

A Three-Stage Mixed-Integer Convex Optimization Framework for Capacitor Bank Allocation in Distribution Systems

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Abstract This paper presents a mixed-integer convex optimization framework for the optimal placement and sizing of capacitor banks in radial distribution networks. The proposed formulation employs a linearized power flow representation with a convex quadratic objective and linear and second-order cone constraints in the complex domain, implemented through a three-stage methodology: initial exact power flow, convex optimization for capacitor placement, and exact validation via power flow. The framework is evaluated on the 24-bus, 33-bus, and 69-bus test systems, demonstrating loss reductions of up to 179.49 kW (26.8%) for the 24-bus system, 72.65 kW (34.4%) for the 33-bus system, and 79.73 kW (35.4%) for the 69-bus system. The proposed method consistently matches or outperforms state-of-the-art metaheuristic approaches including the gravitational search algorithm, the two-stage method, the flower pollination algorithm, and the discrete vortex search algorithm. In all test systems, voltage profiles are improved to satisfy the 7% regulation criterion, with minimum voltages raised above 0.93 p.u. All cases are solved in less than 10 seconds on the 24-bus, 33-bus, and 69-bus systems, indicating the computational efficiency of the approach on the tested benchmarks.

Keywords Capacitor placement; reactive power compensation; mixed-integer convex optimization; radial distribution networks; loss reduction; voltage profile improvement

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Nomenclature

Binary Variables

$x_{k,c}$ Binary variable; 1 if a capacitor of size c is installed at node k , 0 otherwise

$x_{k,c}^*$ Optimal binary decision for capacitor installation

Parameters

ΔP_{loss} Absolute error between exact and approximated losses (p.u.)

δ_k Auxiliary shunt term at node k (p.u.)

$A_{k,l}$ Incidence matrix element

N_{cap} Maximum number of capacitor banks allowed in the network

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P_k^D	Active power demand at node k (p.u.)
P_{loss}	Total real power losses (p.u.)
$P_{\text{loss}}^{(0)}$	Initial power losses from exact power flow (p.u.)
$P_{\text{loss}}^{\text{approx}}$	Approximated power losses from convex model (p.u.)
$P_{\text{loss}}^{\text{exact}}$	Exact power losses from validation power flow (p.u.)
Q_c	Reactive power injection of capacitor size c (p.u.)
Q_k^D	Reactive power demand at node k (p.u.)
R_l	Resistance of branch l (p.u.)
s	Index of the slack node (substation)
S_b	Base apparent power (kVA)
$S_k^{D,\text{mod}}$	Modified complex demand at node k with capacitors (p.u.)
S_l^{max}	Maximum apparent power magnitude on branch l (p.u.)
V_b	Base voltage (kV)
X_l	Reactance of branch l (p.u.)
Y_{km}	Element of the nodal admittance matrix

Sets and Indices

$\mathcal{C}$	Set of available discrete capacitor sizes, indexed by c
$\mathcal{L}$	Set of branches (lines), indexed by l
$\mathcal{N}$	Set of nodes (buses), indexed by k

Continuous Variables

P_k^G	Real part of the complex power injected by the substation at node k (p.u.)
P_l	Real part of the complex power flow on branch l (p.u.)
Q_k^G	Imaginary part of the complex power injected by the substation at node k (p.u.)
Q_l	Imaginary part of the complex power flow on branch l (p.u.)
S_k^G	Complex power injected by the substation at node k (p.u.)
$S_k^{G,\text{exact}}$	Exact substation power injection from validation power flow (p.u.)
S_l	Complex power flow on branch l (p.u.)
V_k	Complex voltage at node k (p.u.)
V_k^{exact}	Exact voltage at node k from validation power flow (p.u.)

1. Introduction

The global energy landscape is undergoing an unprecedented transformation driven by the urgent need to decarbonize electricity generation, enhance energy security, and mitigate the adverse effects of climate change [1, 2]. The Paris Agreement and subsequent international commitments have accelerated the transition from fossil-fuel-based power generation to renewable energy sources, with many countries setting ambitious targets for carbon neutrality by mid-century [3, 4]. Among the key strategies to achieve these objectives, the integration of distributed energy resources (DERs) into active distribution networks (ADNs) has emerged as a promising solution. Photovoltaic systems, wind turbines, and energy storage systems are being increasingly deployed at the distribution level, enabling greater local generation and reducing transmission losses [5, 6]. However, this integration introduces new operational challenges, including increased network losses, voltage deviations, reverse power flows, and power quality issues [7, 8]. The intermittent and variable nature of renewable generation further complicates network operation, requiring advanced planning and control strategies to maintain system stability and reliability [9].

In this context, capacitor banks play a fundamental role in reactive power compensation, loss reduction, and voltage profile improvement [9, 10]. By providing localized reactive power support, capacitor banks reduce the current flowing through distribution lines, thereby decreasing I^2R losses and alleviating voltage drops along feeders [11]. Additionally, they contribute to power factor correction, relieving stress on substation transformers and improving overall system efficiency. The strategic placement and sizing of capacitor banks is therefore a critical planning problem for distribution system operators, with significant implications for operational costs, asset utilization, and service quality [12, 13].

The optimal placement and sizing of capacitor banks in distribution networks has been extensively studied in the literature over the past several decades [14, 15]. Various optimization techniques have been proposed, ranging from classical analytical methods to metaheuristic and mathematical programming approaches. Early works employed analytical methods based on loss sensitivity indices and heuristic rules to identify candidate locations for capacitor installation [16]. While computationally efficient, these methods often fail to capture the global interactions between multiple capacitors and may lead to suboptimal solutions. Classical optimization techniques, including dynamic programming and mixed-integer nonlinear programming, have also been applied to the problem, but they suffer from high computational complexity and convergence issues, particularly for large-scale networks.

The advent of metaheuristic algorithms has provided new avenues for solving the capacitor placement problem. Population-based methods such as the gravitational search algorithm (GSA) [17], the flower pollination algorithm (FPA) [18], and the discrete vortex search algorithm (DVSA) [16] have been successfully applied to capacitor placement problems, demonstrating their ability to explore complex search spaces and identify near-optimal solutions. Other metaheuristics, including genetic algorithms, bacterial foraging optimization [19], and differential evolution, have also been widely employed. Two-stage approaches have been proposed to improve solution quality, where a first stage identifies candidate locations using sensitivity analysis and a second stage optimizes the sizing of selected capacitors [20]. However, metaheuristic methods do not guarantee global optimality and may require extensive parameter tuning to achieve satisfactory performance. Their stochastic nature also makes results difficult to reproduce across different runs, which is a significant drawback for practical applications where reliability and consistency are paramount.

To overcome these limitations, convex relaxation techniques have gained significant attention in recent years. By reformulating the nonlinear power flow equations into convex forms, these methods enable the use of efficient deterministic solvers that can guarantee global optimality of the relaxed problem. Second-order cone programming (SOCP) and semidefinite programming (SDP) relaxations of the optimal power flow problem have been successfully applied to various distribution system optimization problems, including reactive power dispatch and capacitor placement [21]. However, these approaches typically require the incorporation of integer variables to model the discrete nature of capacitor banks, resulting in mixed-integer convex formulations that remain challenging to solve despite their theoretical advantages. Furthermore, many existing convex formulations rely on linearized loss approximations that introduce errors in the power flow calculations. In particular, the approximation $R_l|S_l|^2$ used to estimate line losses assumes near-unity voltage magnitudes, which may not hold under heavy loading or significant voltage drops. These approximation errors can lead to suboptimal capacitor placement

decisions and inaccurate loss estimates if not properly addressed. Consequently, any solution obtained from such convex formulations requires post-processing validation through exact nonlinear power flow analysis to ensure practical feasibility and accurate performance assessment.

The main motivation of this work is therefore to provide a computationally tractable planning tool that combines the efficiency and deterministic convergence of convex optimization with the physical accuracy of the exact nonlinear power flow. Rather than proposing a fundamentally new convex relaxation—since branch-flow SOCP models have been studied for over a decade—we build on these established formulations and add a validation stage that explicitly addresses their approximation error. This positions the paper as an application-oriented engineering contribution, where the novelty lies in the systematic integration of the three stages and in the explicit quantification of the approximation error, rather than in the convex formulation itself. We also note that the present study is a planning tool under steady-state balanced conditions; extensions to unbalanced phases, time-varying loads, and uncertainty are identified as future work rather than claims of the present paper.

The main contributions of this paper are as follows:

1. A mixed-integer convex optimization formulation for capacitor placement and sizing, expressed directly in the complex domain using branch flow variables, resulting in a model with a convex quadratic objective and linear and second-order cone constraints. The formulation builds on well-established SOCP relaxations and is presented here in a form that is convenient for implementation in modern modeling languages.
2. A three-stage solution methodology comprising: (i) an initial exact power flow to establish base case conditions; (ii) a convex optimization stage to determine the capacitor placement and sizing that minimizes the approximate losses; and (iii) a final exact power flow validation to quantify and account for the approximation error introduced by the convex model. The three-stage structure is the principal engineering contribution of this work.
3. An explicit quantification of the approximation error between the convex model and the exact power flow. The error is reported for each test case, and it is shown that the exact validation stage provides an accurate estimate of the true losses and voltage profiles for the selected capacitor configuration.
4. Numerical evaluation on the 24-bus, 33-bus, and 69-bus test systems, demonstrating loss reductions of up to 26.8% for the 24-bus system with five capacitors, 34.4% for the 33-bus system, and 35.4% for the 69-bus system. These values are consistent with the results reported in the literature for the same test feeders.
5. Comparison with state-of-the-art metaheuristic methods, including GSA, FPA, DVSA, and a two-stage method, showing that the proposed approach achieves results that match or slightly improve upon the best reported values while providing deterministic convergence and reproducible solutions.
6. Voltage profile improvements satisfying the 7% voltage regulation criterion across the tested systems, with minimum voltages raised above the 0.93 p.u. threshold.
7. Demonstrated computational efficiency on the tested systems, with all cases solved in less than 10 seconds. This indicates the potential of the approach for planning studies, although scalability to very large networks has not been demonstrated and is identified as a direction for future work.

The scope of this paper is limited to radial distribution networks under steady-state balanced conditions. The proposed formulation assumes balanced three-phase systems and uses a single-phase equivalent representation. Capacitor banks are modeled as discrete reactive power injections with a finite set of commercially available sizes. The methodology is designed for planning studies, where the objective is to determine the optimal location and sizing of permanent capacitor installations. Dynamic or time-varying operation, including switching of capacitor banks, unbalanced three-phase operation, and uncertainty in load and generation, is not considered in this work. While the framework is validated on standard test feeders, its applicability to larger and more complex networks remains an area for future investigation. The three-stage methodology ensures that the approximations inherent in the convex formulation do not compromise the practical feasibility of the solutions, as the final exact validation stage confirms that the selected capacitor configuration satisfies the full nonlinear power flow equations.

The remainder of this paper is organized as follows. Section 2 presents the mathematical formulation of the mixed-integer convex optimization model, including the objective function, constraints, and a discussion of its convexity properties. Section 3 describes the proposed three-stage solution methodology. Section 4 introduces the

test systems used for numerical evaluation. Section 5 presents and discusses the numerical results. Finally, Section 6 concludes the paper with a summary of findings and directions for future research.

2. Mathematical Formulation

The optimal placement and sizing of capacitor banks in a radial distribution network is formulated as a mixed-integer convex optimization problem [21]. All quantities are expressed in per-unit on a common base apparent power S_b and base voltage V_b . The proposed model employs complex power flow variables directly, preserving the natural structure of the power flow equations while maintaining linearity in the constraints [22].

2.1. Objective Function

The objective is to minimize the total real power losses in the network. These losses are approximated using the branch flow variables, under the assumption that nodal voltages remain close to 1.0 p.u., so that the current magnitude on branch l is approximately $|S_l|$ and the loss is $R_l |I_l|^2 \approx R_l |S_l|^2$:

$$\min \sum_{l \in \mathcal{L}} R_l |S_l|^2. \quad (1)$$

The quantity $|S_l|^2$ is real-valued and the objective is therefore a convex quadratic function of the complex variables. It is the only strictly quadratic expression in the formulation. We emphasize that this expression is an approximation of the real line losses, and that the exact losses are computed in the third stage of the methodology.

2.2. Constraints

2.2.1. Nodal Power Balance For every node $k \in \mathcal{N}$, the net complex power leaving the node through branch flows and substation injection must equal the local demand minus the reactive power injected by the capacitor installed at that node, plus the auxiliary shunt term [22]:

$$\sum_{l \in \mathcal{L}} A_{k,l} S_l + S_k^G = P_k^D + j \left(Q_k^D - \sum_{c \in \mathcal{C}} Q_c x_{k,c} \right) + \delta_k, \quad (2)$$

where $j^2 = -1$. The sign convention is as follows. Positive S_l denotes power flowing from the sending end to the receiving end of branch l . Positive S_k^G denotes power injected into the network by the substation. The capacitor term $-jQ_c x_{k,c}$ represents reactive power injected into the network at node k , which reduces the net reactive demand. This convention is consistent with the modification applied in the validation stage, where the capacitor is treated as a negative reactive load.

The auxiliary term δ_k is a small positive constant added to the active power balance for numerical conditioning. It represents a fictitious shunt conductance at nodes with zero active power demand, and it does not have a physical interpretation in the network. Its value is set to $1/S_b$ in per-unit terms, which is small relative to the total system demand and does not affect the optimal capacitor decisions. This term can be removed at the cost of a slight degradation of the numerical conditioning of the solver.

2.2.2. Substation Injection Only the slack node is permitted to inject power from the external grid; all other nodes have zero generation [18]:

$$S_k^G = 0 \quad \forall k \in \mathcal{N} \setminus \{s\}. \quad (3)$$

The slack node is connected to the infinite bus; its voltage magnitude and angle are fixed, but its complex power injection S_s^G remains a free variable to be determined by the optimization.

2.2.3. Capacitor Installation Limits At most one capacitor can be installed at each node, and no capacitor can be placed at the slack node [21]:

$$\sum_{c \in \mathcal{C}} x_{k,c} \leq 1 \quad \forall k \in \mathcal{N} \setminus \{s\}, \quad (4)$$

$$\sum_{c \in \mathcal{C}} x_{s,c} = 0. \quad (5)$$

These constraints are linear inequality and equality restrictions on the binary variables, ensuring that each node can host at most one capacitor bank.

2.2.4. Total Number of Capacitors The total number of installed capacitor banks is bounded by a predefined limit, reflecting practical budget or physical constraints [16]:

$$\sum_{k \in \mathcal{N}} \sum_{c \in \mathcal{C}} x_{k,c} \leq N_{\text{cap}}. \quad (6)$$

2.2.5. Branch Power Flow Limits To prevent line overloading, the magnitude of the apparent power flow on each branch is constrained [22]:

$$|S_l| \leq S_l^{\max} \quad \forall l \in \mathcal{L}. \quad (7)$$

This constraint is equivalent to $\sqrt{\text{Re}(S_l)^2 + \text{Im}(S_l)^2} \leq S_l^{\max}$, which defines a second-order cone. This constraint is nonlinear but convex. We therefore clarify that, contrary to the statement in the abstract of the original submission, the constraints of the model are not all linear: the branch flow limits are second-order cone constraints. The linearity statement applies only to the nodal power balance and the capacitor-related constraints.

2.3. Real-Valued Formulation for Standard Solvers

For solvers that do not natively support complex variables, the formulation can be transformed into an equivalent real-valued representation. Let $S_l = P_l + jQ_l$ and $S_k^G = P_k^G + jQ_k^G$, where $P_l, Q_l \in \mathbb{R}$ and $P_k^G, Q_k^G \in \mathbb{R}$. The nodal power balance in Eq. (2) can then be decomposed into two real linear constraints [23]:

$$\sum_{l \in \mathcal{L}} A_{k,l} P_l + P_k^G = P_k^D + \delta_k, \quad (8)$$

$$\sum_{l \in \mathcal{L}} A_{k,l} Q_l + Q_k^G = Q_k^D - \sum_{c \in \mathcal{C}} Q_c x_{k,c}. \quad (9)$$

The objective function in Eq. (1) becomes:

$$\min \sum_{l \in \mathcal{L}} R_l (P_l^2 + Q_l^2). \quad (10)$$

The branch flow limits in Eq. (7) become:

$$\sqrt{P_l^2 + Q_l^2} \leq S_l^{\max} \quad \forall l \in \mathcal{L}. \quad (11)$$

In this real-valued form, the problem is a Mixed-Integer Second-Order Cone Program (MISOCP), since the objective is a convex quadratic function and the branch flow limits are second-order cone constraints. The remaining constraints are linear.

2.4. Discussion

The convexity structure of the proposed model can be summarized as follows:

- The nodal power balance in Eqs. (8)–(9) and the capacitor-related constraints in Eqs. (4)–(6) are strictly linear.
- The branch flow limits in Eq. (11) are convex second-order cone constraints.
- The objective function in Eq. (10) is a convex quadratic expression.

Consequently, the overall problem is a mixed-integer convex optimization model with a quadratic objective and linear and second-order cone constraints [21]. In its complex-domain form, it is a Mixed-Integer Quadratically Constrained Program (MIQCP). When transformed into real variables, it becomes a Mixed-Integer Second-Order Cone Program (MISOCP) [24]. In either representation, the problem benefits from the branch-and-cut algorithms implemented in commercial solvers such as Gurobi or CPLEX [25].

We clarify that the global-optimality guarantee provided by these solvers applies to the mixed-integer convex approximation of Eq. (10)–(11), and not to the original nonlinear capacitor placement problem. The original problem remains non-convex, and the convex model introduces an approximation in the loss calculation. The purpose of the third stage of the methodology is to quantify the error introduced by this approximation and to ensure that the selected capacitor configuration is physically feasible under the exact nonlinear power flow.

After the optimal binary decisions $x_{k,c}$ are determined, a full nonlinear AC power flow is executed to verify the exact voltage profile and compute the true losses for the selected capacitor configuration. In the numerical implementation, the auxiliary shunt term δ_k is set to $1/S_b$ for nodes with zero active power demand to improve numerical stability. The line flow limits $S_l^{\max}$ are enforced through the parameter $y0_l$ in the code, which serves as a placeholder for the line thermal rating.

3. Proposed Solution Methodology

The proposed solution strategy for the optimal placement and sizing of capacitor banks in distribution networks consists of three sequential stages. The first stage computes the initial operating point of the network using the exact nonlinear power flow formulation [26]. The second stage solves the mixed-integer convex optimization model to determine the optimal capacitor placement and sizing [25]. Finally, the third stage validates the obtained solution by performing an exact power flow with the selected capacitors installed.

3.1. Stage 1: Initial Exact Power Flow

The first stage establishes the base case operating conditions of the distribution network using the full nonlinear AC power flow equations [27]. This stage is essential for obtaining the initial losses and voltage profile, which serve as reference values for evaluating the effectiveness of the capacitor placement.

The exact power flow is formulated as a system of complex equations representing the nodal power balance:

$$S_k^G - S_k^D = V_k \sum_{m \in \mathcal{N}} Y_{km}^* V_m^* \quad \forall k \in \mathcal{N}, \quad (12)$$

where $V_k \in \mathbb{C}$ is the voltage at node k , $S_k^G \in \mathbb{C}$ is the power injection from the substation (nonzero only at the slack node), $S_k^D = P_k^D + jQ_k^D$ is the demand at node k , and Y_{km} is the element of the nodal admittance matrix corresponding to nodes k and m . The slack node voltage is fixed at $V_s = 1.0 \angle 0^\circ$ p.u.

The power losses are computed as:

$$P_{\text{loss}} = \text{Re} \left\{ \sum_{k \in \mathcal{N}} V_k \sum_{m \in \mathcal{N}} Y_{km}^* V_m^* \right\}. \quad (13)$$

This nonlinear system is solved using the primal-dual interior point method implemented in Ipopt. The initial point for the solver is a flat start with all voltage magnitudes set to 1.0 p.u. and all angles set to zero. The

convergence tolerance is the default Ipopt tolerance. Since the base case is a well-conditioned power flow problem, no failure handling is required in the tested cases; in a general implementation, the solver would return an error flag if convergence is not achieved within the iteration limit.

3.2. Stage 2: Convex Optimization for Capacitor Placement

The second stage solves the mixed-integer convex optimization model formulated in Section 2. This model determines the location and size of capacitor banks that minimize the approximated power losses while satisfying all operational constraints [22].

The optimization problem is implemented in Julia using the JuMP modeling language and solved with the Gurobi optimizer. The complete formulation is:

$$\begin{aligned}
 \min \quad & \sum_{l \in \mathcal{L}} R_l |S_l|^2 \\
 \text{s.t.} \quad & \sum_{l \in \mathcal{L}} A_{k,l} S_l + S_k^G = P_k^D + j \left(Q_k^D - \sum_{c \in \mathcal{C}} Q_c x_{k,c} \right) + \delta_k, \quad \forall k \in \mathcal{N}, \\
 & S_k^G = 0, \quad \forall k \in \mathcal{N} \setminus \{s\}, \\
 & \sum_{c \in \mathcal{C}} x_{k,c} \leq 1, \quad \forall k \in \mathcal{N} \setminus \{s\}, \\
 & \sum_{c \in \mathcal{C}} x_{s,c} = 0, \\
 & \sum_{k \in \mathcal{N}} \sum_{c \in \mathcal{C}} x_{k,c} \leq N_{\text{cap}}, \\
 & |S_l| \leq S_l^{\max}, \quad \forall l \in \mathcal{L}, \\
 & x_{k,c} \in \{0, 1\}, \quad \forall k \in \mathcal{N}, c \in \mathcal{C}.
 \end{aligned} \tag{14}$$

In this formulation, the binary variables $x_{k,c}$ determine the installation decisions, while the continuous variables S_l and S_k^G represent the branch flows and substation injection, respectively. The objective function is a convex quadratic approximation of the real power losses, and all constraints are linear except for the branch flow limits, which are second-order cone constraints. When expressed in real variables, this problem is a Mixed-Integer Second-Order Cone Program (MISOCP), which can be solved using Gurobi's branch-and-cut algorithm [28].

The Gurobi solver is used with its default settings. The default optimality tolerance and feasibility tolerance are used for the mixed-integer optimization. No custom branching rules, cuts, or time limits are imposed. Under these settings, Gurobi returns a solution and reports whether the optimality gap has been closed; in the tested cases, the solver terminates with the default optimality tolerance and reports that the solution is optimal within that tolerance. For practical use, we note that solver settings such as the MIP gap and the time limit can be adjusted to trade off optimality against computation time, and that a systematic study of these settings is identified as a direction for future work.

The solution of this stage provides:

- The location of capacitor banks (nodes k where $x_{k,c} = 1$).
- The sizing of each installed capacitor (value Q_c corresponding to the selected size).
- The approximated power losses $P_{\text{loss}}^{\text{approx}}$.
- The optimal branch power flows S_l .

3.3. Stage 3: Exact Validation via Power Flow

The third stage validates the solution obtained from the convex optimization model by performing an exact nonlinear AC power flow with the selected capacitor banks installed. This stage does not feed back into the optimization; it is a validation step that provides the exact losses and voltage profile for the selected configuration,

and that quantifies the approximation error introduced by the convex model. A closed-loop iterative scheme in which the validation results are used to update the convex model is identified as a direction for future work.

The capacitor banks are modeled as constant reactive power injections at their respective nodes. The modified nodal demand at node k becomes:

$$S_k^{D,\text{mod}} = S_k^D - j \sum_{c \in \mathcal{C}} Q_c x_{k,c}^*, \quad (15)$$

where $x_{k,c}^*$ are the optimal binary decisions obtained from Stage 2.

The exact power flow is then solved using the same nonlinear formulation as in Stage 1:

$$S_k^G - S_k^{D,\text{mod}} = V_k \sum_{m \in \mathcal{N}} Y_{km}^* V_m^* \quad \forall k \in \mathcal{N}, \quad (16)$$

with the slack node voltage fixed at $V_s = 1.0 \angle 0^\circ$ p.u.

The solution of this final power flow provides:

- The exact voltage profile V_k^{exact} for the network with capacitors installed.
- The exact power losses $P_{\text{loss}}^{\text{exact}}$.
- The actual substation power injection $S_k^{G,\text{exact}}$.

The difference between the approximated losses from Stage 2 and the exact losses from Stage 3 quantifies the accuracy of the convex approximation:

$$\Delta P_{\text{loss}} = |P_{\text{loss}}^{\text{exact}} - P_{\text{loss}}^{\text{approx}}|. \quad (17)$$

3.4. Discussion of the Methodology

The proposed three-stage methodology offers the following properties.

- The initial exact power flow provides a reference for evaluating the performance improvement achieved by the capacitor installation.
- The convex optimization stage determines the capacitor placement and sizing using a computationally tractable model that can be solved to global optimality within the convex approximation.
- The final exact validation guarantees that the solution is consistent with the full nonlinear power flow equations, ensuring practical feasibility and providing an accurate estimate of the true losses and voltage profile.

In the Julia implementation, the first and third stages are solved using Ipopt, which handles the nonlinear power flow equations effectively [29]. The second stage is solved using Gurobi, which provides state-of-the-art performance for mixed-integer convex optimization problems [25]. All stages are implemented using the JuMP modeling language, which allows seamless integration of different solvers within a unified framework.

Remark 1

In Julia, the optimization problem in Stage 2 can be implemented directly in the complex domain using JuMP. The nodal power balance constraints are linear in the complex variables S_l and S_k^G , while the objective function $\sum R_l |S_l|^2$ is a real-valued convex quadratic function. Therefore, when the branch flow limits are expressed as $|S_l|^2 \leq (S_l^{\text{max}})^2$, the problem is treated as a Mixed-Integer Quadratic Program (MIQP) with linear and second-order cone constraints, which can be efficiently solved using standard solvers such as Gurobi or CPLEX.

The overall methodology ensures that the capacitor placement solution is computationally efficient on the tested benchmarks and physically feasible under the exact power flow. The main limitation of the methodology is that the validation stage does not feed back into the convex optimization. Consequently, if the approximation error is large, the capacitor configuration selected by the convex model may not be optimal for the original nonlinear problem. In the tested systems, the approximation error is small, as reported in Section 5, and therefore the configuration selected by the convex model is expected to be close to the optimal configuration of the nonlinear problem. A closed-loop scheme that iteratively updates the convex model based on the validation results would provide a stronger guarantee and is identified as a direction for future work.

4. Test Feeders

The proposed optimization framework is evaluated on three well-known radial distribution test feeders: the 24-bus system [22], the 33-bus system [18], and the 69-bus system [16]. These benchmarks are widely used in the literature for validation of power system optimization methods. They do not represent real distribution networks with unbalanced phases or time-varying loads, and the results should be interpreted accordingly. All systems are tested under balanced steady-state conditions, consistent with the scope of this work. The following subsections describe each test feeder in detail.

4.1. The 24-Bus Test Feeder

The 24-bus system, illustrated in Figure 1, is a standard benchmark whose topology and parameters were previously reported in [22]. It comprises one substation (bus 1) and 23 load buses, with a nominal line-to-line voltage of 11.4 kV. The network includes 41 candidate branches: 23 normally closed branches and 18 normally open tie branches. The complete line and load data are reproduced in Table 1 for self-containment. The node numbering follows the convention used in the source publication.

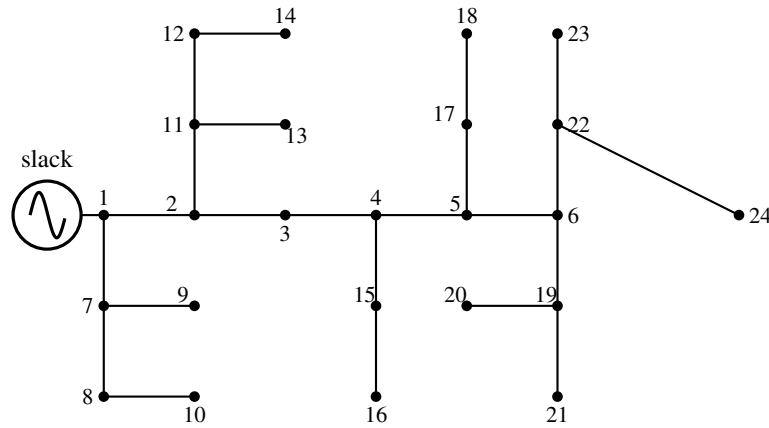


Figure 1. Topology of the 24-bus test feeder

Table 1. Line and load data for the 24-bus test feeder

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0922	0.0470	145	100	12	14	0.3536	0.3333	700	625
2	3	0.3930	0.2512	175	120	4	15	0.2125	0.3895	600	500
3	4	0.2661	0.1864	400	325	15	16	0.3654	0.3456	650	600
4	5	0.1725	0.1435	700	600	5	17	0.2224	0.2550	500	400
5	6	0.1899	0.4114	325	200	17	18	0.3587	0.3356	600	500
1	7	0.2565	0.3624	0	-900	6	19	0.3025	0.3486	750	500
7	8	0.1416	0.1359	500	400	19	20	0.2986	0.1457	450	300
7	9	0.1598	0.1624	800	750	19	21	0.1954	0.1854	0	-1750
8	10	0.3158	0.2886	900	700	6	22	0.1489	0.1875	600	750
2	11	0.2689	0.3154	850	600	22	23	0.3589	0.1752	850	800
11	12	0.2416	0.2548	0	-1250	22	24	0.2222	0.2758	650	400
11	13	0.3050	0.2159	350	300	—	—	—	—	—	—

4.2. The 33-Bus Test Feeder

The 33-bus system is a widely used radial distribution test feeder originally proposed in [18]. It consists of 33 nodes (1 substation and 32 load buses) and 32 branches, all normally closed. The single-line diagram is shown in Figure 2, and the complete line and load data are provided in Table 2. This system has a total load of 3,715 kW and 2,300 kvar.

4.3. The 69-Bus Test Feeder

The 69-bus system is a larger radial distribution test feeder with 69 nodes (1 substation and 68 load buses) and 68 branches. The single-line diagram is shown in Figure 2, and the complete line and load data are provided in Table 2. This system has a total load of approximately 3,801 kW and 2,694 kvar.

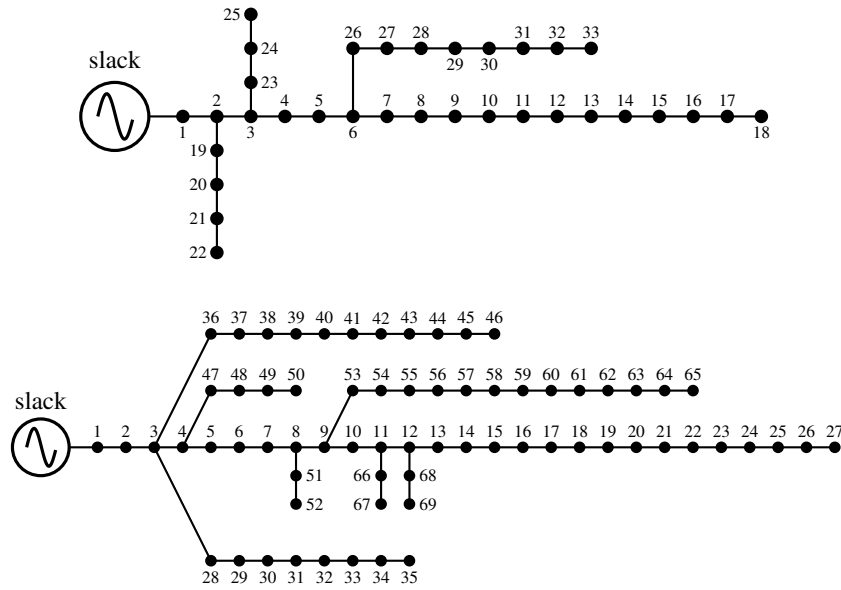


Figure 2. Single-line diagrams of the 33-bus (top) and 69-bus (bottom) test feeders

5. Numerical Results

The proposed mixed-integer convex optimization framework for optimal capacitor placement and sizing is numerically evaluated using the three test feeders described in Section 4. All simulations are executed in Julia 1.10 with the JuMP modeling framework. The convex problems are solved using Gurobi 11.0, and the nonlinear power flow problems of Stages 1 and 3 are solved using Ipopt. The computational hardware consists of an AMD Ryzen 7 3700 processor (2.3 GHz) and 16 GB of RAM; the operating system is Windows 10 64-bit. These details are reported for reproducibility. Gurobi is used with its default settings; no custom tolerances, cuts, or branching rules are imposed.

The available capacitor sizes are discretized into 20 options. For the 24-bus system, sizes range from 150 kvar to 3,000 kvar in increments of 150 kvar. For the 33-bus and 69-bus systems, sizes range from 100 kvar to 2,000 kvar in increments of 100 kvar, consistent with the literature used for comparison.

5.1. Analysis of the 24-Bus Test Feeder

The 24-bus system is used to illustrate the effectiveness of the proposed approach by evaluating the impact of the maximum number of installed capacitor banks on system performance. The parameter N_{cap} is varied from 0 to 5,

Table 2. Branch and load data for the 33-bus and 69-bus test feeders

The 33-bus grid											
Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0922	0.0477	100	60	17	18	0.7320	0.5740	90	40
2	3	0.4930	0.2511	90	40	2	19	0.1640	0.1565	90	40
3	4	0.3660	0.1864	120	80	19	20	1.5042	1.3554	90	40
4	5	0.3811	0.1941	60	30	20	21	0.4095	0.4784	90	40
5	6	0.8190	0.7070	60	20	21	22	0.7089	0.9373	90	40
6	7	0.1872	0.6188	200	100	3	23	0.4512	0.3083	90	50
7	8	1.7114	1.2351	200	100	23	24	0.8980	0.7091	420	200
8	9	1.0300	0.7400	60	20	24	25	0.8960	0.7011	420	200
9	10	1.0400	0.7400	60	20	6	26	0.2030	0.1034	60	25
10	11	0.1966	0.0650	45	30	26	27	0.2842	0.1447	60	25
11	12	0.3744	0.1238	60	35	27	28	1.0590	0.9337	60	20
12	13	1.4680	1.1550	60	35	28	29	0.8042	0.7006	120	70
13	14	0.5416	0.7129	120	80	29	30	0.5075	0.2585	200	600
14	15	0.5910	0.5260	60	10	30	31	0.9744	0.9630	150	70
15	16	0.7463	0.5450	60	20	31	32	0.3105	0.3619	210	100
16	17	1.2860	1.7210	60	20	32	33	0.3410	0.5302	60	40
The 69-bus grid											
Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0005	0.0012	0.00	0.00	3	36	0.0044	0.0108	26.00	18.55
2	3	0.0005	0.0012	0.00	0.00	36	37	0.0640	0.1565	26.00	18.55
3	4	0.0015	0.0036	0.00	0.00	37	38	0.1053	0.1230	0.00	0.00
4	5	0.0251	0.0294	0.00	0.00	38	39	0.0304	0.0355	24.00	17.00
5	6	0.3660	0.1864	2.60	2.20	39	40	0.0018	0.0021	24.00	17.00
6	7	0.3810	0.1941	40.40	30.00	40	41	0.7283	0.8509	1.20	1.00
7	8	0.0922	0.0470	75.00	54.00	41	42	0.3100	0.3623	0.00	0.00
8	9	0.0493	0.0251	30.00	22.00	42	43	0.0410	0.0478	6.00	4.30
9	10	0.8190	0.2707	28.00	19.00	43	44	0.0092	0.0116	0.00	0.00
10	11	0.1872	0.0619	145.00	104.00	44	45	0.1089	0.1373	39.22	26.30
11	12	0.7114	0.2351	145.00	104.00	45	46	0.0009	0.0012	29.22	26.30
12	13	1.0300	0.3400	8.00	5.00	4	47	0.0034	0.0084	0.00	0.00
13	14	1.0440	0.3450	8.00	5.50	47	48	0.0851	0.2083	79.00	56.40
14	15	1.0580	0.3496	0.00	0.00	48	49	0.2898	0.7091	384.70	274.50
15	16	0.1966	0.0650	45.50	30.00	49	50	0.0822	0.2011	384.70	274.50
16	17	0.3744	0.1238	60.00	35.00	8	51	0.0928	0.0473	40.50	28.30
17	18	0.0047	0.0016	60.00	35.00	51	52	0.3319	0.1114	3.60	2.70
18	19	0.3276	0.1083	0.00	0.00	9	53	0.1740	0.0886	4.35	3.50
19	20	0.2106	0.0690	1.00	0.60	53	54	0.2030	0.1034	26.40	19.00
20	21	0.3416	0.1129	114.00	81.00	54	55	0.2842	0.1447	24.00	17.20
21	22	0.0140	0.0046	5.00	3.50	55	56	0.2813	0.1433	0.00	0.00
22	23	0.1591	0.0526	0.00	0.00	56	57	1.5900	0.5337	0.00	0.00
23	24	0.3463	0.1145	28.00	20.00	57	58	0.7837	0.2630	0.00	0.00
24	25	0.7488	0.2475	0.00	0.00	58	59	0.3042	0.1006	100.00	72.00
25	26	0.3089	0.1021	14.00	10.00	59	60	0.3861	0.1172	0.00	0.00
26	27	0.1732	0.0572	14.00	10.00	60	61	0.5075	0.2585	1244.00	888.00
3	28	0.0044	0.0108	26.00	18.60	61	62	0.0974	0.0496	32.00	23.00
28	29	0.0640	0.1565	26.00	18.60	62	63	0.1450	0.0738	0.00	0.00
29	30	0.3978	0.1315	0.00	0.00	63	64	0.7105	0.3619	227.00	162.00
30	31	0.0702	0.0232	0.00	0.00	64	65	1.0410	0.5302	59.00	42.00
31	32	0.3510	0.1160	0.00	0.00	11	66	0.2012	0.0611	18.00	13.00
32	33	0.8390	0.2816	14.00	10.00	66	67	0.0470	0.0140	18.00	13.00
33	34	1.7080	0.5646	19.50	14.00	12	68	0.7394	0.2444	28.00	20.00
34	35	1.4740	0.4873	6.00	4.00	68	69	0.0047	0.0016	28.00	20.00

and for each case, the capacitor placement and sizing are determined. Table 3 summarizes the results for each case, including the capacitor locations and sizes, the total installed reactive power, the exact losses from the validation power flow, and the loss reduction relative to the base case. We note that the actual number of capacitors that appear in the solution may be lower than N_{cap} if the optimizer determines that additional capacitors are not economically beneficial in the convex approximation, or if the solution reaches the specified N_{cap} .

The base case losses for the 24-bus system are 668.70 kW. The installation of a single capacitor of 3,000 kvar at node 22 reduces losses by 145.19 kW (21.7%). The marginal reductions for subsequent capacitors are 21.77 kW,

Table 3. Results for the 24-bus test feeder with varying number of capacitor banks

Number of capacitors	Capacitor placement node (size in kvar)	Total size (kvar)	Losses (kW)	Loss reduction (kW)
Base case (no capacitors)	None	0	668.70	—
1	22 (3000)	3000	523.50	145.19
2	15 (1800), 22 (2250)	4050	501.74	166.96
3	15 (1500), 17 (1350), 23 (1350)	4200	495.71	172.97
4	10 (900), 15 (1500), 17 (1350), 23 (1350)	5100	491.33	177.36
5	5 (1500), 10 (900), 16 (900), 23 (1200), 28 (600)	5100	489.21	179.49

6.01 kW, 4.39 kW, and 2.13 kW, showing diminishing returns. This behavior confirms that the model prioritizes the most effective compensation points first.

The evolution of the placement as N_{cap} increases reveals the following patterns. Node 15 appears in all configurations with two or more capacitors. Node 22 is selected only when one or two capacitors are available, and is replaced by nodes 17 and 23 when additional installation points become accessible, indicating that the model favors distributed compensation when multiple capacitors are allowed. These patterns are consistent with the physical interpretation that reactive compensation is most effective when distributed along the main feeder.

The voltage profile improvement achieved by the capacitor installation is illustrated in Figure 3. In the base case, the minimum voltage is 0.9049 p.u. at node 23, which violates the 7% voltage regulation criterion (i.e., voltages should remain above 0.93 p.u.). After the installation of the five capacitors, the minimum voltage increases to 0.9346 p.u. at node 24, satisfying the regulation requirement. This indicates that the proposed approach not only reduces losses but also improves voltage quality on the tested system.

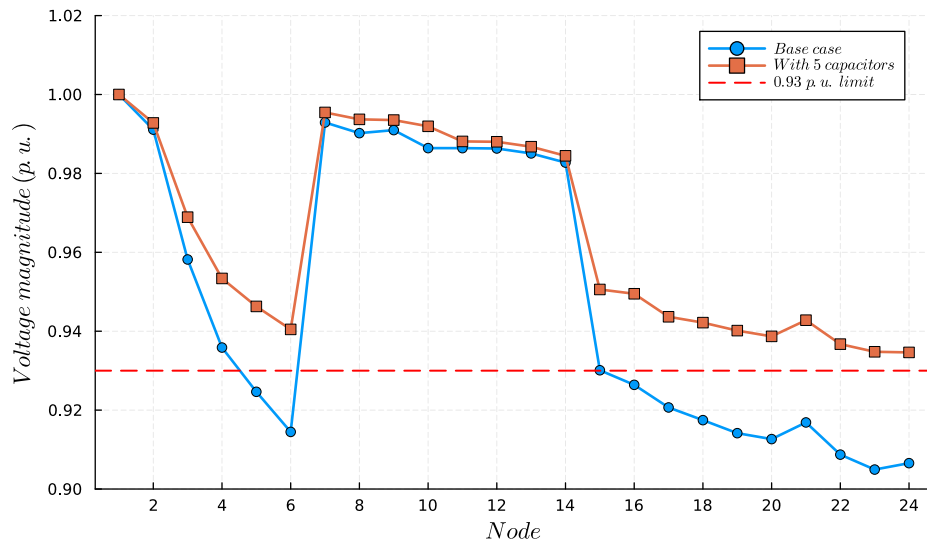


Figure 3. Voltage profile performance for the 24-bus grid comparing the base case with the five-capacitor solution

The diminishing returns observed have practical implications. The first capacitor provides a loss reduction of 145.19 kW, while the fifth adds only 2.13 kW. This information can be incorporated into cost-benefit analysis to determine the economically optimal number of capacitors. The consistency of selected nodes across configurations—particularly nodes 15, 17, and 23—is a positive indicator of the robustness of the optimization results on this system. All cases are solved in less than 5 seconds.

5.2. Comparison with Literature on the 33-Bus Test Feeder

The 33-bus system is used to compare the proposed approach against results reported in the literature. Table 4 compares the capacitor placement and sizing obtained by the proposed method with those reported in previous

studies that proposed the gravitational search algorithm (GSA) [17], the two-stage method (TSM) [20], the flower pollination algorithm (FPA) [18], and the discrete vortex search algorithm (DVSA) [16]. A maximum of three capacitors ($N_{\text{cap}} = 3$) is considered, consistent with the studies used for comparison. We note that the comparison is performed using the values reported in those references, which were obtained with their own capacitor size sets, node limits, and tuning procedures; a re-run of the comparison methods under a unified experimental protocol is identified as a direction for future work.

Table 4. Comparison of capacitor placement results for the 33-bus system

Method	Capacitor placement	Total size (kvar)	Losses (kW)	Loss reduction (kW)
Base case (no capacitors)	None	0	210.98	–
GSA [17]	Nodes 9, 29, 30: 450, 800, 900 kvar	2150	171.78	39.20
TSM [20]	Nodes 7, 29, 30: 850, 25, 900 kvar	1775	144.04	66.94
FPA [18]	Nodes 13, 24, 30: 450, 450, 900 kvar	1800	139.08	71.90
DVSA [16]	Nodes 12, 24, 30: 450, 450, 1050 kvar	1950	138.42	72.56
Proposed method (MICP)	Nodes 13, 24, 30: 400, 500, 1000 kvar	1900	138.33	72.65

The proposed MICP approach achieves a loss reduction of 72.65 kW (34.4%), which matches or slightly improves upon the best reported value among the compared methods. The GSA shows the poorest performance (39.20 kW) in this comparison. The TSM, FPA, and DVSA achieve progressively better results (66.94, 71.90, and 72.56 kW, respectively), with the proposed method slightly surpassing the DVSA.

The proposed method identifies nodes 13, 24, and 30 as optimal locations, with sizes of 400, 500, and 1,000 kvar, totaling 1,900 kvar. This placement is consistent with the FPA and DVSA, which also select nodes 24 and 30, supporting the reasonableness of these critical locations.

The voltage profile improvement is illustrated in Figure 4. In the base case, the minimum voltage is 0.9038 p.u. at node 18, violating the 7% regulation criterion. After capacitor installation, the minimum voltage increases to 0.9318 p.u. at the same node, satisfying the regulation requirement. The problem is solved in less than 2 seconds.

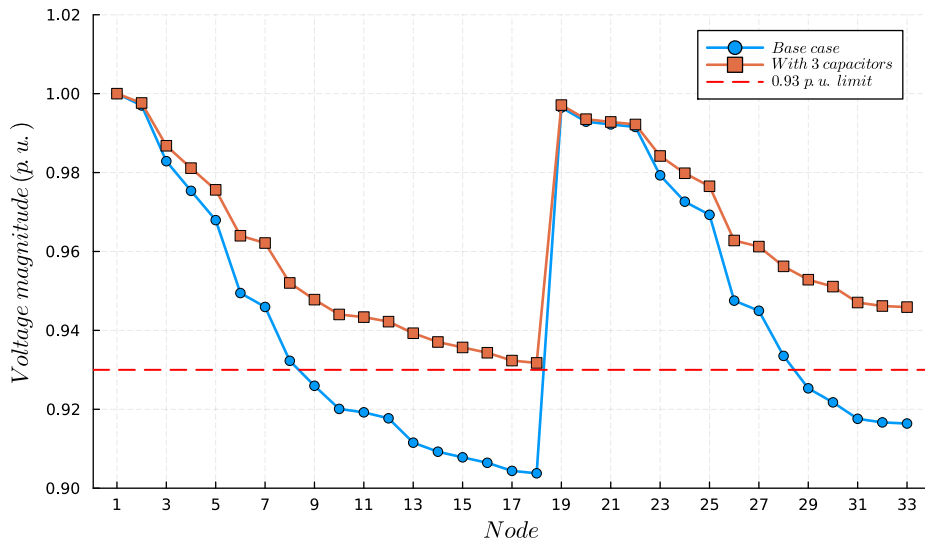


Figure 4. Voltage profile performance for the 33-bus grid comparing the base case with the three-capacitor solution

5.3. Comparison with Literature on the 69-Bus Test Feeder

The 69-bus system provides a larger test case. Table 5 compares the results obtained by the proposed method with those reported in the literature [17, 20, 18, 16], again considering $N_{\text{cap}} = 3$. As in the 33-bus case, the comparison is based on the values reported in the original references and not on a unified re-run.

Table 5. Comparison of capacitor placement results for the 69-bus system

Method	Capacitor placement	Total size (kvar)	Losses (kW)	Loss reduction (kW)
Base case (no capacitors)	None	0	225.07	—
GSA [17]	Nodes 11, 29, 60: 900, 1050, 450 kvar	2400	163.28	61.79
TSM [20]	Nodes 19, 62, 63: 225, 900, 225 kvar	1350	148.91	76.16
FPA [18]	Nodes 11, 22, 61: 450, 150, 1350 kvar	1950	145.86	79.21
DVSA [16]	Nodes 11, 18, 61: 300, 300, 1200 kvar	2000	145.40	79.67
Proposed method (MICP)	Nodes 11, 21, 61: 400, 200, 1200 kvar	2000	145.34	79.73

The proposed MICP approach achieves a loss reduction of 79.73 kW (35.4%), which matches or slightly improves upon the best reported value among the compared methods. The GSA shows the poorest performance (61.79 kW) despite using the largest total capacity (2,400 kvar). The TSM, FPA, and DVSA achieve 76.16, 79.21, and 79.67 kW, respectively.

The proposed method identifies nodes 11, 21, and 61 as optimal locations, with sizes of 400, 200, and 1,200 kvar, totaling 2,000 kvar. Nodes 11 and 61 appear in multiple literature solutions (GSA, FPA, DVSA), supporting the reasonableness of these locations.

The voltage profile improvement is illustrated in Figure 5. In the base case, the minimum voltage is 0.9092 p.u. at node 65, violating the 7% regulation criterion. After capacitor installation, the minimum voltage increases to 0.9308 p.u. at the same node, satisfying the requirement. The problem is solved in less than 5 seconds.

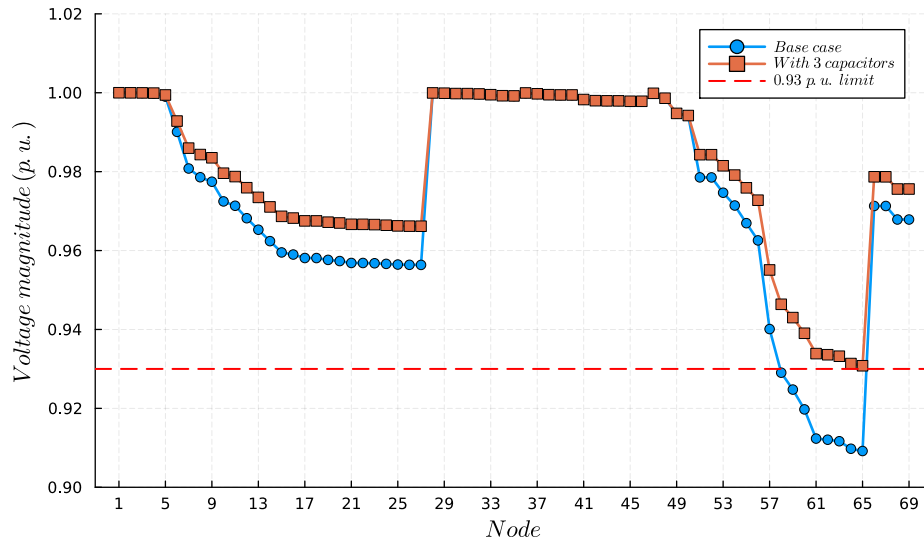


Figure 5. Voltage profile performance for the 69-bus grid comparing the base case with the three-capacitor solution

5.4. Discussion of Results

The numerical results on the three test systems allow the following observations.

The 24-bus system analysis shows that the proposed approach identifies the trade-off between the number of installed capacitors and loss reduction. The diminishing returns observed for larger N_{cap} values provide information

for economic decision-making, while the voltage profile improvements confirm that the approach can satisfy regulatory voltage requirements on the tested systems.

The comparison with the literature on the 33-bus and 69-bus systems shows that the proposed approach achieves results that match or slightly improve upon the best values reported for the compared metaheuristic methods on these test cases. The match with state-of-the-art values is expected, since the underlying problem is well studied and the convex approximation tends to identify the same compensation nodes as the exact problem when the network operates close to nominal voltage. The contribution of the proposed approach lies not in a substantially better objective value, but in the deterministic convergence and reproducible solution that the convex formulation provides.

The computational times on the tested systems are under 10 seconds, which is practical for planning studies on networks of the tested size. However, scalability to networks with hundreds or thousands of buses has not been demonstrated in this work. The computational cost of the mixed-integer convex model is expected to grow with the number of binary variables (i.e., with the number of nodes and the number of discrete capacitor sizes), while the cost of each exact power flow evaluation grows with the size of the network. Decomposition strategies, warm-start strategies, and parallel evaluation of candidate configurations are identified as directions for future work.

The voltage profile improvements observed across the tested systems indicate that the capacitor placement not only reduces losses but also improves voltage quality. In all cases, the minimum voltage was raised above the 0.93 p.u. threshold, satisfying the 7% regulation criterion.

Finally, we note that the loss reductions reported here are consistent with those reported in the literature for the same test systems, and the differences relative to the best-performing methods are small (on the order of 0.1 kW in the 33-bus and 69-bus cases). This is expected because the capacitor placement problem is well studied and the objective landscape is relatively smooth. The practical value of the proposed approach is therefore in its deterministic convergence and in the explicit quantification of the approximation error, rather than in achieving a substantial improvement in the objective value.

The proposed approach has not been tested on real distribution feeders, unbalanced three-phase systems, or networks with high penetration of distributed generation. These are identified as limitations and directions for future work.

6. Conclusions and Future Works

This paper has presented a mixed-integer convex optimization framework for the optimal placement and sizing of capacitor banks in radial distribution networks. The formulation uses branch flow variables in the complex domain, with a convex quadratic objective and linear and second-order cone constraints. A three-stage methodology was implemented: an initial exact power flow to establish base case conditions, a convex optimization stage to determine the capacitor placement and sizing, and a final exact validation via power flow to compute the exact losses and voltage profile for the selected configuration. The framework was evaluated on the 24-bus, 33-bus, and 69-bus test systems.

The 24-bus analysis revealed diminishing returns as the number of installed capacitors increased, with the first capacitor providing the most significant loss reduction of 145.19 kW and the fifth adding only 2.13 kW. This behavior is consistent with the physical expectation that reactive compensation is most effective at the first few strategically located nodes.

The comparison with the literature on the 33-bus and 69-bus systems showed that the proposed approach achieves results that match or slightly improve upon the best reported values among the compared metaheuristic methods, including the gravitational search algorithm, the two-stage method, the flower pollination algorithm, and the discrete vortex search algorithm. On the 33-bus system, the proposed method achieved a loss reduction of 72.65 kW (34.4%). On the 69-bus system, the proposed method achieved a loss reduction of 79.73 kW (35.4%). In all test systems, the minimum voltage was raised above the 0.93 p.u. threshold, satisfying the 7% voltage regulation criterion. The computational times were under 10 seconds on the tested systems.

We emphasize that the global-optimality guarantee of the solver applies to the mixed-integer convex approximation, not to the original nonlinear capacitor placement problem. The third stage of the methodology quantifies the approximation error and ensures that the selected configuration is physically feasible under the exact nonlinear power flow. The proposed approach is positioned as a planning tool under balanced steady-state conditions; it has not been tested on real distribution feeders, unbalanced three-phase systems, or networks with high penetration of distributed generation.

Future work will focus on several directions. First, a closed-loop iterative scheme will be explored, in which the exact validation results are used to update the convex model and to iterate until the approximation error falls below a prescribed threshold. Second, the framework will be extended to incorporate uncertainty in load and renewable generation through stochastic or robust optimization. Third, the formulation will be extended to consider unbalanced three-phase systems and time-varying load profiles. Fourth, the scalability of the approach to larger networks will be investigated, using decomposition and parallel evaluation strategies. Fifth, an economic analysis will be conducted to compare the annual energy loss savings against the capital cost of the capacitor banks, including payback period and net present value. Sixth, the proposed approach will be compared with recently published methods from the last five years under a unified experimental protocol. Finally, the application of the proposed methodology to real distribution networks will be pursued to validate its practical applicability.

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