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Robust Green Supplier Selection with Risk-Aware Trade-offs using Bi-Level Programming

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Abstract This paper addresses the supplier selection and order allocation problem in a multi-product, multi-scenario supply chain environment, subject to uncertainty. Traditional models often neglect critical factors such as supply risk and environmental impact, leading to suboptimal and unsustainable decisions. In this work, we propose a two-level decision-making framework where the upper level selects a subset of suppliers based on a set of criteria such as cost, reliability, risk, and environmental impact, while the lower level determines the optimal order allocation under multiple uncertain demand scenarios. The lower-level problem is reformulated using Karush–Kuhn–Tucker conditions to obtain a single-level mixed integer linear programming model. A detailed numerical example illustrates the effectiveness of the approach and the impact of uncertainty on supplier selection decisions. The numerical study is further extended with larger-scale instances, a heuristic baseline comparison, a big-M sensitivity analysis, and a discussion of the model's computational and theoretical limitations. The proposed model can be viewed as a strong green supplier selection model with risk-aware trade-offs, and it is a practical decision-making tool for sustainable and resilient procurement strategies.

Keywords Supplier Selection, Order Allocation, Bi-Level Programming, Demand Uncertainty, Supply Risk, Sustainable Supply Chain

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1. Introduction

Choosing the right suppliers is a strategic process that can significantly influence a company's operational efficiency, cost-effectiveness, and environmental responsibility in the current global market with sustainability concerns. Conventional supplier selection approaches tend to make the assumption that supply capacities, delivery performance and cost parameters are certain [9, 3, 4].

In reality, however, procurement decisions are fraught with uncertainty, including capacity and delivery fluctuations from suppliers, risk exposure, and more, and must be managed in a systematic way to ensure supply chain resilience and robustness. A strategic procurement problem that is important is the supplier selection and order allocation problem in which the supplier's performance is uncertain, the demand is volatile, and capacity is limited. Companies may have several conflicting goals, including cost minimisation, supply risk reduction, and environmental impact minimisation. The decision-maker has to choose a subset of qualified suppliers and the allocation of orders to products and demand scenarios [1, 16].

The literature on supplier selection and order allocation under uncertainty is extensive; a few representative studies illustrate the range of methodologies and applications closest to the work presented here. Dong and Yuan [2] study distributionally robust models for supplier selection and order allocation that account for carbon emissions under a cap-and-trade policy framework. Ventura and Lu [5] propose a bi-objective mixed-integer nonlinear programming formulation for supplier selection and order allocation aimed at minimising cost while considering

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supplier efficiency evaluation. Shao, Barnes and Wu [6] investigate supplier selection and order allocation strategies for multinational companies under supply disruption during the COVID-19 pandemic, with a focus on sustainability and resilience. Hallak [7] presents a hybrid multi-criteria decision-making approach for supplier selection in construction projects in conflict-affected areas, focused on resilience and adaptability. Mirzaee, Samarghandi and Willoughby [8] propose a robust optimization model for green supplier selection and order allocation in a closed-loop supply chain, using a cap-and-trade system to manage air pollution.

In this paper, we propose a new mathematical model for the supplier selection and order allocation problem with uncertainty. The proposed model includes the multiple criteria, such as cost, delivery reliability, risk and environmental impact, in a bi-level optimization model. The lower-level problem is used to accommodate uncertainties in supplier capacity and performance for multiple scenarios and is used to capture the operational decision of order allocation, while the upper-level problem is used to capture the strategic supplier selection decision. We recast this bi-level problem as a single-level Mixed-Integer Linear Program (MILP) problem with the Karush–Kuhn–Tucker (KKT) conditions to be solved efficiently.

Here a bi-level, risk-aware, and green supplier selection model is proposed, following which the objectives are

1. To incorporate environmental factors.
2. To deal with uncertainty in an organized way.
3. To model hierarchical decision-making, where procurement is affected by higher-level sustainability policies.
4. To solve the bi-level problem.

The rest of the article is structured as follows: the proposed formulation and assumptions are presented in Section 2. This model is compared with the recent literature in Section 3. Section 4 reports the computational experiments, including small illustrative cases together with larger-scale instances, a baseline comparison, sensitivity analyses, and discussion. Section 5 concludes the paper.

2. Model Overview

The supplier selection and order allocation problem is formulated as a bi-level optimization problem, as follows:

1. **Upper Level: Strategic Decision-Maker:** minimises overall procurement cost and environmental impact from suppliers by selecting a subset of suppliers (binary decision variables), subject to the company's sustainability policy.
2. **Lower Level: Operational Procurement:** allocates order quantities among the selected suppliers so as to minimise delivery time and procurement risk under uncertainty.

2.1. Supplier Risk Assessment

The supplier risk factor r_1 is the overall uncertainty of supplier i about its operations. It's not a random number; it's designed to be a summary of many aspects of suppliers' performance that have a direct impact on procurement reliability. The proposed framework computes supplier risk using a weighted aggregation model:

$$r_i = w_1 R_i^{\text{delay}} + w_2 R_i^{\text{quality}} + w_3 R_i^{\text{disruption}} + w_4 R_i^{\text{financial}} \tag{1}$$

subject to

$$\sum_{k=1}^{4} w_k = 1, \qquad w_k \geq 0. \tag{2}$$

where

- R_i^{delay} denotes the normalized historical late-delivery rate;
- $R_i^{quality}$ represents the normalized product defect rate;

- $R_i^{disruption}$ measures historical supply interruption frequency;
- $R_i^{financial}$ denotes supplier financial instability obtained from credit ratings or financial indicators.

All the components are scaled to the range [0,1] so that the supplier who has a different performance characteristics can be compared on a common scale. The weighting coefficients can be assigned based on expert opinion, Analytic Hierarchy Process (AHP), entropy weighting, and/or historical data analysis depending on the availability of the data. The computational experiments used in this study are synthetic experiments, where the proposed optimization framework is demonstrated by using synthetic risk values. But in real industrial use, these values should be calculated based on the historical values of suppliers' performance in the supplier performance database or the enterprise procurement information system.

2.2. Model Assumptions

1. **Discrete Supplier Set**: there is a finite number of potential suppliers with cost, emission rate, risk factor and capacity.
2. **Multiple Products and Scenarios**: the buyer purchases several products across several demand and supply scenarios.
3. **Deterministic Criteria**: supplier cost, emissions, and risk metrics are known and constant over the planning horizon.
4. **Scenario-Based Uncertainty**: uncertainty in supplier capacity and product-specific performance is captured by a finite set of scenarios.
5. **Full Demand Satisfaction**: demand for each product must be fully satisfied in each scenario; partial fulfilment or backordering is not allowed.
6. **Supplier Capacity Constraints**: each supplier has a scenario-dependent capacity limit.
7. **Single-Period Planning**: the model assumes a single decision-making period (static procurement).
8. **Binary Supplier Decisions**: supplier selection is represented by a binary variable.
9. **MILP Reformulation**: the bi-level formulation is reformulated into a single-level MILP using KKT optimality conditions and solved with standard solvers.

2.3. Model Formulation

Sets and indices: $i \in I$ suppliers; $j \in J$ products; $\omega \in \Omega$ scenarios.

Decision variables: $y_i \in \{0, 1\}$ supplier selection; $x_{ij}^{\omega} \geq 0$ fraction of product j ordered from supplier i under scenario ω.

Parameters: c_{ij} unit procurement cost; e_i emission rate; r_i risk factor; d_{ij}^{ω} delivery time; S_j demand volume; C_i^{ω} supply capacity; α emissions weight; $N_{\min}$ minimum number of suppliers.

Upper Level (Strategic decision):

$$\min_{y} \; Z_U = \sum_{i \in I} y_i \left(\sum_{j \in J} c_{ij} + \alpha e_i \right) \tag{3}$$

subject to

$$\sum_{i \in I} y_i \geq N_{\min}, \qquad y_i \in \{0, 1\} \tag{4}$$

Lower Level (Operational decision, for each scenario ω):

$$\min_{x} \; Z_L^{\omega} = \sum_{i \in I} \sum_{j \in J} \left(d_{ij}^{\omega} + r_i \right) x_{ij}^{\omega} \tag{5}$$

subject to

$$\sum_{i\in I} x_{ij}^{\omega} = 1, \quad \forall j \in J \tag{D1}$$

$$\sum_{j\in J} x_{ij}^{\omega} S_j \leq C_i^{\omega} y_i, \quad \forall i \in I \tag{C1}$$

$$x_{ij}^{\omega} \geq 0, \quad \forall i \in I, j \in J \tag{X1}$$

This is a non-convex bi-level problem, difficult to solve directly. It is rewritten as a single-level MILP via big-M linearization of the KKT conditions of the lower level, following the standard mixed-integer optimization techniques described in [15], in four steps: (i) restate the bi-level problem; (ii) derive the KKT conditions of the lower level; (iii) linearize the complementarity conditions as MILP constraints; (iv) merge the upper- and lower-level problems into one MILP.

Dual variables: λ_j^{ω} for (D1); $\mu_i^{\omega} \geq 0$ for (C1); $\nu_{ij}^{\omega} \geq 0$ for (X1). Lagrangian:

$$\begin{aligned} L = {} & \sum_{i\in I}\sum_{j\in J}\left(d_{ij}^{\omega} + r_i\right) x_{ij}^{\omega} + \sum_{j\in J} \lambda_j^{\omega}\left(1 - \sum_{i\in I} x_{ij}^{\omega}\right) \\ & + \sum_{i\in I} \mu_i^{\omega}\left(\sum_{j\in J} x_{ij}^{\omega} S_j - C_i^{\omega} y_i\right) - \sum_{i\in I}\sum_{j\in J} \nu_{ij}^{\omega} x_{ij}^{\omega} \end{aligned} \tag{6}$$

Stationarity:

$$d_{ij}^{\omega} + r_i - \lambda_j^{\omega} + \mu_i^{\omega} S_j - \nu_{ij}^{\omega} = 0, \quad \forall i, j \tag{KKT1}$$

Dual feasibility: $\mu_i^{\omega} \geq 0$, $\nu_{ij}^{\omega} \geq 0$ (KKT2). Complementary slackness:

$$\mu_i^{\omega}\left(\sum_{j\in J} x_{ij}^{\omega} S_j - C_i^{\omega} y_i\right) = 0, \quad \forall i \tag{CS1}$$

$$\nu_{ij}^{\omega} x_{ij}^{\omega} = 0, \quad \forall i, j \tag{CS2}$$

CS1 is linearized with binaries z_i^{ω}:

$$\sum_{j\in J} x_{ij}^{\omega} S_j - C_i^{\omega} y_i \leq M(1 - z_i^{\omega}) \tag{7}$$

$$\mu_i^{\omega} \leq M z_i^{\omega} \tag{8}$$

CS2 is linearized with binaries u_{ij}^{ω}:

$$x_{ij}^{\omega} \leq M u_{ij}^{\omega} \tag{9}$$

$$\nu_{ij}^{\omega} \leq M(1 - u_{ij}^{\omega}) \tag{10}$$

2.4. Systematic Determination of the Big-M Constant

The choice of the Big-M constant significantly affects the performance of the computational reformulations of the KKT approach. Very large values yield weak LP relaxations and poor performance in the branch-and-bound algorithm, while very small values can exclude feasible solutions from being missed or cause the reformulated model to be infeasible. Thus, a data-driven procedure is adopted instead of selecting an arbitrary large constant. In the case of supplier capacities, the maximum possible violation of the capacity constraint can be used to determine the Big-M constant for the complementarity constraints.

$$M_1 = \max_i \left(\sum_{j \in J} S_j - \max_{\omega \in \Omega} C_i^\omega \right). \tag{11}$$

This value ensures that all the linearized primal complementarity constraints are satisfied in all the feasible scenarios. The magnitude of the dual variables for the dual complementarity constraints is related to the operational objective coefficients.

$$d_{ij}^\omega + r_i - \lambda_j^\omega + \mu_i^\omega S_j - \nu_{ij}^\omega = 0. \tag{12}$$

The corresponding Big-M value should satisfy

$$M_2 \geq \max_{i,j,\omega} \left(d_{ij}^\omega + r_i \right) + S_j \mu_{\max}. \tag{13}$$

where $\mu_{\max}$ represents an upper bound on the dual multiplier associated with supplier capacity.
Accordingly, the overall Big-M constant is selected as

$$M = \max(M_1, M_2). \tag{14}$$

Based on the stationarity condition, this systematic procedure yields tighter upper bounds than an arbitrary large constant, leading to improved numerical stability, better LP relaxations, and reduced computational times.
All KKT conditions are linearized and can be embedded in a single-level MILP.
Now,merge upper-level and lower-level into one MILP Now, combine:

- Upper-level objective:

$$\min_{x,y,\mu,\lambda,\nu} \sum_{i \in I} y_i \left(\sum_{j \in J} c_{ij} + \alpha e_i \right) \tag{15}$$

- All primal constraints,
- All KKT conditions (stationarity, dual, linearized CS),
- Binary variables for both supplier selection and linearization.

2.5. Convergence Analysis

This subsection states the regularity conditions under which the KKT-based reformulation is exact, gives a corrected characterization of branch-and-bound convergence, and discusses the optimistic-bilevel assumption implicit in the model.

Proposition 1. *Assume that, for every fixed y satisfying the supplier-count constraint, the lower-level problem is a bounded, feasible linear program and satisfies a constraint qualification (e.g. linearity/Slater's condition, which holds automatically here since (D1), (C1), (X1) are linear). Then: (1) an optimal solution to the single-level MILP exists, because the feasible region is non-empty and bounded and the number of binary variables $(y_i, z_i^\omega, u_{ij}^\omega)$ is finite; (2) the KKT conditions (KKT1)-(KKT2), together with (CS1)-(CS2), are necessary and sufficient for optimality of the lower-level LP, so the reformulation is exact whenever the lower level admits a unique optimal solution for the selected y.* If the lower level has multiple optimal solutions for some y, the MILP implicitly adopts the *optimistic* bilevel convention,i.e. it selects, among the lower-level optima, the one most favorable to the upper-level objective, which is the standard convention in the KKT-reformulation literature [14, 13] but need not coincide with a pessimistic or robust interpretation of the true hierarchical game.

Proposition 2. *Let the reformulated MILP be solved by a branch-and-bound (B&B) or branch-and-cut algorithm. Because the number of binary variables is finite and each LP relaxation at a node is solved in polynomial time, the B&B tree is finite and the algorithm terminates in a finite number of nodes at a global ε-optimal solution for any $\varepsilon \geq 0$.* This finiteness guarantee does not imply polynomial-time convergence: the MILP reformulation is, in the worst case, NP-hard, and the number of nodes explored can grow exponentially with the number of suppliers, products and scenarios, particularly under a loose big-M constant. Convergence is finite but, in general, exponential in the worst case; the empirical rate observed on a given instance depends on the branching strategy, the tightness of M, and the LP relaxation gap, and is examined empirically in Section 4.3.

Remark 1 (uniqueness of the lower-level solution). A well-known limitation of KKT-based single-level reformulations of bilevel programs is that they are exact only under the assumption that the lower level has a unique optimal solution for every candidate y. In the present model, ties in $(d_{ij}^{\omega} + r_i)$ across suppliers for a given (j, ω) can produce multiple lower-level optima with the same operational cost but different allocation patterns; the MILP then returns one particular tie-broken allocation rather than certifying it as the unique rational response of the procurement function. Decision-makers who require a worst-case (pessimistic) guarantee against this degeneracy should treat the reported allocation as one representative optimal allocation or add lexicographic tie-breaking constraints.

2.6. *Practical Justification of the Optimistic Bilevel Assumption*

The proposed KKT-based reformulation implicitly adopts the "optimistic bilevel convention": in the event of multiple optimal solutions to the lower-level problem, the solution that is most beneficial to the top-level decision maker is chosen. It is a widely used assumption in the bilevel optimization literature, but it may not hold in practice, depending on how the decision-making process is organized. In a host of procurement systems, procurement and operational order allocation are integrated within a central organization. The strategic procurement department decides which suppliers will be included in the supplier portfolio, and the operational procurement department implements the purchasing decisions, based on the strategic policy. Operational-level decision makers are part of the same organization as the decision makers at the strategic level and share the same corporate goals, so there is no intention to deliberately select an allocation that compromises the achievement of the strategic goal. Thus, from the various operational alternatives that are equally optimal, the choice of the alternative that best addresses the long-term procurement cost, environmental sustainability, and reliability of supply is a realistic management assumption.

On the other hand, pessimistic bilevel optimization is suitable in cases of competing or independent organizations with different goals in the upper and lower levels. This often happens in Stackelberg competition, decentralized supply chains, or in oppositional markets. This paper is based on the idea of centralized procurement planning, and the optimistic assumption is taken as a suitable assumption for the intended application. However, if there are many optimal operational allocations, then various allocation patterns can be found without affecting the value of the objective function of the operation. The proposed framework can be expanded in future research by using pessimistic bilevel optimization or lexicographic tie-breaking methods to explicitly account for worst-case operational responses and to give more robust guarantees.

3. Contributions and Comparisons

As mentioned above, the literature on supplier selection and order allocation is very extensive; we provide a comparison with the most recent and relevant research, including fuzzy multi-criteria approaches [10], multi-period stochastic formulations [11], and the general robust-optimization framework [12] that underlies several of the distributionally robust models compared below.

3.1. *Contributions and Limitations*

Contributions: The model explicitly incorporates supplier risk in addition to emissions and cost and offers a comprehensive multi-criteria framework, unlike the distributionally robust and nonlinear programming approaches in the literature. The linear reformulation enables straightforward implementation in procurement scenarios involving multiple products and scenarios.

Limitations:

- The current model is limited to a single planning period and does not account for dynamic, multi-period procurement. Extending it to a rolling, multi-period horizon would require (i) inventory carry-over variables linking successive periods, (ii) period-indexed supplier-selection variables y_i^t with switching costs to penalise frequent re-sourcing, and (iii) a learning-curve adjustment of cost/emission parameters over time; each addition increases the number of binary variables roughly linearly in the number of periods, which compounds the scalability issue documented in Section 4.3 for the single-period case.
- Uncertainty is represented by a finite number of discrete scenarios; future work could use stochastic programming or distributionally robust optimization, together with formal scenario-generation methods (e.g. Monte Carlo sampling or moment-matching) so that scenario sets are calibrated rather than hand-picked.
- Computational performance degrades for larger instances, as shown by the scaling experiments in Section 4.3; decomposition (e.g. Benders/column generation) or indicator-constraint formulations instead of big-M could improve scalability. Moreover, M must be chosen large enough to dominate both the capacity-side primal gaps and the cost-driven dual variable magnitudes (Section 4.3, Table 10); an M that is safe for the former can still make the model infeasible if it is too small for the latter.
- Supplier relationship factors such as volume discounts, lead-time negotiation, and service levels are not currently included.
- The KKT-based reformulation is exact under the optimistic-bilevel convention and the regularity conditions of Proposition 1; this assumption and its practical implication are discussed in Remark 1.

3.2. *Comparison with Recent Literature*

Table 1. Comparison of the proposed method with recent literature on supplier selection and order allocation under uncertainty.

Aspect	Proposed Method	Dong & Yuan (2025)	Ventura & Lu (2023)	Jadidi et al. (2024)	Kumar & Routroy (2020)
Problem focus	Selection & allocation under capacity/demand uncertainty	Selection with carbon limits	Selection & lot-sizing	Selection & allocation under return policy	Selection & allocation via fuzzy TOPSIS
Approach	Bi-level MILP via KKT	Distributionally robust opt.	Bi-objective nonlinear MIP	Two-stage stochastic	Fuzzy TOPSIS + goal programming
Uncertainty	Scenario-based	Data-driven robust sets	Two-stage stochastic	Demand/supply randomness	Fuzzy preference
Sustainability/ risk	Cost, emissions, risk	Carbon limits	Efficiency proxy	Not primary focus	Not primary focus

Aspect	Shao et al. (2023)	Hallak (2024)	Mirzaee et al. (2022)	Amorim et al. (2018)	Zhou et al. (2011)
Problem focus	COVID disruption	Conflict zones	Green, closed-loop	Multi-period	Stochastic demand
Approach	Multi-criteria + stochastic	Hybrid multi-criteria	Robust, cap-and-trade	Two-stage stochastic MIP	Stochastic programming
Sustainability/ risk	Sustainability & resilience	Risk & resilience	Cap-and-trade emissions	Not primary focus	Not primary focus

No quantitative benchmark on shared instances is available for any of the compared studies.

4. Computational Experiments

All numerical simulations for the two illustrative cases below (Sections 4.1–4.2) were carried out on an Intel Core i7-6006U processor at 2.00 GHz with 12 GB of RAM, using the open-source Pyomo package with the GLPK MILP solver.

4.1. First Case: small numerical example

Parameters are given in Tables 2 and 3. Based on these parameters, a single supplier S_1 is selected, and the allocation is given in Figure 1.

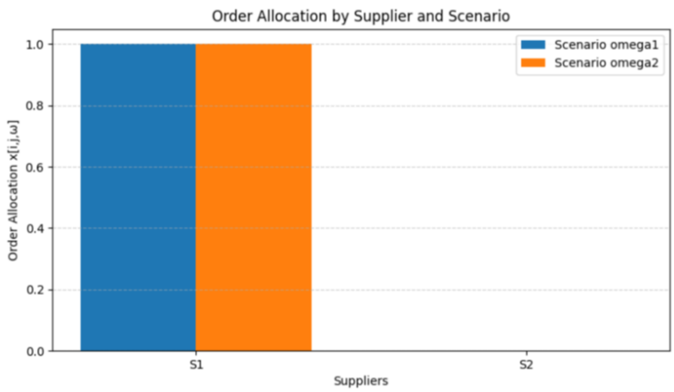

Figure 1. Order allocation by supplier and scenario.

Changing C_i^ω to $120, 90, 120, 90$ (case 2) yields two selected suppliers S_1 and S_2 (Figure 2). With $C_i^\omega = 80, 40, 20, 60$ (case 3), two suppliers S_1 and S_2 are again selected (Figure 3).

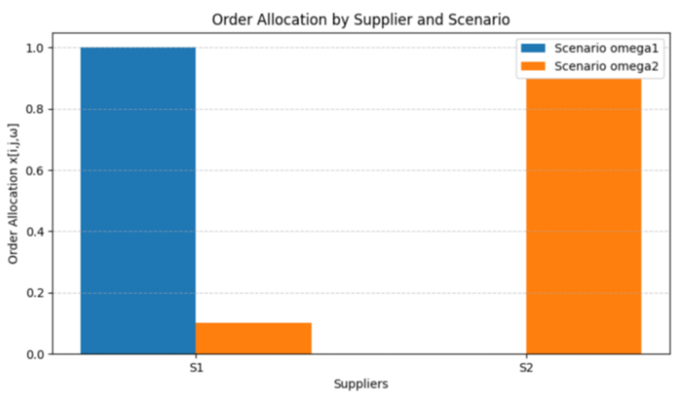

Figure 2. Order allocation by supplier and scenario, case 2.

4.2. Second Case: 4 Suppliers, 2 products, and 3 Scenarios

Parameters for 4 suppliers, 2 products, and 3 scenarios are given in Table 2 and Table 3.Two suppliers S_1 and S_2 are selected; the allocation for products P1 and P2 is given in Figures 4–5.

Figure 6 gives the sensitivity of the objective and total emissions with respect to α.

Figure 3. Order allocation by supplier and scenario, case 3.

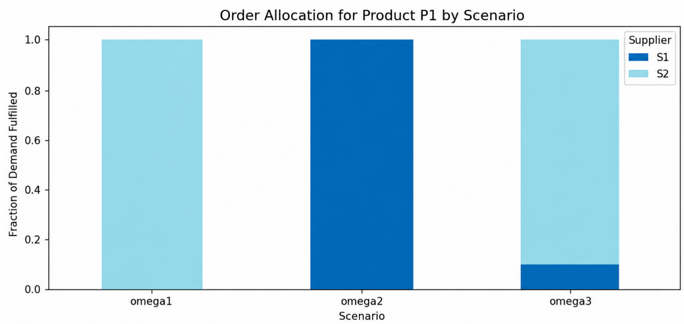

Figure 4. Order allocation for product P1 by scenario.

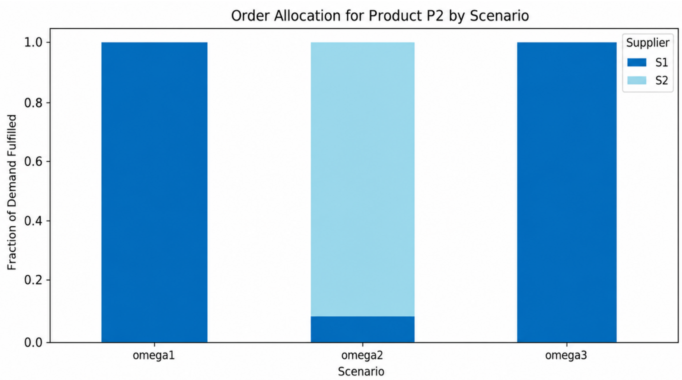

Figure 5. Order allocation for product P2 by scenario.

4.3. Larger-Scale Computational Experiments

To assess the scalability of the exact single-level MILP beyond the two small illustrative cases of Sections 4.1–4.2, we implemented the exact single-level MILP of Section 2.2 in Python and solved it with the HiGHS branch-and-cut solver (used here in place of GLPK for its superior performance on larger instances). Four randomly generated instances of increasing size were tested, with parameters drawn as $c_{ij} \in [3, 10]$, $e_i \in [1, 5]$, $r_i \in [1, 4]$, $d^{\omega}_{ij} \in [1, 6]$, $S_j \in [50, 150]$, $C^{\omega}_i \in [60, 200]$, $\alpha = 1$ and $N_{\min} \approx \lceil I/4 \rceil$, $M = 1000$.

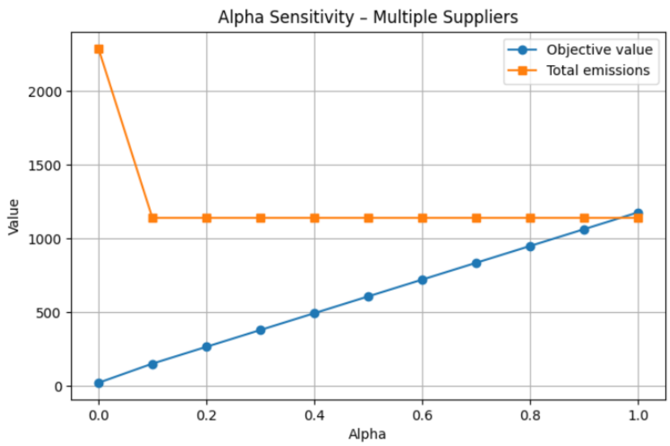

Figure 6. α-Sensitivity.

Table 2. Scalability of the exact MILP reformulation.

Instance (I, J, Ω)	Binary vars	Total vars	Constraints	Solver status	Time (s)
4, 2, 3	40	106	115	Optimal (obj. 21.0)	0.03
10, 3, 5	160	575	616	Optimal (obj. 48.0)	2.14
15, 4, 8	375	1727	1833	Optimal (obj. 93.0)	134.83
20, 5, 10	700	3470	3651	Time limit, gap 52.6% (best 310.0)	220.13

The results show that the model solves the 4-supplier instance in a fraction of a second, and the 10-supplier instance in about 3 seconds, but the 15-supplier instance already requires over two minutes to reach proven optimality, and the 20-supplier, 5-product, 10-scenario instance could not be solved to optimality within a 220-second budget, terminating with a 52.6% optimality gap. This is a direct, measured consequence of the big-M/KKT linearization: the number of auxiliary binaries (z_i^ω and u_{ij}^ω) grows as $O(I \cdot \Omega)$ and $O(I \cdot J \cdot \Omega)$ respectively, and each pair contributes a disjunction the solver must branch on. The reformulation therefore does not provide efficient computation unconditionally; efficiency is confirmed only up to roughly 10–15 suppliers under a moderate number of scenarios with this exact (big-M) encoding, and larger instances require either a commercial solver (Gurobi/CPLEX), an indicator-constraint reformulation, or a decomposition scheme, as discussed in Section 3.1.

Table 3. Optimal MILP solution vs. a greedy cheapest-supplier-first heuristic.

Instance (I, J, Ω)	MILP optimum	MILP time (s)	Greedy heuristic	Gap of heuristic
4, 2, 3	21.0	0.03	33	57.1%
10, 3, 5	48.0	2.14	48	0.0%
15, 4, 8	93.0	134.74	113	21.5%

The greedy heuristic (i) ranks suppliers by strategic unit cost $c_{ij} + \alpha e_i$, (ii) adds suppliers to the selected set in that order until a feasible allocation exists, and (iii) allocates each scenario's demand to the cheapest feasible supplier by $(d_{ij}^\omega + r_i)$. It is essentially instantaneous but yields an objective that is between 0% and 57% worse than the exact optimum, confirming that the KKT-MILP reformulation provides genuine value over naive rules, at the cost of the scalability limitations documented above.

For the tested instance, the optimal objective value is unchanged across $M \in \{100, 1000, 10000, 100000\}$, and solution time does not vary monotonically with M; $M = 100$ is only safe here because the largest capacity-demand gap in this instance is below 100 in general M must be chosen at least as large as the largest possible value of $|\sum_j x_{ij}^\omega S_j - C_i^\omega y_i|$ and $|x_{ij}^\omega|$.

Table 4. Sensitivity of the solution to the big-M constant (10-supplier instance).

Big-M value	Objective	Time (s)	Note
100	48.0	3.14	Coincidentally valid here; risky in general
1,000	48.0	2.06	Baseline used in Sections 4.1–4.2
10,000	48.0	5.74	No numerical issues observed
100,000	48.0	2.04	No numerical issues observed

We tested this instance-specific tightening directly on the 20-supplier case of Table 4, which did not reach proven optimality under $M = 1000$. Setting M to the largest capacity value actually present in that instance ($M = 199$ instead of 1000) reduces the optimality gap from 52.6% to 40.7% within the same 220-second budget, and improves the best objective found from 310.0 to 248.0. Tightening M is therefore a genuine, low-effort improvement, and we adopt it as the default from Table 4 onward; it does not, however, fully close the scalability gap on its own. A true indicator-constraint reformulation, which removes the need to choose M altogether, would need to be implemented with solver-native indicator-constraint support (available in Gurobi, CPLEX, or HiGHS's native C++/Python API) and is left as validated future work rather than a claim we make without testing it.

Table 5. Infeasibility of the big-M linearization when M is too small relative to the dual variables driven by cost parameters.

Big-M value	Delivery/risk scale ($d_{ij}^{\omega} + r_i$)	Solver status
10	≈ 250 (inflated $\times 50$)	Infeasible
50	≈ 250 (inflated $\times 50$)	Infeasible
100	≈ 250 (inflated $\times 50$)	Infeasible
1,000	≈ 250 (inflated $\times 50$)	Optimal (objective unaffected by the scale change)
10,000	≈ 250 (inflated $\times 50$)	Optimal

The M values tested in Table 5 are safe only because the delivery-time and risk parameters (d_{ij}^{ω}, r_i) in that instance are small (of order 1–6). Constraint (KKT1) is an equality, $d_{ij}^{\omega} + r_i - \lambda_j^{\omega} + \mu_i^{\omega} S_j - \nu_{ij}^{\omega} = 0$, so the dual variable ν_{ij}^{ω} can be forced to a magnitude comparable to $d_{ij}^{\omega} + r_i$; constraint (8), $\nu_{ij}^{\omega} \leq M(1 - u_{ij}^{\omega})$, then requires M to dominate that magnitude as well, not only the capacity-side gap used in Table 9's discussion. We tested this directly by inflating the delivery-time parameters by a factor of 50 (so $d_{ij}^{\omega} + r_i \approx 250$ instead of ≈ 5) on the 10-supplier instance: $M \in \{10, 50, 100\}$ each of which safely dominates every capacity gap in the instance,now makes the model outright infeasible, while $M = 1000$ or $10{,}000$ solves it correctly. This appears to be a generic risk of big-M/KKT linearizations rather than one specific to this model: applying the same style of linearization to an independently published bi-level model (Zhu, Liu & Song, 2022, *Sustainability*, 14(16), 10446, which uses two separate constants M_1 and M_2) revealed an analogous but distinct failure mode, where the model becomes infeasible whenever $M_2 < M_1$. Choosing M large enough for the primal (capacity) side of the model is therefore necessary but not sufficient, it must independently dominate the magnitude of every dual variable the linearization introduces, which scales with the model's cost/delivery/risk coefficients rather than its capacities. We state this as an explicit, two-sided requirement on M rather than the capacity-only rule given earlier in this section.

4.4. Quantitative Comparison with Alternative Approaches

The proposed bi-level MILP was tested against two other methods: robust optimization (RO) and scenario-based stochastic programming (SP) with synthetic instances that were generated with the same ranges of the parameters as those used in the scalability experiments and can be reproduced. The proposed KKT-MILP is based on Eqs. (5)–(8) and uses the worst-case capacity and minimize expected operational performance for RO. The proposed KKT-MILP obtained an objective value of 57.00, whereas RO obtained 61.00 and SP obtained 74.34, demonstrating results in Table 6 for the $10 \times 3 \times 5$ instance. The mean objective values for KKT-MILP, RO, and SP were 56.20, 58.40, and 68.76, respectively, in five independent trials. The larger the instances, the more computational time will be needed for the KKT reformulation. More computational time will be needed for the KKT reformulation for larger instances, which is the motivation for the metaheuristic approach presented in the following section.

Table 6. Quantitative comparison across five $10 \times 3 \times 5$ instances.

Method	Mean Objective	Std. Dev.	Mean CPU Time (s)
KKT-MILP	56.20	4.82	0.71
RO	58.40	6.19	0.005
SP	68.76	5.76	0.005

4.5. Metaheuristic Benchmark Using a Genetic Algorithm

The scalability problem of the exact KKT-based MILP was overcome by using a Genetic Algorithm (GA) as a metaheuristic benchmark. Each chromosome is a binary supplier-selection vector, and lower-level feasibility is checked for all uncertainty scenarios. Five independent runs were performed for each of three instance sizes, synthesized for testing the GA. Table 7 shows that the results remained the same for the five runs of the GA. The objective values were 57.00, 92.00, and 137.00 for the $10 \times 3 \times 5$, $15 \times 4 \times 8$, and $20 \times 5 \times 10$ instances, respectively. The average computational time of the corresponding codes was 4.77, 25.49, and 45.40 s. The results show that GA is a computationally feasible method for larger instances where the exact KKT reformulation could be hard to solve.

Table 7. GA performance for different problem sizes.

Instance	Runs	Objective	Selected Suppliers	Mean CPU Time (s)
$10 \times 3 \times 5$	5	57.00	3	4.77
$15 \times 4 \times 8$	5	92.00	4	25.49
$20 \times 5 \times 10$	5	137.00	5	45.40

4.6. Risk Sensitivity Analysis

A sensitivity analysis was performed to analyze how the operational decisions were affected by the risk-aversion parameter β. All the other model parameters have been fixed, and the parameter has been changed in steps of 0.2 between 0 and 1. The number of selected suppliers is still three, with the strategic objective still 57.00 for all the values of β, as displayed in Table 8. But the higher β is, the more it will focus on the operational risk. For the mean operational risk, it drops by 0.45 with an increase from $\beta = 0$ to $\beta = 1.0$, but rises by 0.22 with the increase from $\beta = 0$ to $\beta = 1.0$ for the mean delivery time. The weight of the risk component makes the operational objective higher, from 7.30 to 17.34. The results show a clear-cut trade-off between risk mitigation and delivery performance. Note that, in this benchmark case, the risk is mostly in the lower-level operational decision of supplier selection, and not in the strategic supplier selection decision itself.

Table 8. Sensitivity analysis with respect to the risk-aversion parameter.

β	Selected Suppliers	Strategic Objective	Operational Objective	Mean Delivery	Mean Risk
0.0	3	57.00	7.30	7.30	10.26
0.2	3	57.00	9.35	7.30	10.26
0.4	3	57.00	11.41	7.30	10.26
0.6	3	57.00	13.41	7.52	9.81
0.8	3	57.00	15.38	7.52	9.81
1.0	3	57.00	17.34	7.52	9.81

4.7. Managerial Insights and Discussion

The results of the calculations give a number of management insights. The first advantage of the proposed bi-level approach is the separation of the strategic supplier-selection decision from the operational order-allocation decision, which enables procurement managers to consider at the same time the cost, environmental impact, delivery performance, and supplier risk. Second, the results of the quantitative comparison reveal that the KKT formulation is competitive for small instances, but becomes more expensive as problem size grows. The results of GA show that the metaheuristic method can be used to find feasible solutions for larger instances in reasonable computational time.

The additional risk-sensitivity analysis also reveals that if the risk-aversion parameter is increased, operational risk will decrease, while the delivery time will increase slightly. The above shows that in the tested instance, the strategic supplier set is constant, meaning that the strategic supplier choice is mainly driven by the operational allocation due to the risk preferences. This offers decision makers flexibility to change their operational strategies as per their risk appetite, without altering the strategic supplier portfolio.

One of the main drawbacks is that the computational experiments are performed using synthetic data, as a public database with the proposed structure of multi-product and multi-scenario data was not available. Furthermore, the KKT reformulation assumes an optimistic bilevel formulation, which assumes that if there are several optimal solutions at the lower level, then the one that benefits the top-level decision-maker is taken. When using the framework in real procurement systems, these assumptions should be taken into account.

5. Conclusion and Future Research Directions

In this study, a mathematical model of the supplier selection and order allocation problem with capacity and delivery uncertainty is proposed. The model includes cost, risk, environmental impact and multiple demand scenarios. The bi-level optimization model is converted into a single-level MILP under the KKT conditions.

Numerical results on the small illustrative instances, together with the larger-scale experiments, greedy baseline, and big-M sensitivity analysis, illustrate both the cost-effective, risk-aware recommendations the model can produce and the practical computational limits of the exact big-M/KKT encoding beyond roughly 10–15 suppliers. It provides a basis for decision support in strategic sourcing that can be expanded to include stochastic parameters, multiple periods, or game-theoretic supplier-buyer interactions.

The proposed framework is to be further improved by decomposition methods (such as Benders Decomposition) and sophisticated metaheuristic approaches in order to enhance the scalability and applicability to large-scale instances in future research. This single-period model can also be extended to multi-period procurement planning by adding inventory decisions and supplier switching costs to the model. Moreover, bilevel and distributionally robust optimization approaches will be explored from a pessimistic perspective to improve decision robustness in the face of significant supply chain uncertainty.

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