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Fuzzy Statistical Inference for Managing Imprecision in Hypothesis Testing: An Empirical Study of Medical Samples

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Abstract This study addresses the limitations of traditional statistical hypothesis testing, which relies on binary decision-making (accept/reject) and may not adequately represent the uncertainty inherent in real-world clinical data. To overcome these limitations, a fuzzy statistical framework is proposed by incorporating the concepts of possibility and necessity into the decision-making process. The proposed approach employs membership functions to replace rigid decision boundaries with necessary and possible acceptance/rejection regions, allowing more gradual and flexible statistical decisions. In addition, the study represents observations using hybrid numbers that integrate both randomness and imprecision, providing a more realistic representation of uncertain clinical phenomena.For empirical evaluation, real medical data collected from Mosul General Hospital were analyzed. The Z-test for two proportions and the one-sample t-test were implemented using both classical and fuzzy approaches, and their results were comparatively examined. The findings showed general consistency between the classical and fuzzy methods in cases characterized by clear statistical evidence. Moreover, the results demonstrated that the fuzzy expanstion coefficient η influences the width of the acceptance and rejection regions and consequently affects the associated fuzzy error measures, provided an improved balance between decision flexibility and statistical precision within the analyzed data. Overall, the proposed fuzzy approach offers a more flexible and interpretable framework for statistical inference and may provide practical advantages when dealing with uncertain medical datasets.

Keywords Fuzzy hypothesis testing, Hybrid numbers, Membership functions, Necessary and possible decision regions

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1. Introduction

Fuzzy set theory was first introduced by Lotfi Zadeh at the University of California in 1965 as a mathematical framework to address complex systems characterized by high uncertainty and incomplete information [1]. Fuzzification has since emerged as a viable real-world solution for such problem domains. This theory has witnessed remarkable advancements across diverse disciplines, including artificial intelligence, computer science, medicine, and engineering. Within the realm of statistical sciences, fuzzy hypothesis testing represents a vital application, utilizing fuzzy set theory to arrive at more flexible and realistic decisions . [2] Fuzzy logic operates as a multi-valued logic that accommodates intermediate truth values between traditional binary assessments, such as true/false, yes/no, or high/low [3].In complex environments, crisp binary logic is often insufficient to represent logical uncertainties, especially in medical diagnostics where boundaries are naturally imprecise; thus, natural human languages form the foundation for constructing intuitive fuzzy systems [4].
Numerous recent studies have focused on advancing hypothesis testing via fuzzy logic to optimize statistical inference [5]. For instance, Haktan*ı*r and Kahraman [6] developed a robust hypothesis testing framework utilizing Z-fuzzy numbers, where data ambiguity is modeled via membership functions coupled with a reliability score, significantly improving statistical decisions under data imprecision. Similarly, Mylonas and Papadopoulos

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[7] introduced unbiased fuzzy estimators in fuzzy hypothesis testing based on the probability distribution of parameters, where fuzzy test statistics were constructed from traditional confidence intervals, showing structural alignment with classical tests. Furthermore, Kalpanapriya et al. [12] proposed a novel statistical method utilizing fractional statistical analysis to handle imprecise data, relying on lacunarity measures to evaluate uncertain data intervals. More recently, Elsherif et al. [13] presented an innovative methodology for fuzzy hypothesis testing designed to minimize the probabilities of false alarms and missed detections in noise-contaminated systems. To extend these advancements, this study proposes a comprehensive fuzzy hypothesis testing approach that evaluates hypotheses progressively through continuous membership functions rather than binary partitions, thereby enhancing the precision and clinical utility of test outcomes.

To extend these advancements and address the limitations of binary statistical decisions in clinical environments, this study proposes a comprehensive fuzzy hypothesis testing framework tailored for medical diagnostics. Specifically, we implement the Z-test for two proportions and the one-sample t-test using real clinical datasets collected from Mosul General Hospital, focusing on patients diagnosed with diabetes, angina pectoris, and anemia. Unlike classical sharp inference, our approach models sample parameters as hybrid fuzzy-random numbers and partitions the critical regions into Necessary and Possible acceptance/rejection zones via continuous membership functions. Furthermore, we systematically evaluate the behavior of statistical decision errors specifically Type I $\alpha$and Type II β fuzzy risks by adjusting the fuzzy expansion coefficient η up to a high-confidence threshold of 0.8. This practical implementation demonstrates how integrating non-probabilistic imprecision into standard tests provides healthcare professionals with a safer, more flexible, and highly descriptive decision-making tool for critical patient evaluations.

Main Contributions of the Study

1. Development of a fuzzy hypothesis testing framework that replaces rigid binary decisions with necessary and possible acceptance/rejection regions.
2. Integration of hybrid fuzzy-random numbers to simultaneously represent sampling variability and measurement imprecision in medical data.
3. Application of the proposed framework to real clinical datasets from Mosul General Hospital using the fuzzy Z-test for two proportions and the fuzzy one-sample t-test.
4. Evaluation of fuzzy Type I and Type II error behavior under uncertainty, providing a more flexible interpretation of statistical decisions.

2. Materials and methods

2.1. Traditional Hypothesis Testing

Statistical inference is one of the most important fundamental goals of statistics, as it is used to deduce the characteristics of statistical populations based on information derived from samples drawn from them. Mathematical statistics is based on two main axes: the first is the estimation of unknown statistical parameters, such as the mean and variance [24, 25], while the second is the testing of statistical hypotheses. A statistical test is based on two basic hypotheses: the null hypothesis (H_0), which always assumes no effect or no significant difference, and the alternative hypothesis (H_1), which is the claim the researcher attempts to prove. It assumes the existence of a real effect or a significant difference.

The decision regarding the two hypotheses depends on the value of the test statistic, which is extracted from the sample data. This value is then compared to the appropriate tabular values according to the type of statistical test used and the specified significance level. The most common test statistics used in traditional statistics include the F χ^2z and T tests. Based on this comparison, the final statistical decision is made by accepting one hypothesis and rejecting the other.

By studying the characteristics of traditional statistical tests, we can observe that they are characterized by two main features:

First, the decision made is a sharp or rigid one, with a clear dividing line between the acceptance and rejection zones. For example, in a two-tailed Z-test, at a significance level of $\alpha = 0.05$, the critical value is $Z_0 = 1.96$.

Therefore, if the absolute value of the calculated test statistic exceeds this critical value, the null hypothesis H_0 is rejected and the alternative hypothesis H_1 is accepted. Conversely, the null hypothesis is accepted if the calculated value falls within the acceptance zone. Critical values in traditional tests are fixed and do not change with variations in data or sample characteristics, which may lead to inconsistent results when applied to different data sets. For example, if the calculated test statistic is greater than the critical value, the null hypothesis is rejected, and the alternative hypothesis is accepted when the statistic is less than the critical value. In this case, a sharp and clear boundary exists between the acceptance and rejection zones, without any transitional zone between them [26, 27, 28].

Second, the power of the test: The power of a statistical test is defined as the probability of rejecting the null hypothesis when it is false, and it is represented mathematically $(1-\beta)$ Where β indicates the probability of a Type II error. Test power is an important indicator of the effectiveness of a statistical test; power increases as the test is better able to detect true differences between hypotheses. However, traditional test power is affected by several factors, including sample size, significance level, and true parameter value, and may decrease significantly in some practical cases [15, 16, 17].

2.2. Fuzzy Hypothesis Testing

This is a development of traditional statistical tests aimed at dealing with cases where scientific or practical claims are not defined with absolute numerical precision, but rather with linguistic or approximate formulations. This makes it difficult to definitively determine the truth or falsehood of the hypothesis. For this reason, a sample representative of the population under study is used, and the necessary data are then extracted from it to estimate the validity of the hypothesis [18]. If the statistical population follows a normal distribution of $\mathrm{N}(\mu,\sigma^2)$ when $\sigma^2$is known, the test statistical $\bar{\mathrm{X}}$ also follows a normal distribution $N\left(\mu,\sigma^2/\mathrm{n}\right)$. then the formulation of the fuzzy hypothesis test for both sides is as follows:

$$\tilde{H}_0 : \mu = \tilde{\mu}_0$$

$$\tilde{H}_1 : \mu \neq \tilde{\mu}_0$$

Both $\tilde{H}_0$ and $\tilde{H}_1$ are represented using Membership Functions, based on Fuzzy Sets Theory. The sample mean $\overline{X}$ represents a hybrid number $\overline{X}_H$, defined as a combination of a random number and a fuzzy number, as previously defined by Kaufmann [19], and its equation is written as follows:

$$\bar{X}_H = \bar{X}_0 + \tilde{\mu}_0 \tag{1}$$

$$\overline{\bar{X}}_0 = \bar{X} - \mu_0 \tag{2}$$

Where $\bar{X}_0$: represents the random number distributed $N\left(0,\sigma^2/\sqrt{n}\right)$ and $\tilde{\mu}_0$ represents the fuzzy mean e of the Triangular Type and can be calculated as shown in the following figure [13]:

$$\tilde{\mu}_0 = \left(\mu_0 - \eta\frac{\sigma}{\sqrt{n}}\ ,\ \mu_0,\ \mu_0 + \eta\frac{\sigma}{\sqrt{n}}\right) \tag{3}$$

By using the fuzzy Z_H test, which consists of the Z statistic added to$(-\eta,\ 0, \eta)$ where η represents the fuzzy expansion coefficient of the allowed range.In the original fuzzy hypothesis testing approach proposed by Kato et al. (2000), the fuzzy expansion coefficient was taken as $\eta = 1$. In the present study, different values of η emerged according to the characteristics of each dataset and the corresponding uncertainty level. Therefore, η should be regarded as an application-dependent parameter rather than a universal constant [20, 21, 29, 30].

$$Z_H = \frac{\bar{X}_H - \mu_0}{\sigma/\sqrt{n}} \tag{4}$$

In the conventional method, the rejection region R is defined as the set of values at which the test statistic exceeds the predetermined critical value, i.e. [21, 22].

$$R = \{z : |z| \geq z_\alpha\}$$

In fuzzy logic, the rejection zone is not as sharp as in the traditional case, but rather is characterized by a degree of fuzziness, in fuzzy statistical models, the expansion coefficient η is defined within the interval [0,1] and controls the width of the fuzzy decision bounds. While η can be adapted as an application-dependent parameter based on dataset characteristics, it was systematically fixed at 0.8 in this study. This choice serves as a representative level of flexibility to ensure structural consistency across all medical analyses and maintain an optimal balance between decision flexibility and statistical power. This fuzziness leads to the division of the decision space into two distinct regions.

1. The necessary rejection region is the region where the rejection of the null hypothesis is absolutely certain, and it is denoted by the abbreviation $\mathrm{NesR}\,(\mathrm{z})$.

1. The possible rejection region is the region where the test statistics indicate a probability of rejecting the null hypothesis with partial degrees of belonging, and it is denoted by the abbreviation $PosR\,(z)$.

It can be represented as follows:

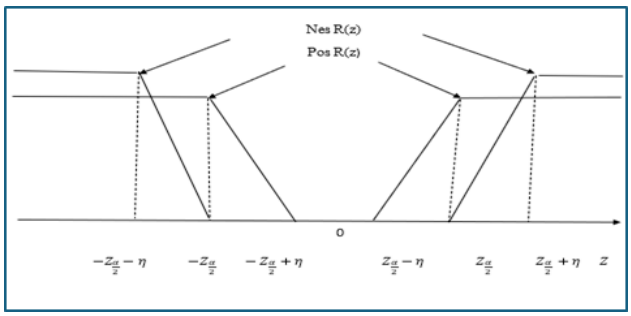

Figure 1. represents of the two rejection regions.

Following the same approach, two areas of acceptance can be identified:
1. Necessary Acceptance Region, denoted by the symbol $Nes\overline{R}\,(z)$

$$Nes\bar{R}\,(z)=\left(-z_{\frac{\alpha}{2}}\ ,-z_{\frac{\alpha}{2}}+\eta\ ,z_{\frac{\alpha}{2}}-\eta\ ,z_{\frac{\alpha}{2}}\right)$$

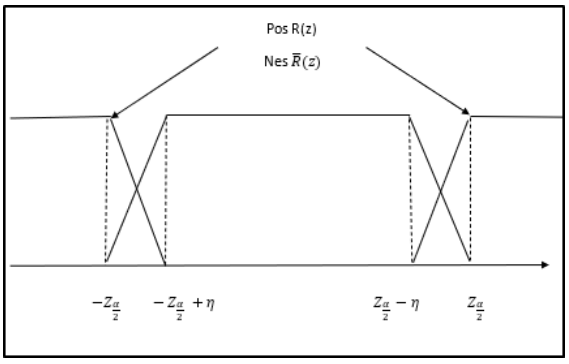

Figure 2. illustrates the Necessary Acceptance regions $Nes\overline{R}\,(z)$ and the Possible Rejection $Pos\ R\,(z)$.

2. Possible Acceptance Interval, denoted by the symbol $Pos\overline{R}\,(z)$.

$$Pos\bar{R}\,(z)=\left(-z_{\frac{\alpha}{2}}-\eta\ ,-z_{\frac{\alpha}{2}},z_{\frac{\alpha}{2}}\ ,z_{\frac{\alpha}{2}}+\eta\right)$$

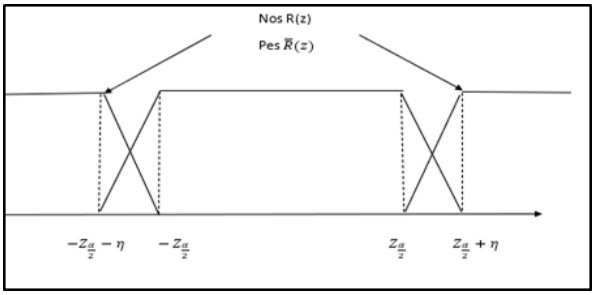

Figure 3. illustrates the Possible Acceptance regions $Pos\overline{R}(z)$ and Necessary Rejection $\mathrm{NesR(z)}$.

The necessary and possible areas of rejection can be found from the following relationship:

$$PosR(z) = \overline{Nes\overline{\mathrm{R}}(z)}$$

,

$$NesR(z) = \overline{Pos\overline{\mathrm{R}}(z)}$$

Accepting or rejecting the null hypothesis exposes us to two types of errors

1. Type I Error: We reject the null hypothesis H_0 even though it is true.
2 Type II Error: We accept the null hypothesis H_0even though it is false.

The Probability of a Type II Error

To study the Probability of a type II error when the membership function is:

$$m_{\mu=\tilde{\mu}_0}(\varepsilon)$$

$$m_{\mu\neq\tilde{\mu}_0}(\varepsilon)$$

where m represents the membership function. The degree H_1 at $\mu = \mu_1$ is given by:

$$m_{\mu=\tilde{\mu}_0}(\varepsilon = \mu_1) = \mathrm{g}_1$$

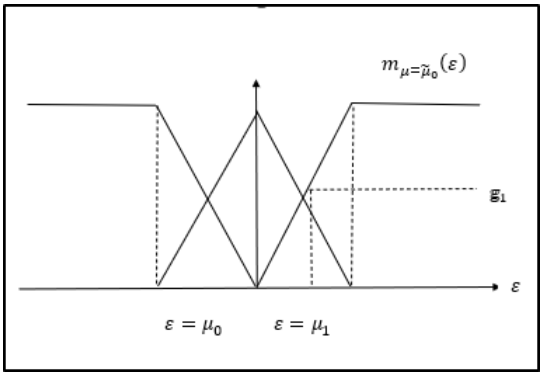

Figure 4. represents the membership function $m_{\mu\neq\tilde{\mu}_0}(\varepsilon)$ and its degree $\mu = \mu_1$.

The necessary region $Nes\ \bar{r}(\bar{x})$and the possible region $Pos\ \bar{r}(\bar{x})$ are two fuzzy regions:

$$Nes\ \bar{r}(\bar{x}) = \left(\mu_0 - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \mu_0 - \left(Z_{\frac{\alpha}{2}} - \eta\right)\frac{\sigma}{\sqrt{n}}, \mu_0 + \left(Z_{\frac{\alpha}{2}} - \eta\right)\frac{\sigma}{\sqrt{n}}, \mu_0 + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) \tag{5}$$

$$Pos\ \bar{r}(\bar{x}) = \left(\mu_0, -\left(Z_{\frac{\alpha}{2}} + \eta\right)\frac{\sigma}{\sqrt{n}}, \mu_0\ - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \mu_0 + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \mu_0 + \left(Z_{\frac{\alpha}{2}} + \eta\right)\frac{\sigma}{\sqrt{n}}\right) \tag{6}$$

The two regions, $Nes\ \bar{r}\,(\bar{x})$ and $Pos\ \bar{r}\,(\bar{x})$, represent a fuzzy trapezoidal number.
The membership function $m(x)$ can be calculated as follows:

$$m\,(x) = \begin{cases} 0 & \mathrm{x} < \mathrm{a} \\ \frac{\mathrm{x}-\mathrm{a}}{\mathrm{b}-\mathrm{a}} & \mathrm{a} \leq \mathrm{x} < \mathrm{b} \\ 1 & \mathrm{b} \leq \mathrm{x} \leq \mathrm{c} \\ \frac{\mathrm{d}-\mathrm{x}}{\mathrm{d}-\mathrm{c}} & \mathrm{c} \leq \mathrm{x} \leq \mathrm{d} \\ 0 & \mathrm{x} > \mathrm{d} \end{cases} \tag{7}$$

Where: a ,b,c , and d denote the four parameters of the trapezoidal fuzzy number, and the interval $[b, c]$represents the core region with full membership degree equal to 1.It should be noted that the fuzzy expansion coefficient η is fixed throughout the study and is used to construct the fuzzy decision regions. In contrast, the membership degree m(x) is calculated separately for each application from the corresponding membership function and may therefore take different values depending on the location of the observed statistic within the fuzzy region.
To calculate μ_1 for both sides of the test, we substitute the value of $m(x)$:

$$\mu_1 = \mu_0 \ \pm m(x)\ \frac{\sigma}{\sqrt{n}} \tag{8}$$

There are two types of type II errors that relate to a region that is acceptable and a region that is necessary to accept[20].
1- The error is of type 2 in a necessary region $\beta_{Nes}\,(\varepsilon = \mu_1)$ which is:

$$\beta_{Nes}\,(\varepsilon = \mu_1) \ = P\left(\bar{X} \ \ Nes\ \bar{r}\left(\bar{X}/\mu\right) = \mu_1\right) \tag{9}$$

$$= P(\frac{\bar{X} - \mu_1}{\frac{\sigma}{\sqrt{n}}} \ \ \frac{Nes\ \bar{r}\,(\bar{x})\ \mu_1}{\frac{\sigma}{\sqrt{n}}} |\ P - \mu_1) \tag{10}$$

$$= P\left(Z \in Nes\bar{R}\,(z) + \ \frac{\mu_0 - \mu_1}{\frac{\sigma}{\sqrt{n}}}\ \right) \tag{11}$$

2- The type II error for the acceptable region $\beta_{pos}\,(\varepsilon = \mu_1)$ is:

$$\beta_{Pos}\,(\varepsilon = \mu_1) \ = P\left(\bar{X} \ \ Pos\ \bar{r}\left(\bar{X}/\mu\right) = \mu_1\right) \tag{12}$$

$$= P\left(\frac{\bar{X} - \mu_1}{\frac{\sigma}{\sqrt{n}}} \ \ \frac{Pos\ \bar{r}\,(\bar{x}) - \ \mu_1}{\frac{\sigma}{\sqrt{n}}}\right) \tag{13}$$

$$\mathrm{P}\left(\mathrm{Z} \in \mathrm{pos}\bar{\mathrm{R}}\,(\mathrm{z}) + \ \frac{\mu_0 - \mu_1}{\frac{\sigma}{\sqrt{\mathrm{n}}}}\right) \tag{14}$$

A type I error is necessary α_{Nes} and possible α_{Pos}as follows*:*

$$1 - P\left(\bar{X} \in Nes\ \bar{r}\,(\varepsilon)\right)$$

$$1 - P\left(\bar{X} \in Pos\ \bar{r}\,(\varepsilon)\right)$$

3. Practical Aspect

3.1. Statistical Analysis of the Z-test for Two Ratios

Statistical data for this study were collected from Mosul General Hospital (Statistics Unit) during the period from May to September 2025. The total study sample consisted of 428 participants, including 264 males and 164 females.

Table 1. Classification of patients according to their clinical condition and health status

According to patients	Their classification:	Males	Females	Male percentage	females percentage
Heart attack	Healthy	25	38	0.397	0.603
	Diabetic	9	7	0.529	0.412
	High blood pressure	21	5	0.808	0.192
Angina	Healthy	62	13	0.827	0.173
	Diabetic	27	16	0.628	0.372
	High blood pressure	23	21	0.523	0.477
Heart rhythm disorders	Healthy	52	23	0.693	0.307
	Diabetic	18	11	0.621	0.379
	High blood pressure	27	30	0.474	0.526

The participants were classified according to their primary clinical condition (heart attack, angina pectoris, and cardiac arrhythmia) and further categorized according to their health status (healthy, diabetic, and hypertensive). Table 1 presents the distribution of the study sample across these categories.

As shown in Table 1, the study sample included participants with different clinical conditions and health characteristics. These data were subsequently used in the classical and fuzzy hypothesis testing procedures.

3.1.1. Classical Hypothesis Testing To examine whether there is a statistically significant difference between the proportion of female patients with diabetes who have angina pectoris (P_1) and those who have cardiac arrhythmia (P_2) ,the classical two-tailed Z-test for two independent proportions was conducted at a significance level of $\alpha = 0.05$. The hypotheses are formulated as follows:

$$H_0 : P_1 = P_2$$

$$H_1 : P_1 \neq P_2$$

The standard test statistic Z_{cal} is computed using the pooled proportion $\hat{P}$:

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \tag{15}$$

$$\widehat{\mathrm{P}} = \frac{\mathrm{X}_1 + \mathrm{X}_2}{\mathrm{n}_1 + \mathrm{n}_2} \tag{16}$$

$$= 0.237$$

$$\mathrm{Z} = \frac{0.148}{0.08026}$$

Z= 1.844

Given that the critical values for a two-tailed at $\alpha = 0.05$ are $\pm Z_{\frac{\alpha}{2}} = \pm 1.96$, the calculated test statistic$(Z_{cal} = 1.844)$ falls within the non-rejection (acceptance) region under the standard normal curve$(-1.96 < 1.844 < 1.96)$ .Consequently, the null hypothesis H_0 cannot be rejected, indicating no statistically significant difference between the two proprtions under the classical approach.

3.1.2. Performing the fuzzy test The percentage of female patients with angina who also have diabetes was compared to the percentage of female patients with cardiac arrhythmias who also have diabetes, as follows::

$$\tilde{H}_0 : P_1 = P_2 = \tilde{P}_0$$

$$\tilde{H}_0{:}P_1 \neq P_2 \neq \tilde{P}_0$$

The random ratio $\overline{P}_0$ was calculated as follows:

$$\overline{P}_0 = \left(\hat{P}_1 - \hat{P}_2\right) - P_0 \tag{17}$$

0.148 =

Whereas: $P_0 = 0$, $\hat{P}_1 = 0.32$, $\hat{P}_2 = 0.172$, $\widehat{\mathrm{P}} = 0.237$

The fuzzy rate $\tilde{P}_0$ was calculated from the following relationship

$$P_0 = \left(P_0 - \eta\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\ ,\ P_0\ , P_0 + \eta\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right) = (-0.06421, 0, 0.06421) \tag{18}$$

The hybrid number $\hat{P}_H$is calculated by adding and subtracting the random ratio $\bar{P}_0$ to the elements of the fuzzy average $\tilde{P}_0$

$$\hat{P}_H = \tilde{P}_0 \pm \bar{P}_0 \tag{19}$$

Table 2. shows the two ends of the hybrid number $\hat{P}_H$

left side	**right side**
$\hat{P}_{H_1} = -0.21221$	$\hat{P}_{H_1} = 0.08379$
$\hat{P}_{H_2} = -0.148$	$\hat{P}_{H_2} = 0.148$
$\hat{P}_{H_3} = -0.08379$	$\hat{P}_{H_3} = 0.21221$

The fuzzy $Z_H$test is applied as follows:

$$Z_H = \frac{\hat{P}_H - P_0}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \tag{20}$$

Table 3. shows the two ends of the Z_H

left side	**right side**
$Z_{H_1} = -2.644$	$Z_{H_1} = 1.044$
$Z_{H_2} = -1.844$	$Z_{H_2} = 1.844$
$Z_{H_3} = -1.044$	$Z_{H_3} = 2.644$

The acceptance and rejection zones at a significance level of $\alpha = 0.05$ are*:*

1- Necessary Acceptance Region $Nes\bar{R}(z)$

$$Nes\bar{R}(z) = \left(-Z_{\frac{\alpha}{2}}\ , -Z_{\frac{\alpha}{2}} + \eta\ , Z_{\frac{\alpha}{2}} - \eta\ , Z_{\frac{\alpha}{2}}\right)$$
$$= (-1.96, -1.16, 1.16, 1.96)$$

2- Possible acceptance Region $Pos\bar{R}(z)$

$$Pos\bar{R}(z) = \left(-Z_{\frac{\alpha}{2}} - \eta\ , -Z_{\frac{\alpha}{2}}, Z_{\frac{\alpha}{2}}\ , Z_{\frac{\alpha}{2}} + \eta\right)$$
$$= (-2.76, -1.96, 1.96, 2.76)$$

Since all Z_H lies within the regions $Nes\bar{R}(z)$ and $Pos\bar{R}(z)$, then the fuzzy hypothesis $\tilde{H}_0$ is accepted and $\tilde{H}_1$ is rejected. This result is consistent with the result obtained using the traditional method. Furthermore, two regions

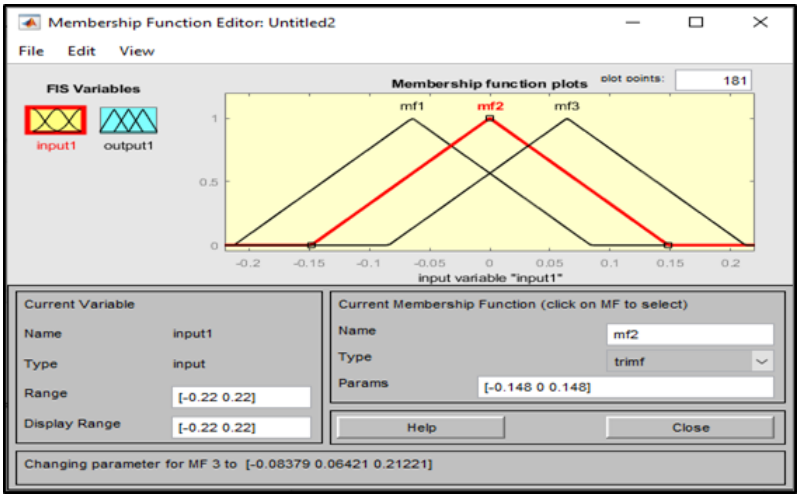

Figure 5. shows the membership function of $\hat{P}_H$

of acceptance can be defined according to $\hat{P}_H$ and the significance level $\alpha = 0.05$ by applying the following two relationships:

$$\text{Nes}\overline{\text{r}}\left(\widehat{\text{P}}\right) = (\text{P}_0 - \text{Z}_{\frac{\alpha}{2}}\sqrt{\widehat{\text{P}}\widehat{\text{q}}\left(\frac{1}{\text{n}_1}+\frac{1}{\text{n}_2}\right)}\,,\ \text{P}_0 - (\text{Z}_{\frac{\alpha}{2}} - \eta)\sqrt{\widehat{\text{P}}\widehat{\text{q}}\left(\frac{1}{\text{n}_1}+\frac{1}{\text{n}_2}\right)}, \text{P}_0 +$$

$$(\text{Z}_{\frac{\alpha}{2}} - \eta)\sqrt{\widehat{\text{P}}\widehat{\text{q}}\left(\frac{1}{\text{n}_1}+\frac{1}{\text{n}_2}\right)}\,,\ \text{P}_0 + \text{Z}_{\frac{\alpha}{2}}\sqrt{\widehat{\text{P}}\widehat{\text{q}}\left(\frac{1}{\text{n}_1}+\frac{1}{\text{n}_2}\right)} \qquad (21)$$

$$= (-0.157314, -0.093104, 0.093104, 0.157314)$$

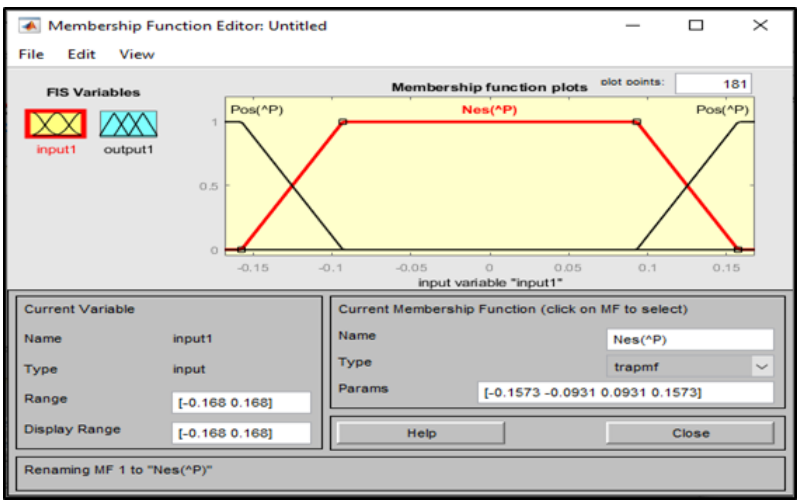

Figure 6. the necessary acceptance $Nes\overline{r}(\hat{P})$ and the possible rejection $Pos\overline{r}(\hat{P})$.

$$Pos\bar{r}\left(\hat{P}\right) = (P_0 - \left(Z_{\frac{\alpha}{2}} + \eta\right)\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\ ,\ P_0 - Z_{\frac{\alpha}{2}}\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)},$$
$$P_0 + Z_{\frac{\alpha}{2}}\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\ ,\ P_0 + \left(Z_{\frac{\alpha}{2}} + \eta\right)\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\ ) \quad (22)$$
$$= (-0.22152, -0.15731, 0.15731, 0.22152)$$

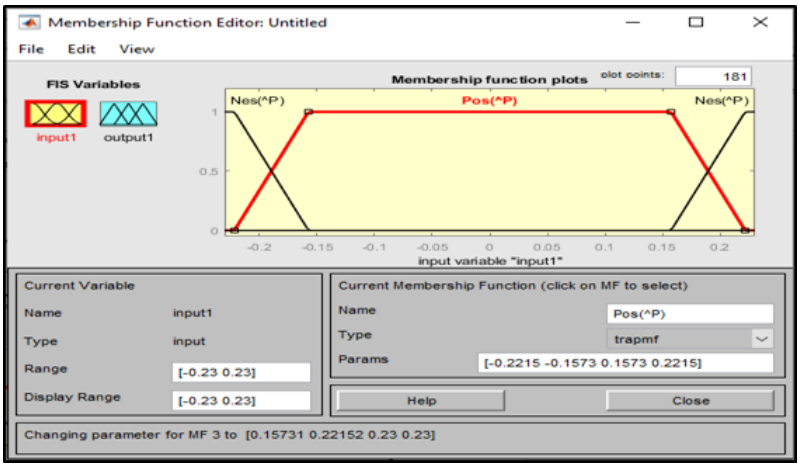

Figure 7. illustrates the area of possible acceptance $Pos\bar{r}(\hat{P})$ and the area of necessary rejection $Nes\bar{r}(\hat{P})$.

The membership degre $m(x)$ was calculated from the right side $Nes\bar{r}(\hat{P})$of the membership function defined in Eq. (7) as follows:

$$\text{where, c} = 0.093104\ ,\quad \text{d} = 0.157314,\quad \text{x} = 0.148$$

$$m(x) = \frac{\text{d} - \text{x}}{\text{d} - \text{c}}$$
$$m(x) = 0.145$$

Similarly, the left-hand side of $Pos\bar{r}(\hat{P})$

$$\text{c} = 0.157314\ ,\ \text{d} = 0.22152\ ,\quad \text{x} = 0.2122$$

$$m(x) = \frac{d - x}{d - c}$$
$$m(x) = 0.145$$

$$\hat{P}_1 = 0.00931$$

3- Type II Error

The Type II error was calculated using the membership function as shown in Figure (4)

$$m_{P_1 = P_2 = \bar{P}_0(x)}$$

$$m_{P_1 \neq P_2 \neq \bar{P}_0(x)}$$

To study the probability of type II error in the case of a health score of $\tilde{H}_1$ at$(P_1 \neq P_2) \neq \bar{P}_0$ which is given by $\text{m}_{(\text{P}_1 \neq \text{P}_2) \neq \bar{\text{P}}_0} = 0.145$

As we have previously explained, there are two types of error of the second type:
1-Necessary Acceptance Region $\beta_{Nes}\left(\varepsilon=\hat{P}_1\right)$

$$\beta_{Nes}\left(\varepsilon=P_1'\right)=P\left(\hat{P}\ Nes\ \bar{r}\left(\hat{P}\right)/\ P_1\neq P_2=\hat{P}_1\right)$$

$$=P(\frac{\hat{P}-\hat{P}_1}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}\ \frac{Nes\ \bar{r}\left(\hat{P}\right)-\hat{P}_1}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$=P(\frac{\hat{P}-\hat{P}_1}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}\frac{0.148-0.00931}{0.08026}$$

$$=0.958$$

$$=P\left(\begin{array}{c} Z\ Nes\bar{R}\left(z\right)+\\ \frac{\hat{P}-\hat{P}_1}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}\end{array}\right)$$

$$=\mathrm{P}\left(\mathrm{Z}\quad 1.728\ \right)-\mathrm{P}\left(\mathrm{Z}\quad -1.96\right)$$

= 0.933
2- Possible acceptance Region $\beta_{pos}\left(\varepsilon=\hat{P}_1\right)$

$$\beta_{P\mathrm{os}}\left(\varepsilon=\hat{P}_1\right)=P\left(\hat{\mathrm{P}}\ \mathrm{pos}\ \bar{\mathrm{r}}\left(\hat{\mathrm{P}}\right)/\ \mathrm{P}_1\neq \mathrm{P}_2=\hat{P}_1\right)$$

$$=\mathrm{P}(\frac{\hat{\mathrm{P}}-\hat{P}_1}{\sqrt{\hat{\mathrm{P}}\hat{\mathrm{q}}\left(\frac{1}{\mathrm{n}_1}+\frac{1}{\mathrm{n}_2}\right)}}\ \frac{\mathrm{Pos}\ \bar{\mathrm{r}}\left(\hat{\mathrm{P}}\right)-\hat{P}_1}{\sqrt{\hat{\mathrm{P}}\hat{\mathrm{q}}\left(\frac{1}{\mathrm{n}_1}+\frac{1}{\mathrm{n}_2}\right)}}$$

$$=P(\frac{\hat{P}-\hat{P}_1}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}\frac{0.2122-0.00931}{0.08026}$$

= 0.994

$$=P\left(\begin{array}{c} Z\ Nes\bar{R}\left(z\right)+\\ \frac{\hat{P}-\hat{P}_1}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}\end{array}\right)$$

$$=\mathrm{P}\left(\mathrm{Z}\ \ 2.528\ \right)-\mathrm{P}\left(\mathrm{Z}\quad -2.76\right)$$

= 0.99
The necessary and possible type 1 of error was calculated as follows:

$$\alpha_{Nes}\ =1-P\left(\hat{P}\ Nes\bar{r}\left(\varepsilon\right)\right)$$

$$=0.067$$

$$\alpha_{\mathrm{Pos}}\ =1-\mathrm{P}\left(\hat{\mathrm{P}}\ \mathrm{Pos}\bar{\mathrm{r}}\left(\varepsilon\right)\right)$$

$$=0.01$$

3.2. One-Sample T-Test

Iron is an essential mineral in the human body, playing a vital role in the formation of hemoglobin, which is responsible for transporting oxygen to body tissues. It also contributes to energy production, immune system function, and cognitive performance. Iron deficiency may lead to iron deficiency anemia, resulting in symptoms such as fatigue, weakness, dizziness, and poor concentration. Therefore, maintaining normal iron levels is critically important for overall health and proper physiological functioning. The concentration of iron in human blood varies between healthy individuals and patients, and measurement units may differ according to medical laboratories and the diagnostic systems adopted. The mean iron concentration in the blood of healthy individuals $(138.3\ \mu\mathrm{g/dL})$, reported in the study conducted by [22] on healthy subjects, was adopted as the reference value for the population mean iron concentration. For the one-sample t-test implementation, the null hypothesis evaluates the population mean of iron levels against the reference value of $(138.3\ \mathrm{g/dl})$. The selection of $(138.3\ \mathrm{g/dl})$ as the reference point is highly justified from a clinical standpoint. According to contemporary hematological protocols documented in Wintrobe's Clinical Hematology [23], the standard reference intervals for normal serum iron concentrations are established between $(50-170\ \mathrm{g/dl})$ for adult females and $(65-175\ \mathrm{g/dl}\ )$for adult males. Since the baseline parameter of $(138.3\ \mathrm{g/dl})$ falls strictly within the core of these universally accepted physiological ranges, it serves as a robust, non-arbitrary standard criterion (Baseline Reference Value). In contrast, the acceptable concentration for patients $(40\ \mu\mathrm{g/dL})$ was applied at the sample level. The traditional and fuzzy hypothesis tests were applied to two populations:A healthy population (without the disease) and a population of patients with anemia. A sample was taken from each population and the tests were performed to find the relationship between human blood iron concentration and the probability of developing the disease.

3.2.1. Classical and fuzzy hypothesis testing for healthy individuals : Classical and fuzzy hypothesis tests were performed on a sample of 28 healthy individuals without anemia, aged between 14 and 62 years. The sample serum iron concentration was $(133.3250\ \mu g/dL)$, compared with the population mean of the serum iron concentration for healthy individusls, which was assumed to be $(138.3\ \mu g/dL)$, The sample standard deviation was S= 16.53863 as follows:

$$H_0 : \mu = \mu_0 = 138.3$$
$$H_1 : \mu \neq \mu_0 = 138.3$$

Using the statistical laboratory t:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \qquad (23)$$
$$= -1.592$$

Whereas: $t_{\left(\frac{\alpha}{2}\ ,\ n-1\right)} = 2.052$

We accept the null hypothesis H_0. When testing the traditional hypothesis for healthy individuals, the result of the test was $t = -1.592$. Therefore, there is no significant difference between the sample mean and the population mean at a significance level of $\alpha = 0.05$.

Fuzzy logic test

$$\tilde{H}_0 : \mu = \widetilde{\mu}_0 \ = \ \tilde{138.3})$$
$$\tilde{H}_1 : \mu \neq \widetilde{\mu}_0 \ \neq \ \tilde{138.3})$$

The random number $\overline{X}_0$ was calculated from the relationship as follows:

$$\overline{X}_0 = \overline{X} - \mu_0$$
$$= -4.975$$

Calculating the fuzzy ratio $\widetilde{\mu}_0$ is as follows:

$$\widetilde{\mu}_0 = \left(\mu_0 - \eta\frac{S}{\sqrt{n}}\ ,\ \mu_0,\ \mu_0 + \eta\frac{S}{\sqrt{n}}\right)$$

$$\widetilde{\mu}_0 = (135.7995941, 138.3\ , 140.8004058)$$

Then calculate the fuzzy $\overline{X}_H$ for both sides, and the results were as follows:

Table 4. shows the hybrid number$\overline{X}_H$

left side	right side
$\overline{X}_{H_1} = 130.8215941$	$\overline{X}_{H_1} = 140.7775941$
$\overline{X}_{H_2} = 133.322$	$\overline{X}_{H_2} = 143.278$
$\overline{X}_{H_3} = 135.8224058$	$\overline{X}_{H_3} = 145.7784058$

By applying the fuzzy t_H test to both ends, where:

$$t_H = \frac{\overline{X}_H - \mu_0}{S/\sqrt{n}} \tag{24}$$

The results were as follows:

Table 5. shows the t_H test for both sides

left side	right side
$t_{H_1} = -2.392$	$t_{H_1} = 0.792$
$t_{H_2} = -1.592$	$t_{H_2} = 1.592$
$t_{H_3} = -0.792$	$t_{H_3} = 2.392$

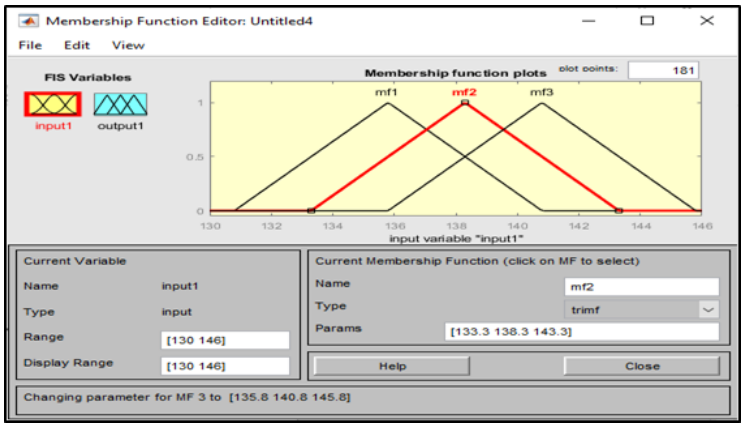

Figure 8. illustrates the membership function of $\overline{X}_H$.

It is noted that the fuzzy $t_H$defines two regions of acceptance according to the significance level $\alpha = 0.05$ and degrees of freedom $(\mathrm{n}-1)$are.
1-Necessary Acceptance Region to t_H

$$Nes\bar{R}(t) = \left(-t_{\frac{\alpha}{2}}\ , -t_{\frac{\alpha}{2}} + \eta\ , t_{\frac{\alpha}{2}} - \eta\ , t_{\frac{\alpha}{2}}\right)$$

= (-2.052, -1.252, 1.252, 2.052)
2-Possible acceptance Region to t_H

$$Pos\bar{R}(t) = \left(-t_{\frac{\alpha}{2}} - \eta\ , -t_{\frac{\alpha}{2}}, t_{\frac{\alpha}{2}}\ , t_{\frac{\alpha}{2}} + \eta\right)$$

= (-2.852, -2.052, 2.052 ,2.852)

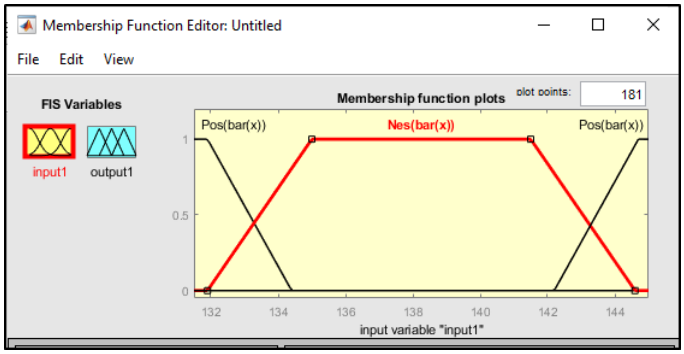

Figure 9. illustrates the Necessary Acceptance Region $Nes\bar{r}(\bar{x})$ and Possible Rejection $Pos\bar{r}(\bar{x})$.

Two acceptance zones can also be defined according to $\bar{X}_H$ at a significance level of $\alpha = 0.05$

$$Nes\bar{r}\,(\bar{x}) = (131.873957\,, 134.374362\,, 142.225637\,, 144.726042)$$

$$Pos\bar{r}\,(\bar{x}) = \left(\mu_0 - \left(t_{\frac{\alpha}{2}} + \eta\right)\frac{S}{\sqrt{n}}\,,\ \mu_0 - t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}\,, \mu_0 + t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}\,,\ \mu_0 + \left(t_{\frac{\alpha}{2}} + \eta\right)\frac{S}{\sqrt{n}}\right)$$
$$= (129.3735512, 131.8739570, 144.726042, 147.226448)$$

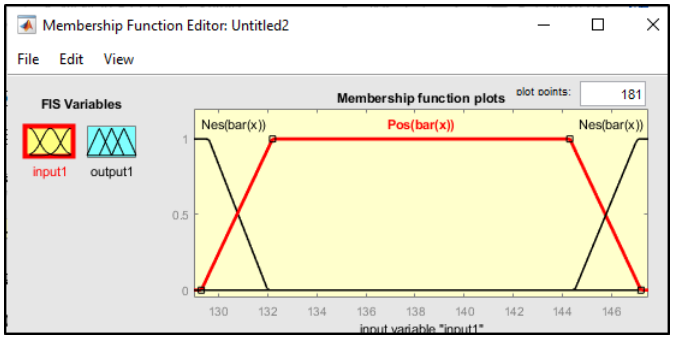

Figure 10. illustrates the possible Acceptance Region $Pos\bar{r}(\bar{x})$ and Necessary Rejection $Nes\bar{r}(\bar{x})$.

$m\,(x)$ and μ_1 were calculated as follows:

$$m(x) = 0.575323$$

$$\mu_1 = \mu_0 + m(x)\,\frac{S}{\sqrt{n}}$$
$$\mu_1 = 140.110053$$
$$\mu_1 = \mu_0 - m(x)\,\frac{S}{\sqrt{n}}$$
$$\mu_1 = 136.489946$$

Type II Error

The type II error for the fuzzy hypothesis was calculated using the membership function

$$m_{\mu=\tilde{\mu}_0(\varepsilon)}$$

$$m_{\mu \neq \tilde{\mu}_0(\varepsilon)}$$

To study the probability of type II error in the case of a health score of $\tilde{H}_1$ at$\mu = \mu_1$ which is given by $m_{\mu \neq \tilde{\mu}_0(\varepsilon)} = 0.579123$
1-Necessary Acceptance Region $\beta_{Nes}(\varepsilon = \mu_1)$

$$\beta_{Nes}(\varepsilon = \mu_1) = P\left(\bar{X}\ Nes\ \bar{r}(\bar{x}) / \mu = \mu_1\right)$$

$$= P\left(\frac{\bar{X} - \mu_1}{S/\sqrt{n}}\right)\ \frac{Nes\ \bar{r}(\bar{x}) - \mu_1}{S/\sqrt{n}} | \ \mu = \mu_1)$$

$$= P\left(\frac{\bar{X} - \mu_1}{S/\sqrt{n}}\ 1.013\right)$$

= 0.8399

$$= P\left(\ t\ \ Nes\bar{R}(t) + \frac{\mu_0 - \mu_1}{S/\sqrt{n}}\right)$$

$$= \mathrm{P}(\mathrm{t}\ 1.013\ \ ) - \mathrm{P}(\mathrm{t}\ \ -2.052)$$

$$= 0.8149$$

2- Possible acceptance Region $\beta_{Pos}(\varepsilon = \mu_1)$

$$\beta_{Pos}(\varepsilon = \mu_1) = P\left(\bar{X}\ Pos\ \bar{r}(\bar{x}) / \mu = \mu_1\right)$$

$$= P\left(\frac{\bar{X} - \mu_1}{S/\sqrt{n}}\ \ 2.971\right)$$

$$= 0.997$$

$$= P\left(\ \mathrm{t}\ \ \mathrm{Pos\bar{R}}(\mathrm{t}) + \frac{\mu_0 - \mu_1}{\mathrm{S}/\sqrt{\mathrm{n}}}\right)$$

$$= \mathrm{P}(\mathrm{t}\ \ 2.971\ \ ) - \mathrm{P}(\mathrm{t}\ \ -2.852)$$

$$= 0.986$$

The necessary type 1 error α_{Nes} and the possible α_{Pos} are as follows:

$$\alpha_{\mathrm{Nes}} = 1 - \mathrm{P}\left(\bar{\mathrm{X}}\ \mathrm{Nes\bar{r}}(\varepsilon)\right) = 0.1601$$

$$\alpha_{\mathrm{Pos}} = 1 - \mathrm{P}\left(\bar{\mathrm{X}}\ \ \mathrm{Pos\bar{r}}(\varepsilon)\right) = 0.003$$

3.2.2. Classical and fuzzy hypothesis testing for patients : The classical and fuzzy hypothesis test for the sample of patients with anemia is Eq. (17) samples, and the sample mean for serum iron concentration is 37.4882 $(\mu g/dl)$and S=5.85308. The mean of those affected was compared to the population mean of 40 $(\mu g/dl)$at a significance level of $\alpha = 0.05$ as follows*:*

$$H_0 : \mu = \mu_0 = 40$$

$$H_1 : \mu \neq \mu_0 = 40$$

Using the statistical laboratory t:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= -1.77$$

Where: $t_{\left(\frac{\alpha}{2}\ ,\ n-1\right)} = 2.120$ We accept the null hypothesis H_0 meaning there is no significant difference between the means at a significance level of $\alpha = 0.05$.

Testing the fuzzy hypothesis

$$\tilde{H}_0 : \mu = \widetilde{40})$$
$$\tilde{H}_1 : \mu \neq \widetilde{40})$$

The random number $\bar{X}_0$ was calculated as follows:

$$\bar{X}_0 = \bar{X} - \mu_0 = -2.5118$$

Calculating the fuzzy ratio $\tilde{\mu}_0$ is as follows:

$$\tilde{\mu}_0 = \left(\mu_0 - \eta\frac{S}{\sqrt{n}}\ ,\ \mu_0\ , \mu_0 + \eta\frac{S}{\sqrt{n}}\right)$$

$$\tilde{\mu}_0 = (38.8643357,\ 40\ ,\ 41.1356643)$$

The $\bar{X}_H$ values were calculated using a fuzzy t_H and the results are as follows:

Table 6. shows the hybrid number$\overline{X}_H$and t_H

Right side $\overline{\boldsymbol{X}}_{\boldsymbol{H}}$	**left side $\overline{\boldsymbol{X}}_{\boldsymbol{H}}$**
$\overline{X}_{H_1} = 41.376136$	$\overline{X}_{H_1} = 36.352536$
$\overline{X}_{H_2} = 42.5118$	$\overline{X}_{H_2} = 37.4882$
$\overline{X}_{H_3} = 43.6474643$	$\overline{X}_{H_3} = 38.6238643$

By applying Eq. (24),we obtain the results shown in Table (7)

Table 7. shows the t_H test for both sides

Right side t_H	**left side t_H**
$t_{H_1} = 0.97$	$t_{H_1} = -2.57$
t_{H_2} =1.77	$t_{H_2} = -1.77$
$t_{H_3} = 2.57$	$t_{H_3} = -0.97$

It is noted that the fuzzy $t_H$defines two regions of acceptance according to the significance level $\alpha = 0.05$ and degrees of freedom $(\mathrm{n}-1)$ are:
1- Necessary Acceptance Region of $Nes\bar{R}\,(t)$

$$Nes\bar{R}\,(t) = \left(-t_{\frac{\alpha}{2}}\ , -t_{\frac{\alpha}{2}} + \eta\ , t_{\frac{\alpha}{2}} - \eta\ , t_{\frac{\alpha}{2}}\right)$$

Whereas $\eta = 0.8$
= (-2.120, -1.32, 1.32, 2.120)
2- Possible acceptance Region of $Pos\bar{R}\,(t)$

$$Pos\bar{R}\,(t) = \left(-t_{\frac{\alpha}{2}} - \eta\ , -t_{\frac{\alpha}{2}}, t_{\frac{\alpha}{2}}\ , t_{\frac{\alpha}{2}} + \eta\right)$$

= (-2.92, -2.120, 2.120,2.92)
Two acceptance zones can also be defined according to $\bar{X}_H$ at a significance level of $\alpha = 0.05$

$$Nes\overline{r}\,(\overline{x}) = (36.9904895,\ 38.1261539,\ 41.8738461,\ 43.0095105)$$

$$Pos\overline{r}(\overline{x}) = (35.8548251, 36.9904895, 43.0095105, 44.1451748($$

Since the calculated values of $\bar{X}_H$ lie within $Pos\bar{r}\,(\bar{x})$ and $Nes\bar{r}\,(\bar{x})$and the test values t_H lie within the necessary acceptance region $Nes\bar{R}\,(t)$ and the possible acceptance region $Pos\bar{R}\,(t)$, and the fuzzy values of $t_H$lie within the necessary and possible acceptance regions, then the hypothesis $\tilde{H}_0$ is acceptable at the significance level $\alpha = 0.05$.

$m\left(x\right)$ and μ were calculated, and the results are as follows:

$$m(x) = 0.4382549$$

$$\mu_1 = \mu_0 + m(x)\ \frac{S}{\sqrt{n}}$$

$$\mu_1 = \mu_0 + m(x)\ \frac{S}{\sqrt{n}} = 40.6221381$$

$$\mu_1 = \mu_0 - m(x)\ \frac{S}{\sqrt{n}} = 39.3778619$$

The type II error for the fuzzy hypothesis was calculated using the membership function

$$m_{\mu=\tilde{\mu}_0(\varepsilon)}$$

$$m_{\mu\neq\tilde{\mu}_0(\varepsilon)}$$

To study the probability of type II error in the case of a health score of $\tilde{H}_1$ at $\mu = \mu_1$ which is given by $\mathrm{m}_{\mu\neq 40} = 0.4382549$

1-Necessary Acceptance Region $\beta_{Nes}\left(\varepsilon = \mu_1\right)$

$$\beta_{\mathrm{Nes}}\left(\varepsilon = \mu_1\right) = \mathrm{P}\left(\bar{\mathrm{X}}\ \mathrm{Nes}\ \bar{\mathrm{r}}\left(\bar{\mathrm{x}}\right) / \ \mu = \mu_1\right)$$

$$= 0.899$$

$$= P\left(\ t\ \ Nes\bar{R}\left(t\right) + \frac{\mu_0 - \mu_1}{S/\sqrt{n}}\right)$$

$$= \mathrm{P}\left(\mathrm{t}\ \ 1.331\right) - \mathrm{P}\left(\mathrm{t}\ \ -2.120\right)$$

= 0.874

2- Possible acceptance Region $\beta_{Pos}\left(\varepsilon = \mu_1\right)$

$$\beta_{Pos}\left(\varepsilon = \mu_1\right) = P\left(\bar{X}\ Pos\ \bar{r}\left(\bar{x}\right) / \ \mu = \mu_1\right)$$

$$= P\left(\frac{\bar{X} - \mu_1}{S/\sqrt{n}}\ \ 3.008\right)$$

$$= 0.995$$

$$= \mathrm{P}\left(\ \mathrm{t}\ \ \mathrm{Pos}\bar{\mathrm{R}}\left(\mathrm{t}\right) + \frac{\mu_0 - \mu_1}{\mathrm{S}/\sqrt{\mathrm{n}}}\right)$$

$$= \mathrm{P}\left(\mathrm{t}\ \ 3.008\ \ \right) - \mathrm{P}\left(\mathrm{t}\ \ -2.92\right)$$

$$= 0.99$$

The necessary type 1 error α_{Nes} and the possible α_{Pos} are as follows:

$$\alpha_{\mathrm{Nes}}\ = 1 - \mathrm{P}\left(\bar{\mathrm{X}}\ \mathrm{Nes}\bar{\mathrm{r}}\left(\varepsilon\right)\right) = 0.11$$

$$\alpha_{\mathrm{Pos}}\ = 1 - \mathrm{P}\left(\bar{\mathrm{X}}\ \ \mathrm{Pos}\bar{\mathrm{r}}\left(\varepsilon\right)\right) = 0.005$$

4. Conclusion

Integrating fuzzy logic into statistical hypothesis testing frameworks represents a significant advancement over the limitations of classical binary inference (t and Z tests). Based on the empirical analysis of medical datasets from Mosul General Hospital, the core conclusions can be summarized as follows:

1. **Enhanced Decision-Making Flexibility:** The proposed fuzzy framework replaces rigid binary boundaries with adaptive, continuous decision spaces. By partitioning criteria into Necessary Acceptance/Rejection and Possible Acceptance/Rejection regions, the model offers clinicians a realistic mechanism to interpret borderline or ambiguous clinical cases that fall into standard mathematical "gray areas."
2. **Robustness of Hybrid Numerical Models:** Representing the sample mean and variance as hybrid fuzzy-random numbers bridges the gap between stochastic variability and human measurement imprecision. This hybrid formulation successfully preserves the underlying sampling distributions while capturing the non-probabilistic uncertainty inherent in real-world clinical environments.
3. **Effect of the Fuzzy Expansion Coefficient**: The empirical analysis using $\eta = 0.8$ demonstrated that the fuzzy expansion coefficient controls the width of the fuzzy decision regions and influences the corresponding Type I and Type II fuzzy error measures. The selected value provided a reasonable balance between uncertainty representation and statistical decision stability in the analyzed medical data.Thereforwe, the selection of η should be guided by the degree of uncertainty and the characteristics of the analyzed dataset rather than adopting a universal fixed value

5. Recommendations:

1. We recommend developing software packages in R or Python that integrate these fuzzy tests to facilitate their use by researchers who are not specialists in complex mathematics.
2. This model should be tested on larger and more complex datasets in other fields, such as radiological diagnostics or sensitive chemical analysis.
3. We call for a reconsideration of the traditional significance levels (= 0.05) when using fuzzy logic and their replacement with "degree of belonging" criteria to determine the strength of the decision.

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