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New Fixed point Results for Interpolative Gamma-k Contractions

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Abstract The main aim of this paper is to introduce several new and innovative fixed point results. We use the concept of gamma-k distance mappings together with interpolative contractions and the H-simulation function to develop important new types of contractions. In addition, to highlight the practical relevance of our work, we present several numerical examples to demonstrate the application of our findings.

Keywords Gamma-k Distance Mapping, Interpolative Contraction, Fixed Point Theory

AMS 2010 subject classifications Primary 47H10; Secondary 54H25

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1. Introduction

Due to the fundamental theorem of Banach [7] being later named as Banach Contraction Principle (BCP), the importance of fixed point theory cannot be underestimated within the realm of analysis and applied mathematics. As a consequence, many authors contributed to extending the BCP in numerous aspects. Such extensions include but not limited to metric spaces endowed with additional structures, like G-metric spaces [5, 8], quasi-metric spaces [18, 1], b-metric spaces [29], extended quasi b-metric spaces [35, 42], complex-valued b-metric spaces [4], and modular spaces [20]. Moreover, many new types of contractions have been considered, namely Geraghty-type contractions [9], (H, Ω_b)-interpolative contractions [36], and weakly Chatterjea-type cyclic contractions [21]. Additionally, the development of ω-distance mappings [37]. Moreover, many new types of contractions have been considered, namely the Gauss–Seidel fixed-point approach for maximum likelihood estimation in Epanechnikov–Burr XII distributions [34] and the fixed point maximum likelihood estimation for the Epanechnikov-Pareto distribution [19].

For more development on fixed point theory, we refer the reader to [10, 11, 12, 22, 23, 24, 25, 26, 27, 43, 44, 45].

The study of generalized distance spaces was taken in account by many authors, for instance, Jleli and Samet, [28] introduced a new generalization for many distance settings as metric, b-metric spaces, and studied some fixed point results. Abodayeh et al. [2] introduced generalized Ω-distance mappings and studied some fixed point theorems, and subsequently Abu-Irwaq et al. proved some fixed point theorems in generalized Ω-distance mappings [3]. More work on generalized distance spaces can be also found in [13, 14]

The idea of gamma-k distance mapping was formulated by Bataihah et al. [15] as an extended idea of gamma distance mapping [16], recently, several authors extended the use of this idea to present several fixed point theorems; see, for example [38].

The concept of gamma-k distance was articulated in the following manner.

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Definition 1.1. [15] Let d be a metric on $\mathcal{V}$. A function $\gamma : [0,\infty) \times \mathcal{V} \times \mathcal{V} \to [0,\infty)$ is said to be $\gamma(k)$-distance over $(\mathcal{V}, d)$ if γ satisfying:

(γ_1) For all $t \geq 0$, $\gamma(t, v_1, v_2) \geq t^k d(v_1, v_2)$, $k \geq 1$.
(γ_2) for each sequences $(v_n), (w_n)$ in $\mathcal{V}$ and (t_n) in $(0,\infty)$, we have $d(v_n, w_n) \to \xi > 0$ and $t_n \to t > 0$ imply $\liminf\limits_{n\to\infty} \gamma(t_n, v_n, w_n) > \xi t^k$.

Throughout the paper, the parameter $k \geq 1$ is fixed, and the notation $\gamma(k)$ indicates that the function γ which satisfies conditions γ_1 and γ_2 with this exponent.

In the following examples, the functions $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$ are established as mappings from the domain $[0,+\infty) \times \mathcal{V} \times \mathcal{V}$ to the co-domain $[0,+\infty)$.

Example 1.2. [15] On $(\mathcal{V}, d)$, let $\phi : [0,\infty) \to [0,\infty)$ is continuous function with $\phi(a) > 0$ for $a > 0$. Then, $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ and γ_5 are γ-distance over $(\mathcal{V}, d)$.

1. $\gamma_1(t, v_1, v_2) = t^k\, d(v_1, v_2) + \phi(d(v_1, v_2))$,
2. $\gamma_2(t, v_1, v_2) = t^k\, d(v_1, v_2) + \phi(t)$,
3. $\gamma_3(t, v_1, v_2) = e^{tk} d(v_1, v_2)$,
4. $\gamma_4(t, v_1, v_2) = \dfrac{(t+\varepsilon)^{2k} + [d(v_1, v_2)]^2}{2}$, $\varepsilon > 0$,
5. $\gamma_6(t, v_1, v_2) = \dfrac{t^{2k}+1}{2}\, d(v_1, v_2)$,

We see that Bataihah et al. [16, 15] introduced the basics of γ-distance and $\gamma(k)$-distance mappings, along with standard contraction conditions and their uses in fractional boundary differential equations. Our work takes this in two important directions. First, we came up with weak product-type $\gamma(k)$-contractions (defined in 2.1), which use a product of distance terms with exponents α and $2-\alpha$, where α is between 1 and 2. This wasn't looked at before. Second, we created M-$\gamma(k)$-contractions (defined in 4.1), which involve a max-term structure using $d(x, y)$, $d(x, fx)$, and $d(y, fy)$. These new types give us unique fixed-point theorems that build on the earlier work.

The idea of $\mathcal{H}$-simulation function was formulated by Bataihah et al. [17], and they utilized this idea to prove innovative fixed points results. Interpolative mappings generalize classical contractions by taking into account the product of distances raised to powers that are not necessarily equal to one. Interpolative Kannan contractions were proposed for the first time by Karapinar [31]. The concept was generalized further by Aydi et al. [6] to ω-interpolative Ćirić–Reich–Rus contractions, by Karapinar and Fulga [32] to b-metric spaces, and Karapinar, Alqahtani, and Aydi [33] to Hardy–Rogers type contractions. These works show interpolative conditions unify and generalize many known fixed-point results. On the other hand, Several pioneers extended this idea to presented many fixed points results for example [39, 40, 41]

Definition 1.3. [17] A function $\mathcal{H} : [1,+\infty) \times [1,+\infty) \to \mathbb{R}$ is called $\mathcal{H}$-simulation function if $\mathcal{H}(v_1, v_2) \leq \dfrac{v_2}{v_1}\ \forall\, v_1, v_2 \in [1,+\infty)$.

Remark 1.4. [17] Let $\mathcal{H} \in \mathcal{H}$. If (v_n) and (v'_n) are sequences in $[1,+\infty)$ with $1 \leq \lim\limits_{n\to\infty} v'_n < \lim\limits_{n\to\infty} v_n$, then $\limsup\limits_{n\to\infty} \mathcal{H}(v_n, v'_n) < 1$.

Following this, we will present several examples of $\mathcal{H}$-simulation functions.

Example 1.5. [17] Examples of $\mathcal{H}$-simulation functions are given below:

1. $\mathcal{H}_1(v_1, v_2) = \frac{k\, v_2^{\,r}}{v_1}, \qquad k, r \in (0,1]$,
2. $\mathcal{H}_2(v_1, v_2) = \frac{\min\{v_1,v_2\}}{\max\{v_1,v_2\}}$.
3. $\mathcal{H}_3(v_1, v_2) = \frac{v_2}{v_1+\ln\left(\frac{v_2}{v_1}\right)}$.
4. $\mathcal{H}_4(v_1, v_2) = \frac{v_2}{v_1+\sqrt{v_2}}$.

5. $\mathcal{H}_5(v_1, v_2) = \frac{v_2}{1+v_1 v_2}$.
6. Let $f_1, f_2 : (0, +\infty) \to (0, +\infty)$ be continuous functions such that:
   $f_1(r) < r$ and $f_2(r) \geq r$, for each $r \in (0, +\infty)$. Define $\mathcal{H}_6(v_1, v_2) = \frac{f_1(v_2)}{f_2(v_1)}$.

In the subsequent of this manuscript, we will refer to $\mathcal{V}$ as a non-empty set, and $(\mathcal{V}, d)$ will denote a metric space and F_f refers to the set of all fixed points.

2. Fixed point results for Weak Interpolative $\gamma(k)$-Contraction

Definition 2.1. Suppose there is $\gamma(k)$-distance γ over $(\mathcal{V}, d)$. A mapping $f : \mathcal{V} \to \mathcal{V}$ is called a weak interpolative $\gamma(k)$-contraction if there exist $\alpha \in [1, 2)$ such that for all $v_1, v_2 \in \mathcal{V}$ we have:

$$v_1 \neq v_2 \Longrightarrow \gamma(d(fv_1, fv_2), fv_1, fv_2) \leq [d(v_1, fv_1)^{(\frac{k+1}{2})\alpha}.d(v_2, fv_2)^{\frac{k+1}{2}(2-\alpha)}], \ k \geq 1. \tag{2.1}$$

Remark 2.2. If $f : \mathcal{V} \to \mathcal{V}$ is weak interpolative $\gamma(k)$-contraction, then $\forall\, v_1, v_2 \in \mathcal{V}$, we have:

$$d(fv_1, fv_2)^2 \leq [d(v_1, fv_1)^{\alpha}.d(v_2, fv_2)^{2-\alpha}].$$

Proof
Using the definition of $\gamma(k)$ with weak interpolative $\gamma(k)$-contraction, we get the following:

$$d^{k+1}(fv_1, fv_2) \leq \gamma(d(fv_1, fv_2), fv_1, fv_2) \leq [d^{\alpha}(v_1, fv_1).d^{2-\alpha}(v_2, fv_2)]^{\frac{k+1}{2}}.$$

By taking the power $\frac{2}{k+1}$ to both sides we get

$$d^2(fv_1, fv_2) \leq \gamma(d(fv_1, fv_2), fv_1, fv_2) \leq [d^{\alpha}(v_1, fv_1).d^{2-\alpha}(v_2, fv_2)].$$

Hence, the desired result. □

Lemma 2.3
Suppose that the self mapping f on $\mathcal{V}$ is a weak interpolative $\gamma(k)$-contraction, for any $t_1, t_2 \in F_f$, then $t_1 = t_2$.

Proof
Assume that there are $t_1, t_2 \in F_f$ such that $t_1 \neq t_2$, by using remark 2.2 we get the following:

$$d^2(t_1, t_2) = d^2(ft_1, ft_2) \leq [d^{\alpha}(t_1, t_1).d^{2-\alpha}(t_2, t_2)] = 0.$$

Consequently, we get that $t_1 = t_2$. □

Lemma 2.4
Suppose that f is a weak interpolative $\gamma(k)$-contraction, and $v_0 \in \mathcal{V}$. Then for the Picard sequence $(v_n) \in \mathcal{V}$ generated by the self mapping f at v_0, for all $v_n \neq v_{n+1}$ and for each $n \in \mathbb{N}$, we have

$$\lim_{n\to\infty} d(v_n, v_{n+1}) = 0.$$

Proof
Since $v_n \neq v_{n+1}$, from remark 2.2, we have

$$d(v_n, v_{n+1}) \leq d(v_{n-1}, v_n).$$

Therefore, $(d(v_n, v_{n+1}) : \ n \in \mathbb{N})$ is a non-increasing sequence in $(0, \infty)$. So, there is $\beta \geq 0$ such that $\lim_{n\to\infty} d(v_n, v_{n+1}) = \beta$. Suppose to the contrary; that is $\beta > 0$. From Condition 2.1, we get

$$\gamma(d(v_{n-1}, v_n), v_n, v_{n+1}) \leq d(v_{n-1}, v_n)^{\frac{k+1}{2}\alpha} d(v_n, v_{n+1})^{\frac{k+1}{2}(2-\alpha)}.$$

Taking the limit inferior as $n \to \infty$, we get

$$\liminf_{n\to\infty} \gamma(d(v_{n-1}, v_n), v_n, v_{n+1}) \le \beta^{\frac{k+1}{2}\alpha}\beta^{\frac{k+1}{2}(2-\alpha)} = \beta^{k+1}.$$

Let $= d(v_{n-1}, v_n) = s_n$. Then $s_n \to \beta$. Therefore, by γ_2 of Definition 1.1, we have:

$$\beta^{k+1} < \liminf_{n\to\infty} \gamma(d(v_{n-1}, v_n), v_n, v_{n+1}) \le \beta^{k+1},$$

a contradiction. Hence, we get the desired the result. □

Theorem 2.5
Suppose that $(\mathcal{V}, d)$ is complete and there is $\gamma(k)$-distance γ on $(\mathcal{V}, d)$. Assume that $f : \mathcal{V} \to \mathcal{V}$ is weak interpolative $\gamma(k)$-contraction. Then F_f has only one element.

Proof
By choosing an arbitrary element $v_0 \in \mathcal{V}$, we construct the Picard sequence (v_n) generated by f at v_0. If there is $s \in \mathbb{N}$ such that $v_s = v_{s+1}$, then $v_s \in F_f$. So we assume that for each $n \in \mathbb{N}$, $v_n \neq v_{n+1}$. Now, suppose to the contrary; that (v_n) is not Cauchy. Therefore, depending on Lemma 2.4 there is $\epsilon > 0$ and two sub-sequences (v_{n_k}) and (v_{m_k}) of (v_n) such that

$$\lim_{k\to\infty} d(v_{n_k}, v_{m_k}) = \epsilon. \tag{2.2}$$

and

$$\lim_{k\to\infty} d(v_{n_k-1}, v_{m_k-1}) = \epsilon. \tag{2.3}$$

Now, set $t_k = d(v_{n_k-1}, v_{m_k-1}),\ a_k = v_{n_k}$ and $b_k = v_{m_k}$.
Then, $\lim_{k\to\infty} t_k = \lim_{k\to\infty} d(a_k, b_k) = \epsilon > 0$. So, using (γ_2) and condition 2.1, we get

$$\epsilon^{k+1} < \liminf_{k\to\infty} \gamma(t_k, a_k, b_k) \le \epsilon^{k+1},$$

a contradiction. Consequently, (v_n) is Cauchy, so there is $t_1 \in \mathcal{V}$ such that (v_n) converges to t_1. Now, by Remark 2.2, we have

$$d^2(ft_1, fv_n) \le [d(t_1, ft_1)^\alpha . d(v_n, fv_n)^{2-\alpha}].$$

Now, by letting $n \to +\infty$, we get $v_n \to ft_1$, and so by the uniqueness of limit, we get $t_1 = ft_1$.
Lemma 2.3 ensures the uniqueness. □

3. M-$\gamma(k)$-contraction

Definition 3.1. Suppose there is $\gamma(k)$-distance γ over $(\mathcal{V}, d)$. A self mapping $f : \mathcal{V} \to \mathcal{V}$ is said to be $M - \gamma(k)$-contraction if there exist $\alpha \in [1, 2)$ such that for all $v_, v_2 \in \mathcal{V}$ we have:

$$\gamma(d(fv_1, fv_2), fv_1, fv_2) < \max\{d^{k+1}(v_1, v_2), [d^\alpha(v_1, fv_1).d^{2-\alpha}(v_2, fv_2)]^{(\frac{k+1}{2})}\},\ k \ge 1. \tag{3.1}$$

Remark 3.2. If $f : \mathcal{V} \to \mathcal{V}$ is $M - \gamma(k)$ contraction, then $\forall\ v_1, v_2 \in \mathcal{V}$, we have:

$$d(fv_1, fv_2)^2 < \max\{d^2(v_1, v_2), [d(v_1, fv_1)^\alpha.d(v_2, fv_2)^{2-\alpha}].$$

Proof
Applying Condition 3.1, we obtain the following:

$$d^{k+1}(fx, fy) = \gamma(d(fv_1, fv_2), fv_1, fv_2) < \max\{d^{k+1}(v_, v_2), [d^\alpha(v_1, fv_1).d^{2-\alpha}(v_2, fv_2)]^{(\frac{k+1}{2}}\}.$$

Hence the desired result. □

Lemma 3.3
Suppose that the self mapping f on $\mathcal{V}$ is a $M-\gamma(k)$ contraction, for any $t_1, t_2 \in F_f$, then $t_1 = t_2$.

Proof
Employing remark 3.2, we obtain the following:

$$d^2(t_1,t_2) = d^2(ft_1, ft_2) < \max\{d^2(t_1,t_2), [d^\alpha(t_1,t_1).d^{2-\alpha}(t_2,t_2)].$$

Consequently, we get that $t_1 = t_2$. □

Lemma 3.4
Suppose that f is a $M-\gamma(k)$ contraction, and $v_0 \in \mathcal{V}$. Then for the Picard sequence $(v_n) \in \mathcal{V}$ generated by the self mapping f at v_0, for all $v_n \neq v_{n+1}$ and for each $n \in \mathbb{N}$, we have

$$\lim_{n\to\infty} d(v_n, v_{n+1}) = 0.$$

Proof
Since $v_n \neq v_{n+1}$, from remark 3.2, we obtain the following:

$$d^2(v_n,v_{n+1}) < \max\{d^2(v_{n-1},v_n), d^\alpha(v_{n-1},v_n)d^{2-\alpha}(v_n,v_{n+1})\}$$

Therefore, $(d(v_n, v_{n+1}):\ n \in \mathbb{N})$ is a non-increasing sequence in $(0,\infty)$. So, there is $\beta \geq 0$ such that $\lim\limits_{n\to\infty} d(v_n,v_{n+1}) = \beta$. Suppose on the contrary; that is $\beta > 0$. Let $= d(v_{n-1}, v_n) = s_n$. Then $s_n \to \beta$. Therefore, by γ_2 of Definition 1.1, we have:

$$\beta^{k+1} < \liminf_{n\to\infty} \gamma(d(v_{n-1},v_n), v_n, v_{n+1}) \leq \beta^{k+1},$$

a contradiction. Hence, we get the desired result. □

Theorem 3.5
Suppose that $(\mathcal{V}, d)$ is complete and suppose that there is $\gamma(k)$-distance γ on $(\mathcal{V}, d)$. Assume that $f: \mathcal{V} \to \mathcal{V}$ is a $M-\gamma(k)$ contraction. Then F_f has only one element.

Proof
By choosing an arbitrary element $v_0 \in \mathcal{V}$, we construct the Picard sequence (v_n) generated by f at v_0. We claim that (v_n) is a Cauchy sequence. Suppose to the contrary that (v_n) is not Cauchy. Therefore, depending on Lemma 2.4 there is $\epsilon > 0$ and two sub-sequences (v_{n_k}) and (v_{m_k}) of (v_n) such that

$$\lim_{k\to\infty} d(v_{n_k}, v_{m_k}) = \epsilon. \tag{3.2}$$

and

$$\lim_{k\to\infty} d(v_{n_k-1}, v_{m_k-1}) = \epsilon. \tag{3.3}$$

Now, set $t_k = d(v_{n_k-1}, v_{m_k-1}),\ a_k = v_{n_k}$ and $b_k = v_{m_k}$. Then

$$\lim_{k\to\infty} t_k = \lim_{k\to\infty} d(a_k, b_k) = \epsilon > 0.$$

So, by (γ_2) and condition 3.1, we get

$$\epsilon^{k+1} < \liminf_{k\to\infty} \gamma(t_k, a_k, b_k) \leq \epsilon^{k+1},$$

a contradiction. Consequently, (v_n) is Cauchy, so there is $t_1 \in \mathcal{V}$ such that (v_n) converges to t_1. Now, by Remark 3.2, we have

$$d^2(ft_1, fv_n) < \max\{d^2(t_1,v_n), [d(t_1,ft_1)^\alpha.d(v_n,fv_n)^{2-\alpha}]\}.$$

Now, by letting $n \to +\infty$, we get $v_n \to ft_1$, and so by the uniqueness of limit, we get $t_1 = ft_1$.
Lemma 3.4 ensures the uniqueness. □

In the next example, we employ Theorem 3.5 to prove that the following self mapping

$$fx = \frac{1-x^m}{B+x^m}, \quad \frac{B+1}{B^2} < \frac{1}{\sqrt[k+1]{2}\, m}. \tag{3.4}$$

has only one element in F_f in the unit interval $[0,1]$.

Example 3.6. Let $\mathcal{V} = [0,1]$ and let $f : \mathcal{V} \to \mathcal{V}$ be defined by

$$fx = \frac{1-x^m}{B+x^m}, \quad \frac{B+1}{B^2} < \frac{1}{\sqrt[k+1]{2}\, m}. \tag{3.5}$$

Then f has a unique fixed point in $[0,1]$.

Proof
To show this, let $d(x,y) = |x-y|$ be the usual metric and assume that$\gamma(k)$ is defined by $\gamma : [0,\infty) \times \mathcal{V} \times \mathcal{V} \to [0,\infty)$ via $\gamma(t,x,y) = 2t^{k+1} d(x,y), \; k \geq 1$.
Now, $\forall\, x, y \in \mathcal{V}$, we obtain the following:

$$\begin{aligned} d(fx,fy) &= |fx - fy| \\ &= \left| \frac{1-x^m}{B+x^m} - \frac{1-y^m}{B+y^m} \right| \\ &= \frac{B+1}{(B+x^m)(B+y^m)} |x^m - y^m| \\ &\leq \frac{(B+1)m}{B^2} |x-y| \\ &= \beta m |x-y|, \; \beta = \frac{(B+1)}{B^2} \end{aligned}$$

Observe that

$$\frac{(B+1)}{B^2} < \frac{1}{\sqrt[k+1]{2}m} \Longrightarrow \frac{(B+1)^{k+1}}{B^{2k+2}} < \frac{1}{2m^{k+1}} \Longrightarrow 2(\beta m)^{k+1} = \frac{2(B+1)^{k+1} m^{k+1}}{B^{2k+2}} < 1.$$

Therefore,

$$\begin{aligned} \gamma(d(fx,fy), fx, fy) &= 2d^{k+1}(fx,fy) \\ &\leq 2(\beta m |x-y|)^{k+1} \\ &< |x-y|^{k+1} \\ &= [d(x,y)]^{k+1} \\ &\leq \max\{(d(x,y)^{k+1}, (d^{\alpha}(x,fx) d^{2-\alpha}(y,fy))^{\frac{k+1}{2}}\}. \end{aligned}$$

Consequently, the mapping f fulfills all the requirements outlined in of
Theorem 3.5 which ensures that F_f has only one element. □

For specific values $B = 100$, $k = 3$, and $m = 50$, the function

$$f(x) = \frac{1-x^{50}}{100+x^{50}}$$

has a unique fixed point in $[0,1]$. Note that

$$\frac{B+1}{B^2} = 0.0101 \; < \; \frac{1}{\sqrt[4]{2} \cdot 50} \approx 0.0168,$$

and the unique fixed point is approximately 0.01.

4. Application

To construct our application, we will refer to Example 3.6. Let $m \in \mathbb{Z},\ B \in \mathbb{R}$ with $\frac{B+1}{B^2} < \frac{1}{\sqrt[k+1]{2}\ m}$. Then the following equation

$$x^{m+1} + x^m + Bx - 1 = 0, \tag{4.1}$$

has exactly one solution in $[0,1]$.
As a typical proof, it can be demonstrated that the mapping below admits a unique fixed point contained in $[0,1]$.

$$fx = \frac{1 - x^m}{B + x^m},\ \frac{B+1}{B^2} < \frac{1}{\sqrt[k+1]{2}\ m}. \tag{4.2}$$

As demonstrated in Example 3.6, the set F_f contains exactly one element in the interval $[0,1]$. Consequently, equation 4.1 admits a unique solution.

5. Conclusion

In this article, we have successfully merged the concepts of the $\gamma(k)$-distance mapping, interpolative contraction, and $\mathcal{H}$-simulation function to obtain several novel fixed point results. Specifically, we introduced three new types of contractions: weak interpolative $\gamma(k)$-contractions, weak interpolative $(\gamma(k) - \mathcal{H})$-contractions, and M-$\gamma(k)$-contractions. For each class, we established existence and uniqueness theorems for fixed points in complete metric spaces. The numerical example provided verified the correctness of our newly introduced contractions, and an application to a polynomial equation demonstrated their practical applicability. Our results generalize and unify several existing results in the literature, particularly those concerning γ-distance mappings and interpolative-type contractions.

Despite the generality of our results, several limitations should be acknowledged. Our results are confined to complete metric spaces, and extending them to more general structures such as b-metric spaces, partial metric spaces, or quasi-metric spaces remains an open question. The $\gamma(k)$-distance used throughout this paper requires the parameter $k \geq 1$, while the case $0 < k < 1$ was not investigated and may lead to different contractive conditions. The application provided is limited to a specific algebraic equation, and more complex applications, particularly those involving integral or differential equations, were not explored. Additionally, the uniqueness proofs rely heavily on the contraction conditions holding for all points in the space, whereas local contractions (where the condition holds only on a subset) were not considered.

Based on the work presented, several promising directions for future research emerge. An immediate extension would be to investigate $\gamma(k)$-contractions in b-metric spaces, extended b-metric spaces, or modular spaces, where the triangular inequality is modified, which could broaden the applicability to a wider class of problems. Fixed point theorems are fundamental tools for proving existence and uniqueness of solutions to fractional differential equations, and future work could apply our contractions to problems of the form $D^\alpha x(t) = f(t, x(t))$ with $x(0) = x_0$, where D^α is the Caputo or Riemann-Liouville fractional derivative of order $\alpha \in (0,1)$. The $\gamma(k)$-distance could provide new contractive conditions suited to the weakly singular kernels appearing in fractional calculus. Similarly, many physical phenomena are modeled by Fredholm or Volterra integral equations of the form $x(t) = \int_a^b K(t,s,x(s))\,ds + g(t)$, and our results could be applied to establish existence and uniqueness of solutions under new interpolative-type conditions on the kernel K, potentially relaxing standard Lipschitz assumptions. Extending our $\gamma(k)$-contractions to coupled, tripled, or n-tuple fixed point theorems could handle systems of equations arising in mathematical biology, economics, and engineering. Incorporating the structure of a graph into the metric space could lead to interpolative contractions defined only on edges of the graph, generalizing recent work on metric spaces with graphs. Developing iterative algorithms based on Picard sequences for approximating fixed points under our contractions, along with convergence rate analysis, would enhance the practical utility of our results. Finally, connecting fixed point theory with optimization problems, where the fixed point of a contraction corresponds to a minimizer of a functional, represents another promising avenue.

In summary, while our results contribute meaningful generalizations to fixed point theory, substantial opportunities remain for extending them to more complex mathematical structures and real-world applications, particularly in the realms of fractional differential equations and integral equations.

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