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Fuzzy Parameter Modeling and Survival Function Estimation for Inverse Power Maxwell Data

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Abstract This paper investigates the problem of estimating the survival function for lifetimes modeled by the fuzzy inverse power Maxwell distribution, motivated by reliability and survival data subject to vagueness and imprecision in observations and parameters. The classical inverse power Maxwell model is first reviewed and its main distributional properties relevant to survival analysis are derived, including the probability density function, cumulative distribution function, survival function, and hazard rate. To capture epistemic uncertainty inherent in practical lifetime studies, the model parameters are then represented as fuzzy numbers, and the corresponding fuzzy survival function is constructed via the extension principle under appropriate $\alpha - cut$ formulations. Several estimation strategies for the fuzzy survival function are developed, including plug-in estimation based on maximum likelihood estimators of the crisp parameters, fuzzy least-squares-type procedures, and a possibility-based approach; explicit computational forms and algorithmic steps are provided for each method. A comprehensive Monte Carlo simulation study is conducted to compare the finite-sample behavior of the proposed estimators in terms of bias, mean squared error, and coverage characteristics across different degrees of fuzziness, sample sizes, and parameter configurations. The practical usefulness of the methodology is illustrated with a real data set from survival studies, where the inverse power Maxwell model with fuzzy parameters yields more flexible survival function estimates than its classical counterpart and better accounts for expert judgement and imprecise measurements. The results indicate that the proposed fuzzy inference framework provides a robust and informative tool for survival analysis when ambiguity in lifetime data cannot be ignored.

Keywords Fuzzy Inverse Power Maxwell Distribution, fuzzy numbers, neutrosophic statistics, survival analysis, hazard function

AMS 2010 subject classifications 62Exx, 03E72

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1. Introduction

Survival analysis play a central role in a wide range of disciplines, including biomedical research, engineering, industrial quality control, and risk assessment. In many of these applications, the primary object of interest is the survival function, which quantifies the probability that a system, component, or individual survives beyond a given time [1, 2, 3, 4, 21, 32, 33]. Accurate estimation of the survival function is therefore essential for understanding lifetime behavior, comparing treatments or designs, and making informed decisions about maintenance, replacement, or intervention strategies [5, 6, 7, 8, 22, 34]. Classical survival analysis, however, is typically formulated within a purely probabilistic framework in which lifetimes and model parameters are assumed to be precisely known, an assumption that may be overly restrictive in real-world situations where information is often vague, incomplete, or linguistically expressed [9, 10, 11, 23, 24, 35, 36].

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In practical survival studies, uncertainty arises not only from random variability but also from epistemic imprecision in expert knowledge, measurement error, and incomplete or subjective information [37, 38, 39]. When failure times are derived from expert opinion, field reports, or heterogeneous data sources, it may be difficult to specify model parameters as crisp values [12, 13, 14, 25, 40]. Fuzzy set theory offers a natural framework to model such imprecision by representing uncertain quantities as fuzzy numbers rather than fixed scalars. Incorporating fuzziness into lifetime models allows one to capture a family of plausible parameter values and, consequently, a family of plausible survival functions, providing a richer description of uncertainty than classical point estimates alone [26, 27, 28, 29, 41].

Among the various lifetime distributions proposed in the literature, the inverse power Maxwell distribution has attracted attention due to its flexibility in modeling diverse failure patterns, including non-monotone hazard rates and heavy-tailed behaviors that frequently arise in mechanical and industrial systems [42, 43, 44, 45, 46]. Its shape properties make it a suitable candidate for analyzing complex reliability data where standard models such as the exponential or Weibull may be inadequate [15, 16, 17]. However, existing work on the inverse power Maxwell distribution has largely focused on crisp, non-fuzzy settings, and to the best of our knowledge, a systematic treatment of survival function estimation under fuzzy parameterization for this family is still lacking.

Motivated by these considerations, this paper develops a comprehensive framework for survival function estimation based on the fuzzy inverse power Maxwell distribution. We first recall the main structural properties of the inverse power Maxwell model that are relevant to survival analysis and then extend the model by allowing its parameters to be represented as fuzzy numbers. Using the extension principle and -cut representations, we derive the corresponding fuzzy survival function, which yields an interval-valued description of survival probabilities at each time point. Building on this formulation, we propose estimation procedures for the fuzzy survival function, primarily through maximum likelihood–based methods adapted to the fuzzy setting, and we investigate their theoretical and numerical properties.

To assess the finite-sample performance of the proposed estimators, we conduct an extensive Monte Carlo simulation study under a variety of parameter configurations and degrees of fuzziness. The simulations are designed to examine important criteria such as bias, mean squared error, and the ability of the fuzzy survival bands to cover the true survival curve. The results demonstrate that incorporating fuzziness into the inverse power Maxwell model can lead to more informative and robust survival inferences, particularly in settings where parameter imprecision is non-negligible. The proposed methodology thus contributes to the growing interface between fuzzy set theory and survival analysis, providing practitioners with a flexible tool for modeling and inference in the presence of both random variability and epistemic uncertainty.

2. Fuzzy Inverse Power Maxwell Distribution

A random variable Y is said to be distribution according to Maxwell distribution with scale parameter θ, if its probability density function(pdf) is given by

$$f\left(y;\lambda\right)=\frac{4}{\sqrt{\pi}}\lambda^{3/2}y^{2}exp\left[-\lambda y^{2}\right] \tag{1}$$

Where y$\geq 0, \lambda > 0$, and the cumulative distribution function (cdf) is given by

$$F\left(y;\lambda\right)=\ \frac{2}{\sqrt{\pi}}\gamma(3/2,\lambda y^{2}) \tag{2}$$

Where $\gamma\left(a_1,a_2\right)=\int_0^{a_2} y^{a_1-1}e^{-y}$dy is the lower incomplete gamma function.

The Inverse Maxwell distribution, proposed by [18] from the inverse of a random variable having Maxwell distribution. In other words, if a random variable Y has Maxwell distribution, then the random variable Y=1/q follows the inverse Maxwell distribution with pdf and cdf defined, respectively, by

$$f\left(q;\lambda\right)=\frac{4}{\sqrt{\pi}}\lambda^{3/2}q^{-4}\exp[-\lambda q^{-2}] \tag{3}$$

$$F\left(q;\lambda\right)=\frac{2}{\sqrt{\pi}}\Gamma(\frac{3}{2},\lambda q^{-2}) \tag{4}$$

where $q>0,\lambda>0\ and\ \Gamma\left(a_1,a_2\right)=\int_{a_2}^{\infty}q^{a_1-1}\ e^{-q}dq$ is the upper incomplete gamma function.

Consider Y be a random variable having pdf, Eq.(1), then the random variable X=$Y^{-1/\delta}$ follows the IPM(δ,λ) distribution with pdf given by

$$f\left(x;\delta,\lambda\right)=\frac{4}{\sqrt{\pi}}\delta\lambda^{\frac{3}{2}}x^{-3\alpha-1}\exp[-\lambda x^{-2\delta}] \tag{5}$$

Where $x>0\ and\ \delta,\lambda>0$. The corresponding cumulative distribution function is given by

$$F\left(X;\delta,\lambda\right)=\frac{2}{\sqrt{\pi}}\Gamma(\frac{3}{2},\lambda x^{-2\delta}) \tag{6}$$

It can be noticed that the inverse Maxwell distribution is a special case of the IMP (δ,λ) when δ=1.

From pdf, Eq.(5), it can be observed that the behavior of the density function of the IPM (δ,λ) at x= 0 and x= ∞, are give by

$$\lim_{x\to0}f\left(x\right)=\lim_{x\to\infty}f\left(x\right)=0$$

Theorem1: The probability density function f(x) of the inverse power Maxwell distribution is log-concave for $x<(2\delta+1)/(3\delta+1))^{1/2\delta}$ and upside-down bathtub function in x.

Proof: f(x) is log-concave if (f '(x) / f(x))'= (lnf(x))" <0 on its interval$<$ by choosing η(x)= $\frac{d}{dx}lnf(x)$ we have

$$\eta\left(x\right)=-\frac{3\delta+1}{x}+2\delta\lambda x^{-2\delta-1}$$

it follows that

$$\eta'\left(x\right)=\frac{(3\delta+1)-2\delta\lambda(2\delta+1)x^{-2\delta}}{x^2}$$

$\eta'\left(x\right)=0$ at the point

$$x_0=(\frac{2\delta\lambda(2\delta+1)}{3\delta+1})^{1/2\delta}$$

It implies that, f(x) is log-concave if x$<$ x_0. after that the first derivative of f(x) is given by

$$f'\left(x\right)=-\frac{\psi\left(x\right)}{x}f(x)$$

Where $\psi\left(x\right)=1+(3-2\lambda x^{-2\delta})\delta$

1. If $\psi(x)<0$, then f'(x) >0 which implies that f(x) is increasing.
2. If $\psi(x)$> 0, then f'(x) <0 which implies that f(x) is decreasing .
3. If $\psi\left(x\right)$ = 0, then f'(x) = 0 which occurs at the point.

$$x_1=(\frac{2\delta\lambda}{3\delta+1})^{1/2\delta}$$

The second derivative of f(x) given by
f"(x) = $-\frac{1}{x}([1+\psi\left(x\right)]f'\left(x\right)+\psi'\left(x\right)f\left(x\right))$
where $\psi'\left(x\right)=4\lambda\delta^2x^{-2\delta-1}$. Clearly, $\psi(x)$ is a unimodal function with maximum value at the point x_1 since, $f''\left(x_1\right)=-\frac{1}{x_1}\psi'\left(x_1\right)f\left(x_1\right)<0$, f(x) has global maximum at x_1; hence, the mode of f(x) is given by

$$x_1=(\frac{2\delta\lambda}{3\delta+1})^{\frac{1}{2\delta}}$$

The hazard (failure)rate function h(x), also known as the (instantaneous) rate of failure for the survivors to time x during the next instant of time [19, 20, 30, 31]. The hazard rate function is given by

$$h\left(x\right)=\frac{2\delta\lambda^{\frac{3}{2}}x^{-3\delta-1}\exp[-\lambda x^{-2\delta}]}{\gamma(\frac{3}{2},\lambda x^{-2\delta})}$$

where x >0, δ, λ >0

the behavior of h(x) has the same meanings as behavior of -$\eta(x)$ where $\eta(x)$=(lnf(x))'. The following theorem shows the shape of the hazard rate function of the IPM (δ ,λ).

The survival function is defined as:

$$S\left(x;\delta,\lambda\right)=(\frac{x^{-\delta}}{\sqrt{2}\ \delta})+\frac{\sqrt{2}}{\sqrt{\pi}}\frac{x^{-\delta}}{\delta}\exp(\frac{x^{-2\lambda}}{2\delta^{2}})$$

Theorem2: The hazard rate function h(x) of the invers power Maxwell distribution is upside-down bathtub function in x.

Proof: from theorem 1 it follows that the second derivative of -$\eta(x)$ given by

$$-\eta''\left(x\right)=\frac{2\left(3\delta+1\right)-4\delta\lambda(2\delta+1)(\delta+1)x^{-2\delta}}{x^{3}}$$

Clearly $-\eta''\left(x_0\right)=\ -\frac{2\delta(3\delta+1)}{x_0^3}<0,\ -\eta(x)$ has a global maximum at the point x_0 . therefore -$\eta(x)$ is unimodal shaped and lead to the conclusion that h(x) is also unimodal shaped.

Depending on the maximum likelihood method, the distribution parameters can be estimated efficiently.

Regarding the fuzzy dataset, some definitions are needed.

Definition 1 [18]

A membership function mapping the elements of nonempty set $X$to the interval $[0,\ 1]$ defines a fuzzy set$\tilde{A}$. $\mu_{\tilde{A}}:X\rightarrow[0,1]$, is called the degree of membership function of the fuzzy set $\tilde{A}$. $\mu_{\tilde{A}}(x)$ is called the membership value of $x\in X$in the fuzzy set $\tilde{A}$

Definition 2 [19]

A fuzzy number $\tilde{A}$ is a fuzzy set on real line $\mathbb{R}$, if satisfy the following conditions:

1- $\mu_{\tilde{A}}\ \ (z)$ is piecewise continuous, where$z\in\mathbb{R}$.

2- $\exists\ z\in\mathbb{R}$ such that $\mu_{\tilde{A}}\ \ (z)\ \ =\ 1$, (i.e. $\tilde{A}$ is said to be normal)

3- $\tilde{A}$ is convex $(i.e.\ \mu_{\tilde{A}}(\alpha_{x_1}+(1-\alpha)_{x_2})\geq\min\{\mu_{\tilde{A}}(x_1),\mu_{\tilde{A}}(x_2)\}$,for all $x_1,x_2\in\mathbb{R}$, and all $\alpha\in[0,\ 1]\,,\ \ [5])$.

Definition 4 [20]

The $\alpha-cut$ set of a fuzzy set $\tilde{A}$ is crisp set defined by$A_\alpha=\ \{x\in X|\mu_{\tilde{A}}\ (x)\ \geq\ \alpha\}$.

A fuzzy set is convex $\leftrightarrow$ all its $\alpha-cut$ are convex.

Regarding to this, the fuzzy Inverse power Maxwell distribution is defined as:

$$\tilde{F}\left(\tilde{x}_{A^{(a)}}\right)=\ \int_0^{\tilde{x}_{A^{(a)}}}f\left(u\right)du$$

$$=\int_0^{\tilde{x}_{A^{(a)}}}\ \ \frac{4}{\sqrt{\pi}}\delta\lambda^{\frac{3}{2}}x^{-3\delta-1}\exp\left[-\lambda x^{-2\delta}\ \right]du$$

$$=\frac{2}{\sqrt{\delta}}\Gamma\left(\frac{3}{2},\ \lambda{\tilde{x}_{A^{(a)}}}^{-2\delta}\right)$$

and

$$\tilde{f}\left(\tilde{x}_{A^{(a)}}\right)=\frac{4}{\sqrt{\pi}}\delta\lambda^{\frac{3}{2}}\,{\tilde{x}_{A^{(a)}}}^{-3\delta-1}\exp\left[-\lambda{\tilde{x}_{A^{(a)}}}^{-2\delta}\right] \tag{7}$$

The fuzzy survival and fuzzy hazard functions are, respectively

$$\tilde{S}\left(\tilde{x}_{A^{(a)}}\right)=1-\ \tilde{F}\left(\tilde{x}_{A^{(a)}}\right)$$

$$= 1 - \frac{2}{\sqrt{\delta}}\,\Gamma\left(\frac{3}{2},\ \lambda\tilde{x}_{A^{(a)}}{}^{-2\delta}\right)$$

$$= \left(\frac{\tilde{x}_{A^{(a)}}{}^{-\delta}}{\sqrt{2}\,\delta}\right) + \frac{\sqrt{2}}{\sqrt{\pi}}\frac{\tilde{x}_{A^{(a)}}{}^{-\delta}}{\delta}\exp\left(\frac{\tilde{x}_{A^{(a)}}{}^{-2\lambda}}{2\delta^2}\right)$$

$$\tilde{h}\left(\tilde{x}_{A^{(a)}}\right) = \frac{\tilde{f}\left(\tilde{x}_{A^{(a)}};\delta,\lambda\right)}{\tilde{S}\left(\tilde{x}_{A^{(a)}}\right)} = \frac{\frac{4}{\sqrt{\pi}}\delta\lambda^{\frac{3}{2}}\,\tilde{x}_{A^{(a)}}{}^{-3\delta-1}\exp\left[-\lambda\tilde{x}_{A^{(a)}}{}^{-2\delta}\right]}{\left(\frac{\tilde{x}_{A^{(a)}}{}^{-\delta}}{\sqrt{2}\,\delta}\right) + \frac{\sqrt{2}}{\sqrt{\pi}}\frac{\tilde{x}_{A^{(a)}}{}^{-\delta}}{\delta}\exp\left(\frac{\tilde{x}_{A^{(a)}}{}^{-2\lambda}}{2\delta^2}\right)}$$

3. Simulation Results

In this section, we conduct simulation to assess how well the proposed approach. The Akaike information criterion (AIC), Bayesian information criteria (BIC), and the mean time to failure (MTTF) are assessed. The sample size, n, in the simulation is considered: 30, 50, 150, respectively. We considered the initial values of the IPM distribution parameters, (δ,λ) as: (2,1.5) and (3,5). Further, the value of the cut (CC) for the fuzzy set is: 0.1, 0.5, and 0.7. The simulation is repeated 1000 times for each sample size. We reported all results in Tables 1–6.

Based on the obtained results Tables 1–6, several observations can be obtained as follows:

1. Generally, the behavior of the fuzzy IPM distribution is better than that of IPM distribution, regarding the survival and hazard functions.
2. Clearly, in terms of AIC and BIC, fuzzy IPM distribution improved the performance of the survival and hazard functions compared to IPM distribution in all the cases of simulation by getting the small values of AIC and BIC.
3. In terms of MTTF, fuzzy IPM distribution again improved the performance of the survival and hazard functions compared to IPM distribution in all the cases of simulation in yielding the high MTTF.
4. In terms of the membership (mem) values, there is increasing in the AIC and BIC values regardless the value of n with the priority for fuzzy IPM distribution.
5. With respect to the value of n, the AIC and BIC values decrease when n increases, regardless the value of the membership degree CC.

Table 1. The results when ($\delta = 2$,$\lambda = 1.5$) and CC=0.1

		Real Time	*Fuzzy Time*
n=30	*AIC*	*147.0202*	*142.7938*
	BIC	*144.3414*	*142.127*
	MTTF	*5.99631*	*6.009489*
n=50	*AIC*	*146.0102*	*140.7698*
	BIC	*144.2723*	*140.061*
	MTTF	*6.00631*	*6.009489*
n=150	*AIC*	*145.0102*	*138.7595*
	BIC	*143.0309*	*137.386*
	MTTF	*6.0105*	*6.029489*

4. Real Application

From our study, we have used premature infant staying time data that we gathered from Kirkuk hospital, Iraq for about six months to apply our proposed IPM distribution. The time is equated with the number of days that the

Table 2. The results when ($\delta = 2$,$\lambda = 1.5$) and CC=0.5

		Real Time	*Fuzzy Time*
n=30	*AIC*	*141.7621*	*196.6384*
	BIC	*143.1633*	*197.7739*
	MTTF	*2.73821*	*2.810379*
n=50	*AIC*	*141.7621*	*196.6384*
	BIC	*143.6742*	*197.7739*
	MTTF	*2.73821*	*2.810379*
n=150	*AIC*	*141.7621*	*196.6384*
	BIC	*144.7728*	*197.7739*
	MTTF	*2.73821*	*2.810379*

Table 3. The results when ($\delta = 2$,$\lambda = 1.5$) and CC=0.7

		Real Time	*Fuzzy Time*
n=30	*AIC*	*143.7862*	*198.6625*
	BIC	*145.1874*	*199.798*
	MTTF	*4.76231*	*4.834479*
n=50	*AIC*	*143.7862*	*198.6625*
	BIC	*145.6983*	*199.798*
	MTTF	*4.76231*	*4.834479*
n=150	*AIC*	*143.7862*	*198.6625*
	BIC	*146.7969*	*199.798*
	MTTF	*4.76231*	*4.834479*

Table 4. The results when ($\delta = 3$,$\lambda = 5$) and CC=0.1

		Real Time	*Fuzzy Time*
n=30	*AIC*	*586.009*	*517.694*
	BIC	*585.4102*	*515.6776*
	MTTF	*6.585502*	*6.601041*
n=50	*AIC*	*585.009*	*516.584*
	BIC	*582.921*	*513.6996*
	MTTF	*6.597502*	*6.620617*
n=150	*AIC*	*649.4041*	*466.4484*
	BIC	*644.4148*	*465.5839*
	MTTF	*6.892914*	*7.10749*

Table 5. The results when ($\delta = 3$,$\lambda = 5$) and CC=0.5

		Real Time	*Fuzzy Time*
n=30	*AIC*	*585.48*	*632.2417*
	BIC	*586.8812*	*635.0539*
	MTTF	*7.056502*	*7.113207*
n=50	*AIC*	*649.87513*	*71.02347*
	BIC	*651.78715*	*71.79605*
	MTTF	*7.363914*	*7.107857*
n=150	*AIC*	*649.87513*	*71.02347*
	BIC	*652.88577*	*71.79605*
	MTTF	*7.363914*	*7.107857*

premature baby is alive since the hospital discharge date. The study has a subject population of 220 premature

Table 6. The results when ($\delta = 3$,$\lambda = 5$) and CC=0.7

		Real Time	*Fuzzy Time*
n=30	*AIC*	*585.9551*	*531.0057*
	BIC	*587.3563*	*533.3751*
	MTTF	*7.531602*	*7.528625*
n=50	*AIC*	*650.35023*	*32.61429*
	BIC	*652.26225*	*32.91688*
	MTTF	*7.839014*	*7.373118*
n=150	*AIC*	*650.35023*	*32.61429*
	BIC	*653.36087*	*32.91688*
	MTTF	*7.839014*	*7.373118*

babies. But the times of premature infants are not counted, the member states must come up with clear and precise time definitions. Thus, it is always problematic to supervise unpredictable, insufficient and incongruent data by defining the number of days that a premature infant with alive discharge should have to be born but there is no explicit information available to define the number of days. The data related to exploratory factor analysis is given in Table 7.

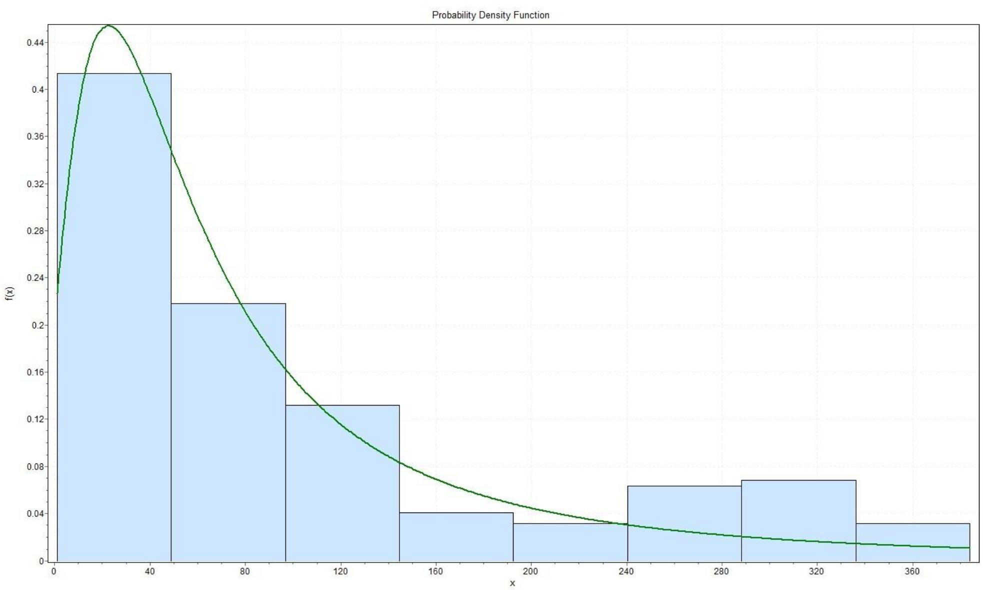

Figure 1. The pdf of the IPM distribution for the real data

An informal graphical technique has been utilized to show that the IPM distribution is one of the plausible models for explaining the premature infant staying time data. Figure 7 displays a visual fit of the IPM distribution. Further, the χ^2 test for the goodness of fit shows that the premature infant staying time data follows IPM distribution with p-value=0.781. A descriptive assessment of the premature infant staying time data using IPM is shown in Table 8. Table 8 makes it abundantly evident that uncertainties taken into account in the observed sample are the cause of discrepancies in a number of the critical numerical statistics of the failure times data. Further, it is more clearly shown from Table 8 that there are high varies among the estimation methods in estimating the IPM distribution parameters $\delta = 1.6409, \lambda = 74.472$.

For the real data application, the fuzzy-time model clearly outperforms the classical real-time model across all considered values of the cut of point (CC = 0.1, 0.5, 0.7), as evidenced by consistently and substantially lower AIC

Table 7. Real data values

4	168	2	44	117	149	289	8	2
3	192	24	28	172	153	1	314	4
14	216	16	31	169	138	359	313	1
18	264	18	41	9	129	333	9	23
2	270	12	34	2	155	377	374	227
11	269	30	47	5	258	310	15	22
8	288	17	39	5	254	344	12	48
13	260	32	37	7	290	2	220	45
15	6	2	46	6	299	23	24	38
22	19	1	40	4	291	21	1	35
20	29	103	42	140	297	12	250	47
144	26	30	118	128	236	365	13	43
335	255	52	83	75	101	133	48	58
270	302	60	91	107	108	104	26	55
312	168	54	86	89	100	210	13	72
34	98	72	93	84	109	280	23	50
35	384	62	77	97	116	261	14	65
48	177	59	78	106	120	21	16	72
37	61	63	81	90	113	257	7	71
39	56	49	94	76	123	32	24	50
42	67	72	82	110	119	25	21	324
245	51	80	102	73	99	29	3	381
222	70	96	95	105	126	36	28	320
227	57	74	87	79	122	27	33	323

and BIC values in Table 8. At CC = 0.1, both approaches provide very similar estimates of the mean time to failure (MTTF), with the fuzzy-time model yielding a slightly higher expected lifetime, while still achieving markedly better information criteria, indicating a more efficient representation of the data. When CC increases to 0.5 and 0.7, the superiority of the fuzzy-time framework becomes even more pronounced, with AIC and BIC values dropping to very low levels and remaining stable, suggesting robustness of the fuzzy specification to changes in CC. In this range, the real-time model produces larger MTTF values than the fuzzy-time model, which may indicate a tendency of the crisp model to overestimate the expected lifetime, whereas the fuzzy-time model offers a more parsimonious and statistically well-supported prediction. Overall, these results demonstrate that incorporating fuzzy time leads to a better-fitting and more stable model, while providing realistic estimates of MTTF under different levels of the correction coefficient.

Furthermore, the survival function plot for both the real time and the fuzzy time of the real data set for the three values of the CC are summarized in Figures 2, 3, and 4, respectively. As can be seen that the survival time for the fuzzy IPM distribution is better than the real-time IPM distribution.

5. Conclusion

In this study, we developed a comprehensive inferential framework for survival function estimation under the fuzzy inverse power Maxwell distribution, motivated by lifetime and reliability data characterized by both stochastic variation and epistemic uncertainty. The theoretical properties of the proposed model were investigated in detail, and closed-form expressions for the survival function and related reliability measures were derived under fuzzy parameterization. On this basis, we constructed estimation procedures that explicitly incorporate fuzziness into the model, leading to survival function estimators that are capable of reflecting imprecise prior information and

Table 8. The performance results for the real data application

		Real Time	Fuzzy Time
CC=0.1	AIC	625.281	570.3316
	BIC	626.6822	572.701
	MTTF	6.487502	6.514525
CC=0.5	AIC	689.67613	71.94019
	BIC	691.58815	72.24278
	MTTF	6.814914	6.249018
CC=0.7	AIC	689.67613	71.94019
	BIC	692.68677	72.24278
	MTTF	6.824914	6.309018

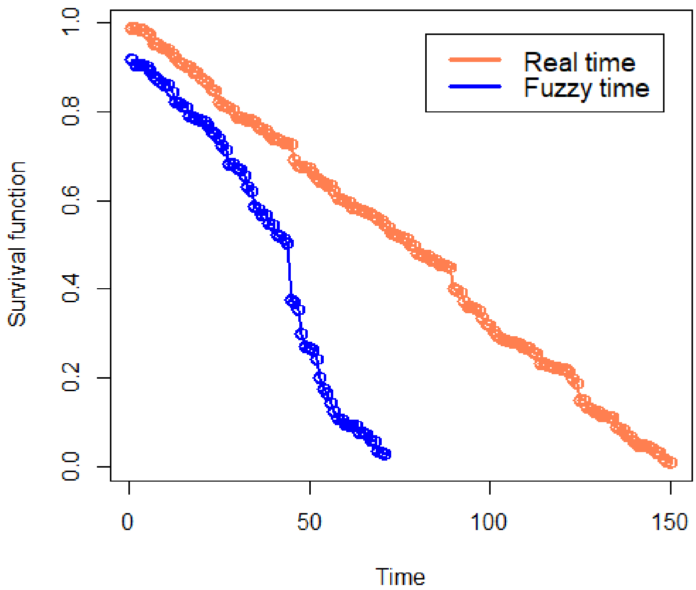

Figure 2. The survival function for the real data when CC=0.1

vague expert judgments more realistically than classical, purely probabilistic approaches. Extensive Monte Carlo simulations were conducted to examine the finite-sample performance of the proposed estimators. Across a wide range of parameter settings and degrees of fuzziness, the results indicated that the estimators are numerically stable and exhibit satisfactory behavior in terms of bias and mean squared error. In particular, the fuzzy-based estimators tend to outperform their crisp counterparts when the underlying system is subject to pronounced ambiguity, confirming the practical value of integrating fuzzy concepts into survival analysis. Overall, the proposed framework offers a flexible and robust tool for modeling survival data when uncertainty cannot be fully captured by traditional probabilistic models. Despite these contributions, the present work has several limitations that should be acknowledged. First, the analysis is restricted to the inverse power Maxwell family, and the generalizability of the proposed fuzzy estimation strategy to broader classes of lifetime distributions has not been formally established. Second, the study focuses primarily on parametric estimation under correctly specified models; issues such as model misspecification, outliers, and censoring mechanisms were not explored in depth. Third, all numerical investigations were based on simulated data, which, although informative, cannot fully represent the complexity and heterogeneity of real-world applications. Finally, the computational aspects of the algorithms, including convergence behavior and scalability to large data sets, were only assessed indirectly through simulation

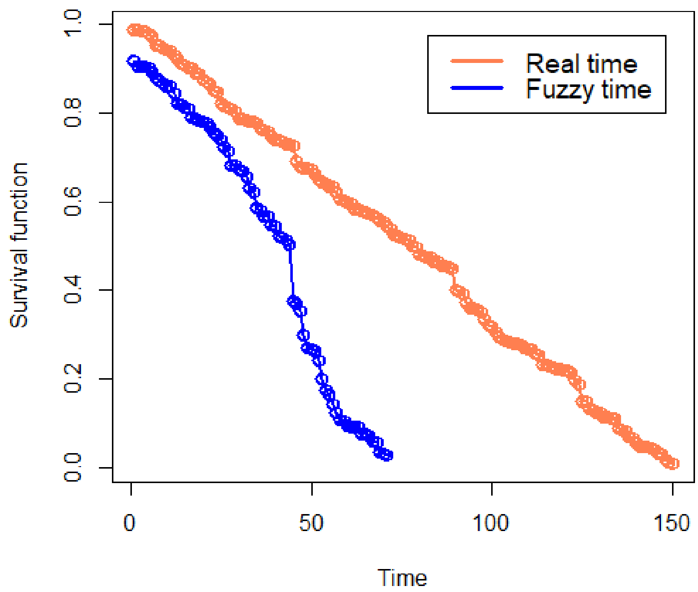

Figure 3. The survival function for the real data when CC=0.5

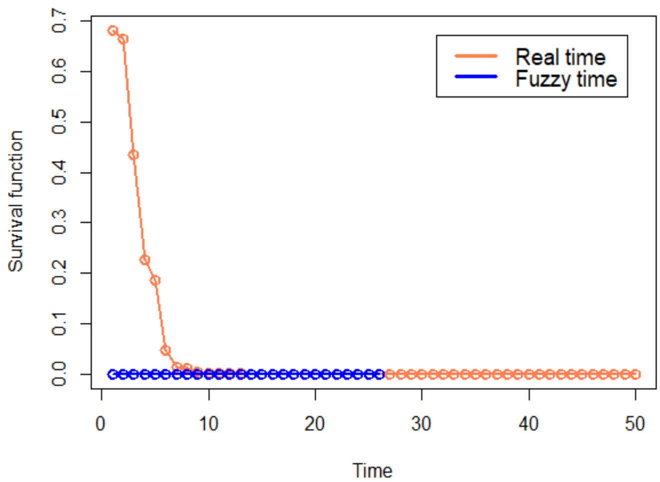

Figure 4. The survival function for the real data when CC=0.7

performance metrics. The main numerical findings of the simulation study can be summarized as follows. Under correctly specified models and moderate to large sample sizes, the proposed fuzzy survival function estimators exhibit small bias and competitive mean squared error relative to classical estimators that ignore fuzziness. When the degree of fuzziness increases, the classical (crisp) estimators become less reliable and may underestimate the true uncertainty, whereas the fuzzy-based estimators maintain more accurate coverage of the underlying survival behavior. The simulations further reveal that the performance of the proposed methods is robust across a range of parameter values, suggesting that the approach is not overly sensitive to specific shape characteristics of the inverse power Maxwell distribution. These results collectively demonstrate that the proposed framework offers both improved interpretability and enhanced inferential performance in settings where imprecision is inherent. Several directions for future research arise naturally from this study. An immediate extension is to incorporate censoring schemes, such as right, left, or interval censoring, into the fuzzy inverse power Maxwell framework and to derive corresponding estimation procedures for the survival function. Another promising direction is to generalize the methodology to broader families of fuzzy lifetime models, including other heavy-tailed or bathtub-shaped distributions that are widely used in reliability and biomedical applications. From a methodological standpoint, it would be valuable to investigate Bayesian or hybrid Bayesian–fuzzy approaches for survival function estimation, which could facilitate the incorporation of prior expert knowledge and provide a richer quantification of uncertainty. Finally, applications to real data sets from engineering, medical survival studies, or environmental reliability would further validate the practical merits of the proposed methods and may highlight additional modeling refinements, such as regression structures, covariate effects, or high-dimensional extensions.

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