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Modeling and Optimal Control Analysis of Lassa Fever with Lockdown and Treatment Interventions

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Abstract Lassa fever is an acute zoonotic viral disease caused by the Lassa virus and remains a major public health concern in endemic regions. In this study, a mathematical model is proposed to investigate the transmission dynamics of Lassa fever incorporating hospital quarantine and control interventions in the form of lockdown measures and treatment strategies. The model considers both human to human and rodent to human transmission pathways. The model admits two equilibrium points, namely the disease free equilibrium and the endemic equilibrium. The existence and local stability of these equilibria depend on the basic reproduction number, R_0. The disease free equilibrium is locally asymptotically stable when $R_0 < 1$, whereas the endemic equilibrium exists and becomes locally asymptotically stable when $R_0 > 1$. Sensitivity analysis is performed to identify the parameters that most strongly influence disease transmission. The results indicate that the human to human transmission rate, the recovery rate of infectious individuals, and the treatment uptake rate are among the most influential factors affecting disease spread. An optimal control problem is formulated and analyzed using the Pontryagin Maximum Principle to determine effective intervention strategies that minimize both the infected population and implementation costs. Numerical simulations demonstrate that treatment interventions significantly reduce the number of infected individuals, while the combined implementation of lockdown and treatment controls provides additional benefits in reducing disease transmission. These findings provide qualitative insight into the role of quarantine and intervention strategies in controlling Lassa fever transmission.

Keywords Mathematical model, Stability, Optimal control, Lassa fever, Lockdown, Treatment.

AMS 2010 subject classifications 34A34, 34A45, 92C60, 92D30, 93C15

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1. Introduction

In 1969, Lassa fever was discovered in West Africa, specifically in Nigeria [1]. Lassa fever is an acute virus or zoonosis, in humans due to contact with infected animals. The reservoir host (source of the carrier agent) of the Lassa virus is animals of the genus Mastomys, species Mastomys natalensis known as multimammate rats [2]. The virus that causes Lassa fever is Lassa Virus (LASV) or a virus that is a class of arbovirus with the family arenaviridae [2]. Family arenaviridae is a virus whose common members are diseases transmitted by rodents to humans [3].

The incubation period for Lassa fever is 6-21 days [4]. Infection with this disease has gradual symptoms starting with fever and fatigue. After a few days, sore throat, muscle pain, nausea, vomiting, diarrhea and abdominal pain appear [5]. In severe cases, swelling of the face, fluid in the lung cavity, bleeding from the mouth or nose, and low blood pressure may occur [4].

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About 80% of people infected with this virus show symptoms that result in severe disease by attacking several organs such as the liver, spleen, and kidneys [6]. According to [4], from January 1 to February 9, 2020 there were 472 laboratory confirmed death cases (fatality ratio of 14.8%) that 26 out of 36 states of Nigeria have cases annually. There are many interactions that occur in humans and the environment so that the Lassa fever virus is transmitted to humans, one of which is urine released carelessly by multimmamate rats. In the Western tropics, the risk of Lassa virus infection is high due to lack of rainfall and multimammate rats breed more during the dry season [7].

Mathematical modeling is a field of mathematics that seeks to represent and explain physical systems or problems in the real world, so that an understanding of these real-world problems becomes more precise [8]. Mathematical models are used to simplify mathematical forms to make them easier to solve. In the Lassa fever mathematical model, [9] presents the analysis of mathematical models with Lassa fever evaluation using static quantities. [10] examines the mathematical model of Lassa fever by examining disease transmission and optimal control models with control variables, namely fogging the environment with pesticides, using condoms, using sprays in the room. In the Lassa fever Mathematical model, [11, 12] studied about analyzing asymptotic stability at a non-endemic equilibrium point.

Based on the explanation above, the author is interested in studying the mathematical model of the spread of Lassa fever disease written by [11] which consists of five compartments, namely the vulnerable human population, infected human population, recovered human population, vulnerable rodent population, and infected rodent population. Therefore, the author modified the model by adding a quarantined human population in the hospital.

In this study, the authors will also add optimal control variables, namely lockdown and treatment. Lockdown has been implemented in the West Africa region of Nigeria to reduce the rate of spread of the virus and limit social interaction between humans. Treatment is carried out as an effort to cure infected humans. Furthermore, to see the effectiveness of the given control variables, numerical simulations are carried out.

2. Formulation of a Lassa fever Model with Hospital Quarantine

In this section we will explain the mathematical model of the spread of Lassa fever which refers to the paper written by [11], discussing the mathematical model of Lassa fever with five compartments, namely the vulnerable human population $S(t)$, the infected human population $I(t)$, the recovered human population $R(t)$, the vulnerable rodent $P(t)$ and the infected rodent $Q(t)$. The author modifies the model by adding the human population quarantining the hospital $H(t)$.

The compartments and parameters used in the mathematical model of the spread of Lassa fever are presented in Table 1 and Table 2.

Table 1. Definition and Notation of Compartments in the Mathematical Model of Lassa fever Disease Spread

Compartment	Description
$S(t)$	Susceptible human population at time t
$I(t)$	Infected human population at time t
$R(t)$	Recovered human population at time t
$H(t)$	Hospital quarantined human population at time t
$N(t)$	Total human population at time t
$P(t)$	Susceptible rodent population at time t
$Q(t)$	Infected rodent population at time t
$Z(t)$	Total rodent population at time t

The transmission dynamics of Lassa fever disease can be visualized through a compartmental diagram that illustrates the flow of individuals between different epidemiological states. Figure 1 presents the transmission

Table 2. Parameter definition in mathematical model of Lassa fever disease spread

Parameters	Description
β	Rate of susceptible humans infected by infected humans
α	Rate of susceptible humans infected by rodents
μ	Natural death rate of humans
ρ	Rate at which infected humans recover
ξ	Rodent natural mortality rate
ϕ	Rate of susceptible rodents infected with other infected rodents
ω	Rate of infected humans recovering while receiving treatment
τ	Rate of infected humans receiving treatment
γ	Recovery rate due to treatment

diagram of the proposed model, showing the interactions between human and rodent populations, as well as the various pathways through which the disease spreads within and between these populations.

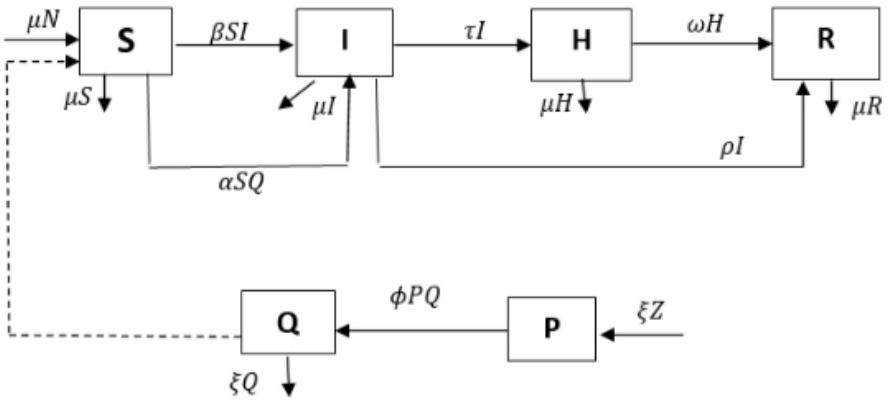


Figure 1. Transmission diagram of Lassa fever disease spread model

The mathematical model of Lassa fever disease spread is given by the following system of differential equations:

$$\frac{dS}{dt} = \mu N - \beta SI - \alpha SQ - \mu S, \tag{1}$$

$$\frac{dI}{dt} = \beta SI + \alpha SQ - (\mu + \rho + \tau)I, \tag{2}$$

$$\frac{dH}{dt} = \tau I - (\mu + \omega)H, \tag{3}$$

$$\frac{dR}{dt} = \rho I + \omega H - \mu R, \tag{4}$$

$$\frac{dP}{dt} = \xi Z - \phi PQ - \xi P, \tag{5}$$

$$\frac{dQ}{dt} = \phi PQ - \xi Q. \tag{6}$$

The total rodent population Z is assumed to remain approximately constant over the epidemic time horizon. Under this assumption, the recruitment term ξZ represents a constant inflow into the susceptible rodent population,

maintaining the reservoir size throughout the simulation period. This assumption is commonly adopted in zoonotic disease models when the reservoir population is relatively large and the infection does not significantly alter the overall rodent population dynamics. Nevertheless, we recognize that real rodent populations may exhibit density-dependent growth, seasonal variation, and environmental influences that could affect disease transmission. The present simplification is adopted to preserve analytical tractability and to focus primarily on the interaction between human infection dynamics and the rodent reservoir.

2.1. Equilibrium Point of the Mathematical Model

An equilibrium state is a condition when the total population over time is zero or no change occurs. The mathematical model of the spread of Lassa fever disease has an equilibrium point if it meets the equation of zero disease infection, then the mathematical model meets an equilibrium state when

$$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dH}{dt} = \frac{dR}{dt} = \frac{dP}{dt} = \frac{dQ}{dt} = 0.$$

Based on this equation, two equilibrium points are obtained, namely the non-endemic and the endemic equilibrium point. Non-endemic (disease-free) equilibrium point is an equilibrium point where the condition when there is no spread of disease in the population. This situation occurs when the population of infected humans and animals is zero $(I = Q = 0)$, substituting the value in the equation then the equilibrium point is non endemic E_0 obtained

$$E_0 = (S, I, R, H, P, Q) = (N, 0, 0, 0, Z, 0).$$

Next, the Basic Reproduction Number (R_0) will be determined which is used to determine the threshold of a situation that indicates that a disease is spreading or not. In calculating R_0, using the NGM (Next Generation Matrix) method with the approach of [13]. From the method obtained the value of R_0.

$$R_0 = \max\left[R_{0m}, R_{0h}\right] \tag{7}$$

with $R_{0m} = \frac{\beta N}{(\mu+\tau+\rho)}$ and $R_{0h} = \frac{\phi Z}{\xi}$.

The endemic equilibrium point is a condition that occurs when the spread of disease in the susceptible human population, infected human population, treated human population, recovered human population, susceptible animal population, and infected animal population can be expressed as $S \neq 0, I \neq 0, R \neq 0, H \neq 0, P \neq 0$, and $Q \neq 0$. The endemic equilibrium point of the mathematical model of the spread of Lassa fever disease is obtained which is expressed as $E_1 = (S^*, I^*, R^*, H^*, P^*, Q^*)$, by

$$S^* = \frac{\mu N}{\beta I^* + \alpha\left(Z - \frac{\xi}{\phi}\right) + \mu}, \tag{8}$$

$$R^* = \frac{I^*}{\mu}\left(\rho + \frac{\omega\tau}{\mu+\omega}\right), \tag{9}$$

$$H^* = \frac{I^*\tau}{\mu+\omega}, \tag{10}$$

$$P^* = \frac{\xi}{\phi}, \tag{11}$$

$$Q^* = Z - \frac{\xi}{\phi}. \tag{12}$$

Furthermore, to get the value of I^*, a substitution is made in the equation, I so that it is obtained

$$I^2\beta(\mu+\rho+\tau) + I\left((\mu+\rho+\tau)\left(\alpha Z - \frac{\alpha\xi}{\phi} + \mu\right) - \beta\mu N\right) - \alpha\mu N\left(Z - \frac{\xi}{\phi}\right) = 0$$

and I^* is the positive root of the equation

$$aI^{*2} + bI^* + c = 0 \tag{13}$$

with

$$\begin{aligned} a &= \beta(\mu + \rho + \tau), \\ b &= (\mu + \rho + \tau)\left(\alpha Z - \frac{\alpha\xi}{\phi} + \mu\right) - \beta\mu N, \\ c &= -\alpha\mu N\left(Z - \frac{\xi}{\phi}\right). \end{aligned}$$

Endemic equilibrium point $E_1 = (S^*, I^*, R^*, H^*, P^*, Q^*)$ exists if:
(a) The discriminant is positive when it satisfies

$$b^2 - 4ac > 0 \Longleftrightarrow b^2 > 4ac.$$

(b) The product of two roots is negative when $I^*{}_1, I^*{}_2 < 0$, where $I^*{}_1$ and $I^*{}_2$ are the roots of equation (13). In quadratic equations, the property $I^*{}_1 I^*{}_2 = \frac{c}{a}$ where $a > 0$ so that $I^*{}_1 I^*{}_2 < 0$ is satisfied only if c is negative. The coefficient c becomes negative when the following conditions are met::

$$Z\phi > \xi \Leftrightarrow \frac{Z\phi}{\xi} > 1 \Leftrightarrow R_{0h} > 1.$$

The endemic equilibrium point will exist if $R_{0h} > 1$ with $R_{0h} = \frac{Z\phi}{\xi}$.

2.2. Analysis of Local Stability of the Equilibrium Point

The mathematical model for the spread of Lassa fever disease in this paper is formulated as a system of nonlinear differential equations. Therefore, to analyze the local stability of the previously obtained equilibrium points, the system must be linearized using the Jacobian matrix. The Jacobian matrix of the model is given by:

$$J = \begin{pmatrix} -\beta I - \alpha Q - \mu & -\beta S & 0 & 0 & 0 & -\alpha S \\ \beta I + \alpha Q & \beta S - (\mu + \rho + \tau) & 0 & 0 & 0 & \alpha S \\ 0 & \tau & -(\mu + \omega) & 0 & 0 & 0 \\ 0 & \rho & \omega & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & -\phi Q - \xi & -\phi P \\ 0 & 0 & 0 & 0 & \phi Q & \phi P - \xi \end{pmatrix}$$

2.2.1. Stability of Non-Endemic Equilibrium Point (E_0) Jacobian matrix of non-endemic equilibrium point $E_0 = (S, I, R, H, P, Q) = (N, 0, 0, 0, Z, 0)$ as follows:

$$J_{E_0} = \begin{bmatrix} -\mu & -\beta N & 0 & 0 & 0 & -\alpha N \\ 0 & \beta N - (\mu + \rho + \tau) & 0 & 0 & 0 & \alpha N \\ 0 & \tau & -(\mu + \omega) & 0 & 0 & 0 \\ 0 & \rho & \omega & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & -\xi & -\phi Z \\ 0 & 0 & 0 & 0 & 0 & \phi Z - \xi \end{bmatrix}$$

Based on the Jacobian matrix, a characteristic equation can be formed using $\det(J_{E_0} - \lambda I) = 0$, obtained

$$(-\mu - \lambda)(\phi Z - \xi - \lambda)(-\xi - \lambda)(-\mu - \lambda)(\beta N - (\mu + \rho + \tau) - \lambda)(-(\mu + \omega) - \lambda) = 0$$

Explicitly obtained eigenvalues $\lambda_1 = -\mu$, $\lambda_3 = -\xi$, $\lambda_4 = -\mu$ and $\lambda_6 = -(\mu + \omega)$. The others are roots of the equation $\lambda^2 + p_1\lambda + p_2 = 0$, with

$$p_1 = \xi - \phi Z,$$
$$p_2 = \mu + \rho + \tau - \beta N.$$

The sign of the real part of the roots of the equation is found using the Routh-Hurwitz Criterion. Based on the criterion, it is known that the characteristic equation has degree two when $p_1, p_2 > 0$ has negative roots, so the following conditions are obtained:

(i) $R_{0m} = \frac{\beta N}{\mu+\rho+\tau} < 1.$
(ii) $R_{0h} = \frac{\phi Z}{\xi} < 1.$

2.2.2. Stability of Endemic Equilibrium Point (E_1) The stability analysis of the endemic equilibrium point follows similar procedure using the Jacobian matrix evaluated at E_1. Through Routh-Hurwitz criterion, it can be shown that the endemic equilibrium point is locally asymptotically stable when $R_{0h} > 1$.

3. Parameter Sensitivity Analysis

Sensitivity analysis is carried out to find out which parameters have an effect on the stability conditions of the equilibrium point of the model. The sensitivity index at R_0 is $\frac{\partial R_0}{\partial m}\frac{m}{R_0}$, where m is the parameter to be analyzed [14]. R_0 used in the parameter sensitivity analysis is $R_{0m} = \frac{\beta N}{\mu+\rho+\tau}$.

The following are the parameter values used to calculate the parameter sensitivity index.

Table 3. Parameter Value of Mathematical Model on the Spread of Lassa fever Disease

Parameters	**Value**	**Unit**	**Source**
β (Non Endemic)	0.000002	week^{-1}	Assumptions
β (Endemic)	0.00002	week^{-1}	[11]
α	0.00001	week^{-1}	[11]
μ	0.0000457	week^{-1}	[11]
ρ	0.476	week^{-1}	[11]
ξ	0.2	week^{-1}	[7]
ϕ	0.05	week^{-1}	[7]
ω	0.118	week^{-1}	[15]
τ	0.07	week^{-1}	[11]
γ	0.35	week^{-1}	Assumptions

Several parameters used in this study, including the transmission coefficient β and the treatment recovery rate γ, were selected based on biologically plausible ranges reported in previous epidemiological studies. Due to the limited availability of reliable empirical estimates specific to the modeled setting, these parameters were treated as baseline assumptions to facilitate the investigation of the proposed control strategies.

The transmission coefficient β was assigned different values for the non endemic and endemic scenarios in order to represent distinct epidemiological conditions. In the non endemic scenario, a relatively smaller value of β was used to describe situations where human to human transmission is infrequent. Conversely, the larger value of β in the endemic scenario reflects settings characterized by sustained interpersonal transmission, increased contact rates, delayed case detection, or limited infection prevention measures. This distinction enables the model to capture different transmission environments commonly observed in Lassa fever outbreaks.

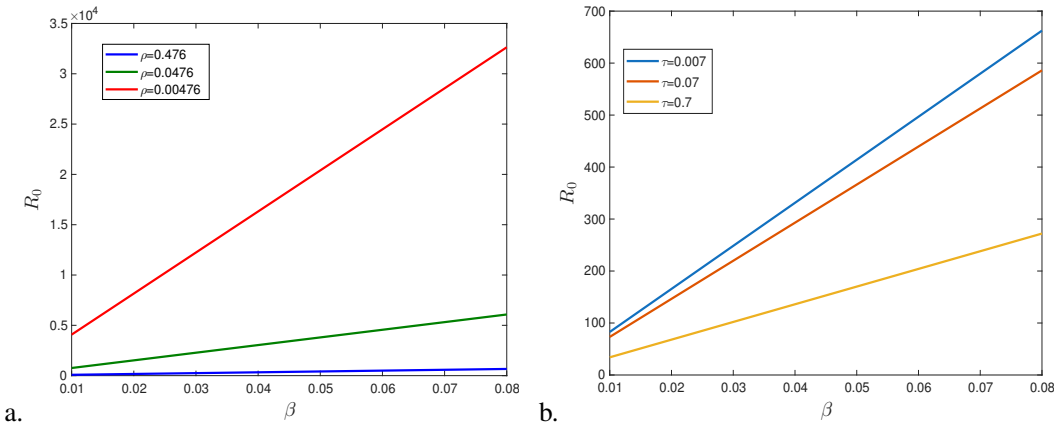
Moreover, the calculation of the sensitivity index of the parameters of the mathematical model of the spread of Lassa fever disease results are listed in Table 4. In the present study, the sensitivity analysis primarily focused on parameters related to the human transmission component, particularly β, ρ, and τ, because the implemented optimal control strategies mainly target human related interventions such as treatment and reduction of interpersonal contact. Nevertheless, the rodent reservoir remains a fundamental component of Lassa fever transmission

dynamics. Parameters associated with rodent ecology and transmission, including the rodent transmission rate ϕ, the recruitment rate ξ, and the total rodent population Z, may substantially influence disease persistence and the overall reproduction number.

Table 4. Parameter Sensitivity Index Value at number R_0 Human Population

Parameters	**Sensitivity Index**
β	1
μ	-0.00086
ρ	-0.90415
τ	-0.094974
ϕ	1
Z	1
ξ	-1

Next, we will simulate the sensitivity of the parameters β, ρ, and τ. In the first simulation, three parameter values of ρ were chosen, namely $\rho = 0.476, \rho = 0.0476, \rho = 0.00476$ with β on the interval $0.01 \leq \beta \leq 0.08$. In the second simulation, three values of the parameter τ were chosen, namely $\tau = 0.007, \tau = 0.07$, and $\tau = 0.7$ with β on the interval $0.001 \leq \beta \leq 0.008$. The simulation results are presented in Figures 2a and 2b.



a. b.

Figure 2. Sensitivity of β to R_0 (a) with three different values of ρ, (b) with three different values of τ. All parameter values are in Table 3.

Based on the sensitivity analysis, the parameter value of ρ is getting smaller at R_0 for all values of ρ. This is because the sensitivity index value of ρ is negative. Similarly, the parameter value of τ is getting smaller at R_0 for all values of τ. This is because the sensitivity index value of τ is negative. These results indicate that increasing the recovery rate (ρ) and the treatment rate (τ) are effective strategies to reduce the basic reproduction number and control the spread of Lassa fever disease.

4. Application of Optimal Control

In this section, we will determine the optimal control of the mathematical model of Lassa fever disease by constructing the model by adding control variables u_1 and u_2, each of which is in the form of lockdown and treatment efforts. The control variable u_1 is introduced to represent public health interventions aimed at reducing human to human transmission, including movement restrictions, social distancing, and limitations on public gatherings. Accordingly, the control acts on the human to human transmission parameter β. In the context of Lassa fever, zoonotic transmission from infected rodents, particularly Mastomys natalensis, remains an important

transmission pathway. However, lockdown related interventions do not necessarily reduce rodent to human exposure and may even maintain or increase such exposure due to prolonged indoor stay. Therefore, in the present model, u_1 is intentionally applied only to the interpersonal transmission pathway and does not directly affect the rodent to human transmission parameter α. Additional interventions such as environmental sanitation, rodent-proofing, and food storage management could potentially reduce rodent to human transmission, but these measures are not explicitly incorporated into the current modeling framework. The mathematical model of Lassa fever disease spread that has been constructed by adding control variables is as follows:

$$\frac{dS}{dt} = \mu N - \beta SI\left(1-u_1\right) - \alpha SQ - \mu S, \tag{14}$$

$$\frac{dI}{dt} = \beta SI\left(1-u_1\right) + \alpha SQ - \left(\mu+\rho+\tau\right)I - \gamma u_2 I, \tag{15}$$

$$\frac{dH}{dt} = \tau I - \left(\mu+\omega\right)H, \tag{16}$$

$$\frac{dR}{dt} = \rho I + \omega H - \mu R + \gamma u_2 I, \tag{17}$$

$$\frac{dP}{dt} = \xi Z - \phi PQ - \xi P, \tag{18}$$

$$\frac{dQ}{dt} = \phi PQ - \xi Q. \tag{19}$$

For the optimal control in this study, a cost function was devised to minimize the human population exposed to Lassa fever, while also minimizing the cost associated with implementing control measures. Therefore, the cost function can be formulated as follows:

The performance index $\left(MinJ\left(u_1,u_2\right)\right)$ that can be formed based on the above explanation is as follows:

$$\int_0^{t_f}\left(A_1 I(t) + \frac{B_1}{2}\left(u_1\right)^2(t) + \frac{B_2}{2}\left(u_2\right)^2(t)\right)dt$$

where $0 \le u_1, u_2 \le 1$ and $A_1, B_1, B_2 > 0$. A_1 is a weighting constant of I. Meanwhile, B_1 and B_2 are weighting constants of u_1 and u_2. The weighting parameters in the objective functional represent the relative importance assigned to minimizing the infected population and the implementation costs associated with the control strategies. Specifically, the parameter A_1 measures the importance of reducing the number of infected individuals, whereas B_1 and B_2 correspond to the economic and social costs associated with implementing lockdown and treatment interventions, respectively. Optimal control time out is at an interval $t_0 \le t \le t_f$ that expresses the time of observation made, which is the time when the control is given to the end time of the control. The quadratic forms of the control terms are adopted to reflect the increasing marginal cost associated with stronger intervention efforts, which is a standard assumption in optimal control theory. The quadratic function of the control cost is adopted, as stated in [16, 17, 18].

Based on Pontryagin's Maximum Principle [19], the first step carried out in the analysis of the optimal control problem is to form a Hamiltonian (H) function, that is:

$$H = A_1 I(t) + \frac{B_1}{2}\left(u_1\right)^2(t) + \frac{B_2}{2}\left(u_2\right)^2(t) + \theta^T(t)\left(f\left(x\left(t\right),u\left(t\right),t\right)\right)$$

$\left(f\left(x\left(t\right),u\left(t\right),t\right)\right)$ is the right segment model of Lassa fever spread accompanied by a control variable, while $\theta^T(t)$ is co-state variable.

Furthermore, in order to obtain optimal conditions, the Hamiltonian function above must meet stationary conditions, namely $\frac{\partial H}{\partial u_1} = 0$ and $\frac{\partial H}{\partial u_2} = 0$. So that the optimal controllers u_1 and u_2 are obtained

$$u_1^* = \min\left\{1, \max\left(0, \frac{(\theta_2 - \theta_1)\,\beta SI}{B_1}\right)\right\},$$
$$u_2^* = \min\left\{1, \max\left(0, \frac{(\theta_4 - \theta_2)\,\gamma I}{B_2}\right)\right\}.$$

The controller forms of u_1^* and u_2^* depend on state and co-state variables. The state equations are as follows:

$$\begin{aligned}
\frac{dS}{dt} &= \frac{\partial H}{\partial \theta_1} = \mu N - \beta SI\,(1-u_1) - \alpha SQ - \mu S,\\
\frac{dI}{dt} &= \frac{\partial H}{\partial \theta_2} = \beta SI\,(1-u_1) + \alpha SQ - (\mu + \rho + \tau)\,I - \gamma u_2 I,\\
\frac{dH}{dt} &= \frac{\partial H}{\partial \theta_3} = \tau I - (\mu + \omega)\,H,\\
\frac{dR}{dt} &= \frac{\partial H}{\partial \theta_4} = \rho I + \omega H - \mu R + \gamma u_2 I,\\
\frac{dP}{dt} &= \frac{\partial H}{\partial \theta_5} = \xi Z - \phi PQ - \xi P,\\
\frac{dQ}{dt} &= \frac{\partial H}{\partial \theta_6} = \phi PQ - \xi Q.
\end{aligned} \tag{20}$$

Meanwhile, the co-state equations are as follows :

$$\begin{aligned}
\dot{\theta_1} &= -\frac{\partial H}{\partial S} = (\theta_1 - \theta_2)\,(\beta I\,(1-u_1) + SQ)\,,\\
\dot{\theta_2} &= -\frac{\partial H}{\partial I} = -A_1 + (\theta_1 - \theta_2)\,\beta S\,(1-u_1) + \theta_2\,(\mu + \rho + \tau - \gamma u_2) - \theta_3\tau - (\rho + \gamma u_2 I)\\
\dot{\theta_3} &= -\frac{\partial H}{\partial H} = \theta_3\,(\mu + \omega) - \theta_4\omega,\\
\dot{\theta_4} &= -\frac{\partial H}{\partial R} = \theta_4\mu,\\
\dot{\theta_5} &= -\frac{\partial H}{\partial P} = (\theta_5 - \theta_6)\,(\phi Q) + \theta_5\xi,\\
\dot{\theta_6} &= -\frac{\partial H}{\partial Q} = (\theta_5 - \theta_6)\,(\phi P) + \theta_6\xi.
\end{aligned} \tag{21}$$

Based on the description above, to get the values of S, I, H, R, P and Q from the optimal form u_1^* and u_2^* then it is necessary to solve the non-linear state and co-state equations. The non-linear equation system is hard to be solved analytically, so it will be solved numerically.

5. Numerical Results

The numerical simulation involves comparing a mathematical model of Lassa fever spread without control variables to one integrating control variables. The aim is to assess the effectiveness of the control efforts in achieving the objectives outlined by the presented cost function. For solving the optimal control strategy, we employ the fourth-order Runge-Kutta (RK4) scheme. Initially, we utilize the forward RK4 technique to solve the state system. Subsequently, the backward RK4 scheme is applied to solve the co-state system. The initial values for all population compartments in this simulation are set as follows: $S(0) = 1500$, $I(0) = 1000$, $H(0) = 700$,

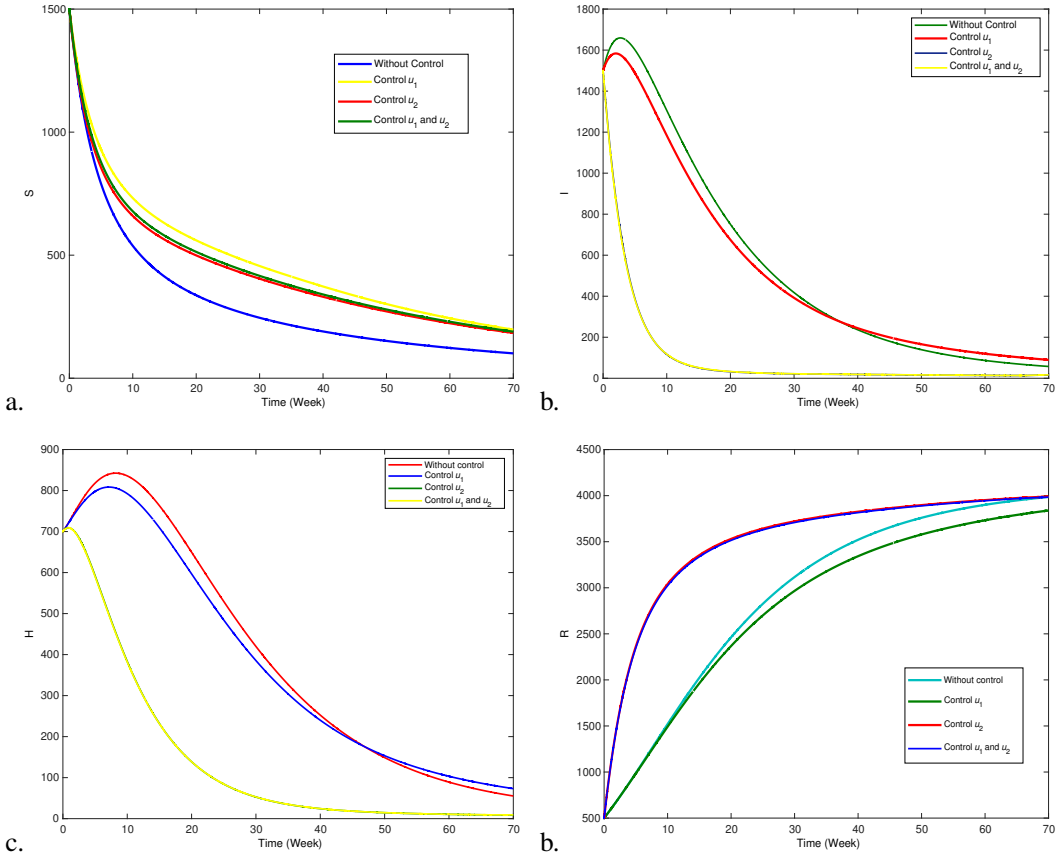


Figure 3. Comparison between the number population (a) Susceptible S, (b) Infected of Lassa fever I, (c) Hospital quarantine H, (d) Recovered R without and with control. All parameter values are in Table 3.

$R(0) = 500$, $P(0) = 1000$, and $Q(0) = 12000$, with the simulation conducted from $t_0 = 0$ to $t_f = 70$. Parameter values in this numerical simulation correspond to those listed in Table 3.

Figure 3 shows that the simultaneous implementation of lockdown control (u_1) and treatment control (u_2) results in a substantial reduction in both the infected population (I) and the hospitalized (quarantined) population (H) compared with the scenario without any control measures. In the absence of controls, the values of I and H remain high; however, once u_1 and u_2 are applied, both populations exhibit a marked decline.

Table 5. Comparison of total population S, I, H, R with and without control strategies

Condition	Total population up to seventy weeks				Cost value
	S	I	H	R	
No Control	101	58	55	3986	–
Control u_1	197	90	73	3839	36725
Control u_2	184	16	9	3990	7279
Control u_1 and u_2	190	16	9	3990	7274

Furthermore, it can be seen from Table 5 that the application of controls u_1 and u_2 simultaneously is the most effective in reducing the infected population I by 72%, and the hospitalized (quarantined) population H by 84%. Also, the application of both control simultaneously gives the smallest cost value compared to the application of control u_1 or u_2 alone. This observation suggests that, under the selected parameter values and weighting

assumptions, treatment plays a dominant role in reducing the final number of infected individuals. Consequently, the additional contribution of the lockdown control u_1 to the final infection count appears relatively limited in the current simulation setting.

6. Conclusion

This study provides a comprehensive analysis of the transmission dynamics of Lassa fever incorporating the rodent and human transmission pathways. The model yields two equilibrium points, namely disease free and endemic equilibrium. Disease free equilibrium is shown to be locally asymptotically stable when the effective reproduction number is less than one. Sensitivity analysis of the model parameters highlights the key factors that most strongly influence the spread of Lassa fever, offering valuable insights for designing effective intervention strategies.

Optimal control strategies were introduced in the form of lockdown measures and treatment interventions. Numerical simulations reveal that applying both controls simultaneously leads to a significant reduction in the number of infected individuals (I), and hospitalized (quarantined) individuals (H) when compared to scenarios without controls. These findings demonstrate that a combined approach restricting movement to limit transmission while enhancing treatment availability can substantially mitigate the spread of Lassa fever within affected communities.

Despite the useful insights provided by the model, several limitations should be acknowledged. Future research on Lassa fever modelling could introduce additional complexities such as environmental reservoirs, rodent population dynamics, or fractional-order differential equations to capture memory effects in disease progression. Including fractional-order dynamics may provide richer insights into delayed immune responses, prolonged infectious stages, and persistent environmental contamination characteristics commonly associated with Lassa virus transmission. Extensions of the model could also explore spatial heterogeneity, co-infections, and socio-economic determinants that influence disease outcomes.

By identifying effective combinations of lockdown and treatment strategies, this work highlights the value of evidence-based interventions that can reduce infection prevalence and prevent outbreaks. Continued research will be essential to refine these strategies and support global efforts to reduce the burden of emerging and re-emerging infectious diseases.

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