha)}}{(1)^2 + \left[\dfrac{q(\alpha)}{\eta(\alpha)} lng(v)\right]^2 - 2i\dfrac{q(\alpha)}{\eta(\alpha)} lng(\alpha)\ - 1} e^z$$

So,

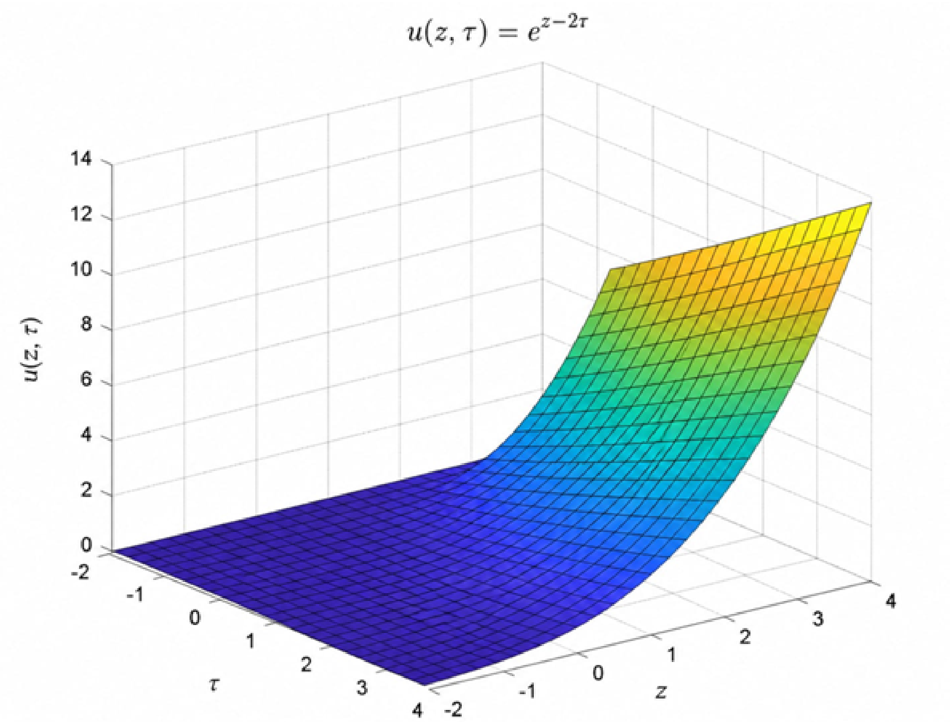
$$U(z,\alpha)=\frac{-i\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ \dfrac{p(\alpha)}{\eta(\alpha)}}{(1)^2+\left[\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2-2i\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ -1}e^z$$

After computations, we have:

$$U(z,s)=\frac{\dfrac{p(\alpha)}{\eta(\alpha)}}{i\dfrac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\ +2}e^z$$

Take inverse of EMG technique we obtain:

$$u(z,\tau)=\ e^{-2\tau}e^z=e^{z-2\tau}.$$



Example 5.4. *Solve the Klein-Gordon Equation:*

$$v_{\tau\tau}=v_{xx}+v_x+2v \qquad -\infty<x<\infty \qquad t>0$$

With conditions:

$$u(x,0)=e^x, \qquad u_t(x,0)=0.$$

We applying EMG technique, we get:

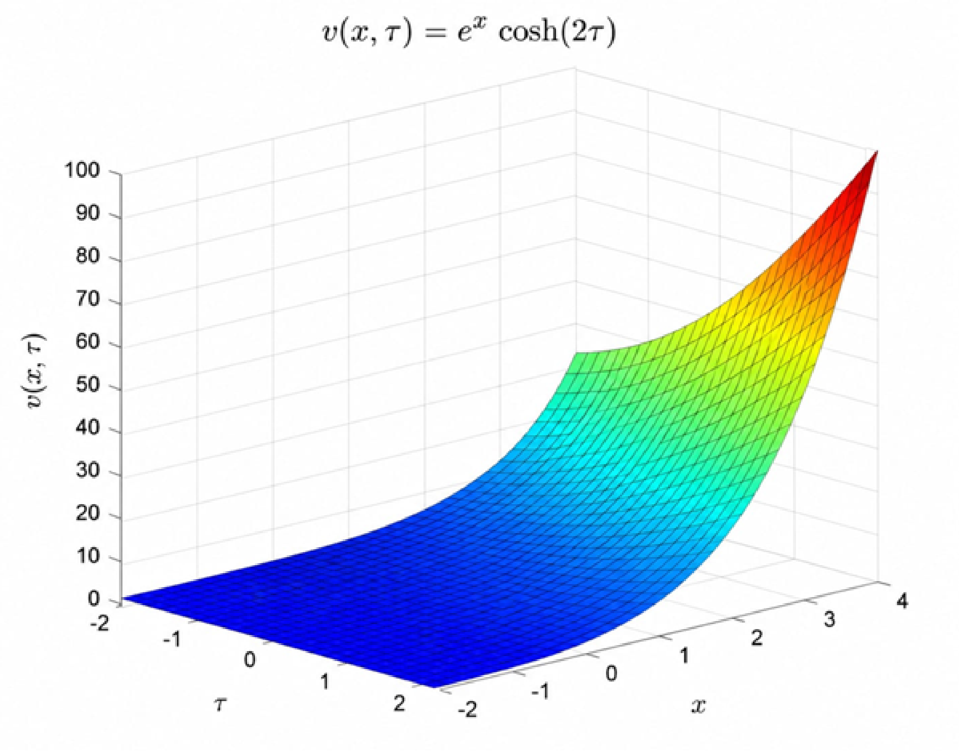
$$-\left[\frac{q(v)}{\eta(\alpha)}lng(v)\right]^2 V(x,\alpha)-i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,\frac{p(\alpha)}{\eta(\alpha)}v(x,0)-\frac{\beta(\alpha)}{\eta(\alpha)}v_\tau(x,0)=v''(x,\alpha)+V'(x,s)+2V(x,\alpha)$$

$$-\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2 V(x,\alpha)-i\frac{q(s)}{\eta(s)}lng(s)\,\frac{\beta(\alpha)}{\eta(s)}e^x=V''(x,\alpha)+V'(x,\alpha)+2V(x,\alpha)$$

$$V(x,\alpha)=\frac{-i\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\,\frac{\beta(\alpha)}{\eta(\alpha)}}{D^2+D+2+\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2}e^x=\frac{-i\frac{q(\alpha)}{\eta(v)}lng(v)\,\frac{\beta(\alpha)}{\eta(\alpha)}}{\left[\frac{q(\alpha)}{\eta(\alpha)}lng(\alpha)\right]^2+4}e^x$$

By taking the inverse of EMG technique, we get:

$$v(x,\tau)=e^x cosh(2\tau)\,.$$



6. Conclusion and Future work

In the current paper, a novel approach i.e. "EMG transform" has been used to study the solution of LODE's and LPDE's. Additionally, its applicability demonstrated using some partial differential operator equations (wave, heat, Laplace and others), we check the particular solutions of these operator equations. We observe that this approach is more general than the majority of the transforms obtained by Laplace transform, and that is because of the functions of parameter α, we observed that this method is simplified from another methods as a domain decomposition method and This transform is a highly generalized, multi-parameter integral transform. While classical transforms like Laplace, Aboodh, or Elzaki are simpler to compute for standard problems and it offers distinct mathematical and practical advantages in specific scenarios. In the future work, we will design solutions for the fractional linear and nonlinear ODEs and linear and nonlinear PDEs in the EMG technique and try to solve numeric solution as homotopy permutation method.

Acknowledgement

This research is funded by the Deanship of Scientific Research at Zarqa University.

REFERENCES

1. E. A. Mansour, and E. A. Kuffi, *The Complex SEE Transform Technique in Difference Equations and Differential Difference Equations*, International Journal of Neutrosophic Science, vol. 23, no. 1, pp. 323–334, 2024.
2. G. Gharib, and R. Saadeh, *Reduction of the self dual Yang Mills equations to sinh poisson equation and exact solutions*, WSEAS Transactions on Mathematics, 2021.
3. G. Gharib, M. Alsoudi, and I. Benkemache, *Solving Non-Linear Fractional Newell Whitehead Equation By Atomic Solution*, WSEAS Transactions on Mathematics, vol. 22, pp. 114–119, 2023.
4. R.Saadeh, A.K. Sedeeg, B. Ghazal,and G. Gharib, *Double Formable Integral Transform for Solving Heat Equations*, Symmetry, vol. 15, no. 218, pp. 1-16, 2023.
5. K. S. Aboodh, *Application of new transform 'Aboodh Transform' to partial differential equations*, Global Journal of Pure and Applied Mathematics, vol. 10, no. 2, pp. 249–254, 2014.
6. K. S. Aboodh, *The New Integral Transform 'Aboodh Transform'*, Global Journal of Pure and Applied Mathematics, vol. 9, no. 1, pp. 35–43, 2013.
7. C. Constanda, *Solution techniques for elementary partial differential equations*, Chapman and Hall/CRC, 2002.
8. D. G. Duffy, *Transform methods for solving partial differential equations*, CRC Press, 2004.
9. T. M. Elzaki, *The new integral transform 'Elzaki transform'*, Global Journal of Pure and Applied Mathematics, vol. 7, no. 1, pp. 57–64, 2011.
10. T. M. Elzaki, *Application of new transform 'Elzaki transform' to partial differential equations*, Global Journal of Pure and Applied Mathematics, vol. 7, no. 1, pp. 65–70, 2011.
11. G. M. Gharib, E. A. Kuffi, M. Alsaoudi, and M. A. Labeeb, *Solution of Integral Equations and Mayan Transform*, International Journal of Applied Mathematics, vol. 38, no. 11s, pp. 2267–2282, 2025.
12. M. A. M. Mahgoub, *The new integral transform 'Mahgoub Transform'*, Advances in Theoretical and Applied Mathematics, vol. 11, no. 4, pp. 391–398, 2016.
13. M. A. M. Mahgoub, and A. A. Alshikh, *An application of new transform 'Mahgoub Transform' to partial differential equations*, Mathematical Theory and Modeling, vol. 7, no. 1, pp. 7–9, 2017.
14. M. Mohand, and A. Mahgoub, *The new integral transform 'Mohand Transform'*, Advances in Theoretical and Applied Mathematics, vol. 12, no. 2, pp. 113–120, 2017.
15. T. Salameh, G. M. Gharib, M. Alsaoudi, and M. A. Labeeb, *Using Conformable Fractional Laplace Transform to Solve Fractional System*, Statistics, Optimization and Information Computing, vol. 15, no. 5, pp. 4277–4285, 2026.
16. A. H. Mohammed, A. H. Battor, E. Kuffi, and A. Kuffi, *Battor-Al-Zughair Transform and Its Applications*, AIP Conference Proceedings, vol. 2591, 050022, 2023.
17. T. J. Aldhlki, E. A. Kuffi, R. Saadeh, A. Qazza, L. S. Wong, and M. A. Labeeb, *Mayan Technique for Solving First Order Complex Differential Operator Equation in Engineering Problems*, 2025 Asian Conference on Communication and Networks (ASIANComNet), IEEE, pp. 5, 2025. DOI: 10.1109/ASIANComNet68615.2025.11579609.
18. A. H. Mohammed, A. H. Battor, E. Kuffi, Battor-Al-Zughair , *Transform and Its Applications*, AIP Conference Proceedings vol. 2591, 050022, 2023.
19. S. F. Maktoof, E. Kuffi, and E. S. Abbas, *Emad-Sara Transform: a new integral transform*, Journal of Interdisciplinary Mathematics, vol. 24, no. 4, 2021.
20. E. A. Mansour, S. Mehdi, and E. A. Kuffi, *The New integral transform and Its Applications*, International Journal of Nonlinear Analysis and Applications, vol. 12, no. 2, 2021.
21. E. A. Mansour, E. A. Kuffi, and S. A. Mehdi, *The new integral transform "SEE transform" and Its Applications*, Periodicals of Engineering and Natural Sciences, vol. 9, no. 2, 2021.
22. S. A. Mahdi, E. A. Kuffi, and J. A. Jasim, *Solving Ordinary Differential Equations Using a New General Complex Integral transform*, Journal of Interdisciplinary Mathematics, 2022.
23. E. A. Kuffi, A. H. Mohammed, A. Q. Majde, and E. S. Abbas, *Applying Al-Zughair transform on nuclear physics*, International Journal of Engineering and Technology, vol. 9, no. 1, 2020.
24. E. A. Kuffi, N. K. Meftin, E. S. Abbas, and E. A. Mansour, *A Review on the integral Transforms*, Eurasian Journal of Physics, Chemistry and Mathematics, vol. 1, 2021.
25. T. M. Elzaki, *The new integral transform 'Elzaki transform'*, Global Journal of Pure and Applied Mathematics, vol. 7, no. 1, pp. 57–64, 2011.
26. W. Kurt, *Integral transforms in science and engineering*, Springer Science & Business Media, vol. 11, 2013.

27. A. Kamal, and H. Sedeeg, *The New Integral Transform Kamal Transform*, Advances in Theoretical and Applied Mathematics, vol. 11, no. 4, pp. 451–458, 2016.
28. E. Kuffi, and A. Moazzam, *Solution of population growth rate linear differential models via two parametric SEE transformation*, IHJPAS, vol. 36, no. 2, 2023.
29. A. H. Mohammed, A. H. Battor, and E. Kuffi, *Kuffi-Al-Tememe Transform and Its Applications*, 2023.
30. M. Mohand, and A. Mahgoub, *The new integral transform Mohand Transform*, Advances in Theoretical and Applied Mathematics, vol. 12, no. 2, pp. 113–120, 2017.
31. I. A. Mansour, E. Kuffi, and S. A. Mehdi, *Applying SEE Transform to Solve Linear Volterra Integro differential Equation of the Second Kind*, AIP Conference Proceedings, vol. 2591, 050008, 2023.
32. S. A. Mehdi, E. Kuffi, and J. Adel, *Using a New General Complex Integral Transform for Solving Population Growth and Decay Problems*, IHJPAS, vol. 36, no. 1, 2023.
33. Zamil, and E. Kuffi, *A Novel Integral Transform: INEM-Transform*, Journal of Kufa for Mathematics and Computer, vol. 10, no. 2, pp. 109–113, 2023.
34. S. A. Mehdi, E. Kuffi, and J. Adel, *Solving the Delay Differential Equations by Using the SEJI Integral Transform*, ICCITM, 2022.
35. S. A. Mehdi, E. Kuffi, J. Adel, and J. A. Jasim, *Solving the partial differential equations by using SEJI integral transform*, Journal of Interdisciplinary Mathematics, 2023.
36. E. Kuffi, and A. Moazzam, *Solution of population growth rate linear differential models via two parametric SEE transformation*, IHJPAS, vol. 36, no. 2, 2023.
37. A. Moazzam, E. Kuffi, Z. Ul Abideen, and A. Anjum, *Two parametric SEE transformation and its applications in solving differential equations*, Al-Qadisiyah Journal for Engineering Sciences, vol. 16, pp. 116–120, 2023.
38. R. A. Deshmukh, V. S. Goswami, and D. P. Patil, *The new integral transform DPR Transform and its Application*, Statistics, Optimization and Information Computing, vol. 16, pp. 1561–1578, 2026.