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A Novel Possibility Complex Spherical Fuzzy Soft Set Approach for Supplier Selection in Multi-Attribute Decision-Making

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Abstract One aspect of supplier selection that is extremely important to the supply chain's performance and organizational efficiency is supplier selection early in the process. Implementing intelligent decision-support system in the real life supplier evaluation is, however, difficult, since multiple criteria related to supplier's quality, delivery time, cost effectiveness, reliability, sustainability, flexibility and technological competence are involved. In order to overcome this, in this article a novel idea named the Possibility Complex Spherical Fuzzy Soft Set (PCSpFSS) is introduced for effective Multi-Attribute Decision-Making (MADM). The structure PCSpFSS is intuitively explained in this article through an illustrative example and the fundamental set-theoretical properties of PCSpFSS, such as De Morgan's law, are discussed. After that, a PCSpFSS based MADM algorithm is subsequently developed and provided in the form of pseudo code for selecting the best supplier in uncertain and vague environment. A supplier selection case study is provided to illustrate the practicality of the model and the proposed algorithm is implemented in Python. Apart from supporting decision making process, the python code in the model can also be used to perform the sensitivity analysis using different statistical tools, thus, verifying the robustness and reliability of the model. Moreover, a comparative study is carried out between the proposed model and the available models of MADM and the upgraded decision-making ability of the model presented in this paper has been well explained. Finally this article presented the synthesis of the main results obtained and indicated the possible directions for further a study of integration between soft computing technologies and intelligent supply chain management systems for decision making.

Keywords Supplier selection; Supply chain management; MADM; Fuzzy logics; Algorithm; Uncertainty; Decision support system.

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1. Introduction

In recent times, intelligent decision-support systems and Artificial Intelligence (AI)-based technologies have gained the status of transformative tools in modern industry and commerce, which have facilitated faster, efficient and reliable decision making systems in these sectors [1, 2, 3]. With global markets growing and getting more competitive, there is a growing pressure on organizations to choose suppliers who can offer them quality products and services at competitive prices and can sustain, remain reliable and flexible. In these complex environments new innovations, powered by AI, that include machine learning (ML), deep learning, optimization techniques, and soft computing methods have proven to be promising and useful in enhancing supplier evaluation and selection procedures. These clever approaches come in handy when handling large and uncertain collection of supplier data like delivery performance, product quality, operational efficiency, technological capability, environmental

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sustainability, risks etc. With the support of AI, some decision-support systems help provide strategic insights to improve traditional supplier selection and boost company competitiveness [4, 5, 6, 7, 8].
In order to go beyond such restrictions, different researchers used the combination of each framework with the concepts of fuzzy set theory, intuitionistic fuzzy sets, Pythagorean fuzzy sets, spherical fuzzy sets and soft set theory with MADM. These advanced extensions allow bounded vagueness and imprecise preferences of evaluations which are inherent in supplier selection and supply chain management problems as well as uncertain evaluations and uncertain decision parameters. In these methods, the fuzzy soft set based MADM approaches have received significant attention since they are capable of addressing the parametric uncertainties and the vague decision making simultaneously in the complex industrial environments [9, 10, 11, 12].

1.1. Literature review

The classical set is defined as a collection of elements, where each element is well defined and different from the other elements; it doesn't define any specific properties of the elements themselves. The number of students in a class, for instance, constitutes a set, but not the participation of each student (e.g. teacher partially, fully or not at all). This restriction was overcome by Zadeh [13] by introducing the Fuzzy Set (FS) where the elements have a membership degree that describes how much they belong. But FS is incapable to present the non-membership degree that is needed to control uncertainty more precisely. However, to overcome this limitation, Atanassov [14] developed the Intuitionistic Fuzzy Set (IFS) which assigns to each element of the set a membership degree and a non-membership degree such that their sum is in the unit interval $[0, 1]$. However, the IFS does have a drawback in that they only accept a sum of degrees within this unit interval and therefore lack flexibility.
In order to overcome this fact, Yager [15] proposed the Pythagorean Fuzzy Set (PyFS) which has a weaker constraint, namely the sum of the squares of membership and non-membership degrees is in the unit interval. Such existing structures are extensions of the fuzzy set theory for real-valued functions. However, they do not handle the complex number system, which is compulsory for modeling periodic or phase-based uncertainty. To cover such type of uncertainty, Ramot et al. [16] presented the structure named Complex Fuzzy Set (CFS), by using the complex-valued membership function instead of the real-valued membership function, introducing both magnitude and phase. After this, some extensions of fuzzy set towards complex systems, were also introduced like Complex Intuitionistic Fuzzy Set (CIFS) [17], Complex Picture Fuzzy Set (CPFS) [18] with help of complex valued non-membership and neutral degree functions.
Since the above mentioned structures, are useful in handling the uncertainty due to data either it is linear or periodic form, but on the other hand, to handle the decision-making scenarios dealing with parameters, there was needed a structure. To overcome this gap, Molodtsov [19] introduced the Soft Set (SS), in which each element of the universal set is attached with a some number of parameters. However, SS handles is for parametric uncertainty only, but does not handle data uncertainty.
To simultaneously address both parameter and data uncertainty, researchers proposed hybrid structures. Çağman et al. [20] introduced the Fuzzy Soft Set (FSS) by combining FS and SS. Subsequently, the concept was extended by Thirunavukarasu et al. [21] to Complex Fuzzy Soft Set (CFSS) by incorporating complex-valued membership functions, making it more effective in managing large or periodic datasets. Further advancements led to the Complex Intuitionistic Fuzzy Soft Set (CIFSS) [22] by integrating CIFS with SS. As previously noted, the limitation of CIFS concerning the unit circle constraint was also present in CIFSS. This led to the development of the Complex Pythagorean Fuzzy Soft Set (CPyFSS) [23], which combines the strengths of CPyFS and SS, offering a broader capability to handle uncertainty and vagueness compared to CIFSS.
Despite these advancements, existing structures still lacked a structure for modeling degree of importance, which quantify the degree of importance of parameters. To overcome this drawback, this research introduced an innovative concept of the degree of importance as possibility degree.

1.2. Research Gap and Motivation

As detailed in the literature, each successive framework up to the Complex Spherical Fuzzy Soft Set (CSpFSS) has been developed to address specific limitations of its predecessors. However, although CSpFSS provides a flexible mechanism for representing complex-valued membership, non-membership, and neutral information within a spherical fuzzy soft-set environment, it does not explicitly incorporate a possibility or acceptance degree associated with the decision parameters. Consequently, the framework does not directly distinguish the uncertainty contained in an evaluation from the degree to which a particular parameter is considered possible, relevant, or acceptable in a given decision-making environment. This limitation becomes important in multi-attribute decision-making problems where different parameters may have different levels of relevance or acceptance.

The motivation for introducing the proposed Possibility Complex Spherical Fuzzy Soft Set (PCSpFSS) is therefore based on the complementary roles of its constituent components. The complex-valued representation enables the simultaneous characterization of the magnitude and phase of an assessment, which can be useful when decision information exhibits periodic, cyclic, or phase-dependent characteristics. The spherical fuzzy structure allows membership, non-membership, and neutral information to be represented simultaneously, thereby providing greater flexibility for expressing positive, negative, and neutral or hesitant assessments. The soft-set component provides parameterization of the uncertain information and maintains the relationship between an evaluation and its corresponding decision criterion. Finally, the possibility degree introduces additional information concerning the possibility, relevance, or acceptance associated with the parameters.

These components address different dimensions of uncertainty and are therefore not redundant. A conventional fuzzy representation primarily provides membership information but does not directly incorporate phase information or the simultaneous representation of membership, non-membership, and neutral degrees. A complex fuzzy representation can incorporate phase information, but it does not inherently provide the parameterized structure of soft sets or the spherical representation of membership, non-membership, and neutral information. Similarly, a soft-set representation provides parameterization but does not inherently incorporate complex-valued phase information and possibility degrees. Thus, integrating these complementary components provides a unified representation for decision-making situations in which complex-valued evaluations, spherical fuzzy information, parameter-dependent uncertainty, and different levels of parameter possibility or acceptance coexist.

In the context of supplier selection, for example, a decision maker may evaluate suppliers according to quality, delivery performance, cost, technological capability, and sustainability. Such evaluations may contain positive, negative, and neutral opinions, while some information may exhibit periodic or phase-dependent characteristics. At the same time, the criteria may differ in their relevance or acceptance within a particular decision environment. These characteristics motivate the use of an integrated framework capable of representing the different forms of information simultaneously. Accordingly, PCSpFSS is proposed to provide a more comprehensive representation of heterogeneous uncertain information and is subsequently incorporated into a multi-attribute decision-making framework for supplier selection.

Therefore, the proposed PCSpFSS does not merely combine existing structures; rather, it is intended to address the complementary limitations of these structures by jointly incorporating complex-valued information, spherical fuzzy assessments, soft parameterization, and possibility or acceptance information within a single decision-making framework.

The main contributions of this study are summarized as follows:

- A Possibility Complex Spherical Fuzzy Soft Set (PCSpFSS) is developed to jointly represent complex-valued information, spherical fuzzy assessments, soft parameterization, and possibility/acceptance information.
- The distinct roles of complex magnitude and phase, membership, non-membership and neutral information, parameter uncertainty, and possibility/acceptance degrees are clarified within the proposed framework.
- A PCSpFSS-based multi-attribute decision-making procedure is developed in which criterion weights are incorporated into the aggregation of criterion-level evaluations.
- A hypothetical supplier-selection problem is used to demonstrate the mathematical applicability and computational validity of the proposed framework.
- Comparative, sensitivity, ablation, and scalability analyses are conducted to investigate the behavior and robustness of the proposed decision-making procedure.

1.3. Arrangement of the article

In the first section, the article presents a detailed introduction and literature review to establish the research gap and highlight the motivation behind the study. Section 2 provides the formal definition of the PCSpFSS framework,

along with a real-life example that helps to illustrate the concept clearly. This section also presents several set-theoretical properties of PCSpFSS, each explained with the aid of examples. Section 3 discusses the general problem statement regarding selection of supplier for best supply chain management, followed by a PCSpFSS-based MADM algorithmic approach designed to address this challenge. A case study on supplier selection using AI strategies is also presented, demonstrating how the proposed MADM approach can support the selection of suitable AI-based diagnostic tools.
Furthermore, a sensitivity analysis is conducted to validate the effectiveness and robustness of the proposed algorithm. Section 4 provides a detailed comparative analysis between the newly developed algorithm and existing models to showcase its advantages. This section also provides the limitations and future research direction. Finally, the article concludes with a comprehensive summary of the findings and contributions, presented in Section 5.
For clarity and consistency, the main notations used throughout the article are provided in Table 1.

Notation	Description
$\mathfrak{L} = \{\mathfrak{l}_1, \mathfrak{l}_2, \ldots, \mathfrak{l}_n\}$	Set of alternatives
$\mathfrak{K} = \{\mathfrak{k}_1, \mathfrak{k}_2, \ldots, \mathfrak{k}_m\}$	Set of parameters/criteria
n	Number of alternatives
m	Number of parameters/criteria
$\mathfrak{l}_i$	ith alternative, $i = 1, 2, \ldots, n$
$\mathfrak{k}_j$	jth parameter/criterion, $j = 1, 2, \ldots, m$
$\mathfrak{f}(\mathfrak{l}_{ij})$	Membership magnitude of $\mathfrak{l}_i$ with respect to $\mathfrak{k}_j$
$\mathfrak{g}(\mathfrak{l}_{ij})$	Non-membership magnitude of $\mathfrak{l}_i$ with respect to $\mathfrak{k}_j$
$\mathfrak{h}(\mathfrak{l}_{ij})$	Neutral magnitude of $\mathfrak{l}_i$ with respect to $\mathfrak{k}_j$
$\mathfrak{d}(\mathfrak{l}_{ij})$	Membership phase value
$\mathfrak{e}(\mathfrak{l}_{ij})$	Non-membership phase value
$\mathfrak{c}(\mathfrak{l}_{ij})$	Neutral phase value
$\mathfrak{S}(\mathfrak{k}_j)$	Possibility (acceptance) degree associated with $\mathfrak{k}_j$
χ_{ij}	Evaluation score of alternative $\mathfrak{l}_i$ under criterion $\mathfrak{k}_j$
w_j	Normalized weight of criterion $\mathfrak{k}_j$
$\mathcal{C}(\mathfrak{l}_i)$	Overall evaluation score of alternative $\mathfrak{l}_i$
$L(\mathfrak{l}_i, \mathfrak{l}_j)$	Pairwise leadership value of $\mathfrak{l}_i$ over $\mathfrak{l}_j$
ϵ	Small positive constant used to avoid division by zero

Table 1. Main notations used in the proposed SCSpFSS-based decision-making framework.

2. Possibility complex spherical fuzzy soft set and its set theoretical properties

This section presented the core concept of the proposed model, named Possibility Complex spherical Fuzzy Soft Set (PCSpFSS), along with an illustrative example. Furthermore, it provides several fundamental set-theoretical properties of PCSpFSS.

2.1. *Basic concept*

For a better understanding of the PCSpFSS framework, it is necessary to recall some fundamental definitions, which are outlined below.
Fuzzy Set [13]. The set $\mathfrak{F} = \{(\mathfrak{l}, \zeta(\mathfrak{l})) \mid \zeta(\mathfrak{l}) \in [0,1]\}$ is defined as FS over $\mathfrak{L}$, in which $\mathfrak{l}$ is member of the universe $\mathfrak{L}$.
Soft Set [19]. The set $\{(\mathfrak{t}, \zeta(\mathfrak{l})) \mid \zeta(\mathfrak{t}) \in \mathcal{P}(\mathfrak{L})\}$ is defined as SS over $\mathfrak{L}$, in which attribute $\mathfrak{t}$ is member of the set $\mathfrak{K}$ and $\mathcal{P}(\mathfrak{L})$ is the collection of all subsets of $\mathfrak{L}$.
Complex Fuzzy Soft Set [21]. The set $\{(\mathfrak{t}, \zeta(\mathfrak{l})) \mid \zeta(\mathfrak{t}) \in \mathcal{P}(\mathfrak{L})\}$ is defined as CFSS over $\mathfrak{L}$, in which attribute $\mathfrak{t}$ is member of the set $\mathfrak{K}$ and $\mathcal{P}(\mathfrak{L})$ is the collection of all complex fuzzy subsets of $\mathfrak{L}$.

2.2. Introduction to possibility complex spherical fuzzy soft sets

The extended form of the complex spherical fuzzy soft set to the possibility complex spherical fuzzy soft set is defined as:

Definition 1. Say the set universe is $\mathfrak{L} = \{\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3, \cdots, \mathfrak{l}_n\}$ and $\mathfrak{K} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \cdots, \mathfrak{k}_m\}$ be the set of parameters. Then the Possibility complex spherical fuzzy soft set (PCSpFSS) is defined as:

$$\Psi = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{f}(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}(\mathfrak{l}_j)}, \mathfrak{g}(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}(\mathfrak{l}_j)}, \mathfrak{h}(\mathfrak{l}_j)e^{2\pi i \mathfrak{c}(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\},$$

where $\mathfrak{f}(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}(\mathfrak{l}_j)}$ is membership function and $\mathfrak{g}(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}(\mathfrak{l}_j)}$ is non-membership function, with possibility value $\mathfrak{S}(\mathfrak{l}_j)$ for the element of the set of parameters such that $\mathfrak{f}(\mathfrak{l}_j), \mathfrak{g}(\mathfrak{l}_j), \mathfrak{d}(\mathfrak{l}_j), \mathfrak{e}(\mathfrak{l}_j), \mathfrak{S}(\mathfrak{l}_j) \in [0,1]$ under the following conditions:

$$\mathfrak{f}^2(\mathfrak{l}_j) + \mathfrak{g}^2(\mathfrak{l}_j) + \mathfrak{h}^2(\mathfrak{l}_j) \in [0,1] \text{ and } \mathfrak{d}(\mathfrak{l}_j) + \mathfrak{e}(\mathfrak{l}_j) + \mathfrak{c}(\mathfrak{l}_j) \in [0,1]$$

Interpretation of the possibility degree. For each parameter $\mathfrak{k}_j \in \mathfrak{K}$, the possibility degree $\mathfrak{S}(\mathfrak{l}_j) \in [0,1]$ represents the possibility or acceptance degree associated with the corresponding parameter in the considered decision-making environment. A larger value of $\mathfrak{S}(\mathfrak{l}_j)$ indicates a higher degree to which the parameter is considered possible, applicable, or acceptable in the decision-making process, whereas a smaller value indicates a lower degree of possibility or acceptance. Thus, $\mathfrak{S}(\mathfrak{l}_j)$ provides parameter-level information that is complementary to the complex spherical fuzzy assessment of an alternative under that parameter. More formally, the possibility degree is represented by a mapping $\mathfrak{S} : \mathfrak{K} \to [0,1]$, where $\mathfrak{S}(\mathfrak{l}_j) \in [0,1], \qquad j = 1, 2, \ldots, m$. Here, $\mathfrak{S}(\mathfrak{l}_j) = 0$ represents the minimum degree of possibility or acceptance associated with parameter k_j, whereas $S(k_j) = 1$ represents the maximum degree.

It is important to distinguish the possibility or acceptance degree $\mathfrak{S}(\mathfrak{l}_j)$ from the criterion weight w_j. The possibility degree describes the degree of possibility, applicability, or acceptance associated with a parameter, whereas the criterion weight represents the relative importance assigned to that parameter in the aggregation of the decision evaluations. Therefore, these two quantities convey different types of decision information and should not be interpreted as interchangeable. In the proposed framework, $\mathfrak{S}(\mathfrak{l}_j)$ characterizes parameter-level possibility information, while w_j is used to represent the relative importance of the corresponding criterion in the multi-attribute aggregation process.

Example 1. Let $\mathfrak{L} = \{\text{Hospital } 1 = \mathfrak{l}_1, \text{Hospital } 2 = \mathfrak{l}_2, \text{Hospital } 3 = \mathfrak{l}_3\}$ be the set of universe, from which a person wants to select a hospital on the basis of parameters, given as $\mathfrak{K} = \{\text{Care} = \mathfrak{k}_1, \text{Accessibility} = \mathfrak{k}_2, \text{Cost} = \mathfrak{k}_3\}$. Then the PCSpFSS with help of an expert 1 is given as:

$$\Psi_1 = \left\{ \begin{array}{l} \left(\mathfrak{k}_1, \left\{ \frac{0.7^{0.4}, 0.6^{0.3}, 0.45^{0.2}}{\mathfrak{l}_1}, \frac{0.8^{0.4}, 0.5^{0.5}, 0.35^{0.4}}{\mathfrak{l}_2}, \frac{0.9^{0.6}, 0.4^{0.2}, 0.55^{0.3}}{\mathfrak{l}_3} \right\}; 0.8 \right), \\ \left(\mathfrak{k}_2, \left\{ \frac{0.55^{0.3}, 0.7^{0.6}, 0.25^{0.5}}{\mathfrak{l}_1}, \frac{0.85^{0.6}, 0.4^{0.3}, 0.6^{0.2}}{\mathfrak{l}_2}, \frac{0.95^{0.3}, 0.3^{0.6}, 0.5^{0.4}}{\mathfrak{l}_3} \right\}; 0.9 \right), \\ \left(\mathfrak{k}_3, \left\{ \frac{0.4^{0.4}, 0.8^{0.3}, 0.65^{0.2}}{\mathfrak{l}_1}, \frac{0.3^{0.3}, 0.9^{0.4}, 0.2^{0.6}}{\mathfrak{l}_2}, \frac{0.55^{0.4}, 0.8^{0.3}, 0.75^{0.5}}{\mathfrak{l}_3} \right\}; 0.6 \right) \end{array} \right\}.$$

Moreover, the another PCSpFSS is given as:

$$\Psi_2 = \left\{ \begin{array}{l} \left(\mathfrak{k}_1, \left\{ \frac{0.66^{0.3}, 0.68^{0.3}, 0.47^{0.2}}{\mathfrak{l}_1}, \frac{0.72^{0.3}, 0.63^{0.6}, 0.54^{0.4}}{\mathfrak{l}_2}, \frac{0.81^{0.5}, 0.52^{0.3}, 0.39^{0.6}}{\mathfrak{l}_3} \right\}; 0.7 \right), \\ \left(\mathfrak{k}_2, \left\{ \frac{0.88^{0.2}, 0.45^{0.7}, 0.31^{0.5}}{\mathfrak{l}_1}, \frac{0.69^{0.5}, 0.67^{0.4}, 0.58^{0.2}}{\mathfrak{l}_2}, \frac{0.77^{0.2}, 0.58^{0.7}, 0.44^{0.3}}{\mathfrak{l}_3} \right\}; 0.8 \right), \\ \left(\mathfrak{k}_3, \left\{ \frac{0.79^{0.4}, 0.55^{0.4}, 0.62^{0.3}}{\mathfrak{l}_1}, \frac{0.87^{0.4}, 0.42^{0.5}, 0.29^{0.6}}{\mathfrak{l}_2}, \frac{0.91^{0.3}, 0.36^{0.3}, 0.73^{0.4}}{\mathfrak{l}_3} \right\}; 0.6 \right) \end{array} \right\}.$$

Definition 2. Let $\mathfrak{L} = \{\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3, \cdots, \mathfrak{l}_n\}$ be the set of universe and $\mathfrak{K} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \cdots, \mathfrak{k}_m\}$ be the set of parameters. Then the Possibility complex spherical fuzzy soft set $\Psi_1 = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{f}_1(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}_1(\mathfrak{l}_j)}, \mathfrak{g}_1(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}_1(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}_1(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\}$, is said to be the Possibility complex spherical fuzzy soft subset of $\Psi_2 = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{f}_2(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}_2(\mathfrak{l}_j)}, \mathfrak{g}_2(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}_2(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}_2(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\}$, if and only if $\mathfrak{f}^2{}_1(\mathfrak{l}_j) \leq \mathfrak{f}^2{}_2(\mathfrak{l}_j), \mathfrak{d}_1(\mathfrak{l}_j) \leq \mathfrak{d}_2(\mathfrak{l}_j), \mathfrak{g}^2{}_1(\mathfrak{l}_j) \geq \mathfrak{g}^2{}_2(\mathfrak{l}_j), \mathfrak{e}_1(\mathfrak{l}_j) \geq \mathfrak{e}_2(\mathfrak{l}_j)$ and $\mathfrak{S}_1(\mathfrak{k}_i) \leq \mathfrak{S}_2(\mathfrak{k}_i)$.

Example 2. Based on the data of **Example 1**, by the **Definition 2**, the set PCSpFSS Ψ_2 is the Possibility complex spherical fuzzy soft subset of the set Ψ_1.

Definition 3. Let $\mathfrak{L} = \{\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3, \cdots, \mathfrak{l}_n\}$ be the set of universe and $\mathfrak{K} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \cdots, \mathfrak{k}_m\}$ be the set of parameters. Then the complement of PCSpFSS $\Psi = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{f}(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}(\mathfrak{l}_j)}, \mathfrak{g}(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\}$, is defined as:

$$\Psi' = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{g}(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}(\mathfrak{l}_j)}, \mathfrak{f}(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; (1 - \mathfrak{S}(\mathfrak{k}_i)) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\}.$$

Example 3. Based on the data of **Example 1** , the complement of the PCSpFSS Ψ_1 according to the **Definition 3** is given as:

$$\Psi_1' = \left\{ \begin{array}{l} \left(\mathfrak{k}_1, \left\{ \frac{0.6^{0.3}, 0.7^{0.4}, 0.48^{0.2}}{\mathfrak{l}_1}, \frac{0.5^{0.5}, 0.8^{0.2}, 0.36^{0.6}}{\mathfrak{l}_2}, \frac{0.4^{0.5}, 0.9^{0.4}, 0.57^{0.3}}{\mathfrak{l}_3} \right\}; 0.2 \right), \\ \left(\mathfrak{k}_2, \left\{ \frac{0.7^{0.6}, 0.55^{0.3}, 0.29^{0.5}}{\mathfrak{l}_1}, \frac{0.4^{0.3}, 0.85^{0.4}, 0.63^{0.2}}{\mathfrak{l}_2}, \frac{0.3^{0.6}, 0.95^{0.3}, 0.41^{0.4}}{\mathfrak{l}_3} \right\}; 0.1 \right), \\ \left(\mathfrak{k}_3, \left\{ \frac{0.8^{0.3}, 0.4^{0.4}, 0.52^{0.5}}{\mathfrak{l}_1}, \frac{0.9^{0.4}, 0.3^{0.3}, 0.24^{0.6}}{\mathfrak{l}_2}, \frac{0.9^{0.3}, 0.55^{0.4}, 0.68^{0.2}}{\mathfrak{l}_3} \right\}; 0.4 \right) \end{array} \right\}.$$

Definition 4. Let $\mathfrak{L} = \{\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3, \cdots, \mathfrak{l}_n\}$ be the set of universe and $\mathfrak{K} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \cdots, \mathfrak{k}_m\}$ be the set of parameters. Consider two PCSpFSS $\Psi_1 = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{f}_1(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}_1(\mathfrak{l}_j)}, \mathfrak{g}_1(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}_1(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}_1(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\}$, and $\Psi_2 = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{f}_2(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}_2(\mathfrak{l}_j)}, \mathfrak{g}_2(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}_2(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}_2(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\}$, then their union $\Psi_1 \cup \Psi_2$ is defined as:

$$\Psi_1 \cup \Psi_2 = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{h}_1(\mathfrak{l}_j)e^{2\pi i \mathfrak{c}_1(\mathfrak{l}_j)}, \mathfrak{h}_2(\mathfrak{l}_j)e^{2\pi i \mathfrak{c}_2(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{E}(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\},$$

where $\mathfrak{h}_1(\mathfrak{l}_j) = \max.(\mathfrak{f}_1(\mathfrak{l}_j), \mathfrak{f}_2(\mathfrak{l}_j))$, $\mathfrak{h}_2(\mathfrak{l}_j) = \min.(\mathfrak{g}_1(\mathfrak{l}_j), \mathfrak{g}_2(\mathfrak{l}_j))$ $\mathfrak{c}_1(\mathfrak{l}_j) = \max.(\mathfrak{d}_1(\mathfrak{l}_j), \mathfrak{d}_2(\mathfrak{l}_j))$, $\mathfrak{c}_2(\mathfrak{l}_j) = \min.(\mathfrak{e}_1(\mathfrak{l}_j), \mathfrak{e}_2(\mathfrak{l}_j))$ and $\mathfrak{E}(\mathfrak{k}_i) = \max.(\mathfrak{D}(\mathfrak{k}_i), \mathfrak{G}(\mathfrak{k}_i))$.

Definition 5. Let $\mathfrak{L} = \{\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3, \cdots, \mathfrak{l}_n\}$ be the set of universe and $\mathfrak{K} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \cdots, \mathfrak{k}_m\}$ be the set of parameters. Consider two PCSpFSS $\Psi_1 = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{f}_1(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}_1(\mathfrak{l}_j)}, \mathfrak{g}_1(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}_1(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}_1(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\}$, and $\Psi_2 = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{f}_2(\mathfrak{l}_j)e^{2\pi i \mathfrak{d}_2(\mathfrak{l}_j)}, \mathfrak{g}_2(\mathfrak{l}_j)e^{2\pi i \mathfrak{e}_2(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}_2(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\}$, then their intersection $\Psi_1 \cap \Psi_2$ is defined as:

$$\Psi_1 \cap \Psi_2 = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{\mathfrak{h}_1(\mathfrak{l}_j)e^{2\pi i \mathfrak{c}_1(\mathfrak{l}_j)}, \mathfrak{h}_2(\mathfrak{l}_j)e^{2\pi i \mathfrak{c}_2(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{E}(\mathfrak{k}_i) \right) \mid \mathfrak{k}_i \in \mathfrak{K} \right\},$$

where $\mathfrak{h}_1(\mathfrak{l}_j) = \min.(\mathfrak{f}_1(\mathfrak{l}_j), \mathfrak{f}_2(\mathfrak{l}_j))$, $\mathfrak{h}_2(\mathfrak{l}_j) = \max.(\mathfrak{g}_1(\mathfrak{l}_j), \mathfrak{g}_2(\mathfrak{l}_j))$ $\mathfrak{c}_1(\mathfrak{l}_j) = \min.(\mathfrak{d}_1(\mathfrak{l}_j), \mathfrak{d}_2(\mathfrak{l}_j))$, $\mathfrak{c}_2(\mathfrak{l}_j) = \max.(\mathfrak{e}_1(\mathfrak{l}_j), \mathfrak{e}_2(\mathfrak{l}_j))$ and $\mathfrak{E}(\mathfrak{k}_i) = \min.(\mathfrak{D}(\mathfrak{k}_i), \mathfrak{G}(\mathfrak{k}_i))$.

Example 4. Based on the data of **Example 1**, the union of two SpCFSS Ψ_1 and Ψ_2 according to the **Definition 4** is given as:

$$\Psi_1 \cup \Psi_2 = \left\{ \begin{array}{l} \left(\mathfrak{k}_1, \left\{ \frac{0.7^{0.4}, 0.6^{0.3}, 0.46^{0.2}}{\mathfrak{l}_1}, \frac{0.8^{0.4}, 0.5^{0.5}, 0.33^{0.6}}{\mathfrak{l}_2}, \frac{0.9^{0.6}, 0.4^{0.2}, 0.58^{0.3}}{\mathfrak{l}_3} \right\}; 0.8 \right), \\ \left(\mathfrak{k}_2, \left\{ \frac{0.88^{0.3}, 0.45^{0.6}, 0.27^{0.5}}{\mathfrak{l}_1}, \frac{0.85^{0.6}, 0.4^{0.3}, 0.64^{0.2}}{\mathfrak{l}_2}, \frac{0.95^{0.3}, 0.3^{0.6}, 0.49^{0.4}}{\mathfrak{l}_3} \right\}; 0.9 \right), \\ \left(\mathfrak{k}_3, \left\{ \frac{0.79^{0.4}, 0.55^{0.3}, 0.72^{0.2}}{\mathfrak{l}_1}, \frac{0.87^{0.3}, 0.42^{0.4}, 0.31^{0.5}}{\mathfrak{l}_2}, \frac{0.91^{0.4}, 0.36^{0.3}, 0.67^{0.6}}{\mathfrak{l}_3} \right\}; 0.6 \right) \end{array} \right\}.$$

The intersection of these two SpCFSS Ψ_1 and Ψ_2 according to the Definition **??** is given as:

$$\Psi_1 \cap \Psi_2 = \left\{ \begin{array}{l} \left(\mathfrak{k}_1, \left\{ \frac{0.66^{0.4}, 0.68^{0.3}, 0.43^{0.2}}{\mathfrak{l}_1}, \frac{0.72^{0.4}, 0.63^{0.5}, 0.57^{0.3}}{\mathfrak{l}_2}, \frac{0.81^{0.6}, 0.51^{0.2}, 0.38^{0.4}}{\mathfrak{l}_3} \right\}; 0.8\right), \\ \left(\mathfrak{k}_2, \left\{ \frac{0.55^{0.3}, 0.7^{0.6}, 0.26^{0.5}}{\mathfrak{l}_1}, \frac{0.69^{0.6}, 0.67^{0.3}, 0.61^{0.2}}{\mathfrak{l}_2}, \frac{0.77^{0.3}, 0.58^{0.6}, 0.47^{0.4}}{\mathfrak{l}_3} \right\}; 0.9\right), \\ \left(\mathfrak{k}_3, \left\{ \frac{0.4^{0.4}, 0.8^{0.3}, 0.69^{0.2}}{\mathfrak{l}_1}, \frac{0.3^{0.3}, 0.9^{0.4}, 0.28^{0.6}}{\mathfrak{l}_2}, \frac{0.55^{0.4}, 0.8^{0.3}, 0.74^{0.5}}{\mathfrak{l}_3} \right\}; 0.6\right) \end{array} \right\}.$$

3. A multi-attribute decision-making model based on PCSpFSS

Nowadays, intelligent decision-support systems and artificial intelligence (AI)-based technologies have emerged as transformative tools in modern supply chain management, enabling faster, more efficient, and reliable decision-making processes across industrial and commercial sectors. As global markets continue to expand and competition intensifies, organizations face increasing pressure to select suppliers capable of delivering high-quality products and services at competitive costs while maintaining sustainability, reliability, and flexibility. In such complex environments, AI-driven innovations such as machine learning (ML), deep learning, optimization techniques, and soft computing methods have shown significant promise in improving supplier evaluation and selection procedures. These intelligent techniques are particularly valuable in analyzing large-scale and uncertain supplier-related information, including delivery performance, product quality, operational efficiency, technological capability, environmental sustainability, and risk assessment.

In this regard, an algorithmic approach based on the Possibility Complex Spherical Fuzzy Soft Set (PCSpFSS) framework is presented in the next subsection. In practical multi-attribute decision-making problems, the decision criteria may not have equal importance. Therefore, a normalized criterion-weight vector is introduced as $W = (w_1, w_2, \dots, w_m)$, where $w_j \geq 0, \quad \sum_{j=1}^{m} w_j = 1$. Here, w_j represents the relative importance assigned to criterion k_j in the decision-making process. The criterion weights are conceptually different from the possibility/acceptance degrees $S(k_j)$: w_j determines the contribution of criterion k_j to the overall aggregation, whereas $S(k_j)$ represents the possibility or acceptance associated with that parameter.

3.1. Algorithmic approach based on PCSpFSS

Input:
Let $\mathfrak{L} = \{\mathfrak{l}_1, \mathfrak{l}_2, \dots, \mathfrak{l}_n\}$ {Set of alternatives (universe)}
Let $\mathfrak{K} = \{\mathfrak{k}_1, \mathfrak{k}_2, \dots, \mathfrak{k}_m\}$ {Set of parameters (attributes)}
for each $\mathfrak{k}_i \in \mathfrak{K}$ **do**
  for each $\mathfrak{l}_j \in \mathfrak{L}$ **do**
    Assign $f^2(\mathfrak{l}_j), d(\mathfrak{l}_j)$ {Membership: magnitude and phase}
    Assign $g^2(\mathfrak{l}_j), e(\mathfrak{l}_j)$ {Non-membership: magnitude and phase}
    Assign $h^2(\mathfrak{l}_j), c(\mathfrak{l}_j)$ {Neutral: magnitude and phase}
    Assign $\mathfrak{S}(\mathfrak{k}_i)$ {Possibility (acceptance) value}
  end for
end for
Construct:

$$\Psi = \left\{ \left(\mathfrak{k}_i, \left\{ \frac{f^2(\mathfrak{l}_j)e^{2\pi i d(\mathfrak{l}_j)},\ g^2(\mathfrak{l}_j)e^{2\pi i e(\mathfrak{l}_j)},\ h^2(\mathfrak{l}_j)e^{2\pi i c(\mathfrak{l}_j)}}{\mathfrak{l}_j} : \mathfrak{l}_j \in \mathfrak{L} \right\}; \mathfrak{S}(\mathfrak{k}_i) \right) \middle| \mathfrak{k}_i \in \mathfrak{K} \right\}$$

Construct the tabular form for all $\mathfrak{l}_j \in \mathfrak{L}$ and $\mathfrak{k}_i \in \mathfrak{K}$ as shown in Table **??**
for $i = 1$ to n **do**
  for $j = 1$ to m **do**
    Compute:

$$\chi_{ij} = \left| \mathfrak{S}(\mathfrak{k}_j) + f^2(\mathfrak{l}_{ij}) + h^2(\mathfrak{l}_{ij}) - g^2(\mathfrak{l}_{ij}) \right|$$

$$\cdot |d(\mathfrak{l}_{ij}) + c(\mathfrak{l}_{ij}) - e(\mathfrak{l}_{ij})|$$

end for
end for
for $i = 1$ to n **do**
Compute:

$$C(l_i) = \sum_{j=1}^{m} w_j \chi_{ij}, \qquad i = 1, 2, \ldots, n$$

end for
Here, $C(l_i)$ denotes the overall evaluation score of alternative l_i, χ_{ij} denotes the evaluation score of alternative l_i with respect to criterion k_j, and w_j denotes the normalized weight of criterion k_j. Since, $\sum_{j=1}^{m} w_j = 1$, the overall score is obtained through a weighted aggregation of the criterion-level evaluations.
for all pairs $(\mathfrak{l}_i, \mathfrak{l}_j)$ such that $i \neq j$ **do**
Compute:

$$L(\mathfrak{l}_i, \mathfrak{l}_j) = \frac{\mathcal{C}(\mathfrak{l}_i) - \mathcal{C}(\mathfrak{l}_j)}{\max\{\mathcal{C}(\mathfrak{l}_i), \mathcal{C}(\mathfrak{l}_j)\} + \epsilon}$$

$\{\epsilon$ is a small constant, e.g., $10^{-8}\}$
end for
Output:
for each pair $(\mathfrak{l}_i, \mathfrak{l}_j)$ **do**
if $L(\mathfrak{l}_i, \mathfrak{l}_j) > 0$ **then**
$\mathfrak{l}_i$ leads $\mathfrak{l}_j$
else if $L(\mathfrak{l}_i, \mathfrak{l}_j) < 0$ **then**
$\mathfrak{l}_j$ leads $\mathfrak{l}_i$
else
$\mathfrak{l}_i$ is equivalent to $\mathfrak{l}_j$
end if
end for
Choose the alternative which leads all others in pairwise comparisons

Thus, the proposed weighted formulation generalizes the equal-weight case while allowing the decision maker to incorporate heterogeneous criterion importance. It should be noted that the criterion weight w_j and the possibility degree $\mathfrak{S}(\mathfrak{k}_i)$ represent different types of information. The criterion weight w_j describes the relative importance of parameter $\mathfrak{k}_i$ in the aggregation of the decision evaluations, whereas $\mathfrak{S}(\mathfrak{k}_i)$ represents the possibility or acceptance associated with that parameter within the PCSpFSS framework. Hence, w_j and $\mathfrak{S}(\mathfrak{k}_i)$ are not interchangeable and may provide complementary information in the decision-making process. Note that the step-wise layout of the proposed PCSpFSS-based algorithmic approach is shown in Figure 1.

3.2. Case study

Modern supply chain systems are under continuous pressure due to increasing market competition, fluctuating customer demands, and global economic uncertainty. Organizations must carefully evaluate suppliers to ensure product quality, timely delivery, cost efficiency, sustainability, and operational reliability. In many industries, selecting an inappropriate supplier may lead to production delays, increased operational costs, and reduced customer satisfaction. Intelligent decision-support systems and artificial intelligence (AI)-based approaches offer effective solutions to improve supplier evaluation processes, minimize human bias, and streamline strategic procurement decisions. Authorities and organizations are exploring the ten suppliers for selection, given name as, $\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3, \mathfrak{l}_4, \mathfrak{l}_5, \mathfrak{l}_6, \mathfrak{l}_7, \mathfrak{l}_8$. Evaluation criteria include: $\mathfrak{k}_1$ product quality and reliability, $\mathfrak{k}_2$ delivery time and logistics performance, $\mathfrak{k}_3$ procurement and operational cost, $\mathfrak{k}_4$ technological capability and flexibility, and $\mathfrak{k}_5$ sustainability and environmental compliance. Historical supplier records, performance reports, market analysis, and expert assessments are used to support the supplier selection process.

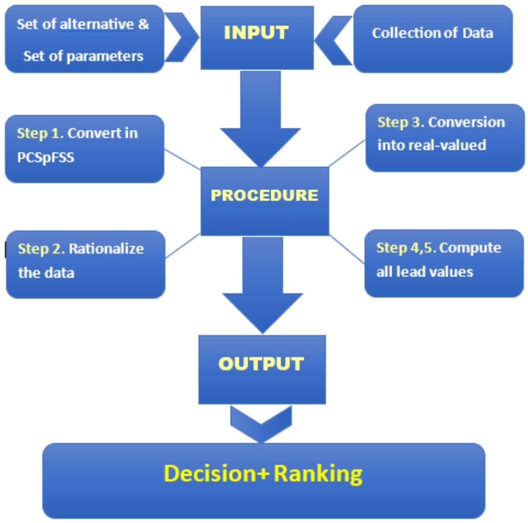

Figure 1. Step-wise layout of the proposed algorithm

To apply the proposed algorithmic approach for selecting the best alternative, consider the set of alternatives $\mathfrak{L} = \{\mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3, \cdots, \mathfrak{l}_8\}$ and the set of properties $\mathfrak{K} = \{\mathfrak{k}_1, \mathfrak{k}_2, \mathfrak{k}_3, \mathfrak{k}_4, \mathfrak{k}_5\}$ on the basis of which the alternatives will be ranked. The dataset is synthetically constructed for methodological illustration. The numerical values are selected to satisfy the admissibility conditions of the proposed PCSpFSS structure and to provide a controlled environment for demonstrating the complete decision-making procedure. The purpose of this hypothetical dataset is to examine whether the proposed framework can represent complex-valued, spherical fuzzy, parameterized, and possibility-based information and subsequently produce a ranking of the supplier alternatives. Therefore, the numerical example should be interpreted as a methodological validation of the proposed framework. The dataset in the form of PCSpFSS is given as:

$$\left\{\begin{array}{l}
\left(\mathfrak{k}_1, \left\{\begin{array}{l} \frac{0.73^{0.4}, 0.64^{0.3}, 0.42^{0.2}}{\mathfrak{l}_1}, \frac{0.76^{0.4}, 0.61^{0.5}, 0.58^{0.3}}{\mathfrak{l}_2}, \frac{0.78^{0.6}, 0.6^{0.2}, 0.37^{0.4}}{\mathfrak{l}_3}, \frac{0.82^{0.1}, 0.55^{0.5}, 0.63^{0.6}}{\mathfrak{l}_4}, \\ \frac{0.63^{0.4}, 0.44^{0.3}, 0.52^{0.2}}{\mathfrak{l}_5}, \frac{0.75^{0.4}, 0.51^{0.5}, 0.48^{0.3}}{\mathfrak{l}_6}, \frac{0.68^{0.6}, 0.52^{0.2}, 0.47^{0.4}}{\mathfrak{l}_7}, \frac{0.72^{0.1}, 0.65^{0.5}, 0.63^{0.6}}{\mathfrak{l}_8}. \end{array}\right\}, 0.81\right) \\
\left(\mathfrak{k}_2, \left\{\begin{array}{l} \frac{0.83^{0.3}, 0.52^{0.6}, 0.49^{0.2}}{\mathfrak{l}_1}, \frac{0.84^{0.6}, 0.5^{0.3}, 0.67^{0.5}}{\mathfrak{l}_2}, \frac{0.86^{0.3}, 0.48^{0.6}, 0.31^{0.4}}{\mathfrak{l}_3}, \frac{0.87^{0.3}, 0.46^{0.2}, 0.74^{0.1}}{\mathfrak{l}_4}, \\ \frac{0.73^{0.4}, 0.42^{0.5}, 0.42^{0.3}}{\mathfrak{l}_5}, \frac{0.64^{0.3}, 0.7^{0.4}, 0.57^{0.4}}{\mathfrak{l}_6}, \frac{0.76^{0.5}, 0.58^{0.3}, 0.41^{0.5}}{\mathfrak{l}_7}, \frac{0.57^{0.4}, 0.66^{0.5}, 0.64^{0.2}}{\mathfrak{l}_8}, \end{array}\right\}; 0.72\right) \\
\left(\mathfrak{k}_3, \left\{\begin{array}{l} \frac{0.79^{0.2}, 0.6^{0.5}, 0.54^{0.3}}{\mathfrak{l}_1}, \frac{0.68^{0.4}, 0.69^{0.3}, 0.28^{0.6}}{\mathfrak{l}_2}, \frac{0.8^{0.1}, 0.57^{0.2}, 0.65^{0.4}}{\mathfrak{l}_3}, \frac{0.99^{0.6}, 0.22^{0.4}, 0.39^{0.5}}{\mathfrak{l}_4} \\ \frac{0.69^{0.23}, 0.56^{0.51}, 0.44^{0.32}}{\mathfrak{l}_5}, \frac{0.64^{0.41}, 0.63^{0.32}, 0.24^{0.61}}{\mathfrak{l}_6}, \frac{0.83^{0.21}, 0.55^{0.23}, 0.24^{0.42}}{\mathfrak{l}_7}, \frac{0.91^{0.26}, 0.24^{0.41}, 0.34^{0.51}}{\mathfrak{l}_8}. \end{array}\right\}; 0.84\right), \\
\left(\mathfrak{k}_4, \left\{\begin{array}{l} \frac{0.6^{0.5}, 0.66^{0.4}, 0.44^{0.2}}{\mathfrak{l}_1}, \frac{0.85^{0.2}, 0.48^{0.6}, 0.71^{0.3}}{\mathfrak{l}_2}, \frac{0.87^{0.3}, 0.42^{0.1}, 0.33^{0.5}}{\mathfrak{l}_3}, \frac{0.59^{0.5}, 0.67^{0.1}, 0.52^{0.4}}{\mathfrak{l}_4} \\ \frac{0.73^{0.2}, 0.62^{0.5}, 0.44^{0.3}}{\mathfrak{l}_5}, \frac{0.58^{0.4}, 0.61^{0.3}, 0.23^{0.6}}{\mathfrak{l}_6}, \frac{0.82^{0.1}, 0.47^{0.2}, 0.64^{0.4}}{\mathfrak{l}_7}, \frac{0.93^{0.6}, 0.24^{0.4}, 0.49^{0.5}}{\mathfrak{l}_8}. \end{array}\right\}; 0.8\right), \\
\left(\mathfrak{k}_5, \left\{\begin{array}{l} \frac{0.66^{0.1}, 0.68^{0.5}, 0.47^{0.6}}{\mathfrak{l}_1}, \frac{0.95^{0.4}, 0.26^{0.2}, 0.62^{0.3}}{\mathfrak{l}_2}, \frac{0.67^{0.6}, 0.7^{0.4}, 0.35^{0.1}}{\mathfrak{l}_3}, \frac{0.45^{0.4}, 0.9^{0.6}, 0.76^{0.2}}{\mathfrak{l}_4} \\ \frac{0.79^{0.2}, 0.6^{0.5}, 0.54^{0.3}}{\mathfrak{l}_5}, \frac{0.68^{0.4}, 0.69^{0.3}, 0.28^{0.6}}{\mathfrak{l}_6}, \frac{0.82^{0.1}, 0.57^{0.2}, 0.65^{0.4}}{\mathfrak{l}_7}, \frac{0.99^{0.6}, 0.22^{0.4}, 0.39^{0.5}}{\mathfrak{l}_8}. \end{array}\right\}; 0.7\right)
\end{array}\right\}.$$

where the representation $\mathfrak{f}(\mathfrak{l}_{nm})e^{2\pi i \mathfrak{d}(\mathfrak{l}_{nm})}$ is used as $\mathfrak{f}(\mathfrak{l}_{nm})^{\mathfrak{d}(\mathfrak{l}_{nm})}$.

This data set is used as input for proposed model and for the five criteria considered in the case study, the weight vector is therefore given by $W = (0.20, 0.20, 0.20, 0.20, 0.20)$. The equal-weight setting is used only for the primary illustrative example. The influence of alternative criterion-weight distributions is examined separately through sensitivity analysis. Then by using Python for compuattional purpose, the output is given in Table 2.

And for the overall evaluation scores is given in Table 3.

A larger C-value indicates that the corresponding alternative leads an alternative having a smaller C-value. Thus,

χ_{ij}	$\mathfrak{k}_1$	$\mathfrak{k}_2$	$\mathfrak{k}_3$	$\mathfrak{k}_4$	$\mathfrak{k}_5$
$\mathfrak{l}_1$	0.3960	0.1520	0.0000	0.3540	0.2300
$\mathfrak{l}_2$	0.3080	1.3840	0.7770	0.1880	1.0050
$\mathfrak{l}_3$	1.0880	0.1410	0.5160	1.1060	0.3060
$\mathfrak{l}_4$	0.3420	0.3740	1.4000	0.9920	0.0000
$\mathfrak{l}_5$	0.4560	0.2900	0.0564	0.0000	0.0000
$\mathfrak{l}_6$	0.3060	0.3690	0.7630	0.7000	0.6790
$\mathfrak{l}_7$	1.1520	0.9170	0.5440	0.5370	0.4800
$\mathfrak{l}_8$	0.3020	0.1270	0.6660	1.3860	1.3020

Table 2. Criterion-level evaluation values χ_{ij} for the supplier alternatives

Alternative	Overall evaluation score $C(\mathfrak{l}_i)$
$\mathfrak{l}_1$	0.226400
$\mathfrak{l}_2$	0.732400
$\mathfrak{l}_3$	0.631400
$\mathfrak{l}_4$	0.621600
$\mathfrak{l}_5$	0.160480
$\mathfrak{l}_6$	0.563400
$\mathfrak{l}_7$	0.726000
$\mathfrak{l}_8$	0.756600

Table 3. Overall evaluation scores of the supplier alternatives

based on the overall evaluation scores, the ranking of the alternatives is given by

$$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5.$$

Therefore, the best alternative is $\mathfrak{l}_8$.

3.3. Sensitivity Analysis

To check the robustness and effectiveness of the proposed PCSpFSS-based decision-making algorithm, a sensitivity analysis is conducted by changing the criterion weights and the possibility degrees. The purpose of this analysis is to examine whether reasonable changes in the decision-maker's preferences and possibility degree substantially affect the final ranking of the alternatives.
For the baseline case, all five parameters are assigned equal weights $W^{(0)} = (0.20, 0.20, 0.20, 0.20, 0.20)$, while the possibility-degree vector is $\mathfrak{S}^{(0)} = (0.81, 0.72, 0.84, 0.80, 0.70)$. Using the proposed algorithm, the baseline overall scores were obtained as

$$\begin{aligned} C(\mathfrak{l}_1) &= 0.184998, \quad C(\mathfrak{l}_2) = 0.668508, \\ C(\mathfrak{l}_3) &= 0.567810, \quad C(\mathfrak{l}_4) = 0.560056, \\ C(\mathfrak{l}_5) &= 0.136707, \quad C(\mathfrak{l}_6) = 0.460490, \\ C(\mathfrak{l}_7) &= 0.644352, \quad C(\mathfrak{l}_8) = 0.705137. \end{aligned} \tag{1}$$

Consequently, the baseline ranking is

$$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5.$$

The comparative of baseline ranking and ranking obtained by proposed algorithm is shown in Figure 2. First, the sensitivity of the ranking to criterion weights is investigated. In each experiment, one criterion was assigned a higher weight of 0.30, while the remaining 0.70 is distributed equally among the other four criteria, resulting in weights of 0.175 for each of them. An additional scenario with $w_1 = w_2 = 0.25$ is also considered. The results

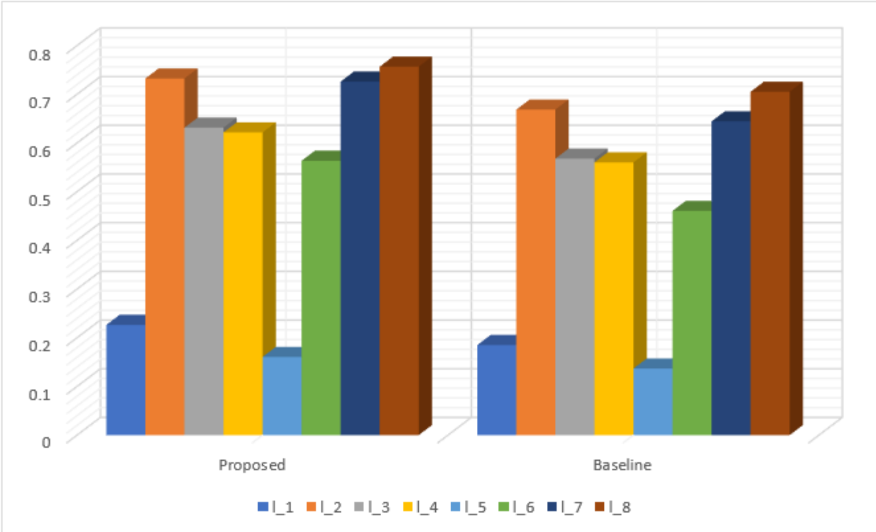

Figure 2. Comparative of baseline ranking and ranking obtained by proposed algorithm

Scenario	Ranking	Spearman's ρ	RS (%)
Baseline	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$w_1 = 0.30$	$\mathfrak{l}_7 \succ \mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	0.9286	62.5
$w_2 = 0.30$	$\mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_8 \succ \mathfrak{l}_4 \succ \mathfrak{l}_3 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	0.9048	37.5
$w_3 = 0.30$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_4 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	0.9286	62.5
$w_4 = 0.30$	$\mathfrak{l}_8 \succ \mathfrak{l}_3 \succ \mathfrak{l}_7 \succ \mathfrak{l}_2 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	0.9048	75.0
$w_5 = 0.30$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$w_1 = w_2 = 0.25$	$\mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_8 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	0.9286	62.5

Table 4. Sensitivity analysis with respect to criterion weights.

are reported in Table 4. The results indicate that the ranking is reasonably robust to changes in the criterion weights. The Spearman rank correlation coefficient remains between 0.9048 and 1.0000. Although increasing the weights of $\mathfrak{k}_1$, $\mathfrak{k}_2$, $\mathfrak{k}_3$, or $\mathfrak{k}_4$ changes some positions in the ranking, the overall ordering remains highly correlated with the baseline result. In particular, $\mathfrak{l}_1$ and $\mathfrak{l}_5$ remain in the seventh and eighth positions, respectively, in all considered weight scenarios. Moreover, increasing the weight of $\mathfrak{k}_5$ to 0.30 does not change the baseline ranking.
Second, the sensitivity of the proposed method to the possibility degrees is investigated. Each possibility degree is independently increased and decreased by 0.10, while the remaining possibility degrees are kept at their baseline values. The results are presented in Table 5. The possibility-degree analysis demonstrates a higher degree of ranking stability than the weight analysis. In eight out of the ten individual perturbation scenarios, the complete ranking remains identical to the baseline ranking, resulting in $\rho = 1.0000$. Only two scenarios, namely decreasing $\mathfrak{S}(\mathfrak{k}_1)$ by 0.10 and increasing $\mathfrak{S}(\mathfrak{k}_3)$ by 0.10, lead to a change in the ordering of $\mathfrak{l}_3$ and $\mathfrak{l}_4$. Even in these cases, the Spearman coefficient remains high at $\rho = 0.9762$.

Furthermore, when all possibility degrees are simultaneously decreased or increased by 0.10, the original ranking remains completely unchanged, yielding $\rho = 1.0000$ and a rank stability of 100%. These results demonstrate that the proposed PCSpFSS-based decision-making procedure is relatively robust to moderate variations in the possibility degrees.

Overall, the sensitivity analysis confirms that the proposed framework produces a stable decision pattern under moderate perturbations of its principal input parameters. Although the criterion weights have a greater influence on the ranking than the possibility degrees, the obtained Spearman coefficients remain high in all investigated

Scenario	Ranking	Spearman's ρ	RS (%)
$S(\mathfrak{k}_1) - 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_4 \succ \mathfrak{l}_3 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	0.9762	75
$S(\mathfrak{k}_1) + 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$S(\mathfrak{k}_2) - 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$S(\mathfrak{k}_2) + 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$S(\mathfrak{k}_3) - 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$S(\mathfrak{k}_3) + 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_4 \succ \mathfrak{l}_3 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	0.9762	75
$S(\mathfrak{k}_4) - 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$S(\mathfrak{k}_4) + 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$S(\mathfrak{k}_5) - 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100
$S(\mathfrak{k}_5) + 0.10$	$\mathfrak{l}_8 \succ \mathfrak{l}_2 \succ \mathfrak{l}_7 \succ \mathfrak{l}_3 \succ \mathfrak{l}_4 \succ \mathfrak{l}_6 \succ \mathfrak{l}_1 \succ \mathfrak{l}_5$	1.0000	100

Table 5. Sensitivity analysis with respect to possibility degrees.

scenarios. Importantly, the alternative $\mathfrak{l}_8$ remains among the top-ranked alternatives in all experiments and remains the best alternative in all possibility-degree sensitivity scenarios. This supports the robustness of the proposed framework for the considered decision-making problem.

3.4. Computational Complexity Analysis

The computational complexity of the proposed PCSpFSS-based decision-making algorithm is analyzed in terms of the number of alternatives n and the number of parameters (criteria) m. The analysis considers the principal computational operations involved in the proposed procedure, including the calculation of the criterion-level evaluation scores, weighted aggregation, and pairwise leadership comparisons. First, the criterion-level score

$$\chi_{ij} = \left|\mathfrak{S}(\mathfrak{k}_j) + f^2(\mathfrak{l}_{ij}) + h^2(\mathfrak{l}_{ij}) - g^2(\mathfrak{l}_{ij})\right| \cdot |d(\mathfrak{l}_{ij}) + c(\mathfrak{l}_{ij}) - e(\mathfrak{l}_{ij})|$$

is calculated for every alternative-parameter pair. Since there are n alternatives and m parameters, the total number of evaluations is nm. Therefore, this step requires $O(nm)$, time complexity. Next, the overall score of each alternative is obtained by $C(\mathfrak{l}_i) = \sum_{j=1}^{m} w_j \chi_{ij}$.

For each of the n alternatives, m criterion-level scores are aggregated. Consequently, the computational complexity of this stage is also $O(nm)$. The final stage of the algorithm evaluates the leadership function for every pair of distinct alternatives:

$$L(\mathfrak{l}_i, \mathfrak{l}_j) = \frac{C(\mathfrak{l}_i) - C(\mathfrak{l}_j)}{\max\{C(\mathfrak{l}_i), C(\mathfrak{l}_j)\} + \epsilon}.$$

The number of unordered pairs of alternatives is $\binom{n}{2} = \frac{n(n-1)}{2}$, and hence the pairwise comparison stage has computational complexity $O(n^2)$. Therefore, the total computational complexity of the proposed algorithm can be expressed as

$$O(nm) + O(nm) + O(n^2), \implies \boxed{O(nm + n^2)}.$$

For the numerical example considered in this study, there are $n = 8$ alternatives and $m = 5$ parameters. Thus, the algorithm requires $8 \times 5 = 40$ criterion-level evaluations, followed by 8 weighted aggregation operations. The pairwise leadership stage involves $\frac{8(8-1)}{2} = 28$ unique alternative pairs.

It should be noted that the computational cost of the proposed framework increases with both the number of alternatives and the number of parameters. In particular, the pairwise leadership stage becomes increasingly important when the number of alternatives is large because its complexity grows quadratically with n. Nevertheless, the criterion-level evaluation and aggregation stages remain linear in the number of alternative–parameter combinations.

The additional computational cost of PCSpFSS is associated with its richer information representation, including complex-valued membership, non-membership and neutral information together with possibility (acceptance) degrees. This additional cost is justified when such heterogeneous information is relevant to the decision problem.

However, for decision environments involving only simple real-valued assessments, conventional fuzzy or soft-set-based methods may provide a more computationally parsimonious alternative.

4. Comparative discussion

In the proposed research, a new mathematical structure is introduced, known as the possibility complex Spherical fuzzy soft set (PCSpFSS), where a new parameter (acceptance degree ”?”) is added into the structure of existing mathematical structure called complex Spherical fuzzy soft set. This development work enables the modelling of uncertainty and hesitations in complex decision making more finely. Furthermore, an algorithmic solution proposed by the PCSpFSS is used to effectively assess and prioritize alternatives that are based on the principles of multi-attribute decision making (MADM) under uncertainty. {In order to prove the applicability and robustness of the proposed model, it is applied to a real-life situation, supply chain management for best supplier selection. Successfully develops a good ranking of alternatives available to them to help them to choose best supplier. This is especially beneficial in resource-constrained settings, where making informed decisions is crucial.}
To validate the same decision making problem is solved with several well-known existing models in the literature proposed by Ranjbar, M., and Effati, S. [24], Das et al. [25], Chachi, J. and Kazemifard, A. [26], Xiao et al. [27], Khan et al. [28], Jan et al. [29] and Asghar et al. [30]. Usually, such models are based on the relationships between parameters and the consequential degrees of the belongingness of alternatives. They are not very good on including the acceptance degree of the parameters in the decision-making process, though, in real-world decision situations not each one of the parameters may be equally relevant or reliable. Anyhow, the PCSpFSS based approach allows us to incorporate the parameters' relationships, as well as the degrees of the alternatives and the acceptance degree of each parameter according to its context-relevance and reliability. This results in a more comprehensive and realistic evaluation framework. The comparative results of the proposed and existing models are presented in Table 6, where the superiority of the proposed approach is evident in terms of both interpretability and decision accuracy. Thus, the proposed model offers both theoretical enhancement and practical utility, bridging a crucial gap in the

Models	$\mathfrak{l}_1$	$\mathfrak{l}_2$	$\mathfrak{l}_3$	$\mathfrak{l}_4$	$\mathfrak{l}_5$	$\mathfrak{l}_6$	$\mathfrak{l}_7$	$\mathfrak{l}_8$
[24]	0.8426	1.1743	0.9638	1.2865	1.0537	0.9174	1.3621	1.6847
[25]	1.9264	2.3187	1.7842	2.5471	1.6395	2.0846	2.7318	3.1642
[26]	0.4387	0.5726	0.6914	0.8163	0.6248	0.7531	0.8927	1.1475
[27]	2.6418	1.8736	2.3184	3.0472	2.1547	2.7861	3.4215	3.8956
[28]	0.7135	0.8462	0.9274	0.7816	0.9583	0.8647	1.1268	1.3842
[29]	1.1846	0.9367	1.2738	1.0524	1.3165	1.1479	1.4386	1.6723
[30]	0.8952	1.0674	0.7846	1.2148	1.1387	0.9735	1.3264	1.5917

Table 6. Comparative analysis

current literature on fuzzy decision-making frameworks. For visualization of the comparative Table 6, the data of [24, 25, 26, 27, 28] is shown graphically in Figure **??**, in which alternative 4, that is, $\mathfrak{l}_4$ shows the highest value all over the remaining alternatives.

4.1. Limitations and future research

Although the proposed PCSpFSS framework provides a flexible mechanism for representing heterogeneous uncertain information, several limitations should be acknowledged. First, the supplier-selection dataset considered in this study is hypothetical and is primarily intended to demonstrate the mathematical applicability and computational behavior of the proposed framework. Consequently, empirical validation using real industrial supplier data remains an important direction for future research.
Second, the use of complex-valued assessments introduces additional parameters, including phase information, which may increase the cognitive burden on decision makers when eliciting evaluations. Third, the proposed framework is mathematically and computationally more involved than simpler fuzzy or classical MADM

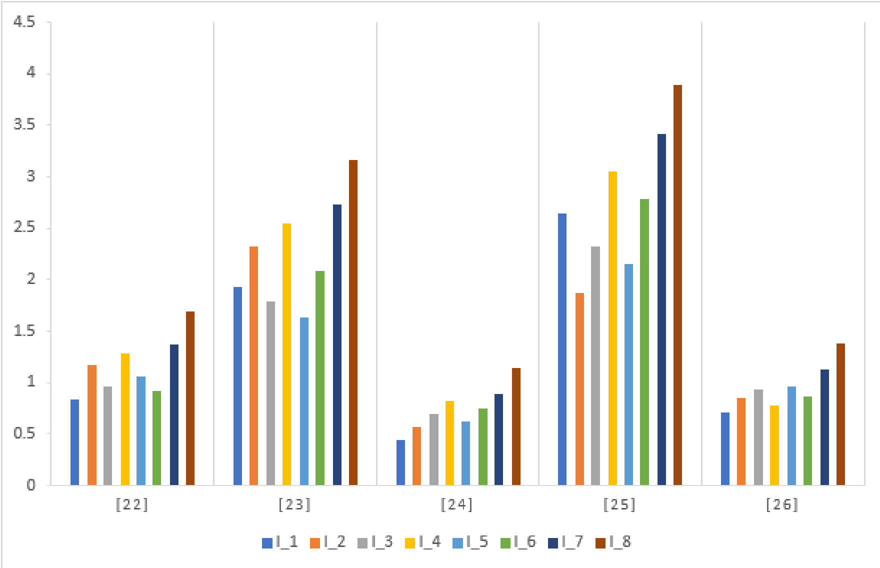

Figure 3. Comparative analysis

approaches. Therefore, its use should be motivated by the characteristics of the available decision information.
Fourth, the final ranking may depend on the criterion weights, possibility degrees, and aggregation functions employed in the decision procedure. Although sensitivity analyses are conducted to investigate ranking robustness, further studies using alternative weight-determination mechanisms and real decision-maker preferences are desirable.
Future research may therefore focus on empirical supplier-selection applications, automated or data-driven determination of criterion weights and possibility degrees, large-scale implementations, and extensions of the PCSpFSS framework to other MADM and group decision-making environments.

5. Conclusion

This paper extends the concept of the complex Spherical fuzzy soft set (CSpFSS) to a more generalized structure, the Possibility Complex Spherical Fuzzy Soft Set (PCSpFSS). It formally defines the PCSpFSS framework and illustrates the concept with a real-life example for better understanding. The study also establishes several set-theoretical properties of PCSpFSS, each supported with illustrative examples. Following the theoretical foundation, a general problem statement regarding the supplier selection in supply chain management is presented. To address this, a PCSpFSS-based Multi-Attribute Decision-Making (MADM) algorithm is proposed. A case study for the suitable supplier selection using AI strategies is presented to demonstrate how the proposed algorithm can effectively assist in selecting appropriate supplier. Furthermore, a sensitivity analysis employing various statistical tools is performed to evaluate the robustness and reliability of the proposed approach. A comparative analysis with existing models is also conducted, highlighting the advantages and improved performance of the developed algorithm. The results indicate that the proposed framework can provide a flexible mechanism for decision environments in which heterogeneous uncertain information needs to be represented simultaneously.
Nevertheless, the present study is limited by the use of a hypothetical supplier-selection dataset. Therefore, the results should be interpreted as a methodological and computational demonstration rather than empirical validation of supplier-selection performance. Future research will focus on real-world supplier data, data-driven determination of criterion weights and possibility degrees, large-scale decision-making problems, and extensions to group decision-making and other application domains.

REFERENCES

1. Duishekeeva, A., Fei, J., Xue, L., Kostiuk, S., & Zhao, B. (2026). Artificial Intelligence in the Production of Multimedia News in Chinese Journalism. Statistics, Optimization & Information Computing.
2. Onwujekwe, G., & Weistroffer, H. R. (2025). Intelligent decision support systems: An analysis of the literature and a framework for development. Information Systems Frontiers, 27(5), 2027-2058.
3. Theotista, G., & Fandi, A. M. (2026). A Hybrid Machine Learning and Rule-Based Recommendation Framework: A Probabilistic Approach for Reliable Customer Next-Transaction Prediction. Statistics, Optimization & Information Computing.
4. Beil, D. R. (2010). Supplier selection. Wiley encyclopedia of operations research and management science.
5. Ali, A. A. M. A. (2026). A Data-Driven Decision Support Framework for Business Intelligence Systems Leveraging BERT-Enhanced LLMs. Statistics, Optimization & Information Computing.
6. Jaboob, A. S., Awain, A. M. B., Ali, K. A. M., & Mohammed, A. M. (2024). Introduction to operation and supply chain management for entrepreneurship. In Applying Business Intelligence and Innovation to Entrepreneurship (pp. 52-80). IGI Global Scientific Publishing.
7. Khedr, A. M. (2024). Enhancing supply chain management with deep learning and machine learning techniques: A review. Journal of Open Innovation: Technology, Market, and Complexity, 10(4), 100379.
8. Ferdous, S. M., Newaz, S. N. E., Mugdha, S. B. S., & Uddin, M. (2025). Sentiment analysis in the transformative era of machine learning: A comprehensive review. Statistics, Optimization & Information Computing, 13(1), 331-346.
9. Chen, T., Ab Jamil, N., Liang, W., Bao, Y., & Ma, H. (2025). Mapping and prioritizing barriers to sustainable green supply chain management: An integrated fuzzy MADM approach in manufacturing enterprises. Journal of applied research on industrial engineering, 12(3), 388-413.
10. Asghar, A., Khan, K. A., Rahman, A. U., & El-Morsy, S. (2025). A novel algorithmic approach to model uncertainties associated with sustainable supply chain management using complex spherical fuzzy soft settings. International Journal of Information Technology, 17(5), 2877-2885.
11. Panpatil, S. S., Prajapati, H., & Kant, R. (2023). Effect of green supply chain practices on sustainable performance indicators: a fuzzy MADM approach. Process Integration and Optimization for Sustainability, 7(3), 475-488.
12. Asghar, A., Khan, K. A., Albahar, M. A., & Alammari, A. (2023). An optimized multi-attribute decision-making approach to construction supply chain management by using complex picture fuzzy soft set. PeerJ Computer Science, 9, e1540.
13. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, **8**(3), 338–353.
14. K. Atanassov, ”Intuitionistic fuzzy sets”, *Fuzzy Sets and Systems*, vol. **20**, pp. 87-96, 1986.
15. Yager, R. R. (2015). Properties and applications of Pythagorean fuzzy sets. In Imprecision and Uncertainty in information representation and processing: new tools based on intuitionistic fuzzy sets and generalized nets (pp. 119-136). Cham: Springer International Publishing.
16. Ramot, D., Milo, R., Friedman, M., and Kandel, A. (2002). Complex fuzzy sets. IEEE transactions on fuzzy systems, 10(2), 171-186.
17. Alkouri, A. M. D. J. S., & Salleh, A. R. (2012, September). Complex intuitionistic fuzzy sets. In AIP conference proceedings (Vol. 1482, No. 1, pp. 464-470). American Institute of Physics.
18. Khan, Z., Hussain, F., Rahim, T., Jan, R., & Boulaaras, S. (2024). Distance measure and its application to decision making, medical diagnosis, and pattern recognition problems under complex picture fuzzy sets. The European Physical Journal Plus, 139(3), 243.
19. Molodtsov, D. (1999). Soft set theoryÂ—first results. Computers & mathematics with applications, 37(4-5), 19-31.
20. Çağman, N., Enginoğlu, S., and Citak, F. (2011). Fuzzy soft set theory and its applications. Iranian journal of fuzzy systems, 8(3), 137-147.
21. Thirunavukarasu, P., Suresh, R., and Ashokkumar, V. (2017). Theory of complex fuzzy soft set and its applications. Int. J. Innov. Res. Sci. Technol, 3(10), 13-18.
22. Kumar, T., and Bajaj, R. K. (2014). On complex intuitionistic fuzzy soft sets with distance measures and entropies. Journal of Mathematics, 2014(1), 972198.
23. Mahmood, T., and Ali, Z. (2022). A method to multiattribute decision making problems under interaction aggregation operators based on complex Pythagorean fuzzy soft settings and their applications. Computational and Applied Mathematics, 41(6), 227.
24. Ranjbar, M., and Effati, S. (2022). A new approach for fuzzy classification by a multiple-attribute decision-making model. Soft Computing, 26(9), 4249-4260.
25. Das, S., Shil, B., and Tripathy, B. C. (2021). Tangent similarity measure based MADM-strategy under SVPNS-environment. Neutrosophic Sets and systems, 43(1), 93-104.
26. Chachi, J., and Kazemifard, A. (2024). A novel extended approach to evaluate criteria weights in MADM problems in fuzzy framework. Iranian Journal of Fuzzy Systems, 21(4), 101-122.
27. Xiao, Y., Asghar, A., Niaz, T., and Ali, R. S. (2025). Mathematical modeling for precise disease diagnosis: Distance and similarity measures in complex neutrosophic soft structures. Neurocomputing, 130620.
28. Khan, K. A., Asghar, A., Rahman, A. U., and Mabela, R. M. (2024). An algorithmic multiple attribute decision-making context to model uncertainty associated with hospital site selection problem using complex sv-neutrosophic soft information. Applied artificial intelligence, 38(1), 2375110.
29. Jan, N., Gwak, J., Pamucar, D., & Kang, H. (2024). An integrated complex T-spherical fuzzy set and soft set model for quantum computing and energy resource planning. Information sciences, 661, 120101.
30. Asghar, A., Khan, K. A., Rahman, A. U., & El-Morsy, S. (2025). A novel algorithmic approach to model uncertainties associated with sustainable supply chain management using complex spherical fuzzy soft settings. International Journal of Information Technology, 17(5), 2877-2885.