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Laplace New Iterative Method for solving time-fractional Sawada-Kotera and Lax KdV Equations

Ali F. Shabta, Raad A. Hameed*

Department of Mathematics, College of Education for Pure Sciences University of Tikrit, Iraq

Abstract This study investigates the use of the Laplace New Iterative Method (LNIM) to compute approximate solutions to the time-fractional Sawada–Kotera equation and the time-fractional Lax equation. The findings highlight that the method is straightforward to implement, especially within mathematical programming environments. Its performance is supported by the observed convergence of the approximate solutions toward the exact ones. In addition, the analysis shows that the solution behavior varies with the fractional order, capturing key features and properties of both equations. Moreover, the derived solutions align with the inherent structure of the two models, which supports the reliability and practicality of the Laplace New Iterative Method for this class of problems. The results also suggest that the approach is not restricted to these two equations and may be extended to other fractional sub-equations. Consequently, LNIM can be regarded as a significant numerical technique with potential applications across several scientific domains, including applied mathematics, physics, and engineering, reinforcing its relevance and motivating further work in this direction.

Keywords Approximate solution; Initial condition; Laplace New Iterative Method (LNIM); Caputo fractional derivative; fractional calculus; time-fractional Sawada–Kotera equation;time-fractional Lax equation.

AMS Mathematics Subject Classification: 65M06, 35K99, 65M12.

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1. Introduction

In light of the interest that non-linear partial differential equations have attracted amongst some researchers in recent decades, owing to their significance in the field of applied sciences where they can describe physical phenomena, many studies have consequently focused on this area [1, 2, 3]. Fractional calculus has long been regarded as the cornerstone for understanding and interpreting the behaviour of many natural phenomena, which has earned it great significance as one of the best and simplest methods for explaining science and engineering [4, 5, 6, 7, 8]. Although fractional calculus has attracted widespread interest from scientists over the past three centuries, it has only recently been addressed in applied sciences and engineering. The fundamental idea behind fractional calculus centers on dealing with integrals and derivatives of various orders. Several important definitions have been proposed in this field, such as the Riemann-Liouville fractional function, the Caputo fractional function, and the Metag-Lifler function. These concepts can be used in semi-analytical methods for solving fractional partial differential equations. This is evident from references [9, 10, 11]. One of the important equations he will discuss in this field is the Lax equation, derived by Lax, which is considered one of the fundamental equations within the KdV family [12]. Subsequently, Sawada and Kotera derived another model, namely the Sawada-Kotera equation, which subsequently came to be regarded as one of the well-known differential equations [13]. in the recent years, a variety of analytical and semi-analytical methods are proposed to solve nonlinear and fractional differential

*Correspondence to: Raad A.Awad (Email: dr.raad.awad@tu.edu.iq). Department of Mathematics, College of Education for Pure Sciences, University of Tikrit, Salah al-Din 34001, Iraq.

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equations. In that regard, Jaafar and Mousaf implemented a new homotopy perturbation method for solving nonlinear problems [14]. Later, Alemu and Mesfin used the transform Adomian decomposition method (TADM) to study fractional differential models [15] In another proposal, Kartizami and Lissisami established an analytical approach to construct approximate solutions of the fractional equations [16] . Also, Boulakour et al. studied the time-fractional nonlinear Kuramoto-Sivashinsky equation with the help of the Laplace Adomian decomposition method (LADM) [17], while Saad et al. analyzed the corresponding fractional model using a different analytical approach [18] , In addition, Humaira et al. have used the ARPSM method based on the Caputo fractional derivative to obtain approximate solutions of the Sawada-Kotera, Kaup-Kupershmidt and Ito equations [19] In the same manner, Eriqat et al. have applied the L-RPSM method to seek approximate solutions of the fractional Lax and Caudrey-Dodd-Gibbon equations [20]. More recently, Ihsan et al. have presented accurate analytical solutions of several KdV-CDG models showing the efficiency of the proposed method [21]. All these works show the continuous trend of development of analytical methods for the case of fractional nonlinear differential equations and create a good basis for further improvements of the proposed methodology. In light of previous studies, the general form of the equations under consideration can be written as follows:[22]:

$$
\begin{aligned}
&{}^{C}D_t^{\alpha}\Omega + \Omega_{\xi\xi\xi\xi\xi} + A\Omega\Omega_{\xi\xi\xi} + B\Omega_{\xi}\Omega_{\xi\xi} + C\Omega^2\Omega_{\xi} = 0, \qquad \tau > 0, 0 < \alpha \leq 1 \qquad (1)\\
&\Omega(\xi, 0) = f(\xi), \qquad (2)
\end{aligned}
$$

where A,B and C are any constant and D_{τ}^{α} denotes Caputo time fractional derivative,When value $A = 15, B = 15$ and $C = 45$ we obtain to Sawada-Kotera equation we can write as below

$$
\begin{aligned}
&{}^{C}D_{\tau}^{\alpha}\Omega + \Omega_{\xi\xi\xi\xi\xi} + 15\Omega\Omega_{\xi\xi\xi} + 15\Omega_{\xi}\Omega_{\xi\xi} + 45\Omega^2\Omega_{\xi} = 0, \qquad \tau > 0, 0 < \alpha \leq 1 \qquad (3)\\
&\Omega(\xi, 0) = 2k^2\mathrm{sech}^2(k\xi), \qquad (4)
\end{aligned}
$$

and when we took the values for each of $A = 10, B = 30$ and $C = 30$, we obtain the Lax equation as shown below.

$$
\begin{aligned}
&{}^{C}D_{\tau}^{\alpha}\Omega + \Omega_{\xi\xi\xi\xi\xi} + 10\Omega\Omega_{\xi\xi\xi} + 30\Omega_{\xi}\Omega_{\xi\xi} + 30\Omega^2\Omega_{\xi} = 0, \qquad \tau > 0, 0 < \alpha \leq 1 \qquad (5)\\
&\Omega(\xi, 0) = 2k^2\left(2 - 3\tanh^2(k\xi)\right), \qquad (6)
\end{aligned}
$$

. As for what we will address in our study, it is the Laplace New Iterative Method (LNIM), which has been noted by some researchers. This is illustrated in references [23, 24]. Research efforts have not been limited to improving these types of methods. This has contributed to achieving better results through various improved methods. Please refer to [25].

This manuscript consists of six sections: the second section reveals the basic concepts and definitions linked directly to fractional calculus, which constitute the theoretical framework of the research. The third section deals with the idea and usage of Laplace New Iterative Method (LNIM). The fourth section explores the convergence of the method and the accuracy of the results acquired by it. In The fifth section, the method is implemented to acquire approximate solutions to the time-dependent fractional Sawada-Kotera and Lax equations and the outcomes are presented and discussed. Finally, the sixth section delivers the conclusions of the work.

2. Preliminaries

In this section, we recall some basic definitions and results dealing with the fractional calculus and Laplace transform which are later used in this paper.

Definition 1.[26] Let $\Im(t)$ be a real function , $\tau > 0$ is called in the space C_{σ}, $\sigma \in \mathbb{R}$ if there exists $p(> \sigma)$ belong a real number , such that $\Im(\tau) = \tau^p\Im_1(\tau)$, where $\Im_1 \in C[0, \infty]$. Clearly $C_{\sigma} \subset C_{\beta}$ if $\beta \leq \sigma$.

Definition 2.[26] Let $\Im(\tau)$ be a function, $\tau > 0$ is called in the space C_{σ}^{m}, $m \in \mathbb{N} \cup \{0\}$, if $\Im^{(m)} \in C_{\sigma}$.

Definition 3 .[26] Let $\alpha > 0$ be the order of $\Im$ and $\Im$ a function belong to C_σ ,$\sigma \geq -1$ The Riemann–Liouville fractional integral is defined as

$$I^{\alpha}\Im(\tau) = \begin{cases} \dfrac{1}{\Gamma(\alpha)} \displaystyle\int_0^{\tau} \frac{\Im(\varphi)}{(\tau-\varphi)^{1-\alpha}}\, d\varphi, & \alpha > 0,\ \tau > 0, \\ \Im(\tau), & \alpha = 0. \end{cases} \tag{7}$$

Definition 4 .[26] The order α of Caputo fractional derivative of $\Im$ such that $\Im \in C_\sigma^m$, $m \in \mathbb{N} \cup \{0\}$ is introduced as follows:

$$D^{\alpha}\Im(t) = \frac{\partial^{\alpha}\Im(\tau)}{\partial\tau^{\alpha}} = \begin{cases} I^{m-\alpha}\left[\dfrac{\partial^m \Im(\tau)}{\partial\tau^m}\right], & m-1 < \alpha < m,\ m \in \mathbb{N}, \\ \dfrac{\partial^m \Im(\tau)}{\partial\tau^m}, & \alpha = m. \end{cases} \tag{8}$$

Note that

$$\text{(i)} \quad I_{\tau}^{\alpha}\Im(\xi,\tau) = \frac{1}{\Gamma(\alpha)} \int_0^{\tau} \frac{\Im(\xi,s)}{(\tau-s)^{1-\alpha}}\, ds, \qquad \alpha > 0,\ \tau > 0. \tag{9}$$

$$\text{(ii)} \quad D_{\tau}^{\alpha}\Im(\xi,\tau) = I_{\tau}^{m-\alpha}\frac{\partial^m \Im(\xi,\tau)}{\partial\tau^m}, \qquad m-1 < \alpha \leq m. \tag{10}$$

Definition 5.[26] The Mittag–Leffler function $E_\alpha(z)$ admits the following series representation:

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+1)}, \qquad \alpha > 0,\ z \in \mathbb{C}. \tag{11}$$

Definition 6.[26] The Laplace transform of a function $\Im(\tau)$ is defined as follows:

$$F(s) = \mathcal{L}[\Im(\tau)] = \int_0^{\infty} e^{-s\tau}\Im(\tau)\, d\tau. \tag{12}$$

Definition 7.[26] The Laplace transform $\mathcal{L}[\Im(\tau)]$ of the Riemann–Liouville fractional integral can be expressed in the form:

$$\mathcal{L}[I^{\alpha}\Im(\tau)] = s^{-\alpha}F(s). \tag{13}$$

Definition 8.[26] The Laplace transform related to the Caputo fractional derivative is given by:

$$\mathcal{L}[D^{\alpha}\Im(\tau)] = s^{\alpha}F(s) - \sum_{k=0}^{n-1} s^{(\alpha-k-1)}\Im^{(k)}(0), \qquad n-1 < \alpha \leq n. \tag{14}$$

3. Map of Laplace New Iterative Method (LNIM)

Consider the following general equation

$$\Omega_t^{\alpha} + \mathcal{N}(\Omega) + \mathfrak{L}(\Omega) = f(\Omega) \tag{15}$$

Based on the fundamental idea of Laplace New Iterative Method (LNIM), the solution of equation (15) is sought in series form, where $f(\Omega)$ is a known function, $\Omega = (\Omega_1, \Omega_2, \Omega_3, \ldots, \Omega_n)$, while $\mathfrak{L}$ and $\mathcal{N}$ are both operators depending on Ω, with $\mathfrak{L}$ representing the linear part and $\mathcal{N}$ the nonlinear part, subject to the initial condition:

$$\Omega(\xi, 0) = f(\xi) \tag{16}$$

Applying the Laplace transform to both sides of equation (15) yields:

$$s^{\alpha}\mathcal{L}[\Omega(\xi,\tau)] - s^{\alpha-1}\Omega(\xi,0) = \mathcal{L}\Big[f\big(\Omega(\xi,\tau)\big)\Big] - \mathcal{L}\Big[\mathcal{N}\big(\Omega(\xi,\tau)\big) + \mathfrak{L}\big(\Omega(\xi,\tau)\big)\Big] \tag{17}$$

which can be rewritten as:

$$\mathcal{L}[\Omega(\xi,\tau)] = \frac{f(x)}{s} - \frac{1}{s^{\alpha}}\mathcal{L}\Big[\mathcal{N}\big(\Omega(\xi,\tau)\big) + \mathfrak{L}\big(\Omega(\xi,\tau)\big) - f\big(\Omega(\xi,\tau)\big)\Big] \tag{18}$$

According to the NIM framework, the solution is expressed as an infinite series:

$$\Omega(\xi,\tau) = \sum_{k=0}^{\infty} \Omega_k(\xi,\tau) \tag{19}$$

The linear operator $\mathfrak{L}$ satisfies the following decomposition property:

$$\sum_{k=0}^{\infty} \mathcal{L}\big(\Omega_k(\xi,\tau)\big) = \mathcal{L}\left(\sum_{k=0}^{\infty} \Omega_k(\xi,\tau)\right) \tag{20}$$

For the nonlinear operator $\mathcal{N}$, the decomposition takes the form:

$$\mathcal{N}\left(\sum_{k=0}^{\infty} \Omega_k\right) = \mathcal{N}\big(\Omega_0(\xi,\tau)\big) + \sum_{k=1}^{\infty}\left\{\mathcal{N}\left(\sum_{i=0}^{k} \Omega_i(\xi,\tau)\right) - \mathcal{N}\left(\sum_{i=0}^{k-1} \Omega_i(\xi,\tau)\right)\right\} \tag{21}$$

Consequently, substituting the above decompositions into equation (18) gives the general iterative formula:

$$\mathcal{L}\left[\sum_{k=0}^{\infty} \Omega_k(\xi,\tau)\right] = \frac{f(x)}{s} - \frac{1}{s^{\alpha}}\mathcal{L}\Big[\mathcal{N}\big(\Omega_k(\xi,\tau)\big) + \mathfrak{L}\big(\Omega_k(\xi,\tau)\big) - f\big(\Omega_k(\xi,\tau)\big)\Big] \tag{22}$$

By implementing the New Iterative Method, the components of the solution are determined through the following recurrence relations. The zeroth component is obtained from:

$$\mathcal{L}[\Omega_0(\xi,\tau)] = \frac{f(\xi)}{s} \tag{23}$$

while each subsequent component satisfies:

$$\mathcal{L}[\Omega_{k+1}(\xi,\tau)] = -\frac{1}{s^{\alpha}}\mathcal{L}\Big[\mathcal{N}\big(\Omega_k(\xi,\tau)\big) + \mathcal{L}\big(\Omega_k(\xi,\tau)\big) - f\big(\Omega_k(\xi,\tau)\big)\Big], \quad k \geq 1 \tag{24}$$

Finally, taking the inverse Laplace transform of equations (23) and (24) respectively gives:

$$\Omega_0(\xi,\tau) = f(\xi) \tag{25}$$

$$\Omega_{k+1}(\xi,\tau) = \mathcal{L}^{-1}\left[-\frac{1}{s^{\alpha}}\mathcal{L}\Big[\mathcal{N}\big(\Omega_k(\xi,\tau)\big) + \mathfrak{L}\big(\Omega_k(\xi,\tau)\big)\Big]\right] \tag{26}$$

Remark 3.1
The Classical New Iterative Method (NIM) is extended to the Laplace New Iterative Method (LNIM) by involving the Laplace transform technique into the iterative scheme before the recursive construction of the components of the solution. In NIM the solution components are constructed directly from the linear and nonlinear operators decomposition. Conversely, LNIM formulates the recursive construction in the Laplace-transformed domain and then applies the inverse Laplace transform to obtain the corresponding solution components.

4. Convergence Analysis of Laplace Transform New Iterative Method

Theorem 1 [17] [existence and uniqueness] Let $\mathbb{X}$ be a Banach space defined by the following formula: $\mathbb{X} = C(\mathbb{R} \times [0,T])$ which contains all continuous functions defined on $\mathbb{R} \times [0,T]$, fitted with the parameter sup norm:

$$\|\Omega\| = \sup_{(\xi,\tau)\in\mathbb{R}\times[0,T]} |\Omega(\xi,\tau)|. \tag{27}$$

Let $\mathbb{H}$ be an operator defined as $\mathbb{H} : \mathbb{X} \to \mathbb{X}$. We can define it on the fractional fKdV equation in the form of a fixed point:

$$\Omega(\xi,\tau) = \mathbb{H}\Omega(\xi,\tau), \tag{28}$$

where,

$$\mathbb{H}\Omega = \Omega_0 - \mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s}) \cdot \mathcal{L}\left[A\Omega\Omega_{\xi\xi\xi} + B\Omega_\xi\Omega_{\xi\xi} + C\Omega^2\Omega_\xi + \Omega_{\xi\xi\xi\xi\xi}\right]\right]. \tag{29}$$

We can define $\mathfrak{F}(\mathfrak{s})$ as the auxiliary function that relates the chosen fractional derivative. We can deal with the nonlinear part of the Laplace New Iterative Method (LNIM) through successive differentiation, as follows:

$$\mathcal{N}(\Omega) - \mathcal{N}(\Theta) = A(\Omega\Omega_{\xi\xi\xi} - \Theta\Theta_{\xi\xi\xi}) + B(\Omega_\xi\Omega_{\xi\xi} - \Omega_\xi\Theta_{\xi\xi}) + C(\Omega^2\Omega_\xi - \Theta^2\Theta_\xi). \tag{30}$$

Assume that:

1. The linear spatial operators and nonlinear terms are Lipschitz continuous. That is, there exist constants $L_A, L_B, L_C, L_N > 0$ such that, for all $\Omega, \Theta \in \mathbb{X}$:

$$\|\Omega\mathcal{D}_\xi^3\Omega - \Theta\mathcal{D}_\xi^3\Theta\| \le L_A\|\Omega - \Theta\| \tag{31}$$
$$\|\mathcal{D}_\xi\Omega \cdot \mathcal{D}_\xi^2\Omega - \mathcal{D}_\xi\Theta \cdot \mathcal{D}_\xi^2\Theta\| \le L_B\|\Omega - \Theta\| \tag{32}$$
$$\|\Omega^2\mathcal{D}_\xi\Omega - \Theta^2\mathcal{D}_\xi\Theta\| \le L_C\|\Omega - \Theta\| \tag{33}$$
$$\|\mathcal{D}_\xi^5\Omega - \mathcal{D}_\xi^5\Theta\| \le L_N\|\Omega - \Theta\| \tag{34}$$

2. Define:

$$L = \sup\left(L_A|A|,\ L_B|B|,\ L_C|C|,\ L_N\right), \qquad \psi = L \cdot \mathcal{L}^{-1}\left[\frac{\mathfrak{F}(\mathfrak{s})}{\mathfrak{s}}\right]. \tag{35}$$

If $0 \le \psi < 1$, then the operator $\mathbb{H}$ is a contraction on $\mathbb{X}$.

Under these assumptions, the operator $\mathbb{H}$ admits a unique fixed point $\Omega^* \in \mathbb{X}$; this fixed point is the unique continuous solution of the fractional fKdV equation on $\mathbb{R} \times [0,T]$:

$${}^C D_t^\alpha\Omega + \Omega_{\xi\xi\xi\xi\xi} + A\Omega\Omega_{\xi\xi\xi} + B\Omega_\xi\Omega_{\xi\xi} + C\Omega^2\Omega_\xi = 0. \tag{36}$$

Proof. Let Ω and Θ be two functions in $\mathbb{X}$. From the definition of the operator $\mathbb{H}$, we have:

$$\begin{aligned}|\mathbb{H}\Omega - \mathbb{H}\Theta| &= \left|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s}) \cdot \mathcal{L}\left[A(\Omega\Omega_{\xi\xi\xi} - \Theta\Theta_{\xi\xi\xi}) + B(\Omega_\xi\Omega_{\xi\xi} - \Theta_\xi\Theta_{\xi\xi}) + C(\Omega^2\Omega_\xi - \Theta^2\Theta_\xi)\right]\right]\right| \\ &\le |A|\,\mathcal{L}^{-1}[\mathfrak{F}(\mathfrak{s}) \cdot \mathcal{L}[|\Omega\Omega_{\xi\xi\xi} - \Theta\Theta_{\xi\xi\xi}|]] + |B|\,\mathcal{L}^{-1}[\mathfrak{F}(\mathfrak{s}) \cdot \mathcal{L}[|\Omega_\xi\Omega_{\xi\xi} - \Theta_\xi\Theta_{\xi\xi}|]] \\ &\quad + |C|\,\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s}) \cdot \mathcal{L}\left[|\Omega^2\Omega_\xi - \Theta^2\Theta_\xi|\right]\right]\end{aligned} \tag{37}$$

Taking sup norms and using the Lipschitz bounds yields:

$$\begin{aligned}\|A\Omega\mathcal{D}_\xi^3\Omega - A\Theta\mathcal{D}_\xi^3\Theta\| &= |A|\|\Omega(\mathcal{D}_\xi^3\Omega - \mathcal{D}_\xi^3\Theta) + (\Omega-\Theta)\mathcal{D}_\xi^3\Theta\| \le |A|\left(\|\Omega\|_\infty\|\mathcal{D}_\xi^3\Omega - \mathcal{D}_\xi^3\Theta\| + \|\mathcal{D}_\xi^3\Theta\|_\infty\|\Omega-\Theta\|\right)\\ &\le |A|\left(L_3\|\Omega\|_\infty + \|\mathcal{D}_\xi^3\Theta\|_\infty\right)\|\Omega-\Theta\| = \underbrace{|A|\left(L_3\|\Omega\|_\infty + \|\mathcal{D}_\xi^3\Theta\|_\infty\right)}_{=:L_A}\|\Omega-\Theta\|\end{aligned}$$

$$\|\Omega\mathcal{D}_\xi^3\Omega - \Theta\mathcal{D}_\xi^3\Theta\| \le L_A\|\Omega-\Theta\| \tag{38}$$

$$\begin{aligned}\|B\mathcal{D}_\xi\Omega\cdot\mathcal{D}_\xi^2\Omega - B\mathcal{D}_\xi\Theta\cdot\mathcal{D}_\xi^2\Theta\| &= |B|\|\mathcal{D}_\xi\Omega(\mathcal{D}_\xi^2\Omega - \mathcal{D}_\xi^2\Theta) + (\mathcal{D}_\xi\Omega - \mathcal{D}_\xi\Theta)\mathcal{D}_\xi^2\Theta\|\\ &\le |B|\left(\|\mathcal{D}_\xi\Omega\|_\infty\|\mathcal{D}_\xi^2\Omega - \mathcal{D}_\xi^2\Theta\| + \|\mathcal{D}_\xi^2\Theta\|_\infty\|\mathcal{D}_\xi\Omega - \mathcal{D}_\xi\Theta\|\right)\\ &\le |B|\left(L_2\|\mathcal{D}_\xi\Omega\|_\infty + L_1\|\mathcal{D}_\xi^2\Theta\|_\infty\right)\|\Omega - v\|\\ &= \underbrace{|B|\left(L_2\|\mathcal{D}_\xi\Omega\|_\infty + L_1\|\mathcal{D}_\xi^2\Theta\|_\infty\right)}_{=:L_B}\|\Omega-\Theta\|\end{aligned}$$

$$\|\mathcal{D}_\xi\Omega\cdot\mathcal{D}_\xi^2\Omega - \mathcal{D}_\xi\Theta\cdot\mathcal{D}_\xi^2\Theta\| \le L_B\|\Omega-\Theta\| \tag{39}$$

$$\begin{aligned}\|C\Omega^2\mathcal{D}_\xi\Omega - C\Theta^2\mathcal{D}_\xi\Theta\| &= |C|\|\Omega^2(\mathcal{D}_\xi\Omega - \Theta_x\mathcal{D}_\xi\Theta) + (\Omega^2-\Theta^2)\mathcal{D}_\xi\Theta\|\\ &\le |C|\left(\|\Omega\|_\infty^2\|\mathcal{D}_\xi\Omega - \mathcal{D}_\xi\Theta\| + \|\Omega^2-\Theta^2\|\,\|\mathcal{D}_\xi\Theta\|_\infty\right)\\ &= |C|\left(\|\Omega\|_\infty^2\|\mathcal{D}_\xi\Omega - \mathcal{D}_\xi\Theta\| + \|(\Omega-\Theta)(\Omega+\Theta)\|\,\|\mathcal{D}_\xi\Theta\|_\infty\right)\\ &\le |C|\left(\|\Omega\|_\infty^2 L_1 + \|\Omega+\Theta\|_\infty\|\mathcal{D}_\xi\Theta\|_\infty\right)\|\Omega-\Theta\|\\ &= \underbrace{|C|\left(L_1\|\Omega\|_\infty^2 + \|\Omega+\Theta\|_\infty\|\mathcal{D}_\xi\Theta\|_\infty\right)}_{=:L_C}\|\Omega-\Theta\|\end{aligned}$$

$$\|\Omega^2\mathcal{D}_\xi\Omega - \Theta^2\mathcal{D}_\xi\Theta\| \le L_C\|\Omega-\Theta\| \tag{40}$$

$$\|\mathcal{D}_\xi^5\Omega - \mathcal{D}_\xi^5\Theta\| \le L_N\|\Omega-\Theta\| \tag{41}$$

$$\begin{aligned}\|\mathbb{H}\Omega - \mathbb{H}\Theta\| &\le L_A|A|\,\mathcal{L}^{-1}[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}[\|\Omega-\Theta\|]] + L_B|B|\,\mathcal{L}^{-1}[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}[\|\Omega-\Theta\|]]\\ &\quad + L_C|C|\,\mathcal{L}^{-1}[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}[\|\Omega-\Theta\|]] + L_N\mathcal{L}^{-1}[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}[\|\Omega-\Theta\|]]\\ &\le L\cdot\mathcal{L}^{-1}[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}[\|\Omega-\Theta\|]] \le L\cdot\mathcal{L}^{-1}\left[\frac{\mathfrak{F}(\mathfrak{s})}{\mathfrak{s}}\right]\|\Omega-\Theta\| \le \psi\|\Omega-\Theta\|\end{aligned}$$

Let us assume that the value of ψ belongs to the interval $[0,1)$. Therefore, the operator $\mathbb{H}$ can be a contraction on the space $\mathbb{X}$. Consequently, the operator $\mathbb{H}$ has a unique fixed point $\Omega^*\in\mathbb{X}$, which represents the only continuous solution to the fractional fKdV equation on $\mathbb{R}\times[0,T]$. The value of the contraction constant ψ can be taken as an explicit value for the continuous function $\mathfrak{F}(\mathfrak{s})$, which can reflect the type of fractional derivative used.

Theorem 2 [17] [convergence]
We can assume the validity of the existence and uniqueness theorem. Let $\mathbb{H}$ be a contraction operator defined as $\mathbb{H}:\mathbb{X}\to\mathbb{X}$ with a contraction constant ψ. The definition of the subtotal sequence:

$$\mathbb{G}_n(\xi,\tau) = \sum_{k=0}^{n}\Omega_k(\xi,\tau) \tag{42}$$

derived by the L-NIM method, from which a Cauchy sequence is formed in Banach space $\mathbb{X}$, is as follows: The L-NIM series converges uniformly to the unique element $\Omega^* \in \mathbb{X}$, which in turn represents the exact continuous solution of the fractional fKdV equation on $\mathbb{R} \times [0, T]$.

Proof. We begin by proving that the sequence $\{\mathbb{G}_n\}$ is a Cauchy sequence in Banach space $\mathbb{X}$. The nonlinear operator in the L-NIM method is treated using successive differences, as follows:

$$\begin{aligned}\mathcal{N}(\mathbb{G}_n) - \mathcal{N}(\mathbb{G}_{n-1}) = A(&\mathbb{G}_n\mathbb{G}_{\xi\xi\xi,n} - \mathbb{G}_{n-1}\mathbb{G}_{\xi\xi\xi,n-1}) \\ &+ B(\mathbb{G}_{\xi,n}\mathbb{G}_{\xi\xi,n} - \mathbb{G}_{\xi,n-1}\mathbb{G}_{\xi\xi,n-1}) \\ &+ C(\mathbb{G}_n^2\mathbb{G}_{\xi,n} - \mathbb{G}_{n-1}^2\mathbb{G}_{\xi,n-1})\end{aligned} \tag{43}$$

According to the standard NIM ,

$$\mathcal{N}\left(\sum_{k=0}^{\infty}\Omega_k\right) = \mathcal{N}(\Omega_0) + \sum_{k=1}^{\infty}\left[\mathcal{N}\left(\sum_{i=0}^{k}\Omega_i\right) - \mathcal{N}\left(\sum_{i=0}^{k-1}\Omega_i\right)\right] \tag{44}$$

Since

$$\mathbb{G}_k = \sum_{i=0}^{k}\Omega_i \tag{45}$$

the above relation can be rewritten as

$$\mathcal{N}\left(\sum_{k=0}^{\infty}\Omega_k\right) = \mathcal{N}(\Omega_0) + \sum_{k=1}^{\infty}\left[\mathcal{N}(\mathbb{G}_k) - \mathcal{N}(\mathbb{G}_{k-1})\right] \tag{46}$$

Now, we compute

$$\begin{aligned}|\mathbb{G}_n - \mathbb{G}_q| &= \left|\sum_{k=q+1}^{n}\Omega_k\right| \\ &= \left|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[\sum_{k=q}^{n-1}\left(\mathcal{N}(\mathbb{G}_k) - \mathcal{N}(\mathbb{G}_{k-1})\right)\right]\right]\right| \\ &= \left|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[\mathcal{N}(\mathbb{G}_{n-1}) - \mathcal{N}(\mathbb{G}_{q-1})\right]\right]\right|\end{aligned} \tag{47}$$

Using the definition of the nonlinear operator yields

$$\begin{aligned}|\mathbb{G}_n - \mathbb{G}_q| = \Bigg|\mathcal{L}^{-1}\Bigg[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\Bigg[&A(\mathbb{G}_{n-1}\mathbb{G}_{\xi\xi\xi,n-1} - \mathbb{G}_{q-1}\mathbb{G}_{\xi\xi\xi,q-1}) \\ &+ B(\mathbb{G}_{\xi,n-1}\mathbb{G}_{\xi\xi,n-1} - \mathbb{G}_{\xi,q-1}\mathbb{G}_{\xi\xi,q-1}) \\ &+ C(\mathbb{G}_{n-1}^2\mathbb{G}_{\xi,n-1} - \mathbb{G}_{q-1}^2\mathbb{G}_{\xi,q-1})\Bigg]\Bigg]\Bigg|\end{aligned} \tag{48}$$

Applying the triangle inequality gives

$$\begin{aligned}|\mathbb{G}_n - \mathbb{G}_q| \le\ &|A|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[|\mathbb{G}_{n-1}\mathbb{G}_{\xi\xi\xi,n-1} - \mathbb{G}_{q-1}\mathbb{G}_{\xi\xi\xi,q-1}|\right]\right] \\ &+ |B|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[|\mathbb{G}_{\xi,n-1}\mathbb{G}_{\xi\xi,n-1} - \mathbb{G}_{\xi,q-1}\mathbb{G}_{\xi\xi,q-1}|\right]\right] \\ &+ |C|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[|\mathbb{G}_{n-1}^2\mathbb{G}_{\xi,n-1} - \mathbb{G}_{q-1}^2\mathbb{G}_{\xi,q-1}|\right]\right]\end{aligned} \tag{49}$$

Applying the sup norms and Lipschitz conditions, we get

$$\begin{aligned}\|\mathbb{G}_n - \mathbb{G}_q\| \leq L_A|A|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[\|\mathbb{G}_{n-1} - \mathbb{G}_{q-1}\|\right]\right] + L_B|B|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[\|\mathbb{G}_{n-1} - \mathbb{G}_{q-1}\|\right]\right] \\ + L_C|C|\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[\|\mathbb{G}_{n-1} - \mathbb{G}_{q-1}\|\right]\right] + L_N\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[\|\mathbb{G}_{n-1} - \mathbb{G}_{q-1}\|\right]\right]\end{aligned}$$

Where

$$L = \sup\left(L_A|A|,\ L_B|B|,\ L_C|C|,\ L_N\right), \qquad \psi = L\cdot\mathcal{L}^{-1}\left[\frac{\mathfrak{F}(\mathfrak{s})}{\mathfrak{s}}\right]. \tag{50}$$

$$\begin{aligned}&\leq L\cdot\mathcal{L}^{-1}\left[\mathfrak{F}(\mathfrak{s})\cdot\mathcal{L}\left[\|\mathbb{G}_{n-1} - \mathbb{G}_{q-1}\|\right]\right] \\ &\leq L\cdot\mathcal{L}^{-1}\left[\frac{\mathfrak{F}(\mathfrak{s})}{\mathfrak{s}}\right]\|\mathbb{G}_{n-1} - \mathbb{G}_{q-1}\| \\ &\leq \psi\|\mathbb{G}_{n-1} - \mathbb{G}_{q-1}\|.\end{aligned} \tag{51}$$

Substituting $n = q + 1$ into the contraction inequality yields the following successive-difference estimate:

$$\begin{aligned}\|\mathbb{G}_{q+1} - \mathbb{G}_q\| &\leq \psi\|\mathbb{G}_q - \mathbb{G}_{q-1}\| \\ &\leq \psi^2\|\mathbb{G}_{q-1} - \mathbb{G}_{q-2}\| \\ &\leq \psi^3\|\mathbb{G}_{q-2} - \mathbb{G}_{q-3}\| \\ &\ \ \vdots \\ &\leq \psi^q\|\mathbb{G}_1 - \mathbb{G}_0\|.\end{aligned} \tag{52}$$

For arbitrary $n > q$, we write

$$\mathbb{G}_n - \mathbb{G}_q = \sum_{k=q}^{n-1}(\mathbb{G}_{k+1} - \mathbb{G}_k). \tag{53}$$

Taking the sup norms and using the geometric bound yields

$$\begin{aligned}\|\mathbb{G}_n - \mathbb{G}_q\| &\leq \sum_{k=q}^{n-1}\|\mathbb{G}_{k+1} - \mathbb{G}_k\| \\ &\leq \sum_{k=q}^{n-1}\psi^k\|\mathbb{G}_1 - \mathbb{G}_0\|.\end{aligned} \tag{54}$$

Letting $n \to \infty$ and summing the geometric series gives

$$\|\mathbb{G}_n - \mathbb{G}_q\| \leq \sum_{k=q}^{\infty}\psi^k\|\mathbb{G}_1 - \mathbb{G}_0\| = \frac{\psi^q}{1-\psi}\|\mathbb{G}_1 - \mathbb{G}_0\|. \tag{55}$$

Since ψ belongs to the interval $[0, 1)$, the right-hand side of the previous sequence approaches zero as $q \to \infty$, and the sequence $\{\mathbb{G}_n\}$ is a Cauchy sequence in $\mathbb{X}$; therefore, it converges uniformly to $\Omega^* \in \mathbb{X}$. Since $\mathbb{H}$ is a continuous operator and the sequence is iterative,

$$\mathbb{H}(\Omega^*) = \Omega^*. \tag{56}$$

can be obtained. Therefore, Ω^* represents the only constant point of the operator $\mathbb{H}$, and thus it is the only continuous solution to the fractional fKdV equation on $\mathbb{R} \times [0, T]$. Consequently, an estimate of the cutoff error corresponding to the partial sum of order N-th can be derived as follows:

$$\|\Omega^* - \mathbb{G}_N\| \leq \frac{\psi^N}{1-\psi}\|\mathbb{G}_1 - \mathbb{G}_0\|. \tag{57}$$

Remark 4.1
The parameter ψ is the contraction factor of the iterative procedure operator H. The condition $0 \leq \psi < 1$ guaranties that the difference of successive approximations falls off geometrically. Hence, the unique fixed point to which the sequence of iterations converges uniformly to. Ω^* The smaller ψ, the faster the convergence, while a value close to 1 suggests slower convergence and may require more iterative parts to obtain a given accuracy. So the The condition $\psi < 1$ is the mathematical guaranty of convergence and the speed of convergence of the proposed LNIM.

5. Numerical Experiments

In this section of the manuscript, we find the semi-analytical solution to both the time fractional Sawada-Kotera equation in and the time fractional Lax equation.

Example 1
[22]

$$^{C}D_{\tau}^{\alpha}\Omega + \Omega_{x\xi\xi\xi\xi} + 15\Omega\Omega_{\xi\xi\xi} + 15\Omega_{\xi}\Omega_{\xi\xi} + 45\Omega^2\Omega_{\xi} = 0, \qquad \tau > 0, 0 < \alpha \leq 1 \tag{58}$$

$$\Omega(\xi, 0) = 2k^2\text{sech}^2(k\xi), \tag{59}$$

Applying the Laplace-Iterative to both sides of Equation (58) augmented with the given initial condition, then, produces that:

$$\Omega = \mathcal{L}^{-1}\left[\frac{1}{\mathfrak{s}^{\alpha}} \cdot \mathcal{L}\left[-\Omega_{\xi\xi\xi\xi\xi} - 15\Omega\Omega_{\xi\xi\xi} - 15\Omega_{\xi}\Omega_{\xi\xi} - 45\Omega^2\Omega_{\xi}\right]\right] \tag{60}$$

Then we have:

$$\Omega_0(\xi, \tau) = 2k^2\text{sech}^2(k\xi), \tag{61}$$

$$\Omega_1(\xi, \tau) = \frac{64k^7\tau^{\alpha}\tanh(k\xi)\text{sech}^2(k\xi)}{\Gamma(\alpha+1)}, \tag{62}$$

$$\begin{aligned}\Omega_2(\xi,\tau) =&\frac{7680k^{12}\tau^{2\alpha}\text{sech}^8(k\xi)}{\Gamma(2\alpha+1)} + \frac{64k^{12}\tau^{2\alpha}(1191\cosh(2k\xi) - 120\cosh(4k\xi) + \cosh(6k\xi) - 1208)\text{sech}^8(k\xi)}{\Gamma(2\alpha+1)}\\ &+ \frac{23592960k^{22}\Gamma(3\alpha+1)\tau^{4\alpha}\tanh^4(k\xi)\text{sech}^6(k\xi)}{\Gamma(\alpha+1)^3\Gamma(4\alpha+1)} + \frac{983040k^{17}\Gamma(2\alpha+1)\tau^{3\alpha}\tanh^5(k\xi)\text{sech}^4(k\xi)}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)}\\ &+ \frac{960k^7\tau^{\alpha}\tanh^3(k\xi)\text{sech}^4(k\xi)}{\Gamma(\alpha+1)} - \frac{92160k^{12}\tau^{2\alpha}\tanh^2(k\xi)\text{sech}^6(k\xi)}{\Gamma(2\alpha+1)} - \frac{480k^7\tau^{\alpha}\tanh(k\xi)\text{sech}^6(k\xi)}{\Gamma(\alpha+1)}\\ &+ \frac{245760k^{17}\Gamma(2\alpha+1)\tau^{3\alpha}\tanh(k\xi)\text{sech}^8(k\xi)}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} - \frac{11796480k^{22}\Gamma(3\alpha+1)\tau^{4\alpha}\tanh^2(k\xi)\text{sech}^8(k\xi)}{\Gamma(\alpha+1)^3\Gamma(4\alpha+1)}\\ &- \frac{1720320k^{17}\Gamma(2\alpha+1)\tau^{3\alpha}\tanh^3(k\xi)\text{sech}^6(k\xi)}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} - \frac{480k^7\tau^{\alpha}(\cosh(2k\xi)-2)\tanh(k\xi)\text{sech}^6(k\xi)}{\Gamma(\alpha+1)}\\ &+ \frac{61440k^{12}\tau^{2\alpha}\tanh^4(k\xi)\text{sech}^4(k\xi)}{\Gamma(2\alpha+1)}\end{aligned} \tag{63}$$

we obtain the approximate solution of Equation (58), as the below:

$$\Omega = \Omega_0 + \Omega_1 + \Omega_2 \tag{64}$$

$$\begin{aligned}\Omega(\xi,\tau) =&2k^2\text{sech}^2(k\xi) + \frac{64k^{12}\tau^{2\alpha}(1191\cosh(2k\xi) - 120\cosh(4k\xi) + \cosh(6k\xi) - 1208)\text{sech}^8(k\xi)}{\Gamma(2\alpha+1)}\\ &+ \frac{960k^7\tau^{\alpha}\tanh^3(k\xi)\text{sech}^4(k\xi)}{\Gamma(\alpha+1)} - \frac{480k^7\tau^{\alpha}\tanh(k\xi)\text{sech}^6(k\xi)}{\Gamma(\alpha+1)} - \frac{92160k^{12}\tau^{2\alpha}\tanh^2(k\xi)\text{sech}^6(k\xi)}{\Gamma(2\alpha+1)}\\ &+ \frac{245760k^{17}\Gamma(2\alpha+1)\tau^{3\alpha}\tanh(k\xi)\text{sech}^8(k\xi)}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} + \frac{7680k^{12}\tau^{2\alpha}\text{sech}^8(k\xi)}{\Gamma(2\alpha+1)} + \frac{64k^7\tau^{\alpha}\tanh(k\xi)\text{sech}^2(k\xi)}{\Gamma(\alpha+1)}\\ &+ \frac{23592960k^{22}\Gamma(3\alpha+1)\tau^{4\alpha}\tanh^4(k\xi)\text{sech}^6(k\xi)}{\Gamma(\alpha+1)^3\Gamma(4\alpha+1)} - \frac{480k^7\tau^{\alpha}(\cosh(2k\xi)-2)\tanh(k\xi)\text{sech}^6(k\xi)}{\Gamma(\alpha+1)}\\ &+ \frac{983040k^{17}\Gamma(2\alpha+1)\tau^{3\alpha}\tanh^5(k\xi)\text{sech}^4(k\xi)}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} - \frac{11796480k^{22}\Gamma(3\alpha+1)\tau^{4\alpha}\tanh^2(k\xi)\text{sech}^8(k\xi)}{\Gamma(\alpha+1)^3\Gamma(4\alpha+1)}\\ &+ \frac{61440k^{12}\tau^{2\alpha}\tanh^4(k\xi)\text{sech}^4(k\xi)}{\Gamma(2\alpha+1)} - \frac{1720320k^{17}\Gamma(2\alpha+1)\tau^{3\alpha}\tanh^3(k\xi)\text{sech}^6(k\xi)}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)}\end{aligned} \tag{65}$$

Note: Symbolic calculations and inverse Laplace transforms were carried out with Mathematica 13.3.

Table 1. Comparison of exact solution and numerical results of Laplace New Iterative Method (LNIM) for $\alpha = 1$ with initial condition $\Omega(\xi, 0) = 2k^2\text{sech}^2(k\xi)$ to Equation (58). For the numerical results presented in Table 1, the parameter k was set to $k = 0.4$.

ξ	τ	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 1.0$	Exact solution	Absolute error	Residual	Absolute residual error
0.01	0.2	0.319533	0.319647	0.319736	0.319735	3.243529×10^{-7}	0.003646	0.003646
0.01	0.4	0.318223	0.318530	0.318792	0.318791	6.321131×10^{-7}	0.029150	0.029150
0.01	0.6	0.316203	0.316712	0.317167	0.317171	4.448502×10^{-6}	0.098496	0.098496
0.01	0.8	0.313534	0.314223	0.314862	0.314888	0.000026	0.234034	0.234034
0.01	1.0	0.310257	0.311086	0.311881	0.311962	0.000081	0.458691	0.458691
0.03	0.2	0.319699	0.319793	0.319863	0.319862	1.461988×10^{-6}	0.003631	0.003631
0.03	0.4	0.318578	0.318858	0.319094	0.319085	9.721592×10^{-6}	0.028887	0.028887
0.03	0.6	0.316751	0.317229	0.317654	0.317628	0.000026	0.097278	0.097278
0.03	0.8	0.314290	0.314945	0.315552	0.315506	0.000046	0.230436	0.230436
0.03	1.0	0.311239	0.312034	0.312795	0.312734	0.000061	0.450296	0.450296
0.05	0.2	0.319824	0.319898	0.319950	0.319948	2.592576×10^{-6}	0.003593	0.003593
0.05	0.4	0.318892	0.319145	0.319356	0.319337	0.000019	0.028441	0.028441
0.05	0.6	0.317260	0.317707	0.318102	0.318046	0.000057	0.095445	0.095445
0.05	0.8	0.315007	0.315628	0.316203	0.316084	0.000118	0.225385	0.225385
0.05	1.0	0.312183	0.312943	0.313670	0.313469	0.000201	0.439074	0.439074
0.07	0.2	0.319908	0.319962	0.319996	0.319993	3.711986×10^{-6}	0.003532	0.003532
0.07	0.4	0.319166	0.319392	0.319577	0.319550	0.000028	0.027816	0.027816
0.07	0.6	0.317729	0.318144	0.318510	0.318424	0.000087	0.093013	0.093013
0.07	0.8	0.315684	0.316272	0.316814	0.316624	0.000189	0.218924	0.218924
0.07	1.0	0.313087	0.313813	0.314506	0.314167	0.000339	0.425117	0.425117
0.10	0.2	0.319958	0.319982	0.319989	0.319983	5.361241×10^{-6}	0.003400	0.003400
0.10	0.4	0.319500	0.319685	0.319832	0.319791	0.000041	0.026555	0.026555
0.10	0.6	0.318356	0.318725	0.319045	0.318915	0.000131	0.088285	0.088285
0.10	0.8	0.316624	0.317161	0.317654	0.317360	0.000294	0.206706	0.206706
0.10	1.0	0.314367	0.315042	0.315683	0.315142	0.000541	0.399311	0.399311

Note. Table 1 presents a comprehensive comparison between the numerical results obtained using the Laplace New Iterative Method (LNIM) and the exact solution to the fractional Sawada-Kotera equation under the initial condition specified in Equation (58). This comparison was carried out for five spatial values, namely ξ = 0.01, 0.03, 0.05, 0.07, 0.1, and five time values: τ = 0.2, 0.4, 0.6, 0.8, 1, as well as four values for the fractional order α = 0.9, 0.95, 1. It is evident from the values obtained from the table that the numerical results converge remarkably closely to the exact solution, particularly as the fractional order α approaches one. When α = 1, the classical differential equation is of exact order. Furthermore, the absolute error for the values of ξ and τ remains within acceptable limits, which reinforces the methods effectiveness and accuracy. From a spatial perspective, as the value of ξ increases, there is a corresponding increase in the absolute error. On the other hand, it remains remarkably stable as time τ varies at the same spatial location This indicates that LNIM exhibits good temporal stability. These results confirm that LNIM is an effective method for modelling fractional partial differential equations.

Table 2. For Example 1, the absolute errors associated with Equation (58) using LNIM are compared with the corresponding results obtained by mVIA-I and MADM. For the numerical results presented in Table 2, the parameter k was set to $k = 0.01$.

ξ/τ	$\tau = 0.2$			$\tau = 0.4$		
	LNIM	mVIA-I [11]	MADM [27]	LNIM	mVIA-I [11]	MADM [27]
2	0	6.23416×10^{-19}	1.54499×10^{-18}	2.71050×10^{-20}	7.04731×10^{-19}	4.52654×10^{-18}
4	2.71050×10^{-20}	4.3368×10^{-19}	5.3668×10^{-18}	2.71050×10^{-20}	4.60785×10^{-19}	1.12757×10^{-17}
6	2.71050×10^{-20}	1.89735×10^{-19}	1.13841×10^{-17}	0	1.6263×10^{-19}	2.02746×10^{-17}
8	8.13151×10^{-20}	7.86046×10^{-19}	1.97054×10^{-17}	5.42101×10^{-20}	8.40255×10^{-19}	3.15232×10^{-17}
10	2.71050×10^{-20}	5.69206×10^{-19}	3.02492×10^{-17}	0	5.42101×10^{-19}	4.50486×10^{-17}

Note. Table 2 shows a comparison of the results obtained by modelling the Sawada-Kotera equation using the Laplace Transform New Iterative Method (LNIM) with those from the mVIA-I and ADM methods, for a selection of different values of the variables ξ and τ. The Laplace Transform New Iterative Method (LNIM) method demonstrated superiority over the other methods by achieving a smaller absolute error and better convergence. This contributed to its efficiency in handling this type of equation.

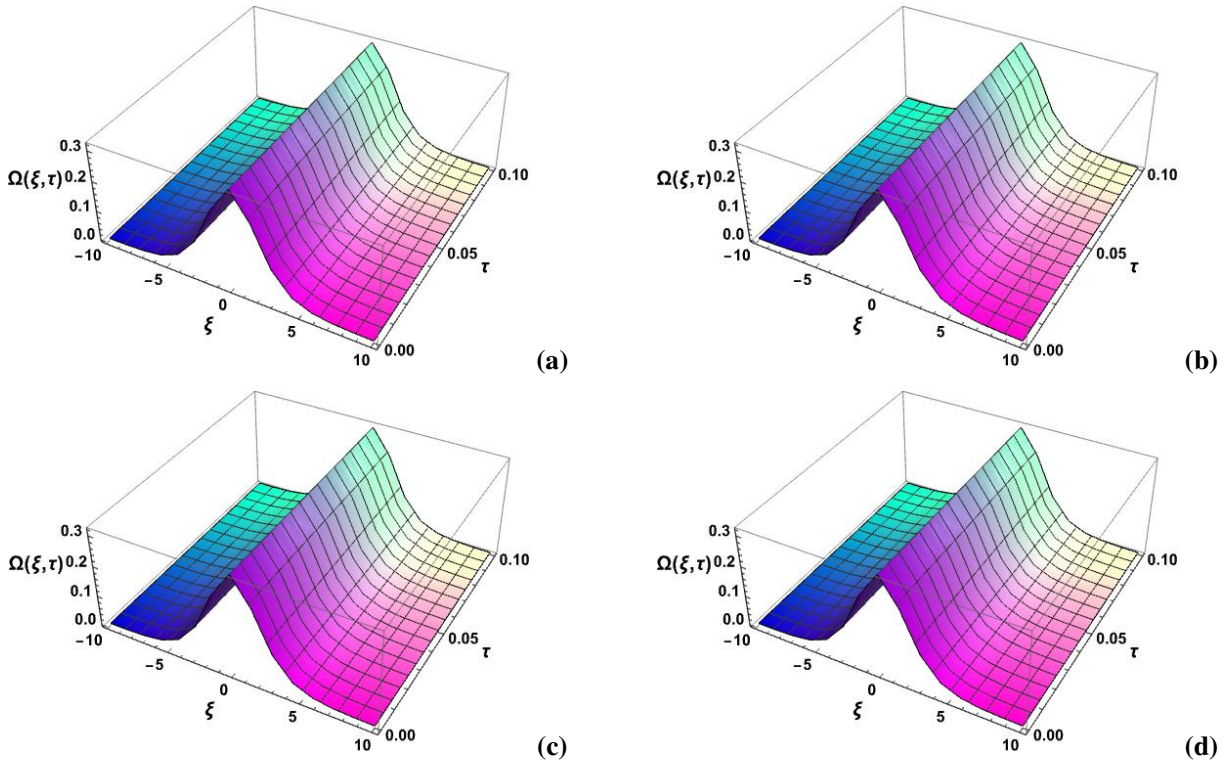
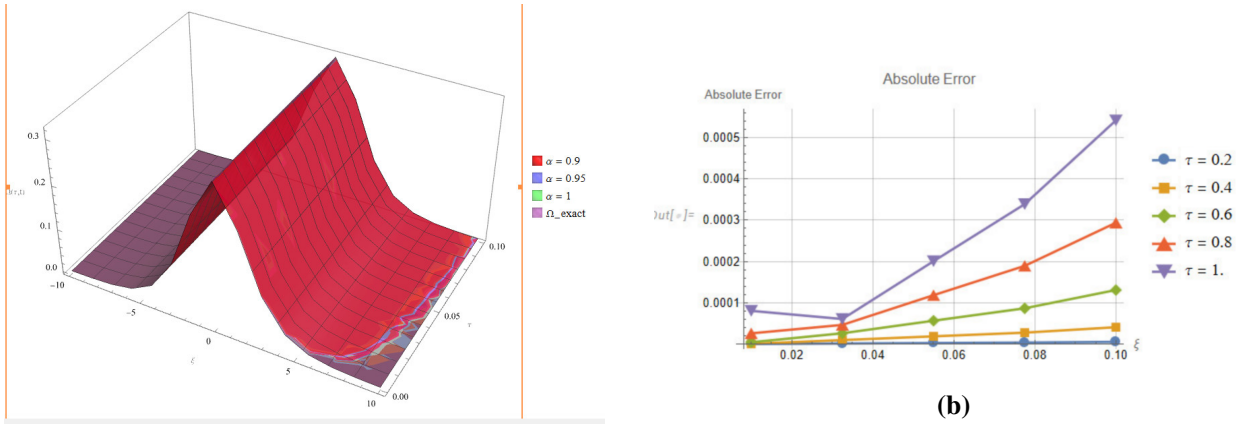


Figure 1. 3D plots of the solution $\Omega(\xi, \tau)$ at Example 1 Generated via Laplace New Iterative Method (LNIM): (a) $alpha = 0.90$, (b) $alpha = 0.95$, (c) $alpha = 1$ and with (d) the exact solution corresponding. The horizontal axs are the spatial variable ξ and time variable τ, and the vertical axis is the solution $\Omega(\xi, \tau)$. Good agreement between approximate solutions and the exact solution demonstrates the accuracy of the proposed LNIM.

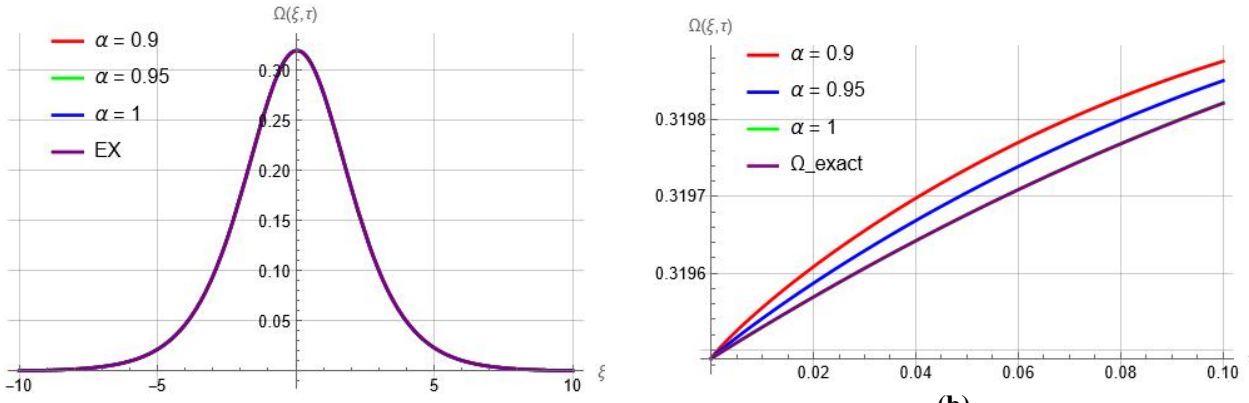
Figure 1 shows three-dimensional plots of the solution $\Omega(\xi, \tau)$ for Example 1 by using Laplace New Iterative Method (LNIM) for different values of the fractional order α. Figures 1(a)–1(c) corresponding to $\alpha = 0.90$, $\alpha = 0.95$ and $\alpha = 1$, respectively, while Figure 1(d) is the corresponding exact solution. The spatial variable ξ and time variable τ are plotted along the vertical axes, and $\Omega(\xi, \tau)$ is shown on the horizontal axis. The approximate solutions display behavior consistent with the exact solution, and their agreement is close enough to show the high accuracy of the suggested LNIM. The overlapping of the solution profiles shows that the method gives a good approximation to the fractional order considered equation.



(a) **(b)**

Figure 2. (a) 3D plots of $\Omega(x,t)$ for $\alpha = 0.9,\ 0.95,\ 1$ and Ω_{exact}, with $-10 \le x \le 10,\ 0 \le t \le 0.10$, showing the convergence to the exact solution and memory induced damping: smaller α values lead to smaller and smoother amplitude. (b) the 2D graph of the Absolute error for any value of τ and $\alpha = 0.90, 0.95, 1, Exactsolution$

(a) The general applicability is shown by considering the fractional order α over the values 0.9, 0.95 and 1 and comparing the obtained surfaces of $\Omega(x,t)$ with the exact solution Ω_{exact} over the domain $-10 \le x \le 10$, $0 \le t \le 0.10$. The method over this range reproduces the expected stable solitary-wave behavior for each α, and the $\alpha = 1$ case recovers the classical (integer-order) solution, confirming consistency with the limiting case. We also note that the Sawada–Kotera and Lax examples considered in the manuscript correspond to different coefficient structures $(45, 15, 15)$ vs $(30, 20, 10)$ and different integrability properties, so the validation is already not for a single representative equation.
(b) Absolute error between the approximate and exact solutions for $\alpha = 0.9, 0.95$ and 1 are shown in Figure 2(b). The error is still small for all the three fractional orders, which shows that the approximate and exact solutions are in good agreement and confirms the accuracy of the proposed method.



(a) **(b)**

Figure 3. (a) The $2D$ graph of the Ω for any value of ξ and $\alpha = 0.90, 0.95, 1, Exactsolution$ (b) the 2D graph of the Ω for any value of τ and $\alpha = 0.90, 0.95, 1, Exactsolution$

Figure 2 shows a high degree of agreement between the numerical solutions obtained for different values of the fractional order α and the exact solution. It can be seen that the curves are almost identical, which highlights the accuracy of the results provided by Laplace New Iterative Method (LNIM). Furthermore, changes in the fractional order of α affect the behaviour of the solution, but only to a slight extent, whilst maintaining the convergence between the numerical solutions and the exact solution. This confirms the effectiveness and stability of the method in modelling fractional partial differential equations of this type

Example 2
[22]

$$ {}^{C}D_t^{\alpha}\Omega + 30\Omega^2\Omega_{\xi} + 30\Omega_{\xi}\Omega_{\xi\xi} + 10\Omega\Omega_{\xi\xi\xi} + \Omega_{\xi\xi\xi\xi\xi} = 0, \qquad t > 0,\ 0 < \alpha \leq 1 \tag{66} $$

$$ \Omega(\xi, 0) = 2k^2\left(2 - 3\tanh^2(k\xi)\right) \tag{67} $$

Applying the Laplace-Iterative to both sides of Equation (66) augmented with the given initial condition, then, produces that:

$$ \Omega = \mathcal{L}^{-1}\left[\frac{1}{\mathfrak{s}^{\alpha}} \cdot \mathcal{L}\left[-30\Omega^2\Omega_{\xi} - 30\Omega_{\xi}\Omega_{\xi\xi} - 10\Omega\Omega_{\xi\xi\xi} - \Omega_{\xi\xi\xi\xi\xi}\right]\right] \tag{68} $$

Then we have:

$$ \Omega_0(\xi, \tau) = 2k^2\left(2 - 3\tanh^2(k\xi)\right), \tag{69} $$

$$ \Omega_1(\xi, \tau) = \frac{12k^7\tau^{\alpha}(148\cosh(2k\xi) + 7\cosh(4k\xi) - 219)\tanh(k\xi)\mathrm{sech}^6(k\xi)}{\Gamma(\alpha+1)}, \tag{70} $$

$$ \begin{aligned} \Omega_2(\xi, \tau) = {} & 3k^{12}\tau^{2\alpha}\mathrm{sech}^{12}(k\xi)\Big(\frac{-5517534\cosh(2k\xi) + 1217808\cosh(4k\xi) - 106707\cosh(6k\xi) + 3556\cosh(8k\xi)}{\Gamma(2\alpha+1)} \\ & + \frac{49\cosh(10k\xi) + 4559244}{\Gamma(2\alpha+1)} + \frac{1}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)}60k^5\Gamma(2\alpha+1)\tau^{\alpha}\mathrm{sech}^5(k\xi)(873542220\sinh(k\xi) \\ & + 15071\sinh(11k\xi) + 49\sinh(13k\xi)) - 449106111\sinh(3k\xi) + 110255375\sinh(5k\xi) \\ & - 10283690\sinh(7k\xi) + 53226\sinh(9k\xi) + \frac{8640k^{10}\tau^{2\alpha}(148\cosh(2k\xi) + 7\cosh(4k\xi) - 219)^2}{\Gamma(\alpha+1)^3\Gamma(4\alpha+1)} \\ & + \Gamma(3\alpha+1)(-2463\cosh(2k\xi) + 240\cosh(4k\xi) + 7\cosh(6k\xi) + 2344)\tanh^2(k\xi)\mathrm{sech}^8(k\xi)\Big) \end{aligned} \tag{71} $$

we obtain the approximate solution of Equation (66), as the below:

$$ \Omega = \Omega_0 + \Omega_1 + \Omega_2 \tag{72} $$

$$ \begin{aligned} \Omega(\xi, \tau) = {} & \frac{3k^{12}\tau^{2\alpha}(-5517534\cosh(2k\xi) + 1217808\cosh(4k\xi) - 106707\cosh(6k\xi) + 3556\cosh(8k\xi)}{\Gamma(2\alpha+1)} \\ & + \frac{49\cosh(10k\xi) + 4559244)\mathrm{sech}^{12}(k\xi)}{\Gamma(2\alpha+1)} + \frac{180k^{17}\Gamma(2\alpha+1)\tau^{3\alpha}\mathrm{sech}^{17}(k\xi)}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)}(873542220\sinh(k\xi) \\ & - 449106111\sinh(3k\xi) + 110255375\sinh(5k\xi) - 10283690\sinh(7k\xi) + 53226\sinh(9k\xi) \\ & + 15071\sinh(11k\xi) + 49\sinh(13k\xi))2k^2\Big(\frac{3k^5\tau^{\alpha}(-586\sinh(k\xi) + 141\sinh(3k\xi) + 7\sinh(5k\xi))\mathrm{sech}^7(k\xi)}{\Gamma(\alpha+1)} \\ & - 3\tanh^2(k\xi) + 2 + \frac{12960k^{20}\tau^{4\alpha}(148\cosh(2k\xi) + 7\cosh(4k\xi) - 219)^2}{\Gamma(\alpha+1)^3\Gamma(4\alpha+1)} + \Gamma(3\alpha+1)\Big(-2463\cosh(2k\xi) \\ & + 240\cosh(4k\xi) + 7\cosh(6k\xi) + 2344\Big)\tanh^2(k\xi)\mathrm{sech}^{20}(k\xi)\Big) \end{aligned} \tag{73} $$

Table 3. Comparison of exact solution and numerical results of Laplace New Iterative Method (LNIM) for $\alpha = 1$ with initial condition $\Omega(\xi, 0) = 2k^2\left(2 - 3\tanh^2(k\xi)\right)$ to Equation (66). For the numerical results presented in Table 3, the parameter k was set to $k = 0.1$.

ξ	τ	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 1.0$	Exact solution	Absolute error	Residual	Absolute residual error
0.01	0.2	0.0399999	0.0399999	0.0399999	0.0200000	0.0200000	-1.2716861×10^{-11}	1.2716861×10^{-11}
0.01	0.4	0.0400000	0.0400000	0.0399999	0.0200000	0.0200000	-1.0068153×10^{-11}	1.0068153×10^{-11}
0.01	0.6	0.0400000	0.0400000	0.0400000	0.0200000	0.0200000	6.9145253×10^{-11}	6.9145253×10^{-11}
0.01	0.8	0.0400001	0.0400000	0.0400000	0.0200000	0.0200000	2.8612265×10^{-10}	2.8612265×10^{-10}
0.01	1.0	0.0400001	0.0400001	0.0400001	0.0200000	0.0200001	7.0206340×10^{-10}	7.0206340×10^{-10}
0.03	0.2	0.0399994	0.0399994	0.0399994	0.0199998	0.0199996	-5.8546629×10^{-11}	5.8546629×10^{-11}
0.03	0.4	0.0399994	0.0399994	0.0399994	0.0199998	0.0199996	-1.9342392×10^{-10}	1.9342392×10^{-10}
0.03	0.6	0.0399994	0.0399994	0.0399994	0.0199998	0.0199996	-3.4348720×10^{-10}	3.4348720×10^{-10}
0.03	0.8	0.0399995	0.0399994	0.0399994	0.0199998	0.0199996	-4.4759085×10^{-10}	4.4759085×10^{-10}
0.03	1.0	0.0399995	0.0399995	0.0399995	0.0199998	0.0199996	-4.4458839×10^{-10}	4.4458839×10^{-10}
0.05	0.2	0.0399984	0.0399984	0.0399984	0.0199995	0.0199989	-1.0434732×10^{-10}	1.0434732×10^{-10}
0.05	0.4	0.0399984	0.0399984	0.0399984	0.0199995	0.0199989	-3.7670042×10^{-10}	3.7670042×10^{-10}
0.05	0.6	0.0399984	0.0399984	0.0399984	0.0199995	0.0199988	-7.5602467×10^{-10}	7.5602467×10^{-10}
0.05	0.8	0.0399984	0.0399984	0.0399983	0.0199995	0.0199988	-1.1812837×10^{-9}	1.1812837×10^{-9}
0.05	1.0	0.0399984	0.0399984	0.0399984	0.0199995	0.0199988	-1.5914395×10^{-9}	1.5914395×10^{-9}
0.07	0.2	0.0399969	0.0399970	0.0399970	0.0199990	0.0199979	-1.5009338×10^{-10}	1.5009338×10^{-10}
0.07	0.4	0.0399969	0.0399969	0.0399969	0.0199990	0.0199978	-5.5979538×10^{-10}	5.5979538×10^{-10}
0.07	0.6	0.0399968	0.0399968	0.0399968	0.0199990	0.0199978	-1.1682369×10^{-9}	1.1682369×10^{-9}
0.07	0.8	0.0399968	0.0399968	0.0399968	0.0199991	0.0199977	-1.9145462×10^{-9}	1.9145462×10^{-9}
0.07	1.0	0.0399968	0.0399968	0.0399968	0.0199991	0.0199977	-2.7378492×10^{-9}	2.7378492×10^{-9}
0.10	0.2	0.0399938	0.0399938	0.0399939	0.0199980	0.0199958	-2.1855423×10^{-10}	2.1855423×10^{-10}
0.10	0.4	0.0399937	0.0399937	0.0399937	0.0199980	0.0199957	-8.3387394×10^{-10}	8.3387394×10^{-10}
0.10	0.6	0.0399936	0.0399936	0.0399936	0.0199980	0.0199956	-1.7854418×10^{-9}	1.7854418×10^{-9}
0.10	0.8	0.0399935	0.0399935	0.0399935	0.0199981	0.0199955	-3.0127369×10^{-9}	3.0127369×10^{-9}
0.10	1.0	0.0399935	0.0399935	0.0399935	0.0199981	0.0199954	-4.4552346×10^{-9}	4.4552346×10^{-9}

Note. Table 3 presents a comparison between the numerical results obtained using the Laplace New Iterative Method (LNIM) and the exact solution to the Lax equation. The results are shown for a number of different values of the spatial variable ξ and the time variable τ, as well as for three values of the fractional order α. A close examination of the results presented in the table reveals that the numerical results converge towards the exact solution as the value of α approaches one. This is intuitive, since when α = 1, this corresponds to a return to the ordinary differential equation. It should be noted that, in all the cases studied, the absolute error is small, which reinforces the accuracy and efficiency of the method. As for the spatial aspect, the higher the value of ξ, the slightly higher the absolute error becomes. However, it changes to a limited extent when the value of τ changes at the same spatial location. This indicates the stability of the method over time. In conclusion, the results obtained confirm that the Laplace New Iterative Method (LNIM) is capable of providing accurate and reliable results in modelling fractional partial differential equations, particularly in cases where no direct analytical solution is available.

Table 4. For Example 2, the absolute errors associated with Equation (66) using LNIM are compared with the corresponding results obtained by mVIA-I and MADM. For the numerical results presented in Table 4, the parameter k was set to $k = 0.01$.

ξ/τ	$t = 0.2$			$t = 0.8$		
	LNIM	mVIA-I [11]	MADM [27]	LNIM	mVIA-I [11]	MADM [27]
2	5.74771×10^{-14}	5.74774×10^{-14}	5.76197×10^{-14}	2.29908×10^{-13}	2.29909×10^{-13}	2.30342×10^{-13}
4	1.14220×10^{-13}	1.14220×10^{-13}	1.15280×10^{-13}	4.56881×10^{-13}	4.56882×10^{-13}	4.60727×10^{-13}
6	1.69509×10^{-13}	1.69509×10^{-13}	1.72985×10^{-13}	6.78036×10^{-13}	6.78036×10^{-13}	6.19153×10^{-13}
8	2.22649×10^{-13}	2.22649×10^{-13}	2.3073×10^{-13}	8.90596×10^{-13}	8.90596×10^{-13}	9.21621×10^{-13}
10	2.72984×10^{-13}	2.72984×10^{-13}	2.88518×10^{-13}	1.09193×10^{-12}	1.09193×10^{-12}	1.15213×10^{-12}

Note. Table 4 presents a comparison of the results obtained using the Laplace Transform New Iterative Method (LNIM) by modelling the Lax equation for various values of the variables ξ and τ, with the results of the mVIA-I and ADM methods. The absolute error in the Laplace Transform New Iterative Method (LNIM) was found to be small compared with the other methods. This reinforces its efficiency in handling this type of equation.

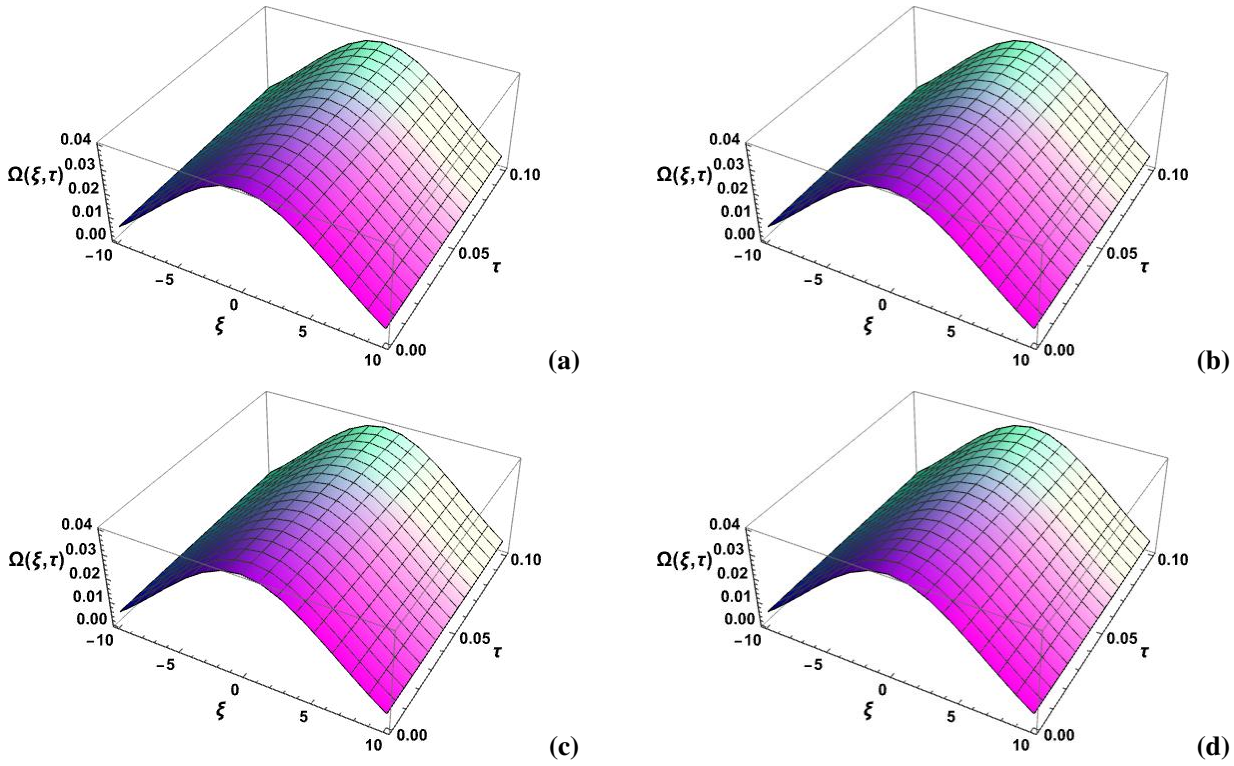
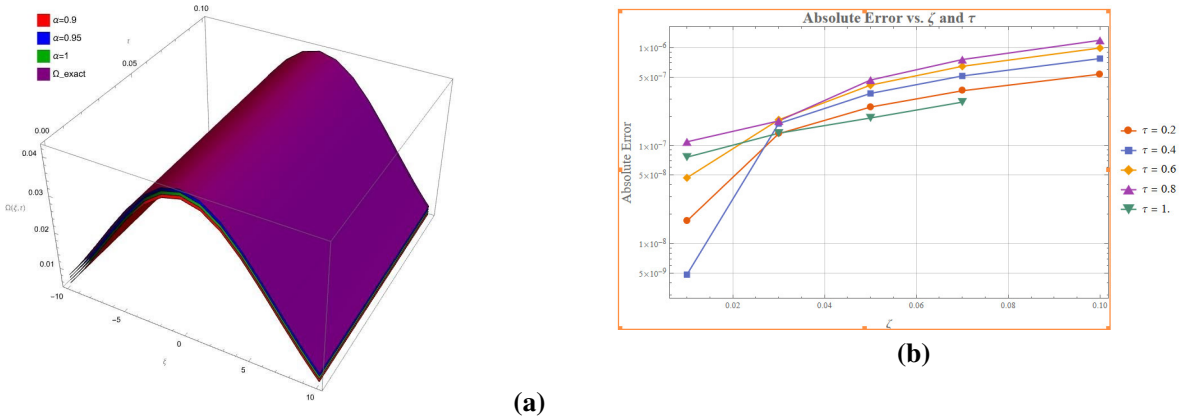


Figure 4. The graphs for Example 2 Three-dimensional representations of the approximate solutions Laplace New Iterative Method (LNIM) for $\Omega(\xi,\tau)$ for Example 2: (a) $\alpha = 0.90$, (b) $\alpha = 0.95$ and (c) $\alpha = 1$ as against (d) the corresponding exact solution. The axs of the horizontal represent the spatial variable ξ and the time variable τ, respectively. the vertical axis is the solution $\Omega(\xi,\tau)$. The near agreement of the approximate and exact solutions indicates the accuracy of the proposed LNIM.

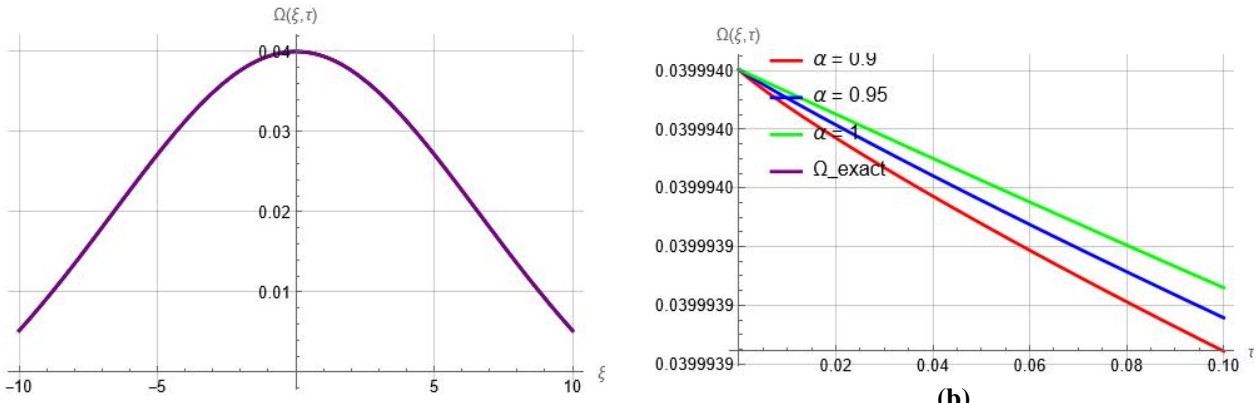
Figure 4 shows the three-dimensional plots of the solution $\Omega(\xi,\tau)$ Example 2 using the Laplace New Iterative Method (LNIM) for different values of fractional order α. Figures 3(a)–3(c) correspond to $\alpha = 0.90$, $\alpha = 0.95$ and $\alpha = 1$, respectively, The exact solution corresponding to this is shown in Fig. 3(d). The space variable ξ and time variable τ are represented along the horizontal axs and $\Omega(\xi,\tau)$ is on the vertical axis. The approximate solutions suggest behavior consistent with the exact solution, and the close agreement between the two shows the high accuracy of the suggested LNIM. In particular the overlap of the solution profiles shows that the method provides a reliable approximation to the fractional equation considered.



(a) **(b)**

Figure 5. (a) 3D plots of $\Omega(x,t)$ for $\alpha = 0.9,\ 0.95,\ 1$ and Ω_{exact}, with $-10 \le x \le 10,\ 0 \le t \le 0.10$, showing the convergence to the exact solution and memory induced damping: smaller α values lead to smaller and smoother amplitude. (b) the 2D graph of the Absolute error for any value of τ and $\alpha = 0.90, 0.95, 1, Exactsolution$

(a) The general applicability is shown by considering the fractional order α over the values 0.9, 0.95 and 1 and comparing the obtained surfaces of $\Omega(x,t)$ with the exact solution Ω_{exact} over the domain $-10 \le x \le 10$, $0 \le t \le 0.10$. The method over this range reproduces the expected stable solitary-wave behavior for each α, and the $\alpha = 1$ case recovers the classical (integer-order) solution, confirming consistency with the limiting case. We also note that the Sawada–Kotera and Lax examples considered in the manuscript correspond to different coefficient structures $(45, 15, 15)$ vs $(30, 20, 10)$ and different integrability properties, so the validation is already not for a single representative equation.
(b) Absolute error between the approximate and exact solutions for $\alpha = 0.9, 0.95$ and 1 are shown in Figure 2(b). The error is still small for all the three fractional orders, which shows that the approximate and exact solutions are in good agreement and confirms the accuracy of the proposed method.



(a) **(b)**

Figure 6. (a) The $2D$ graph of the Ω for any value of ξ and $\alpha = 0.90, 0.95, 1, Exactsolution$ (b) the 2D graph of the Ω for any value of τ and $\alpha = 0.90, 0.95, 1, Exactsolution$

Figure 4 illustrates the solutions obtained by modelling the LAX equation using Laplace New Iterative Method (LNIM). For different values of the fractional order α = 0.90, α = 0.95, and α = 1 they correspond very closely to the exact solutions. It can be observed from the plots that the curves follow the same trend. This demonstrates a high degree of convergence with the exact solutions. This convergence indicates the efficiency of the Laplace New Iterative Method (LNIM) method and its ability to produce accurate and stable results when studying the equation in question.

6. Conclusions

In conclusion, we are pleased to share several findings with the reader. The method employed to solve the nonlinear fractional partial differential equations over time has proven successful. Experience confirms that the values of the initial and approximate solutions are remarkably close for both equations. The approximate solution using Laplace-New Iterative Method(LNIM) for both the Sawada-Kotera and Lax equations was found to be in agreement with the mathematical conditions for each equation. This includes a small absolute ratio and remarkably close results. Future studies may focus on extending this work to other types of nonlinear fractional differential equations and systems of fractional equations. Additionally, the method's effectiveness in modeling more complex models and higher-dimensional problems may be investigated.

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