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A New Unit Generated Family of Distributions: Classical Estimation Methods and Applications to Real Data

Alaa R. El-Alosey [1], Mohammed Elgarhy [1,2,*], Ahmed M. Gemeay [1]

[1] *Department of Mathematics, Faculty of Science, Tanta University, Tanta 31527, Egypt; alaa.abdelmaksoud@science.tanta.edu.eg; mohammed.elgarhy@ecu.edu.eg; ahmed.gemeay@science.tanta.edu.eg*
[2] *Faculty of Computers and Information Systems, Egyptian Chinese University, Nasr City, Egypt*

Abstract In this article, a new unit-generated family of distributions called the unit inverse Lindley family is investigated and discussed. Four sub-models of the suggested truncated family are discussed, such as unit inverse Lindley- exponential, unit inverse Lindley- Lomax, unit inverse Lindley- Topp Leone, and unit inverse Lindley- Kumaraswamy distributions. Some important Statistical features of the new unit-generated family are computed, such as quantiles, moments, and moment generating function. Different types of entropies, such as Rényi entropy, Tsallis entropy, Havrda and Charvat entropy, and Arimoto entropy, are computed. Sixteen different approaches to estimation, such as maximum likelihood, least-squares, maximum product of spacing, weighted least squares, Cramér-von Mises, and Anderson-Darling, are discussed to estimate the parameters. Monte Carlo simulations are used to investigate the performance of the estimation methodologies. Finally, two real-world datasets are examined to show the practical applicability and relevance of the suggested family.

Keywords Unit inverse Lindley distribution, Generated family, Maximum likelihood, Maximum product of spacings, Anderson-Darling method, Simulation.

AMS 2010 subject classifications 62G05, 62H10, 62E15, 62N01

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1. Introduction

The performance of the estimating methodologies is improved. The importance of probability distributions in modern data processing has had a significant impact on the development of new distribution theories in recent decades. To select the optimal parametric model, one must first understand the underlying data distribution. This selection strategy increases inference accuracy while minimizing information loss. Despite the fact that numerous unbounded models have been developed, there remains a significant paucity of bounded models for specific real-world events. Unbounded models usually fail to represent the most important restrictions of real-world data. Distributions having support on the unit interval [0, 1] are especially useful in modeling data like rates, fractions, and percentages. Such data is regularly generated in a variety of sectors, including risk analysis, psychology, economics, medicine, and engineering. Distributions having desirable characteristics, such as hazard rate functions (HRFs) with bathtub, declining, and growing shapes, are particularly desirable. Only a limited number of distributions are available for modeling data on the unit interval. Some notable instances of newly bounded distributions are as follows: the unit exponentiated Fréchet [1], unit half-normal [7], unit Zeghdoudi [8], unit Maxwell Boltzmann [9], unit inverse exponentiated Lomax [16], unit exponentiated Lomax [19], unit-power Burr X [20], power unit inverse Lindley [22], unit inverse Gaussian [23], unit Johnson S_U [25], unit Xgamma [27],

*Correspondence to: Mohammed Elgarhy (Email: mohammed.elgarhy@ecu.edu.eg). Department of Mathematics, Faculty of Science, Tanta University, Tanta 31527, Egypt.

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unit inverse exponentiated Pareto [28], unit Gompertz [29, 46], trigonometric unit [10], unit inverse exponentiated Weibull [30], unit exponentiated half logistic [31], unit power Lomax [32], unit new power function [36], unit Chen [38], unit Burr-XII [39], unit Teissier [40], unit Muth [42], unit Birnbaum-Saunders [43], unit generalized half-normal [44], unit Weibull [45], unit Lindley [47], unit gamma [48], unit logistic [49], unit Burr-III [51], unit extended exponential [56], unit half-logistic geometric [57], unit inverse Weibull [59], unit logistic exponential [60], unit inverse exponentiated Lomax [17], unit Burr-Hatke [61], and unit Bilal [65] distributions.

On the other hand, statistical models can be used to describe and forecast real-world events. Numerous extended distributions have been extensively used in data modeling over the past few years. Recent improvements have concentrated on building new families that broaden well-known distributions while also giving enormous flexibility in modeling data in practice. This is particularly relevant in disciplines such as biology, environmental science, engineering, and economics. Several researchers have suggested some of the generated families of distributions in this regard (see, for instance, [13, 14, 37, 63]). Our focus here is on the same approach developed for the beta-G family studied in [18]. The cumulative distribution function (CDF) beta-G family is given by

$$F(x) = \int_0^{G(x)} r(t)dt, \tag{1}$$

where $r(t)$ is the probability density function (PDF) of the beta distribution and $G(x)$ is a CDF for any continuous distribution. To establish a new family, simply take another PDF for $r(t)$ with support $[0, 1]$ (see [18]).

In fact, the literature contains only a few references to the unit-G family. Reference [6] proposed the unit Garima-G family, [24] explored the unit extended Weibull-G family, and [54] proposed a unit Nadarajah-Haghighi-G family.

The fundamental goal of this study is to introduce the unit inverse Lindley-G (UIL-G) family. The following arguments provide sufficient motivation for studying it.

- The UIL-G family of distributions is very flexible and simple.
- The UIL-G family of distributions includes several new continuous statistical distributions.
- The shapes of the PDFs of the generated distributions can be unimodal, decreasing, bathtub, right-skewed, and near-symmetric. Also, the hazard rate function (HRF) shapes for these distributions can be increasing, decreasing, U-shaped, upside-down-shaped, N-shaped and J- shaped.
- Various important statistical and mathematical properties of the UIL-G family are computed.
- Sixteen different approaches of estimation are used to estimate the UIL- Lindley (UILLi) distribution parameters based on complete samples.
- Two real data sets are used to illustrate the flexibility of the UILLi distribution. The application results indicate that the UILLi distribution can fit the data more accurately than other comparable distributions.

The rest of this article is organized as follows: Section 2 defines the important functions of the UIL-G family. Four sub-models of the UIL-G family are provided in Section 3. In Section 4, some important statistical properties of the UIL-G family are computed. Section 5 discusses sixteen different estimation methods for the unknown parameters of the UIL-G family. A simulation study to examine the theoretical performance of different estimation methods for the UILLi distribution is investigated in Section 6. The practicality and goodness of fit of the suggested models using two actual data sets are presented in Section 7. Finally, Section 8 provides concluding remarks throughout the article.

2. Formulation of the UIL-G Family

In this section, we suggest a new unit family based on the unit inverse Lindley (UIL) distribution [22]. The CDF and PDF of the UIL distribution are given by

$$G(x;\delta) = \frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{x}\right)e^{-\frac{\delta}{x}}, \quad 0 < x \leq 1,\ \delta > 0, \tag{2}$$

and

$$g(x;\delta)=\frac{\delta^2e^{\delta}}{1+\delta}x^{-3}e^{-\frac{\delta}{x}},\quad 0<x\leq 1,\ \delta>0. \tag{3}$$

To formulate the UIL-G family, let $G(x;\varphi)$ and $g(x;\varphi)$ be the baseline CDF and PDF of any continuous distribution, and φ is a vector of parameters. The CDF of the UIL-G family is given by employing the generalized version of the B-G generator stated in (1) and utilizing the UIL distribution (3) as below:

$$F\left(x;\delta,\varphi\right)=\frac{\delta^2e^{\delta}}{1+\delta}\int_0^{G(x;\varphi)}t^{-3}e^{-\frac{\delta}{t}}dt=\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{G\left(x;\varphi\right)}\right)e^{-\frac{\delta}{G(x;\varphi)}},\quad x\in\mathbb{R},\ \ \delta>0, \tag{4}$$

where δ is a scale parameter. Therefore, the PDF of the UIL-G family is provided via

$$f\left(x;\delta,\varphi\right)=\frac{\delta^2e^{\delta}}{1+\delta}g\left(x;\varphi\right)G\left(x;\varphi\right)^{-3}e^{-\frac{\delta}{G(x;\varphi)}},\quad x\in\mathbb{R},\ \ \delta>0. \tag{5}$$

The survival function (SF), HRF, reversed HRF, and cumulative HRF of the UIL-G family are provided, respectively, via

$$S\left(x;\delta,\varphi\right)=1-\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{G\left(x;\varphi\right)}\right)e^{-\frac{\delta}{G(x;\varphi)}},\quad x\in\mathbb{R},\ \ \delta>0,$$

$$h\left(x;\delta,\varphi\right)=\frac{\frac{\delta^2e^{\delta}}{1+\delta}g\left(x;\varphi\right)G\left(x;\varphi\right)^{-3}e^{-\frac{\delta}{G(x;\varphi)}}}{1-\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{G(x;\varphi)}\right)e^{-\frac{\delta}{G(x;\varphi)}}},\quad x\in\mathbb{R},\ \ \delta>0, \tag{6}$$

$$\tau\left(x;\delta,\varphi\right)=\frac{\delta^2g\left(x;\varphi\right)}{G\left(x;\varphi\right)^2\left(\delta+G\left(x;\varphi\right)\right)},\quad x\in\mathbb{R},\ \ \delta>0,$$

and

$$H\left(x;\delta,\varphi\right)=-\log\left(1-\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{G\left(x;\varphi\right)}\right)e^{-\frac{\delta}{G(x;\varphi)}}\right),\quad x\in\mathbb{R},\ \ \delta>0.$$

3. Four Sub-models

In this section, we formulate four new sub-models of the UIL-G family, such as UIL-exponential (UILE), UIL-Lomax (UILLo), UIL-Lindley (UILTLi), and UIL-Kumaraswamy (UILK) distributions.

3.1. The UIL- exponential distribution

The CDF, PDF and HRF of the UILE distribution are constructed from (4), (5) and (6) by taking $G\left(x;\theta\right)=1-e^{-\theta x},\theta,x>0$, and $g\left(x;\theta\right)=\theta e^{-\theta x}$, as follows:

$$F\left(x;\delta,\theta\right)=\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{1-e^{-\theta x}}\right)e^{-\frac{\delta}{1-e^{-\theta x}}},\quad x>0,\ \ \delta,\theta>0,$$

$$f\left(x;\delta,\theta\right)=\frac{\theta\delta^2e^{\delta}}{1+\delta}e^{-\theta x}\left(1-e^{-\theta x}\right)^{-3}e^{-\frac{\delta}{1-e^{-\theta x}}},\quad x>0,\ \ \delta,\theta>0,$$

and

$$h\left(x;\delta,\theta\right)=\frac{\frac{\theta\delta^2e^{\delta}}{1+\delta}e^{-\theta x}\left(1-e^{-\theta x}\right)^{-3}e^{-\frac{\delta}{1-e^{-\theta x}}}}{1-\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{1-e^{-\theta x}}\right)e^{-\frac{\delta}{1-e^{-\theta x}}}},\quad x>0,\ \ \delta,\theta>0.$$

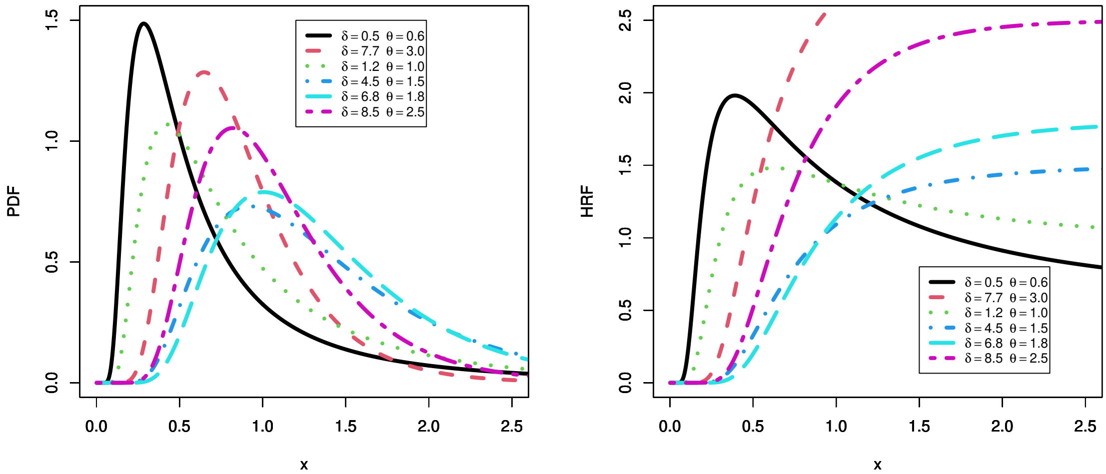

Figure 1. 2D plots of PDF and HRF for the UILE distribution

3.2. *The UIL- Lomax distribution*

The CDF, PDF and HRF of the UILLo distribution are investigated from (4), (5) and (6) by taking $G\left(x;\alpha,\beta\right)=1-\left(1+\frac{x}{\beta}\right)^{-\alpha}$, $\alpha,\beta,x>0$, and $g\left(x;\alpha,\beta\right)=\frac{\alpha}{\beta}\left(1+\frac{x}{\beta}\right)^{-\alpha-1}$, as follows:

$$F\left(x;\delta,\alpha,\beta\right)=\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}\right)e^{-\frac{\delta}{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}},\quad x>0,\ \ \delta,\alpha,\beta>0,$$

$$f\left(x;\delta,\alpha,\beta\right)=\frac{\alpha\delta^{2}e^{\delta}}{\beta(1+\delta)}\left(1+\frac{x}{\beta}\right)^{-\alpha-1}\left(1-\left(1+\frac{x}{\beta}\right)^{-\alpha}\right)^{-3}e^{-\frac{\delta}{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}},x>0,\ \ \delta,\alpha,\beta>0,$$

and

$$h\left(x;\delta,\alpha,\beta\right)=\frac{\frac{\alpha\delta^{2}e^{\delta}}{\beta(1+\delta)}\left(1+\frac{x}{\beta}\right)^{-\alpha-1}\left(1-\left(1+\frac{x}{\beta}\right)^{-\alpha}\right)^{-3}e^{-\frac{\delta}{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}}}{1-\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}\right)e^{-\frac{\delta}{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}}},x>0,\ \ \delta,\alpha,\beta>0.$$

3.3. *The UIL- Lindley distribution*

The CDF, PDF and HRF of the UILLi distribution are constructed from (4), (5) and (6) by taking $G\left(x;\lambda\right)=1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}$, $\lambda,x>0$, and $g\left(x;\lambda\right)=\frac{\lambda^{2}}{\lambda+1}\left(1+x\right)e^{-\lambda x}$, as follows:

$$F\left(x;\delta,\lambda\right)=\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}}\right)e^{-\frac{\delta}{1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}}},\quad x>0,\ \ \delta,\lambda>0,$$

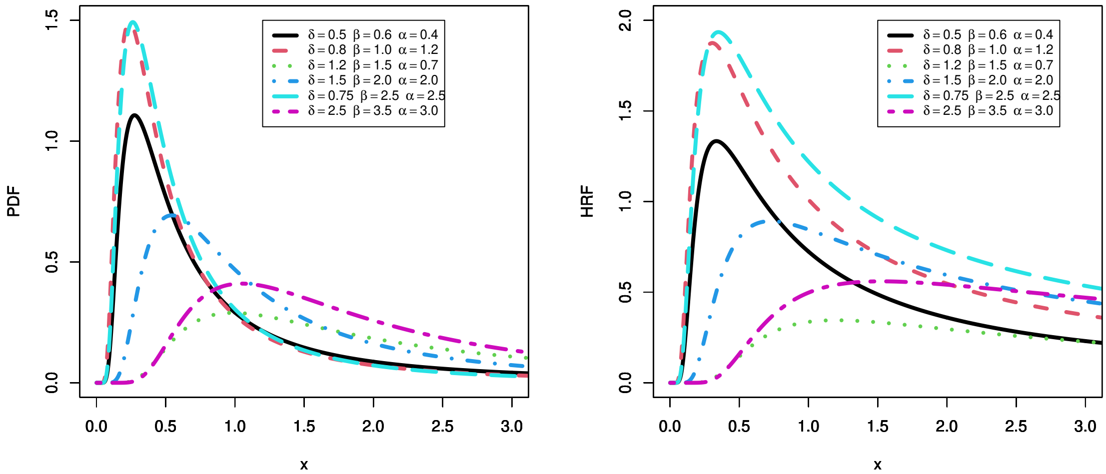

Figure 2. 2D plots of PDF and HRF for the UILLo distribution

$$f\left(x;\delta,\lambda\right)=\frac{\lambda^{2}\delta^{2}e^{\delta}(1+x)\,e^{-\lambda x}}{(1+\delta)(1+\lambda)}\left(1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}\right)^{-3}e^{-\frac{\delta}{1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}}},\quad x>0,\ \ \delta,\lambda>0,$$

and

$$h\left(x;\delta,\lambda\right)=\frac{\frac{\lambda^{2}\delta^{2}e^{\delta}(1+x)e^{-\lambda x}}{(1+\delta)(1+\lambda)}\left(1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}\right)^{-3}e^{-\frac{\delta}{1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}}}}{1-\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}}\right)e^{-\frac{\delta}{1-\left(1+\frac{\lambda x}{\lambda+1}\right)e^{-\lambda x}}}},\quad x>0,\ \ \delta,\lambda>0.$$

3.4. The UIL- Kumaraswamy distribution

The CDF, PDF and HRF of the UILK distribution are formulated from (4), (5) and (6) by taking $G\left(x;a,b\right)=1-\left(1-x^{a}\right)^{b}, a,b>0, x\in(0,1)$ and $g\left(x;a,b\right)=abx^{a-1}\left(1-x^{a}\right)^{b-1}$, as follows:

$$F\left(x;\delta,a,b\right)=\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{1-\left(1-x^{a}\right)^{b}}\right)e^{-\frac{\delta}{1-(1-x^{a})^{b}}},\quad x\in(0,1),\ \ \delta,a,b>0,$$

$$f\left(x;\delta,a,b\right)=\frac{\delta^{2}e^{\delta}abx^{a-1}\left(1-x^{a}\right)^{b-1}}{1+\delta}\left(1-\left(1-x^{a}\right)^{b}\right)^{-3}e^{-\frac{\delta}{1-(1-x^{a})^{b}}},\quad x\in(0,1),\ \ \delta,a,b>0,$$

and

$$h\left(x;\delta,a,b\right)=\frac{\frac{\delta^{2}e^{\delta}abx^{a-1}(1-x^{a})^{b-1}}{1+\delta}\left(1-\left(1-x^{a}\right)^{b}\right)^{-3}e^{-\frac{\delta}{1-(1-x^{a})^{b}}}}{1-\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{1-(1-x^{a})^{b}}\right)e^{-\frac{\delta}{1-(1-x^{a})^{b}}}},\quad x\in(0,1),\ \ \delta,a,b>0.$$

Figures 1-4 show the 2D plots of PDF and HRF for the UILE, the UILLo, the UILLi and the UILK distributions. From these figures, we can note that the PDF can be right-skewed, unimodal, heavy-tailed, left-skewed, and U-shaped. In addition, the HRF can be increasing, upside-down, J-shaped, and bathtub.

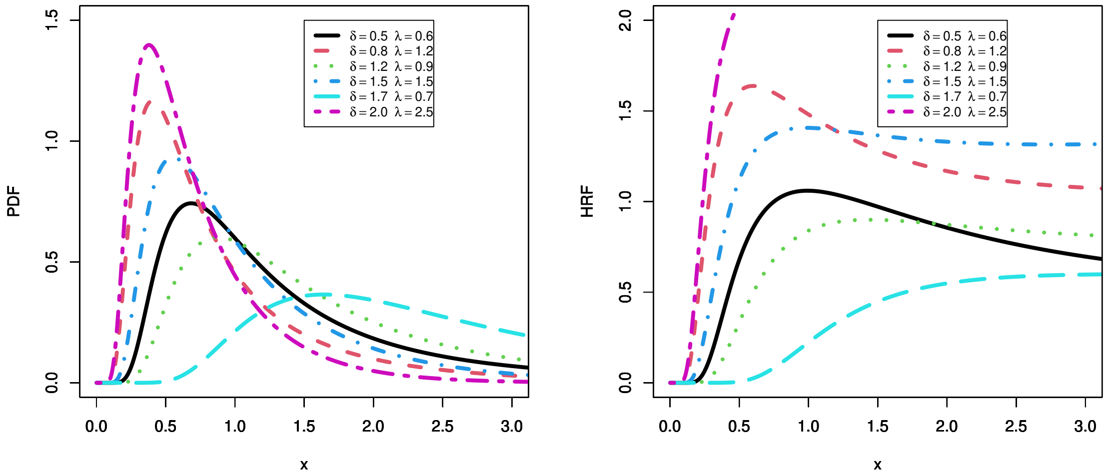

Figure 3. 2D plots of PDF and HRF for the UILLi distribution

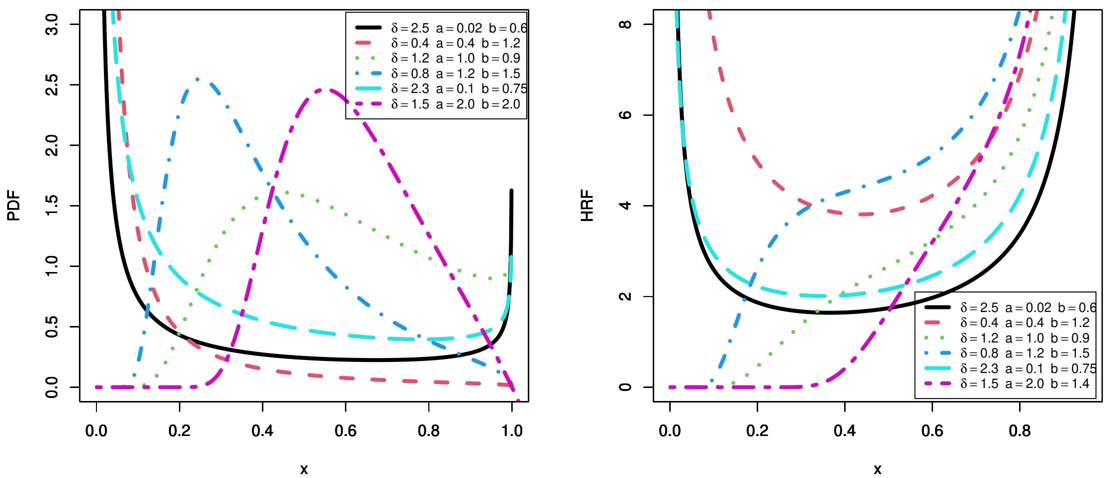

Figure 4. 2D plots of PDF and HRF for the UILK distribution

4. Statistical Properties

In this section, we explore the mathematical properties of the suggested UIL-G family of distributions.

4.1. Linear representation

A helpful expansion of the PDF for the UIL-G family is given in this subsection. Given that the exponential series is

$$e^{-az} = \sum_{i=0}^{\infty} \frac{(-1)^i a^i}{i!} z^i, \tag{7}$$

By applying Equation (7) in the last term of Equation (5), we get

$$f\left(x;\delta,\varphi\right)=\frac{e^{\delta}}{1+\delta}\sum_{i=0}^{\infty}\frac{(-1)^{i}\delta^{i+2}}{i!}g\left(x;\varphi\right)G\left(x;\varphi\right)^{-i-3},\quad x\in\mathbb{R},\ \ \delta>0.$$

By adding and subtracting 1, we can rewrite the above equation as follows

$$f\left(x;\delta,\varphi\right)=\frac{e^{\delta}}{1+\delta}\sum_{i=0}^{\infty}\frac{(-1)^{i}\delta^{i+2}}{i!}g\left(x;\varphi\right)[1-[1-G\left(x;\varphi\right)]]^{-i-3},\quad x\in\mathbb{R},\ \ \delta>0. \tag{8}$$

Using the next binomial series

$$(1-z)^{-\beta}=\sum_{j=0}^{\infty}\binom{\beta+j-1}{j}z^{j},\ \ |z|<1, \tag{9}$$

Applying the binomial expansion in Equation (9) into the last term of Equation (8), we get

$$f\left(x;\delta,\varphi\right)=\frac{e^{\delta}}{1+\delta}\sum_{i,j=0}^{\infty}\frac{(-1)^{i}\delta^{i+2}}{i!}\binom{i+j+2}{j}g\left(x;\varphi\right)[1-G\left(x;\varphi\right)]^{j},\quad x\in\mathbb{R},\ \ \delta>0. \tag{10}$$

Again, using the binomial expansion

$$(1-z)^{\beta}=\sum_{k=0}^{\infty}(-1)^{k}\binom{\beta}{k}z^{k},\ \ |z|<1, \tag{11}$$

By inserting the binomial expansion (11) in Equation (10), then the PDF of the UIL-G family is given by

$$f\left(x;\delta,\varphi\right)=\sum_{i,j,k=0}^{\infty}\Omega_{i,j,k}g\left(x;\varphi\right)G\left(x;\varphi\right)^{k},\quad x\in\mathbb{R},\ \ \delta>0, \tag{12}$$

where $\Omega_{i,j,k}=\frac{(-1)^{i+k}e^{\delta}\delta^{i+2}}{(1+\delta)i!}\binom{i+j+2}{j}\binom{j}{k}$.

Now, we need to get the expansion of $[f\left(x;\delta,\varphi\right)]^{\kappa}$

$$[f\left(x;\delta,\varphi\right)]^{\kappa}=\frac{\delta^{2\kappa}e^{\delta\kappa}}{(1+\delta)^{\kappa}}g\left(x;\varphi\right)^{\kappa}G\left(x;\varphi\right)^{-3\kappa}e^{-\frac{\delta\kappa}{G(x;\varphi)}}. \tag{13}$$

By inserting (7), (9) and (11) into (13), then

$$[f\left(x;\delta,\varphi\right)]^{\kappa}=\sum_{i,j,m=0}^{\infty}\zeta_{i,j,m}g\left(x;\varphi\right)^{\kappa}G\left(x;\varphi\right)^{m}, \tag{14}$$

where $\zeta_{i,j,m}=\frac{(-1)^{i+m}e^{\delta\kappa}\delta^{i+2}}{(1+\delta)^{\kappa}i!}\binom{i+3\kappa+j-1}{j}\binom{j}{m}$.

4.2. *Quantile function*

The quantile function of the UIL-G family is given as $Q\left(p;\delta,\varphi\right)=F^{-1}\left(p;\delta,\varphi\right)$, with $p\in(0,1)$. It is thus computed by inverting the CDF in Equation (4) as below:

$$\frac{e^{\delta}}{1+\delta}\left(1+\frac{\delta}{G\left(x;\varphi\right)}\right)e^{-\frac{\delta}{G(x;\varphi)}}=p,$$

that provides

$$\left(1+\frac{\delta}{G\left(x;\varphi\right)}\right)e^{-\frac{\delta}{G(x;\varphi)}}=\left(1+\delta\right)e^{-\delta}p.$$

Multiplying both sides of the above equation by $-e^{-1}$ yields the Lambert-type equation:

$$-\left(1+\frac{\delta}{G\left(x;\varphi\right)}\right)e^{-\left(1+\frac{\delta}{G(x;\varphi)}\right)}=-\left(1+\delta\right)e^{-(1+\delta)}p.$$

Using the negative Lambert W function of the real argument, written as $W_{-1}\left(.\right)$, we establish that

$$Q(p;\delta,\varphi)=G^{-1}\left[\frac{-\delta}{1+W_{-1}\left[-\left(1+\delta\right)e^{-(1+\delta)}p\right]}\right].$$

We may readily manipulate this quantile function for computation purposes because the Lambert W function is implemented in the majority of scientific software.

Tables 1–4 show the quantiles, Bowley skewness (BS), and Moors kurtosis (MK) for UILLi, UILLo, UILE, and UILK distributions. Tables 1–4 show that quartiles increase systematically with δ and decrease with λ for UILLi, with BS ranging between 0.288 and 0.339 and MK between 1.438 and 1.613, both decreasing as δ increases. UILLo exhibits positive BS ranging from 0.537 to 0.640 and MK from 2.51 to 3.39, confirming its heavy-tailed nature, while UILE shows nearly constant BS (~ 0.31–0.35) and MK below 1.65. Most notably, the UILK distribution demonstrates negative, zero, and positive BS values (ranging from -0.104 to 0.042), proving its exceptional flexibility in modeling bounded data on the unit interval with various skewness patterns.

Table 1. Quantiles, BS, and MK for the UILLi distribution

δ	λ	Q_1	Q_2	Q_3	BS	MK
0.3	1.5	0.1256	0.2029	0.3573	0.3326	1.5941
0.3	1.8	0.0984	0.1595	0.2825	0.3366	1.6047
0.3	2.1	0.0804	0.1306	0.2323	0.3392	1.6125
0.5	1.5	0.2087	0.3354	0.5820	0.3211	1.5387
0.5	1.8	0.1641	0.2651	0.4635	0.3252	1.5488
0.5	2.1	0.1344	0.2178	0.3829	0.3284	1.5565
0.7	1.5	0.2892	0.4609	0.7831	0.3049	1.4833
0.7	1.8	0.2282	0.3658	0.6267	0.3095	1.4921
0.7	2.1	0.1872	0.3014	0.5196	0.3129	1.4989
0.9	1.5	0.3664	0.5777	0.9600	0.2883	1.4377
0.9	1.8	0.2899	0.4600	0.7709	0.2929	1.4448
0.9	2.1	0.2384	0.3800	0.6408	0.2962	1.4508

4.3. Moments

Moments are essential and crucial in any statistical study, particularly in applications. As a result, we calculate the rth moment for the UIL-G family. If X has the PDF (12), the rth moment is derived as follows:

$$\mu_r'=\sum_{i,j,k=0}^{\infty}\Omega_{i,j,k}\int_{-\infty}^{\infty}x^r g\left(x;\varphi\right)G\left(x;\varphi\right)^k dx=\sum_{i,j,k=0}^{\infty}\Omega_{i,j,k}\tau_{r,k},\tag{15}$$

Table 2. Quantiles, BS, and MK for the UILLo distribution

δ	α	β	Q_1	Q_2	Q_3	BS	MK
4	0.6	5	33.2068	86.0710	319.0717	0.6301	3.3218
4	0.6	6	39.8482	103.2852	382.8860	0.6301	3.3218
4	0.7	5	23.5742	55.1619	173.5792	0.5788	2.8245
4	0.7	6	28.2890	66.1943	208.2951	0.5788	2.8245
4	0.8	5	17.9799	39.0836	109.2150	0.5374	2.5093
4	0.8	6	21.5759	46.9004	131.0581	0.5374	2.5093
5	0.6	5	46.9777	125.7309	480.0785	0.6363	3.3608
5	0.6	6	56.3732	150.8770	576.0942	0.6363	3.3608
5	0.7	5	32.2009	77.0146	247.3350	0.5834	2.8459
5	0.7	6	38.6410	92.4175	296.8020	0.5834	2.8459
5	0.8	5	23.9471	52.8131	149.5616	0.5404	2.5205
5	0.8	6	28.7365	63.3757	179.4739	0.5404	2.5205

Table 3. Quantiles, BS, and MK for the UILE distribution

δ	θ	Q_1	Q_2	Q_3	BS	MK
0.3	1.5	0.0772	0.1263	0.2278	0.3480	1.6422
0.3	1.8	0.0643	0.1053	0.1898	0.3474	1.6422
0.3	2.1	0.0551	0.0902	0.1627	0.3472	1.6423
0.5	1.5	0.1300	0.2132	0.3823	0.3408	1.5934
0.5	1.8	0.1084	0.1777	0.3186	0.3405	1.5934
0.5	2.1	0.0929	0.1523	0.2731	0.3408	1.5938
0.7	1.5	0.1825	0.2981	0.5264	0.3276	1.5365
0.7	1.8	0.1521	0.2484	0.4386	0.3279	1.5363
0.7	2.1	0.1304	0.2129	0.3760	0.3278	1.5365
0.9	1.5	0.2339	0.3793	0.6567	0.3125	1.4865
0.9	1.8	0.1950	0.3161	0.5473	0.3125	1.4865
0.9	2.1	0.1671	0.2709	0.4691	0.3124	1.4863

Table 4. Quantiles, BS, and MK for the UILK distribution

δ	a	b	Q_1	Q_2	Q_3	BS	MK
1.5	1.5	0.8	0.6364	0.7673	0.8891	-0.0363	1.0759
1.5	1.5	1.2	0.5214	0.6504	0.7909	0.0425	1.1163
1.5	2.0	0.8	0.7125	0.8198	0.9156	-0.0566	1.0867
1.5	2.0	1.2	0.6136	0.7243	0.8386	0.0165	1.1186
1.7	1.5	0.8	0.6687	0.7948	0.9058	-0.0632	1.0781
1.7	1.5	1.2	0.5519	0.6797	0.8131	0.0211	1.1106
1.7	2.0	0.8	0.7395	0.8417	0.9285	-0.0820	1.0903
1.7	2.0	1.2	0.6403	0.7486	0.8562	-0.0030	1.1154
1.9	1.5	0.8	0.6965	0.8173	0.9188	-0.0867	1.0834
1.9	1.5	1.2	0.5787	0.7046	0.8311	0.0026	1.1084
1.9	2.0	0.8	0.7624	0.8596	0.9385	-0.1038	1.0964
1.9	2.0	1.2	0.6635	0.7691	0.8705	-0.0200	1.1147

where $\tau_{r,k} = \int_{-\infty}^{\infty} x^r g\left(x;\varphi\right) G\left(x;\varphi\right)^k dx$.
For a random variable X, the moment generating function is specified as:

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r' = \sum_{r,i,j,k=0}^{\infty} \Omega_{i,j,k}\frac{t^r}{r!}\tau_{r,k}.$$

Tables 5–8 demonstrate that the first four raw moments, variance (σ^2), standard deviation (σ), skewness, kurtosis, and coefficient of variation a(CV) re highly sensitive to the parameter values across the four sub-models. For the UILLi distribution, increasing δ (while holding λ fixed) raises the mean and reduces both skewness and kurtosis, whereas UILLo exhibits substantially larger moment values with CV frequently exceeding one, reflecting its heavy-tailed behavior. UILE shows the highest skewness and kurtosis at $\delta = 0.3$, which gradually decrease as δ increases, while UILK allows for both negative and positive skewness with moments confined to the unit interval $(0, 1)$, making it particularly suitable for modeling proportions and rates.

Table 5. Moments and related properties for the UILLi distribution

δ	λ	μ_1'	μ_2'	μ_3'	μ_4'	σ^2	σ	Skewness	Kurtosis	CV
0.3	1.5	0.3170	0.2352	0.3780	0.9945	0.1347	0.3670	4.4123	34.5494	1.1577
0.3	1.8	0.2522	0.1527	0.2035	0.4450	0.0891	0.2985	4.5163	36.0303	1.1835
0.3	2.1	0.2083	0.1062	0.1206	0.2254	0.0628	0.2505	4.6007	37.2753	1.2028
0.5	1.5	0.4930	0.4824	0.8643	2.3512	0.2393	0.4892	3.3366	20.4839	0.9922
0.5	1.8	0.3942	0.3148	0.4665	1.0532	0.1594	0.3993	3.4044	21.1952	1.0130
0.5	2.1	0.3267	0.2197	0.2770	0.5339	0.1130	0.3361	3.4604	21.8024	1.0289
0.7	1.5	0.6446	0.7444	1.4331	4.0003	0.3289	0.5735	2.8068	15.1924	0.8896
0.7	1.8	0.5170	0.4874	0.7749	1.7935	0.2202	0.4692	2.8589	15.6464	0.9076
0.7	2.1	0.4295	0.3411	0.4609	0.9097	0.1566	0.3957	2.9024	16.0386	0.9213
0.9	1.5	0.7766	1.0065	2.0442	5.8286	0.4033	0.6351	2.4837	12.4765	0.8177
0.9	1.8	0.6243	0.6607	1.1070	2.6152	0.2709	0.5205	2.5269	12.8075	0.8337
0.9	2.1	0.5196	0.4632	0.6591	1.3272	0.1932	0.4396	2.5634	13.0962	0.8460

Table 6. Moments and related properties for the UILLo distribution

δ	α	β	μ_1'	μ_2'	μ_3'	μ_4'	σ^2	σ	Skewness	Kurtosis	CV
4	0.6	5	22.0379	1010.3550	82711.84	6155318	524.6843	22.9060	3.1052	3.9983	1.0394
4	0.6	6	21.6884	1018.1894	85303.81	6395705	547.8017	23.4052	3.0776	4.0160	1.0792
4	0.7	5	23.8084	1047.6511	81901.04	5999660	480.8136	21.9275	3.2309	3.4567	0.9210
4	0.7	6	24.0695	1088.7420	87056.61	6424008	509.4032	22.5700	3.1598	3.1600	0.9377
4	0.8	5	24.2584	1023.0100	76656.09	5533049	434.5404	20.8456	3.3955	3.5377	0.8593
4	0.8	6	25.0708	1090.9866	83612.33	6079133	462.4422	21.5045	3.3257	2.9152	0.8578
5	0.6	5	20.5361	979.9606	84066.59	6353242	558.2275	23.6268	3.1097	4.4726	1.1505
5	0.6	6	19.5922	954.2767	84035.67	6404698	570.4225	23.8835	3.1554	4.8396	1.2190
5	0.7	5	23.6357	1088.1525	88804.03	6598926	529.5048	23.0110	3.1232	3.2608	0.9736
5	0.7	6	23.2900	1099.1805	92021.59	6895408	556.7559	23.5957	3.0820	3.2820	1.0131
5	0.8	5	25.1646	1114.9329	86944.59	6359401	481.6769	21.9471	3.2773	2.7623	0.8721
5	0.8	6	25.4553	1161.0051	92862.95	6849467	513.0319	22.6502	3.2005	2.4629	0.8898

Table 7. Moments and related properties for the UILE distribution

δ	θ	μ_1'	μ_2'	μ_3'	μ_4'	σ^2	σ	Skewness	Kurtosis	CV
0.3	1.5	0.2091	0.1178	0.1599	0.3670	0.0741	0.2721	5.1762	47.1081	1.3015
0.3	1.8	0.1743	0.0818	0.0925	0.1770	0.0514	0.2268	5.1762	47.1081	1.3015
0.3	2.1	0.1494	0.0601	0.0583	0.0955	0.0378	0.1944	5.1761	47.1081	1.3015
0.5	1.5	0.3329	0.2481	0.3707	0.8727	0.1373	0.3705	3.8671	26.9022	1.1131
0.5	1.8	0.2774	0.1723	0.2146	0.4209	0.0953	0.3088	3.8671	26.9022	1.1131
0.5	2.1	0.2378	0.1266	0.1351	0.2272	0.0701	0.2647	3.8671	26.9023	1.1131
0.7	1.5	0.4422	0.3898	0.6212	1.4923	0.1943	0.4408	3.2345	19.4993	0.9968
0.7	1.8	0.3685	0.2707	0.3595	0.7197	0.1349	0.3673	3.2345	19.4993	0.9968
0.7	2.1	0.3159	0.1989	0.2264	0.3885	0.0991	0.3149	3.2345	19.4994	0.9968
0.9	1.5	0.5391	0.5343	0.8938	2.1841	0.2436	0.4936	2.8531	15.7518	0.9155
0.9	1.8	0.4493	0.3710	0.5172	1.0533	0.1692	0.4113	2.8531	15.7518	0.9155
0.9	2.1	0.3851	0.2726	0.3257	0.5685	0.1243	0.3526	2.8531	15.7518	0.9155

Table 8. Moments and related properties for the UILK distribution

δ	a	b	μ_1'	μ_2'	μ_3'	μ_4'	σ^2	σ	Skewness	Kurtosis	CV
1.5	1.5	0.8	0.7550	0.5951	0.4858	0.4080	0.0250	0.1582	-0.3324	2.2118	0.2096
1.5	1.5	1.2	0.6556	0.4595	0.3403	0.2635	0.0297	0.1723	0.0289	2.1256	0.2628
1.5	2.0	0.8	0.8064	0.6671	0.5640	0.4858	0.0167	0.1293	-0.4308	2.3554	0.1603
1.5	2.0	1.2	0.7237	0.5447	0.4242	0.3403	0.0210	0.1448	-0.0909	2.1791	0.2000
1.7	1.5	0.8	0.7783	0.6282	0.5225	0.4453	0.0225	0.1499	-0.4352	2.3258	0.1926
1.7	1.5	1.2	0.6803	0.4905	0.3712	0.2922	0.0278	0.1667	-0.0602	2.1378	0.2451
1.7	2.0	0.8	0.8256	0.6964	0.5984	0.5225	0.0148	0.1217	-0.5291	2.4901	0.1474
1.7	2.0	1.2	0.7446	0.5738	0.4556	0.3712	0.0193	0.1390	-0.1742	2.2162	0.1867
1.9	1.5	0.8	0.7978	0.6568	0.5548	0.4786	0.0202	0.1422	-0.5261	2.4525	0.1782
1.9	1.5	1.2	0.7014	0.5179	0.3991	0.3186	0.0260	0.1612	-0.1374	2.1675	0.2298
1.9	2.0	0.8	0.8415	0.7213	0.6282	0.5548	0.0132	0.1147	-0.6163	2.6338	0.1363
1.9	2.0	1.2	0.7625	0.5992	0.4835	0.3991	0.0178	0.1336	-0.2467	2.2657	0.1752

4.4. Entropy

Entropy estimates the amount of data required to characterize a random variable by assessing the uncertainty or unpredictability associated with a probability distribution. In practice, entropy measurements are extremely useful for evaluating various lifetime models. Numerous entropy metrics are discussed here, such as Rényi (R), Tsallis (T), Havrda and Charvat (HC), and Arimoto (A). Table 9 provides the formulas for these entropy metrics.

To acquire the measurements in Table 9, the integral $I = \int_{-\infty}^{\infty} \left[f\left(x;\delta,\varphi\right)\right]^{\kappa} dx$ must be determined first. By inserting the PDF from Equation (14) into the expression (ϵ_1), the calculation I simplifies to

$$I = \sum_{i,j,m=0}^{\infty} \zeta_{i,j,m} \int_0^{\infty} g\left(x;\varphi\right)^{\kappa} G\left(x;\varphi\right)^{m} dx, \tag{16}$$

Finally, the entropies R, T, CH, and A of the UIL-G family are calculated by inserting Equation (16) in formulas ϵ_1, ϵ_2, ϵ_3, and ϵ_4 given in Table 9.

Tables 10–13 show some numerical values of entropy measures for UILLi, UILLo, UILE, and UILK distributions. Tables 10–13 reveal that Rényi entropy increases (indicating lower predictability) as δ increases

Table 9. Formulas for various entropy metrics

	Entropy	Expression	Authors
1	R	$\epsilon_1 = \frac{1}{1-\kappa} \log \left\{ \int_{-\infty}^{\infty} [f(x;\delta,\varphi)]^{\kappa} \, dx \right\}; \quad \kappa > 0, \ \kappa \neq 1,$	[58]
2	T	$\epsilon_3 = \frac{1}{\kappa-1} \left\{ 1 - \int_{-\infty}^{\infty} [f(x;\delta,\varphi)]^{\kappa} \, dx \right\}; \quad \kappa > 0, \ \kappa \neq 1,$	[69]
3	HC	$\epsilon_2 = \frac{1}{2^{1-\kappa}-1} \left\{ \int_{-\infty}^{\infty} [f(x;\delta,\varphi)]^{\kappa} \, dx - 1 \right\}; \quad \kappa > 0, \ \kappa \neq 1,$	[33]
4	A	$\epsilon_4 = \frac{\kappa}{1-\kappa} \left\{ \left(\int_{-\infty}^{\infty} [f(x;\delta,\varphi)]^{\kappa} \, dx \right)^{1/\kappa} - 1 \right\}; \quad \kappa > 0, \ \kappa \neq 1,$	[5]

and λ decreases for the UILLi distribution, while Tsallis entropy remains remarkably stable around 0.832 for UILLo despite parameter variations, suggesting low sensitivity of this measure. All entropy values are negative and substantial in magnitude for UILE, especially at $\delta = 0.3$, reflecting the high concentration of this distribution, whereas Rényi entropy for UILK ranges from -0.4575 to -1.0663, becoming less negative (higher uncertainty) when $b = 0.8$ compared to $b = 1.2$. These findings confirm the ability of the proposed family to generate varying degrees of randomness depending on the selected sub-model and parameter configurations.

Table 10. Entropy measures for the UILLi distribution at κ=2

δ	λ	Rényi	Tsallis	Havrda	Arimoto
0.3	1.5	-0.8230	-1.4041	-2.9836	-1.0388
0.3	1.8	-1.0587	-2.1353	-4.5374	-1.4328
0.3	2.1	-1.2553	-2.9252	-6.2159	-1.8025
0.5	1.5	-0.3306	-0.4058	-0.8623	-0.3623
0.5	1.8	-0.5589	-0.7963	-1.6921	-0.6535
0.5	2.1	-0.7505	-1.2176	-2.5873	-0.9274
0.7	1.5	-0.0295	-0.0300	-0.0637	-0.0297
0.7	1.8	-0.2520	-0.2942	-0.6252	-0.2701
0.7	2.1	-0.4396	-0.5789	-1.2301	-0.4967
0.9	1.5	0.1765	0.1591	0.3381	0.1683
0.9	1.8	-0.0414	-0.0425	-0.0903	-0.0419
0.9	2.1	-0.2257	-0.2593	-0.5509	-0.2402

5. Estimation Methods

This section outlines traditional methods for estimating the parameters λ and δ of UILLi distribution. The estimating process primarily entails maximizing an objective function, either by minimizing, to get our proposed model estimators.

5.1. Method of maximum likelihood

Maximum likelihood estimation ([21, 71, 15, 26]) is the predominant method for parameter estimation. It determines the parameter values that optimize the likelihood function, which quantifies the probability of witnessing the specified sample data under the presumed UILLi distribution. This approach is appealing because of its known asymptotic qualities, including consistency and efficiency, which make it fundamental to parametric inference.

Table 11. Entropy measures for the UILLo distribution at κ=2

δ	α	β	Rényi	Tsallis	Havrda	Arimoto
4	0.6	5	5.4314	0.8321	1.7682	1.7386
4	0.6	6	5.6138	0.8323	1.7687	1.7475
4	0.7	5	4.9949	0.8313	1.7664	1.7131
4	0.7	6	5.1772	0.8317	1.7672	1.7245
4	0.8	5	4.6432	0.8302	1.7640	1.6877
4	0.8	6	4.8255	0.8308	1.7654	1.7015
5	0.6	5	5.8079	0.8325	1.7691	1.7562
5	0.6	6	5.9903	0.8327	1.7694	1.7635
5	0.7	5	5.3312	0.8319	1.7678	1.7333
5	0.7	6	5.5135	0.8322	1.7684	1.7427
5	0.8	5	4.9485	0.8311	1.7661	1.7100
5	0.8	6	5.1308	0.8316	1.7670	1.7217

Table 12. Entropy measures for the UILE distribution at κ=2

δ	θ	Rényi	Tsallis	Havrda	Arimoto
0.3	1.5	-1.2782	-3.0301	-6.4388	-1.8483
0.3	1.8	-1.4606	-3.9750	-8.4465	-2.2332
0.3	2.1	-1.6147	-4.9520	-10.5227	-2.5900
0.5	1.5	-0.7560	-1.2313	-2.6163	-0.9358
0.5	1.8	-0.9384	-1.7362	-3.6892	-1.2253
0.5	2.1	-1.0925	-2.2583	-4.7987	-1.4936
0.7	1.5	-0.4296	-0.5621	-1.1945	-0.4841
0.7	1.8	-0.6119	-0.9034	-1.9197	-0.7265
0.7	2.1	-0.7661	-1.2563	-2.6696	-0.9510
0.9	1.5	-0.2021	-0.2288	-0.4861	-0.2137
0.9	1.8	-0.3845	-0.4885	-1.0380	-0.4277
0.9	2.1	-0.5386	-0.7571	-1.6088	-0.6261

Table 13. Entropy measures for the UILK distribution at κ=2

δ	a	b	Rényi	Tsallis	Havrda	Arimoto
1.5	1.5	0.8	-0.6316	-0.9449	-2.0078	-0.7540
1.5	1.5	1.2	-0.4575	-0.6096	-1.2954	-0.5197
1.5	2.0	0.8	-0.8610	-1.5083	-3.2051	-1.0989
1.5	2.0	1.2	-0.6341	-0.9502	-2.0191	-0.7576
1.7	1.5	0.8	-0.7252	-1.1562	-2.4568	-0.8895
1.7	1.5	1.2	-0.4913	-0.6693	-1.4221	-0.5634
1.7	2.0	0.8	-0.9647	-1.8186	-3.8645	-1.2695
1.7	2.0	1.2	-0.6810	-1.0535	-2.2386	-0.8247
1.9	1.5	0.8	-0.8191	-1.3936	-2.9613	-1.0327
1.9	1.5	1.2	-0.5297	-0.7401	-1.5727	-0.6141
1.9	2.0	0.8	-1.0663	-2.1627	-4.5955	-1.4464
1.9	2.0	1.2	-0.7300	-1.1678	-2.4816	-0.8968

Examine a basic random sample of size n extracted from the UILLi distribution. The log-likelihood function for the model parameters is formulated as:

$$\log L = \sum_{i=1}^{n}\left(\frac{\delta\left(\lambda x_i+\lambda+1\right)}{\lambda x_i-(\lambda+1)e^{\lambda x_i}+\lambda+1}+2\lambda x_i\right)-3\sum_{i=1}^{n}\log\left((\lambda+1)e^{\lambda x_i}-\lambda\left(x_i+1\right)-1\right)+\sum_{i=1}^{n}\log\left(x_i+1\right)$$
$$+n\log\left(\frac{\delta^2\left(\lambda^2+\lambda\right)^2}{\delta+1}\right). \quad (17)$$

The nonlinear nature of Equation (17) prevents an analytical solution by straight differentiation. Therefore, we use numerical optimization procedures to calculate the maximum likelihood estimates (MLEs). The derivatives of Equation (17) concerning the parameters λ and δ are ascertained as follows:

$$\frac{\partial l}{\partial\lambda} = \sum_{i=1}^{n}\frac{x_i\left(e^{\lambda x_i}\left((\delta-4)\lambda(\lambda+2)+(\delta-4)\lambda(\lambda+1)x_i-4\right)+2(\lambda+1)^2e^{2\lambda x_i}+2\left(\lambda x_i+\lambda+1\right)^2\right)}{\left(\lambda x_i-(\lambda+1)e^{\lambda x_i}+\lambda+1\right)^2}$$
$$-3\sum_{i=1}^{n}\frac{(\lambda+1)x_ie^{\lambda x_i}+e^{\lambda x_i}-x_i-1}{(\lambda+1)e^{\lambda x_i}-\lambda\left(x_i+1\right)-1}+\frac{2(2\lambda+1)n}{\lambda^2+\lambda},$$

$$\frac{\partial l}{\partial\delta} = \sum_{i=1}^{n}\frac{\lambda x_i+\lambda+1}{\lambda x_i-(\lambda+1)e^{\lambda x_i}+\lambda+1}+\frac{(\delta+2)n}{\delta(\delta+1)}.$$

5.2. Anderson-Darling estimation methods

The Anderson-Darling (A)D technique [4] is a minimum distance estimator recognized for its increased sensitivity to deviations in the tails of a distribution. For UILLi distribution, we derive estimators for λ and δ by minimizing the statistic:

$$A(x_i) = -n-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\log\eta(i,n)+\log[1-\eta(n-i-1,n)]\right],$$

where $\eta(i,n)=\frac{\left(\frac{\delta}{1-e^{\lambda\left(-x_{i:n}\right)}\left(\frac{\lambda x_{i:n}}{\lambda+1}+1\right)}+1\right)\exp\left(\delta-\frac{\delta}{1-e^{\lambda\left(-x_{i:n}\right)}\left(\frac{\lambda x_{i:n}}{\lambda+1}+1\right)}\right)}{\delta+1}$ is the CDF evaluated at the i-th ordered sample.

Two variations of the AD statistic are used to highlight certain tail behavior. The right-tail Anderson-Darling (RTAD) approach [66] improves sensitivity in the higher tail and minimizes the following equation to obtain our proposed model estimators.

$$R(x_i) = \frac{n}{2}-2\sum_{i=1}^{n}\eta(i,n)-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\log[1-\eta(i,n)].$$

The left tail Anderson-Darling method (LTAD) method [52] focuses on the lower tail, and proposed model estimators obtained by optimizing the following:

$$L(x_i) = -\frac{3}{2}n+2\sum_{i=1}^{n}\eta(i,n)-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\log\eta(i,n).$$

The Anderson-Darling second-order left-tail estimation (ADSOE) [2] refines left-tail analysis by incorporating a second-order term and optimizing the following equation:

$$LTS = 2\sum_{i=1}^{n}\log\eta(i,n)+\frac{1}{n}\sum_{i=1}^{n}\frac{(2i-1)}{\eta(i,n)}.$$

5.3. Method of Cramer von Mises

The Cramér von Mises (CVM) criterion [11] minimizes an unweighted quadratic distance between the empirical and theoretical cumulative distribution functions, making it especially adept at detecting deviations in the core of the distribution. The estimators are obtained by minimizing:

$$C(x_i) = \frac{1}{12n} + \sum_{i=1}^{n} \left[\eta(i,n) - \frac{2i-1}{2n}\right]^2.$$

5.4. Method of maximum product of spacings

The maximum product of spacings (MPS) estimate approach, introduced by [35], provides a reliable alternative to MLE, particularly for distributions with intricate bounds. The method optimizes the geometric mean of the intervals between successive order statistics. The goal is to maximize the objective function:

$$\delta\left(x_i\right) = \frac{1}{n+1} \sum_{i=1}^{n+1} \log I_i(x_i),$$

where $I_i(x_i) = \eta(i,n) - \eta(i-1,n)$, $\eta(0,n) = 0$ and $\eta(n+1,n) = 1$.

5.5. Estimation methods of squares

The least squares (LS) technique [66] minimizes the aggregate of squared discrepancies between the theoretical cumulative distribution function and the actual plotting positions. The proposed model estimators are obtained by minimizing the following equation

$$V(x_i) = \sum_{i=1}^{n} \left[\eta(i,n) - \frac{i}{n+1}\right]^2.$$

The weighted version, referred to as weighted least squares (WLS) [66], integrates weights to address the non-constant variance of the order statistics, resulting in the minimization criterion:

$$W(x_i) = \sum_{i=1}^{n} \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[\eta(i,n) - \frac{i}{n+1}\right]^2.$$

5.6. Kolmogorov Method

The Kolmogorov estimation (KE) technique [2] minimizes the supremum norm between the empirical and theoretical cumulative distribution functions, yielding a direct measure of the maximum vertical discrepancy.

$$KM = \underset{1\leq i\leq n}{MAX} \left[\frac{i}{n} - \eta(i,n), \eta(i,n) - \frac{i-1}{n}\right].$$

5.7. Minimum distance estimation methods based on spacings

A class of estimators minimizes different measures of discrepancy between the spacings $\Phi_i(\boldsymbol{y})$ and their expected value $(n+1)^{-1}$. The minimum spacing absolute distance (MSAD) method [68] minimizes absolute deviations:

$$\zeta(\boldsymbol{y}) = \sum_{i=1}^{n+1} \left|\Phi_i(\boldsymbol{y}) - \frac{1}{n+1}\right|.$$

The minimum spacing absolute-log distance (MSALD) method [68] applies a log transformation to these deviations:

$$\Upsilon(\boldsymbol{y}) = \sum_{i=1}^{n+1} \left| \log \Phi_i(\boldsymbol{y}) - \log\left(\frac{1}{n+1}\right) \right|.$$

Similarly, the minimum spacing square distance (MSSD) and minimum spacing square-log distance (MSSLDE) estimation methods are minimized:

$$\phi(\boldsymbol{y}) = \sum_{i=1}^{n+1} \left(\Phi_i(\boldsymbol{y}) - \frac{1}{n+1} \right)^2,$$
$$\Psi(\boldsymbol{y}) = \sum_{i=1}^{n+1} \left(\log \Phi_i(\boldsymbol{y}) - \log\left(\frac{1}{n+1}\right) \right)^2.$$

The minimum spacing Linex distance (MSLD) estimation employs an asymmetric function:

$$\Lambda(\boldsymbol{y}) = \sum_{i=1}^{n+1} \left[\exp\left(\Phi_i(\boldsymbol{y}) - \frac{1}{n+1} \right) - \left(\Phi_i(\boldsymbol{y}) - \frac{1}{n+1} \right) - 1 \right].$$

6. Simulation Study

To assess the estimators of the UILLi model, the flexibility of the UILLi distribution, and its efficacy across different sample sizes, we conducted an extensive Monte Carlo simulation study. To determine the success of the UILLi distribution application, this was executed. All computations were performed according to the settings established in the R software environment. During the simulation we conducted, we used the following specific measurements, detailed as follows:

- Average bias (BIAS), $|Bias(\widehat{\boldsymbol{C}})| = \frac{1}{L}\sum_{i=1}^{L} |\widehat{\boldsymbol{C}} - \boldsymbol{C}|$.
- Mean squared errors (MSE), $MSE = \frac{1}{L}\sum_{i=1}^{L} (\widehat{\boldsymbol{C}} - \boldsymbol{C})^2$.
- Mean relative errors (MRE), $MRE = \frac{1}{L}\sum_{i=1}^{L} |\widehat{\boldsymbol{C}} - \boldsymbol{C}|/\boldsymbol{C}$.
- Average absolute difference (D_{abs}), $D_{abs} = \frac{1}{nH}\sum_{i=1}^{H}\sum_{j=1}^{n} |F(x_{ij};\boldsymbol{C}) - F(x_{ij};\widehat{\boldsymbol{C}})|$.
- Maximum absolute difference (D_{max}), $D_{max} = \frac{1}{H}\sum_{i=1}^{H} \max_{j=1,\ldots,n} |F(x_{ij};\boldsymbol{C}) - F(x_{ij};\widehat{\boldsymbol{C}})|$.
- Average squared absolute error, $ASAE = \frac{1}{n}\sum_{i=1}^{n} \frac{|x_i - \hat{x_i}|}{x_n - x_1}$, $\boldsymbol{C} = (\delta,\ \lambda)$.

Tables 14 to 18 provide the outcomes derived from the Monte Carlo simulation for the UILLi model. These tables provide diverse sample sizes and 1,000 iterations for each metric. Figures 5 through 13 graphically display the data in Table 18. The Monte Carlo simulation studies indicate that, with increasing sample size, all metrics show greater stability and lower values. This pattern, in conjunction with fundamental asymptotic theory, shows that all evaluated estimators are consistent. The results of the simulation ranking study, shown in Table 19, demonstrate that the KE regularly surpasses all other estimation methods. The MLE is ranked second; Table 19 presents the rankings of each method.

Table 14. Numerical values of simulation measures for $\delta = 0.9,\ \lambda = 2.0$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$0.30765^{\{3\}}$	$0.44305^{\{13\}}$	$0.41555^{\{7\}}$	$0.42717^{\{10\}}$	$0.43022^{\{11\}}$	$0.41992^{\{8\}}$	$0.45478^{\{14\}}$	$0.38004^{\{4\}}$	$0.24657^{\{2\}}$	$0.42439^{\{9\}}$	$0.38253^{\{5\}}$	$0.09807^{\{1\}}$	$0.45641^{\{15\}}$	$0.43934^{\{12\}}$	$0.40801^{\{6\}}$
25	BIAS($\hat{\lambda}$)	$0.50192^{\{3\}}$	$0.79311^{\{14\}}$	$0.70265^{\{6\}}$	$0.75547^{\{11\}}$	$0.74100^{\{10\}}$	$0.72247^{\{8\}}$	$0.79761^{\{15\}}$	$0.64867^{\{5\}}$	$0.37383^{\{2\}}$	$0.73055^{\{9\}}$	$0.58853^{\{4\}}$	$0.08113^{\{1\}}$	$0.77705^{\{12\}}$	$0.77884^{\{13\}}$	$0.71168^{\{7\}}$
25	MSE($\hat{\delta}$)	$0.14443^{\{3\}}$	$0.24907^{\{12\}}$	$0.21865^{\{7\}}$	$0.24191^{\{11\}}$	$0.22554^{\{8\}}$	$0.23523^{\{10\}}$	$0.25823^{\{14\}}$	$0.18787^{\{4\}}$	$0.10466^{\{2\}}$	$0.23254^{\{9\}}$	$0.19900^{\{5\}}$	$0.02098^{\{1\}}$	$0.25112^{\{13\}}$	$0.26522^{\{15\}}$	$0.20362^{\{6\}}$
25	MSE($\hat{\lambda}$)	$0.38906^{\{3\}}$	$0.81432^{\{13\}}$	$0.63235^{\{6\}}$	$0.77763^{\{12\}}$	$0.69040^{\{8\}}$	$0.73334^{\{10\}}$	$0.82548^{\{14\}}$	$0.55752^{\{4\}}$	$0.30628^{\{2\}}$	$0.72695^{\{9\}}$	$0.58711^{\{5\}}$	$0.01831^{\{1\}}$	$0.76623^{\{11\}}$	$0.88052^{\{15\}}$	$0.63279^{\{7\}}$
25	MRE($\hat{\delta}$)	$0.34184^{\{3\}}$	$0.49228^{\{13\}}$	$0.46173^{\{7\}}$	$0.47463^{\{10\}}$	$0.47802^{\{11\}}$	$0.46657^{\{8\}}$	$0.50531^{\{14\}}$	$0.42227^{\{4\}}$	$0.27397^{\{2\}}$	$0.47155^{\{9\}}$	$0.42504^{\{5\}}$	$0.10897^{\{1\}}$	$0.50712^{\{15\}}$	$0.48815^{\{12\}}$	$0.45334^{\{6\}}$
25	MRE($\hat{\lambda}$)	$0.25096^{\{3\}}$	$0.39655^{\{14\}}$	$0.35132^{\{6\}}$	$0.37773^{\{11\}}$	$0.37050^{\{10\}}$	$0.36124^{\{8\}}$	$0.39881^{\{15\}}$	$0.32433^{\{5\}}$	$0.18691^{\{2\}}$	$0.36528^{\{9\}}$	$0.29427^{\{4\}}$	$0.04057^{\{1\}}$	$0.38853^{\{12\}}$	$0.38942^{\{13\}}$	$0.35584^{\{7\}}$
25	D_{abs}	$0.02977^{\{1\}}$	$0.04679^{\{9\}}$	$0.04599^{\{7\}}$	$0.04051^{\{2\}}$	$0.04662^{\{8\}}$	$0.04785^{\{11\}}$	$0.04532^{\{5\}}$	$0.04348^{\{3\}}$	$0.05284^{\{13\}}$	$0.04697^{\{10\}}$	$0.04554^{\{6\}}$	$0.04833^{\{12\}}$	$0.06072^{\{15\}}$	$0.04449^{\{4\}}$	$0.05554^{\{14\}}$
25	D_{max}	$0.04514^{\{1\}}$	$0.06881^{\{7\}}$	$0.06894^{\{9\}}$	$0.06009^{\{2\}}$	$0.06837^{\{6\}}$	$0.07107^{\{12\}}$	$0.06687^{\{5\}}$	$0.06498^{\{3\}}$	$0.07582^{\{13\}}$	$0.06883^{\{8\}}$	$0.06932^{\{11\}}$	$0.06912^{\{10\}}$	$0.08964^{\{15\}}$	$0.06637^{\{4\}}$	$0.08152^{\{14\}}$
25	ASAE	$0.04525^{\{8\}}$	$0.03926^{\{3\}}$	$0.04160^{\{5\}}$	$0.04447^{\{6\}}$	$0.04059^{\{4\}}$	$0.03690^{\{1\}}$	$0.03833^{\{2\}}$	$0.04481^{\{7\}}$	$0.04886^{\{12\}}$	$0.04566^{\{9\}}$	$0.05330^{\{13\}}$	$0.04810^{\{11\}}$	$0.05338^{\{14\}}$	$0.04699^{\{10\}}$	$0.05477^{\{15\}}$
25	$\sum Ranks$	$28^{\{1\}}$	$98^{\{13\}}$	$60^{\{6\}}$	$75^{\{7\}}$	$76^{\{8.5\}}$	$76^{\{8.5\}}$	$98^{\{13\}}$	$39^{\{2.5\}}$	$50^{\{4\}}$	$81^{\{10\}}$	$58^{\{5\}}$	$39^{\{2.5\}}$	$122^{\{15\}}$	$98^{\{13\}}$	$82^{\{11\}}$
85	BIAS($\hat{\delta}$)	$0.26417^{\{3\}}$	$0.33260^{\{9\}}$	$0.38586^{\{15\}}$	$0.37933^{\{14\}}$	$0.35000^{\{10\}}$	$0.31555^{\{6\}}$	$0.36589^{\{12\}}$	$0.37449^{\{13\}}$	$0.22233^{\{2\}}$	$0.30881^{\{5\}}$	$0.33188^{\{8\}}$	$0.05073^{\{1\}}$	$0.32680^{\{7\}}$	$0.35098^{\{11\}}$	$0.30813^{\{4\}}$
85	BIAS($\hat{\lambda}$)	$0.43619^{\{3\}}$	$0.55136^{\{9\}}$	$0.64658^{\{14\}}$	$0.66295^{\{15\}}$	$0.58891^{\{10\}}$	$0.51199^{\{7\}}$	$0.60181^{\{12\}}$	$0.60618^{\{13\}}$	$0.35130^{\{2\}}$	$0.51165^{\{5\}}$	$0.50717^{\{4\}}$	$0.04140^{\{1\}}$	$0.54420^{\{8\}}$	$0.59898^{\{11\}}$	$0.51196^{\{6\}}$
85	MSE($\hat{\delta}$)	$0.11117^{\{3\}}$	$0.15767^{\{8\}}$	$0.20256^{\{14\}}$	$0.20970^{\{15\}}$	$0.16104^{\{9\}}$	$0.15134^{\{7\}}$	$0.18789^{\{12\}}$	$0.18414^{\{11\}}$	$0.08951^{\{2\}}$	$0.14420^{\{6\}}$	$0.16163^{\{10\}}$	$0.00443^{\{1\}}$	$0.13911^{\{5\}}$	$0.18988^{\{13\}}$	$0.12500^{\{4\}}$
85	MSE($\hat{\lambda}$)	$0.29656^{\{3\}}$	$0.43511^{\{9\}}$	$0.55588^{\{13\}}$	$0.67761^{\{15\}}$	$0.45895^{\{10\}}$	$0.41315^{\{6\}}$	$0.53035^{\{12\}}$	$0.47905^{\{11\}}$	$0.25250^{\{2\}}$	$0.42625^{\{7\}}$	$0.42791^{\{8\}}$	$0.00442^{\{1\}}$	$0.39505^{\{5\}}$	$0.59673^{\{14\}}$	$0.35499^{\{4\}}$
85	MRE($\hat{\delta}$)	$0.29352^{\{3\}}$	$0.36955^{\{9\}}$	$0.42873^{\{15\}}$	$0.42147^{\{14\}}$	$0.38889^{\{10\}}$	$0.35061^{\{6\}}$	$0.40654^{\{12\}}$	$0.41610^{\{13\}}$	$0.24704^{\{2\}}$	$0.34313^{\{5\}}$	$0.36876^{\{8\}}$	$0.05637^{\{1\}}$	$0.36311^{\{7\}}$	$0.38998^{\{11\}}$	$0.34237^{\{4\}}$
85	MRE($\hat{\lambda}$)	$0.21810^{\{3\}}$	$0.27568^{\{9\}}$	$0.32329^{\{14\}}$	$0.33148^{\{15\}}$	$0.29445^{\{10\}}$	$0.25599^{\{7\}}$	$0.30090^{\{12\}}$	$0.30309^{\{13\}}$	$0.17565^{\{2\}}$	$0.25582^{\{5\}}$	$0.25359^{\{4\}}$	$0.02070^{\{1\}}$	$0.27210^{\{8\}}$	$0.29949^{\{11\}}$	$0.25598^{\{6\}}$
85	D_{abs}	$0.02074^{\{1\}}$	$0.02638^{\{8\}}$	$0.02751^{\{11\}}$	$0.02549^{\{4\}}$	$0.02647^{\{9\}}$	$0.02606^{\{6\}}$	$0.02576^{\{5\}}$	$0.02451^{\{2\}}$	$0.03147^{\{13\}}$	$0.02624^{\{7\}}$	$0.02979^{\{12\}}$	$0.02454^{\{3\}}$	$0.03465^{\{15\}}$	$0.02654^{\{10\}}$	$0.03301^{\{14\}}$
85	D_{max}	$0.03153^{\{1\}}$	$0.03966^{\{9\}}$	$0.04159^{\{11\}}$	$0.03845^{\{4\}}$	$0.03941^{\{7\}}$	$0.03926^{\{6\}}$	$0.03912^{\{5\}}$	$0.03776^{\{3\}}$	$0.04570^{\{12\}}$	$0.03943^{\{8\}}$	$0.04607^{\{13\}}$	$0.03532^{\{2\}}$	$0.05067^{\{15\}}$	$0.04022^{\{10\}}$	$0.04865^{\{14\}}$
85	ASAE	$0.01782^{\{5\}}$	$0.01622^{\{3\}}$	$0.01785^{\{6\}}$	$0.01676^{\{4\}}$	$0.01830^{\{7\}}$	$0.01540^{\{1\}}$	$0.01611^{\{2\}}$	$0.01952^{\{9\}}$	$0.02097^{\{12\}}$	$0.02002^{\{10\}}$	$0.02479^{\{15\}}$	$0.02013^{\{11\}}$	$0.02332^{\{14\}}$	$0.01875^{\{8\}}$	$0.02314^{\{13\}}$
85	$\sum Ranks$	$25^{\{2\}}$	$73^{\{7\}}$	$113^{\{15\}}$	$100^{\{14\}}$	$82^{\{8.5\}}$	$52^{\{4\}}$	$84^{\{10.5\}}$	$88^{\{12\}}$	$49^{\{3\}}$	$58^{\{5\}}$	$82^{\{8.5\}}$	$22^{\{1\}}$	$84^{\{10.5\}}$	$99^{\{13\}}$	$69^{\{6\}}$
145	BIAS($\hat{\delta}$)	$0.21904^{\{3\}}$	$0.31101^{\{10\}}$	$0.34119^{\{15\}}$	$0.28124^{\{8\}}$	$0.32821^{\{13\}}$	$0.27724^{\{6\}}$	$0.31124^{\{11\}}$	$0.32885^{\{14\}}$	$0.20303^{\{2\}}$	$0.28597^{\{9\}}$	$0.32470^{\{12\}}$	$0.04352^{\{1\}}$	$0.27926^{\{7\}}$	$0.25439^{\{4\}}$	$0.26940^{\{5\}}$
145	BIAS($\hat{\lambda}$)	$0.36141^{\{3\}}$	$0.51234^{\{12\}}$	$0.54234^{\{15\}}$	$0.48038^{\{9\}}$	$0.53189^{\{13\}}$	$0.43985^{\{5\}}$	$0.50624^{\{11\}}$	$0.53339^{\{14\}}$	$0.31831^{\{2\}}$	$0.46669^{\{8\}}$	$0.49346^{\{10\}}$	$0.03755^{\{1\}}$	$0.45630^{\{6\}}$	$0.41785^{\{4\}}$	$0.46149^{\{7\}}$
145	MSE($\hat{\delta}$)	$0.08411^{\{3\}}$	$0.13787^{\{10\}}$	$0.16725^{\{15\}}$	$0.13087^{\{9\}}$	$0.15572^{\{14\}}$	$0.12112^{\{8\}}$	$0.14412^{\{11\}}$	$0.15103^{\{12\}}$	$0.07488^{\{2\}}$	$0.12052^{\{7\}}$	$0.15425^{\{13\}}$	$0.00353^{\{1\}}$	$0.10408^{\{4\}}$	$0.10543^{\{6\}}$	$0.10410^{\{5\}}$
145	MSE($\hat{\lambda}$)	$0.21769^{\{3\}}$	$0.37472^{\{9\}}$	$0.42061^{\{14\}}$	$0.42538^{\{15\}}$	$0.39975^{\{13\}}$	$0.30285^{\{5\}}$	$0.38995^{\{10\}}$	$0.39400^{\{12\}}$	$0.19095^{\{2\}}$	$0.33895^{\{8\}}$	$0.39149^{\{11\}}$	$0.00429^{\{1\}}$	$0.28980^{\{4\}}$	$0.31005^{\{6\}}$	$0.31335^{\{7\}}$
145	MRE($\hat{\delta}$)	$0.24338^{\{3\}}$	$0.34556^{\{10\}}$	$0.37910^{\{15\}}$	$0.31249^{\{8\}}$	$0.36468^{\{13\}}$	$0.30804^{\{6\}}$	$0.34582^{\{11\}}$	$0.36539^{\{14\}}$	$0.22559^{\{2\}}$	$0.31775^{\{9\}}$	$0.36078^{\{12\}}$	$0.04836^{\{1\}}$	$0.31029^{\{7\}}$	$0.28266^{\{4\}}$	$0.29934^{\{5\}}$
145	MRE($\hat{\lambda}$)	$0.18071^{\{3\}}$	$0.25617^{\{12\}}$	$0.27117^{\{15\}}$	$0.24019^{\{9\}}$	$0.26594^{\{13\}}$	$0.21992^{\{5\}}$	$0.25312^{\{11\}}$	$0.26670^{\{14\}}$	$0.15915^{\{2\}}$	$0.23335^{\{8\}}$	$0.24673^{\{10\}}$	$0.01877^{\{1\}}$	$0.22815^{\{6\}}$	$0.20892^{\{4\}}$	$0.23074^{\{7\}}$
145	D_{abs}	$0.01560^{\{1\}}$	$0.02062^{\{7\}}$	$0.01991^{\{4\}}$	$0.01840^{\{2\}}$	$0.02097^{\{9\}}$	$0.02146^{\{10\}}$	$0.01914^{\{3\}}$	$0.02004^{\{5\}}$	$0.02390^{\{14\}}$	$0.02159^{\{11\}}$	$0.02350^{\{12.5\}}$	$0.02049^{\{6\}}$	$0.02350^{\{12.5\}}$	$0.02075^{\{8\}}$	$0.02508^{\{15\}}$
145	D_{max}	$0.02431^{\{1\}}$	$0.03152^{\{8\}}$	$0.03149^{\{7\}}$	$0.02810^{\{2\}}$	$0.03217^{\{9\}}$	$0.03284^{\{11\}}$	$0.02984^{\{4\}}$	$0.03096^{\{5\}}$	$0.03535^{\{13\}}$	$0.03264^{\{10\}}$	$0.03695^{\{15\}}$	$0.02950^{\{3\}}$	$0.03515^{\{12\}}$	$0.03112^{\{6\}}$	$0.03693^{\{14\}}$
145	ASAE	$0.01216^{\{5\}}$	$0.01148^{\{2\}}$	$0.01256^{\{6\}}$	$0.01167^{\{4\}}$	$0.01269^{\{7\}}$	$0.01100^{\{1\}}$	$0.01161^{\{3\}}$	$0.01371^{\{9\}}$	$0.01443^{\{12\}}$	$0.01380^{\{10\}}$	$0.01771^{\{15\}}$	$0.01326^{\{8\}}$	$0.01643^{\{14\}}$	$0.01389^{\{11\}}$	$0.01579^{\{13\}}$
145	$\sum Ranks$	$25^{\{2\}}$	$80^{\{10.5\}}$	$106^{\{14\}}$	$66^{\{6\}}$	$104^{\{13\}}$	$57^{\{5\}}$	$75^{\{8\}}$	$99^{\{12\}}$	$51^{\{3\}}$	$80^{\{10.5\}}$	$110.5^{\{15\}}$	$23^{\{1\}}$	$72.5^{\{7\}}$	$53^{\{4\}}$	$78^{\{9\}}$
190	BIAS($\hat{\delta}$)	$0.20985^{\{3\}}$	$0.26799^{\{9\}}$	$0.29955^{\{12\}}$	$0.26828^{\{10\}}$	$0.31619^{\{14\}}$	$0.25449^{\{7\}}$	$0.27077^{\{11\}}$	$0.32251^{\{15\}}$	$0.18032^{\{2\}}$	$0.24770^{\{6\}}$	$0.30532^{\{13\}}$	$0.03559^{\{1\}}$	$0.26071^{\{8\}}$	$0.23884^{\{5\}}$	$0.23413^{\{4\}}$
190	BIAS($\hat{\lambda}$)	$0.33778^{\{3\}}$	$0.42526^{\{8\}}$	$0.48969^{\{13\}}$	$0.45223^{\{11\}}$	$0.51341^{\{14\}}$	$0.41925^{\{7\}}$	$0.43088^{\{9\}}$	$0.51969^{\{15\}}$	$0.28218^{\{2\}}$	$0.41321^{\{6\}}$	$0.48042^{\{12\}}$	$0.03404^{\{1\}}$	$0.43351^{\{10\}}$	$0.39850^{\{5\}}$	$0.39233^{\{4\}}$
190	MSE($\hat{\delta}$)	$0.07356^{\{3\}}$	$0.10949^{\{9\}}$	$0.13209^{\{12\}}$	$0.11752^{\{11\}}$	$0.14272^{\{14\}}$	$0.10025^{\{8\}}$	$0.11263^{\{10\}}$	$0.14923^{\{15\}}$	$0.06551^{\{2\}}$	$0.09826^{\{7\}}$	$0.14033^{\{13\}}$	$0.00250^{\{1\}}$	$0.09422^{\{5\}}$	$0.09657^{\{6\}}$	$0.08143^{\{4\}}$
190	MSE($\hat{\lambda}$)	$0.18794^{\{3\}}$	$0.28553^{\{7\}}$	$0.35800^{\{11\}}$	$0.36460^{\{12\}}$	$0.36896^{\{13\}}$	$0.26606^{\{6\}}$	$0.28588^{\{8\}}$	$0.37824^{\{14\}}$	$0.16103^{\{2\}}$	$0.28595^{\{9\}}$	$0.38904^{\{15\}}$	$0.00380^{\{1\}}$	$0.26371^{\{5\}}$	$0.28744^{\{10\}}$	$0.23916^{\{4\}}$
190	MRE($\hat{\delta}$)	$0.23317^{\{3\}}$	$0.29777^{\{9\}}$	$0.33283^{\{12\}}$	$0.29809^{\{10\}}$	$0.35132^{\{14\}}$	$0.28277^{\{7\}}$	$0.30086^{\{11\}}$	$0.35834^{\{15\}}$	$0.20036^{\{2\}}$	$0.27522^{\{6\}}$	$0.33924^{\{13\}}$	$0.03955^{\{1\}}$	$0.28967^{\{8\}}$	$0.26538^{\{5\}}$	$0.26014^{\{4\}}$
190	MRE($\hat{\lambda}$)	$0.16889^{\{3\}}$	$0.21263^{\{8\}}$	$0.24485^{\{13\}}$	$0.22612^{\{11\}}$	$0.25670^{\{14\}}$	$0.20963^{\{7\}}$	$0.21544^{\{9\}}$	$0.25985^{\{15\}}$	$0.14109^{\{2\}}$	$0.20660^{\{6\}}$	$0.24021^{\{12\}}$	$0.01702^{\{1\}}$	$0.21675^{\{10\}}$	$0.19925^{\{5\}}$	$0.19616^{\{4\}}$
190	D_{abs}	$0.01445^{\{1\}}$	$0.01827^{\{7\}}$	$0.01838^{\{8\}}$	$0.01636^{\{2\}}$	$0.01857^{\{11\}}$	$0.01841^{\{9\}}$	$0.01850^{\{10\}}$	$0.01665^{\{3\}}$	$0.02100^{\{13\}}$	$0.01814^{\{5\}}$	$0.02006^{\{12\}}$	$0.01722^{\{4\}}$	$0.02189^{\{14\}}$	$0.01816^{\{6\}}$	$0.02290^{\{15\}}$
190	D_{max}	$0.02247^{\{1\}}$	$0.02802^{\{8\}}$	$0.02858^{\{9\}}$	$0.02504^{\{3\}}$	$0.02880^{\{10\}}$	$0.02798^{\{7\}}$	$0.02885^{\{11\}}$	$0.02650^{\{4\}}$	$0.03096^{\{12\}}$	$0.02744^{\{5\}}$	$0.03203^{\{13\}}$	$0.02480^{\{2\}}$	$0.03266^{\{14\}}$	$0.02759^{\{6\}}$	$0.03404^{\{15\}}$
190	ASAE	$0.00998^{\{5\}}$	$0.00972^{\{3\}}$	$0.01080^{\{6\}}$	$0.00976^{\{4\}}$	$0.01093^{\{7\}}$	$0.00897^{\{1\}}$	$0.00945^{\{2\}}$	$0.01176^{\{11\}}$	$0.01211^{\{12\}}$	$0.01130^{\{8\}}$	$0.01541^{\{15\}}$	$0.01157^{\{10\}}$	$0.01358^{\{14\}}$	$0.01131^{\{9\}}$	$0.01341^{\{13\}}$
190	$\sum Ranks$	$25^{\{2\}}$	$68^{\{8\}}$	$96^{\{12\}}$	$74^{\{9\}}$	$111^{\{14\}}$	$59^{\{6\}}$	$81^{\{10\}}$	$107^{\{13\}}$	$49^{\{3\}}$	$58^{\{5\}}$	$118^{\{15\}}$	$22^{\{1\}}$	$88^{\{11\}}$	$57^{\{4\}}$	$67^{\{7\}}$
245	BIAS($\hat{\delta}$)	$0.17528^{\{2\}}$	$0.23916^{\{10\}}$	$0.28367^{\{12\}}$	$0.23385^{\{8\}}$	$0.28574^{\{13\}}$	$0.23300^{\{6\}}$	$0.23427^{\{9\}}$	$0.29923^{\{15\}}$	$0.17873^{\{3\}}$	$0.23307^{\{7\}}$	$0.28720^{\{14\}}$	$0.02941^{\{1\}}$	$0.23968^{\{11\}}$	$0.21952^{\{4\}}$	$0.22830^{\{5\}}$
245	BIAS($\hat{\lambda}$)	$0.28470^{\{3\}}$	$0.38925^{\{9\}}$	$0.46799^{\{13\}}$	$0.39509^{\{10\}}$	$0.47600^{\{14\}}$	$0.37502^{\{5\}}$	$0.38437^{\{8\}}$	$0.47820^{\{15\}}$	$0.27553^{\{2\}}$	$0.38100^{\{7\}}$	$0.44009^{\{12\}}$	$0.02631^{\{1\}}$	$0.40394^{\{11\}}$	$0.36631^{\{4\}}$	$0.37973^{\{6\}}$
245	MSE($\hat{\delta}$)	$0.04984^{\{2\}}$	$0.08949^{\{10\}}$	$0.12366^{\{13\}}$	$0.09578^{\{11\}}$	$0.12180^{\{12\}}$	$0.08586^{\{8\}}$	$0.08564^{\{7\}}$	$0.13518^{\{15\}}$	$0.06119^{\{3\}}$	$0.08915^{\{9\}}$	$0.12580^{\{14\}}$	$0.00180^{\{1\}}$	$0.08095^{\{5\}}$	$0.08356^{\{6\}}$	$0.07792^{\{4\}}$
245	MSE($\hat{\lambda}$)	$0.13122^{\{2\}}$	$0.24197^{\{8\}}$	$0.33116^{\{13\}}$	$0.30115^{\{11\}}$	$0.33693^{\{14\}}$	$0.22346^{\{5\}}$	$0.22483^{\{6\}}$	$0.34075^{\{15\}}$	$0.15694^{\{3\}}$	$0.24554^{\{9\}}$	$0.32996^{\{12\}}$	$0.00248^{\{1\}}$	$0.23138^{\{7\}}$	$0.24711^{\{10\}}$	$0.22238^{\{4\}}$
245	MRE($\hat{\delta}$)	$0.19475^{\{2\}}$	$0.26574^{\{10\}}$	$0.31519^{\{12\}}$	$0.25983^{\{8\}}$	$0.31748^{\{13\}}$	$0.25889^{\{6\}}$	$0.26030^{\{9\}}$	$0.33247^{\{15\}}$	$0.19859^{\{3\}}$	$0.25897^{\{7\}}$	$0.31911^{\{14\}}$	$0.03267^{\{1\}}$	$0.26631^{\{11\}}$	$0.24391^{\{4\}}$	$0.25367^{\{5\}}$
245	MRE($\hat{\lambda}$)	$0.14235^{\{3\}}$	$0.19462^{\{9\}}$	$0.23400^{\{13\}}$	$0.19754^{\{10\}}$	$0.23800^{\{14\}}$	$0.18751^{\{5\}}$	$0.19218^{\{8\}}$	$0.23910^{\{15\}}$	$0.13777^{\{2\}}$	$0.19050^{\{7\}}$	$0.22004^{\{12\}}$	$0.01316^{\{1\}}$	$0.20197^{\{11\}}$	$0.18316^{\{4\}}$	$0.18986^{\{6\}}$
245	D_{abs}	$0.01232^{\{1\}}$	$0.01589^{\{4\}}$	$0.01691^{\{8\}}$	$0.01463^{\{3\}}$	$0.01755^{\{11\}}$	$0.01721^{\{9\}}$	$0.01686^{\{7\}}$	$0.01622^{\{5\}}$	$0.01813^{\{12\}}$	$0.01671^{\{6\}}$	$0.01894^{\{14\}}$	$0.01444^{\{2\}}$	$0.01879^{\{13\}}$	$0.01726^{\{10\}}$	$0.02063^{\{15\}}$
245	D_{max}	$0.01908^{\{1\}}$	$0.02428^{\{4\}}$	$0.02600^{\{8\}}$	$0.02235^{\{3\}}$	$0.02719^{\{12\}}$	$0.02623^{\{10\}}$	$0.02558^{\{5\}}$	$0.02572^{\{7\}}$	$0.02702^{\{11\}}$	$0.02566^{\{6\}}$	$0.03027^{\{14\}}$	$0.02086^{\{2\}}$	$0.02806^{\{13\}}$	$0.02621^{\{9\}}$	$0.03084^{\{15\}}$
245	ASAE	$0.00827^{\{4\}}$	$0.00826^{\{3\}}$	$0.00926^{\{6\}}$	$0.00846^{\{5\}}$	$0.00950^{\{7\}}$	$0.00782^{\{1\}}$	$0.00787^{\{2\}}$	$0.01046^{\{11\}}$	$0.01076^{\{12\}}$	$0.01006^{\{9\}}$	$0.01301^{\{15\}}$	$0.01007^{\{10\}}$	$0.01129^{\{13\}}$	$0.00968^{\{8\}}$	$0.01141^{\{14\}}$
245	$\sum Ranks$	$20^{\{1.5\}}$	$67^{\{7.5\}}$	$98^{\{12\}}$	$69^{\{9\}}$	$110^{\{13\}}$	$55^{\{4\}}$	$61^{\{6\}}$	$113^{\{14\}}$	$51^{\{3\}}$	$67^{\{7.5\}}$	$121^{\{15\}}$	$20^{\{1.5\}}$	$95^{\{11\}}$	$59^{\{5\}}$	$74^{\{10\}}$
320	BIAS($\hat{\delta}$)	$0.16533^{\{3\}}$	$0.22153^{\{11\}}$	$0.25921^{\{13\}}$	$0.19081^{\{4\}}$	$0.25469^{\{12\}}$	$0.20342^{\{7\}}$	$0.20789^{\{9\}}$	$0.28553^{\{15\}}$	$0.15051^{\{2\}}$	$0.20030^{\{6\}}$	$0.27160^{\{14\}}$	$0.02847^{\{1\}}$	$0.20703^{\{8\}}$	$0.19323^{\{5\}}$	$0.20812^{\{10\}}$
320	BIAS($\hat{\lambda}$)	$0.26640^{\{3\}}$	$0.35748^{\{11\}}$	$0.42623^{\{13\}}$	$0.31928^{\{5\}}$	$0.41978^{\{12\}}$	$0.32703^{\{6\}}$	$0.33881^{\{8\}}$	$0.45252^{\{15\}}$	$0.23027^{\{2\}}$	$0.33434^{\{7\}}$	$0.42690^{\{14\}}$	$0.02428^{\{1\}}$	$0.34793^{\{10\}}$	$0.30980^{\{4\}}$	$0.34633^{\{9\}}$
320	MSE($\hat{\delta}$)	$0.04531^{\{2\}}$	$0.07813^{\{11\}}$	$0.10014^{\{13\}}$	$0.06286^{\{4\}}$	$0.09481^{\{12\}}$	$0.06492^{\{8\}}$	$0.06643^{\{10\}}$	$0.12212^{\{15\}}$	$0.04695^{\{3\}}$	$0.06439^{\{6\}}$	$0.11593^{\{14\}}$	$0.00140^{\{1\}}$	$0.06592^{\{9\}}$	$0.06444^{\{7\}}$	$0.06375^{\{5\}}$
320	MSE($\hat{\lambda}$)	$0.11503^{\{3\}}$	$0.20121^{\{11\}}$	$0.26843^{\{13\}}$	$0.18904^{\{10\}}$	$0.25593^{\{12\}}$	$0.16443^{\{4\}}$	$0.17849^{\{6\}}$	$0.30054^{\{14\}}$	$0.11296^{\{2\}}$	$0.18508^{\{8\}}$	$0.30271^{\{15\}}$	$0.00132^{\{1\}}$	$0.18606^{\{9\}}$	$0.17389^{\{5\}}$	$0.18177^{\{7\}}$
320	MRE($\hat{\delta}$)	$0.18371^{\{3\}}$	$0.24614^{\{11\}}$	$0.28801^{\{13\}}$	$0.21201^{\{4\}}$	$0.28299^{\{12\}}$	$0.22602^{\{7\}}$	$0.23099^{\{9\}}$	$0.31726^{\{15\}}$	$0.16723^{\{2\}}$	$0.22256^{\{6\}}$	$0.30178^{\{14\}}$	$0.03163^{\{1\}}$	$0.23004^{\{8\}}$	$0.21470^{\{5\}}$	$0.23124^{\{10\}}$
320	MRE($\hat{\lambda}$)	$0.13320^{\{3\}}$	$0.17874^{\{11\}}$	$0.21312^{\{13\}}$	$0.15964^{\{5\}}$	$0.20989^{\{12\}}$	$0.16352^{\{6\}}$	$0.16941^{\{8\}}$	$0.22626^{\{15\}}$	$0.11514^{\{2\}}$	$0.16717^{\{7\}}$	$0.21345^{\{14\}}$	$0.01214^{\{1\}}$	$0.17396^{\{10\}}$	$0.15490^{\{4\}}$	$0.17316^{\{9\}}$
320	D_{abs}	$0.01048^{\{1\}}$	$0.01389^{\{6\}}$	$0.01528^{\{13\}}$	$0.01369^{\{3\}}$	$0.01428^{\{7\}}$	$0.01453^{\{10\}}$	$0.01355^{\{2\}}$	$0.01381^{\{5\}}$	$0.01527^{\{12\}}$	$0.01439^{\{9\}}$	$0.01487^{\{11\}}$	$0.01374^{\{4\}}$	$0.01841^{\{15\}}$	$0.01436^{\{8\}}$	$0.01689^{\{14\}}$
320	D_{max}	$0.01645^{\{1\}}$	$0.02171^{\{5\}}$	$0.02370^{\{12\}}$	$0.02076^{\{3\}}$	$0.02248^{\{9\}}$	$0.02214^{\{8\}}$	$0.02080^{\{4\}}$	$0.02250^{\{10\}}$	$0.02296^{\{11\}}$	$0.02179^{\{6\}}$	$0.02443^{\{13\}}$	$0.01993^{\{2\}}$	$0.02732^{\{15\}}$	$0.02192^{\{7\}}$	$0.02537^{\{14\}}$
320	ASAE	$0.00716^{\{5\}}$	$0.00704^{\{4\}}$	$0.00777^{\{7\}}$	$0.00693^{\{3\}}$	$0.00770^{\{6\}}$	$0.00627^{\{1\}}$	$0.00671^{\{2\}}$	$0.00908^{\{12\}}$	$0.00877^{\{11\}}$	$0.00848^{\{10\}}$	$0.01060^{\{15\}}$	$0.00820^{\{9\}}$	$0.00988^{\{14\}}$	$0.00787^{\{8\}}$	$0.00953^{\{13\}}$
320	$\sum Ranks$	$24^{\{2\}}$	$81^{\{9\}}$	$110^{\{13\}}$	$41^{\{3\}}$	$94^{\{11\}}$	$57^{\{6\}}$	$58^{\{7\}}$	$116^{\{14\}}$	$47^{\{4\}}$	$65^{\{8\}}$	$124^{\{15\}}$	$21^{\{1\}}$	$98^{\{12\}}$	$53^{\{5\}}$	$91^{\{10\}}$

Table 15. Numerical values of simulation measures for $\delta = 1.25,\ \lambda = 1.5$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$0.45606^{\{2\}}$	$0.56129^{\{4\}}$	$0.59721^{\{9\}}$	$0.61057^{\{11\}}$	$0.64146^{\{13\}}$	$0.56454^{\{5\}}$	$0.64437^{\{14\}}$	$0.58878^{\{8\}}$	$0.45851^{\{3\}}$	$0.59857^{\{10\}}$	$0.56834^{\{6\}}$	$0.07944^{\{1\}}$	$0.66415^{\{15\}}$	$0.58616^{\{7\}}$	$0.64082^{\{12\}}$
25	BIAS($\hat{\lambda}$)	$0.36225^{\{2\}}$	$0.49021^{\{5\}}$	$0.49731^{\{6\}}$	$0.55589^{\{12\}}$	$0.54972^{\{10\}}$	$0.50634^{\{8\}}$	$0.56042^{\{13\}}$	$0.50205^{\{7\}}$	$0.41672^{\{3\}}$	$0.55011^{\{11\}}$	$0.46276^{\{4\}}$	$0.10313^{\{1\}}$	$0.58639^{\{15\}}$	$0.51213^{\{9\}}$	$0.57164^{\{14\}}$
25	MSE($\hat{\delta}$)	$0.30559^{\{2\}}$	$0.42180^{\{5\}}$	$0.46450^{\{8\}}$	$0.49588^{\{12\}}$	$0.50208^{\{13\}}$	$0.44797^{\{7\}}$	$0.51571^{\{14\}}$	$0.43173^{\{6\}}$	$0.31716^{\{3\}}$	$0.49222^{\{10\}}$	$0.41150^{\{4\}}$	$0.01550^{\{1\}}$	$0.53105^{\{15\}}$	$0.47723^{\{9\}}$	$0.49572^{\{11\}}$
25	MSE($\hat{\lambda}$)	$0.19067^{\{2\}}$	$0.34327^{\{7\}}$	$0.33312^{\{6\}}$	$0.44023^{\{14\}}$	$0.38296^{\{9\}}$	$0.37577^{\{8\}}$	$0.40981^{\{12\}}$	$0.32864^{\{5\}}$	$0.27293^{\{3\}}$	$0.43479^{\{13\}}$	$0.29559^{\{4\}}$	$0.02198^{\{1\}}$	$0.44622^{\{15\}}$	$0.40533^{\{10\}}$	$0.40802^{\{11\}}$
25	MRE($\hat{\delta}$)	$0.36485^{\{2\}}$	$0.44903^{\{4\}}$	$0.47777^{\{9\}}$	$0.48846^{\{11\}}$	$0.51317^{\{13\}}$	$0.45163^{\{5\}}$	$0.51550^{\{14\}}$	$0.47102^{\{8\}}$	$0.36681^{\{3\}}$	$0.47886^{\{10\}}$	$0.45467^{\{6\}}$	$0.06355^{\{1\}}$	$0.53132^{\{15\}}$	$0.46893^{\{7\}}$	$0.51266^{\{12\}}$
25	MRE($\hat{\lambda}$)	$0.24150^{\{2\}}$	$0.32680^{\{5\}}$	$0.33154^{\{6\}}$	$0.37059^{\{12\}}$	$0.36648^{\{10\}}$	$0.33756^{\{8\}}$	$0.37361^{\{13\}}$	$0.33470^{\{7\}}$	$0.27781^{\{3\}}$	$0.36674^{\{11\}}$	$0.30850^{\{4\}}$	$0.06875^{\{1\}}$	$0.39093^{\{15\}}$	$0.34142^{\{9\}}$	$0.38110^{\{14\}}$
25	D_{abs}	$0.03957^{\{1\}}$	$0.04891^{\{10\}}$	$0.04628^{\{8\}}$	$0.04506^{\{5\}}$	$0.04487^{\{4\}}$	$0.04948^{\{11\}}$	$0.04560^{\{6\}}$	$0.04456^{\{3\}}$	$0.05565^{\{13\}}$	$0.05105^{\{12\}}$	$0.04375^{\{2\}}$	$0.04611^{\{7\}}$	$0.05644^{\{14\}}$	$0.04785^{\{9\}}$	$0.05827^{\{15\}}$
25	D_{max}	$0.06069^{\{1\}}$	$0.07238^{\{9\}}$	$0.07040^{\{8\}}$	$0.06690^{\{3\}}$	$0.06749^{\{4\}}$	$0.07308^{\{11\}}$	$0.06827^{\{6\}}$	$0.06843^{\{7\}}$	$0.08083^{\{13\}}$	$0.07445^{\{12\}}$	$0.06782^{\{5\}}$	$0.06511^{\{2\}}$	$0.08452^{\{14\}}$	$0.07248^{\{10\}}$	$0.08654^{\{15\}}$
25	ASAE	$0.04282^{\{7\}}$	$0.03899^{\{4\}}$	$0.03895^{\{3\}}$	$0.04028^{\{5\}}$	$0.04082^{\{6\}}$	$0.03686^{\{2\}}$	$0.03666^{\{1\}}$	$0.04522^{\{12\}}$	$0.04496^{\{11\}}$	$0.04387^{\{9\}}$	$0.05176^{\{13\}}$	$0.04464^{\{10\}}$	$0.05260^{\{15\}}$	$0.04339^{\{8\}}$	$0.05220^{\{14\}}$
25	$\sum Ranks$	$21^{\{1\}}$	$53^{\{4\}}$	$63^{\{6.5\}}$	$85^{\{11\}}$	$82^{\{10\}}$	$65^{\{8\}}$	$93^{\{12\}}$	$63^{\{6.5\}}$	$55^{\{5\}}$	$98^{\{13\}}$	$48^{\{3\}}$	$25^{\{2\}}$	$133^{\{15\}}$	$78^{\{9\}}$	$118^{\{14\}}$
85	BIAS($\hat{\delta}$)	$0.33635^{\{2\}}$	$0.41015^{\{5\}}$	$0.45145^{\{10\}}$	$0.41935^{\{6\}}$	$0.49652^{\{14\}}$	$0.37532^{\{4\}}$	$0.42194^{\{8\}}$	$0.47896^{\{13\}}$	$0.35401^{\{3\}}$	$0.45903^{\{11\}}$	$0.49859^{\{15\}}$	$0.05859^{\{1\}}$	$0.46425^{\{12\}}$	$0.42157^{\{7\}}$	$0.44664^{\{9\}}$
85	BIAS($\hat{\lambda}$)	$0.27742^{\{2\}}$	$0.33375^{\{5\}}$	$0.36523^{\{8\}}$	$0.36938^{\{9\}}$	$0.40465^{\{15\}}$	$0.29617^{\{3\}}$	$0.34939^{\{6\}}$	$0.39797^{\{13\}}$	$0.30269^{\{4\}}$	$0.39825^{\{14\}}$	$0.39332^{\{11\}}$	$0.06971^{\{1\}}$	$0.39627^{\{12\}}$	$0.34959^{\{7\}}$	$0.38952^{\{10\}}$
85	MSE($\hat{\delta}$)	$0.18055^{\{2\}}$	$0.25932^{\{5\}}$	$0.29318^{\{11\}}$	$0.28216^{\{8\}}$	$0.33837^{\{15\}}$	$0.21140^{\{4\}}$	$0.25934^{\{6\}}$	$0.33132^{\{14\}}$	$0.20490^{\{3\}}$	$0.31827^{\{12\}}$	$0.32475^{\{13\}}$	$0.01057^{\{1\}}$	$0.29188^{\{10\}}$	$0.28312^{\{9\}}$	$0.26858^{\{7\}}$
85	MSE($\hat{\lambda}$)	$0.11909^{\{2\}}$	$0.17340^{\{5\}}$	$0.19754^{\{7\}}$	$0.24172^{\{14\}}$	$0.23518^{\{13\}}$	$0.14166^{\{3\}}$	$0.18713^{\{6\}}$	$0.23486^{\{12\}}$	$0.14956^{\{4\}}$	$0.25943^{\{15\}}$	$0.21099^{\{9\}}$	$0.01175^{\{1\}}$	$0.22458^{\{11\}}$	$0.21553^{\{10\}}$	$0.20948^{\{8\}}$
85	MRE($\hat{\delta}$)	$0.26908^{\{2\}}$	$0.32812^{\{5\}}$	$0.36116^{\{10\}}$	$0.33548^{\{6\}}$	$0.39722^{\{14\}}$	$0.30026^{\{4\}}$	$0.33755^{\{8\}}$	$0.38317^{\{13\}}$	$0.28321^{\{3\}}$	$0.36723^{\{11\}}$	$0.39887^{\{15\}}$	$0.04687^{\{1\}}$	$0.37140^{\{12\}}$	$0.33726^{\{7\}}$	$0.35731^{\{9\}}$
85	MRE($\hat{\lambda}$)	$0.18494^{\{2\}}$	$0.22250^{\{5\}}$	$0.24349^{\{8\}}$	$0.24625^{\{9\}}$	$0.26977^{\{15\}}$	$0.19745^{\{3\}}$	$0.23292^{\{6\}}$	$0.26531^{\{13\}}$	$0.20179^{\{4\}}$	$0.26550^{\{14\}}$	$0.26221^{\{11\}}$	$0.04648^{\{1\}}$	$0.26418^{\{12\}}$	$0.23306^{\{7\}}$	$0.25968^{\{10\}}$
85	D_{abs}	$0.02310^{\{1\}}$	$0.02628^{\{3\}}$	$0.02724^{\{7\}}$	$0.02648^{\{4\}}$	$0.02848^{\{10\}}$	$0.02842^{\{9\}}$	$0.02559^{\{2\}}$	$0.02685^{\{6\}}$	$0.03211^{\{13\}}$	$0.03089^{\{12\}}$	$0.02862^{\{11\}}$	$0.02653^{\{5\}}$	$0.03309^{\{14\}}$	$0.02795^{\{8\}}$	$0.03413^{\{15\}}$
85	D_{max}	$0.03553^{\{1\}}$	$0.04053^{\{5\}}$	$0.04215^{\{7\}}$	$0.04035^{\{4\}}$	$0.04439^{\{10\}}$	$0.04337^{\{9\}}$	$0.03983^{\{3\}}$	$0.04189^{\{6\}}$	$0.04811^{\{13\}}$	$0.04684^{\{12\}}$	$0.04540^{\{11\}}$	$0.03749^{\{2\}}$	$0.05020^{\{14\}}$	$0.04283^{\{8\}}$	$0.05139^{\{15\}}$
85	ASAE	$0.01750^{\{6\}}$	$0.01617^{\{2\}}$	$0.01742^{\{5\}}$	$0.01663^{\{4\}}$	$0.01767^{\{7\}}$	$0.01551^{\{1\}}$	$0.01657^{\{3\}}$	$0.01935^{\{10\}}$	$0.02150^{\{12\}}$	$0.02039^{\{11\}}$	$0.02501^{\{15\}}$	$0.01918^{\{9\}}$	$0.02315^{\{13\}}$	$0.01902^{\{8\}}$	$0.02345^{\{14\}}$
85	$\sum Ranks$	$20^{\{1\}}$	$40^{\{3.5\}}$	$73^{\{9\}}$	$64^{\{7\}}$	$113^{\{15\}}$	$40^{\{3.5\}}$	$48^{\{5\}}$	$100^{\{11\}}$	$59^{\{6\}}$	$112^{\{14\}}$	$111^{\{13\}}$	$22^{\{2\}}$	$110^{\{12\}}$	$71^{\{8\}}$	$97^{\{10\}}$
145	BIAS($\hat{\delta}$)	$0.25717^{\{2\}}$	$0.36127^{\{9\}}$	$0.39061^{\{12\}}$	$0.33626^{\{5\}}$	$0.40439^{\{13\}}$	$0.34101^{\{6\}}$	$0.33620^{\{4\}}$	$0.44882^{\{14\}}$	$0.26411^{\{3\}}$	$0.36570^{\{10\}}$	$0.46671^{\{15\}}$	$0.04928^{\{1\}}$	$0.37936^{\{11\}}$	$0.34862^{\{7\}}$	$0.35995^{\{8\}}$
145	BIAS($\hat{\lambda}$)	$0.21061^{\{2\}}$	$0.29201^{\{8\}}$	$0.32017^{\{11\}}$	$0.28941^{\{7\}}$	$0.32229^{\{13\}}$	$0.26703^{\{4\}}$	$0.26898^{\{5\}}$	$0.36408^{\{14.5\}}$	$0.21911^{\{3\}}$	$0.31173^{\{10\}}$	$0.36408^{\{14.5\}}$	$0.05509^{\{1\}}$	$0.32092^{\{12\}}$	$0.28006^{\{6\}}$	$0.30932^{\{9\}}$
145	MSE($\hat{\delta}$)	$0.11494^{\{2\}}$	$0.20645^{\{10\}}$	$0.22856^{\{12\}}$	$0.19247^{\{7\}}$	$0.24423^{\{13\}}$	$0.18379^{\{5\}}$	$0.17897^{\{4\}}$	$0.28650^{\{14\}}$	$0.11863^{\{3\}}$	$0.21233^{\{11\}}$	$0.29678^{\{15\}}$	$0.00763^{\{1\}}$	$0.20342^{\{8\}}$	$0.20361^{\{9\}}$	$0.18598^{\{6\}}$
145	MSE($\hat{\lambda}$)	$0.07425^{\{2\}}$	$0.13440^{\{6\}}$	$0.15893^{\{12\}}$	$0.15620^{\{11\}}$	$0.15604^{\{10\}}$	$0.10963^{\{4\}}$	$0.11090^{\{5\}}$	$0.18765^{\{15\}}$	$0.08253^{\{3\}}$	$0.16531^{\{13\}}$	$0.18527^{\{14\}}$	$0.00685^{\{1\}}$	$0.15295^{\{9\}}$	$0.14990^{\{8\}}$	$0.14459^{\{7\}}$
145	MRE($\hat{\delta}$)	$0.20574^{\{2\}}$	$0.28902^{\{9\}}$	$0.31249^{\{12\}}$	$0.26901^{\{5\}}$	$0.32352^{\{13\}}$	$0.27281^{\{6\}}$	$0.26896^{\{4\}}$	$0.35906^{\{14\}}$	$0.21129^{\{3\}}$	$0.29256^{\{10\}}$	$0.37337^{\{15\}}$	$0.03943^{\{1\}}$	$0.30349^{\{11\}}$	$0.27890^{\{7\}}$	$0.28796^{\{8\}}$
145	MRE($\hat{\lambda}$)	$0.14041^{\{2\}}$	$0.19467^{\{8\}}$	$0.21345^{\{11\}}$	$0.19294^{\{7\}}$	$0.21486^{\{13\}}$	$0.17802^{\{4\}}$	$0.17932^{\{5\}}$	$0.24272^{\{14.5\}}$	$0.14607^{\{3\}}$	$0.20782^{\{10\}}$	$0.24272^{\{14.5\}}$	$0.03673^{\{1\}}$	$0.21394^{\{12\}}$	$0.18670^{\{6\}}$	$0.20621^{\{9\}}$
145	D_{abs}	$0.01880^{\{2\}}$	$0.02076^{\{5\}}$	$0.02183^{\{9\}}$	$0.01801^{\{1\}}$	$0.02110^{\{7\}}$	$0.02184^{\{10\}}$	$0.01972^{\{3\}}$	$0.02147^{\{8\}}$	$0.02333^{\{12\}}$	$0.02231^{\{11\}}$	$0.02383^{\{13\}}$	$0.02098^{\{6\}}$	$0.02517^{\{15\}}$	$0.02021^{\{4\}}$	$0.02464^{\{14\}}$
145	D_{max}	$0.02898^{\{2\}}$	$0.03255^{\{6\}}$	$0.03447^{\{10\}}$	$0.02782^{\{1\}}$	$0.03333^{\{7\}}$	$0.03379^{\{8\}}$	$0.03077^{\{4\}}$	$0.03450^{\{11\}}$	$0.03492^{\{12\}}$	$0.03427^{\{9\}}$	$0.03885^{\{15\}}$	$0.02981^{\{3\}}$	$0.03846^{\{14\}}$	$0.03164^{\{5\}}$	$0.03762^{\{13\}}$
145	ASAE	$0.01238^{\{6\}}$	$0.01139^{\{4\}}$	$0.01301^{\{7\}}$	$0.01135^{\{2\}}$	$0.01236^{\{5\}}$	$0.01125^{\{1\}}$	$0.01136^{\{3\}}$	$0.01445^{\{11\}}$	$0.01513^{\{12\}}$	$0.01426^{\{9\}}$	$0.01777^{\{15\}}$	$0.01434^{\{10\}}$	$0.01673^{\{13\}}$	$0.01357^{\{8\}}$	$0.01679^{\{14\}}$
145	$\sum Ranks$	$22^{\{1\}}$	$65^{\{8\}}$	$96^{\{12\}}$	$46^{\{4\}}$	$94^{\{11\}}$	$48^{\{5\}}$	$37^{\{3\}}$	$116^{\{14\}}$	$54^{\{6\}}$	$93^{\{10\}}$	$131^{\{15\}}$	$25^{\{2\}}$	$105^{\{13\}}$	$60^{\{7\}}$	$88^{\{9\}}$
190	BIAS($\hat{\delta}$)	$0.24916^{\{3\}}$	$0.30113^{\{7\}}$	$0.36075^{\{12\}}$	$0.28568^{\{4\}}$	$0.37248^{\{13\}}$	$0.30040^{\{6\}}$	$0.33215^{\{10\}}$	$0.39854^{\{14\}}$	$0.24859^{\{2\}}$	$0.31764^{\{8\}}$	$0.43759^{\{15\}}$	$0.03736^{\{1\}}$	$0.34364^{\{11\}}$	$0.29561^{\{5\}}$	$0.32321^{\{9\}}$
190	BIAS($\hat{\lambda}$)	$0.20257^{\{3\}}$	$0.24156^{\{6\}}$	$0.27272^{\{10\}}$	$0.24614^{\{7\}}$	$0.29851^{\{13\}}$	$0.23117^{\{4\}}$	$0.25164^{\{8\}}$	$0.32193^{\{14\}}$	$0.19918^{\{2\}}$	$0.25494^{\{9\}}$	$0.34723^{\{15\}}$	$0.04120^{\{1\}}$	$0.28383^{\{12\}}$	$0.23832^{\{5\}}$	$0.27604^{\{11\}}$
190	MSE($\hat{\delta}$)	$0.10316^{\{2\}}$	$0.13823^{\{4\}}$	$0.20855^{\{13\}}$	$0.14448^{\{6\}}$	$0.20147^{\{12\}}$	$0.14146^{\{5\}}$	$0.17428^{\{11\}}$	$0.22488^{\{14\}}$	$0.10877^{\{3\}}$	$0.15793^{\{8\}}$	$0.26977^{\{15\}}$	$0.00470^{\{1\}}$	$0.16920^{\{10\}}$	$0.15102^{\{7\}}$	$0.15810^{\{9\}}$
190	MSE($\hat{\lambda}$)	$0.06677^{\{2\}}$	$0.09168^{\{5\}}$	$0.11946^{\{11\}}$	$0.11791^{\{9\}}$	$0.13034^{\{13\}}$	$0.08087^{\{4\}}$	$0.09744^{\{6\}}$	$0.14753^{\{14\}}$	$0.07160^{\{3\}}$	$0.10879^{\{7\}}$	$0.17468^{\{15\}}$	$0.00404^{\{1\}}$	$0.12233^{\{12\}}$	$0.11404^{\{8\}}$	$0.11807^{\{10\}}$
190	MRE($\hat{\delta}$)	$0.19933^{\{3\}}$	$0.24090^{\{7\}}$	$0.28860^{\{12\}}$	$0.22854^{\{4\}}$	$0.29798^{\{13\}}$	$0.24032^{\{6\}}$	$0.26572^{\{10\}}$	$0.31883^{\{14\}}$	$0.19887^{\{2\}}$	$0.25411^{\{8\}}$	$0.35007^{\{15\}}$	$0.02989^{\{1\}}$	$0.27491^{\{11\}}$	$0.23649^{\{5\}}$	$0.25857^{\{9\}}$
190	MRE($\hat{\lambda}$)	$0.13505^{\{3\}}$	$0.16104^{\{6\}}$	$0.18181^{\{10\}}$	$0.16410^{\{7\}}$	$0.19900^{\{13\}}$	$0.15411^{\{4\}}$	$0.16776^{\{8\}}$	$0.21462^{\{14\}}$	$0.13279^{\{2\}}$	$0.16996^{\{9\}}$	$0.23148^{\{15\}}$	$0.02747^{\{1\}}$	$0.18922^{\{12\}}$	$0.15888^{\{5\}}$	$0.18403^{\{11\}}$
190	D_{abs}	$0.01618^{\{2\}}$	$0.01711^{\{3\}}$	$0.01813^{\{7\}}$	$0.01736^{\{4\}}$	$0.01765^{\{5\}}$	$0.01813^{\{7\}}$	$0.01816^{\{9\}}$	$0.01813^{\{7\}}$	$0.01917^{\{10\}}$	$0.02112^{\{13\}}$	$0.01966^{\{12\}}$	$0.01602^{\{1\}}$	$0.02205^{\{14\}}$	$0.01930^{\{11\}}$	$0.02322^{\{15\}}$
190	D_{max}	$0.02522^{\{2\}}$	$0.02686^{\{4\}}$	$0.02942^{\{10\}}$	$0.02670^{\{3\}}$	$0.02837^{\{6\}}$	$0.02834^{\{5\}}$	$0.02891^{\{7\}}$	$0.02916^{\{8\}}$	$0.02927^{\{9\}}$	$0.03242^{\{12\}}$	$0.03243^{\{13\}}$	$0.02281^{\{1\}}$	$0.03442^{\{14\}}$	$0.03019^{\{11\}}$	$0.03537^{\{15\}}$
190	ASAE	$0.00999^{\{3\}}$	$0.01029^{\{5\}}$	$0.01086^{\{6\}}$	$0.01024^{\{4\}}$	$0.01108^{\{7\}}$	$0.00936^{\{1\}}$	$0.00968^{\{2\}}$	$0.01185^{\{10\}}$	$0.01269^{\{12\}}$	$0.01209^{\{11\}}$	$0.01560^{\{15\}}$	$0.01164^{\{9\}}$	$0.01431^{\{14\}}$	$0.01152^{\{8\}}$	$0.01405^{\{13\}}$
190	$\sum Ranks$	$23^{\{2\}}$	$47^{\{5\}}$	$91^{\{10\}}$	$48^{\{6\}}$	$95^{\{11\}}$	$42^{\{3\}}$	$71^{\{8\}}$	$109^{\{13\}}$	$45^{\{4\}}$	$85^{\{9\}}$	$130^{\{15\}}$	$17^{\{1\}}$	$110^{\{14\}}$	$65^{\{7\}}$	$102^{\{12\}}$
245	BIAS($\hat{\delta}$)	$0.22246^{\{2\}}$	$0.27246^{\{8\}}$	$0.31962^{\{13\}}$	$0.25069^{\{5\}}$	$0.31274^{\{12\}}$	$0.26056^{\{7\}}$	$0.25913^{\{6\}}$	$0.35011^{\{14\}}$	$0.23585^{\{3\}}$	$0.28375^{\{9\}}$	$0.40949^{\{15\}}$	$0.03717^{\{1\}}$	$0.28973^{\{10\}}$	$0.24315^{\{4\}}$	$0.29151^{\{11\}}$
245	BIAS($\hat{\lambda}$)	$0.18120^{\{2\}}$	$0.21667^{\{8\}}$	$0.25253^{\{13\}}$	$0.20235^{\{5\}}$	$0.24704^{\{12\}}$	$0.20315^{\{6\}}$	$0.20509^{\{7\}}$	$0.28538^{\{14\}}$	$0.19197^{\{4\}}$	$0.22949^{\{9\}}$	$0.33290^{\{15\}}$	$0.03923^{\{1\}}$	$0.24333^{\{11\}}$	$0.19006^{\{3\}}$	$0.23827^{\{10\}}$
245	MSE($\hat{\delta}$)	$0.07356^{\{2\}}$	$0.11601^{\{8\}}$	$0.16488^{\{13\}}$	$0.10217^{\{5\}}$	$0.15549^{\{12\}}$	$0.11394^{\{7\}}$	$0.10769^{\{6\}}$	$0.18577^{\{14\}}$	$0.09883^{\{4\}}$	$0.11962^{\{9\}}$	$0.24362^{\{15\}}$	$0.00393^{\{1\}}$	$0.12771^{\{10\}}$	$0.09212^{\{3\}}$	$0.13253^{\{11\}}$
245	MSE($\hat{\lambda}$)	$0.04942^{\{2\}}$	$0.07431^{\{8\}}$	$0.10394^{\{13\}}$	$0.07388^{\{7\}}$	$0.09898^{\{12\}}$	$0.06705^{\{6\}}$	$0.06647^{\{5\}}$	$0.12458^{\{14\}}$	$0.06478^{\{4\}}$	$0.08161^{\{9\}}$	$0.17430^{\{15\}}$	$0.00380^{\{1\}}$	$0.09336^{\{10\}}$	$0.05819^{\{3\}}$	$0.09464^{\{11\}}$
245	MRE($\hat{\delta}$)	$0.17796^{\{2\}}$	$0.21796^{\{8\}}$	$0.25570^{\{13\}}$	$0.20055^{\{5\}}$	$0.25019^{\{12\}}$	$0.20845^{\{7\}}$	$0.20730^{\{6\}}$	$0.28009^{\{14\}}$	$0.18868^{\{3\}}$	$0.22700^{\{9\}}$	$0.32759^{\{15\}}$	$0.02974^{\{1\}}$	$0.23179^{\{10\}}$	$0.19452^{\{4\}}$	$0.23321^{\{11\}}$
245	MRE($\hat{\lambda}$)	$0.12080^{\{2\}}$	$0.14445^{\{8\}}$	$0.16836^{\{13\}}$	$0.13490^{\{5\}}$	$0.16469^{\{12\}}$	$0.13543^{\{6\}}$	$0.13673^{\{7\}}$	$0.19025^{\{14\}}$	$0.12798^{\{4\}}$	$0.15299^{\{9\}}$	$0.22193^{\{15\}}$	$0.02615^{\{1\}}$	$0.16222^{\{11\}}$	$0.12671^{\{3\}}$	$0.15885^{\{10\}}$
245	D_{abs}	$0.01492^{\{2\}}$	$0.01545^{\{5\}}$	$0.01542^{\{4\}}$	$0.01425^{\{1\}}$	$0.01720^{\{11\}}$	$0.01643^{\{8\}}$	$0.01624^{\{7\}}$	$0.01649^{\{9\}}$	$0.01822^{\{12\}}$	$0.01685^{\{10\}}$	$0.01839^{\{13\}}$	$0.01541^{\{3\}}$	$0.02130^{\{15\}}$	$0.01571^{\{6\}}$	$0.01971^{\{14\}}$
245	D_{max}	$0.02296^{\{3\}}$	$0.02432^{\{4\}}$	$0.02468^{\{6\}}$	$0.02251^{\{2\}}$	$0.02706^{\{11\}}$	$0.02570^{\{8\}}$	$0.02524^{\{7\}}$	$0.02670^{\{10\}}$	$0.02752^{\{12\}}$	$0.02643^{\{9\}}$	$0.03020^{\{13\}}$	$0.02192^{\{1\}}$	$0.03232^{\{15\}}$	$0.02455^{\{5\}}$	$0.03063^{\{14\}}$
245	ASAE	$0.00863^{\{4\}}$	$0.00824^{\{2\}}$	$0.00939^{\{7\}}$	$0.00883^{\{5\}}$	$0.00909^{\{6\}}$	$0.00792^{\{1\}}$	$0.00828^{\{3\}}$	$0.01056^{\{12\}}$	$0.01025^{\{11\}}$	$0.01020^{\{10\}}$	$0.01366^{\{15\}}$	$0.00972^{\{8\}}$	$0.01227^{\{14\}}$	$0.00994^{\{9\}}$	$0.01193^{\{13\}}$
245	$\sum Ranks$	$21^{\{2\}}$	$59^{\{8\}}$	$95^{\{10\}}$	$40^{\{3.5\}}$	$100^{\{11\}}$	$56^{\{6\}}$	$54^{\{5\}}$	$115^{\{14\}}$	$57^{\{7\}}$	$83^{\{9\}}$	$131^{\{15\}}$	$18^{\{1\}}$	$106^{\{13\}}$	$40^{\{3.5\}}$	$105^{\{12\}}$
320	BIAS($\hat{\delta}$)	$0.17583^{\{2\}}$	$0.23455^{\{7\}}$	$0.29716^{\{13\}}$	$0.20680^{\{3\}}$	$0.28068^{\{11\}}$	$0.22184^{\{5\}}$	$0.24577^{\{10\}}$	$0.30611^{\{14\}}$	$0.20957^{\{4\}}$	$0.23952^{\{8\}}$	$0.40895^{\{15\}}$	$0.03524^{\{1\}}$	$0.28936^{\{12\}}$	$0.22266^{\{6\}}$	$0.24399^{\{9\}}$
320	BIAS($\hat{\lambda}$)	$0.14629^{\{2\}}$	$0.18404^{\{7\}}$	$0.23434^{\{12\}}$	$0.16647^{\{3\}}$	$0.22947^{\{11\}}$	$0.17365^{\{5\}}$	$0.19350^{\{8\}}$	$0.25148^{\{14\}}$	$0.17148^{\{4\}}$	$0.19558^{\{9\}}$	$0.32950^{\{15\}}$	$0.03424^{\{1\}}$	$0.24240^{\{13\}}$	$0.17879^{\{6\}}$	$0.20610^{\{10\}}$
320	MSE($\hat{\delta}$)	$0.04945^{\{2\}}$	$0.09310^{\{8\}}$	$0.14157^{\{13\}}$	$0.06931^{\{3\}}$	$0.12408^{\{11\}}$	$0.07826^{\{5\}}$	$0.09705^{\{10\}}$	$0.14767^{\{14\}}$	$0.07537^{\{4\}}$	$0.08523^{\{7\}}$	$0.23836^{\{15\}}$	$0.00379^{\{1\}}$	$0.12578^{\{12\}}$	$0.08185^{\{6\}}$	$0.09363^{\{9\}}$
320	MSE($\hat{\lambda}$)	$0.03375^{\{2\}}$	$0.05649^{\{7\}}$	$0.08572^{\{12\}}$	$0.04462^{\{3\}}$	$0.08169^{\{11\}}$	$0.04930^{\{4\}}$	$0.05802^{\{8\}}$	$0.10072^{\{14\}}$	$0.05012^{\{5\}}$	$0.05722^{\{8\}}$	$0.16074^{\{15\}}$	$0.00269^{\{1\}}$	$0.09222^{\{13\}}$	$0.05439^{\{6\}}$	$0.06956^{\{10\}}$
320	MRE($\hat{\delta}$)	$0.14067^{\{2\}}$	$0.18764^{\{7\}}$	$0.23773^{\{13\}}$	$0.16544^{\{3\}}$	$0.22454^{\{11\}}$	$0.17747^{\{5\}}$	$0.19661^{\{10\}}$	$0.24489^{\{14\}}$	$0.16766^{\{4\}}$	$0.19162^{\{8\}}$	$0.32716^{\{15\}}$	$0.02819^{\{1\}}$	$0.23149^{\{12\}}$	$0.17813^{\{6\}}$	$0.19519^{\{9\}}$
320	MRE($\hat{\lambda}$)	$0.09753^{\{2\}}$	$0.12269^{\{7\}}$	$0.15623^{\{12\}}$	$0.11098^{\{3\}}$	$0.15298^{\{11\}}$	$0.11576^{\{5\}}$	$0.12900^{\{8\}}$	$0.16765^{\{14\}}$	$0.11432^{\{4\}}$	$0.13038^{\{9\}}$	$0.21967^{\{15\}}$	$0.02283^{\{1\}}$	$0.16160^{\{13\}}$	$0.11919^{\{6\}}$	$0.13740^{\{10\}}$
320	D_{abs}	$0.01315^{\{2\}}$	$0.01431^{\{6\}}$	$0.01510^{\{9\}}$	$0.01325^{\{3\}}$	$0.01485^{\{8\}}$	$0.01548^{\{12\}}$	$0.01401^{\{4\}}$	$0.01516^{\{10\}}$	$0.01543^{\{11\}}$	$0.01477^{\{7\}}$	$0.01607^{\{13\}}$	$0.01262^{\{1\}}$	$0.01784^{\{15\}}$	$0.01430^{\{5\}}$	$0.01668^{\{14\}}$
320	D_{max}	$0.02017^{\{2\}}$	$0.02256^{\{6\}}$	$0.02415^{\{11\}}$	$0.02066^{\{3\}}$	$0.02356^{\{8\}}$	$0.02390^{\{10\}}$	$0.02202^{\{4\}}$	$0.02424^{\{12\}}$	$0.02376^{\{9\}}$	$0.02297^{\{7\}}$	$0.02746^{\{14\}}$	$0.01805^{\{1\}}$	$0.02790^{\{15\}}$	$0.02212^{\{5\}}$	$0.02558^{\{13\}}$
320	ASAE	$0.00734^{\{5\}}$	$0.00717^{\{4\}}$	$0.00771^{\{6\}}$	$0.00710^{\{3\}}$	$0.00800^{\{7\}}$	$0.00685^{\{1\}}$	$0.00697^{\{2\}}$	$0.00902^{\{11\}}$	$0.00904^{\{12\}}$	$0.00870^{\{10\}}$	$0.01220^{\{15\}}$	$0.00833^{\{9\}}$	$0.01028^{\{14\}}$	$0.00805^{\{8\}}$	$0.01018^{\{13\}}$
320	$\sum Ranks$	$21^{\{2\}}$	$59^{\{7\}}$	$101^{\{12\}}$	$27^{\{3\}}$	$89^{\{10\}}$	$52^{\{4\}}$	$65^{\{8\}}$	$117^{\{13\}}$	$57^{\{6\}}$	$73^{\{9\}}$	$132^{\{15\}}$	$17^{\{1\}}$	$119^{\{14\}}$	$54^{\{5\}}$	$97^{\{11\}}$

Table 16. Numerical values of simulation measures for $\delta = 1.75,\ \lambda = 0.95$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$0.65205^{\{3\}}$	$0.74561^{\{6\}}$	$0.71866^{\{5\}}$	$0.82595^{\{12\}}$	$0.83626^{\{13\}}$	$0.77168^{\{8\}}$	$0.79506^{\{10\}}$	$0.76491^{\{7\}}$	$0.42067^{\{2\}}$	$0.77746^{\{9\}}$	$0.82163^{\{11\}}$	$0.03776^{\{1\}}$	$1.02953^{\{15\}}$	$0.70393^{\{4\}}$	$0.88238^{\{14\}}$
25	BIAS($\hat{\lambda}$)	$0.20279^{\{3\}}$	$0.27092^{\{7\}}$	$0.25984^{\{5\}}$	$0.31526^{\{13\}}$	$0.31501^{\{12\}}$	$0.28066^{\{9\}}$	$0.28672^{\{10\}}$	$0.26896^{\{6\}}$	$0.17502^{\{2\}}$	$0.29988^{\{11\}}$	$0.27690^{\{8\}}$	$0.07239^{\{1\}}$	$0.41019^{\{15\}}$	$0.25017^{\{4\}}$	$0.33847^{\{14\}}$
25	MSE($\hat{\delta}$)	$0.61860^{\{3\}}$	$0.77669^{\{6\}}$	$0.70333^{\{4\}}$	$0.93329^{\{13\}}$	$0.89489^{\{12\}}$	$0.81824^{\{8\}}$	$0.83374^{\{10\}}$	$0.78216^{\{7\}}$	$0.40594^{\{2\}}$	$0.85288^{\{11\}}$	$0.82741^{\{9\}}$	$0.01046^{\{1\}}$	$1.25929^{\{15\}}$	$0.73346^{\{5\}}$	$0.98085^{\{14\}}$
25	MSE($\hat{\lambda}$)	$0.06113^{\{3\}}$	$0.10963^{\{8\}}$	$0.09671^{\{4\}}$	$0.15425^{\{14\}}$	$0.13719^{\{12\}}$	$0.12061^{\{10\}}$	$0.11630^{\{9\}}$	$0.09980^{\{6\}}$	$0.05625^{\{2\}}$	$0.13514^{\{11\}}$	$0.09832^{\{5\}}$	$0.00889^{\{1\}}$	$0.20703^{\{15\}}$	$0.10640^{\{7\}}$	$0.15036^{\{13\}}$
25	MRE($\hat{\delta}$)	$0.37260^{\{3\}}$	$0.42606^{\{6\}}$	$0.41066^{\{5\}}$	$0.47197^{\{12\}}$	$0.47786^{\{13\}}$	$0.44096^{\{8\}}$	$0.45432^{\{10\}}$	$0.43709^{\{7\}}$	$0.24038^{\{2\}}$	$0.44426^{\{9\}}$	$0.46950^{\{11\}}$	$0.02158^{\{1\}}$	$0.58830^{\{15\}}$	$0.40225^{\{4\}}$	$0.50422^{\{14\}}$
25	MRE($\hat{\lambda}$)	$0.21346^{\{3\}}$	$0.28518^{\{7\}}$	$0.27352^{\{5\}}$	$0.33185^{\{13\}}$	$0.33159^{\{12\}}$	$0.29544^{\{9\}}$	$0.30181^{\{10\}}$	$0.28311^{\{6\}}$	$0.18423^{\{2\}}$	$0.31566^{\{11\}}$	$0.29147^{\{8\}}$	$0.07620^{\{1\}}$	$0.43177^{\{15\}}$	$0.26333^{\{4\}}$	$0.35628^{\{14\}}$
25	D_{abs}	$0.04114^{\{1\}}$	$0.04492^{\{4.5\}}$	$0.04845^{\{9\}}$	$0.04695^{\{6\}}$	$0.04966^{\{11\}}$	$0.04832^{\{8\}}$	$0.04879^{\{10\}}$	$0.04308^{\{2\}}$	$0.05651^{\{13\}}$	$0.05265^{\{12\}}$	$0.04331^{\{3\}}$	$0.04492^{\{4.5\}}$	$0.05795^{\{15\}}$	$0.04706^{\{7\}}$	$0.05781^{\{14\}}$
25	D_{max}	$0.06345^{\{2\}}$	$0.06847^{\{4\}}$	$0.07293^{\{8\}}$	$0.07066^{\{6\}}$	$0.07384^{\{11\}}$	$0.07304^{\{9\}}$	$0.07373^{\{10\}}$	$0.06689^{\{3\}}$	$0.08202^{\{13\}}$	$0.07828^{\{12\}}$	$0.06852^{\{5\}}$	$0.06180^{\{1\}}$	$0.08825^{\{15\}}$	$0.07107^{\{7\}}$	$0.08736^{\{14\}}$
25	ASAE	$0.04302^{\{7\}}$	$0.03672^{\{3\}}$	$0.03987^{\{6\}}$	$0.03737^{\{5\}}$	$0.03719^{\{4\}}$	$0.03421^{\{1\}}$	$0.03558^{\{2\}}$	$0.04303^{\{8\}}$	$0.04962^{\{12\}}$	$0.04624^{\{10\}}$	$0.05129^{\{14\}}$	$0.04652^{\{11\}}$	$0.05145^{\{15\}}$	$0.04372^{\{9\}}$	$0.05120^{\{13\}}$
25	$\sum Ranks$	$28^{\{2\}}$	$51.5^{\{6\}}$	$51^{\{4.5\}}$	$94^{\{11\}}$	$100^{\{13\}}$	$70^{\{8\}}$	$81^{\{10\}}$	$52^{\{7\}}$	$50^{\{3\}}$	$96^{\{12\}}$	$74^{\{9\}}$	$22.5^{\{1\}}$	$135^{\{15\}}$	$51^{\{4.5\}}$	$124^{\{14\}}$
85	BIAS($\hat{\delta}$)	$0.46208^{\{3\}}$	$0.51622^{\{6\}}$	$0.56617^{\{12\}}$	$0.56005^{\{11\}}$	$0.54999^{\{9\}}$	$0.52726^{\{7\}}$	$0.52898^{\{8\}}$	$0.64165^{\{14\}}$	$0.31944^{\{2\}}$	$0.50132^{\{4\}}$	$0.71381^{\{15\}}$	$0.02796^{\{1\}}$	$0.55463^{\{10\}}$	$0.51586^{\{5\}}$	$0.57755^{\{13\}}$
85	BIAS($\hat{\lambda}$)	$0.14096^{\{3\}}$	$0.16980^{\{5\}}$	$0.17335^{\{7.5\}}$	$0.20270^{\{12\}}$	$0.18231^{\{10\}}$	$0.16237^{\{4\}}$	$0.17423^{\{9\}}$	$0.20778^{\{14\}}$	$0.11347^{\{2\}}$	$0.17335^{\{7.5\}}$	$0.23858^{\{15\}}$	$0.04048^{\{1\}}$	$0.19166^{\{11\}}$	$0.17283^{\{6\}}$	$0.20275^{\{13\}}$
85	MSE($\hat{\delta}$)	$0.33943^{\{3\}}$	$0.42140^{\{7\}}$	$0.49567^{\{12\}}$	$0.49760^{\{13\}}$	$0.44258^{\{9\}}$	$0.43403^{\{8\}}$	$0.41836^{\{5\}}$	$0.59239^{\{14\}}$	$0.22374^{\{2\}}$	$0.40866^{\{4\}}$	$0.67706^{\{15\}}$	$0.00649^{\{1\}}$	$0.45698^{\{10\}}$	$0.42099^{\{6\}}$	$0.47378^{\{11\}}$
85	MSE($\hat{\lambda}$)	$0.03020^{\{3\}}$	$0.04748^{\{7\}}$	$0.04696^{\{6\}}$	$0.07740^{\{15\}}$	$0.05123^{\{8\}}$	$0.04221^{\{4\}}$	$0.04615^{\{5\}}$	$0.06424^{\{13\}}$	$0.02421^{\{2\}}$	$0.05597^{\{9\}}$	$0.07647^{\{14\}}$	$0.00299^{\{1\}}$	$0.05757^{\{11\}}$	$0.05601^{\{10\}}$	$0.06065^{\{12\}}$
85	MRE($\hat{\delta}$)	$0.26405^{\{3\}}$	$0.29498^{\{6\}}$	$0.32352^{\{12\}}$	$0.32003^{\{11\}}$	$0.31428^{\{9\}}$	$0.30129^{\{7\}}$	$0.30227^{\{8\}}$	$0.36666^{\{14\}}$	$0.18254^{\{2\}}$	$0.28647^{\{4\}}$	$0.40789^{\{15\}}$	$0.01598^{\{1\}}$	$0.31693^{\{10\}}$	$0.29478^{\{5\}}$	$0.33003^{\{13\}}$
85	MRE($\hat{\lambda}$)	$0.14838^{\{3\}}$	$0.17874^{\{5\}}$	$0.18248^{\{8\}}$	$0.21337^{\{12\}}$	$0.19191^{\{10\}}$	$0.17092^{\{4\}}$	$0.18340^{\{9\}}$	$0.21871^{\{14\}}$	$0.11944^{\{2\}}$	$0.18247^{\{7\}}$	$0.25113^{\{15\}}$	$0.04261^{\{1\}}$	$0.20175^{\{11\}}$	$0.18192^{\{6\}}$	$0.21342^{\{13\}}$
85	D_{abs}	$0.02454^{\{1\}}$	$0.02790^{\{6\}}$	$0.02850^{\{9\}}$	$0.02607^{\{2\}}$	$0.02678^{\{4\}}$	$0.02854^{\{10\}}$	$0.02686^{\{5\}}$	$0.02839^{\{8\}}$	$0.03080^{\{13\}}$	$0.02820^{\{7\}}$	$0.02943^{\{12\}}$	$0.02611^{\{3\}}$	$0.03112^{\{14\}}$	$0.02872^{\{11\}}$	$0.03406^{\{15\}}$
85	D_{max}	$0.03856^{\{2\}}$	$0.04343^{\{6\}}$	$0.04526^{\{11\}}$	$0.04066^{\{3\}}$	$0.04239^{\{5\}}$	$0.04506^{\{10\}}$	$0.04221^{\{4\}}$	$0.04500^{\{9\}}$	$0.04565^{\{12\}}$	$0.04378^{\{7\}}$	$0.04791^{\{13\}}$	$0.03635^{\{1\}}$	$0.04827^{\{14\}}$	$0.04454^{\{8\}}$	$0.05243^{\{15\}}$
85	ASAE	$0.01766^{\{5\}}$	$0.01674^{\{3\}}$	$0.01771^{\{6\}}$	$0.01700^{\{4\}}$	$0.01777^{\{7\}}$	$0.01571^{\{1\}}$	$0.01646^{\{2\}}$	$0.01985^{\{9\}}$	$0.02184^{\{12\}}$	$0.02058^{\{11\}}$	$0.02515^{\{15\}}$	$0.02033^{\{10\}}$	$0.02343^{\{13\}}$	$0.01983^{\{8\}}$	$0.02483^{\{14\}}$
85	$\sum Ranks$	$26^{\{2\}}$	$51^{\{4\}}$	$83.5^{\{11\}}$	$83^{\{10\}}$	$71^{\{9\}}$	$55^{\{5.5\}}$	$55^{\{5.5\}}$	$109^{\{13\}}$	$49^{\{3\}}$	$60.5^{\{7\}}$	$129^{\{15\}}$	$20^{\{1\}}$	$104^{\{12\}}$	$65^{\{8\}}$	$119^{\{14\}}$
145	BIAS($\hat{\delta}$)	$0.37172^{\{3\}}$	$0.42031^{\{6\}}$	$0.46384^{\{10\}}$	$0.41781^{\{5\}}$	$0.50092^{\{13\}}$	$0.43753^{\{9\}}$	$0.42622^{\{7\}}$	$0.51088^{\{14\}}$	$0.29871^{\{2\}}$	$0.42932^{\{8\}}$	$0.64318^{\{15\}}$	$0.02751^{\{1\}}$	$0.47687^{\{11\}}$	$0.38213^{\{4\}}$	$0.48330^{\{12\}}$
145	BIAS($\hat{\lambda}$)	$0.11257^{\{3\}}$	$0.12600^{\{4\}}$	$0.14462^{\{9\}}$	$0.14048^{\{8\}}$	$0.16316^{\{12\}}$	$0.12919^{\{5\}}$	$0.13381^{\{7\}}$	$0.16879^{\{13\}}$	$0.10468^{\{2\}}$	$0.14480^{\{10\}}$	$0.20896^{\{15\}}$	$0.03037^{\{1\}}$	$0.16140^{\{11\}}$	$0.13158^{\{6\}}$	$0.16959^{\{14\}}$
145	MSE($\hat{\delta}$)	$0.22935^{\{3\}}$	$0.27789^{\{6\}}$	$0.33619^{\{11\}}$	$0.26658^{\{5\}}$	$0.39130^{\{13\}}$	$0.30862^{\{9\}}$	$0.28339^{\{7\}}$	$0.40224^{\{14\}}$	$0.19381^{\{2\}}$	$0.28943^{\{8\}}$	$0.55961^{\{15\}}$	$0.00369^{\{1\}}$	$0.32314^{\{10\}}$	$0.24429^{\{4\}}$	$0.33791^{\{12\}}$
145	MSE($\hat{\lambda}$)	$0.02023^{\{2\}}$	$0.02494^{\{4\}}$	$0.03258^{\{8\}}$	$0.03368^{\{9\}}$	$0.04155^{\{12\}}$	$0.02580^{\{5\}}$	$0.02934^{\{6\}}$	$0.04328^{\{14\}}$	$0.02027^{\{3\}}$	$0.03768^{\{10\}}$	$0.06229^{\{15\}}$	$0.00146^{\{1\}}$	$0.03895^{\{11\}}$	$0.03257^{\{7\}}$	$0.04271^{\{13\}}$
145	MRE($\hat{\delta}$)	$0.21241^{\{3\}}$	$0.24018^{\{6\}}$	$0.26505^{\{10\}}$	$0.23875^{\{5\}}$	$0.28624^{\{13\}}$	$0.25002^{\{9\}}$	$0.24355^{\{7\}}$	$0.29193^{\{14\}}$	$0.17069^{\{2\}}$	$0.24533^{\{8\}}$	$0.36753^{\{15\}}$	$0.01572^{\{1\}}$	$0.27250^{\{11\}}$	$0.21836^{\{4\}}$	$0.27617^{\{12\}}$
145	MRE($\hat{\lambda}$)	$0.11849^{\{3\}}$	$0.13263^{\{4\}}$	$0.15223^{\{9\}}$	$0.14787^{\{8\}}$	$0.17175^{\{12\}}$	$0.13599^{\{5\}}$	$0.14085^{\{7\}}$	$0.17767^{\{13\}}$	$0.11019^{\{2\}}$	$0.15242^{\{10\}}$	$0.21996^{\{15\}}$	$0.03197^{\{1\}}$	$0.16990^{\{11\}}$	$0.13851^{\{6\}}$	$0.17851^{\{14\}}$
145	D_{abs}	$0.02021^{\{4\}}$	$0.02194^{\{7\}}$	$0.02133^{\{6\}}$	$0.01946^{\{2\}}$	$0.02333^{\{11\}}$	$0.02224^{\{9\}}$	$0.02002^{\{3\}}$	$0.02213^{\{8\}}$	$0.02349^{\{12\}}$	$0.02227^{\{10\}}$	$0.02401^{\{13\}}$	$0.01927^{\{1\}}$	$0.02414^{\{14\}}$	$0.02119^{\{5\}}$	$0.02625^{\{15\}}$
145	D_{max}	$0.03174^{\{3\}}$	$0.03470^{\{7\}}$	$0.03420^{\{6\}}$	$0.03090^{\{2\}}$	$0.03736^{\{12\}}$	$0.03541^{\{10\}}$	$0.03226^{\{4\}}$	$0.03545^{\{11\}}$	$0.03536^{\{9\}}$	$0.03508^{\{8\}}$	$0.04004^{\{14\}}$	$0.02701^{\{1\}}$	$0.03810^{\{13\}}$	$0.03303^{\{5\}}$	$0.04087^{\{15\}}$
145	ASAE	$0.01225^{\{4.5\}}$	$0.01160^{\{2\}}$	$0.01225^{\{4.5\}}$	$0.01230^{\{6\}}$	$0.01288^{\{7\}}$	$0.01158^{\{1\}}$	$0.01177^{\{3\}}$	$0.01416^{\{9\}}$	$0.01551^{\{11\}}$	$0.01569^{\{12\}}$	$0.01915^{\{15\}}$	$0.01472^{\{10\}}$	$0.01725^{\{13\}}$	$0.01378^{\{8\}}$	$0.01732^{\{14\}}$
145	$\sum Ranks$	$28.5^{\{2\}}$	$46^{\{4\}}$	$73.5^{\{9\}}$	$50^{\{6\}}$	$105^{\{11.5\}}$	$62^{\{8\}}$	$51^{\{7\}}$	$110^{\{13\}}$	$45^{\{3\}}$	$84^{\{10\}}$	$132^{\{15\}}$	$18^{\{1\}}$	$105^{\{11.5\}}$	$49^{\{5\}}$	$121^{\{14\}}$
190	BIAS($\hat{\delta}$)	$0.33830^{\{4\}}$	$0.35047^{\{6\}}$	$0.42603^{\{11\}}$	$0.32789^{\{3\}}$	$0.41930^{\{10\}}$	$0.36373^{\{8\}}$	$0.35449^{\{7\}}$	$0.50764^{\{14\}}$	$0.27588^{\{2\}}$	$0.40108^{\{9\}}$	$0.60366^{\{15\}}$	$0.02152^{\{1\}}$	$0.44836^{\{13\}}$	$0.34347^{\{5\}}$	$0.43591^{\{12\}}$
190	BIAS($\hat{\lambda}$)	$0.09843^{\{3\}}$	$0.11319^{\{8\}}$	$0.13253^{\{9\}}$	$0.10621^{\{4\}}$	$0.13436^{\{11\}}$	$0.10940^{\{6\}}$	$0.10968^{\{7\}}$	$0.15749^{\{14\}}$	$0.09525^{\{2\}}$	$0.13293^{\{10\}}$	$0.19378^{\{15\}}$	$0.02837^{\{1\}}$	$0.14678^{\{12\}}$	$0.10844^{\{5\}}$	$0.14957^{\{13\}}$
190	MSE($\hat{\delta}$)	$0.20657^{\{6\}}$	$0.19832^{\{4\}}$	$0.27756^{\{11\}}$	$0.16965^{\{3\}}$	$0.27672^{\{10\}}$	$0.22011^{\{8\}}$	$0.21198^{\{7\}}$	$0.38474^{\{14\}}$	$0.16900^{\{2\}}$	$0.24936^{\{9\}}$	$0.48970^{\{15\}}$	$0.00219^{\{1\}}$	$0.29960^{\{13\}}$	$0.20265^{\{5\}}$	$0.27906^{\{12\}}$
190	MSE($\hat{\lambda}$)	$0.01595^{\{2\}}$	$0.02071^{\{7\}}$	$0.02574^{\{9\}}$	$0.01792^{\{4\}}$	$0.02872^{\{10\}}$	$0.01921^{\{6\}}$	$0.01907^{\{5\}}$	$0.03577^{\{14\}}$	$0.01679^{\{3\}}$	$0.03021^{\{11\}}$	$0.05075^{\{15\}}$	$0.00138^{\{1\}}$	$0.03327^{\{12\}}$	$0.02158^{\{8\}}$	$0.03413^{\{13\}}$
190	MRE($\hat{\delta}$)	$0.19331^{\{4\}}$	$0.20027^{\{6\}}$	$0.24344^{\{11\}}$	$0.18736^{\{3\}}$	$0.23960^{\{10\}}$	$0.20784^{\{8\}}$	$0.20257^{\{7\}}$	$0.29008^{\{14\}}$	$0.15764^{\{2\}}$	$0.22919^{\{9\}}$	$0.34495^{\{15\}}$	$0.01230^{\{1\}}$	$0.25620^{\{13\}}$	$0.19627^{\{5\}}$	$0.24909^{\{12\}}$
190	MRE($\hat{\lambda}$)	$0.10361^{\{3\}}$	$0.11915^{\{8\}}$	$0.13951^{\{9\}}$	$0.11180^{\{4\}}$	$0.14143^{\{11\}}$	$0.11515^{\{6\}}$	$0.11545^{\{7\}}$	$0.16578^{\{14\}}$	$0.10026^{\{2\}}$	$0.13992^{\{10\}}$	$0.20398^{\{15\}}$	$0.02986^{\{1\}}$	$0.15451^{\{12\}}$	$0.11415^{\{5\}}$	$0.15744^{\{13\}}$
190	D_{abs}	$0.01707^{\{1\}}$	$0.01820^{\{5\}}$	$0.01945^{\{9\}}$	$0.01852^{\{6\}}$	$0.01961^{\{10\}}$	$0.01780^{\{3\}}$	$0.01752^{\{2\}}$	$0.02075^{\{11\}}$	$0.02102^{\{12\}}$	$0.01944^{\{8\}}$	$0.02136^{\{13\}}$	$0.01786^{\{4\}}$	$0.02162^{\{14\}}$	$0.01911^{\{7\}}$	$0.02204^{\{15\}}$
190	D_{max}	$0.02681^{\{2\}}$	$0.02867^{\{5\}}$	$0.03121^{\{10\}}$	$0.02903^{\{6\}}$	$0.03117^{\{9\}}$	$0.02836^{\{4\}}$	$0.02790^{\{3\}}$	$0.03392^{\{12\}}$	$0.03169^{\{11\}}$	$0.03063^{\{8\}}$	$0.03602^{\{15\}}$	$0.02485^{\{1\}}$	$0.03451^{\{13\}}$	$0.03020^{\{7\}}$	$0.03480^{\{14\}}$
190	ASAE	$0.01075^{\{6\}}$	$0.01003^{\{3\}}$	$0.01048^{\{5\}}$	$0.01010^{\{4\}}$	$0.01094^{\{7\}}$	$0.00977^{\{1\}}$	$0.00978^{\{2\}}$	$0.01225^{\{10\}}$	$0.01278^{\{12\}}$	$0.01251^{\{11\}}$	$0.01608^{\{15\}}$	$0.01212^{\{9\}}$	$0.01451^{\{13\}}$	$0.01190^{\{8\}}$	$0.01467^{\{14\}}$
190	$\sum Ranks$	$31^{\{2\}}$	$52^{\{7\}}$	$84^{\{9\}}$	$37^{\{3\}}$	$88^{\{11\}}$	$50^{\{6\}}$	$47^{\{4\}}$	$117^{\{13\}}$	$48^{\{5\}}$	$85^{\{10\}}$	$133^{\{15\}}$	$20^{\{1\}}$	$115^{\{12\}}$	$55^{\{8\}}$	$118^{\{14\}}$
245	BIAS($\hat{\delta}$)	$0.29806^{\{4\}}$	$0.34225^{\{9\}}$	$0.35976^{\{10\}}$	$0.30111^{\{5\}}$	$0.38394^{\{12\}}$	$0.32490^{\{8\}}$	$0.32327^{\{7\}}$	$0.40676^{\{13\}}$	$0.26248^{\{2\}}$	$0.29599^{\{3\}}$	$0.57481^{\{15\}}$	$0.02010^{\{1\}}$	$0.41142^{\{14\}}$	$0.32079^{\{6\}}$	$0.36775^{\{11\}}$
245	BIAS($\hat{\lambda}$)	$0.09371^{\{3\}}$	$0.10558^{\{9\}}$	$0.11644^{\{10\}}$	$0.09858^{\{5\}}$	$0.12037^{\{11\}}$	$0.10058^{\{7\}}$	$0.09991^{\{6\}}$	$0.12665^{\{12\}}$	$0.09114^{\{2\}}$	$0.09685^{\{4\}}$	$0.18451^{\{15\}}$	$0.02129^{\{1\}}$	$0.13772^{\{14\}}$	$0.10360^{\{8\}}$	$0.12673^{\{13\}}$
245	MSE($\hat{\delta}$)	$0.15223^{\{5\}}$	$0.18624^{\{9\}}$	$0.20203^{\{10\}}$	$0.14760^{\{4\}}$	$0.23572^{\{12\}}$	$0.16497^{\{6\}}$	$0.16832^{\{7\}}$	$0.26881^{\{14\}}$	$0.13959^{\{2\}}$	$0.14414^{\{3\}}$	$0.44178^{\{15\}}$	$0.00131^{\{1\}}$	$0.24707^{\{13\}}$	$0.17248^{\{8\}}$	$0.20739^{\{11\}}$
245	MSE($\hat{\lambda}$)	$0.01396^{\{2\}}$	$0.01761^{\{8\}}$	$0.02071^{\{10\}}$	$0.01648^{\{7\}}$	$0.02330^{\{11\}}$	$0.01546^{\{5\}}$	$0.01541^{\{4\}}$	$0.02555^{\{12\}}$	$0.01468^{\{3\}}$	$0.01614^{\{6\}}$	$0.04476^{\{15\}}$	$0.00076^{\{1\}}$	$0.02865^{\{14\}}$	$0.01871^{\{9\}}$	$0.02583^{\{13\}}$
245	MRE($\hat{\delta}$)	$0.17032^{\{4\}}$	$0.19557^{\{9\}}$	$0.20558^{\{10\}}$	$0.17206^{\{5\}}$	$0.21940^{\{12\}}$	$0.18566^{\{8\}}$	$0.18473^{\{7\}}$	$0.23243^{\{13\}}$	$0.14999^{\{2\}}$	$0.16914^{\{3\}}$	$0.32846^{\{15\}}$	$0.01148^{\{1\}}$	$0.23510^{\{14\}}$	$0.18331^{\{6\}}$	$0.21014^{\{11\}}$
245	MRE($\hat{\lambda}$)	$0.09864^{\{3\}}$	$0.11114^{\{9\}}$	$0.12256^{\{10\}}$	$0.10377^{\{5\}}$	$0.12670^{\{11\}}$	$0.10587^{\{7\}}$	$0.10517^{\{6\}}$	$0.13332^{\{12\}}$	$0.09594^{\{2\}}$	$0.10195^{\{4\}}$	$0.19422^{\{15\}}$	$0.02241^{\{1\}}$	$0.14497^{\{14\}}$	$0.10905^{\{8\}}$	$0.13340^{\{13\}}$
245	D_{abs}	$0.01555^{\{3\}}$	$0.01594^{\{6\}}$	$0.01652^{\{7\}}$	$0.01546^{\{2\}}$	$0.01712^{\{9\}}$	$0.01749^{\{10\}}$	$0.01564^{\{4\}}$	$0.01692^{\{8\}}$	$0.01962^{\{14\}}$	$0.01799^{\{11\}}$	$0.02005^{\{15\}}$	$0.01329^{\{1\}}$	$0.01867^{\{12\}}$	$0.01589^{\{5\}}$	$0.01906^{\{13\}}$
245	D_{max}	$0.02466^{\{3\}}$	$0.02571^{\{6\}}$	$0.02658^{\{7\}}$	$0.02443^{\{2\}}$	$0.02767^{\{10.5\}}$	$0.02725^{\{8\}}$	$0.02491^{\{4\}}$	$0.02744^{\{9\}}$	$0.02984^{\{12\}}$	$0.02767^{\{10.5\}}$	$0.03392^{\{15\}}$	$0.01856^{\{1\}}$	$0.02993^{\{13\}}$	$0.02523^{\{5\}}$	$0.03012^{\{14\}}$
245	ASAE	$0.00901^{\{5\}}$	$0.00828^{\{3\}}$	$0.00918^{\{7\}}$	$0.00879^{\{4\}}$	$0.00907^{\{6\}}$	$0.00804^{\{1\}}$	$0.00824^{\{2\}}$	$0.00987^{\{8\}}$	$0.01124^{\{12\}}$	$0.01063^{\{11\}}$	$0.01458^{\{15\}}$	$0.01033^{\{10\}}$	$0.01230^{\{14\}}$	$0.00997^{\{9\}}$	$0.01193^{\{13\}}$
245	$\sum Ranks$	$32^{\{2\}}$	$68^{\{9\}}$	$81^{\{10\}}$	$39^{\{3\}}$	$94.5^{\{11\}}$	$60^{\{7\}}$	$47^{\{4\}}$	$101^{\{12\}}$	$51^{\{5\}}$	$55.5^{\{6\}}$	$135^{\{15\}}$	$18^{\{1\}}$	$122^{\{14\}}$	$64^{\{8\}}$	$112^{\{13\}}$
320	BIAS($\hat{\delta}$)	$0.24894^{\{3\}}$	$0.26045^{\{4\}}$	$0.34068^{\{13\}}$	$0.26719^{\{5\}}$	$0.33136^{\{12\}}$	$0.28194^{\{7\}}$	$0.27056^{\{6\}}$	$0.37833^{\{14\}}$	$0.20615^{\{2\}}$	$0.29196^{\{8\}}$	$0.51376^{\{15\}}$	$0.01706^{\{1\}}$	$0.32503^{\{11\}}$	$0.29605^{\{9\}}$	$0.32060^{\{10\}}$
320	BIAS($\hat{\lambda}$)	$0.07965^{\{3\}}$	$0.08021^{\{4\}}$	$0.10795^{\{11\}}$	$0.08502^{\{6\}}$	$0.10683^{\{10\}}$	$0.08410^{\{5\}}$	$0.08650^{\{7\}}$	$0.12055^{\{14\}}$	$0.07117^{\{2\}}$	$0.08802^{\{8\}}$	$0.16448^{\{15\}}$	$0.01925^{\{1\}}$	$0.11039^{\{12\}}$	$0.09375^{\{9\}}$	$0.11041^{\{13\}}$
320	MSE($\hat{\delta}$)	$0.09635^{\{3\}}$	$0.10941^{\{5\}}$	$0.18785^{\{12\}}$	$0.10520^{\{4\}}$	$0.18805^{\{13\}}$	$0.13535^{\{8\}}$	$0.11543^{\{6\}}$	$0.22387^{\{14\}}$	$0.09066^{\{2\}}$	$0.13308^{\{7\}}$	$0.37666^{\{15\}}$	$0.00094^{\{1\}}$	$0.16495^{\{11\}}$	$0.13847^{\{9\}}$	$0.15412^{\{10\}}$
320	MSE($\hat{\lambda}$)	$0.00953^{\{3\}}$	$0.01041^{\{4\}}$	$0.01818^{\{10\}}$	$0.01096^{\{5\}}$	$0.01835^{\{11\}}$	$0.01107^{\{6\}}$	$0.01132^{\{7\}}$	$0.02252^{\{14\}}$	$0.00949^{\{2\}}$	$0.01254^{\{8\}}$	$0.03710^{\{15\}}$	6e-04$^{\{1\}}$	$0.01958^{\{13\}}$	$0.01430^{\{9\}}$	$0.01880^{\{12\}}$
320	MRE($\hat{\delta}$)	$0.14225^{\{3\}}$	$0.14883^{\{4\}}$	$0.19467^{\{13\}}$	$0.15268^{\{5\}}$	$0.18935^{\{12\}}$	$0.16111^{\{7\}}$	$0.15461^{\{6\}}$	$0.21619^{\{14\}}$	$0.11780^{\{2\}}$	$0.16684^{\{8\}}$	$0.29358^{\{15\}}$	$0.00975^{\{1\}}$	$0.18573^{\{11\}}$	$0.16917^{\{9\}}$	$0.18320^{\{10\}}$
320	MRE($\hat{\lambda}$)	$0.08384^{\{3\}}$	$0.08443^{\{4\}}$	$0.11363^{\{11\}}$	$0.08950^{\{6\}}$	$0.11245^{\{10\}}$	$0.08853^{\{5\}}$	$0.09105^{\{7\}}$	$0.12690^{\{14\}}$	$0.07491^{\{2\}}$	$0.09265^{\{8\}}$	$0.17313^{\{15\}}$	$0.02026^{\{1\}}$	$0.11620^{\{12\}}$	$0.09868^{\{9\}}$	$0.11622^{\{13\}}$
320	D_{abs}	$0.01408^{\{6\}}$	$0.01394^{\{5\}}$	$0.01584^{\{10\}}$	$0.01308^{\{3\}}$	$0.01649^{\{11\}}$	$0.01493^{\{8\}}$	$0.01287^{\{2\}}$	$0.01482^{\{7\}}$	$0.01702^{\{12\}}$	$0.01574^{\{9\}}$	$0.01710^{\{13\}}$	$0.01279^{\{1\}}$	$0.01834^{\{14\}}$	$0.01389^{\{4\}}$	$0.01849^{\{15\}}$
320	D_{max}	$0.02177^{\{4\}}$	$0.02203^{\{5\}}$	$0.02539^{\{10\}}$	$0.02103^{\{3\}}$	$0.02593^{\{12\}}$	$0.02359^{\{7\}}$	$0.02048^{\{2\}}$	$0.02445^{\{8\}}$	$0.02562^{\{11\}}$	$0.02478^{\{9\}}$	$0.02928^{\{15\}}$	$0.01784^{\{1\}}$	$0.02864^{\{13\}}$	$0.02227^{\{6\}}$	$0.02871^{\{14\}}$
320	ASAE	$0.00728^{\{4\}}$	$0.00713^{\{3\}}$	$0.00790^{\{7\}}$	$0.00756^{\{6\}}$	$0.00750^{\{5\}}$	$0.00685^{\{1\}}$	$0.00695^{\{2\}}$	$0.00861^{\{10\}}$	$0.00953^{\{12\}}$	$0.00916^{\{11\}}$	$0.01240^{\{15\}}$	$0.00860^{\{9\}}$	$0.01050^{\{14\}}$	$0.00847^{\{8\}}$	$0.01008^{\{13\}}$
320	$\sum Ranks$	$32^{\{2\}}$	$38^{\{3\}}$	$97^{\{11\}}$	$43^{\{4\}}$	$96^{\{10\}}$	$54^{\{7\}}$	$45^{\{5\}}$	$109^{\{12\}}$	$47^{\{6\}}$	$76^{\{9\}}$	$133^{\{15\}}$	$17^{\{1\}}$	$111^{\{14\}}$	$72^{\{8\}}$	$110^{\{13\}}$

Table 17. Numerical values of simulation measures for $\delta = 2.0,\ \lambda = 0.65$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$0.71358^{\{3\}}$	$0.80816^{\{5\}}$	$0.81939^{\{6\}}$	$0.94339^{\{13\}}$	$0.86909^{\{10\}}$	$0.76078^{\{4\}}$	$0.83026^{\{9\}}$	$0.89769^{\{11\}}$	$0.34713^{\{2\}}$	$0.82376^{\{7\}}$	$0.91871^{\{12\}}$	$0.02109^{\{1\}}$	$1.07160^{\{14\}}$	$0.82435^{\{8\}}$	$1.09472^{\{15\}}$
25	BIAS($\hat{\lambda}$)	$0.12923^{\{3\}}$	$0.16594^{\{6\}}$	$0.16520^{\{5\}}$	$0.20646^{\{13\}}$	$0.18198^{\{12\}}$	$0.15023^{\{4\}}$	$0.17569^{\{9\}}$	$0.17731^{\{10\}}$	$0.09440^{\{2\}}$	$0.17056^{\{7\}}$	$0.17890^{\{11\}}$	$0.04594^{\{1\}}$	$0.24803^{\{14\}}$	$0.17303^{\{8\}}$	$0.25096^{\{15\}}$
25	MSE($\hat{\delta}$)	$0.74363^{\{3\}}$	$0.94787^{\{7\}}$	$0.93932^{\{6\}}$	$1.23693^{\{13\}}$	$1.03446^{\{10\}}$	$0.84284^{\{4\}}$	$0.93371^{\{5\}}$	$1.07232^{\{12\}}$	$0.31359^{\{2\}}$	$0.98861^{\{8\}}$	$1.04763^{\{11\}}$	$0.00132^{\{1\}}$	$1.41320^{\{14\}}$	$1.02573^{\{9\}}$	$1.44784^{\{15\}}$
25	MSE($\hat{\lambda}$)	$0.02444^{\{3\}}$	$0.04310^{\{7\}}$	$0.04111^{\{5\}}$	$0.06901^{\{13\}}$	$0.04783^{\{10\}}$	$0.03678^{\{4\}}$	$0.04642^{\{9\}}$	$0.04332^{\{8\}}$	$0.01608^{\{2\}}$	$0.04840^{\{11\}}$	$0.04263^{\{6\}}$	$0.00347^{\{1\}}$	$0.07994^{\{15\}}$	$0.05322^{\{12\}}$	$0.07846^{\{14\}}$
25	MRE($\hat{\delta}$)	$0.35679^{\{3\}}$	$0.40408^{\{5\}}$	$0.40970^{\{6\}}$	$0.47170^{\{13\}}$	$0.43454^{\{10\}}$	$0.38039^{\{4\}}$	$0.41513^{\{9\}}$	$0.44884^{\{11\}}$	$0.17357^{\{2\}}$	$0.41188^{\{7\}}$	$0.45936^{\{12\}}$	$0.01054^{\{1\}}$	$0.53580^{\{14\}}$	$0.41217^{\{8\}}$	$0.54736^{\{15\}}$
25	MRE($\hat{\lambda}$)	$0.19881^{\{3\}}$	$0.25529^{\{6\}}$	$0.25415^{\{5\}}$	$0.31763^{\{13\}}$	$0.27997^{\{12\}}$	$0.23112^{\{4\}}$	$0.27029^{\{9\}}$	$0.27278^{\{10\}}$	$0.14524^{\{2\}}$	$0.26241^{\{7\}}$	$0.27523^{\{11\}}$	$0.07067^{\{1\}}$	$0.38159^{\{14\}}$	$0.26620^{\{8\}}$	$0.38609^{\{15\}}$
25	D_{abs}	$0.04248^{\{1\}}$	$0.04551^{\{3\}}$	$0.04876^{\{8\}}$	$0.04750^{\{6\}}$	$0.05054^{\{11\}}$	$0.04910^{\{9\}}$	$0.04807^{\{7\}}$	$0.04648^{\{5\}}$	$0.05536^{\{13\}}$	$0.05171^{\{12\}}$	$0.04614^{\{4\}}$	$0.04477^{\{2\}}$	$0.06202^{\{14\}}$	$0.04953^{\{10\}}$	$0.06208^{\{15\}}$
25	D_{max}	$0.06542^{\{2\}}$	$0.06877^{\{3\}}$	$0.07443^{\{9\}}$	$0.07184^{\{5\}}$	$0.07622^{\{11\}}$	$0.07403^{\{8\}}$	$0.07198^{\{6\}}$	$0.07179^{\{4\}}$	$0.07947^{\{13\}}$	$0.07715^{\{12\}}$	$0.07225^{\{7\}}$	$0.06150^{\{1\}}$	$0.09383^{\{14\}}$	$0.07487^{\{10\}}$	$0.09474^{\{15\}}$
25	ASAE	$0.04164^{\{7\}}$	$0.03778^{\{4\}}$	$0.03814^{\{5\}}$	$0.03876^{\{6\}}$	$0.03642^{\{3\}}$	$0.03495^{\{1\}}$	$0.03638^{\{2\}}$	$0.04313^{\{9\}}$	$0.05219^{\{15\}}$	$0.04361^{\{10\}}$	$0.05093^{\{12\}}$	$0.04705^{\{11\}}$	$0.05192^{\{14\}}$	$0.04267^{\{8\}}$	$0.05129^{\{13\}}$
25	$\sum Ranks$	$28^{\{2\}}$	$46^{\{4\}}$	$55^{\{6\}}$	$95^{\{13\}}$	$89^{\{12\}}$	$42^{\{3\}}$	$65^{\{7\}}$	$80^{\{8\}}$	$53^{\{5\}}$	$81^{\{9.5\}}$	$86^{\{11\}}$	$20^{\{1\}}$	$127^{\{14\}}$	$81^{\{9.5\}}$	$132^{\{15\}}$
85	BIAS($\hat{\delta}$)	$0.50798^{\{4\}}$	$0.60415^{\{11\}}$	$0.69062^{\{14\}}$	$0.55111^{\{5\}}$	$0.65774^{\{13\}}$	$0.56616^{\{6\}}$	$0.57432^{\{7\}}$	$0.64685^{\{12\}}$	$0.33439^{\{2\}}$	$0.59905^{\{10\}}$	$0.81233^{\{15\}}$	$0.01951^{\{1\}}$	$0.45548^{\{3\}}$	$0.57478^{\{8\}}$	$0.59088^{\{9\}}$
85	BIAS($\hat{\lambda}$)	$0.08480^{\{3\}}$	$0.11233^{\{9\}}$	$0.12335^{\{13\}}$	$0.11137^{\{7\}}$	$0.12203^{\{12\}}$	$0.09674^{\{4\}}$	$0.10587^{\{6\}}$	$0.12392^{\{14\}}$	$0.07521^{\{2\}}$	$0.11795^{\{11\}}$	$0.15448^{\{15\}}$	$0.02672^{\{1\}}$	$0.09792^{\{5\}}$	$0.11213^{\{8\}}$	$0.11708^{\{10\}}$
85	MSE($\hat{\delta}$)	$0.44038^{\{4\}}$	$0.54567^{\{10\}}$	$0.70255^{\{14\}}$	$0.51475^{\{8\}}$	$0.63686^{\{13\}}$	$0.50432^{\{6\}}$	$0.51195^{\{7\}}$	$0.61612^{\{12\}}$	$0.28458^{\{2\}}$	$0.55675^{\{11\}}$	$0.88460^{\{15\}}$	$0.00131^{\{1\}}$	$0.37404^{\{3\}}$	$0.53212^{\{9\}}$	$0.48054^{\{5\}}$
85	MSE($\hat{\lambda}$)	$0.01162^{\{3\}}$	$0.02000^{\{8\}}$	$0.02286^{\{10\}}$	$0.02467^{\{14\}}$	$0.02277^{\{9\}}$	$0.01474^{\{4\}}$	$0.01734^{\{5\}}$	$0.02375^{\{12\}}$	$0.01061^{\{2\}}$	$0.02313^{\{11\}}$	$0.03281^{\{15\}}$	$0.00110^{\{1\}}$	$0.01792^{\{6\}}$	$0.02397^{\{13\}}$	$0.01900^{\{7\}}$
85	MRE($\hat{\delta}$)	$0.25399^{\{4\}}$	$0.30207^{\{11\}}$	$0.34531^{\{14\}}$	$0.27555^{\{5\}}$	$0.32887^{\{13\}}$	$0.28308^{\{6\}}$	$0.28716^{\{7\}}$	$0.32342^{\{12\}}$	$0.16719^{\{2\}}$	$0.29953^{\{10\}}$	$0.40617^{\{15\}}$	$0.00975^{\{1\}}$	$0.22774^{\{3\}}$	$0.28739^{\{8\}}$	$0.29544^{\{9\}}$
85	MRE($\hat{\lambda}$)	$0.13047^{\{3\}}$	$0.17282^{\{9\}}$	$0.18977^{\{13\}}$	$0.17134^{\{7\}}$	$0.18774^{\{12\}}$	$0.14883^{\{4\}}$	$0.16288^{\{6\}}$	$0.19065^{\{14\}}$	$0.11571^{\{2\}}$	$0.18147^{\{11\}}$	$0.23765^{\{15\}}$	$0.04110^{\{1\}}$	$0.15064^{\{5\}}$	$0.17251^{\{8\}}$	$0.18013^{\{10\}}$
85	D_{abs}	$0.02558^{\{1\}}$	$0.02684^{\{5\}}$	$0.02742^{\{10\}}$	$0.02717^{\{8\}}$	$0.02738^{\{9\}}$	$0.02698^{\{6\}}$	$0.02602^{\{3\}}$	$0.02615^{\{4\}}$	$0.03220^{\{15\}}$	$0.03011^{\{11\}}$	$0.03089^{\{12\}}$	$0.02583^{\{2\}}$	$0.03210^{\{14\}}$	$0.02713^{\{7\}}$	$0.03134^{\{13\}}$
85	D_{max}	$0.03998^{\{2\}}$	$0.04246^{\{7\}}$	$0.04420^{\{10\}}$	$0.04234^{\{4\}}$	$0.04358^{\{9\}}$	$0.04237^{\{5\}}$	$0.04121^{\{3\}}$	$0.04239^{\{6\}}$	$0.04728^{\{12\}}$	$0.04669^{\{11\}}$	$0.05046^{\{15\}}$	$0.03571^{\{1\}}$	$0.04809^{\{13\}}$	$0.04252^{\{8\}}$	$0.04843^{\{14\}}$
85	ASAE	$0.01730^{\{5.5\}}$	$0.01693^{\{4\}}$	$0.01730^{\{5.5\}}$	$0.01659^{\{2\}}$	$0.01806^{\{7\}}$	$0.01599^{\{1\}}$	$0.01689^{\{3\}}$	$0.01982^{\{8\}}$	$0.02180^{\{12\}}$	$0.02026^{\{9\}}$	$0.02571^{\{15\}}$	$0.02099^{\{11\}}$	$0.02352^{\{13\}}$	$0.02027^{\{10\}}$	$0.02377^{\{14\}}$
85	$\sum Ranks$	$29.5^{\{2\}}$	$74^{\{8\}}$	$103.5^{\{14\}}$	$60^{\{6\}}$	$97^{\{13\}}$	$42^{\{3\}}$	$47^{\{4\}}$	$94^{\{11\}}$	$51^{\{5\}}$	$95^{\{12\}}$	$132^{\{15\}}$	$20^{\{1\}}$	$65^{\{7\}}$	$79^{\{9\}}$	$91^{\{10\}}$
145	BIAS($\hat{\delta}$)	$0.40423^{\{3\}}$	$0.44278^{\{4\}}$	$0.52269^{\{11\}}$	$0.45894^{\{9\}}$	$0.54444^{\{13\}}$	$0.45586^{\{8\}}$	$0.45263^{\{7\}}$	$0.54418^{\{12\}}$	$0.24469^{\{2\}}$	$0.49645^{\{10\}}$	$0.74795^{\{15\}}$	$0.01702^{\{1\}}$	$0.44735^{\{5\}}$	$0.44971^{\{6\}}$	$0.57235^{\{14\}}$
145	BIAS($\hat{\lambda}$)	$0.06933^{\{3\}}$	$0.07795^{\{4\}}$	$0.09536^{\{11\}}$	$0.08601^{\{8\}}$	$0.10007^{\{12\}}$	$0.07818^{\{5\}}$	$0.07951^{\{6\}}$	$0.10252^{\{13\}}$	$0.05572^{\{2\}}$	$0.09098^{\{10\}}$	$0.13735^{\{15\}}$	$0.02115^{\{1\}}$	$0.08660^{\{9\}}$	$0.08074^{\{7\}}$	$0.10746^{\{14\}}$
145	MSE($\hat{\delta}$)	$0.26268^{\{3\}}$	$0.30922^{\{5\}}$	$0.43011^{\{11\}}$	$0.33732^{\{8\}}$	$0.45790^{\{12\}}$	$0.33494^{\{7\}}$	$0.32577^{\{6\}}$	$0.47654^{\{14\}}$	$0.14422^{\{2\}}$	$0.40196^{\{10\}}$	$0.74991^{\{15\}}$	$0.00093^{\{1\}}$	$0.30347^{\{4\}}$	$0.33798^{\{9\}}$	$0.46482^{\{13\}}$
145	MSE($\hat{\lambda}$)	$0.00772^{\{3\}}$	$0.00959^{\{4\}}$	$0.01457^{\{11\}}$	$0.01337^{\{9\}}$	$0.01537^{\{12\}}$	$0.00977^{\{5\}}$	$0.00990^{\{6\}}$	$0.01700^{\{14\}}$	$0.00580^{\{2\}}$	$0.01425^{\{10\}}$	$0.02568^{\{15\}}$	$0.00067^{\{1\}}$	$0.01128^{\{7\}}$	$0.01159^{\{8\}}$	$0.01647^{\{13\}}$
145	MRE($\hat{\delta}$)	$0.20212^{\{3\}}$	$0.22139^{\{4\}}$	$0.26135^{\{11\}}$	$0.22947^{\{9\}}$	$0.27222^{\{13\}}$	$0.22793^{\{8\}}$	$0.22632^{\{7\}}$	$0.27209^{\{12\}}$	$0.12234^{\{2\}}$	$0.24822^{\{10\}}$	$0.37397^{\{15\}}$	$0.00851^{\{1\}}$	$0.22368^{\{5\}}$	$0.22486^{\{6\}}$	$0.28617^{\{14\}}$
145	MRE($\hat{\lambda}$)	$0.10667^{\{3\}}$	$0.11993^{\{4\}}$	$0.14671^{\{11\}}$	$0.13232^{\{8\}}$	$0.15396^{\{12\}}$	$0.12027^{\{5\}}$	$0.12233^{\{6\}}$	$0.15772^{\{13\}}$	$0.08572^{\{2\}}$	$0.13998^{\{10\}}$	$0.21130^{\{15\}}$	$0.03253^{\{1\}}$	$0.13324^{\{9\}}$	$0.12421^{\{7\}}$	$0.16532^{\{14\}}$
145	D_{abs}	$0.01985^{\{1\}}$	$0.02088^{\{4\}}$	$0.02343^{\{11\}}$	$0.02085^{\{3\}}$	$0.02269^{\{10\}}$	$0.02136^{\{6\}}$	$0.02184^{\{8\}}$	$0.02168^{\{7\}}$	$0.02500^{\{13\}}$	$0.02244^{\{9\}}$	$0.02507^{\{14\}}$	$0.02035^{\{2\}}$	$0.02638^{\{15\}}$	$0.02122^{\{5\}}$	$0.02468^{\{12\}}$
145	D_{max}	$0.03138^{\{2\}}$	$0.03320^{\{4\}}$	$0.03694^{\{12\}}$	$0.03302^{\{3\}}$	$0.03621^{\{10\}}$	$0.03386^{\{6\}}$	$0.03431^{\{7\}}$	$0.03514^{\{8\}}$	$0.03661^{\{11\}}$	$0.03548^{\{9\}}$	$0.04203^{\{15\}}$	$0.02819^{\{1\}}$	$0.04032^{\{14\}}$	$0.03334^{\{5\}}$	$0.03972^{\{13\}}$
145	ASAE	$0.01252^{\{6\}}$	$0.01181^{\{3\}}$	$0.01241^{\{5\}}$	$0.01229^{\{4\}}$	$0.01275^{\{7\}}$	$0.01133^{\{1\}}$	$0.01168^{\{2\}}$	$0.01422^{\{9\}}$	$0.01601^{\{12\}}$	$0.01473^{\{10\}}$	$0.01902^{\{15\}}$	$0.01476^{\{11\}}$	$0.01759^{\{13\}}$	$0.01398^{\{8\}}$	$0.01762^{\{14\}}$
145	$\sum Ranks$	$27^{\{2\}}$	$36^{\{3\}}$	$94^{\{11\}}$	$61^{\{7.5\}}$	$101^{\{12\}}$	$51^{\{5\}}$	$55^{\{6\}}$	$102^{\{13\}}$	$48^{\{4\}}$	$88^{\{10\}}$	$134^{\{15\}}$	$20^{\{1\}}$	$81^{\{9\}}$	$61^{\{7.5\}}$	$121^{\{14\}}$
190	BIAS($\hat{\delta}$)	$0.32903^{\{3\}}$	$0.37480^{\{4\}}$	$0.48429^{\{12\}}$	$0.38348^{\{6\}}$	$0.48902^{\{13\}}$	$0.39627^{\{7\}}$	$0.37888^{\{5\}}$	$0.47154^{\{11\}}$	$0.23847^{\{2\}}$	$0.42553^{\{9\}}$	$0.67694^{\{15\}}$	$0.01640^{\{1\}}$	$0.44195^{\{10\}}$	$0.42066^{\{8\}}$	$0.50139^{\{14\}}$
190	BIAS($\hat{\lambda}$)	$0.05637^{\{3\}}$	$0.06787^{\{5\}}$	$0.08509^{\{11\}}$	$0.07237^{\{7\}}$	$0.08822^{\{13\}}$	$0.06819^{\{6\}}$	$0.06724^{\{4\}}$	$0.08805^{\{12\}}$	$0.05225^{\{2\}}$	$0.07600^{\{9\}}$	$0.12010^{\{15\}}$	$0.01828^{\{1\}}$	$0.08243^{\{10\}}$	$0.07523^{\{8\}}$	$0.09295^{\{14\}}$
190	MSE($\hat{\delta}$)	$0.18548^{\{3\}}$	$0.22217^{\{4\}}$	$0.37089^{\{13\}}$	$0.23788^{\{6\}}$	$0.38335^{\{14\}}$	$0.25901^{\{7\}}$	$0.22653^{\{5\}}$	$0.34895^{\{11\}}$	$0.13924^{\{2\}}$	$0.29036^{\{10\}}$	$0.65612^{\{15\}}$	$0.00084^{\{1\}}$	$0.28827^{\{9\}}$	$0.27047^{\{8\}}$	$0.35999^{\{12\}}$
190	MSE($\hat{\lambda}$)	$0.00517^{\{3\}}$	$0.00723^{\{5\}}$	$0.01123^{\{11\}}$	$0.00949^{\{8\}}$	$0.01184^{\{12\}}$	$0.00733^{\{6\}}$	$0.00696^{\{4\}}$	$0.01189^{\{13\}}$	$0.00513^{\{2\}}$	$0.00959^{\{9\}}$	$0.01983^{\{15\}}$	$0.00054^{\{1\}}$	$0.01029^{\{10\}}$	$0.00897^{\{7\}}$	$0.01234^{\{14\}}$
190	MRE($\hat{\delta}$)	$0.16451^{\{3\}}$	$0.18740^{\{4\}}$	$0.24215^{\{12\}}$	$0.19174^{\{6\}}$	$0.24451^{\{13\}}$	$0.19814^{\{7\}}$	$0.18944^{\{5\}}$	$0.23577^{\{11\}}$	$0.11924^{\{2\}}$	$0.21277^{\{9\}}$	$0.33847^{\{15\}}$	$0.00820^{\{1\}}$	$0.22097^{\{10\}}$	$0.21033^{\{8\}}$	$0.25069^{\{14\}}$
190	MRE($\hat{\lambda}$)	$0.08673^{\{3\}}$	$0.10442^{\{5\}}$	$0.13092^{\{11\}}$	$0.11134^{\{7\}}$	$0.13572^{\{13\}}$	$0.10491^{\{6\}}$	$0.10344^{\{4\}}$	$0.13547^{\{12\}}$	$0.08039^{\{2\}}$	$0.11693^{\{9\}}$	$0.18477^{\{15\}}$	$0.02813^{\{1\}}$	$0.12682^{\{10\}}$	$0.11574^{\{8\}}$	$0.14300^{\{14\}}$
190	D_{abs}	$0.01615^{\{1\}}$	$0.01834^{\{6\}}$	$0.01779^{\{5\}}$	$0.01748^{\{4\}}$	$0.01945^{\{9\}}$	$0.02002^{\{11\}}$	$0.01747^{\{3\}}$	$0.01880^{\{7\}}$	$0.02138^{\{13\}}$	$0.01924^{\{8\}}$	$0.02106^{\{12\}}$	$0.01742^{\{2\}}$	$0.02191^{\{14\}}$	$0.01967^{\{10\}}$	$0.02404^{\{15\}}$
190	D_{max}	$0.02534^{\{2\}}$	$0.02897^{\{5\}}$	$0.02922^{\{6\}}$	$0.02765^{\{3\}}$	$0.03134^{\{10\}}$	$0.03157^{\{11\}}$	$0.02779^{\{4\}}$	$0.03034^{\{7\}}$	$0.03186^{\{12\}}$	$0.03049^{\{8\}}$	$0.03585^{\{14\}}$	$0.02418^{\{1\}}$	$0.03429^{\{13\}}$	$0.03122^{\{9\}}$	$0.03815^{\{15\}}$
190	ASAE	$0.01061^{\{5\}}$	$0.00996^{\{2\}}$	$0.01074^{\{6\}}$	$0.01035^{\{4\}}$	$0.01078^{\{7\}}$	$0.00935^{\{1\}}$	$0.00998^{\{3\}}$	$0.01191^{\{8\}}$	$0.01346^{\{12\}}$	$0.01234^{\{11\}}$	$0.01575^{\{15\}}$	$0.01218^{\{10\}}$	$0.01426^{\{13\}}$	$0.01205^{\{9\}}$	$0.01534^{\{14\}}$
190	$\sum Ranks$	$26^{\{2\}}$	$40^{\{4\}}$	$87^{\{10\}}$	$51^{\{6\}}$	$104^{\{13\}}$	$62^{\{7\}}$	$37^{\{3\}}$	$92^{\{11\}}$	$49^{\{5\}}$	$82^{\{9\}}$	$131^{\{15\}}$	$19^{\{1\}}$	$99^{\{12\}}$	$75^{\{8\}}$	$126^{\{14\}}$
245	BIAS($\hat{\delta}$)	$0.31584^{\{3\}}$	$0.36762^{\{7\}}$	$0.38148^{\{9\}}$	$0.35306^{\{5\}}$	$0.40139^{\{11\}}$	$0.35026^{\{4\}}$	$0.36943^{\{8\}}$	$0.43835^{\{13\}}$	$0.21067^{\{2\}}$	$0.36151^{\{6\}}$	$0.66379^{\{15\}}$	$0.01571^{\{1\}}$	$0.43106^{\{12\}}$	$0.38469^{\{10\}}$	$0.44716^{\{14\}}$
245	BIAS($\hat{\lambda}$)	$0.05477^{\{3\}}$	$0.06268^{\{5\}}$	$0.06602^{\{7\}}$	$0.06529^{\{6\}}$	$0.07006^{\{11\}}$	$0.05998^{\{4\}}$	$0.06617^{\{8\}}$	$0.07854^{\{12\}}$	$0.04299^{\{2\}}$	$0.06747^{\{9\}}$	$0.11814^{\{15\}}$	$0.01537^{\{1\}}$	$0.08080^{\{13\}}$	$0.06855^{\{10\}}$	$0.08383^{\{14\}}$
245	MSE($\hat{\delta}$)	$0.16374^{\{3\}}$	$0.22037^{\{7\}}$	$0.24843^{\{10\}}$	$0.18107^{\{4\}}$	$0.26117^{\{11\}}$	$0.18804^{\{5\}}$	$0.22042^{\{8\}}$	$0.29841^{\{14\}}$	$0.11485^{\{2\}}$	$0.22005^{\{6\}}$	$0.60455^{\{15\}}$	$8e\text{-}04^{\{1\}}$	$0.27337^{\{12\}}$	$0.22970^{\{9\}}$	$0.29157^{\{13\}}$
245	MSE($\hat{\lambda}$)	$0.00480^{\{3\}}$	$0.00650^{\{6\}}$	$0.00701^{\{8\}}$	$0.00616^{\{5\}}$	$0.00787^{\{10\}}$	$0.00552^{\{4\}}$	$0.00695^{\{7\}}$	$0.00963^{\{12\}}$	$0.00377^{\{2\}}$	$0.00824^{\{11\}}$	$0.01882^{\{15\}}$	$0.00038^{\{1\}}$	$0.00988^{\{13\}}$	$0.00733^{\{9\}}$	$0.01103^{\{14\}}$
245	MRE($\hat{\delta}$)	$0.15792^{\{3\}}$	$0.18381^{\{7\}}$	$0.19074^{\{9\}}$	$0.17653^{\{5\}}$	$0.20070^{\{11\}}$	$0.17513^{\{4\}}$	$0.18471^{\{8\}}$	$0.21918^{\{13\}}$	$0.10533^{\{2\}}$	$0.18075^{\{6\}}$	$0.33189^{\{15\}}$	$0.00786^{\{1\}}$	$0.21553^{\{12\}}$	$0.19235^{\{10\}}$	$0.22358^{\{14\}}$
245	MRE($\hat{\lambda}$)	$0.08427^{\{3\}}$	$0.09644^{\{5\}}$	$0.10157^{\{7\}}$	$0.10045^{\{6\}}$	$0.10779^{\{11\}}$	$0.09227^{\{4\}}$	$0.10180^{\{8\}}$	$0.12083^{\{12\}}$	$0.06614^{\{2\}}$	$0.10380^{\{9\}}$	$0.18175^{\{15\}}$	$0.02364^{\{1\}}$	$0.12430^{\{13\}}$	$0.10546^{\{10\}}$	$0.12897^{\{14\}}$
245	D_{abs}	$0.01506^{\{2\}}$	$0.01605^{\{4\}}$	$0.01714^{\{9\}}$	$0.01519^{\{3\}}$	$0.01669^{\{7\}}$	$0.01621^{\{5\}}$	$0.01657^{\{6\}}$	$0.01742^{\{11\}}$	$0.01733^{\{10\}}$	$0.01813^{\{12\}}$	$0.01945^{\{13\}}$	$0.01456^{\{1\}}$	$0.02001^{\{14\}}$	$0.01699^{\{8\}}$	$0.02058^{\{15\}}$
245	D_{max}	$0.02377^{\{2\}}$	$0.02592^{\{5\}}$	$0.02744^{\{10\}}$	$0.02426^{\{3\}}$	$0.02702^{\{8\}}$	$0.02581^{\{4\}}$	$0.02646^{\{7\}}$	$0.02822^{\{11\}}$	$0.02612^{\{6\}}$	$0.02836^{\{12\}}$	$0.03400^{\{15\}}$	$0.02020^{\{1\}}$	$0.03178^{\{13\}}$	$0.02712^{\{9\}}$	$0.03279^{\{14\}}$
245	ASAE	$0.00892^{\{6\}}$	$0.00837^{\{3\}}$	$0.00883^{\{4.5\}}$	$0.00883^{\{4.5\}}$	$0.00899^{\{7\}}$	$0.00818^{\{1\}}$	$0.00820^{\{2\}}$	$0.01007^{\{8\}}$	$0.01151^{\{12\}}$	$0.01102^{\{11\}}$	$0.01444^{\{15\}}$	$0.01032^{\{9\}}$	$0.01255^{\{13\}}$	$0.01044^{\{10\}}$	$0.01285^{\{14\}}$
245	$\sum Ranks$	$28^{\{2\}}$	$49^{\{6\}}$	$73.5^{\{8\}}$	$41.5^{\{5\}}$	$87^{\{11\}}$	$35^{\{3\}}$	$62^{\{7\}}$	$106^{\{12\}}$	$40^{\{4\}}$	$82^{\{9\}}$	$133^{\{15\}}$	$17^{\{1\}}$	$115^{\{13\}}$	$85^{\{10\}}$	$126^{\{14\}}$
320	BIAS($\hat{\delta}$)	$0.27708^{\{4\}}$	$0.32444^{\{8\}}$	$0.36121^{\{10\}}$	$0.27533^{\{3\}}$	$0.39388^{\{13\}}$	$0.32406^{\{7\}}$	$0.30114^{\{5\}}$	$0.42086^{\{14\}}$	$0.20111^{\{2\}}$	$0.32117^{\{6\}}$	$0.58551^{\{15\}}$	$0.01452^{\{1\}}$	$0.37808^{\{12\}}$	$0.33359^{\{9\}}$	$0.37567^{\{11\}}$
320	BIAS($\hat{\lambda}$)	$0.04853^{\{3\}}$	$0.05706^{\{7\}}$	$0.06342^{\{10\}}$	$0.04924^{\{4\}}$	$0.06955^{\{11\}}$	$0.05595^{\{6\}}$	$0.05559^{\{5\}}$	$0.07518^{\{14\}}$	$0.04081^{\{2\}}$	$0.05997^{\{9\}}$	$0.10488^{\{15\}}$	$0.01420^{\{1\}}$	$0.07043^{\{12\}}$	$0.05819^{\{8\}}$	$0.07083^{\{13\}}$
320	MSE($\hat{\delta}$)	$0.13395^{\{4\}}$	$0.17194^{\{8\}}$	$0.21129^{\{10\}}$	$0.11502^{\{3\}}$	$0.25583^{\{13\}}$	$0.16816^{\{7\}}$	$0.14882^{\{5\}}$	$0.29733^{\{14\}}$	$0.09197^{\{2\}}$	$0.16229^{\{6\}}$	$0.49890^{\{15\}}$	$0.00057^{\{1\}}$	$0.23142^{\{12\}}$	$0.17575^{\{9\}}$	$0.21572^{\{11\}}$
320	MSE($\hat{\lambda}$)	$0.00398^{\{4\}}$	$0.00504^{\{7\}}$	$0.00634^{\{10\}}$	$0.00375^{\{3\}}$	$0.00765^{\{11\}}$	$0.00482^{\{5\}}$	$0.00489^{\{6\}}$	$0.00909^{\{14\}}$	$0.00321^{\{2\}}$	$0.00598^{\{9\}}$	$0.01563^{\{15\}}$	$0.00031^{\{1\}}$	$0.00786^{\{13\}}$	$0.00544^{\{8\}}$	$0.00785^{\{12\}}$
320	MRE($\hat{\delta}$)	$0.13854^{\{4\}}$	$0.16222^{\{8\}}$	$0.18061^{\{10\}}$	$0.13767^{\{3\}}$	$0.19694^{\{13\}}$	$0.16203^{\{7\}}$	$0.15057^{\{5\}}$	$0.21043^{\{14\}}$	$0.10055^{\{2\}}$	$0.16059^{\{6\}}$	$0.29276^{\{15\}}$	$0.00726^{\{1\}}$	$0.18904^{\{12\}}$	$0.16679^{\{9\}}$	$0.18784^{\{11\}}$
320	MRE($\hat{\lambda}$)	$0.07466^{\{3\}}$	$0.08778^{\{7\}}$	$0.09757^{\{10\}}$	$0.07575^{\{4\}}$	$0.10700^{\{11\}}$	$0.08608^{\{6\}}$	$0.08552^{\{5\}}$	$0.11566^{\{14\}}$	$0.06279^{\{2\}}$	$0.09227^{\{9\}}$	$0.16136^{\{15\}}$	$0.02185^{\{1\}}$	$0.10836^{\{12\}}$	$0.08952^{\{8\}}$	$0.10897^{\{13\}}$
320	D_{abs}	$0.01296^{\{1\}}$	$0.01452^{\{6\}}$	$0.01480^{\{8\}}$	$0.01352^{\{3\}}$	$0.01453^{\{7\}}$	$0.01450^{\{5\}}$	$0.01408^{\{4\}}$	$0.01597^{\{10\}}$	$0.01629^{\{12\}}$	$0.01605^{\{11\}}$	$0.01733^{\{14\}}$	$0.01322^{\{2\}}$	$0.01707^{\{13\}}$	$0.01482^{\{9\}}$	$0.01784^{\{15\}}$
320	D_{max}	$0.02058^{\{2\}}$	$0.02314^{\{5\}}$	$0.02374^{\{7\}}$	$0.02136^{\{3\}}$	$0.02407^{\{9\}}$	$0.02320^{\{6\}}$	$0.02223^{\{4\}}$	$0.02607^{\{12\}}$	$0.02460^{\{10\}}$	$0.02516^{\{11\}}$	$0.02988^{\{15\}}$	$0.01834^{\{1\}}$	$0.02698^{\{13\}}$	$0.02390^{\{8\}}$	$0.02837^{\{14\}}$
320	ASAE	$0.00754^{\{5\}}$	$0.00748^{\{4\}}$	$0.00787^{\{6\}}$	$0.00744^{\{3\}}$	$0.00793^{\{7\}}$	$0.00683^{\{1\}}$	$0.00727^{\{2\}}$	$0.00860^{\{9\}}$	$0.00940^{\{12\}}$	$0.00858^{\{8\}}$	$0.01270^{\{15\}}$	$0.00878^{\{10\}}$	$0.01065^{\{14\}}$	$0.00883^{\{11\}}$	$0.01049^{\{13\}}$
320	$\sum Ranks$	$30^{\{3\}}$	$60^{\{7\}}$	$81^{\{10\}}$	$29^{\{2\}}$	$95^{\{11\}}$	$50^{\{6\}}$	$41^{\{4\}}$	$115^{\{14\}}$	$46^{\{5\}}$	$75^{\{8\}}$	$134^{\{15\}}$	$19^{\{1\}}$	$113^{\{12.5\}}$	$79^{\{9\}}$	$113^{\{12.5\}}$

Table 18. Numerical values of simulation measures for $\delta = 3.0,\ \lambda = 3.5$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$1.03289^{\{4\}}$	$1.05438^{\{8\}}$	$1.00506^{\{3\}}$	$1.30533^{\{13\}}$	$1.04821^{\{6\}}$	$1.04802^{\{5\}}$	$1.05150^{\{7\}}$	$1.16968^{\{11\}}$	$0.82270^{\{2\}}$	$1.10650^{\{10\}}$	$1.20462^{\{12\}}$	$0.22503^{\{1\}}$	$1.31251^{\{14\}}$	$1.06361^{\{9\}}$	$1.31721^{\{15\}}$
25	BIAS($\hat{\lambda}$)	$0.70417^{\{4\}}$	$0.73806^{\{6\}}$	$0.75094^{\{7\}}$	$1.01541^{\{13\}}$	$0.73458^{\{5\}}$	$0.79347^{\{8\}}$	$0.70216^{\{3\}}$	$0.89401^{\{11\}}$	$0.62081^{\{2\}}$	$0.82666^{\{10\}}$	$0.94967^{\{12\}}$	$0.27383^{\{1\}}$	$1.06404^{\{15\}}$	$0.82084^{\{9\}}$	$1.06320^{\{14\}}$
25	MSE($\hat{\delta}$)	$1.55406^{\{5\}}$	$1.65853^{\{8\}}$	$1.49253^{\{3\}}$	$2.36411^{\{15\}}$	$1.54210^{\{4\}}$	$1.57724^{\{6\}}$	$1.68462^{\{9\}}$	$1.91117^{\{11\}}$	$1.17985^{\{2\}}$	$1.82323^{\{10\}}$	$2.05985^{\{12\}}$	$0.17124^{\{1\}}$	$2.26748^{\{14\}}$	$1.62117^{\{7\}}$	$2.17709^{\{13\}}$
25	MSE($\hat{\lambda}$)	$0.78358^{\{4\}}$	$0.84053^{\{6\}}$	$0.92333^{\{7\}}$	$1.80705^{\{15\}}$	$0.80832^{\{5\}}$	$1.03214^{\{9\}}$	$0.74831^{\{3\}}$	$1.18548^{\{11\}}$	$0.70793^{\{2\}}$	$1.14221^{\{10\}}$	$1.34446^{\{12\}}$	$0.15446^{\{1\}}$	$1.62873^{\{14\}}$	$1.00301^{\{8\}}$	$1.57128^{\{13\}}$
25	MRE($\hat{\delta}$)	$0.34430^{\{4\}}$	$0.35146^{\{8\}}$	$0.33502^{\{3\}}$	$0.43511^{\{13\}}$	$0.34940^{\{6\}}$	$0.34934^{\{5\}}$	$0.35050^{\{7\}}$	$0.38989^{\{11\}}$	$0.27423^{\{2\}}$	$0.36883^{\{10\}}$	$0.40154^{\{12\}}$	$0.07501^{\{1\}}$	$0.43750^{\{14\}}$	$0.35454^{\{9\}}$	$0.43907^{\{15\}}$
25	MRE($\hat{\lambda}$)	$0.20119^{\{4\}}$	$0.21087^{\{6\}}$	$0.21455^{\{7\}}$	$0.29012^{\{13\}}$	$0.20988^{\{5\}}$	$0.22671^{\{8\}}$	$0.20062^{\{3\}}$	$0.25543^{\{11\}}$	$0.17737^{\{2\}}$	$0.23619^{\{10\}}$	$0.27133^{\{12\}}$	$0.07824^{\{1\}}$	$0.30401^{\{15\}}$	$0.23453^{\{9\}}$	$0.30377^{\{14\}}$
25	D_{abs}	$0.04207^{\{2\}}$	$0.04589^{\{3\}}$	$0.05528^{\{11\}}$	$0.05214^{\{9\}}$	$0.05007^{\{6\}}$	$0.05366^{\{10\}}$	$0.04201^{\{1\}}$	$0.05622^{\{12\}}$	$0.05946^{\{14\}}$	$0.04758^{\{4\}}$	$0.05100^{\{8\}}$	$0.05086^{\{7\}}$	$0.05651^{\{13\}}$	$0.04993^{\{5\}}$	$0.06039^{\{15\}}$
25	D_{max}	$0.06728^{\{2\}}$	$0.07204^{\{4\}}$	$0.08472^{\{11\}}$	$0.08181^{\{9\}}$	$0.07775^{\{7\}}$	$0.08235^{\{10\}}$	$0.06564^{\{1\}}$	$0.08728^{\{12\}}$	$0.08795^{\{13\}}$	$0.07427^{\{5\}}$	$0.08043^{\{8\}}$	$0.07049^{\{3\}}$	$0.08838^{\{14\}}$	$0.07652^{\{6\}}$	$0.09393^{\{15\}}$
25	ASAE	$0.04008^{\{8\}}$	$0.03661^{\{4\}}$	$0.03802^{\{5\}}$	$0.03915^{\{6\}}$	$0.03627^{\{3\}}$	$0.03464^{\{1\}}$	$0.03493^{\{2\}}$	$0.03975^{\{7\}}$	$0.04571^{\{10\}}$	$0.04577^{\{11\}}$	$0.05131^{\{13\}}$	$0.04616^{\{12\}}$	$0.05512^{\{15\}}$	$0.04229^{\{9\}}$	$0.05191^{\{14\}}$
25	$\sum Ranks$	$37^{\{3\}}$	$53^{\{6\}}$	$57^{\{7\}}$	$106^{\{13\}}$	$47^{\{4\}}$	$62^{\{8\}}$	$36^{\{2\}}$	$97^{\{11\}}$	$49^{\{5\}}$	$80^{\{10\}}$	$101^{\{12\}}$	$28^{\{1\}}$	$128^{\{14.5\}}$	$71^{\{9\}}$	$128^{\{14.5\}}$
85	BIAS($\hat{\delta}$)	$0.69012^{\{5\}}$	$0.73437^{\{8\}}$	$0.82952^{\{13\}}$	$0.59565^{\{3\}}$	$0.71669^{\{6\}}$	$0.73074^{\{7\}}$	$0.75992^{\{10\}}$	$0.74559^{\{9\}}$	$0.51891^{\{2\}}$	$0.80004^{\{11\}}$	$1.06709^{\{15\}}$	$0.08989^{\{1\}}$	$0.82850^{\{12\}}$	$0.68064^{\{4\}}$	$0.83033^{\{14\}}$
85	BIAS($\hat{\lambda}$)	$0.43330^{\{5\}}$	$0.48740^{\{8\}}$	$0.54142^{\{11\}}$	$0.41958^{\{3\}}$	$0.50604^{\{10\}}$	$0.42828^{\{4\}}$	$0.47738^{\{7\}}$	$0.50076^{\{9\}}$	$0.38443^{\{2\}}$	$0.57203^{\{12\}}$	$0.78287^{\{15\}}$	$0.12355^{\{1\}}$	$0.61914^{\{14\}}$	$0.44614^{\{6\}}$	$0.60680^{\{13\}}$
85	MSE($\hat{\delta}$)	$0.84119^{\{8\}}$	$0.76578^{\{5\}}$	$1.01991^{\{11\}}$	$0.56327^{\{2\}}$	$0.80338^{\{6\}}$	$0.82645^{\{7\}}$	$0.89308^{\{10\}}$	$0.86475^{\{9\}}$	$0.57320^{\{3\}}$	$1.03684^{\{14\}}$	$1.69339^{\{15\}}$	$0.02318^{\{1\}}$	$1.02188^{\{12\}}$	$0.73369^{\{4\}}$	$1.03676^{\{13\}}$
85	MSE($\hat{\lambda}$)	$0.30824^{\{6\}}$	$0.34011^{\{7\}}$	$0.41455^{\{11\}}$	$0.27043^{\{2\}}$	$0.37497^{\{9\}}$	$0.27902^{\{3\}}$	$0.35655^{\{8\}}$	$0.38919^{\{10\}}$	$0.27960^{\{4\}}$	$0.50939^{\{12\}}$	$0.88696^{\{15\}}$	$0.02679^{\{1\}}$	$0.59585^{\{14\}}$	$0.29655^{\{5\}}$	$0.57717^{\{13\}}$
85	MRE($\hat{\delta}$)	$0.23004^{\{5\}}$	$0.24479^{\{8\}}$	$0.27651^{\{13\}}$	$0.19855^{\{3\}}$	$0.23890^{\{6\}}$	$0.24358^{\{7\}}$	$0.25331^{\{10\}}$	$0.24853^{\{9\}}$	$0.17297^{\{2\}}$	$0.26668^{\{11\}}$	$0.35570^{\{15\}}$	$0.02996^{\{1\}}$	$0.27617^{\{12\}}$	$0.22688^{\{4\}}$	$0.27678^{\{14\}}$
85	MRE($\hat{\lambda}$)	$0.12380^{\{5\}}$	$0.13926^{\{8\}}$	$0.15469^{\{11\}}$	$0.11988^{\{3\}}$	$0.14458^{\{10\}}$	$0.12237^{\{4\}}$	$0.13639^{\{7\}}$	$0.14307^{\{9\}}$	$0.10984^{\{2\}}$	$0.16344^{\{12\}}$	$0.22368^{\{15\}}$	$0.03530^{\{1\}}$	$0.17690^{\{14\}}$	$0.12747^{\{6\}}$	$0.17337^{\{13\}}$
85	D_{abs}	$0.02253^{\{1\}}$	$0.02945^{\{9\}}$	$0.02679^{\{4\}}$	$0.02655^{\{3\}}$	$0.02918^{\{7\}}$	$0.02863^{\{6\}}$	$0.02924^{\{8\}}$	$0.02797^{\{5\}}$	$0.03254^{\{11\}}$	$0.03460^{\{14\}}$	$0.03337^{\{12\}}$	$0.02592^{\{2\}}$	$0.03656^{\{15\}}$	$0.03008^{\{10\}}$	$0.03425^{\{13\}}$
85	D_{max}	$0.03684^{\{2\}}$	$0.04734^{\{9\}}$	$0.04441^{\{4\}}$	$0.04155^{\{3\}}$	$0.04595^{\{7\}}$	$0.04589^{\{6\}}$	$0.04727^{\{8\}}$	$0.04533^{\{5\}}$	$0.04965^{\{11\}}$	$0.05485^{\{12\}}$	$0.05557^{\{14\}}$	$0.03610^{\{1\}}$	$0.05809^{\{15\}}$	$0.04798^{\{10\}}$	$0.05493^{\{13\}}$
85	ASAE	$0.01713^{\{5\}}$	$0.01648^{\{3\}}$	$0.01774^{\{7\}}$	$0.01697^{\{4\}}$	$0.01745^{\{6\}}$	$0.01601^{\{2\}}$	$0.01590^{\{1\}}$	$0.01841^{\{8\}}$	$0.02225^{\{12\}}$	$0.02139^{\{11\}}$	$0.03082^{\{15\}}$	$0.02020^{\{10\}}$	$0.02483^{\{13\}}$	$0.01935^{\{9\}}$	$0.02673^{\{14\}}$
85	$\sum Ranks$	$42^{\{3\}}$	$65^{\{7\}}$	$85^{\{11\}}$	$26^{\{2\}}$	$67^{\{8\}}$	$46^{\{4\}}$	$69^{\{9\}}$	$73^{\{10\}}$	$49^{\{5\}}$	$109^{\{12\}}$	$131^{\{15\}}$	$19^{\{1\}}$	$121^{\{14\}}$	$58^{\{6\}}$	$120^{\{13\}}$
145	BIAS($\hat{\delta}$)	$0.47576^{\{3\}}$	$0.52218^{\{6\}}$	$0.63048^{\{13\}}$	$0.48924^{\{4\}}$	$0.59118^{\{10\}}$	$0.67769^{\{14\}}$	$0.53225^{\{7\}}$	$0.53708^{\{8\}}$	$0.46669^{\{2\}}$	$0.58617^{\{9\}}$	$0.95187^{\{15\}}$	$0.08176^{\{1\}}$	$0.60900^{\{12\}}$	$0.49327^{\{5\}}$	$0.60189^{\{11\}}$
145	BIAS($\hat{\lambda}$)	$0.32799^{\{3\}}$	$0.33225^{\{4\}}$	$0.39616^{\{11\}}$	$0.34779^{\{7\}}$	$0.36443^{\{9\}}$	$0.39499^{\{10\}}$	$0.33744^{\{5\}}$	$0.36413^{\{8\}}$	$0.34451^{\{6\}}$	$0.42074^{\{12\}}$	$0.68011^{\{15\}}$	$0.09087^{\{1\}}$	$0.42215^{\{13\}}$	$0.30123^{\{2\}}$	$0.42651^{\{14\}}$
145	MSE($\hat{\delta}$)	$0.34331^{\{2\}}$	$0.47563^{\{6\}}$	$0.64289^{\{13\}}$	$0.36157^{\{3\}}$	$0.57534^{\{12\}}$	$0.78830^{\{14\}}$	$0.48292^{\{8\}}$	$0.57307^{\{11\}}$	$0.36917^{\{4\}}$	$0.47975^{\{7\}}$	$1.32334^{\{15\}}$	$0.01632^{\{1\}}$	$0.52178^{\{9\}}$	$0.42730^{\{5\}}$	$0.56057^{\{10\}}$
145	MSE($\hat{\lambda}$)	$0.16912^{\{3\}}$	$0.18102^{\{6\}}$	$0.25650^{\{12\}}$	$0.17947^{\{4\}}$	$0.20779^{\{8\}}$	$0.23216^{\{9\}}$	$0.17991^{\{5\}}$	$0.24964^{\{11\}}$	$0.18849^{\{7\}}$	$0.24891^{\{10\}}$	$0.71947^{\{15\}}$	$0.01543^{\{1\}}$	$0.29572^{\{14\}}$	$0.15837^{\{2\}}$	$0.29383^{\{13\}}$
145	MRE($\hat{\delta}$)	$0.15859^{\{3\}}$	$0.17406^{\{6\}}$	$0.21016^{\{13\}}$	$0.16308^{\{4\}}$	$0.19706^{\{10\}}$	$0.22590^{\{14\}}$	$0.17742^{\{7\}}$	$0.17903^{\{8\}}$	$0.15556^{\{2\}}$	$0.19539^{\{9\}}$	$0.31729^{\{15\}}$	$0.02725^{\{1\}}$	$0.20300^{\{12\}}$	$0.16442^{\{5\}}$	$0.20063^{\{11\}}$
145	MRE($\hat{\lambda}$)	$0.09371^{\{3\}}$	$0.09493^{\{4\}}$	$0.11319^{\{11\}}$	$0.09937^{\{7\}}$	$0.10412^{\{9\}}$	$0.11286^{\{10\}}$	$0.09641^{\{5\}}$	$0.10404^{\{8\}}$	$0.09843^{\{6\}}$	$0.12021^{\{12\}}$	$0.19432^{\{15\}}$	$0.02596^{\{1\}}$	$0.12061^{\{13\}}$	$0.08607^{\{2\}}$	$0.12186^{\{14\}}$
145	D_{abs}	$0.01874^{\{1\}}$	$0.02147^{\{5\}}$	$0.02220^{\{7\}}$	$0.02013^{\{3\}}$	$0.02308^{\{9\}}$	$0.02361^{\{10\}}$	$0.02056^{\{4\}}$	$0.02206^{\{6\}}$	$0.02486^{\{12\}}$	$0.02479^{\{11\}}$	$0.02533^{\{14\}}$	$0.01880^{\{2\}}$	$0.02566^{\{15\}}$	$0.02293^{\{8\}}$	$0.02526^{\{13\}}$
145	D_{max}	$0.02966^{\{2\}}$	$0.03389^{\{5\}}$	$0.03609^{\{8\}}$	$0.03232^{\{3\}}$	$0.03725^{\{9\}}$	$0.03835^{\{11\}}$	$0.03276^{\{4\}}$	$0.03484^{\{6\}}$	$0.03822^{\{10\}}$	$0.03969^{\{12\}}$	$0.04376^{\{15\}}$	$0.02663^{\{1\}}$	$0.04129^{\{14\}}$	$0.03591^{\{7\}}$	$0.04067^{\{13\}}$
145	ASAE	$0.01278^{\{5\}}$	$0.01252^{\{4\}}$	$0.01299^{\{6.5\}}$	$0.01230^{\{3\}}$	$0.01299^{\{6.5\}}$	$0.01128^{\{1\}}$	$0.01216^{\{2\}}$	$0.01380^{\{8\}}$	$0.01620^{\{12\}}$	$0.01527^{\{11\}}$	$0.02092^{\{15\}}$	$0.01472^{\{10\}}$	$0.01783^{\{14\}}$	$0.01409^{\{9\}}$	$0.01732^{\{13\}}$
145	$\sum Ranks$	$25^{\{2\}}$	$46^{\{5\}}$	$94.5^{\{12\}}$	$38^{\{3\}}$	$82.5^{\{9\}}$	$93^{\{10.5\}}$	$47^{\{6\}}$	$74^{\{8\}}$	$61^{\{7\}}$	$93^{\{10.5\}}$	$134^{\{15\}}$	$19^{\{1\}}$	$116^{\{14\}}$	$45^{\{4\}}$	$112^{\{13\}}$
190	BIAS($\hat{\delta}$)	$0.43618^{\{4\}}$	$0.46970^{\{9\}}$	$0.50766^{\{11\}}$	$0.44702^{\{5\}}$	$0.45113^{\{6\}}$	$0.50695^{\{10\}}$	$0.43488^{\{3\}}$	$0.50814^{\{12\}}$	$0.40644^{\{2\}}$	$0.46849^{\{8\}}$	$0.70527^{\{15\}}$	$0.07045^{\{1\}}$	$0.59016^{\{14\}}$	$0.45684^{\{7\}}$	$0.57101^{\{13\}}$
190	BIAS($\hat{\lambda}$)	$0.29135^{\{3\}}$	$0.29588^{\{6\}}$	$0.31500^{\{11\}}$	$0.31176^{\{10\}}$	$0.30193^{\{9\}}$	$0.29305^{\{5\}}$	$0.28028^{\{2\}}$	$0.33815^{\{12\}}$	$0.29185^{\{4\}}$	$0.30015^{\{8\}}$	$0.52520^{\{15\}}$	$0.08570^{\{1\}}$	$0.41369^{\{14\}}$	$0.29918^{\{7\}}$	$0.40674^{\{13\}}$
190	MSE($\hat{\delta}$)	$0.31988^{\{6\}}$	$0.37051^{\{9\}}$	$0.40542^{\{10\}}$	$0.31672^{\{4\}}$	$0.33256^{\{8\}}$	$0.44643^{\{12\}}$	$0.30863^{\{2\}}$	$0.42726^{\{11\}}$	$0.31601^{\{3\}}$	$0.32876^{\{7\}}$	$0.76976^{\{15\}}$	$0.01192^{\{1\}}$	$0.50707^{\{13\}}$	$0.31903^{\{5\}}$	$0.50905^{\{14\}}$
190	MSE($\hat{\lambda}$)	$0.13438^{\{3\}}$	$0.14056^{\{4\}}$	$0.15320^{\{10\}}$	$0.14517^{\{7\}}$	$0.15379^{\{11\}}$	$0.14129^{\{6\}}$	$0.11855^{\{2\}}$	$0.18933^{\{12\}}$	$0.15013^{\{8\}}$	$0.15042^{\{9\}}$	$0.45050^{\{15\}}$	$0.01353^{\{1\}}$	$0.27161^{\{14\}}$	$0.14096^{\{5\}}$	$0.25683^{\{13\}}$
190	MRE($\hat{\delta}$)	$0.14539^{\{4\}}$	$0.15657^{\{9\}}$	$0.16922^{\{11\}}$	$0.14901^{\{5\}}$	$0.15038^{\{6\}}$	$0.16898^{\{10\}}$	$0.14496^{\{3\}}$	$0.16938^{\{12\}}$	$0.13548^{\{2\}}$	$0.15616^{\{8\}}$	$0.23509^{\{15\}}$	$0.02348^{\{1\}}$	$0.19672^{\{14\}}$	$0.15228^{\{7\}}$	$0.19034^{\{13\}}$
190	MRE($\hat{\lambda}$)	$0.08324^{\{3\}}$	$0.08454^{\{6\}}$	$0.09000^{\{11\}}$	$0.08908^{\{10\}}$	$0.08626^{\{9\}}$	$0.08373^{\{5\}}$	$0.08008^{\{2\}}$	$0.09661^{\{12\}}$	$0.08339^{\{4\}}$	$0.08576^{\{8\}}$	$0.15006^{\{15\}}$	$0.02448^{\{1\}}$	$0.11820^{\{14\}}$	$0.08548^{\{7\}}$	$0.11621^{\{13\}}$
190	D_{abs}	$0.01446^{\{1\}}$	$0.01955^{\{8\}}$	$0.01841^{\{6\}}$	$0.01717^{\{3.5\}}$	$0.01941^{\{7\}}$	$0.01762^{\{5\}}$	$0.01650^{\{2\}}$	$0.02095^{\{11\}}$	$0.02346^{\{14\}}$	$0.01998^{\{9\}}$	$0.02183^{\{12\}}$	$0.01717^{\{3.5\}}$	$0.02271^{\{13\}}$	$0.02061^{\{10\}}$	$0.02363^{\{15\}}$
190	D_{max}	$0.02348^{\{1\}}$	$0.03128^{\{8\}}$	$0.02965^{\{6\}}$	$0.02750^{\{4\}}$	$0.03095^{\{7\}}$	$0.02889^{\{5\}}$	$0.02673^{\{3\}}$	$0.03369^{\{11\}}$	$0.03609^{\{12\}}$	$0.03225^{\{9\}}$	$0.03661^{\{13\}}$	$0.02429^{\{2\}}$	$0.03709^{\{14\}}$	$0.03259^{\{10\}}$	$0.03802^{\{15\}}$
190	ASAE	$0.01022^{\{4\}}$	$0.01011^{\{3\}}$	$0.01098^{\{7\}}$	$0.01025^{\{5\}}$	$0.01060^{\{6\}}$	$0.00973^{\{1\}}$	$0.01001^{\{2\}}$	$0.01200^{\{9\}}$	$0.01446^{\{12\}}$	$0.01273^{\{11\}}$	$0.01759^{\{15\}}$	$0.01214^{\{10\}}$	$0.01514^{\{14\}}$	$0.01196^{\{8\}}$	$0.01482^{\{13\}}$
190	$\sum Ranks$	$29^{\{3\}}$	$62^{\{7\}}$	$83^{\{11\}}$	$53.5^{\{4\}}$	$69^{\{9\}}$	$59^{\{5\}}$	$21^{\{1\}}$	$102^{\{12\}}$	$61^{\{6\}}$	$77^{\{10\}}$	$130^{\{15\}}$	$21.5^{\{2\}}$	$124^{\{14\}}$	$66^{\{8\}}$	$122^{\{13\}}$
245	BIAS($\hat{\delta}$)	$0.38358^{\{5\}}$	$0.44841^{\{11\}}$	$0.42368^{\{9\}}$	$0.35278^{\{2\}}$	$0.43067^{\{10\}}$	$0.40274^{\{6\}}$	$0.36661^{\{3\}}$	$0.48479^{\{14\}}$	$0.37382^{\{4\}}$	$0.40898^{\{7\}}$	$0.64964^{\{15\}}$	$0.05429^{\{1\}}$	$0.47624^{\{12\}}$	$0.41835^{\{8\}}$	$0.48094^{\{13\}}$
245	BIAS($\hat{\lambda}$)	$0.25601^{\{6\}}$	$0.26108^{\{7\}}$	$0.27285^{\{10\}}$	$0.20423^{\{2\}}$	$0.28624^{\{11\}}$	$0.24636^{\{5\}}$	$0.23753^{\{3\}}$	$0.33094^{\{12\}}$	$0.24127^{\{4\}}$	$0.26678^{\{8\}}$	$0.45330^{\{15\}}$	$0.07534^{\{1\}}$	$0.33795^{\{13\}}$	$0.26952^{\{9\}}$	$0.35080^{\{14\}}$
245	MSE($\hat{\delta}$)	$0.23317^{\{5\}}$	$0.35692^{\{12\}}$	$0.28446^{\{9\}}$	$0.17997^{\{2\}}$	$0.28543^{\{10\}}$	$0.27489^{\{8\}}$	$0.19667^{\{3\}}$	$0.34362^{\{11\}}$	$0.22589^{\{4\}}$	$0.25405^{\{7\}}$	$0.69784^{\{15\}}$	$0.00821^{\{1\}}$	$0.36801^{\{13\}}$	$0.25284^{\{6\}}$	$0.38181^{\{14\}}$
245	MSE($\hat{\lambda}$)	$0.10490^{\{6\}}$	$0.11577^{\{7\}}$	$0.12615^{\{11\}}$	$0.06559^{\{2\}}$	$0.11939^{\{10\}}$	$0.09616^{\{4\}}$	$0.08497^{\{3\}}$	$0.15843^{\{12\}}$	$0.09749^{\{5\}}$	$0.11716^{\{9\}}$	$0.33129^{\{15\}}$	$0.01126^{\{1\}}$	$0.19290^{\{14\}}$	$0.11581^{\{8\}}$	$0.19248^{\{13\}}$
245	MRE($\hat{\delta}$)	$0.12786^{\{5\}}$	$0.14947^{\{11\}}$	$0.14123^{\{9\}}$	$0.11759^{\{2\}}$	$0.14356^{\{10\}}$	$0.13425^{\{6\}}$	$0.12220^{\{3\}}$	$0.16160^{\{14\}}$	$0.12461^{\{4\}}$	$0.13633^{\{7\}}$	$0.21655^{\{15\}}$	$0.01810^{\{1\}}$	$0.15875^{\{12\}}$	$0.13945^{\{8\}}$	$0.16031^{\{13\}}$
245	MRE($\hat{\lambda}$)	$0.07315^{\{6\}}$	$0.07459^{\{7\}}$	$0.07796^{\{10\}}$	$0.05835^{\{2\}}$	$0.08178^{\{11\}}$	$0.07039^{\{5\}}$	$0.06787^{\{3\}}$	$0.09456^{\{12\}}$	$0.06893^{\{4\}}$	$0.07622^{\{8\}}$	$0.12951^{\{15\}}$	$0.02153^{\{1\}}$	$0.09656^{\{13\}}$	$0.07701^{\{9\}}$	$0.10023^{\{14\}}$
245	D_{abs}	$0.01399^{\{1\}}$	$0.01505^{\{3\}}$	$0.01734^{\{10\}}$	$0.01661^{\{8\}}$	$0.01651^{\{7\}}$	$0.01618^{\{6\}}$	$0.01521^{\{4\}}$	$0.01690^{\{9\}}$	$0.01609^{\{5\}}$	$0.01870^{\{11\}}$	$0.01957^{\{13\}}$	$0.01479^{\{2\}}$	$0.02061^{\{14\}}$	$0.01929^{\{12\}}$	$0.02290^{\{15\}}$
245	D_{max}	$0.02263^{\{2\}}$	$0.02496^{\{4\}}$	$0.02775^{\{9\}}$	$0.02628^{\{7\}}$	$0.02665^{\{8\}}$	$0.02622^{\{6\}}$	$0.02427^{\{3\}}$	$0.02810^{\{10\}}$	$0.02567^{\{5\}}$	$0.02950^{\{11\}}$	$0.03316^{\{14\}}$	$0.02080^{\{1\}}$	$0.03289^{\{13\}}$	$0.03071^{\{12\}}$	$0.03652^{\{15\}}$
245	ASAE	$0.00886^{\{3\}}$	$0.00838^{\{1\}}$	$0.00900^{\{5\}}$	$0.00888^{\{4\}}$	$0.00906^{\{6\}}$	$0.00866^{\{2\}}$	$0.00912^{\{7\}}$	$0.01027^{\{8\}}$	$0.01099^{\{12\}}$	$0.01054^{\{11\}}$	$0.01487^{\{15\}}$	$0.01033^{\{9\}}$	$0.01259^{\{13\}}$	$0.01035^{\{10\}}$	$0.01306^{\{14\}}$
245	$\sum Ranks$	$39^{\{4\}}$	$63^{\{7\}}$	$82^{\{9.5\}}$	$31^{\{2\}}$	$83^{\{11\}}$	$48^{\{6\}}$	$32^{\{3\}}$	$102^{\{12\}}$	$47^{\{5\}}$	$79^{\{8\}}$	$132^{\{15\}}$	$18^{\{1\}}$	$117^{\{13\}}$	$82^{\{9.5\}}$	$125^{\{14\}}$
320	BIAS($\hat{\delta}$)	$0.34200^{\{6\}}$	$0.37921^{\{10\}}$	$0.39552^{\{11\}}$	$0.29850^{\{2\}}$	$0.41606^{\{13\}}$	$0.32180^{\{4\}}$	$0.30144^{\{3\}}$	$0.42440^{\{14\}}$	$0.36404^{\{7\}}$	$0.33391^{\{5\}}$	$0.59717^{\{15\}}$	$0.05141^{\{1\}}$	$0.40285^{\{12\}}$	$0.36798^{\{8\}}$	$0.36900^{\{9\}}$
320	BIAS($\hat{\lambda}$)	$0.22630^{\{6\}}$	$0.25257^{\{10\}}$	$0.23325^{\{7\}}$	$0.19654^{\{2\}}$	$0.27008^{\{12\}}$	$0.20935^{\{4\}}$	$0.19778^{\{3\}}$	$0.27981^{\{13\}}$	$0.21264^{\{5\}}$	$0.23414^{\{8\}}$	$0.43627^{\{15\}}$	$0.06904^{\{1\}}$	$0.28501^{\{14\}}$	$0.24086^{\{9\}}$	$0.26747^{\{11\}}$
320	MSE($\hat{\delta}$)	$0.18070^{\{4\}}$	$0.23232^{\{10\}}$	$0.27333^{\{14\}}$	$0.14258^{\{3\}}$	$0.25615^{\{11\}}$	$0.18666^{\{5\}}$	$0.13859^{\{2\}}$	$0.27265^{\{13\}}$	$0.21358^{\{8\}}$	$0.19629^{\{6\}}$	$0.59339^{\{15\}}$	$0.00770^{\{1\}}$	$0.26286^{\{12\}}$	$0.21940^{\{9\}}$	$0.20731^{\{7\}}$
320	MSE($\hat{\lambda}$)	$0.07956^{\{5\}}$	$0.09949^{\{10\}}$	$0.08206^{\{7\}}$	$0.06336^{\{3\}}$	$0.10383^{\{11\}}$	$0.07404^{\{4\}}$	$0.06105^{\{2\}}$	$0.11446^{\{13\}}$	$0.07968^{\{6\}}$	$0.09287^{\{8\}}$	$0.32135^{\{15\}}$	$0.00936^{\{1\}}$	$0.13768^{\{14\}}$	$0.09544^{\{9\}}$	$0.11358^{\{12\}}$
320	MRE($\hat{\delta}$)	$0.11400^{\{6\}}$	$0.12640^{\{10\}}$	$0.13184^{\{11\}}$	$0.09950^{\{2\}}$	$0.13869^{\{13\}}$	$0.10727^{\{4\}}$	$0.10048^{\{3\}}$	$0.14147^{\{14\}}$	$0.12135^{\{7\}}$	$0.11130^{\{5\}}$	$0.19906^{\{15\}}$	$0.01714^{\{1\}}$	$0.13428^{\{12\}}$	$0.12266^{\{8\}}$	$0.12300^{\{9\}}$
320	MRE($\hat{\lambda}$)	$0.06466^{\{6\}}$	$0.07216^{\{10\}}$	$0.06664^{\{7\}}$	$0.05615^{\{2\}}$	$0.07717^{\{12\}}$	$0.05981^{\{4\}}$	$0.05651^{\{3\}}$	$0.07995^{\{13\}}$	$0.06076^{\{5\}}$	$0.06690^{\{8\}}$	$0.12465^{\{15\}}$	$0.01973^{\{1\}}$	$0.08143^{\{14\}}$	$0.06882^{\{9\}}$	$0.07642^{\{11\}}$
320	D_{abs}	$0.01303^{\{2\}}$	$0.01492^{\{7\}}$	$0.01575^{\{11\}}$	$0.01272^{\{1\}}$	$0.01551^{\{9\}}$	$0.01438^{\{4\}}$	$0.01333^{\{3\}}$	$0.01501^{\{8\}}$	$0.01574^{\{10\}}$	$0.01636^{\{12\}}$	$0.01885^{\{14\}}$	$0.01441^{\{5\}}$	$0.01769^{\{13\}}$	$0.01444^{\{6\}}$	$0.01886^{\{15\}}$
320	D_{max}	$0.02111^{\{4\}}$	$0.02394^{\{7\}}$	$0.02520^{\{10\}}$	$0.02033^{\{2\}}$	$0.02498^{\{9\}}$	$0.02249^{\{5\}}$	$0.02108^{\{3\}}$	$0.02453^{\{8\}}$	$0.02539^{\{11\}}$	$0.02541^{\{12\}}$	$0.03134^{\{15\}}$	$0.02026^{\{1\}}$	$0.02801^{\{13\}}$	$0.02317^{\{6\}}$	$0.02932^{\{14\}}$
320	ASAE	$0.00743^{\{5\}}$	$0.00720^{\{2\}}$	$0.00766^{\{6\}}$	$0.00738^{\{4\}}$	$0.00793^{\{7\}}$	$0.00709^{\{1\}}$	$0.00736^{\{3\}}$	$0.00800^{\{8\}}$	$0.00937^{\{12\}}$	$0.00869^{\{10\}}$	$0.01300^{\{15\}}$	$0.00859^{\{9\}}$	$0.01107^{\{14\}}$	$0.00875^{\{11\}}$	$0.01073^{\{13\}}$
320	$\sum Ranks$	$44^{\{5\}}$	$76^{\{9\}}$	$84^{\{10\}}$	$21^{\{1.5\}}$	$97^{\{11\}}$	$35^{\{4\}}$	$25^{\{3\}}$	$104^{\{13\}}$	$71^{\{6\}}$	$74^{\{7\}}$	$134^{\{15\}}$	$21^{\{1.5\}}$	$118^{\{14\}}$	$75^{\{8\}}$	$101^{\{12\}}$

Table 19. Partial and overall ranks for all estimation methods of our proposed model.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
$\delta = 0.9,\ \lambda = 2.0$	25	1.0	13.0	6.0	7.0	8.5	8.5	13.0	2.5	4.0	10.0	5.0	2.5	15.0	13.0	11.0
	85	2.0	7.0	15.0	14.0	8.5	4.0	10.5	12.0	3.0	5.0	8.5	1.0	10.5	13.0	6.0
	145	2.0	10.5	14.0	6.0	13.0	5.0	8.0	12.0	3.0	10.5	15.0	1.0	7.0	4.0	9.0
	190	2.0	8.0	12.0	9.0	14.0	6.0	10.0	13.0	3.0	5.0	15.0	1.0	11.0	4.0	7.0
	245	1.5	7.5	12.0	9.0	13.0	4.0	6.0	14.0	3.0	7.5	15.0	1.5	11.0	5.0	10.0
	320	2.0	9.0	13.0	3.0	11.0	6.0	7.0	14.0	4.0	8.0	15.0	1.0	12.0	5.0	10.0
$\delta = 1.25,\ \lambda = 1.5$	25	1.0	4.0	6.5	11.0	10.0	8.0	12.0	6.5	5.0	13.0	3.0	2.0	15.0	9.0	14.0
	85	1.0	3.5	9.0	7.0	15.0	3.5	5.0	11.0	6.0	14.0	13.0	2.0	12.0	8.0	10.0
	145	1.0	8.0	12.0	4.0	11.0	5.0	3.0	14.0	6.0	10.0	15.0	2.0	13.0	7.0	9.0
	190	2.0	5.0	10.0	6.0	11.0	3.0	8.0	13.0	4.0	9.0	15.0	1.0	14.0	7.0	12.0
	245	2.0	8.0	10.0	3.5	11.0	6.0	5.0	14.0	7.0	9.0	15.0	1.0	13.0	3.5	12.0
	320	2.0	7.0	12.0	3.0	10.0	4.0	8.0	13.0	6.0	9.0	15.0	1.0	14.0	5.0	11.0
$\delta = 1.75,\ \lambda = 0.95$	25	2.0	6.0	4.5	11.0	13.0	8.0	10.0	7.0	3.0	12.0	9.0	1.0	15.0	4.5	14.0
	85	2.0	4.0	11.0	10.0	9.0	5.5	5.5	13.0	3.0	7.0	15.0	1.0	12.0	8.0	14.0
	145	2.0	4.0	9.0	6.0	11.5	8.0	7.0	13.0	3.0	10.0	15.0	1.0	11.5	5.0	14.0
	190	2.0	7.0	9.0	3.0	11.0	6.0	4.0	13.0	5.0	10.0	15.0	1.0	12.0	8.0	14.0
	245	2.0	9.0	10.0	3.0	11.0	7.0	4.0	12.0	5.0	6.0	15.0	1.0	14.0	8.0	13.0
	320	2.0	3.0	11.0	4.0	10.0	7.0	5.0	12.0	6.0	9.0	15.0	1.0	14.0	8.0	13.0
$\delta = 2.0,\ \lambda = 0.65$	25	2.0	4.0	6.0	13.0	12.0	3.0	7.0	8.0	5.0	9.5	11.0	1.0	14.0	9.5	15.0
	85	2.0	8.0	14.0	6.0	13.0	3.0	4.0	11.0	5.0	12.0	15.0	1.0	7.0	9.0	10.0
	145	2.0	3.0	11.0	7.5	12.0	5.0	6.0	13.0	4.0	10.0	15.0	1.0	9.0	7.5	14.0
	190	2.0	4.0	10.0	6.0	13.0	7.0	3.0	11.0	5.0	9.0	15.0	1.0	12.0	8.0	14.0
	245	2.0	6.0	8.0	5.0	11.0	3.0	7.0	12.0	4.0	9.0	15.0	1.0	13.0	10.0	14.0
	320	3.0	7.0	10.0	2.0	11.0	6.0	4.0	14.0	5.0	8.0	15.0	1.0	12.5	9.0	12.5
$\delta = 3.0,\ \lambda = 3.5$	25	3.0	6.0	7.0	13.0	4.0	8.0	2.0	11.0	5.0	10.0	12.0	1.0	14.5	9.0	14.5
	85	3.0	7.0	11.0	2.0	8.0	4.0	9.0	10.0	5.0	12.0	15.0	1.0	14.0	6.0	13.0
	145	2.0	5.0	12.0	3.0	9.0	10.5	6.0	8.0	7.0	10.5	15.0	1.0	14.0	4.0	13.0
	190	3.0	7.0	11.0	4.0	9.0	5.0	1.0	12.0	6.0	10.0	15.0	2.0	14.0	8.0	13.0
	245	4.0	7.0	9.5	2.0	11.0	6.0	3.0	12.0	5.0	8.0	15.0	1.0	13.0	9.5	14.0
	320	5.0	9.0	10.0	1.5	11.0	4.0	3.0	13.0	6.0	7.0	15.0	1.5	14.0	8.0	12.0
$\sum$ Ranks		64.5	196.5	305.5	184.5	325.5	169.0	186.0	344.0	141.0	279.0	406.5	36.5	377.0	222.5	362.0
Overall Rank		2	7	10	5	11	4	6	12	3	9	15	1	14	8	13

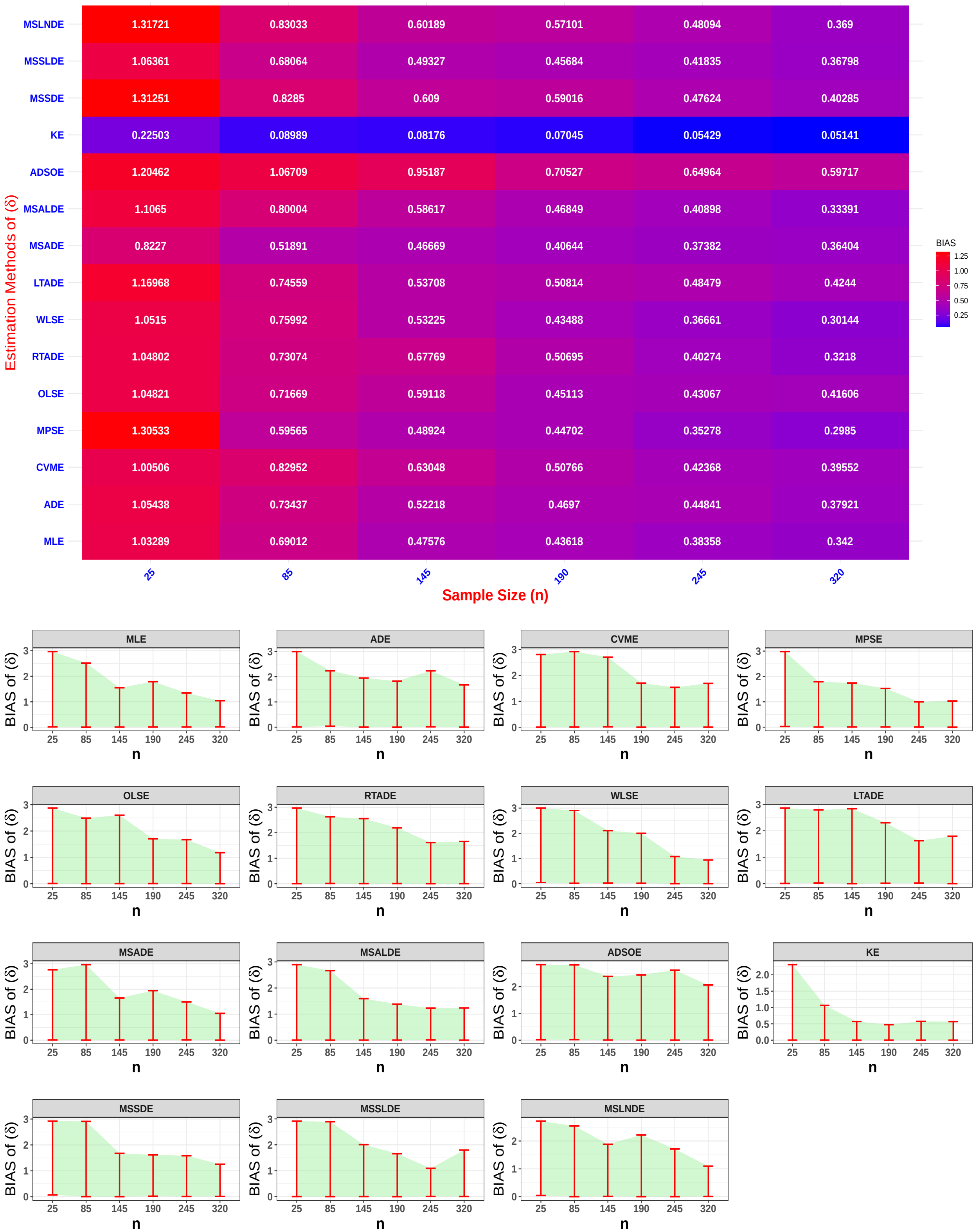

Figure 5. A graphical representation for BIAS values of parameter δ presented in Table 18.

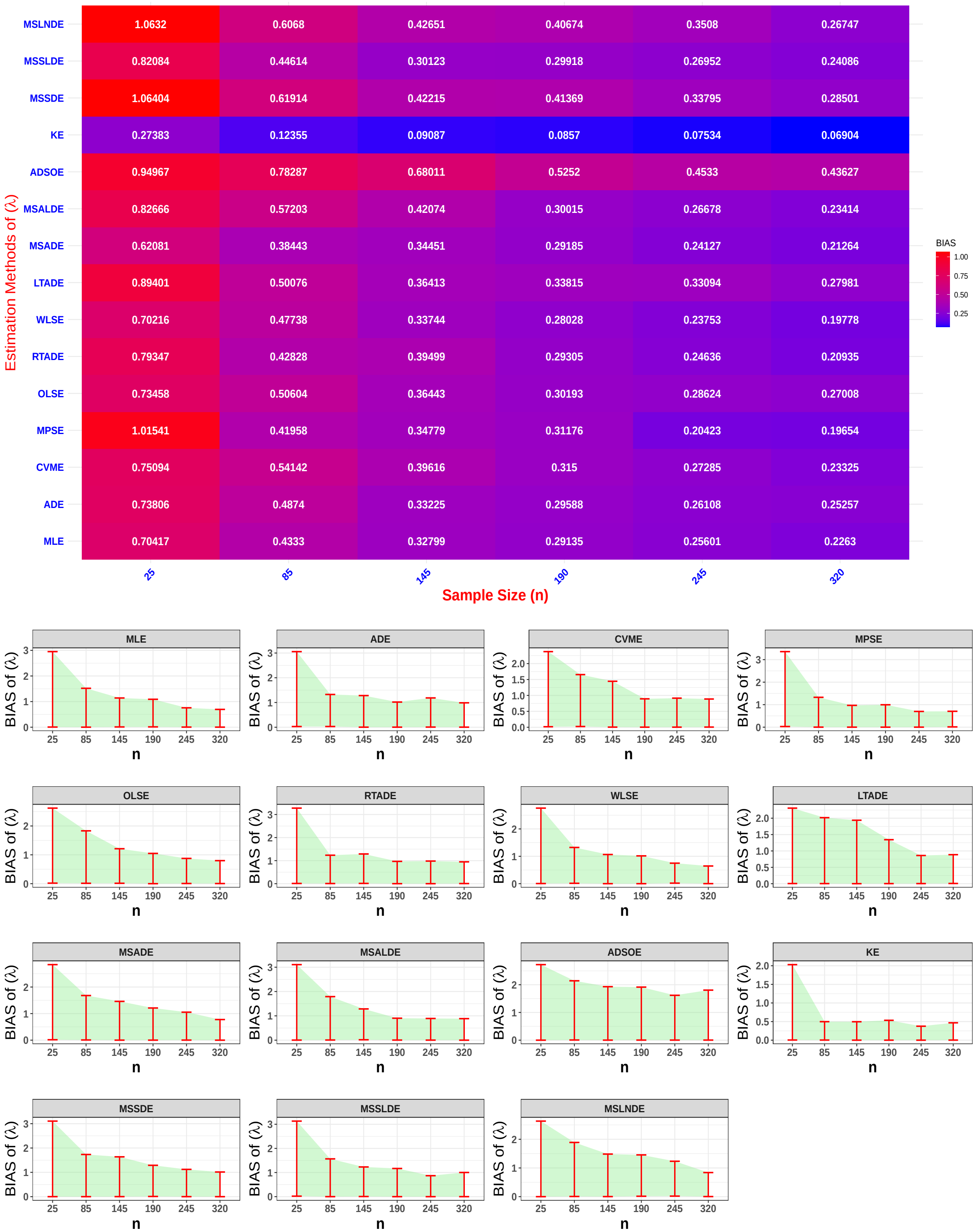

Figure 6. A graphical representation for BIAS values of parameter λ presented in Table 18.

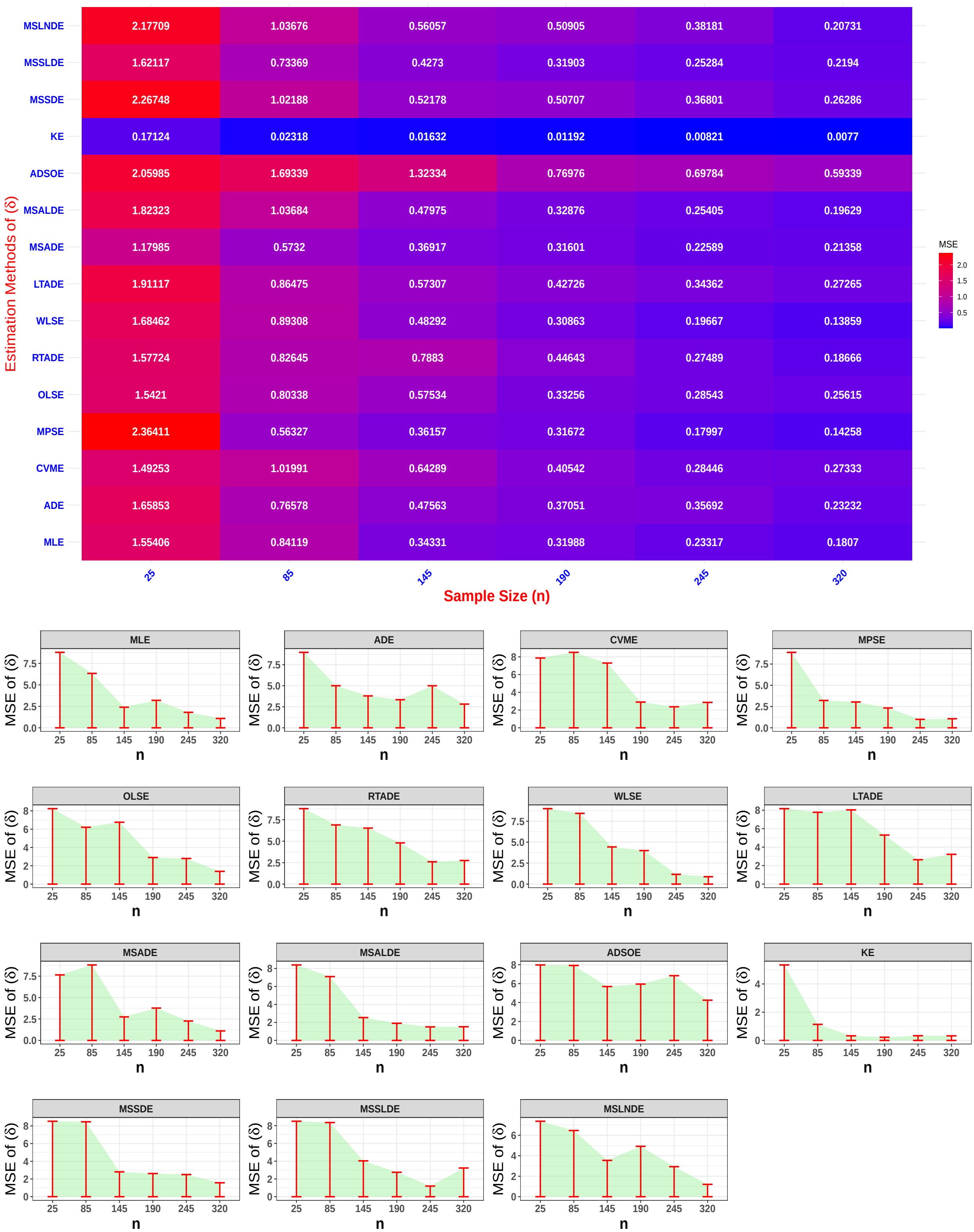

Figure 7. A graphical representation for MSE values of parameter δ presented in Table 18.

Figure 8. A graphical representation for MSE values of parameter λ presented in Table 18.

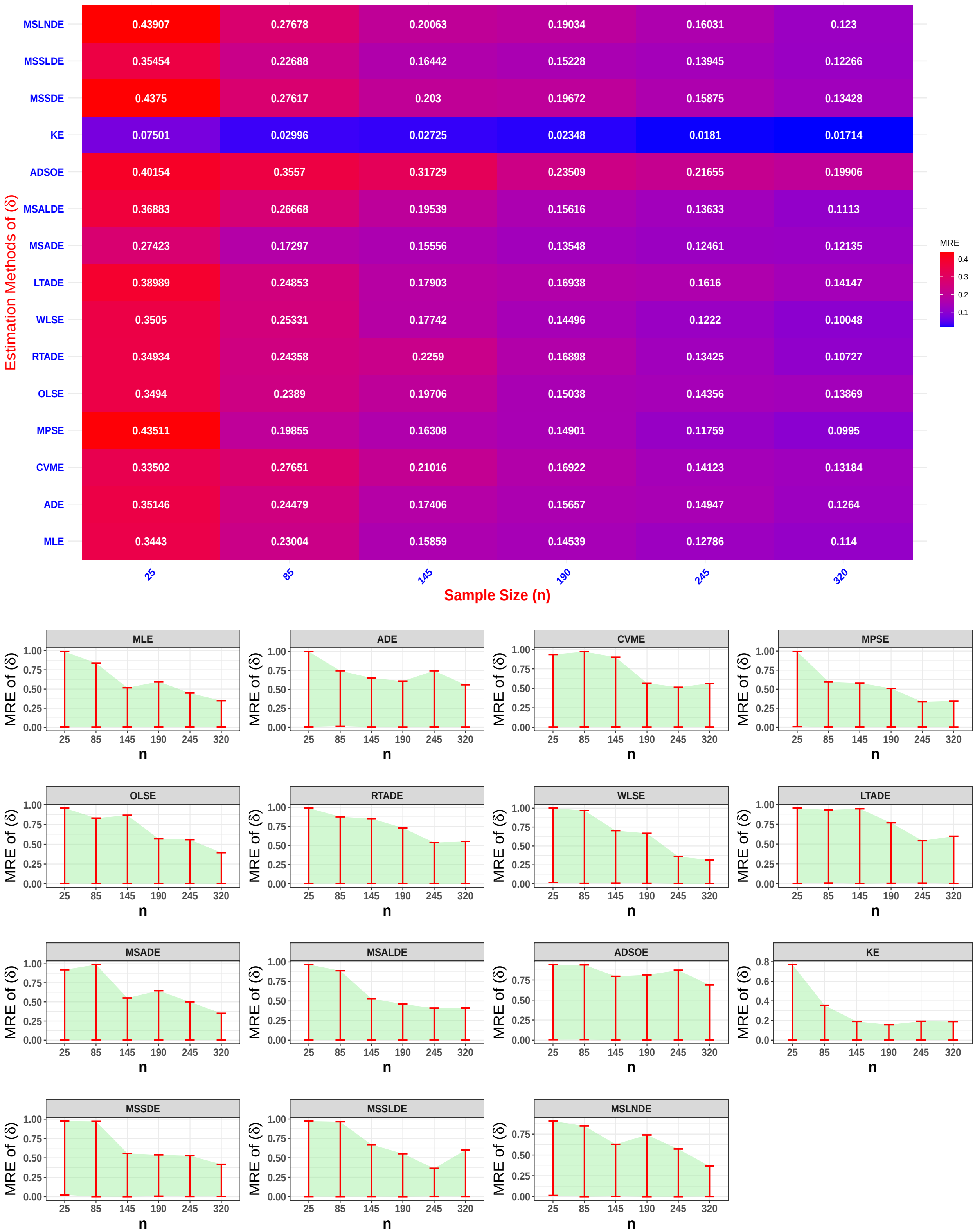

Figure 9. A graphical representation for MRE values of parameter δ presented in Table 18.

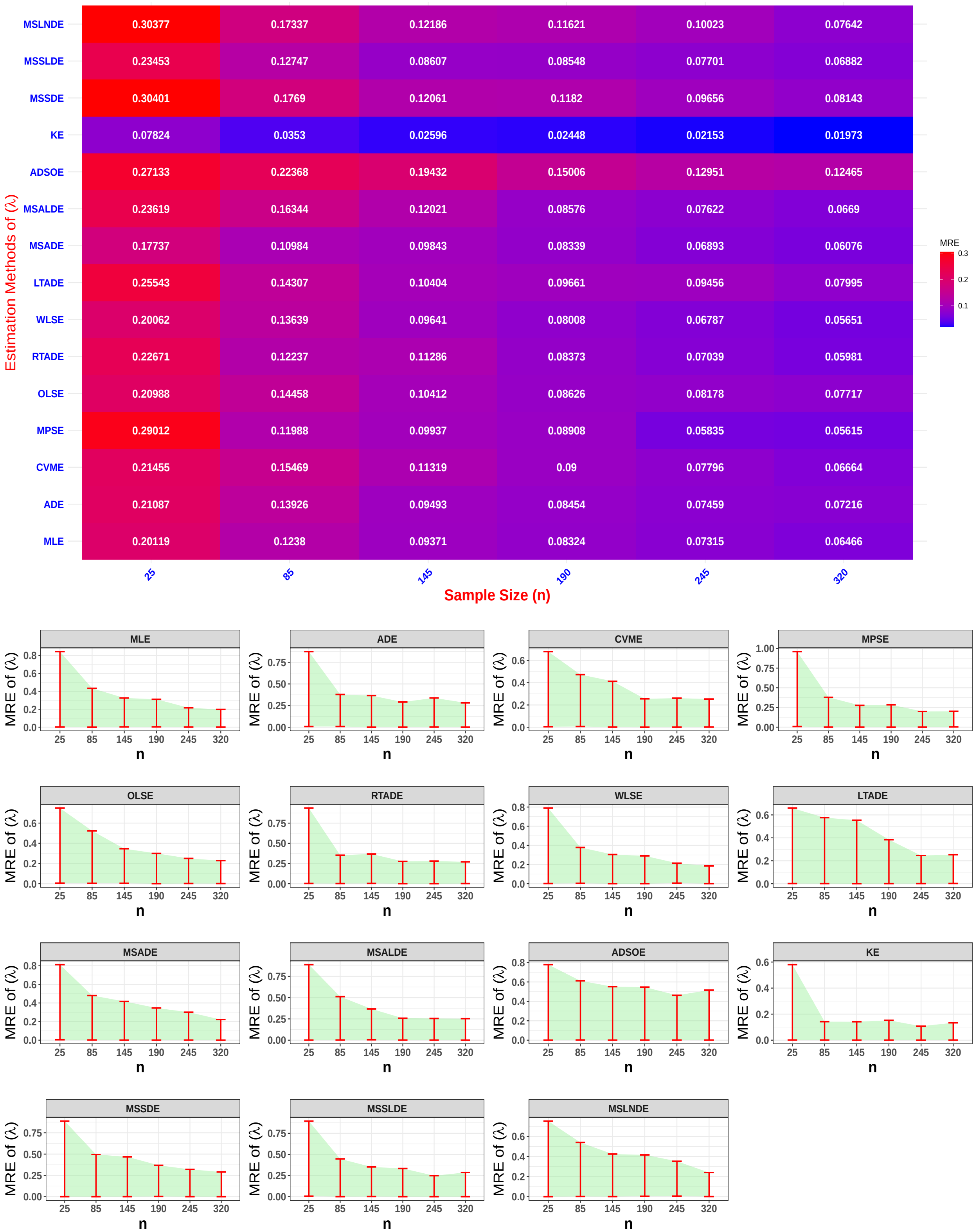

Figure 10. A graphical representation for MRE values of parameter λ presented in Table 18.

Figure 11. A graphical representation for D_{abs} values presented in Table 18.

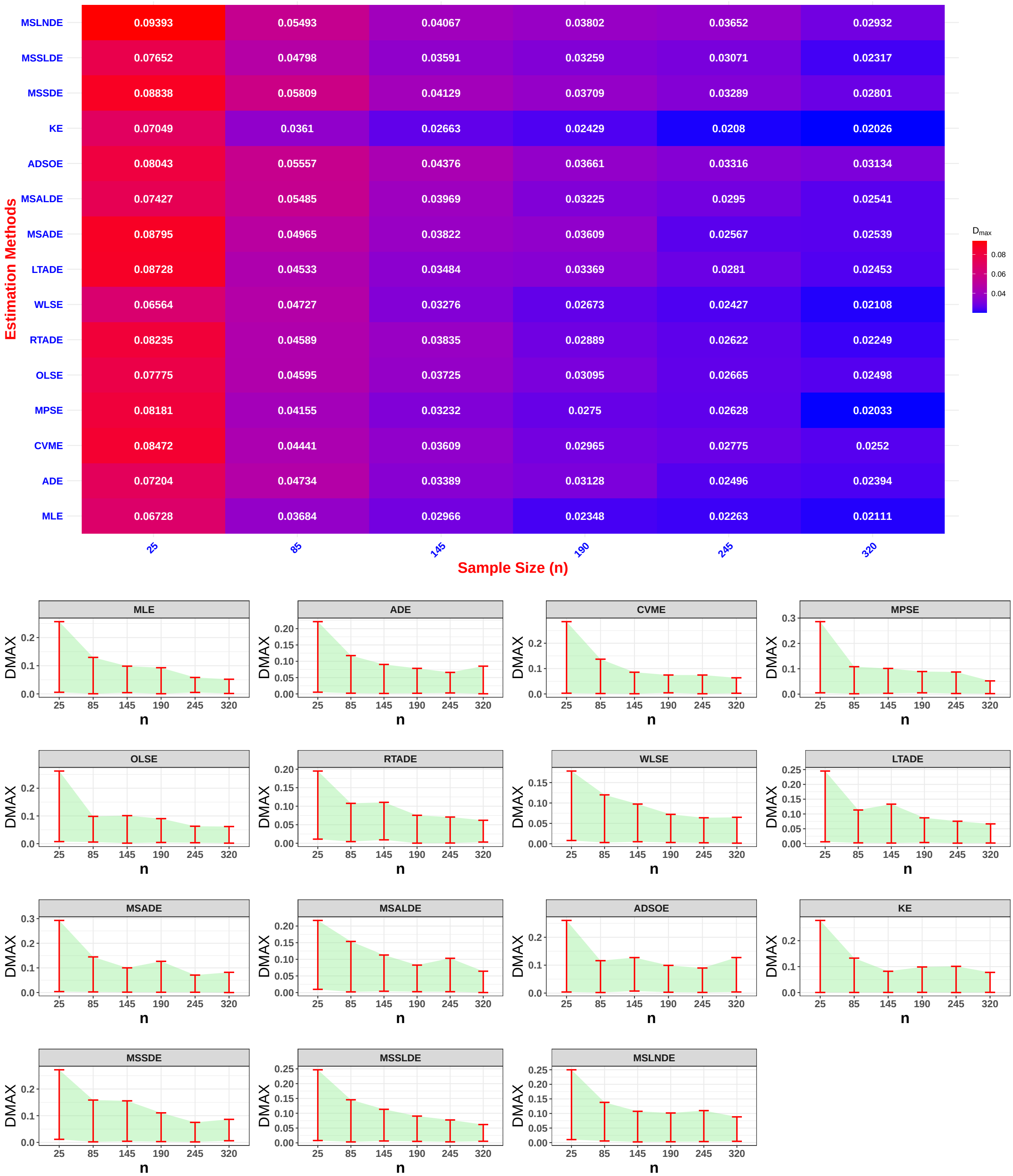

Figure 12. A graphical representation for D_{max} values presented in Table 18.

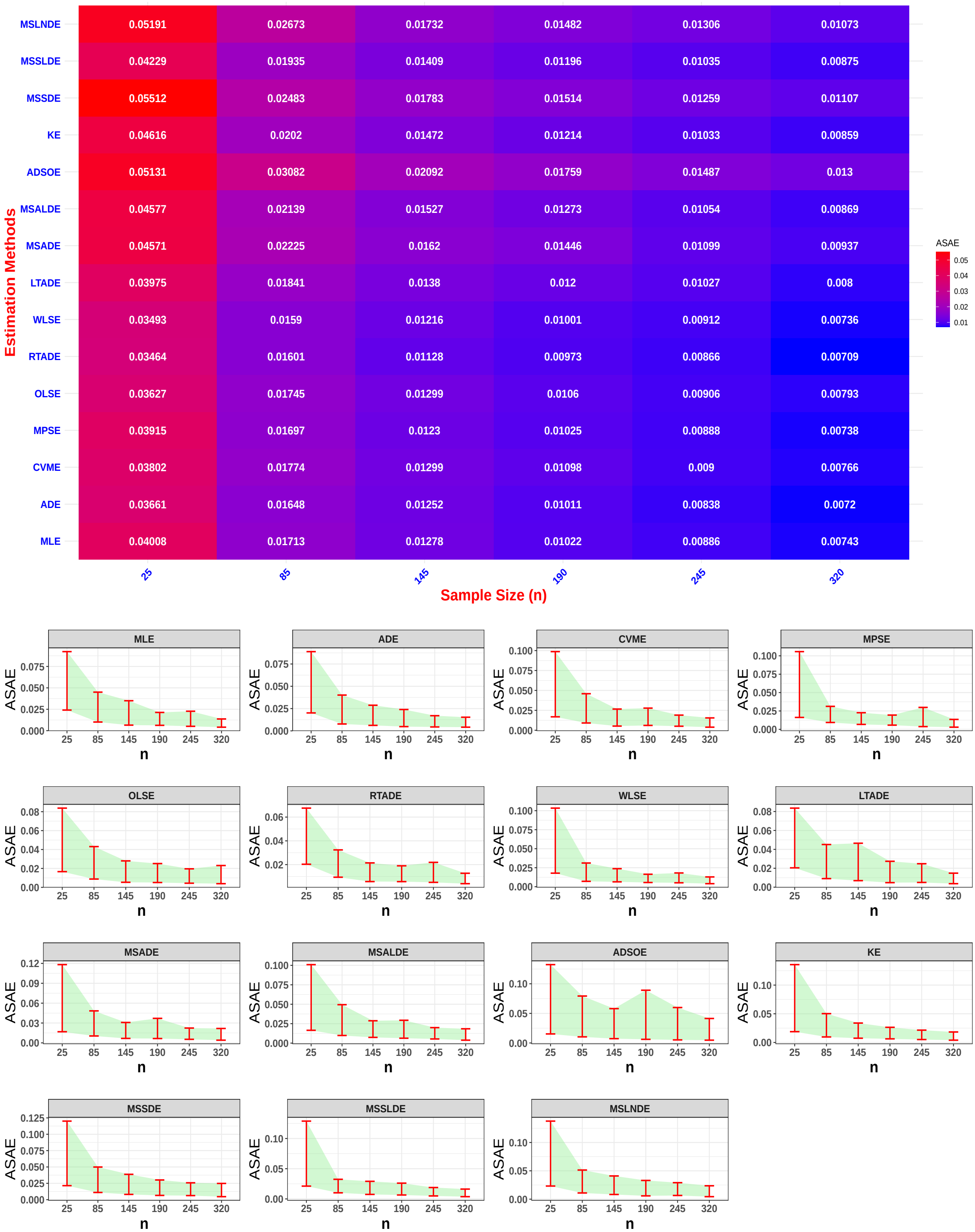

Figure 13. A graphical representation for ASAE values presented in Table 18.

7. Applications to Real Data

This section evaluates the practical applicability and empirical performance of the proposed model using two real-world data sets. The fitting performance of the proposed UILLi distribution is rigorously compared with several well-established competing models to examine its flexibility, robustness, and capability in capturing diverse data characteristics. The set of competing models includes the Lindley distribution (LD) [41], Shanker distribution (ShD) [64], Xgamma distribution (XgD) [62], Chris-Jerry distribution (CJD) [55], XLindley distribution (XLD) [12], exponentiated generalized XLindley distribution (EGXLD) [53], exponentiated generalized power Lindley distribution (EGLD) [50], as well as the classical Weibull (WD) [70] and gamma (GD) [34] distributions.

This comprehensive comparison framework ensures a fair and rigorous assessment of the proposed model across different distributional shapes and tail behaviors. In particular, it allows for evaluating the ability of the UILLi model to provide improved goodness-of-fit relative to both classical and recently developed generalized distributions.

The first data set consists of the monthly actual taxes revenue in Egypt from January 2006 to November 2010. Recently, it has been used by Ali et al. [3]. The observed values are the following:

5.9	20.4	14.9	16.2	17.2	7.8	6.1	9.2	10.2	9.6	13.3	8.5
21.6	18.5	5.1	6.7	17.0	8.6	9.7	39.2	35.7	15.7	9.7	10.0
4.1	36.0	8.5	8.0	9.2	26.2	21.9	16.7	21.3	35.4	14.3	8.5
10.6	19.1	20.5	7.1	7.7	18.1	16.5	11.9	7.0	8.6	12.5	10.3
11.2	6.1	8.4	11.0	11.6	11.9	5.2	6.8	8.9	7.1	10.8	

Descriptive statistics are presented in Table 20.

Table 20. Descriptive Statistics of the (Dataset I)

n	Min	Q_1	Median	Mean	Q_3	Max
59	4.10	8.45	10.60	13.49	16.85	39.20

The second data set represents the maximum annual flood discharges (in units of 1000 cubic feet per second) of the North Saskachevan River at Edmonton, over 48 years. Recently, it was used by Tahir et al [67]. The observed values are the following:

19.885	20.940	21.820	23.700	24.888	25.460	25.760	26.720	27.500	28.100	28.600	30.200
30.380	31.500	32.600	32.680	34.400	35.347	35.700	38.100	39.020	39.200	40.000	40.400
40.400	42.250	44.020	44.730	44.900	46.300	50.330	51.442	57.220	58.700	58.800	61.200
61.740	65.440	65.597	66.000	74.100	75.800	84.100	106.600	109.700	121.970	121.970	185.560

Descriptive statistics are presented in Table 21.

Table 21. Descriptive Statistics of the (Dataset II)

n	Min	Q_1	Median	Mean	Q_3	Max
48	19.89	30.34	40.40	51.50	61.34	185.56

Maximum likelihood estimates (MLEs) and their standard errors (SEs) for all candidate models are summarized in Tables 22 and 25 for Datasets I and II, respectively. The proposed UILLi model demonstrates competitive parameter estimation stability compared with classical and other generalized distributions. Goodness-of-fit statistics, including Log-Likelihood, AIC, BIC, CAIC, HQIC, KS test, and ASAE, are reported in Tables 23 and 26. Additional tests using Cramér–von Mises (CvM) and Anderson–Darling (AD) statistics are presented in Tables 24 and 27. Across both data sets, the UILLi models consistently exhibit superior fit, reflected by higher p-values in KS, CvM, and AD tests and lower AIC/BIC measures relative to competing models.

Table 22. Maximum Likelihood Estimates (MLEs) and Standard Errors (SE) (Dataset I)

Model	$\hat{\lambda}$	SE($\hat{\lambda}$)	$\hat{\delta}$	SE($\hat{\delta}$)	$\hat{\theta}$	SE($\hat{\theta}$)
UILLi	0.1304	0.0510	1.0143	0.6770	–	–
LD	0.1392	0.0129	–	–	–	–
ShD	0.1462	0.0133	–	–	–	–
XgD	0.2029	0.0158	–	–	–	–
CJD	0.2117	0.0162	–	–	–	–
XLD	0.1318	0.0122	–	–	–	–
WD	1.8404	0.1712	15.3065	1.1514	–	–
GD	3.6787	0.6489	3.6665	0.6929	–	–
EGXLD	0.1954	0.0259	0.9745	0.0033	4.8339	1.2152
EGLD	1.0042	0.0026	0.1818	0.0245	4.0253	0.9622

Table 23. Goodness-of-Fit Statistics for Competing Models (Dataset I)

Model	LogLik	AIC	BIC	CAIC	HQIC	KS	KS-pval	ASAE
UILLi	187.905	379.810	383.965	380.025	381.432	0.0691	0.9406	0.2344
LD	200.629	403.259	405.336	403.329	404.070	0.1922	0.0256	0.2971
ShD	198.530	399.061	401.138	399.131	399.872	0.1706	0.0645	0.2888
XgD	199.284	400.568	402.645	400.638	401.379	0.1666	0.0756	0.2792
CJD	196.825	395.649	397.727	395.720	396.460	0.1362	0.2238	0.2701
XLD	202.639	407.279	409.356	407.349	408.090	0.2106	0.0107	0.3056
WD	197.291	398.581	402.736	398.795	400.203	0.1432	0.1779	0.2616
GD	193.082	390.164	394.319	390.378	391.786	0.1336	0.2427	0.2472
EGXLD	191.319	388.638	394.871	389.074	391.071	0.1163	0.4025	0.2400
EGLD	191.124	388.248	394.481	388.685	390.681	0.1193	0.3707	0.2436

Table 24. Additional Goodness-of-Fit Tests (Dataset I)

Model	CvM-Stat	CvM-pval	AD-Stat	AD-pval
UILLi	0.0469	0.8967	0.2765	0.9545
LD	0.4582	0.0506	2.9343	0.0297
ShD	0.3595	0.0924	2.3971	0.0563
XgD	0.3281	0.1127	2.2548	0.0670
CJD	0.2391	0.2031	1.7114	0.1333
XLD	0.5599	0.0278	3.4633	0.0161
WD	0.2804	0.1536	1.8406	0.1128
GD	0.2030	0.2621	1.2308	0.2560
EGXLD	0.1647	0.3484	0.9999	0.3570
EGLD	0.1640	0.3502	0.9694	0.3734

Table 25. Maximum Likelihood Estimates (MLEs) and Standard Errors (SE) (Dataset II)

Model	$\hat{\lambda}$	SE($\hat{\lambda}$)	$\hat{\delta}$	SE($\hat{\delta}$)	$\hat{\theta}$	SE($\hat{\theta}$)
UILLi	0.0244	0.0126	0.5212	0.4519	–	–
LD	0.0381	0.0039	–	–	–	–
ShD	0.0388	0.0040	–	–	–	–
XgD	0.0567	0.0048	–	–	–	–
CJD	0.0575	0.0048	–	–	–	–
XLD	0.0375	0.0038	–	–	–	–
WD	1.7728	0.1777	58.3906	5.0593	–	–
GD	3.6546	0.7145	14.0904	2.9531	–	–
EGXLD	0.2351	0.0361	0.2137	0.0028	3.9594	1.0658
EGLD	0.2148	0.0027	0.2313	0.0358	3.7233	0.9876

Table 26. Goodness-of-Fit Statistics for Competing Models (Dataset II)

Model	LogLik	AIC	BIC	CAIC	HQIC	KS	KS-pval	ASAE
UILLi	215.023	434.047	437.789	434.313	435.461	0.0559	0.9983	0.0153
LD	226.140	454.280	456.151	454.367	454.987	0.1892	0.0642	0.0430
ShD	225.541	453.082	454.953	453.169	453.789	0.1815	0.0846	0.0416
XgD	223.446	448.892	450.763	448.978	449.599	0.1359	0.3377	0.0395
CJD	222.745	447.490	449.362	447.577	448.198	0.1244	0.4480	0.0385
XLD	226.723	455.447	457.318	455.534	456.154	0.1966	0.0489	0.0444
WD	225.707	455.413	459.155	455.680	456.827	0.1404	0.3003	0.0474
GD	221.515	447.031	450.773	447.298	448.445	0.1356	0.3405	0.0395
EGXLD	219.388	444.777	450.390	445.322	446.898	0.1222	0.4710	0.0357
EGLD	219.315	444.630	450.243	445.175	446.751	0.1230	0.4625	0.0356

Table 27. Additional Goodness-of-Fit Tests (Dataset II)

Model	CvM-Stat	CvM-pval	AD-Stat	AD-pval
UILLi	0.0219	0.9953	0.1472	0.9988
LD	0.3418	0.1033	2.2337	0.0688
ShD	0.3152	0.1224	2.0897	0.0823
XgD	0.1955	0.2770	1.3786	0.2083
CJD	0.1814	0.3073	1.2694	0.2424
XLD	0.3687	0.0872	2.3769	0.0578
WD	0.2769	0.1572	1.7729	0.1231
GD	0.1857	0.2977	1.1666	0.2804
EGXLD	0.1433	0.4118	0.9213	0.4008
EGLD	0.1450	0.4063	0.9190	0.4022

Figures 14–16 illustrate the estimated probability density functions (PDFs), cumulative distribution functions (CDFs), and probability–probability (PP) plots for Dataset I, while Figures 17–19 provide the corresponding visual assessments for Dataset II. These graphical evaluations confirm that the proposed UILLi model closely follows the empirical distributions, capturing both central tendencies and tail behavior effectively.

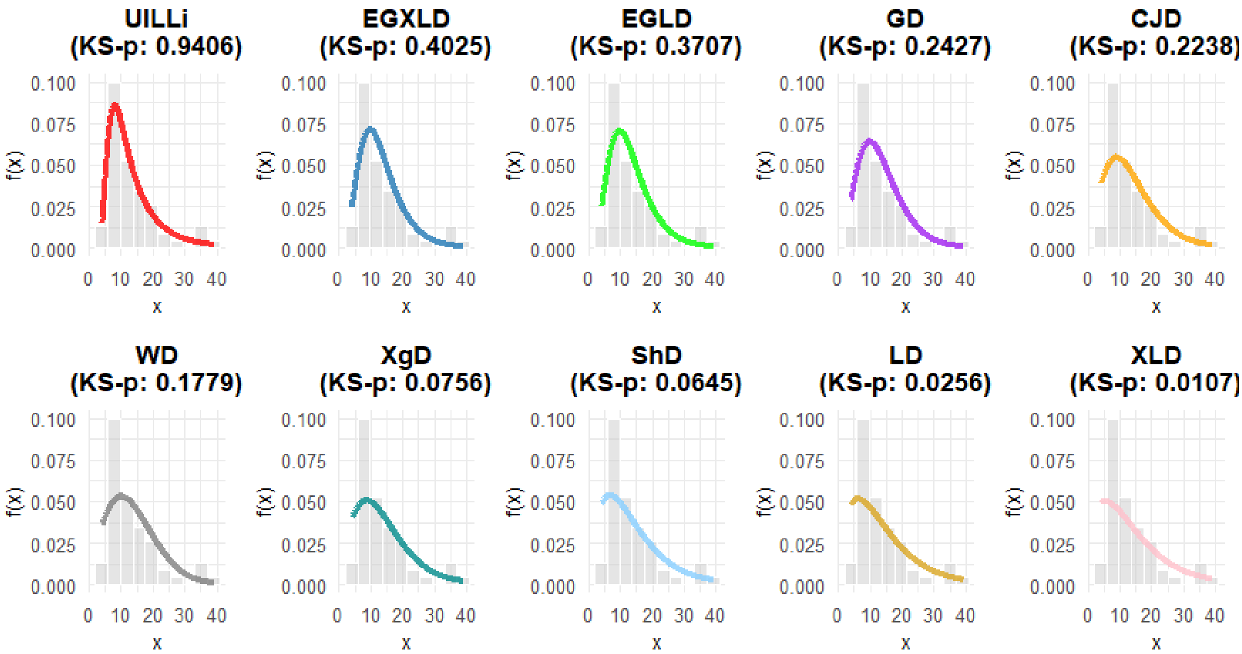

Figure 14. Estimated PDFs for the competing models (Dataset I).

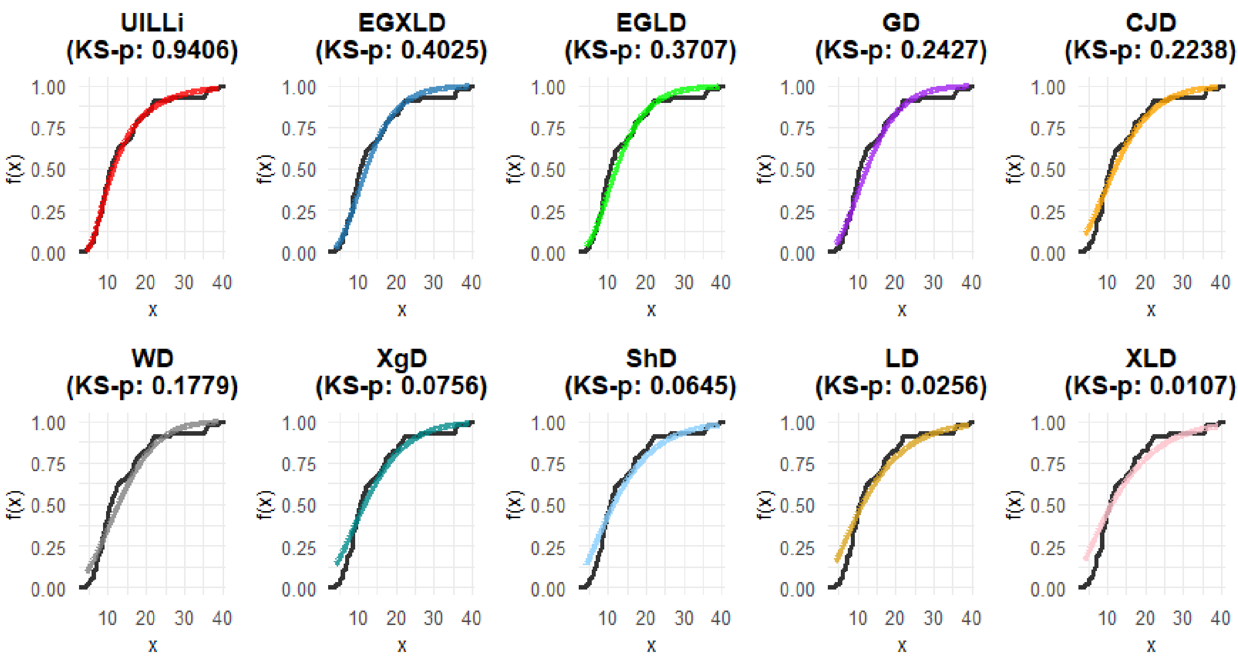

Figure 15. Estimated CDFs for the competing models (Dataset I).

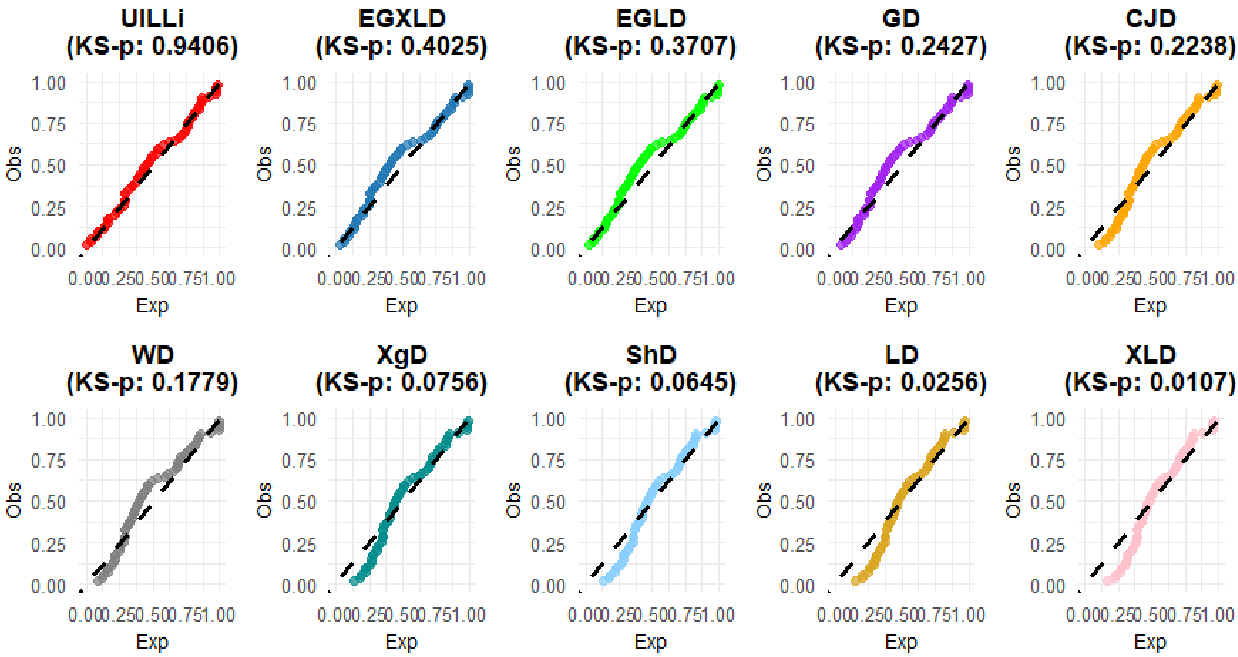

Figure 16. PP for the competing models (Dataset I).

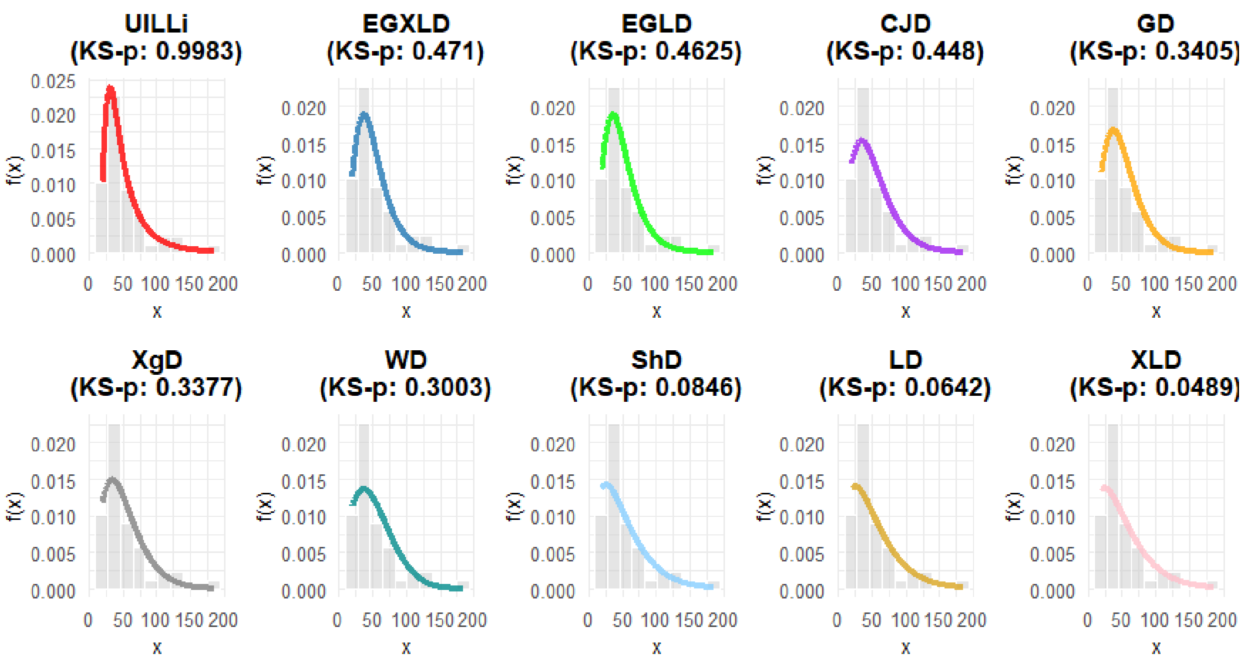

Figure 17. Estimated PDFs for the competing models (Dataset II).

8. Concluding Remarks

In conclusion, this study developed and investigated a new unit-generated family of distributions, the unit inverse Lindley family, emphasizing its flexibility and application in statistical modeling. Several sub-models were

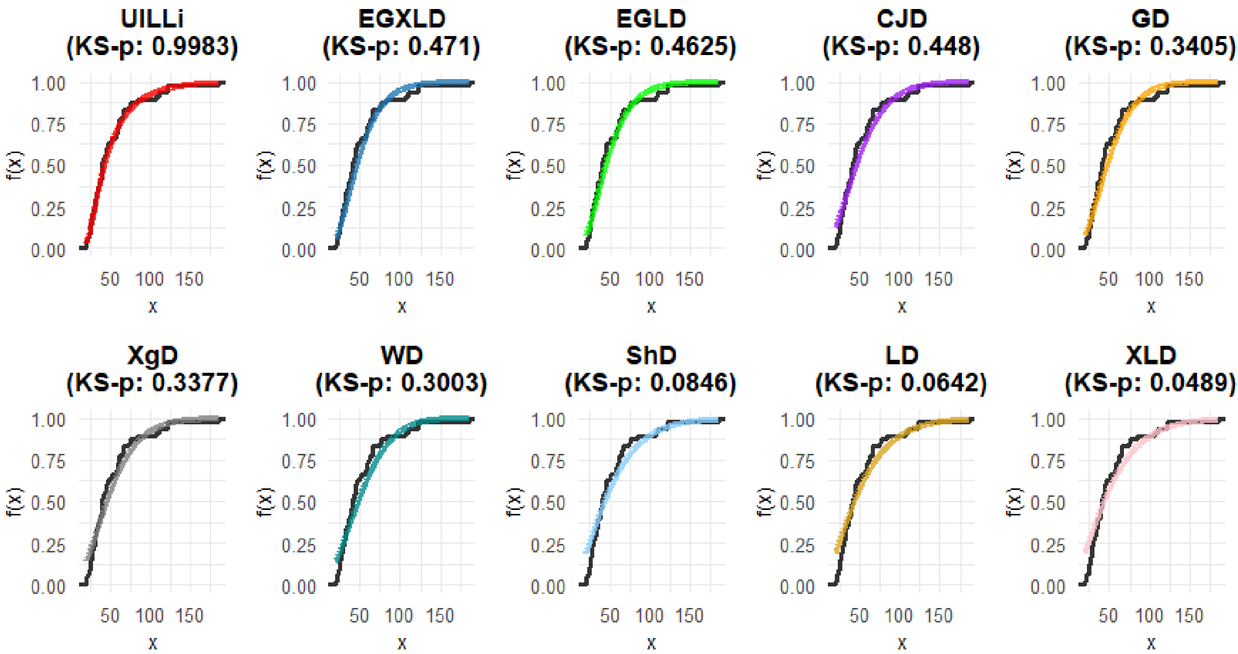

Figure 18. Estimated CDFs for the competing models (Dataset II).

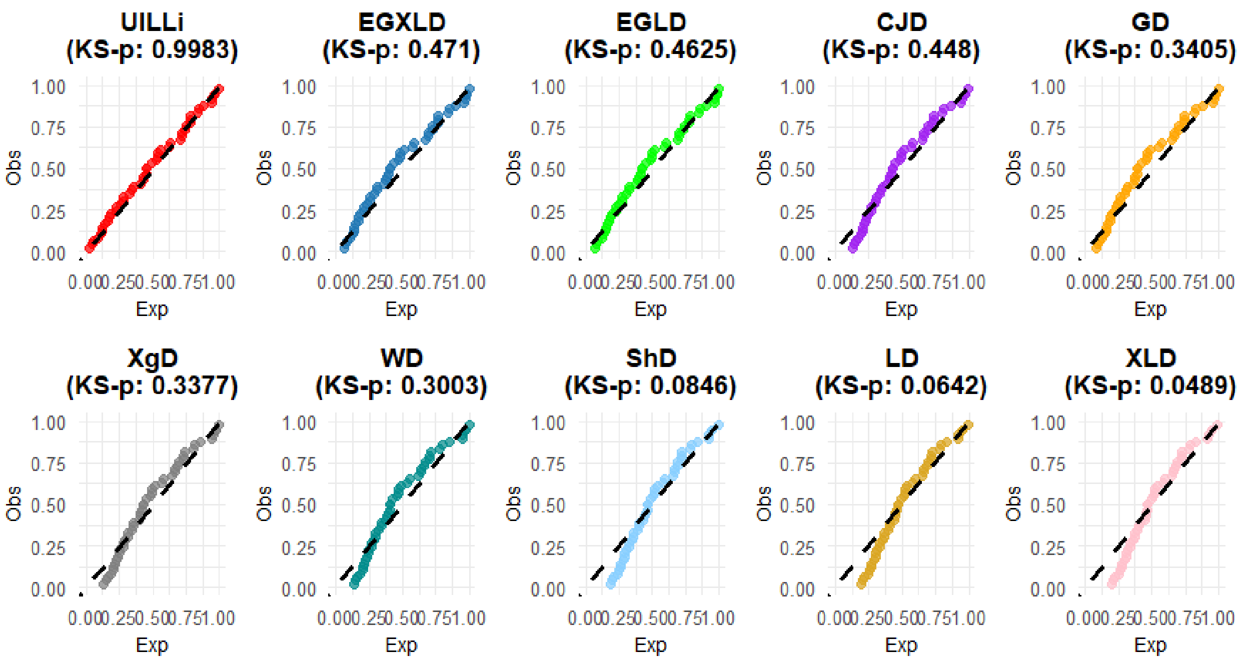

Figure 19. PP for the competing models (Dataset II).

created to demonstrate the flexibility of the proposed family, including the unit inverse Lindley-exponential, unit inverse Lindley-Lomax, unit inverse Lindley-Topp Leone, and unit inverse Lindley-Kumaraswamy distributions. Fundamental statistical aspects of the family, including quantiles, moments, and the moment generating function, were computed. Furthermore, numerous entropy measures—including Rényi, Tsallis, Havrda, and Charvat entropies, as well as Arimoto entropy—were investigated. Several estimation methods, including sixteen different techniques, were implemented and evaluated. Monte Carlo simulations were used to analyze the effectiveness and performance of different estimating strategies.

Finally, the practical relevance and robustness of the proposed family were demonstrated through applications to real-world datasets, confirming its usefulness and potential for modeling diverse types of data.

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