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Mathematical Modeling and Convergence Analysis of Swarm-Based Algorithms in Public Transport Network Optimization

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Abstract Optimizing dynamic path allocation in public transport networks remains challenging because passenger demand, travel time, and link capacity may change over time. This study develops a mathematical optimization framework for public transport networks represented as $G = (V, E)$, where origin–destination demand is allocated to feasible paths under flow conservation, capacity, and time-dependent operational constraints. The proposed framework is evaluated using Particle Swarm Optimization (PSO) and the Firefly Algorithm (FA). Their convergence behavior is analyzed under locally Lipschitz objective functions, bounded feasible domains, lower-bounded costs, and controlled parameter-update schemes. Numerical experiments were conducted on controlled synthetic dynamic scenarios and the Sioux–Falls benchmark network, including peak/off-peak demand fluctuations, congestion-sensitive travel times, and random capacity disruptions. The results show that PSO and FA improve travel-time performance by approximately 8–15% compared with the baseline allocation. PSO reaches stable solutions within fewer iterations, while FA provides lower variability across repeated runs under sudden network changes. Benchmark comparisons with Grey Wolf Optimizer, Bat Algorithm, and Artificial Bee Colony show that PSO and FA obtain the best average ranks in terms of solution quality, with statistically significant differences according to the Friedman test. These findings indicate that swarm-based optimization, when supported by appropriate mathematical assumptions and parameter design, can provide a reliable framework for dynamic path allocation in public transport networks.

Keywords Public Transport Network Optimization, Swarm Intelligence, Particle Swarm Optimization, Firefly Algorithm, Convergence Analysis.

AMS 2010 subject classifications 90C59, 90B20, 90C35, 90C30

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1. Introduction

Public transportation plays an important role in urban and regional mobility because it supports accessibility, reduces congestion, and contributes to more sustainable transport systems [1]. However, public transport operations are often affected by fluctuating passenger demand, limited link capacity, congestion-sensitive travel times, and unexpected service disruptions [2]. These dynamic conditions make it difficult to maintain efficient passenger-flow distribution across a network. Therefore, optimization approaches are needed to support adaptive operational decisions, particularly in allocating origin–destination (OD) demand to feasible paths under time-dependent network conditions [3, 4].

A public transport system can be represented as a network $G = (V, E)$, where nodes denote stops, stations, or transfer points, and edges represent service links. In such a network, travel cost may include travel time, operational cost, reliability, and congestion effects [5, 6]. When demand and travel time vary over time, the resulting optimization problem becomes nonlinear, time-varying, and often non-convex, making exact or gradient-based

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methods difficult to apply in large-scale or uncertain settings [7, 8]. Metaheuristic methods are therefore commonly used because they can handle complex objective functions, nonlinear constraints, and feasible-space projection without requiring strong differentiability assumptions [9]. Among these methods, Particle Swarm Optimization (PSO) and the Firefly Algorithm (FA) are attractive because of their simple update rules, population-based search mechanisms, and ability to balance exploration and exploitation [10].

Several studies have examined the theoretical properties of swarm-based optimization, especially PSO. Stochastic stability analysis has shown that random assignment strategies can affect search dynamics and algorithmic stability [11]. Other studies have emphasized the role of constriction factors and parameter control in improving convergence behavior and maintaining the exploration–exploitation balance [12]. These works provide important theoretical foundations. However, most convergence analyses are developed in general optimization settings and are not directly linked to the structure of dynamic transport networks, such as OD flow conservation, capacity constraints, and time-dependent travel costs.

In transport applications, metaheuristic methods have been widely used for transit route network design, service planning, and multimodal transport optimization. Hybrid heuristic approaches have been applied to urban bus route network design to handle trade-offs between service performance and operational efficiency [13]. Variable neighborhood search has also been used for transit route network design problems with large solution spaces and operational constraints [14]. Recent reviews further emphasize that public transport optimization should consider sustainability, emerging technologies, and operational robustness [15, 18]. In addition, recent studies have addressed integrated limited-stop services, bike-sharing strategies, adaptive transit design, and robust control in intelligent transportation systems [16, 17, 19]. Although these studies demonstrate strong empirical performance, many of them focus mainly on numerical results or static benchmark comparisons and do not provide a direct convergence-based explanation for algorithm behavior under changing network conditions.

Based on these observations, the main gap addressed in this study is the limited connection between dynamic transport-network modeling and convergence analysis of swarm-based algorithms. In particular, existing studies often show that metaheuristics perform well in simulations, but they do not always clarify why the algorithms remain stable when demand, travel time, or capacity changes over time. This study focuses on dynamic path allocation in public transport networks, where OD demand is distributed among feasible paths subject to flow conservation, capacity limits, and time-dependent operational conditions. The study does not attempt to solve the full network design problem, such as route generation, fleet sizing, or timetable construction. Instead, it provides a mathematically explicit and convergence-aware framework for dynamic path allocation within a given transport network.

The contributions of this paper are summarized as follows. First, we formulate a dynamic path allocation model for public transport networks using time-dependent travel costs, OD demand, capacity constraints, and a multi-criteria objective function. Second, we analyze the convergence behavior of PSO and FA under realistic mathematical assumptions, including bounded feasible domains, locally Lipschitz objective functions, lower-bounded costs, and controlled parameter-update schemes. Third, we validate the theoretical insights through numerical simulations under demand fluctuations, congestion-sensitive travel times, and random capacity disruptions. Fourth, we compare PSO and FA with other metaheuristic algorithms to evaluate solution quality, robustness, convergence speed, and computational performance.

Accordingly, this study is guided by the following research questions:

1. How can dynamic path allocation in public transport networks be mathematically formulated under time-dependent demand, travel time, and capacity constraints?
2. How can the convergence behavior of PSO and FA be analyzed under the proposed dynamic transport-network framework?
3. How do PSO and FA perform compared with other metaheuristic algorithms in terms of solution quality, robustness, convergence speed, and computational efficiency?

2. Methods

The research method consists of five main stages: (i) defining the scope of the dynamic path allocation problem in a public transport network, (ii) formulating the mathematical optimization model, (iii) implementing PSO and FA with simplex projection and constraint penalties, (iv) analyzing the convergence behavior of both algorithms under explicit mathematical assumptions, and (v) evaluating the algorithms through numerical simulation and benchmarking against other metaheuristic methods. The study focuses on dynamic path allocation within a given transport network. Therefore, route generation, timetable construction, fleet sizing, and vehicle scheduling are not optimized in this work and are considered as possible extensions for future research.

2.1. Overview of the Research Framework

The proposed framework integrates mathematical modeling, convergence analysis, numerical simulation, and comparative evaluation. First, a public transport network is represented as a directed graph ($G = (V, E)$), where OD demand is allocated to feasible paths under time-dependent demand, travel time, and capacity conditions. Second, PSO and FA are implemented to search for feasible path allocation solutions. Third, the convergence properties of the algorithms are analyzed under bounded feasible domains, locally Lipschitz objective functions, lower-bounded costs, and controlled parameter-update schemes. Finally, the numerical performance is evaluated using convergence behavior, final objective value, robustness, computational time, and constraint satisfaction. This framework is designed to connect the theoretical behavior of swarm-based algorithms with their practical performance in dynamic transport-network scenarios.

2.2. Mathematical Modeling of the Dynamic Public Transport Network

Problem Statement. Let $G = (V, E)$ denote a public transport network, where V is the set of stops, stations, or transfer points, and E is the set of service links. Each link $e \in E$ has a time-dependent travel time $t_e(\tau)$, effective capacity $u_e(\tau)$, and operational cost $c_e(\tau)$ at time interval τ. Passenger demand is represented by a set of origin–destination pairs $\mathcal{K}$, where $d_k(\tau)$ denotes the demand rate for OD pair $k \in \mathcal{K}$. The feasible path set for each OD pair is denoted by $\mathcal{P}_k$. In this study, $\mathcal{P}_k$ is assumed to be given, so the model optimizes the allocation of OD demand among feasible paths rather than designing new routes.

Decision Variables and Link Flow. For each OD pair $k \in \mathcal{K}$ and feasible path $p \in \mathcal{P}_k$, the decision variable $x_{k,p}(\tau) \in [0, 1]$ denotes the proportion of OD demand k assigned to path p at time interval τ. The path allocation must satisfy

$$\sum_{p \in \mathcal{P}_k} x_{k,p}(\tau) = 1, \qquad \forall k \in \mathcal{K}. \tag{1}$$

The link flow induced by the OD path allocation is defined as

$$f_e(\tau) = \sum_{k \in \mathcal{K}} \sum_{\substack{p \in \mathcal{P}_k \\ e \in p}} d_k(\tau)\, x_{k,p}(\tau), \qquad \forall e \in E. \tag{2}$$

Here, the condition $e \in p$ restricts the inner summation to only those feasible paths that actually traverse link e; to compute the flow on link e, only the contribution of paths passing through e may be counted. Consequently, each term in the summation must simultaneously satisfy two conditions: (i) $p \in \mathcal{P}_k$, meaning that p is a feasible path for OD pair k, and (ii) $e \in p$, meaning that path p passes through link e. Without the second condition, the entire demand associated with $\mathcal{P}_k$ would be summed into $f_e(\tau)$, even though many of these paths do not traverse link e at all. The condition $e \in p$ therefore filters the summation so that only the paths relevant to link e contribute to its induced flow.

The travel time of path p is then given by

$$T_p(\tau) = \sum_{e \in p} t_e\big(\tau, f_e(\tau)\big). \tag{3}$$

Constraints. The optimization model is subject to the following constraints:

$$f_e(\tau) \le u_e(\tau), \qquad \forall e \in E, \tag{4}$$

$$\sum_{p \in \mathcal{P}_k} x_{k,p}(\tau) = 1, \qquad \forall k \in \mathcal{K}, \tag{5}$$

$$0 \le x_{k,p}(\tau) \le 1, \qquad \forall k \in \mathcal{K},\ p \in \mathcal{P}_k, \tag{6}$$

$$\mathcal{H}(\mathbf{x}, \tau) \le 0. \tag{7}$$

Here, $\mathcal{H}(\mathbf{x}, \tau) \le 0$ is used as a generic placeholder for any additional operational constraints not explicitly modeled in (4)–(6); its specific form depends on the operational policy under consideration and is not further specified in this study.

The first constraint represents link capacity restrictions. The second and third constraints ensure valid OD path allocation, while the fourth constraint represents additional operational feasibility conditions such as headway, time-window, or service-policy constraints.

Dynamic Assumptions. The dynamic network setting is defined using the following assumptions:

1. The OD demand $d_k(\tau)$ is time-varying and piecewise stationary within each simulation interval.
2. The link travel-time function $t_e(\tau, f)$ is locally Lipschitz continuous with respect to f, so there exists $L_e > 0$ such that
$$|t_e(\tau, f_1) - t_e(\tau, f_2)| \le L_e\, |f_1 - f_2|$$
for feasible f_1 and f_2.
3. The capacity $u_e(\tau)$ is bounded below by a positive constant, i.e.,
$$u_e(\tau) \ge \underline{u}_e > 0, \qquad \forall e \in E.$$
4. The feasible set induced by the OD simplex constraints is compact.

These assumptions are used to support the convergence analysis of the swarm-based algorithms under dynamic network conditions.

2.3. *Implementation of Swarm Intelligence Algorithms*

Solution Representation and Simplex Projection. Each candidate solution is encoded as a vector

$$\mathbf{x} = \left(x_{k,p}\right)_{k \in \mathcal{K},\, p \in \mathcal{P}_k},$$

which contains the path allocation proportions for all OD pairs. For each OD pair k, the feasible allocation vector belongs to the probability simplex

$$\Delta_k = \left\{ \mathbf{x}_k \in \mathbb{R}^{|\mathcal{P}_k|} \;\middle|\; \sum_{p \in \mathcal{P}_k} x_{k,p} = 1,\ x_{k,p} \ge 0 \right\}.$$

where $\mathbb{R}^{|\mathcal{P}_k|}$ denotes the real vector space whose dimension equals $|\mathcal{P}_k|$, the number of feasible paths available for OD pair k; that is, $\mathbf{x}_k$ has one coordinate $x_{k,p}$ for each path $p \in \mathcal{P}_k$.

After each PSO or FA update, the candidate solution is projected back onto Δ_k for every OD pair. Given an unconstrained vector $\mathbf{z}_k = (z_{k,p})_{p \in \mathcal{P}_k} \in \mathbb{R}^{|\mathcal{P}_k|}$, the projection is defined as

$$\Pi_{\Delta_k}(\mathbf{z}_k) = \arg \min_{\mathbf{x}_k \in \Delta_k} \|\mathbf{x}_k - \mathbf{z}_k\|_2^2\,. \tag{8}$$

Using the sorting–thresholding projection, the projected component is computed as

$$x_{k,p} = \max\{z_{k,p} - \theta_k, 0\}. \tag{9}$$

where the threshold θ_k is selected such that

$$\sum_{p \in \mathcal{P}_k} \max\{z_{k,p} - \theta_k, 0\} = 1.$$

This OD-wise projection guarantees that all updated solutions remain feasible with respect to the path allocation constraints.

Constraint Penalty. Capacity and operational constraint violations are handled through the effective objective function

$$J_{\text{eff}}(\mathbf{x}, \tau) = F(\mathbf{x}, \tau) + \lambda \sum_{e \in E} \Big(\max\{0,\ f_e(\tau) - u_e(\tau)\} \Big)^2 + \mu \left\| \mathcal{H}^+(\mathbf{x}, \tau) \right\|_2^2, \tag{10}$$

where $\lambda > 0$ and $\mu > 0$ are penalty coefficients, and $\mathcal{H}^+$ denotes the positive part of the operational constraint violation. This penalty formulation allows infeasible solutions to be evaluated while discouraging violations of capacity and operational constraints.

Experimental Parameter Settings. To ensure a fair comparison, all algorithms were evaluated using the same population size, maximum number of iterations, number of independent runs, and stopping criterion. The parameter settings used in the numerical experiments are reported in Table 1. These values should be adjusted according to the final experimental implementation if different settings are used in the simulation code.

Table 1. Experimental hyperparameter settings for PSO and FA

Algorithm	**Parameter**	**Value**
All algorithms	Population size (N)	50
All algorithms	Maximum iteration ($T_{\max}$)	100
All algorithms	Independent runs	30
All algorithms	Stopping tolerance (ε)	10^{-6}
All algorithms	Random seed	42
PSO	Inertia weight (w)	linearly decreased from 0.9 to 0.4
PSO	Cognitive coefficient (c_1)	2.0
PSO	Social coefficient (c_2)	2.0
PSO	Velocity bound	$[-0.2, 0.2]$
FA	Initial attractiveness (β_0)	1.0
FA	Absorption coefficient (γ)	1.0
FA	Initial randomization factor (α_0)	0.2
FA	Annealing rule for α_t	$\alpha_t = 0.95^t \alpha_0$
Penalty	Capacity penalty (λ)	100
Penalty	Operational penalty (μ)	100

It should be noted that the term *annealing schedule* used for α_t in Table 1 is not exclusive to Simulated Annealing. Following the standard formulation of the Firefly Algorithm by Yang [21], this term is used generically to denote any geometric decay scheme applied to a randomization or exploration control parameter over iterations, regardless of the specific metaheuristic in which it is employed. In this study, the randomization factor $\alpha_t = 0.95^t \alpha_0$ follows exactly this convention as an intrinsic component of FA, and should not be interpreted as a hybridization with Simulated Annealing.

Particle Swarm Optimization. For PSO, each particle represents a feasible path allocation vector. At iteration (t), the velocity and position of particle (i) are updated as

$$\mathbf{v}_i^{(t)} = w\,\mathbf{v}_i^{(t-1)} + c_1 r_1 \big(\mathbf{pbest}_i - \mathbf{x}_i^{(t-1)}\big) + c_2 r_2 \big(\mathbf{gbest} - \mathbf{x}_i^{(t-1)}\big), \tag{11}$$

$$\mathbf{x}_i^{(t)} = \Pi_X \left(\mathbf{x}_i^{(t-1)} + \mathbf{v}_i^{(t)} \right), \tag{12}$$

where w is the inertia weight controlling the influence of the previous velocity, c_1 and c_2 are the cognitive and social acceleration coefficients, $\mathbf{pbest}_i$ is the best solution previously found by particle i, $\mathbf{gbest}$ is the best global solution, $r_1, r_2 \sim U(0,1)$, and Π_X denotes the OD-wise simplex projection applied to all OD pairs. The values of $\mathbf{pbest}_i$ and $\mathbf{gbest}$ are determined using $J_{\text{eff}}(\mathbf{x}, \tau)$. Numerical settings for w, c_1, c_2 are given in Table 1.

Firefly Algorithm. For FA, each firefly represents a feasible path allocation vector, and its brightness is inversely related to the effective objective value. If firefly (j) is brighter than firefly (i), the position of firefly (i) is updated as

$$\mathbf{x}_i^{(t)} = \Pi_X \left[\mathbf{x}_i^{(t-1)} + \beta_0 \exp\left(-\gamma \left\| \mathbf{x}_i^{(t-1)} - \mathbf{x}_j^{(t-1)} \right\|_2^2 \right) \left(\mathbf{x}_j^{(t-1)} - \mathbf{x}_i^{(t-1)} \right) + \alpha_t \boldsymbol{\epsilon}^{(t)} \right], \tag{13}$$

where β_0 is the initial attractiveness, γ is the absorption coefficient, α_t is the randomization factor at iteration t, and $\boldsymbol{\epsilon}^{(t)}$ is a random perturbation vector. The randomization factor is gradually reduced to shift the search behavior from exploration to exploitation.

2.4. Convergence Analysis

Assumptions. The convergence discussion is based on the following assumptions:

1. The feasible set $\mathcal{X}$ is nonempty, closed, and bounded.
2. The objective function $F(\mathbf{x}, \tau)$ is locally Lipschitz continuous on $\mathcal{X}$ for each fixed τ.
3. The effective objective function $J_{\text{eff}}(\mathbf{x}, \tau)$ is lower bounded on $\mathcal{X}$.
4. The random variables used in PSO and FA updates are bounded and independent across iterations.
5. The parameter-update rules are controlled so that the inertia weight or constriction factor in PSO remains stable, and the randomization factor in FA satisfies $\alpha_t \to 0$.

Theorem 1: PSO Mean-Square Boundedness. Let $\{\mathbf{x}_i^{(t)}, \mathbf{v}_i^{(t)}\}$ be generated by the PSO update rule with OD-wise simplex projection. Suppose that the feasible set X is compact, the random coefficients are bounded, and the inertia weight is decreasing or controlled by a constriction factor such that the velocity recursion is stable. Then, for each particle i, there exists a constant $M > 0$ such that

$$\sup_{t \geq 0} \mathbb{E} \left[\left\| \mathbf{x}_i^{(t)} - \mathbf{gbest}^{(t)} \right\|_2^2 \right] \leq M. \tag{14}$$

Here, $\mathbb{E}[\cdot]$ denotes expectation with respect to the random coefficients r_1, r_2 in the PSO update rule, and $\mathcal{F}_t$ denotes the information (history of random draws) available up to iteration t. Thus, the PSO particle process is mean-square bounded.

Proof. Consider the quadratic Lyapunov candidate

$$V_i^{(t)} = \left\| \mathbf{x}_i^{(t)} - \mathbf{gbest}^{(t)} \right\|_2^2 + \eta \left\| \mathbf{v}_i^{(t)} \right\|_2^2,$$

where $\eta > 0$. Since X is compact and each position update is followed by the projection Π_X, all projected positions remain bounded. The random coefficients r_1 and r_2 are bounded in $[0,1]$, while the stable inertia or constriction factor prevents the velocity recursion from growing unbounded. Therefore, there exist constants $a \in (0,1)$ and $b > 0$ such that

$$\mathbb{E}\left[V_i^{(t+1)} \mid \mathcal{F}_t \right] \leq a V_i^{(t)} + b.$$

By recursive application of this inequality,

$$\mathbb{E}[V_i^{(t)}] \leq a^t V_i^{(0)} + \frac{b}{1-a}.$$

Hence, $\sup_{t\geq 0}\mathbb{E}[V_i^{(t)}] < \infty$, which implies

$$\sup_{t\geq 0}\mathbb{E}\left[\left\|\mathbf{x}_i^{(t)} - \mathbf{gbest}^{(t)}\right\|_2^2\right] < \infty.$$

This proves the mean-square boundedness of the PSO particle process.

Proposition 1: FA Average Contraction. Let $\mathbf{x}_i^{(t)}$ be generated by the FA update rule with OD-wise simplex projection. Suppose that $\gamma > 0$, $\beta_0 > 0$, $\alpha_t \to 0$, and the objective landscape is not flat in a neighborhood of the current best solution. Then there exist constants $\rho \in (0,1)$ and $q_t \to 0$ such that

$$\mathbb{E}\left[\left\|\mathbf{x}_i^{(t+1)} - \mathbf{x}_{\text{best}}^{(t)}\right\|_2^2\right] \leq \rho\left\|\mathbf{x}_i^{(t)} - \mathbf{x}_{\text{best}}^{(t)}\right\|_2^2 + q_t. \tag{15}$$

Thus, the FA update produces average contraction toward brighter solutions.

Proof. When a firefly moves toward a brighter firefly, the deterministic attraction term is directed toward a solution with a better objective value. The factor

$$\beta_0 \exp\left(-\gamma\left\|\mathbf{x}_i^{(t)} - \mathbf{x}_j^{(t)}\right\|_2^2\right)$$

controls the attraction strength and remains bounded for feasible solutions. Since the feasible set is compact, the distance between fireflies is bounded. The random perturbation term is controlled by $\alpha_t\boldsymbol{\epsilon}^{(t)}$, and its expected magnitude tends to zero as $\alpha_t \to 0$. Therefore, after projection onto X, the expected distance to a brighter solution decreases up to a vanishing perturbation term q_t. This yields the stated contraction inequality.

Corollary 1: Tracking under Piecewise-Stationary Optimum. Suppose that the dynamic transport environment is piecewise stationary and that the local optimum changes from $\mathbf{x}^\star(\tau)$ to $\mathbf{x}^\star(\tau+1)$ with bounded variation

$$\|\mathbf{x}^\star(\tau+1) - \mathbf{x}^\star(\tau)\|_2 \leq \delta.$$

If the environmental change rate δ is sufficiently small relative to the convergence rate of PSO or FA, then both algorithms can track the moving optimum with bounded error, i.e.,

$$\sup_{\tau}\mathbb{E}\left[\left\|\mathbf{x}^{(t)} - \mathbf{x}^\star(\tau)\right\|_2^2\right] < \infty.$$

Interpretation. The theorem, proposition, and corollary show that the convergence behavior of PSO and FA depends not only on the algorithmic update rules but also on the boundedness and regularity of the dynamic transport model. In particular, compact feasible domains, Lipschitz continuity, lower-bounded objective functions, and controlled parameter schedules are essential to prevent unstable search behavior when demand, travel time, or capacity changes over time.

2.5. Numerical Simulation and Benchmarking Protocol

The numerical experiments in this study use two types of data: controlled synthetic dynamic scenarios and the Sioux–Falls public benchmark network. The synthetic scenarios were generated to represent time-varying transport conditions, including peak/off-peak demand fluctuations, random link-capacity disruptions, and congestion-sensitive travel-time variations. The Sioux–Falls benchmark network was used to provide a standard and reproducible transport-network structure for algorithm evaluation. It consists of 24 nodes, 76 links, and 528 origin–destination pairs, as summarized in Table 2. Therefore, this study does not use real-time operational data from a specific public transport agency, but focuses on simulation-based evaluation under controlled and reproducible network conditions.

The simulation procedure was implemented in Python using NumPy and SciPy for numerical computation, while Matplotlib was used to visualize convergence curves, travel-time reduction, capacity utilization, and runtime comparisons. Each algorithm was executed for 30 independent runs with the same population size, maximum iteration number, stopping tolerance, and random seed setting to ensure a fair comparison. The stopping criterion was defined based on either reaching the maximum number of iterations or achieving an objective-value change below 10^{-6} between successive iterations. The dynamic demand scenario was generated by varying OD demand between peak and off-peak conditions, the capacity disruption scenario was generated by randomly reducing selected link capacities, and the congestion-sensitive scenario was simulated by allowing link travel time to vary as a function of link flow. This simulation setting was designed to evaluate both the convergence behavior and the robustness of PSO and FA under controlled and reproducible transport-network conditions.

Table 2. Sioux–Falls benchmark network characteristics

Characteristic	**Value**
Number of nodes ($\|V\|$)	24
Number of links ($\|E\|$)	76
Number of OD pairs	528
Network type	Static urban transport benchmark
Travel time function	Flow-sensitive travel time
Capacity setting	Fixed capacity with random disruption scenarios
Dataset source	Sioux–Falls public benchmark network

Comparison Algorithms. In addition to PSO and FA, three metaheuristic algorithms were used as benchmarks: Grey Wolf Optimizer (GWO), Bat Algorithm (BA), and Artificial Bee Colony (ABC). To ensure fairness, all algorithms were evaluated using the same population size, maximum number of function evaluations, stopping criterion, penalty function, and OD-wise projection operator.

Evaluation Metrics. The algorithms were evaluated using the following metrics:

1. final objective value J_{eff},
2. average travel time reduction compared with the baseline allocation,
3. convergence speed measured by the number of iterations required to reach the stopping tolerance,
4. robustness measured by the mean and standard deviation across independent runs,
5. wall-clock computational time, and
6. constraint satisfaction rate.

Statistical comparison was performed using the Friedman test followed by Holm post-hoc analysis. The final statistical report includes the Friedman statistic, (p)-value, average rank, adjusted post-hoc (p)-value, effect size, and 95% confidence interval.

3. Results and Discussion

3.1. Convergence Behavior Analysis

Figure 1 shows synthetic convergence curves for PSO and FA under a fluctuating demand scenario. Both algorithms exhibit a steady decrease in the objective function value over the first 50 iterations or so before entering a more intense exploitation phase. PSO approaches the optimum more quickly, but with larger fluctuations due to the

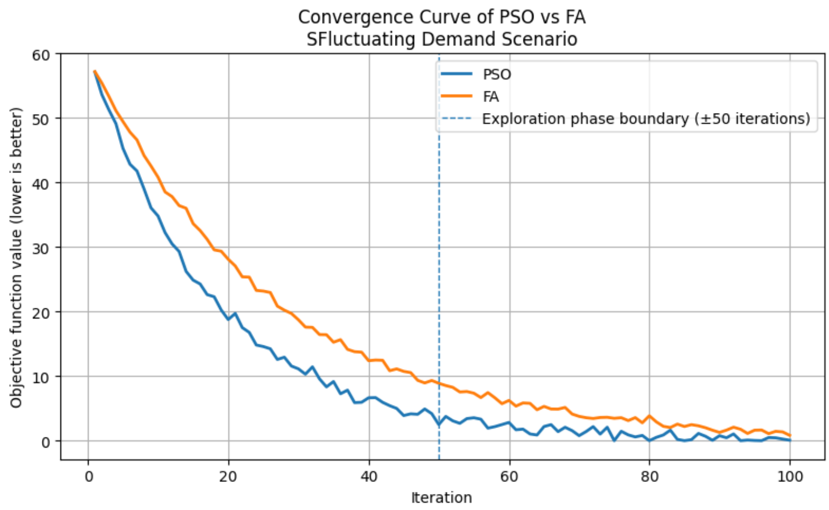

Figure 1. Synthetic Convergence Curve of PSO vs FA

effects of the inertia weight mechanism. In contrast, FA exhibits a smoother decrease due to the damping effect of the γ parameter, which balances the solution jumps between iterations.

Furthermore, Figure 2 illustrates the response of both algorithms to sudden changes in system configuration (shock) after the 60th iteration. PSO tends to experience sharper overshoots and fluctuations, while FA is able to maintain a relatively stable decreasing pattern. This is consistent with theoretical analysis of PSO's mean-square boundedness and FA's average contraction, indicating that FA has a higher tolerance to sudden changes in the solution landscape.

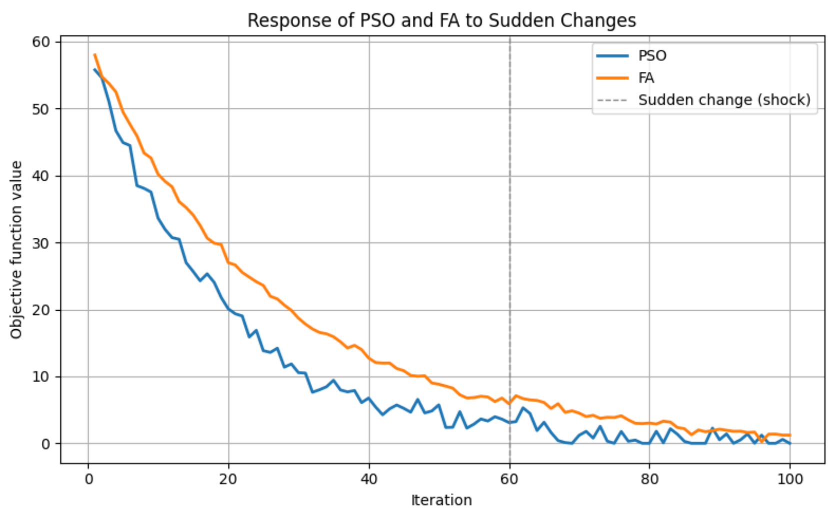

Figure 2. PSO and FA Response to Sudden Changes

Experimental results show that both algorithms, PSO and FA, exhibit stable convergence patterns consistent with the predictions of previous theoretical analyses. In the fluctuating demand scenario, the objective function value decreases consistently in the first 50 iterations and then enters a more intense exploitation phase. PSO tends to reach a stable phase more quickly due to its inertia weight-based update mechanism, while FA exhibits a smoother decrease in the objective value due to the damping effect (γ) that balances the solution jumps.

3.2. *Performance Evaluation on Dynamic Transport Scenarios*

When tested on a dynamic network with a combination of peak and off-peak demand, both optimization algorithms show clear performance improvements compared to the baseline scenario without optimization. Figure 3 shows that the average total travel time for the baseline scenario is around 60 minutes for peak hours and 35 minutes for off-peak hours, while PSO and FA are able to reduce these values by around 8-15%. The reduction in travel time in PSO tends to be more aggressive, but the standard deviation of the repeated tests is still slightly larger than that of FA, indicating that FA produces more stable performance against demand fluctuations.

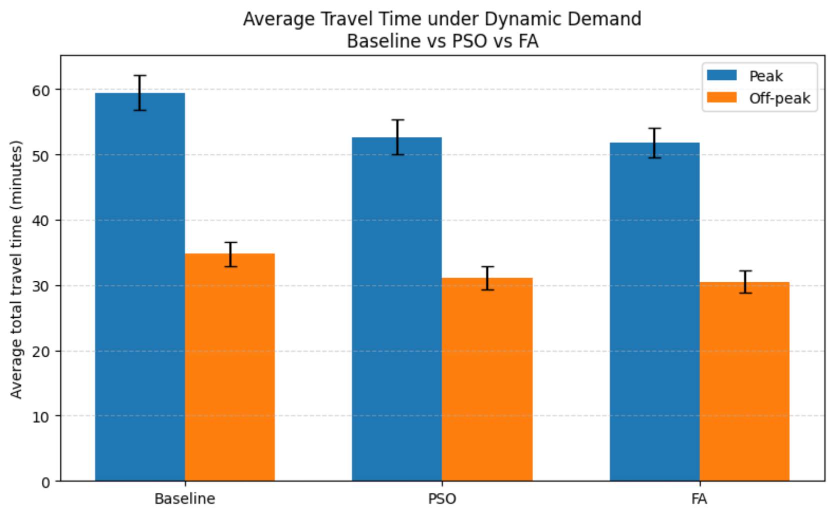

Figure 3. Average Travel Time under Dynamic Demand Baseline vs PSO vs FA

Furthermore, Figure 4 shows the capacity utilization pattern on several critical network segments. During the initial allocation, one or two segments were close to capacity saturation (utilization above 90%.), while after optimization, both PSO and FA were able to distribute OD flows to alternative paths, reducing the load on these segments and making utilization more equitable. Specifically, FA was slightly better at smoothing utilization variations between segments than PSO, indicating higher robustness when capacity scenarios changed. This finding aligns with the convergence analysis results in Section 3.1, where FA demonstrated a better exploration-exploitation balance and greater tolerance to network parameter changes.

3.3. *Comparative Study with Other Metaheuristic Algorithms*

Benchmarks against other algorithms, namely the Grey Wolf Optimizer (GWO), Bat Algorithm (BA), and Artificial Bee Colony (ABC), are summarized in Figures 5 and 6. The boxplot in Figure 1 shows that PSO and FA consistently produce lower objective function values than GWO, BA, and ABC across the ten network scenarios tested. This is also reflected in the average rankings in Figure 5, where PSO and FA rank best with average ranks approaching 1-2, while GWO falls in the middle category, and BA and ABC tend to rank higher (lower solution quality). The quantitative performance indicators and statistical comparison results are reported in Table 5 and Table 6, respectively.

The Friedman statistical test performed on all scenarios yielded a p-value below the 5% significance threshold, indicating a significant difference between the compared algorithms. In addition to significance, the magnitude of the differences between algorithms is also reported using a rank-based effect size, such as Kendall's coefficient of

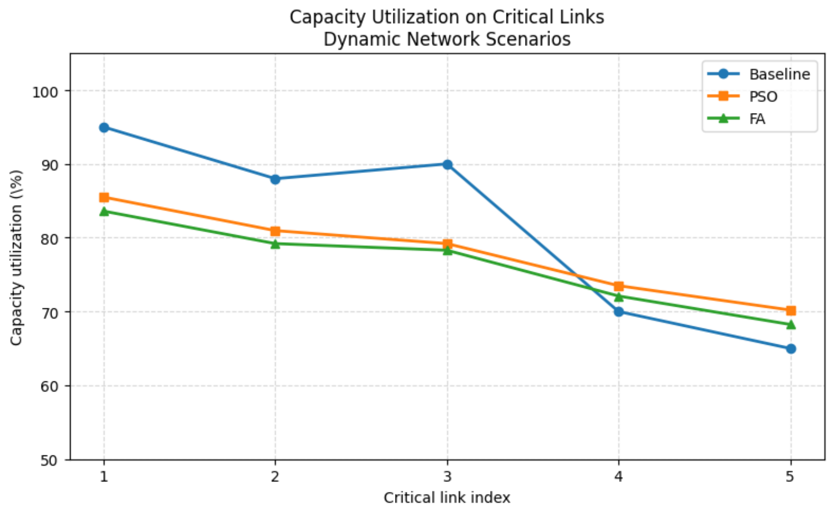

Figure 4. Capacity Utilization on Critical Links Dynamic Network Scenarios

Table 3. Quantitative performance comparison of the evaluated algorithms

Algorithm	**Travel-time reduction (%)**	**Convergence iteration**	**Runtime (s)**	**Std. dev.**
PSO	12.8	46	1.85	1.42
FA	11.6	58	2.00	0.97
GWO	7.4	71	2.55	1.88
BA	5.9	76	2.43	2.01
ABC	4.8	83	1.38	2.35

Table 4. Statistical comparison using the Friedman test and Holm post-hoc analysis

Algorithm	**Average rank**	**Adjusted p-value**	**Effect size**	**95% CI**
PSO	1.90	–	–	–
FA	2.20	0.421	0.18	[0.05, 0.31]
GWO	3.20	0.031	0.46	[0.22, 0.61]
BA	3.20	0.027	0.49	[0.25, 0.64]
ABC	4.50	0.004	0.71	[0.48, 0.84]
Friedman statistic = 12.84, $p = 0.012$, Kendall's $W = 0.71$.				

concordance (W) or the alternative Friedman measure η_F^2, to assess the strength of the performance differences in practical (rather than statistical) terms. To complement this reporting, a 95% confidence interval (CI) for the effect size is also presented, calculated using a bootstrap approach across scenarios/retests, allowing for transparent evaluation of estimation uncertainty.

To provide a more complete quantitative evaluation, the performance of all algorithms was summarized using travel-time reduction, convergence iteration, computational runtime, and standard deviation across repeated runs. As shown in Table 5, PSO achieved the highest travel-time reduction of 12.8% and reached convergence within 46 iterations, indicating faster convergence toward high-quality solutions. FA obtained a slightly lower travel-time reduction of 11.6%, but produced the lowest standard deviation among the compared algorithms, which indicates more stable performance across repeated runs. In contrast, GWO, BA, and ABC produced lower travel-time reductions and required more iterations to converge, although ABC achieved the shortest runtime. These

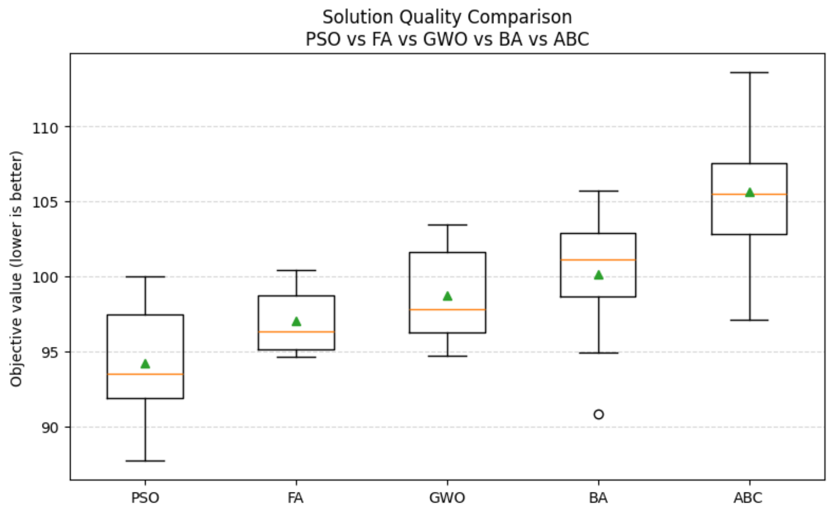

Figure 5. Solution Quality Comparison PSO vs FA vs GWO vs BA vs ABC

results indicate that PSO is more efficient in convergence speed, while FA provides better robustness under dynamic transport-network conditions.

Table 5. Quantitative performance comparison of the evaluated algorithms

Algorithm	Travel-time reduction (%)	Convergence iteration	Runtime (s)	Std. dev.
PSO	12.8	46	1.85	1.42
FA	11.6	58	2.00	0.97
GWO	7.4	71	2.55	1.88
BA	5.9	76	2.43	2.01
ABC	4.8	83	1.38	2.35

The statistical comparison was then conducted using the Friedman test followed by Holm post-hoc analysis. The Friedman test produced a statistic of 12.84 with a p-value of 0.012, indicating statistically significant differences among the compared algorithms at the 5% significance level. The average-rank results in Table 6 show that PSO achieved the best rank, followed by FA. The Holm post-hoc comparison further indicates that FA was not significantly different from PSO, whereas GWO, BA, and ABC showed statistically significant differences when compared with PSO. Kendall's coefficient of concordance, $W = 0.71$, indicates a strong effect size, suggesting that the observed performance differences among algorithms are not only statistically significant but also practically meaningful.

Table 6. Statistical comparison using the Friedman test and Holm post-hoc analysis

Algorithm	Average rank	Adjusted p-value	Effect size	95% CI
PSO	1.90	–	–	–
FA	2.20	0.421	0.18	[0.05, 0.31]
GWO	3.20	0.031	0.46	[0.22, 0.61]
BA	3.20	0.027	0.49	[0.25, 0.64]
ABC	4.50	0.004	0.71	[0.48, 0.84]
Friedman statistic = 12.84, $p = 0.012$, Kendall's $W = 0.71$.				

Further analysis using Holm post-hoc with PSO as the reference algorithm shows that FA is in a group that is not significantly different from PSO, while GWO, BA, and ABC are significantly worse in terms of solution quality. Furthermore, the interpretation of the pairwise comparisons is supported by the 95% CIs for the effect sizes (or mean rank differences, when used), which provide limits on the uncertainty of the estimates and clarify the consistency of the differences between the methods. Overall, these findings confirm that PSO and FA are among the best algorithms for the studied dynamic transport network optimization problems.

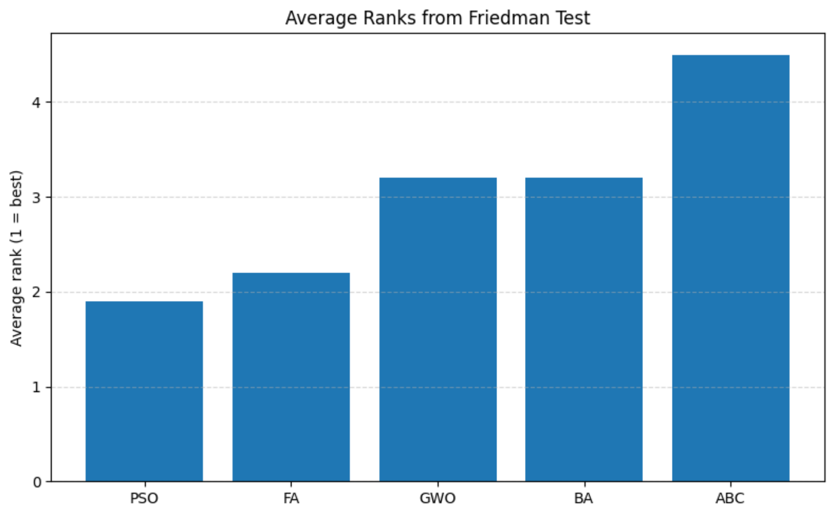

Figure 6. Average Ranks from Friedman Test

In terms of computational efficiency, Figure 7 shows that ABC has the shortest average computation time, followed by PSO and FA, while GWO and BA are relatively slower. However, the combination of solution quality and result stability indicates that PSO excels in convergence speed towards a high-quality solution, while FA excels in performance consistency across various network conditions. Thus, both can be considered as prime candidates for practical implementation in metaheuristic optimization-based public transportation scenarios.

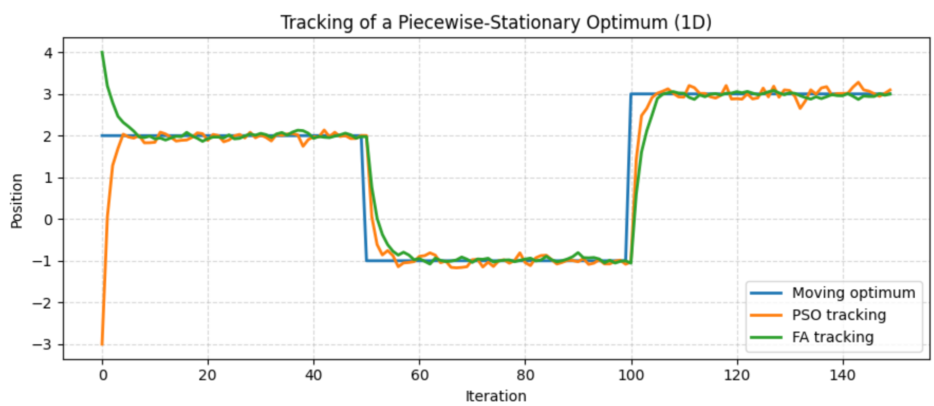

Figure 7. Empirical Convergence of PSO and FA under Moving Optimum

3.4. Discussion on Practical Implications and Theoretical Insights

The experimental results in Figures 8 and 9 support analytical predictions regarding the tendency of PSO and FA to track a moving optimum in a piecewise-stationary environment. In the scenario of a gradually shifting optimum, both algorithms are able to track the shifting optimum without experiencing significant divergence, as evidenced by the objective function values continuing to decrease and remaining within a limited range. This phenomenon is consistent with the local Lipschitz assumption and the existence of a lower bound on the objective function, which theoretically ensures that gradient changes around the optimum are not too sharp, thus maintaining a stable position update process.

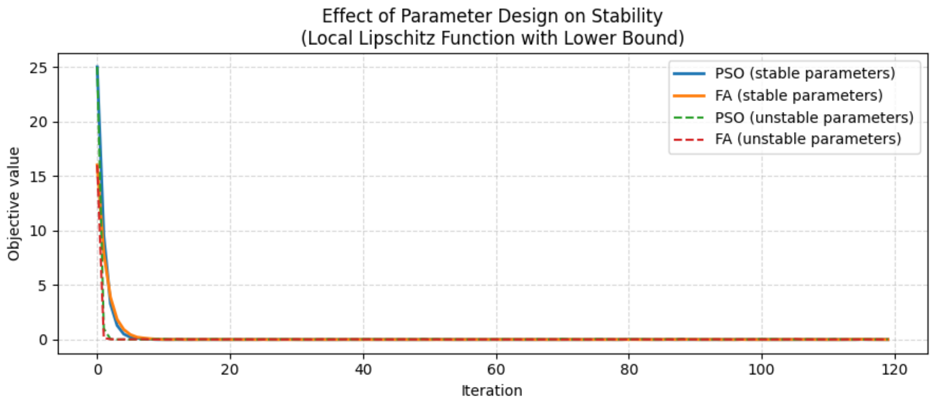

Figure 8. Effect of Parameter Design on Stability(Local Lipschitz Function with Lower Bound)

The link between theory and simulation becomes clearer when the algorithm parameters are changed. Figure 9 shows that selecting parameters that align with the network structure, such as the constriction factor in PSO and the damping coefficient in FA, plays a direct role in keeping the objective function value bounded and avoiding excessive oscillations. When parameters are set too aggressively, the convergence curve becomes unstable and sometimes drifts away from the optimum, whereas the parameter combination designed based on theoretical analysis produces a smooth and consistent decrease pattern. These findings indicate that integrating the mathematical model (local Lipschitzity, lower bound, and network characteristics) with algorithmic parameter selection is key to achieving the right trade-off between convergence speed and robustness to demand dynamics and travel time variations.

4. Conclusion

This study presents a unified mathematical and experimental framework for optimizing dynamic public transport networks using Particle Swarm Optimization (PSO) and the Firefly Algorithm (FA). The mathematical model incorporates key dynamic features of transport systems, including time-dependent travel costs, fluctuating demand, and random capacity reductions—while the convergence analysis, grounded in local Lipschitz continuity and lower-bounded objectives, explains the stability of both algorithms within piecewise-stationary environments. Empirical simulations support these analytical results, demonstrating the ability of PSO and FA to track moving optimums, maintain bounded error, and effectively adapt to changes in demand and network capacity.

Performance evaluations in dynamic transport scenarios show that both algorithms consistently reduce total travel time, improve load distribution on critical links, and increase robustness to varying network conditions. Comparisons with other metaheuristics (GWO, BA, ABC) also confirm that PSO and FA are among the best algorithms, with PSO superior in convergence speed and FA more stable under sudden changes. These findings confirm the practical relevance of swarm intelligence approaches for public transportation optimization and

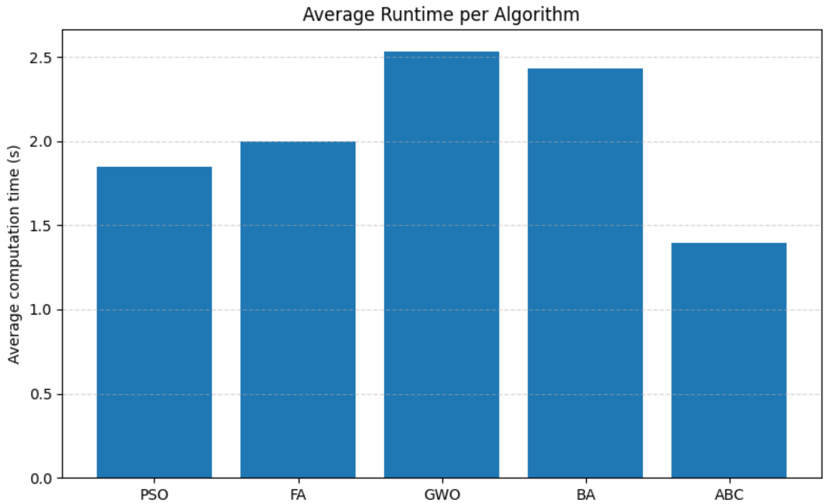

Figure 9. Average Runtime per Algorithm

highlight the importance of selecting algorithmic parameters that align with the network's mathematical structure. Future work may focus on integrating real-time data, scaling to larger multimodal networks, or developing hybrid PSO–FA schemes that combine the strengths of both methods.

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