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# An Intelligent Computational Framework for Multi-Criteria Decision Analysis under Uncertainty

Ahmed A El-Douh[1,5,6,*], Mousa S Janat[2], Myvizhi M[3], Hanadi A Simmak[4]

[1]*College of Informatics, Midocean University, 98123, Moroni, Comoros*
[2]*Technical Electrical Engineering Department, College of Technical Engineering, Ashur University, Baghdad, Iraq*
[3]*Department of Mathematics, KPR Institute of Engineering and Technology, Coimbatore, Tamilnadu, India*
[4]*Biomedical Engineering Department, Engineering Faculty, Ashur University, Baghdad, Iraq*
[5]*Cybersecurity Department, College of Information Technology, Philadelphia University, Amman, Jordan*
[6]*Applied Science Research Center, Applied Science Private University, Amman, Jordan*

**Abstract** Uncertainty, vagueness, and inadequate knowledge sometimes make it hard to make decisions in complicated real-world situations. This research provides an intelligent computational paradigm for multi-criteria decision analysis under uncertainty to address these problems. The suggested approach combines rough set theory with advanced methods for modelling uncertainty to accurately show decision information that is unclear or contradictory. A structured decision-support method is created to assess and prioritize many choices according to a range of criteria that may be conflicting. A sustainable system selection problem, which takes into account technical, environmental, and economic factors all at once, shows how the proposed framework can be used. The experimental findings demonstrate that the suggested methodology improves decision robustness, interpretability, and reliability in comparison to current decision-making models. The suggested intelligent computational framework can be adapted for various decision-support and artificial intelligence applications functioning in uncertain environments, due to its flexibility and universality.

**Keywords** Intelligent Decision Systems, Computational Decision-Making, Multi-Criteria Decision Analysis, Uncertainty Modelling

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## 1. Introduction

*Making decisions* is very important in many real-world situations, especially in engineering, planning for sustainability, and intelligent systems. In these kinds of situations, people who have to make decisions often have to look at a lot of different options and choose the best one based on a number of criteria, some of which may not agree with each other. These problems become much more complicated when the information we have is unclear, incomplete, or vague, which makes traditional decision-making models less useful [1].

Many people use Multi-Criteria Decision Analysis (MCDA) because it is a good way to deal with complicated decisions that have many evaluation attributes [2]. Nonetheless, traditional MCDA methodologies typically presume accurate and clearly delineated input data, a presumption seldom met in real-world contexts. To address this constraint, various computational methodologies have been suggested to represent uncertainty, encompassing fuzzy-based, rough-based, and hybrid uncertainty management techniques [3].

Rough set theory offers a valuable mathematical framework for addressing incomplete and imprecise information through the utilization of lower and upper approximations to depict uncertain knowledge. Advanced uncertainty

*Correspondence to: Ahmed A El-Douh (Email: ahmedeldouh.csis@gmail.com). Department of Cybersecurity, Philadelphia University. Jarash Road, 20 KM out of Amman, Amman Jordan.

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modeling approaches, on the other hand, let decision-makers show different levels of truth, indeterminacy, and inconsistency in evaluation data [2][4]. The amalgamation of these methodologies within a computational framework provides a versatile means to encapsulate the intricacies of real-world decision-making contexts [1].

Inspired by these insights, this research introduces an advanced computational framework for multi-criteria decision analysis in the context of uncertainty. The suggested framework integrates rough-based modeling with sophisticated uncertainty representation techniques to facilitate dependable and comprehensible decision-making. A structured evaluation process is created to look at and rank options based on a number of factors.

A case study of a sustainable system selection problem is used to show how the proposed framework can be used. The results show that the suggested method gives consistent and useful decision results, while still being flexible enough to be used in other areas where decisions are uncertain and complicated.

## 2. Literature Review

Thomason [2], the pioneer of fuzzy matrices, examined the convergence of their powers. Fuzzy matrices are very important for progress in science and engineering. The basic idea behind fuzzy matrices is well known, and most authors focus on the main ideas needed to construct fuzzy models. Instead of presenting detailed and complicated mathematical formulas, examples have been used to illustrate important properties. Numerous studies [3][4][5] have examined the convergence of the power sequence of fuzzy matrices, while Ragab et al. [6] described the properties of the determinant and adjoint of square fuzzy matrices.

Z. Pawlak [1] introduced a new approach for dealing with incomplete knowledge through Rough Set Theory. For many years, philosophers and mathematicians have attempted to address the problem of incomplete or imperfect knowledge. Rough set theory has become increasingly important in computer science, particularly in artificial intelligence, with applications in machine learning, data mining, expert systems, approximate reasoning, and pattern recognition. The theory is based on the assumption that each object in a universe of discourse contains specific information. The indiscernibility relation relies on the fact that objects with identical attribute values cannot be distinguished. The lower and upper approximations represent certain and possible memberships, respectively, and define the boundary region between them.

A decision table consists of condition and decision attributes and represents the most important component of rough set analysis. Each row in the table defines a decision rule that describes an outcome under specific conditions. These rules may be certain or uncertain, corresponding to lower and boundary approximations. The certainty and coverage coefficients are used to measure the reliability of these rules.

Christi DiStefano and her coworkers introduced the concept of matrix energy. Later, Bravo et al. [7] extended this work by establishing several theorems, upper and lower bounds, and other mathematical properties of matrix energy. The matrix energy concept, as an extension of graph energy, explains matrix structure and interrelationships. However, its potential application in neutrosophic and multi-attribute decision-making (MADM) environments has not been fully explored.

Rizvi et al. [8] developed the Rough Intuitionistic Fuzzy Set, and Wei-Zhi Wu et al. [9] introduced Generalized Fuzzy Rough Sets using both axiomatic and constructive methodologies. Gong et al. [10] further extended this concept by introducing interval-valued fuzzy rough sets.

Smarandache [11] proposed the Neutrosophic Set, which enhanced uncertainty representation by defining three independent membership functions, namely truth, indeterminacy, and falsity, each taking values in the interval $(0, 1)$. Zadeh [12] introduced Fuzzy Set Theory as another approach for handling uncertainty.

Otay [13] and Wang [14] introduced neutrosophic fuzzy MCDM models that effectively address ambiguity in complex decision-making contexts.

Broumi et al. [15] later introduced the Rough Neutrosophic Set, combining rough and neutrosophic theories to address partial, vague, and inconsistent information.

Pramanik et al. [16] applied a rough neutrosophic model to engineering selection problems, Biswas and Pramanik [17] studied single-valued neutrosophic decision-making using score and accuracy functions, and Zhang et al. [18] proposed a hybrid rough–fuzzy framework for evaluating water management alternatives. Donbosco and Ganesan

[19] introduced the energy of rough neutrosophic matrices for multi-criteria evaluation, and Martina and Ganesan [20] extended this concept to multi-valued neutrosophic environments.

Multi-Criteria Decision-Making (MCDM) is an important and continuously evolving field of operations research. Fuzzy and neutrosophic approaches have gained popularity because many MCDM problems involve uncertainty, indeterminacy, and incomplete information. Furthermore, several researchers have proposed hybrid MCDM models that combine rough, fuzzy, and neutrosophic environments to support sustainable decision-making.

Notwithstanding the considerable advancements made by hybrid uncertainty modeling methodologies, including rough–fuzzy and rough–neutrosophic frameworks, several limits persist. Most current models depend on triadic neutrosophic structures (T, I, F), which limit their capacity to encompass more intricate and multi-dimensional manifestations of uncertainty, including contradiction, partial truth, and incomplete knowledge. Furthermore, prior research has predominantly concentrated on enhancing uncertainty representation, with little emphasis on integrating structural evaluation metrics, such as matrix energy, to bolster decision stability and robustness.

This paper introduces an Energy-based Rough Heptapartitioned Neutrosophic Set (ERHNS) framework to overcome existing restrictions, expanding traditional neutrosophic representations from a three-dimensional structure to a seven-dimensional membership model. This extension facilitates a more articulate and thorough representation of uncertainty in intricate decision-making contexts.

## 3. Preliminaries

This research expands upon existing theoretical frameworks by proposing an innovative model that integrates the Energy Rough Heptapartitioned Neutrosophic Set (ERHNS) with a Multi-Attribute Decision-Making (MADM) approach. The proposed ERHNS framework combines the structural advantages of Rough Set Theory, the flexibility of Heptapartitioned Neutrosophic Sets, and the analytical power of matrix energy. This integration enables effective modeling of uncertainty, indeterminacy, and inconsistency in complex real-world decision-making scenarios.

### *3.1. Rough sets [1]*

Let $R$ be an equivalence relation on the universal set $U$, known as the indiscernibility relation. The approximation space is defined as the partition of $U$ induced by $R$, denoted by $B = U/R$, which consists of all equivalence (similarity) classes of $U$ with respect to $R$. The lower and upper approximations of a set $S$ in $B$ are denoted by $\underline{B}(S)$ and $\overline{B}(S)$, respectively.

$$\underline{B}(S) = \{\, b \in U : [b]_R \subseteq S \,\}$$

$$\overline{B}(S) = \{\, b \in U : [b]_R \cap S \neq \emptyset \,\}$$

where $[b]_R$ indicate the $R$ equivalence class that has the element $b$ in it.

The pair $B(S) = (\underline{B}(S), \overline{B}(S))$ is named the rough set $S$ in $B$.

### *3.2. Neutrosophic set [11]*

In the universal set $U$, each element $b$ is associated with degrees of truth $T$, indeterminacy $I$, and falsity $F$ in the neutrosophic set, denoted by $D$. Therefore $D = \{(b, T(b), I(b), F(b)) : b \in U\}$.

### *3.3. Rough Neutrosophic set [15]*

Assume that $U$ is a universal set, and that each element $b \in U$ has a degree of $(T, I, F)$ with lower and higher approximations represented by

$$\underline{RN}(S) = \{\, b \in U : (\underline{T}, \underline{I}, \underline{F}),\ [b]_R \subseteq S \,\}$$

$$\overline{RN}(S) = \{\, b \in U : (\overline{T}, \overline{I}, \overline{F}),\ [b]_R \subseteq S \,\}$$

where $0 \leq T + I + F \leq 3$ and $0 \leq \overline{T} + \overline{I} + \overline{F} \leq 3$.
The pair $(\underline{RN}(S), \overline{RN}(S))$ is called the Rough Neutrosophic set.

### 3.4. Energy of a matrix [7]

Let $M$ be the space of $n \times n$ matrices with entries in $\mathbb{B}$ and $P$ be a matrix in $M$ and express the energy as

$$E(P) = \sum_{i=1}^{n} |\beta_i - \gamma|$$

where $\beta_1, \beta_2, \ldots, \beta_n$ are the eigenvalues of $P$ and $\gamma$ is the mean value of eigenvalues.

### 3.5. Energy of Neutrosophic Matrix [19]

Let $P(\mathcal{N})$ be the Neutrosophic matrix with order $n \times n$. It can be stated as three matrices such as truth, indeterminacy and false values.

The Energy of Neutrosophic Matrix is well-defined as

$$E[P(\mathcal{N})] = \langle E[P(T)], E[P(I)], E[P(F)] \rangle$$

$$= \left\langle \sum_{i=1}^{n} |\beta_i - \gamma|, \sum_{i=1}^{n} |\vartheta_i - \gamma|, \sum_{i=1}^{n} |\mu_i - \gamma| \right\rangle$$

This measure reflects the structural stability and discriminative capability of the decision matrix.

where $\beta_i$, $\vartheta_i$ and $\mu_i$ $(i = 1, 2, \ldots, n)$ are the eigenvalues of $T$, $I$ and $F$ values correspondingly and $\gamma$ is denoted as mean values of the given eigenvalues.

The energy of a matrix is defined as the sum of the absolute deviations of its eigenvalues from their mean, which reflects the overall structural distribution of the matrix.

In the context of decision-making, this concept is extended such that matrix energy serves as a quantitative measure of the consistency, stability, and informational richness of the decision data. A higher energy value indicates a more informative and structurally discriminative representation of alternatives.

### 3.6. Heptapartitioned Neutrosophic set [21]

Let $R$ be a non-empty Universe. A Heptapartitioned neutrosophic set (HNS) $A$ over $R$ characterizes each element $p$ in $R$ by an absolute truth-membership function $T_A$, a relative truth membership function $M_A$, a contradiction membership function $C_A$, an ignorance membership function $I_A$, an unknown membership function $U_A$, an absolute falsity membership function $F_A$, and a relative falsity membership function $K_A$ such that for each $p \in R$, $T_A, M_A, C_A, U_A, I_A, K_A, F_A \in [0, 1]$ and

$$A = \left[\, p, T_A(p), M_A(p), C_A(p), U_A(p), I_A(p), K_A(p), F_A(p) : \, p \in R \,\right],$$
$$0 \leq T_A(p) + M_A(p) + C_A(p) + U_A(p) + I_A(p) + K_A(p) + F_A(p) \leq 7.$$

## 4. Proposed Framework

This research expands upon existing theoretical frameworks by presenting an innovative model that integrates the Energy Rough Heptapartitioned Neutrosophic Set (ERHNS) with a Multi-Attribute Decision-Making (MADM) approach. The ERHNS combines the best parts of the structure of Rough Set Theory, the flexibility of

Heptapartitioned Neutrosophic Sets, and the analytical power of matrix energy. This combination lets you model uncertainty, indeterminacy, and inconsistency in real-life situations where you have to make decisions.

The proposed ERHNS–MADM framework is utilized to select the optimal Rainwater Harvesting System by evaluating various technical, environmental, and economic criteria. In the heptapartitioned neutrosophic environment, the model uses an energy-based evaluation system to make decisions more reliable and accurate. The results demonstrate that the ERHNS approach offers a potent, systematic, and rational methodology for addressing complex MADM problems, particularly those related to sustainable system selection and resource management.

## 5. Cross-Domain Generalization of Extended Validation

The ERHNS-MADM framework, which has been presented previously, will be extended to incorporate generalization beyond the particular case study in order to improve applicability. The ERHNS-MADM methodology can also easily be modified for use in other domains including (but not limited to) healthcare (medical diagnosis and treatment options under uncertainty), finance (portfolio construction and risk assessment), and engineering (supplier evaluation and system optimization).

Biswas & Pramanik [17] and Zhang et al. [18] have shown that neutrosophic-based MCDM models are applicable and good performers across multiple domains. Since the ERHNS framework and the neutrosophic-based MCDM models generalize one another, we can infer that this framework will continue to be highly adaptable across multiple domains.

Future directions for research on this method include: applying this method to larger datasets, conducting multi-domain comparative studies, and conducting sensitivity analysis with respect to different degrees of uncertainty.

Table 1 provides a comparison of the proposed HNS based ERHNS-MADM framework's performance compared to other established MCDM techniques like Fuzzy and Neutrosophic classical methods. Traditional methods like TOPSIS and VIKOR are computationally efficient but do not allow for the modelling of more complex uncertainty as do some of the more advanced neutrosophic and fuzzy based approaches, however even this type is still limited to fewer dimensions of membership from which it can choose when making decisions.

## 6. Procedural Framework of Rough Heptapartitioned Neutrosophic Energy Method in MADM Strategy

Suppose a decision-maker aims to select the most suitable alternative from a finite set $D = \{D_1, D_2, \ldots, D_n\}$. Let $B = \{B_1, B_2, \ldots, B_m\}$ denote the finite set of evaluation criteria associated with each alternative. The decision-maker provides assessment information for each alternative $D_i$ $(i = 1, 2, \ldots, n)$ with respect to each attribute $B_j$ $(j = 1, 2, \ldots, m)$ in terms of neutrosophic numbers. All evaluations are organized into a decision matrix, representing the neutrosophic performance of the alternatives under the given attributes. To address the inherent uncertainty, vagueness, and inconsistency in decision-making, each evaluation is represented as a heptapartitioned neutrosophic number consisting of seven independent membership components, capturing different aspects of truth, indeterminacy, and falsity. This enhanced representation provides a richer and more flexible modeling of real-world data compared to traditional fuzzy or intuitionistic fuzzy methods.

A weight vector $W = \{w_1, w_2, \ldots, w_m\}$ is associated with the criteria set $B$, where each weight reflects the relative importance of the corresponding attribute and satisfies the condition $\sum_{j=1}^{m} w_j = 1$.

Using this framework, the aggregated decision matrix is normalized and transformed to enable the computation of an evaluation measure (e.g., matrix energy), which is used to derive an overall performance score and rank all alternatives.

The procedural framework of the proposed Rough Heptapartitioned Neutrosophic Energy Matrix–Multi-Attribute Decision-Making (RHNEM–MADM) method is outlined below (see Figure 1).

**Step 1:** The first part of step one is composed of two distinct stages: (i) collecting evaluations from different decision makers, and (ii) aggregating those evaluations together to form one single decision matrix with all the evaluations for each alternative and attribute.

Table 1. Comparative Analysis of HNS-Based ERHNS–MADM with Existing Methods

| **Criteria / Feature** | **TOPSIS** | **VIKOR** | **Fuzzy MCDM** | **Neutrosophic MCDM (T,I,F)** | **Pythagorean Fuzzy** | **HNS / ERHNS–MADM (Proposed)** |
|---|---|---|---|---|---|---|
| Uncertainty Handling | Low | Low | Moderate | High | High | Very High |
| No. of Membership Components | 1 | 1 | 1 | 3 | 2 | 7 (T, C, U, I, K, F, H) |
| Ability to Model Contradiction | No | No | No | Limited | No | Yes |
| Ability to Model Ignorance | No | No | Partial | Yes | Partial | Yes (explicit) |
| Ability to Capture Hesitation | No | No | Yes | Partial | Yes | Yes (separate component) |
| Handling Incomplete Information | Poor | Poor | Moderate | Good | Good | Excellent |
| Multi-dimensional Uncertainty | No | No | Limited | Moderate | Moderate | High (7-dimensional) |
| Robustness of Ranking | Moderate | Moderate | Moderate | Good | Good | Very High (Energy-based) |
| Sensitivity to Data Variations | High | High | Moderate | Moderate | Moderate | Low (stable rankings) |
| Computational Complexity | $O(nm)$ | $O(nm)$ | $O(nm)$ | $O(nm)$ | $O(nm)$ | $O(n^3)$ |
| Interpretability for Non-Experts | High | High | Moderate | Moderate | Moderate | Moderate (via linguistic scale) |
| Suitability for Large-Scale Problems | High | High | High | Moderate | Moderate | Moderate (needs optimization) |
| Energy-Based Decision Evaluation | No | No | No | No | No | Yes (key contribution) |

To make this easier to understand, I will denote the set of alternatives by: $D = \{D_1, D_2, \ldots, D_n\}$ and the set of attributes by: $B = \{B_1, B_2, \ldots, B_m\}$. The set of decision makers can then be denoted as: $E = \{E_1, E_2, \ldots, E_k\}$.

Each decision maker (i.e., $E_k$), will evaluate $D_i$, for each $B_j$, using a neutrosophic number that has been divided into seven parts. Therefore, based upon the evaluation provided by each decision maker (i.e., $E_k$), a separate decision matrix will be constructed as follows:

$$W^{(k)} = [w_{ij}^{(k)}], \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m$$

where the notation $w_{ij}^{(k)}$ denotes the neutrosophic evaluation that was provided by the decision maker (i.e., $E_k$).

Since there are several decision-makers who are evaluating the same alternatives with respect to the same attribute, the individual decision matrices will be aggregated to produce one collective decision matrix that contains all of the decision-makers' evaluations:

$$W = [w_{ij}]$$

The notation for the collective decision matrix value where $w_{ij}$ will be the aggregation of all of the decision-maker evaluation values for the alternative $D_i$ with respect to attribute $B_j$.

Every alternative is evaluated by decision-makers against each attribute with assigned weighted scores. The resulting data are represented in a matrix $W$ of size $m \times n$, where $m$ attributes are rated by $n$ decision-makers.

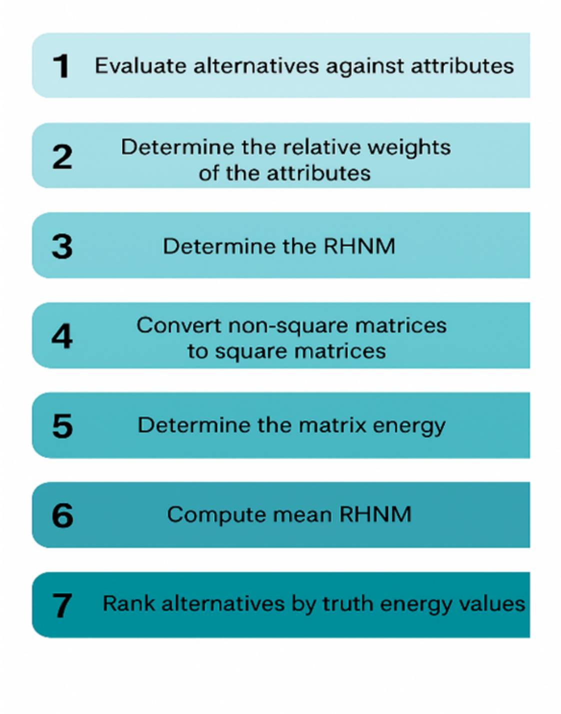


Figure 1. Flow chart of RNEM–MADM strategy.

$$W = \begin{array}{c|cccc} & B_1 & B_2 & \cdots & B_n \\ \hline D_1 & \begin{pmatrix} T_{11}, M_{11}, C_{11}, U_{11} \\ I_{11}, K_{11}, F_{11} \end{pmatrix} & \begin{pmatrix} T_{12}, M_{12}, C_{12}, U_{12} \\ I_{12}, K_{12}, F_{12} \end{pmatrix} & \cdots & \begin{pmatrix} T_{1n}, M_{1n}, C_{1n}, U_{1n} \\ I_{1n}, K_{1n}, F_{1n} \end{pmatrix} \\ D_2 & \begin{pmatrix} T_{21}, M_{21}, C_{21}, U_{21} \\ I_{21}, K_{21}, F_{21} \end{pmatrix} & \begin{pmatrix} T_{22}, M_{22}, C_{22}, U_{22} \\ I_{22}, K_{22}, F_{22} \end{pmatrix} & \cdots & \begin{pmatrix} T_{2n}, M_{2n}, C_{2n}, U_{2n} \\ I_{2n}, K_{2n}, F_{2n} \end{pmatrix} \\ \vdots & \vdots & \vdots & & \vdots \\ D_m & \begin{pmatrix} T_{m1}, M_{m1}, C_{m1}, U_{m1} \\ I_{m1}, K_{m1}, F_{m1} \end{pmatrix} & \begin{pmatrix} T_{m2}, M_{m2}, C_{m2}, U_{m2} \\ I_{m2}, K_{m2}, F_{m2} \end{pmatrix} & \cdots & \begin{pmatrix} T_{mn}, M_{mn}, C_{mn}, U_{mn} \\ I_{mn}, K_{mn}, F_{mn} \end{pmatrix} \end{array}$$

An $n \times m$ matrix may be constructed to represent the ratings of $m$ criteria as assessed by $n$ decision-makers for the available alternatives.

$$\begin{array}{c|cccc} & D_1 & D_2 & \cdots & D_m \\ \hline B_1 & \begin{pmatrix} T_{11}, M_{11}, C_{11}, U_{11} \\ I_{11}, K_{11}, F_{11} \end{pmatrix} & \begin{pmatrix} T_{12}, M_{12}, C_{12}, U_{12} \\ I_{12}, K_{12}, F_{12} \end{pmatrix} & \cdots & \begin{pmatrix} T_{1m}, M_{1m}, C_{1m}, U_{1m} \\ I_{1m}, K_{1m}, F_{1m} \end{pmatrix} \\ B_2 & \begin{pmatrix} T_{21}, M_{21}, C_{21}, U_{21} \\ I_{21}, K_{21}, F_{21} \end{pmatrix} & \begin{pmatrix} T_{22}, M_{22}, C_{22}, U_{22} \\ I_{22}, K_{22}, F_{22} \end{pmatrix} & \cdots & \begin{pmatrix} T_{2m}, M_{2m}, C_{2m}, U_{2m} \\ I_{2m}, K_{2m}, F_{2m} \end{pmatrix} \\ \vdots & \vdots & \vdots & & \vdots \\ B_n & \begin{pmatrix} T_{n1}, M_{n1}, C_{n1}, U_{n1} \\ I_{n1}, K_{n1}, F_{n1} \end{pmatrix} & \begin{pmatrix} T_{n2}, M_{n2}, C_{n2}, U_{n2} \\ I_{n2}, K_{n2}, F_{n2} \end{pmatrix} & \cdots & \begin{pmatrix} T_{nm}, M_{nm}, C_{nm}, U_{nm} \\ I_{nm}, K_{nm}, F_{nm} \end{pmatrix} \end{array}$$

**Step 2:** Determine the relative weights of the attributes as assigned by the decision-makers.

Let $B_1, B_2, \ldots, B_n$ be the decision makers (attributes); each of them has different weights. Assume

$$B_1 = \langle a_1, b_1, c_1, d_1, e_1, f_1, g_1 \rangle, \quad B_2 = \langle a_2, b_2, c_2, d_2, e_2, f_2, g_2 \rangle, \ \ldots, \ \ldots, \ \ldots,$$

$$B_n = \langle a_n, b_n, c_n, d_n, e_n, f_n, g_n \rangle.$$

**Step 3:** The transformation from the aggregated neutrosophic decision matrix to the rough neutrosophic matrix is performed by grouping the evaluations corresponding to each alternative–attribute pair across all decision-makers. For each entry $w_{ij}$, a set of neutrosophic values is formed. The lower approximation is computed by applying minimum operators to the positive components $(T, M, C)$ and maximum operators to the negative components $(U, I, K, F)$, representing a conservative estimate. The upper approximation is obtained using maximum operators for positive components and minimum operators for negative components, reflecting an optimistic estimate.

The lower and upper approximations of each heptapartitioned neutrosophic element are constructed using min–max operators based on rough set theory. The lower approximation represents conservative knowledge by selecting the minimum values for positive components and maximum values for uncertainty-related components. Conversely, the upper approximation captures optimistic knowledge by selecting maximum values for positive components and minimum values for uncertainty components. This formulation ensures that each evaluation is represented as an interval capturing both certainty and possibility, thereby enhancing the robustness of the decision-making process.

Determine the RHNM using the provided attributes and alternative options.
The RHNM is derived by comparing the weighted attributes assigned by the decision-makers.

$$W(D_3B_3) = \left\langle \begin{matrix} \big(\min(a_3, T_{33}), \min(b_3, M_{33}), \min(c_3, C_{33}), \max(d_3, U_{33}), \max(e_3, I_{33}), \max(f_3, K_{33}), \max(g_3, F_{33})\big), \\ \big(\max(a_3, T_{33}), \max(b_3, M_{33}), \max(c_3, C_{33}), \min(d_3, U_{33}), \min(e_3, I_{33}), \min(f_3, K_{33}), \min(g_3, F_{33})\big) \end{matrix} \right\rangle$$

$$W(D_3P_3) = \big\langle (\tilde{T}_{33}, \tilde{M}_{33}, \tilde{C}_{33}, \tilde{U}_{33}, \tilde{I}_{33}, \tilde{K}_{33}, \tilde{F}_{33}), \ (\bar{T}_{33}, \bar{M}_{33}, \bar{C}_{33}, \bar{U}_{33}, \bar{I}_{33}, \bar{K}_{33}, \bar{F}_{33}) \big\rangle$$

$$W = \begin{array}{c|cccc} & B_1 & B_2 & \cdots & B_n \\ \hline D_1 & \begin{pmatrix} \tilde{T}_{11}, \tilde{M}_{11}, \tilde{C}_{11}, \tilde{U}_{11} \\ \tilde{I}_{11}, \tilde{K}_{11}, \tilde{F}_{11} \end{pmatrix} & \begin{pmatrix} \tilde{T}_{12}, \tilde{M}_{12}, \tilde{C}_{12}, \tilde{U}_{12} \\ \tilde{I}_{12}, \tilde{K}_{12}, \tilde{F}_{12} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{T}_{1n}, \tilde{M}_{1n}, \tilde{C}_{1n}, \tilde{U}_{1n} \\ \tilde{I}_{1n}, \tilde{K}_{1n}, \tilde{F}_{1n} \end{pmatrix} \\ D_2 & \begin{pmatrix} \tilde{T}_{21}, \tilde{M}_{21}, \tilde{C}_{21}, \tilde{U}_{21} \\ \tilde{I}_{21}, \tilde{K}_{21}, \tilde{F}_{21} \end{pmatrix} & \begin{pmatrix} \tilde{T}_{22}, \tilde{M}_{22}, \tilde{C}_{22}, \tilde{U}_{22} \\ \tilde{I}_{22}, \tilde{K}_{22}, \tilde{F}_{22} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{T}_{2n}, \tilde{M}_{2n}, \tilde{C}_{2n}, \tilde{U}_{2n} \\ \tilde{I}_{2n}, \tilde{K}_{2n}, \tilde{F}_{2n} \end{pmatrix} \\ \vdots & \vdots & \vdots & & \vdots \\ D_m & \begin{pmatrix} \tilde{T}_{m1}, \tilde{M}_{m1}, \tilde{C}_{m1}, \tilde{U}_{m1} \\ \tilde{I}_{m1}, \tilde{K}_{m1}, \tilde{F}_{m1} \end{pmatrix} & \begin{pmatrix} \tilde{T}_{m2}, \tilde{M}_{m2}, \tilde{C}_{m2}, \tilde{U}_{m2} \\ \tilde{I}_{m2}, \tilde{K}_{m2}, \tilde{F}_{m2} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{T}_{mn}, \tilde{M}_{mn}, \tilde{C}_{mn}, \tilde{U}_{mn} \\ \tilde{I}_{mn}, \tilde{K}_{mn}, \tilde{F}_{mn} \end{pmatrix} \end{array}$$

The RHNM of an alternative reflects the relationship between the attribute weights and the corresponding option.

$$A_3(D_3B_3) = \left\langle \begin{matrix} \big(\min(T_{33}, \tilde{T}_{33}), \min(M_{33}, \tilde{M}_{33}), \min(C_{33}, \tilde{C}_{33}), \max(U_{33}, \tilde{U}_{33}), \max(I_{33}, \tilde{I}_{33})\big), \\ \big(\max(K_{33}, \tilde{K}_{33}), \max(F_{33}, \tilde{F}_{33})\big) \end{matrix} \right\rangle$$

$$= \left\langle \begin{matrix} \big(\max(T_{33}, \bar{T}_{33}), \max(M_{33}, \bar{M}_{33}), \max(C_{33}, \bar{C}_{33}), \min(U_{33}, \bar{U}_{33}), \min(I_{33}, \bar{I}_{33})\big), \\ \big(\min(K_{33}, \bar{K}_{33}), \min(F_{33}, \bar{F}_{33})\big) \end{matrix} \right\rangle$$

$$A_3(D_3B_3) = \big\langle (\tilde{T}_{33}, \tilde{M}_{33}, \tilde{C}_{33}, \tilde{U}_{33}, \tilde{I}_{33}, \tilde{K}_{33}, \tilde{F}_{33}), \ (\bar{T}_{33}, \bar{M}_{33}, \bar{C}_{33}, \bar{U}_{33}, \bar{I}_{33}, \bar{K}_{33}, \bar{F}_{33}) \big\rangle$$

$$A_3 = \begin{array}{c|cc} & D_1 & \cdots \; D_m \\ \hline D_1 & (\tilde{T}_{11}, \tilde{M}_{11}, \tilde{C}_{11}, \tilde{U}_{11}, \tilde{I}_{11}, \tilde{K}_{11}, \tilde{F}_{11}) & \cdots \; (\tilde{T}_{1m}, \tilde{M}_{1m}, \tilde{C}_{1m}, \tilde{U}_{1m}, \tilde{I}_{1m}, \tilde{K}_{1m}, \tilde{F}_{1m}) \\ D_2 & (\tilde{T}_{21}, \tilde{M}_{21}, \tilde{C}_{21}, \tilde{U}_{21}, \tilde{I}_{21}, \tilde{K}_{21}, \tilde{F}_{21}) & \cdots \; (\tilde{T}_{2m}, \tilde{M}_{2m}, \tilde{C}_{2m}, \tilde{U}_{2m}, \tilde{I}_{2m}, \tilde{K}_{2m}, \tilde{F}_{2m}) \\ \vdots & \vdots & \vdots \\ D_n & (\tilde{T}_{n1}, \tilde{M}_{n1}, \tilde{C}_{n1}, \tilde{U}_{n1}, \tilde{I}_{n1}, \tilde{K}_{n1}, \tilde{F}_{n1}) & \cdots \; (\tilde{T}_{nm}, \tilde{M}_{nm}, \tilde{C}_{nm}, \tilde{U}_{nm}, \tilde{I}_{nm}, \tilde{K}_{nm}, \tilde{F}_{nm}) \end{array}$$

**Step 4:** In Step 3, a rough neutrosophic decision matrix was obtained. This decision matrix is usually not square (i.e., $n \times m$). A square matrix is required to calculate energy, and this is done by converting each rough neutrosophic number into a scalar score, using a score function. The result is a crisp matrix that can be multiplied (crisp matrix $\times$ transposition of crisp matrix) with the resulting matrix being a square matrix of order $n \times n$, representing the interrelationship of the alternatives:

$$S = W \cdot W^T$$

This transformation maintains the structural information from the neutrosophic decision matrix and allows for energy values to be calculated via the eigenvalue method in the next step.

**Step 5:** According to the RHNM formulation, determine the matrix energy, resulting in six energies corresponding to the lower and higher approximations.

**Step 6:** The process is repeated for all remaining $k$ alternatives, yielding the mean Rough Heptapartitioned Neutrosophic Matrix (RHNM) energies corresponding to each option.

**Step 7:** In the final stage, the alternatives are ranked according to their truth energy values, with the alternative having the highest truth value identified as the most suitable choice.

## 7. Numerical Demonstration and Application of the Rough Heptapartitioned Neutrosophic Set (RHNS) Matrix

In order to find the best rainwater harvesting system selection in our climatic region, we use the MADM technique to solve the problem. In this problem, the criteria's (attributes) are Cost $(B_1)$, Efficiency $(B_2)$, Environmental Impact $(B_3)$, Adaptability to wet climate $(B_4)$ and Durability $(B_5)$. The following are the alternatives for deciding where we choose: High rainfall $(A_1)$, Solar $(A_2)$, Coastal $(A_3)$, Mixed climate $(A_4)$ and HVAC system $(A_5)$. Among the following options, the decision-makers will be $D_1, D_2, D_3$ and $D_4$. Finally, the best alternative is determined through ranking methods.

Table 2. Linguistic variable for HNN's

| S.No | Linguistic Variable | Hepta-Neutrosophic Numbers |
|---|---|---|
| 1 | Very good $(VG)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ |
| 2 | Good $(G)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ |
| 3 | Moderate $(M)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ |
| 4 | Below Moderate $(BM)$ | $(0.37, 0.1, 0.42, 0.2, 0.2, 0.1, 0.3)$ |
| 5 | Poor $(P)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ |

**Step 1:** As presented in Table 2, linguistic variables are employed by decision-makers to evaluate each alternative's qualities.

**Step 2:** Weights of the decision makers (Attributes) presented in Table 3 and The features are evaluated and rated using linguistic factors prsented in Table 4.

Table 3. Weights of Attributes

| **Attribute** | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ |
|---|---|---|---|---|---|
| $D_1$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ |
| $D_2$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ |
| $D_3$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ |
| $D_4$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ |

Table 4. The features are evaluated and rated using linguistic factors

| **Alternatives / Attributes** | $B$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|---|
| $A_1$ | $B_1$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ |
| | $B_2$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ |
| | $B_3$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ |
| | $B_4$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ |
| | $B_5$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ |
| $A_2$ | $B_1$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ |
| | $B_2$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ |
| | $B_3$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ |
| | $B_4$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ |
| | $B_5$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ |
| $A_3$ | $B_1$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ |
| | $B_2$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ |
| | $B_3$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ |
| | $B_4$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ |
| | $B_5$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ |
| $A_4$ | $B_1$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ |
| | $B_2$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ |
| | $B_3$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ |
| | $B_4$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ |
| | $B_5$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ |
| $A_5$ | $B_1$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ |
| | $B_2$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ |
| | $B_3$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ |
| | $B_4$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ |
| | $B_5$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ |

$$
\begin{aligned}
B_1^* &= M(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4),\\
B_2^* &= VG(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7),\\
B_3^* &= BM(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3),\\
B_4^* &= P(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25),\\
B_5^* &= BM(0.37, 0.1, 0.42, 0.2, 0.1, 0.1, 0.3).
\end{aligned}
$$

**Step 3:** Calculate the Rough Heptapartitioned Neutrosophic Matrix for each attribute.

The relationship among the decision-makers' weights and the options, as presented in Table 5, is formulated as a RNM.

$$
W(D_4B_4) = \left\langle \begin{array}{c} \big(\min(0.25, 0.37), \min(0.1, 0.6), \min(0.2, 0.42), \max(0.2, 0.2), \\ \max(0.3, 0.1), \max(0.1, 0.1), \max(0.3, 0.25)\big), \\ \big(\max(0.25, 0.37), \max(0.1, 0.6), \max(0.2, 0.42), \min(0.2, 0.2), \\ \min(0.3, 0.1), \min(0.1, 0.1), \min(0.3, 0.25)\big) \end{array} \right\rangle
$$

$$
W(D_4B_4) = \langle (0.25, 0.1, 0.2, 0.2, 0.3, 0.1, 0.3),\ (0.37, 0.6, 0.42, 0.2, 0.1, 0.1, 0.25) \rangle
$$

Table 6 illustrates how the relationship between the weights of the qualities and the options forms a Rough Neutrosophic Matrix.

Table 5. Matrix of Rough Heptapartitioned Neutrosophic Attributes

| $D$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ |
|---|---|---|---|---|---|
| $D_1$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$<br>$(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$<br>$(0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$ | $(0.25, 0.1, 0.2, 0.2, 0.3, 0.1, 0.3)$<br>$(0.37, 0.6, 0.42, 0.2, 0.1, 0.1, 0.25)$ | $(0.25, 0.5, 0.2, 0.2, 0.3, 0.2, 0.4)$<br>$(0.6, 0.6, 0.6, 0.1, 0.2, 0.1, 0.25)$ | $(0.25, 0.1, 0.2, 0.2, 0.3, 0.1, 0.3)$<br>$(0.37, 0.6, 0.42, 0.2, 0.1, 0.1, 0.25)$ |
| $D_2$ | $(0.37, 0.1, 0.42, 0.2, 0.2, 0.2, 0.4)$<br>$(0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$<br>$(0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$ | $(0.37, 0.1, 0.4, 0.3, 0.2, 0.3, 0.7)$<br>$(0.4, 0.2, 0.42, 0.1, 0.1, 0.1, 0.3)$ | $(0.25, 0.5, 0.2, 0.2, 0.3, 0.3, 0.6)$<br>$(0.7, 0.6, 0.5, 0.1, 0.1, 0.1, 0.25)$ | $(0.37, 0.1, 0.4, 0.2, 0.2, 0.2, 0.7)$<br>$(0.4, 0.2, 0.42, 0.1, 0.1, 0.1, 0.3)$ |
| $D_3$ | $(0.37, 0.1, 0.42, 0.1, 0.1, 0.1, 0.3)$<br>$(0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$ | $(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$<br>$(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7)$ | $(0.37, 0.1, 0.42, 0.2, 0.2, 0.2, 0.4)$<br>$(0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$ | $(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$<br>$(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$ | $(0.37, 0.1, 0.42, 0.2, 0.2, 0.2, 0.4)$<br>$(0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$ |
| $D_4$ | $(0.6, 0.5, 0.5, 0.1, 0.2, 0.3, 0.6)$<br>$(0.7, 0.5, 0.6, 0.1, 0.1, 0.2, 0.4)$ | $(0.25, 0.2, 0.2, 0.2, 0.3, 0.3, 0.7)$<br>$(0.4, 0.6, 0.4, 0.1, 0.2, 0.1, 0.25)$ | $(0.37, 0.1, 0.42, 0.2, 0.2, 0.2, 0.4)$<br>$(0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$ | $(0.25, 0.1, 0.2, 0.2, 0.3, 0.1, 0.3)$<br>$(0.37, 0.6, 0.42, 0.2, 0.1, 0.1, 0.25)$ | $(0.37, 0.1, 0.4, 0.2, 0.2, 0.3, 0.7)$<br>$(0.4, 0.2, 0.42, 0.1, 0.1, 0.1, 0.3)$ |

$$A_3(D_3B_3) = \left\langle \begin{array}{c} \big(\min(0.37, 0.6), \min(0.2, 0.5), \min(0.42, 0.6), \max(0.2, 0.1), \\ \max(0.1, 0.2), \max(0.1, 0.2), \max(0.3, 0.4)\big), \\ \big(\max(0.37, 0.6), \max(0.2, 0.5), \max(0.42, 0.6), \min(0.2, 0.1), \\ \min(0.1, 0.2), \min(0.1, 0.2), \min(0.3, 0.4)\big) \end{array} \right\rangle$$

$$A_3(D_3B_3) = \langle (0.37, 0.1, 0.41, 0.2, 0.2, 0.2, 0.4),\ (0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3) \rangle$$

Table 6. Alternative 3 – Rough Neutrosophic Matrix

| | |
|---|---|
| $B_1$ | $D_1(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7),\ (0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$<br>$D_2(0.37, 0.1, 0.42, 0.2, 0.2, 0.2, 0.4),\ (0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$<br>$D_3(0.37, 0.1, 0.42, 0.2, 0.2, 0.2, 0.4),\ (0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$<br>$D_4(0.37, 0.1, 0.42, 0.2, 0.2, 0.3, 0.6),\ (0.7, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$ |
| $B_2$ | $D_1(0.37, 0.1, 0.4, 0.2, 0.2, 0.3, 0.7),\ (0.7, 0.5, 0.5, 0.1, 0.1, 0.1, 0.3)$<br>$D_2(0.4, 0.2, 0.4, 0.1, 0.2, 0.3, 0.7),\ (0.6, 0.5, 0.6, 0.1, 0.2, 0.2, 0.4)$<br>$D_3(0.37, 0.1, 0.4, 0.2, 0.2, 0.3, 0.7),\ (0.4, 0.2, 0.42, 0.1, 0.1, 0.1, 0.3)$<br>$D_4(0.25, 0.2, 0.2, 0.2, 0.3, 0.3, 0.7),\ (0.4, 0.6, 0.4, 0.1, 0.2, 0.1, 0.25)$ |
| $B_3$ | $D_1(0.25, 0.1, 0.2, 0.2, 0.3, 0.3, 0.7),\ (0.4, 0.6, 0.42, 0.1, 0.1, 0.1, 0.25)$<br>$D_2(0.37, 0.1, 0.4, 0.2, 0.2, 0.3, 0.7),\ (0.7, 0.5, 0.5, 0.1, 0.1, 0.3, 0.6)$<br>$D_3(0.37, 0.1, 0.42, 0.2, 0.2, 0.2, 0.4),\ (0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$<br>$D_4(0.37, 0.1, 0.42, 0.2, 0.2, 0.2, 0.4),\ (0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$ |
| $B_4$ | $D_1(0.25, 0.5, 0.2, 0.2, 0.3, 0.3, 0.6),\ (0.7, 0.6, 0.6, 0.1, 0.1, 0.1, 0.25)$<br>$D_2(0.25, 0.2, 0.2, 0.2, 0.3, 0.3, 0.7),\ (0.7, 0.6, 0.5, 0.1, 0.1, 0.1, 0.25)$<br>$D_3(0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25),\ (0.25, 0.6, 0.2, 0.2, 0.3, 0.1, 0.25)$<br>$D_4(0.25, 0.1, 0.2, 0.2, 0.3, 0.2, 0.4),\ (0.6, 0.6, 0.6, 0.1, 0.1, 0.1, 0.25)$ |
| $B_5$ | $D_1(0.25, 0.1, 0.2, 0.2, 0.3, 0.1, 0.3),\ (0.37, 0.6, 0.42, 0.2, 0.1, 0.1, 0.25)$<br>$D_2(0.37, 0.1, 0.4, 0.2, 0.2, 0.3, 0.7),\ (0.4, 0.2, 0.42, 0.1, 0.1, 0.1, 0.3)$<br>$D_3(0.37, 0.1, 0.4, 0.2, 0.2, 0.3, 0.7),\ (0.6, 0.5, 0.6, 0.1, 0.1, 0.1, 0.3)$<br>$D_4(0.25, 0.1, 0.2, 0.2, 0.3, 0.3, 0.7),\ (0.4, 0.6, 0.42, 0.1, 0.1, 0.1, 0.25)$ |

**Step 4:** From the non-square matrix, obtain a square matrix.

After translating the matrices from Tables 4 and 5 into six matrices, the truth upper approximation matrices of each table are analyzed.

$$A_3(\overline{T}_{ij})_{n\times m} = \begin{pmatrix} 0.6 & 0.6 & 0.6 & 0.7 \\ 0.7 & 0.6 & 0.4 & 0.4 \\ 0.4 & 0.7 & 0.6 & 0.6 \\ 0.7 & 0.7 & 0.25 & 0.6 \\ 0.37 & 0.4 & 0.6 & 0.4 \end{pmatrix}$$

$$W(\overline{T}_{ij})_{m\times n} = \begin{pmatrix} 0.6 & 0.6 & 0.6 & 0.7 \\ 0.7 & 0.6 & 0.4 & 0.4 \\ 0.37 & 0.4 & 0.6 & 0.6 \\ 0.6 & 0.7 & 0.25 & 0.37 \\ 0.37 & 0.4 & 0.6 & 0.4 \end{pmatrix}$$

$$A_3(\overline{T}_{ij})_{n\times m} \times W(\overline{T}_{ij})_{m\times n} = \begin{pmatrix} 1.57 & 1.3 & 1.242 & 1.189 & 1.102 \\ 1.3 & 1.17 & 0.979 & 1.088 & 0.899 \\ 1.44 & 1.18 & 1.148 & 1.102 & 1.028 \\ 1.14 & 1.25 & 1.049 & 1.1945 & 0.929 \\ 1.102 & 0.899 & 0.8969 & 0.8 & 0.8169 \end{pmatrix}$$

$$A_3(\underline{T}_{ij})_{n\times m} = \begin{pmatrix} 0.4 & 0.37 & 0.37 & 0.37 \\ 0.37 & 0.4 & 0.37 & 0.25 \\ 0.25 & 0.37 & 0.37 & 0.37 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.37 & 0.37 & 0.25 \end{pmatrix}$$

$$W(\underline{T}_{ij})_{m\times n} = \begin{pmatrix} 0.4 & 0.37 & 0.37 & 0.6 \\ 0.4 & 0.4 & 0.4 & 0.25 \\ 0.25 & 0.37 & 0.37 & 0.37 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.37 & 0.37 & 0.37 \end{pmatrix}$$

$$A_3(\underline{T}_{ij})_{n\times m} \times W(\underline{T}_{ij})_{m\times n} = \begin{pmatrix} 0.6558 & 0.5485 & 0.5107 & 0.3775 & 0.5107 \\ 0.5829 & 0.5185 & 0.4699 & 0.3475 & 0.4699 \\ 0.5958 & 0.4885 & 0.4732 & 0.34 & 0.4732 \\ 0.435 & 0.3625 & 0.34 & 0.25 & 0.34 \\ 0.5238 & 0.4585 & 0.4288 & 0.31 & 0.4288 \end{pmatrix}$$

**Step 5:** Computation of the Energy Rough Heptapartitioned Neutrosophic Matrix

The eigenvalues and corresponding energy values must be computed separately for the lower and upper approximation matrices. Let $\underline{T}$ and $\overline{T}$ denote the lower and upper truth matrices, respectively. The energy of a matrix is defined as the sum of the absolute values of its eigenvalues. Accordingly, the eigenvalues of $\underline{T}$ yield an energy value of $8.9958$, while the eigenvalues of $\overline{T}$ result in an energy value of $3.66074$. These values correspond to different matrices and therefore must be interpreted independently.

Eigenvalues of the above matrix for $(\overline{T}_{ij})$ have $0$, $0.0012$, $0.0309$, $0.1896$ and $5.6778$, and the mean value of eigenvalues is $1.1799$.

Eigenvalues of the above matrix for $(\underline{T}_{ij})$ have $0$, $0$, $2.2956$, $0.0153 + 0.0029\,\mathrm{i}$ and $0.0153 - 0.0029\,\mathrm{i}$, and the mean value of eigenvalues is $0.46524$.

$$\begin{aligned} E(\overline{T}_{ij}) &= |0 - 1.1799| + |0.0012 - 1.1799| + |0.0309 - 1.1799| \\ &\quad + |0.1896 - 1.1799| + |5.6778 - 1.1799| = 8.9958. \end{aligned}$$

$$\begin{aligned} E(\underline{T}_{ij}) &= |0 - 0.46524| + |0 - 0.46524| + |2.2956 - 0.46524| \\ &\quad + |0.0153 + 0.0029\,\mathrm{i} - 0.46524| + |0.0153 - 0.0029\,\mathrm{i} - 0.46524| = 3.66074. \end{aligned}$$

**Step 6:** Evaluate the energy for each lower and higher approximation of the $(T, M, C, U, I, K, F)$ matrices.

As mentioned in Figure 1, the energy rough heptapartitioned neutrosophic matrix is given as follows:

$$\text{Place } A_1 = \begin{bmatrix} (4.678,\ 2.43304,\ 4.30724,\ 0.81692,\ 1.19404,\ 1.30868,\ 7.0572), \\ (8.39232,\ 6.8116,\ 7.9006,\ 0.5218,\ 0.66416,\ 0.7082,\ 3.6716) \end{bmatrix}$$

$$\text{Place } A_2 = \begin{bmatrix} (3.61648,\ 1.082,\ 3.56708,\ 0.94284,\ 1.66376,\ 1.67515,\ 9.273548), \\ (9.34888,\ 8.10379,\ 8.44001,\ 0.4448,\ 0.5886,\ 0.5486,\ 5.11542) \end{bmatrix}$$

$$\text{Place } A_3 = \begin{bmatrix} (3.66074,\ 1.5274,\ 3.7218,\ 1.026606,\ 1.2964,\ 4.50331,\ 9.62154), \\ (8.9958,\ 7.8576,\ 8.12276,\ 0.4448,\ 0.60828,\ 0.5188,\ 3.25938) \end{bmatrix}$$

$$\text{Place } A_4 = \begin{bmatrix} (3.6884,\ 1.0402,\ 3.47612,\ 0.9842,\ 1.76264,\ 1.7516,\ 9.4542), \\ (9.13376,\ 7.9397,\ 8.3164,\ 0.4498,\ 0.52300,\ 0.5956,\ 3.3664) \end{bmatrix}$$

$$\text{Place } A_5 = \begin{bmatrix} (3.46308,\ 1.2338,\ 3.26188,\ 0.98624,\ 1.7854,\ 1.83828,\ 9.46626), \\ (10.1982,\ 8.4652,\ 8.8426,\ 0.4448,\ 0.517,\ 0.65584,\ 3.5652) \end{bmatrix}$$

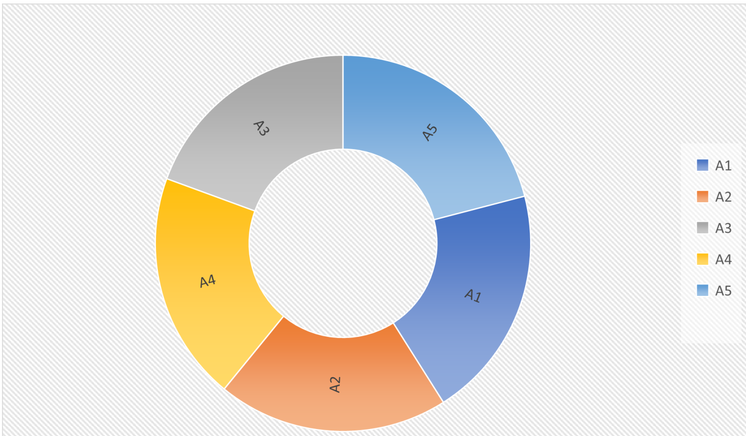


Figure 2. Reality True Energy

**Step 7:** Using the truth values, the order of alternatives is established from the average energy of the lower and higher approximations of the Rough Heptapartitioned Neutrosophic sets.

The average energy for each alternative is shown in the table below:

$$A_1 = [(6.53516,\ 4.62232,\ 6.10392,\ 0.66936,\ 0.9291,\ 1.00844,\ 5.3644)]$$

$$A_2 = [(6.48268,\ 4.59289,\ 6.00354,\ 0.69382,\ 1.12618,\ 1.11188,\ 7.1948)]$$

$$A_3 = [(6.32827,\ 4.6925,\ 5.92228,\ 0.735703,\ 0.95234,\ 2.5111,\ 6.44046)]$$

$$A_4 = [(6.41108,\ 4.48995,\ 5.89626,\ 0.717,\ 1.14282,\ 1.1736,\ 6.4103)]$$

$$A_5 = [(6.83064,\ 4.8495,\ 6.05224,\ 0.71552,\ 1.1512,\ 1.24706,\ 6.51573)]$$

The heptapartitioned neutrosophic model captures multiple dimensions of uncertainty. Truth membership component, however, represents the most direct measure of the desirability of an alternative. The other components $(M, C, U, I, K,$ and $F)$ mainly describe uncertainty, hesitance, and inconsistency. Therefore, truth will be used as the primary-hand ranking indicator of an alternative; the other components will have second-hand influence through the rough set aggregation process. The ranking order of alternatives and average truth energy are shown in Table 7. This computational example shows the procedure using the alternative $A_1$. The truth membership values

Table 7. Average Energy Values of Neutrosophic Components for Each Alternative

| **Alternative** | $E_{\text{avg}}(T)$ | $E_{\text{avg}}(M)$ | $E_{\text{avg}}(C)$ | $E_{\text{avg}}(U)$ | $E_{\text{avg}}(I)$ | $E_{\text{avg}}(K)$ | $E_{\text{avg}}(F)$ |
|---|---|---|---|---|---|---|---|
| $A_1$ | 6.53516 | 4.62232 | 6.10392 | 0.66936 | 0.92910 | 1.00844 | 5.36440 |
| $A_2$ | 6.48268 | 4.59289 | 6.00354 | 0.69382 | 1.12618 | 1.11188 | 7.19480 |
| $A_3$ | 6.32827 | 4.69250 | 5.92228 | 0.73570 | 0.95234 | 2.51110 | 6.44046 |
| $A_4$ | 6.41108 | 4.48995 | 5.89626 | 0.71700 | 1.14282 | 1.17360 | 6.41030 |
| $A_5$ | 6.83064 | 4.84950 | 6.05224 | 0.71552 | 1.15120 | 1.24706 | 6.51573 |

to create square matrices used in two different approximations (lower approximation and upper approximation) by extracting both lower and upper approximation matrix entries are calculated through multiplication of the matrices. Each square matrix's eigenvalues are computed, and their corresponding energy is computed by summing absolute real parts of all eigenvalues. The lower energy for the alternative $A_1$ equals $7.21032$, while its upper energy equals $5.86000$.

A final average energy for truth is calculated as the mean value of both lower and upper energies for the alternative $A_1$ to give:

$$E_{\text{avg}}(A_1) = 6.53516$$

The total of all the positive neutrosophic component energies is calculated by taking the average energy of both the upper approximation and lower approximation energy matrices. More formally, for a component $X \in \{T, M, C, U, I, K, F\}$, the average energy of that component is defined as:

$$E_{\text{avg}}(X_i) = \frac{E(\overline{X}_i) + E(X_i)}{2}$$

The aggregated energy total for each alternative is calculated by summing the average energies of all positive components $(T, M, C)$ and then subtracting the average energies of the uncertainty-related components $(U, I, K, \text{and } F)$ from this total to ensure that all relevant information from both positive and negative perspectives are included in the ranking process. The final ranking of alternatives is presented in Table 8.

Table 8. Organizing in order

| **Alternatives** | **Reality Energy** | **Organizing in order** |
|---|---|---|
| $A_1$ | 6.53516 | II |
| $A_2$ | 6.48268 | III |
| $A_3$ | 6.32827 | V |
| $A_4$ | 6.41108 | IV |
| $A_5$ | 6.83064 | I |

The order in which options are ranked is $A_5 > A_1 > A_2 > A_4 > A_3$.

Finally, the HVAC system $(A_5)$ is identified as the best alternative, achieving the highest rank.

## 8. Conclusion

In this study, the decision-maker aims to identify the most suitable rainwater harvesting system in a climatic region. The evaluation process relies on the energy of the Rough Heptapartitioned Neutrosophic Matrices (RHNMs), which serves as a key factor influencing the weight of each alternative. The energy is computed from the fundamental neutrosophic matrix, where both lower and upper approximations - represented as (T,M,C,U,I,K,F) form the core components of the RHNM. To get the total or resultant energy, you average the lower and higher energy bounds of each approximation. The options are prioritized based on the truth values that were found. So, using RHNM energy consistently and the suggested energy-based approach both help solve Multi-Attribute Decision-Making (MADM) problems when things aren't clear.

## 9. Future Work

The proposed framework has the potential for further expansion in various directions. Future research may explore the integration of alternative uncertainty modeling methodologies or hybrid evaluation strategies to enhance decision-making flexibility and resilience. Moreover, the incorporation of machine learning or optimization methodologies may enable automatic parameter adjustment and improve scalability for larger decision-making challenges. To see how well the framework works in different situations of uncertainty, it can be used in a number of real-world areas, such as smart systems and resource management.

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