STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **16**, September 2026, pp 2200–2214.
Published online in International Academic Press (www.IAPress.org)

Estimating Value at Risk of the Indonesian Sharia Stock Index using Asymmetric GARCH-MIDAS

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Abstract This study estimates the market risk of the Indonesian Sharia Stock Index (ISSI) using a volatility framework that combines short-run asymmetric GARCH dynamics with a long-run GARCH-MIDAS component. The sample covers the period from 13 May 2011 to 17 March 2026, using daily ISSI closing prices and monthly inflation and BI Rate data. The conditional mean is represented as a constant mean process because the ACF and PACF do not show a stable autoregressive or moving-average pattern, additional mean terms do not provide a meaningful improvement, and the Ljung-Box test indicates no remaining linear autocorrelation. Conditional variance is modeled using GARCH(1,1), TGARCH(1,1), GJR-GARCH(1,1), GARCH(1,1)-MIDAS, and GJR-GARCH(1,1)-MIDAS. Model fit is compared using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), while Value at Risk (VaR) at the 5%, 2.5%, and 1% quantiles is assessed using the Kupiec Proportion of Failure test. ISSI returns exhibit volatility clustering, ARCH effects, and significant asymmetric responses to negative shocks. TGARCH(1,1) has the lowest AIC and BIC within the conventional GARCH family, whereas GJR-GARCH-MIDAS with the BI Rate has the lowest criteria within the MIDAS family. Inflation and the BI Rate are nevertheless statistically insignificant in the long-run component, which may reflect the rapid incorporation of publicly available domestic macroeconomic information and the stronger influence of global or market-specific shocks during the sample period. The GJR-GARCH-MIDAS specifications pass the Kupiec POF test at the 2.5% and 1% VaR quantiles. Taken together, the results support the asymmetric GJR structure as the main source of improved downside-risk estimation rather than a statistically significant contribution from the two macroeconomic MIDAS variables.

Keywords Indonesian Sharia Stock Index, Value at Risk, GARCH-MIDAS, GJR-GARCH, asymmetric volatility, Kupiec POF

AMS 2010 subject classifications 62M10, 91G70, 91G10

DOI: 10.19139/soic-2310-5070-3899

1. Introduction

Financial markets have experienced repeated episodes of uncertainty during the last two decades, including the 2008 global financial crisis, the COVID-19 pandemic, geopolitical conflicts, and changes in international monetary policy. These events increased market risk exposure and strengthened the need for accurate risk measurement in portfolio management. Value at Risk (VaR) is one of the most widely used measures of market risk because it summarizes the maximum potential loss over a given horizon and confidence level [26, 19]. However, VaR accuracy depends strongly on the ability of the underlying model to capture time-varying volatility, nonlinearity, and asymmetry in financial returns [9, 4].

Empirical financial return data often exhibit heteroskedasticity, volatility clustering, heavy tails, and asymmetric responses to positive and negative shocks [27, 7]. These characteristics make constant-variance time-series models insufficient for risk estimation. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model introduced by Bollerslev [9] is widely used to model conditional volatility because it allows the variance to change

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ISSN 2310-5070 (online) ISSN 2311-004X (print)

over time and to depend on past squared shocks and past conditional variance. This feature makes GARCH a useful foundation for dynamic VaR estimation.

Nevertheless, the standard GARCH model assumes that positive and negative shocks with the same magnitude have identical effects on volatility. In equity markets, negative returns often increase subsequent volatility more strongly than positive returns of equal size. This empirical regularity is known as the leverage effect or asymmetric volatility. Models such as EGARCH, TGARCH, and GJR-GARCH were developed to address this limitation by allowing volatility to respond differently to bad and good news [31, 24, 35]. Evidence from international equity markets indicates that asymmetric GARCH specifications are useful for capturing leverage effects and crisis-related changes in volatility and market dependence [2, 18].

Although asymmetric GARCH models improve short-run volatility modeling, they do not explicitly separate short-run market fluctuations from long-run volatility components. The GARCH-MIDAS framework proposed by Engle et al. [22] addresses this issue by decomposing conditional variance into a short-run GARCH component and a long-run MIDAS component driven by lower-frequency variables. The Mixed Data Sampling (MIDAS) approach allows variables with different frequencies to be combined in a single model without forcing excessive aggregation [23]. Therefore, GARCH-MIDAS provides a suitable framework for linking daily stock return volatility with monthly macroeconomic variables such as inflation and interest rates.

The Indonesian Sharia Stock Index (ISSI) is a broad benchmark of sharia-compliant stocks listed on the Indonesia Stock Exchange. ISSI represents the performance of the wider Indonesian Islamic equity universe and remains exposed to systemic and external shocks despite sharia screening criteria. Previous studies document increased volatility in Indonesian sharia stocks during COVID-19 waves [1], volatility persistence in Islamic stock indices [14], and the usefulness of ARCH/GARCH models for the Indonesian sharia stock index [13]. The closest related study is Danila [17], which analyzes the Jakarta Islamic Index and primarily examines whether inflation and short-term interest rates explain long-run volatility in a GARCH-MIDAS setting.

This study is distinguished from Danila [17] in its market coverage, research objective, model comparison, and validation strategy. It examines ISSI from the index's initial trading period through March 2026 rather than the narrower Jakarta Islamic Index, directly compares symmetric and asymmetric GARCH and GARCH-MIDAS specifications, and converts the estimated volatility into VaR forecasts. The contribution is therefore not merely the inclusion of mixed-frequency macroeconomic variables, but the evaluation of whether short-run asymmetry improves downside-risk measurement across the 5%, 2.5%, and 1% quantiles using formal Kupiec backtesting.

The contribution of this study is threefold. First, it provides a VaR-focused analysis of the broad ISSI universe over an extended sample beginning at the index launch. Second, it identifies the incremental role of asymmetric short-run dynamics by comparing GARCH, TGARCH, GJR-GARCH, GARCH-MIDAS, and GJR-GARCH-MIDAS specifications. Third, it evaluates model adequacy using both penalized-likelihood criteria (AIC and BIC) and Kupiec POF backtesting at the 5%, 2.5%, and 1% VaR quantiles. This design separates evidence for the asymmetric GJR structure from evidence for the macroeconomic MIDAS variables themselves.

2. Data and Methods

2.1. Data and return

The data used in this study consist of daily closing prices of the Indonesian Sharia Stock Index (ISSI) and monthly macroeconomic variables, namely inflation and the BI Rate. The ISSI data comprise 3,600 daily observations covering the period from 13 May 2011, shortly after the index was launched, to 17 March 2026. The daily closing prices are subsequently transformed into logarithmic returns for further analysis. Meanwhile, the monthly inflation and BI Rate data cover the corresponding period from May 2011 to March 2026. The ISSI closing price is used to construct daily returns, while inflation and the BI Rate are incorporated as low-frequency variables in the MIDAS component. Inflation and policy rates are widely used indicators of price dynamics and the monetary-policy stance [8, 30]. This mixed-frequency structure links short-run financial volatility with longer-run economic conditions without aggregating the daily return series [22, 23].

Let P_t denote the ISSI closing price on day t and P_{t-1} denote the closing price on the previous trading day. The continuously compounded or logarithmic return is calculated as

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right). \tag{1}$$

The transformation from price to log return is required because stock price series often contain long-run trends and are generally nonstationary in level. Return series are more appropriate for volatility modeling because they describe percentage changes in asset value and usually fluctuate around a relatively stable mean [12, 32]. The logarithmic transformation is also a standard member of the broader family of power transformations used to stabilize statistical behavior [10]. In this study, the return transformation is also used to evaluate short-run market risk through VaR.

2.2. Mean model

Before modeling conditional variance, the conditional mean of ISSI returns is considered within the Autoregressive Integrated Moving Average (ARIMA) framework. The general ARIMA(p, d, q) model can be written as

$$\phi_p(B)(1-B)^d r_t = \theta_0 + \theta_q(B)a_t, \tag{2}$$

where p is the autoregressive order, d is the differencing order, q is the moving-average order, B is the backshift operator, and a_t is a white-noise error term. Because financial return series are typically stationary after logarithmic transformation, the differencing order is expected to be $d = 0$. The identification of p and q is supported by the autocorrelation function (ACF), partial autocorrelation function (PACF), information criteria, parameter significance, and residual diagnostics [33, 16, 11, 29].

After candidate mean models are estimated, the residuals are examined using the Ljung-Box test to detect remaining autocorrelation and the Jarque-Bera test to evaluate normality. Even when normality is rejected, the model can still be used as a mean specification because non-normal residuals are common in financial return data due to heavy tails and extreme observations [12]. The most important requirement before volatility modeling is that the residuals no longer contain systematic linear autocorrelation in the mean equation.

2.3. GARCH and Asymmetric GARCH models

The GARCH model is used when the residuals of the mean equation exhibit conditional heteroskedasticity. The presence of ARCH effects indicates that the conditional variance is not constant over time and should be modeled explicitly. The general GARCH(P, Q) model is

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{Q} \beta_j \sigma_{t-j}^2, \tag{3}$$

where σ_t^2 is the conditional variance, ε_{t-i}^2 is the squared shock from previous periods, ω is the variance intercept, α_i is the ARCH parameter, and β_j is the GARCH parameter. The GARCH(1,1) specification is used as a parsimonious benchmark for capturing volatility clustering and persistence while avoiding unnecessary model complexity [9, 34].

The adequacy of GARCH modeling is examined using the ARCH-LM test. The null hypothesis states that there is no ARCH effect in the residuals, while the alternative hypothesis states that at least one lag of squared residuals is significant. A significant ARCH-LM statistic indicates that the variance process contains time-varying dependence and supports the use of GARCH-type models [20, 32].

To account for asymmetric volatility, this study also considers TGARCH and GJR-GARCH. The GJR-GARCH model introduced by Glosten et al. [24] augments the GARCH variance equation with an indicator variable for negative shocks:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{4}$$

where $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $I_{t-1} = 0$ otherwise. The parameter γ captures the asymmetric response of volatility to negative shocks. If γ is positive and statistically significant, negative shocks increase volatility more strongly than positive shocks of the same magnitude. The presence of asymmetric volatility is evaluated with the Sign Bias Test proposed by Engle and Ng [21].

2.4. GARCH-MIDAS and asymmetric GARCH-MIDAS

The GARCH-MIDAS model decomposes volatility into two components: a short-run component driven by daily shocks and a long-run component driven by lower-frequency variables. In this study, the high-frequency variable is the daily ISSI return, while the low-frequency variables are monthly inflation and the BI Rate. Previous evidence shows that macroeconomic variables can improve long-run variance prediction within a GARCH-MIDAS framework in some markets [5]. The general GARCH-MIDAS return equation is

$$r_{i,t} = \mu + \sqrt{\tau_t g_{i,t}}\,\varepsilon_{i,t}, \tag{5}$$

where $r_{i,t}$ is the daily return on day i within month t, μ is the mean return, $g_{i,t}$ is the short-run volatility component, τ_t is the long-run volatility component, and $\varepsilon_{i,t}$ is a standardized innovation. The short-run GARCH component can be written as

$$g_{i,t} = (1-\alpha-\beta) + \alpha\frac{(r_{i-1,t}-\mu)^2}{\tau_t} + \beta g_{i-1,t}. \tag{6}$$

The long-run MIDAS component is specified as

$$\tau_t = \exp\left(m + \theta\sum_{k=1}^{K} w(k;\omega_1,\omega_2)X_{t-k}\right), \tag{7}$$

where X_{t-k} denotes the low-frequency macroeconomic variable at lag k, θ measures the effect of the macroeconomic variable on long-run volatility, and $w(k;\omega_1,\omega_2)$ is the MIDAS weighting function. The beta-polynomial weighting function is

$$w(k;\omega_1,\omega_2) = \frac{(k/K)^{\omega_1-1}(1-k/K)^{\omega_2-1}}{\sum_{j=1}^{K}(j/K)^{\omega_1-1}(1-j/K)^{\omega_2-1}}. \tag{8}$$

This weighting function ensures that lagged macroeconomic information enters the model in a structured and parsimonious way [22].

The asymmetric version used in this study is the GJR-GARCH-MIDAS model, which combines the short-run GJR-GARCH structure and the long-run MIDAS component. This specification allows the model to capture both leverage effects in daily return shocks and the contribution of monthly macroeconomic variables to long-run volatility. Following Amendola et al. [3], this framework is useful when macroeconomic conditions affect volatility asymmetrically or when short-run market stress is not fully captured by symmetric models.

2.5. Akaike Information Criterion and Bayesian Information Criterion

The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are used as complementary measures of model fit. Both criteria balance the maximized likelihood against a penalty for model complexity, so a smaller value indicates a more favorable trade-off between goodness of fit and parsimony among models estimated on the same data and likelihood scale. The AIC is defined as follows [16]:

$$\text{AIC} = -2\log(\widehat{L}) + 2k. \tag{9}$$

The BIC applies a sample-size-dependent penalty and is defined as

$$\text{BIC} = -2\log(\widehat{L}) + k\log(n), \tag{10}$$

where $\widehat{L}$ is the maximized likelihood value, k is the number of estimated parameters in the model, and n is the number of observations used to estimate the model.

2.6. Value at Risk

Value at Risk is a downside-risk measure that estimates the maximum potential loss of an asset or portfolio over a specified horizon and confidence level [26, 25]. In a volatility-based model, VaR is computed from the conditional mean and conditional standard deviation obtained from the selected volatility model. For the lower-tail quantile α, VaR can be expressed as

$$\mathrm{VaR}_t(\alpha) = \mu_t + z_\alpha \sigma_t, \tag{11}$$

where μ_t is the conditional mean, σ_t is the conditional standard deviation, and z_α is the standard normal quantile at level α. In this study, VaR is estimated at the 5%, 2.5%, and 1% quantiles. A VaR violation occurs when the realized return is lower than the estimated VaR threshold. Because VaR is directly affected by conditional volatility, models that better capture volatility clustering and leverage effects are expected to produce more reliable VaR estimates.

2.7. Kupiec Proportion of Failure backtesting

Backtesting is used to evaluate whether estimated VaR is statistically consistent with realized losses. This study applies the Kupiec Proportion of Failure (POF) test, also known as the unconditional coverage test [28]. The null hypothesis states that the observed violation rate is equal to the expected violation probability α. The test statistic is

$$LR_{\mathrm{POF}} = -2\ln\left[\frac{(1-\alpha)^{T-N}\alpha^N}{\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^N}\right] \sim \chi_1^2, \tag{12}$$

where T is the number of observations and N is the number of VaR violations. If the p-value is greater than 0.05, the null hypothesis of correct unconditional coverage is not rejected, indicating that the model produces an acceptable number of violations. If the p-value is less than 0.05, the model is rejected because the realized violation frequency is statistically inconsistent with the expected rate.

3. Empirical Results and Discussion

3.1. Data exploration: closing price and return patterns

The empirical analysis begins with a visual examination of ISSI closing prices. Figure 1 shows that ISSI generally follows an increasing long-run trend from the beginning to the end of the observation period. This upward pattern indicates that the Indonesian Islamic equity market experienced long-term growth. However, the increase was not smooth. The index experienced several correction phases and sharp declines, especially around periods of heightened uncertainty. The most visible sharp decline occurred around 2020, which is consistent with the high market stress associated with the COVID-19 period. The final part of the series also shows a strong increase followed by a short correction, indicating that market risk remains relevant even during periods of long-run growth.

The presence of a clear long-run trend in the closing price confirms that the price series is not appropriate for direct volatility modeling. Therefore, the closing price is transformed into logarithmic return. Figure 2 shows that ISSI returns fluctuate around zero and do not display the long-run trend observed in the price series. This pattern suggests that the return transformation is effective in stabilizing the mean of the series. Nevertheless, the magnitude of return fluctuations is not constant over time. Several periods show large positive and negative shocks, and these large shocks tend to cluster in specific periods. This is a visual indication of volatility clustering.

The return plot shows that periods of high volatility are often followed by further high-volatility observations, while calmer periods are followed by relatively smaller fluctuations. This feature is a key empirical characteristic of financial returns and justifies the use of ARCH/GARCH-type models. The return series also contains several extreme negative observations, indicating that downside risk should be evaluated carefully through VaR estimation. Because the magnitude of fluctuations varies over time, a dynamic volatility model is more appropriate than a constant-variance model.

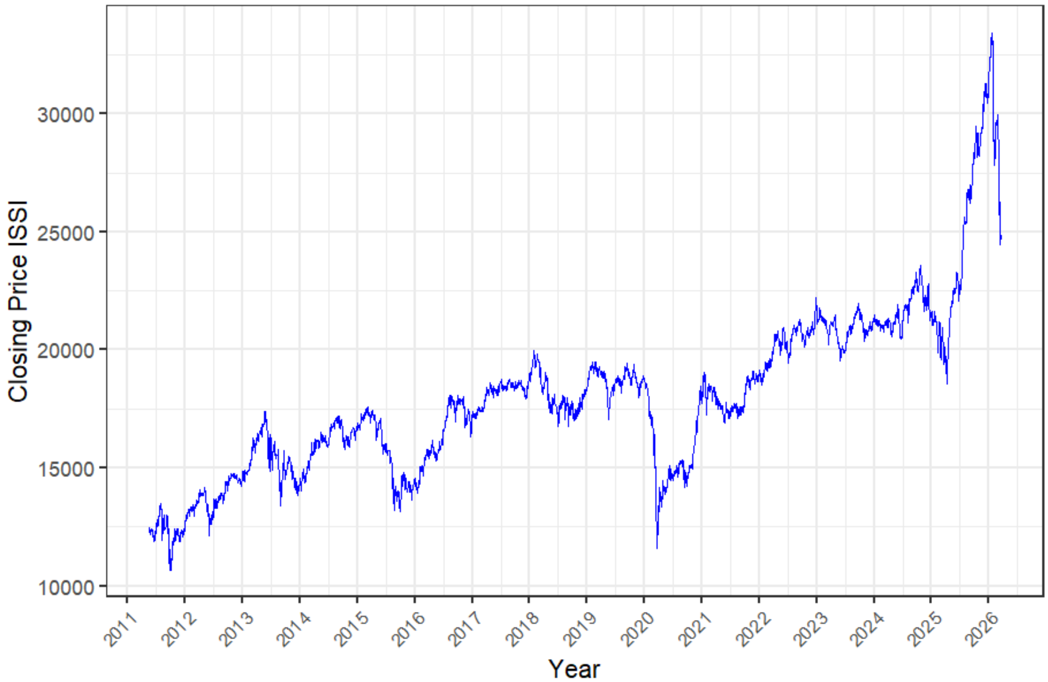

Figure 1. ISSI closing price during the observation period.

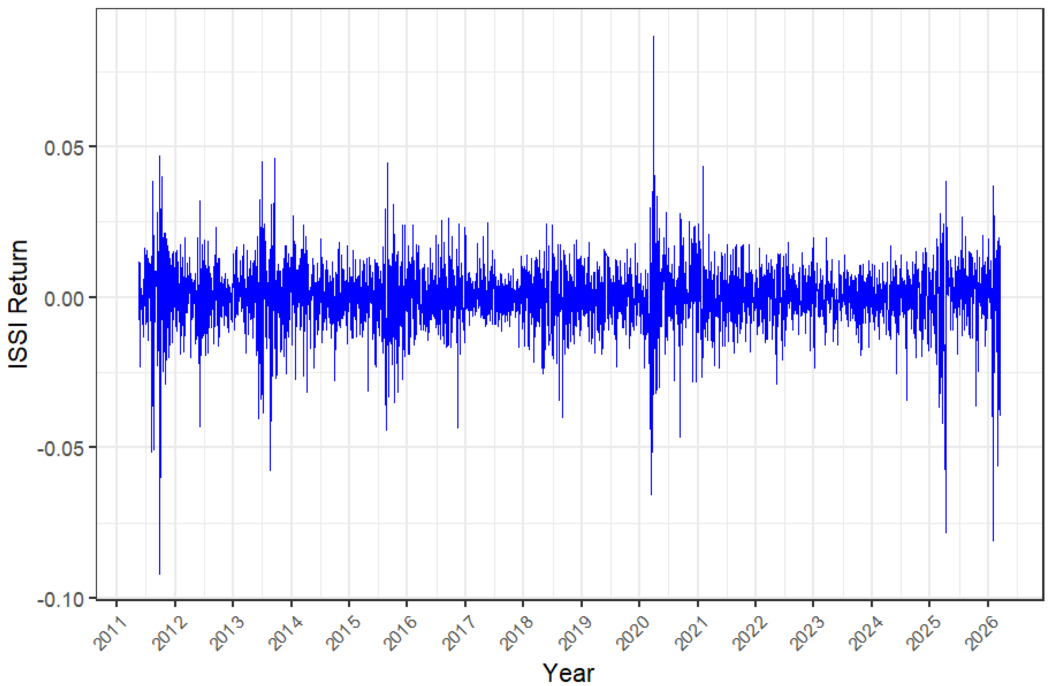

Figure 2. Daily logarithmic return of ISSI during the observation period.

3.2. Mean model

Before estimating volatility models, the return series is examined for stationarity using the Augmented Dickey-Fuller (ADF) test. The ADF statistic is -15.253 with a p-value below 0.01, so ISSI returns are stationary and no differencing is required. The mean equation can therefore be evaluated directly in levels as a stationary return process.

Table 1. ADF stationarity test for ISSI return.

Variable	Test	Statistic	Lag order	p-value	Decision
ISSI return	ADF	-15.253	15	< 0.01	Stationary

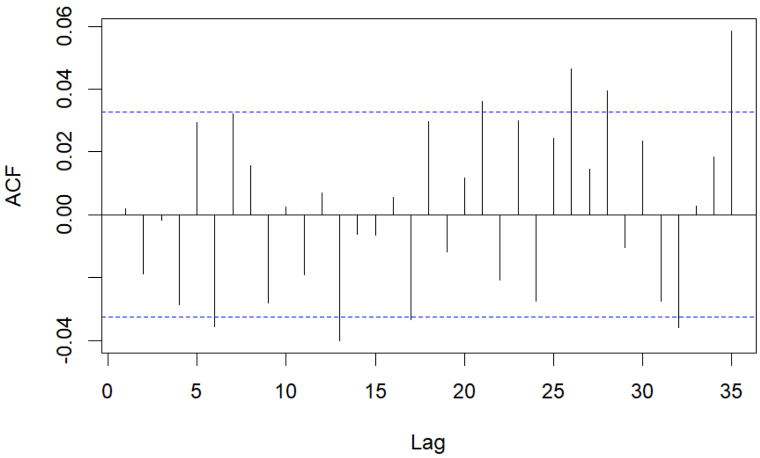

Figure 3. ACF plot of ISSI return.

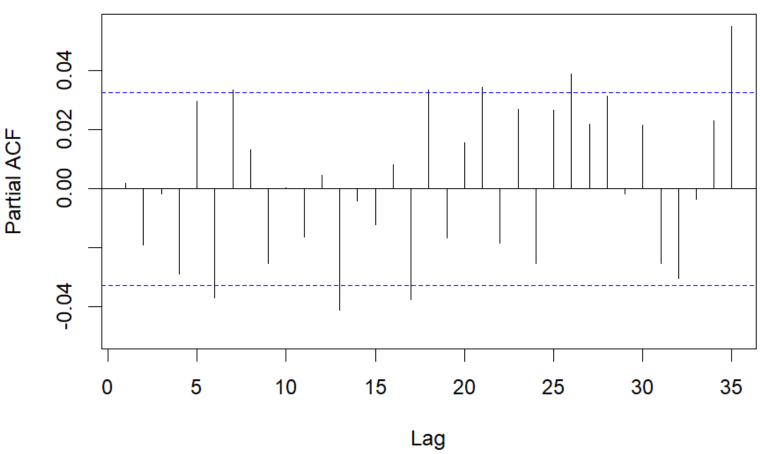

Figure 4. PACF plot of ISSI return.

The ACF and PACF plots of ISSI returns are shown in Figures 3 and 4. Although several isolated spikes exceed the significance bounds, they do not form a stable cut-off or decay pattern that would justify a systematic AR or MA structure. Candidate mean models with additional terms do not provide a meaningful improvement in the information criteria, and the selected residuals pass the Ljung-Box test. The mean is therefore written directly as the constant mean process

$$r_t = 0.0002 + \varepsilon_t, \tag{13}$$

rather than using redundant ARIMA order notation. This parsimonious specification avoids unnecessary parameters while retaining the small positive average return.

The Ljung–Box test produces a p-value of 0.1976, which is greater than the 5% significance level. Therefore, the null hypothesis of no residual autocorrelation cannot be rejected, indicating that the residuals of the constant mean process do not exhibit a significant linear dependence. This finding suggests that additional autoregressive or moving-average terms are unnecessary, as no systematic linear pattern remains in the conditional mean. In contrast, the Jarque–Bera test rejects the null hypothesis of normality at the 1% significance level. This result is consistent with the presence of skewness, excess kurtosis, and extreme observations commonly found in financial return data. Consequently, the subsequent analysis should focus on modeling conditional heteroskedasticity rather than introducing redundant terms into the mean equation.

Table 2. Diagnostic tests for the constant mean residuals.

Model	Ljung–Box statistic	Ljung–Box p-value	Jarque–Bera statistic	Jarque–Bera p-value
Constant mean process	7.324	0.1976	9430.5	< 0.01

3.3. Estimation of GARCH and Asymmetric GARCH

The ARCH-LM test is applied to the residuals of the constant mean process. The test statistic is 619.2 with a p-value below 0.001, indicating strong ARCH effects. Thus, although no linear dependence remains in the conditional mean, the residual variance is time-varying and requires GARCH-type modeling. The ACF and PACF plots of squared residuals in Figures 5 and 6 also show significant dependence in the variance process.

Table 3. ARCH-LM test on constant mean residuals.

Variable	Test statistic	p-value	Decision
Constant mean residuals	619.2	< 0.001	ARCH effect exists

The Sign Bias Test is then used to evaluate asymmetric volatility. The joint effect statistic is 11.994241 with p-value 0.007, which is less than 0.05. This result indicates the presence of leverage effects in ISSI returns. In other words, negative and positive shocks do not affect volatility in the same way. This finding supports the estimation of asymmetric volatility models such as TGARCH(1,1), GJR-GARCH(1,1), and GJR-GARCH-MIDAS.

Table 4. Sign Bias test for leverage effect.

Test component	Statistic	p-value	Decision
Joint effect	11.994241	0.007	Asymmetric volatility exists

The rejection of the null hypothesis indicates that asymmetric GARCH specifications are relevant. Table 5 summarizes the GARCH and asymmetric GARCH parameter estimates. In the GARCH(1,1) model, all variance

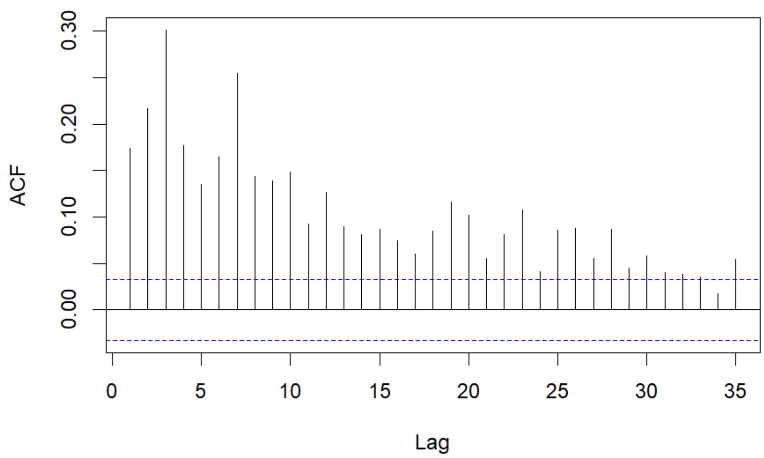

Figure 5. ACF plot of squared constant mean residuals.

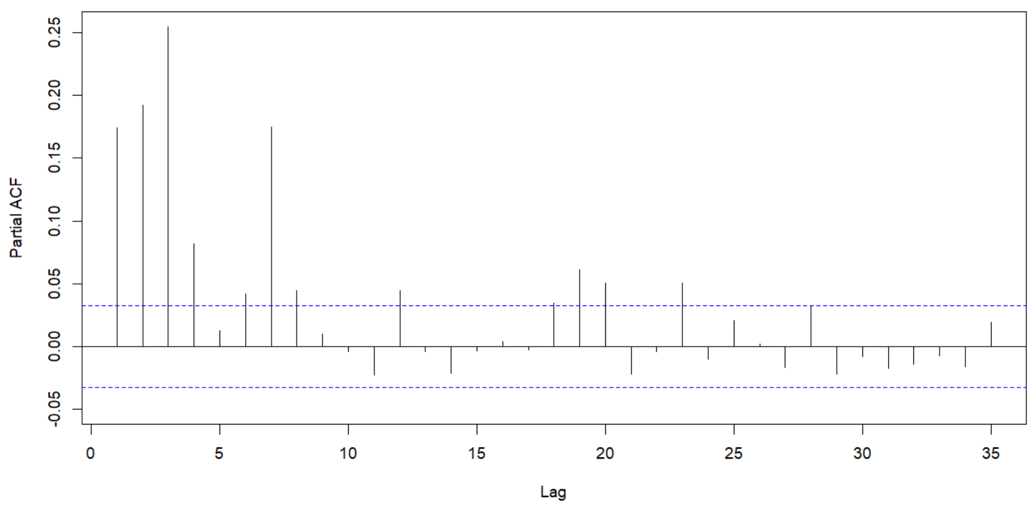

Figure 6. PACF plot of squared constant mean residuals.

parameters are significant at the 5% level. The ARCH parameter indicates that recent shocks affect current volatility, while the GARCH parameter indicates volatility persistence. The sum of the ARCH and GARCH parameters is approximately 0.75, suggesting that volatility shocks are persistent but mean reverting.

The γ parameter in TGARCH and GJR-GARCH captures asymmetric volatility. The TGARCH(1,1) model shows that the asymmetric parameter γ is positive and statistically significant. This means that negative shocks tend to increase volatility more than positive shocks. The GJR-GARCH(1,1) model also shows a positive and significant γ parameter, confirming the leverage effect. Although the mean and variance intercepts in GJR-GARCH are not significant at the 5% level, the main volatility parameters α, β, and γ are all significant. Therefore, GJR-GARCH(1,1) remains a relevant candidate because it captures both volatility persistence and asymmetry.

Table 5. Parameter estimates of GARCH and asymmetric GARCH models.

Model	Parameter	Estimate	p-value
GARCH(1,1)	μ	3.60×10^{-4}	0.006
	ω	3.8×10^{-6}	0.049
	α	0.111	< 0.001
	β	0.639	< 0.001
TGARCH(1,1)	μ	0.000195	0.143
	ω	0.000369	< 0.001
	α	0.107204	< 0.001
	β	0.879672	< 0.001
	γ	0.383421	< 0.001
GJR-GARCH(1,1)	μ	2.4×10^{-4}	0.061
	ω	3.99×10^{-6}	0.064
	α	5.99×10^{-2}	< 0.001
	β	8.57×10^{-1}	< 0.001
	γ	7.85×10^{-2}	< 0.001

3.4. Estimation of GARCH-MIDAS and GJR-GARCH-MIDAS

The next stage estimates GARCH-MIDAS and GJR-GARCH-MIDAS models with inflation and the BI Rate as macroeconomic variables in the long-run volatility component. The purpose is to determine whether monthly macroeconomic information contributes to the long-run volatility of ISSI returns after short-run volatility dynamics are considered. Table 6 summarizes the parameter estimates for the GARCH-MIDAS models.

Table 6. Parameter estimates of GARCH-MIDAS models.

Model	Parameter	Estimate	p-value
GARCH(1,1)-MIDAS (Inflation)	α	0.111814	< 0.001
	β	0.852928	< 0.001
	m	-9.121845	< 0.001
	θ	-0.006537	0.925
	ω	6.615488	< 0.001
GARCH(1,1)-MIDAS (BI Rate)	α	0.112163	< 0.001
	β	0.849035	< 0.001
	m	-9.777896	< 0.001
	θ	0.108397	0.064
	ω	1.001003	< 0.001

The θ parameter measures the effect of the macroeconomic variable on the long-run volatility component. In the GARCH-MIDAS model with inflation, α and β are significant, implying that short-run volatility remains driven by previous shocks and volatility persistence. However, the inflation coefficient θ is not significant, with p-value 0.925. This means that inflation does not provide statistically significant explanatory power for the long-run volatility component during the sample period. In the GARCH-MIDAS model with the BI Rate, θ has a p-value of 0.064. Although this value is closer to the 5% threshold than the inflation model, it is still not significant at the 5% level. Therefore, the BI Rate is also not confirmed as a significant long-run volatility determinant in the symmetric MIDAS specification.

Table 7 reports the GJR-GARCH-MIDAS results. In both inflation and BI Rate specifications, the short-run volatility parameters α and β are significant. More importantly, the asymmetric parameter γ is also significant with p-values below 0.001. This result indicates that even after incorporating macroeconomic variables into the long-run component, the leverage effect remains important in explaining ISSI volatility.

Table 7. Parameter estimates of GJR-GARCH-MIDAS models.

Model	Parameter	Estimate	p-value
GJR-GARCH(1,1)-MIDAS (Inflation)	α	0.058299	< 0.001
	β	0.856821	< 0.001
	m	-9.241989	< 0.001
	θ	-0.002182	0.132
	ω	5.124786	0.709
	γ	0.085380	< 0.001
GJR-GARCH(1,1)-MIDAS (BI Rate)	α	0.05817	< 0.001
	β	0.852566	< 0.001
	m	-9.823574	< 0.001
	θ	0.100242	0.107
	ω	1.001002	0.579
	γ	0.086888	< 0.001

The significant γ parameters show that asymmetric volatility persists in the MIDAS framework. The macroeconomic parameters in the GJR-GARCH-MIDAS models are not significant at the 5% level. Inflation has a p-value of 0.132 and the BI Rate has a p-value of 0.107. The result should not be interpreted as evidence that macroeconomic conditions are generally irrelevant. Rather, after daily shocks, volatility persistence, and leverage effects are included, these two publicly observed domestic variables provide little additional long-run explanatory power in this specification. One plausible explanation is that inflation and policy-rate information is incorporated rapidly into stock prices, while a second is that ISSI volatility over this long sample is dominated by global risk episodes, exchange-rate pressures, pandemic and geopolitical shocks, or market-specific news not captured by the two MIDAS variables. These interpretations are economic explanations rather than direct tests of market efficiency or global-shock dominance.

The evidence therefore supports the asymmetric GJR structure more strongly than it supports the macroeconomic MIDAS enhancement itself. The GJR-GARCH-MIDAS framework remains useful as a testing structure because it separates short-run asymmetric volatility from a long-run component, but the significant γ coefficient and the risk-backtesting performance cannot be attributed to significant inflation or BI Rate effects. Its empirical advantage in this study is primarily the ability to allow negative shocks to affect volatility more strongly than positive shocks.

3.5. Model comparison using AIC and BIC

Table 8. AIC and BIC values for the estimated volatility models.

Model	AIC	BIC
GARCH(1,1)	-6.580274	-6.571678
TGARCH(1,1)	-6.584290	-6.573976
GJR-GARCH(1,1)	-6.583934	-6.573620
GARCH(1,1)-MIDAS (Inflation)	-27801.103745	-27763.971611
GARCH(1,1)-MIDAS (BI Rate)	-27804.096996	-27766.964861
GJR-GARCH(1,1)-MIDAS (Inflation)	-27817.811635	-27774.490811
GJR-GARCH(1,1)-MIDAS (BI Rate)	-27820.969144	-27777.648320

Within the conventional GARCH family, TGARCH(1,1) has the smallest AIC (-6.584290) and BIC (-6.573976), followed very closely by GJR-GARCH(1,1). The relatively small differences indicate that both asymmetric specifications improve penalized fit compared with the standard GARCH(1,1), although the information criteria alone do not evaluate tail-risk calibration.

Within the MIDAS family, GJR-GARCH(1,1)-MIDAS with the BI Rate records the smallest AIC (-27820.969144) and BIC (-27777.648320), followed by the corresponding inflation specification. Because the conventional GARCH and MIDAS results are produced on different reported likelihood scales, their absolute criterion values are interpreted within their respective model families rather than compared directly across families. The lower AIC and BIC of the GJR-MIDAS specifications support the asymmetric short-run variance structure after penalizing model complexity; they do not establish that the BI Rate or inflation is an individually significant long-run determinant, since both θ coefficients remain statistically insignificant.

3.6. Value at Risk and backtesting

After estimating and comparing the volatility models using AIC and BIC, VaR is calculated at the 5%, 2.5%, and 1% quantiles. The estimated VaR is then evaluated using the Kupiec POF test. Table 9 presents the observed violation percentages and p-values for each model and quantile. A model is considered acceptable when the p-value is greater than 0.05, indicating that the observed violation rate is not statistically different from the expected violation probability.

Table 9. Kupiec POF backtesting results for VaR.

Model	Violation rate			p-value		
	$q = 5\%$	$q = 2.5\%$	$q = 1\%$	$q = 5\%$	$q = 2.5\%$	$q = 1\%$
GARCH(1,1)	5.97%	3.08%	1.33%	0.009	0.030	0.055
TGARCH(1,1)	6.08%	3.02%	1.47%	0.003	0.049	0.007
GJR-GARCH(1,1)	6.05%	2.94%	1.30%	0.004	0.096	0.078
GARCH(1,1)-MIDAS (Inflation)	5.92%	3.08%	1.38%	0.014	0.030	0.026
GARCH(1,1)-MIDAS (BI Rate)	5.78%	3.13%	1.41%	0.036	0.018	0.018
GJR-GARCH(1,1)-MIDAS (Inflation)	6.02%	2.88%	1.33%	0.006	0.144	0.055
GJR-GARCH(1,1)-MIDAS (BI Rate)	5.97%	2.91%	1.33%	0.009	0.118	0.055

The null hypothesis of the Kupiec POF test is correct unconditional coverage. P-values greater than 0.05 indicate an acceptable violation frequency. At the 5% quantile, all models have p-values below 0.05. Therefore, all models reject the null hypothesis of correct unconditional coverage at this quantile. This means that none of the estimated VaR models provides an observed violation frequency that matches the expected 5% violation rate. The rejection at the 5% level may indicate that moderate-tail risk is more difficult to capture under the model assumptions or that the distributional structure of returns requires further refinement.

At the 2.5% quantile, the GJR-GARCH(1,1), GJR-GARCH(1,1)-MIDAS with inflation, and GJR-GARCH(1,1)-MIDAS with the BI Rate models have p-values greater than 0.05. These results indicate that GJR-based specifications provide acceptable VaR estimates at the 2.5% quantile. The GJR-GARCH-MIDAS inflation model has the highest p-value at 0.144, followed by the GJR-GARCH-MIDAS BI Rate model with p-value 0.118. This suggests that asymmetric volatility modeling improves the adequacy of VaR estimates in the lower tail.

At the 1% quantile, the null hypothesis of correct unconditional coverage is not rejected for GARCH(1,1), GJR-GARCH(1,1), GJR-GARCH-MIDAS with inflation, and GJR-GARCH-MIDAS with the BI Rate. In contrast, the null hypothesis is rejected for TGARCH and the symmetric GARCH-MIDAS models. The fact that both GJR-GARCH-MIDAS specifications pass the test at the 2.5% and 1% quantiles indicates that they are the most consistent models in this empirical setting. These results support the conclusion that incorporating asymmetric volatility is more important for VaR accuracy than relying only on macroeconomic variables in the MIDAS component.

Overall, the backtesting results show that the GJR-based MIDAS specifications provide the most consistent VaR coverage at the 2.5% and 1% quantiles. This is also consistent with the AIC and BIC ranking within the MIDAS family, where the GJR-GARCH-MIDAS models outperform their symmetric counterparts. However, because the inflation and BI Rate coefficients are not statistically significant, the improvement should be attributed mainly to the asymmetric short-run GJR component rather than to a demonstrated macroeconomic contribution from the MIDAS variables.

3.7. Discussion

The empirical findings confirm several important features of ISSI returns. First, the closing price series follows a long-run upward trend, while the return series fluctuates around zero. This supports the standard practice of transforming prices into log returns before volatility modeling. Second, the return series exhibits volatility clustering, which is consistent with the widely documented stylized facts of financial returns [2, 7]. Third, the presence of ARCH effects indicates that conditional variance changes over time and that GARCH-type volatility models are necessary.

The Sign Bias Test and the significant γ parameters in GJR-GARCH and GJR-GARCH-MIDAS show that ISSI volatility responds asymmetrically to shocks. This means that negative market information has a stronger impact on risk than positive information. Such a response is economically plausible because negative returns can increase perceived downside risk, trigger portfolio rebalancing, and raise uncertainty among investors. The finding is consistent with the leverage-effect literature [21, 24, 27].

The insignificance of inflation and the BI Rate in the MIDAS component deserves careful interpretation. It does not imply that macroeconomic conditions never affect ISSI. Inflation and policy rates are conventional indicators of macroeconomic conditions and monetary-policy stance [8, 30], while prior GARCH-MIDAS evidence shows that macroeconomic variables can improve variance prediction in other settings [5]. Public inflation and policy-rate announcements may nevertheless be incorporated rapidly into prices, leaving limited incremental information once daily volatility persistence and leverage effects are modeled. In addition, the sample includes major global and market-wide disturbances, such as the COVID-19 shock, changes in global monetary conditions, geopolitical uncertainty, exchange-rate pressure, and economic-policy uncertainty [6], which may dominate the two domestic variables. Because the present model does not test market efficiency or global-shock channels directly, these mechanisms should be viewed as plausible economic explanations rather than definitive causal conclusions.

The backtesting results further suggest that VaR performance is sensitive to quantile choice. All models fail at the 5% quantile, but GJR-based models perform better at the 2.5% and 1% quantiles. This indicates that the model may be more reliable for more extreme downside risk than for moderate downside risk. Since VaR is commonly used for tail-risk monitoring, the acceptance of GJR-GARCH-MIDAS at the 2.5% and 1% quantiles provides useful evidence that asymmetric volatility modeling improves risk measurement for ISSI.

For academic contribution, this study shows how asymmetric volatility, mixed-frequency variables, penalized-likelihood criteria, and multi-quantile VaR backtesting can be evaluated in a single ISSI risk framework. The evidence does not demonstrate a statistically significant macroeconomic MIDAS effect; instead, it shows that the asymmetric GJR component is the most consistently supported feature. For practical contribution, the results indicate that risk managers should account for the stronger effect of negative shocks when estimating ISSI downside risk and should avoid interpreting the inclusion of macroeconomic variables as useful unless their incremental contribution is supported statistically.

4. Conclusion

This study estimates Value at Risk for the Indonesian Sharia Stock Index using GARCH, asymmetric GARCH, GARCH-MIDAS, and GJR-GARCH-MIDAS models over the period 13 May 2011 to 17 March 2026. ISSI prices show a long-run upward trend with sharp corrections, whereas returns fluctuate around a stable mean and exhibit volatility clustering. The ADF test confirms stationarity, and a constant mean process is selected because no stable AR or MA pattern remains and the Ljung-Box test finds no significant residual autocorrelation.

The ARCH-LM test indicates significant conditional heteroskedasticity in the constant mean residuals, supporting GARCH-type variance models. The Sign Bias Test and the significant γ coefficients confirm asymmetric volatility: negative shocks affect subsequent risk more strongly than positive shocks. Among the conventional GARCH models, TGARCH(1,1) records the smallest AIC and BIC, while GJR-GARCH(1,1) remains competitive and provides acceptable VaR coverage at the lower quantiles.

Within the MIDAS family, GJR-GARCH(1,1)-MIDAS with the BI Rate has the smallest AIC and BIC, followed by GJR-GARCH(1,1)-MIDAS with inflation. Nevertheless, inflation and the BI Rate are not statistically significant determinants of the long-run volatility component at the 5% level. The information-criterion ranking therefore supports the asymmetric short-run GJR structure after accounting for complexity, but it does not establish a significant macroeconomic MIDAS effect. The insignificant coefficients may reflect rapid incorporation of domestic macroeconomic information and the stronger influence of global or market-specific shocks during the sample period.

The Kupiec POF backtesting results show that all models are rejected at the 5% VaR quantile. At the 2.5% quantile, the null hypothesis of correct unconditional coverage is not rejected for GJR-GARCH(1,1) and both GJR-GARCH-MIDAS specifications; at the 1% quantile, it is not rejected for GARCH(1,1), GJR-GARCH(1,1), and both GJR-GARCH-MIDAS specifications. The GJR-GARCH-MIDAS models are therefore the most consistent across the two more extreme quantiles. Combined with the AIC and BIC evidence, the findings indicate that improved ISSI downside-risk estimation is driven primarily by asymmetric volatility modeling rather than by statistically significant inflation or BI Rate effects.

Future research may extend this study in several ways. First, alternative innovation distributions such as Student-t or GED can be used to better accommodate heavy tails and non-normal residuals. Second, additional macroeconomic and global variables, including exchange rates, money supply, industrial production, oil prices, gold prices, global stock indices, or economic policy uncertainty, may improve the MIDAS component. Third, future studies may combine GARCH-type volatility models with Extreme Value Theory to model the tail distribution of standardized residuals more explicitly. Fourth, VaR evaluation can be expanded beyond Kupiec POF by using Christoffersen conditional coverage backtesting to assess both the number and independence of violations [15].

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