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A Dawoud–Kibria Shrinkage Estimator for the Waring Regression Model: Methodology, Simulation, and Application

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Abstract Count regression models often suffer from two concurrent problems: overdispersion in the response and multicollinearity among predictors, which together destabilize maximum likelihood estimation in the Waring regression model (WRM). To address this issue, we extend the Dawoud–Kibria two-parameter shrinkage strategy to the WRM and develop a new estimator, termed the Waring Dawoud–Kibria estimator (WRDKE). The proposed estimator introduces two biasing parameters into the WRM score equations and yields a closed-form solution whose bias and variance are characterized analytically via the matrix mean squared error (MMSE) and scalar mean squared error (MSE). We derive conditions under which the WRDKE dominates the classical maximum likelihood estimator (MLE), the Waring ridge regression estimator (WRRE), and the Waring Liu estimator (WRLE) in the MMSE sense. A comprehensive Monte Carlo simulation study is conducted under varying sample sizes, numbers of predictors, pairwise correlations, and overdispersion levels. Predictors are generated following the McDonald–Galarneau scheme to induce controlled multicollinearity, while responses are drawn from the Waring distribution with different overdispersion parameters. Across all configurations, the WRDKE consistently achieves the smallest average MSE (AMSE), with reductions in AMSE relative to MLE that exceed 50% in severe multicollinearity scenarios and remain substantial even for moderate correlations and larger sample sizes. The WRDKE also outperforms WRRE and WRLE uniformly, yielding notably lower AMSE values such as 2.05 versus 5.28 for MLE when the sample size is 500 and correlation is 0.85. Finally, an application to real overdispersed count data illustrates that the WRDKE provides more stable coefficient estimates and improved in-sample fit compared with existing WRM estimators.

Keywords Count data, overdispersion, multicollinearity, shrinkage estimator, Dawoud–Kibria estimator, Waring regression model

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1. Introduction

Count data modeling refers to the analysis of outcomes that are non-negative integers, such as the number of hospital admissions or defaults or accidents or trades in a period of time. The analysis of these data is usually performed in the context of the generalized linear model with the help of the distributions of the Poisson family adapted to the discrete number of events. A key feature of the typical Poisson regression model (PRM) is the equidispersion assumption that the conditional variance of the response is equal to the conditional mean of the response [1, 2, 3, 4, 5, 6]. In reality, the variability of the count data is usually greater or smaller than this bound (i.e. it is overdispersed or underdispersed relative to the PRM).

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The data are considered as overdispersed if the variance of the observed counts is greater than the mean adjusting for the factors [38, 39, 40, 41, 42]. It is common in the applications where the units or individuals are not homogeneous, the events are dependent or if the observations are more variable due to the presence of unobserved risk factors. The causes of overdispersion are; Unobserved variability in exposure, clustering or correlation, misspecified functional forms and zero inflation or heavy-tailed count distribution [7, 8, 9, 43, 44]. Ignoring overdispersion and fitting a PRM when the data are overdispersed has important consequences for statistical inference [45, 46, 47]. Although maximum likelihood estimates of the regression coefficients remain consistent, the standard errors of the estimated regression coefficients are usually biased downwards, resulting in inflated test statistics, underestimated confidence intervals, and a higher probability of committing a Type I error in hypothesis testing [10, 11, 12, 13, 14].

There are many extended models available to fit overdispersed count data. The Waring regression model (WRM) is a count model that explicitly models overdispersed count data, thus overcoming the overdispersion issue that occurs when the variance is smaller than the Poisson variance [15, 16, 17, 18]. In the case of severely overdispersed count data, we can position the WRM as a generalised version of more typical count-regression models. The negative binomial model on most applications relaxes the equidispersion assumption of the Poisson model, but still underestimates extreme counts and thick tails, resulting in a further extra-Poisson variability in the residuals. The Waring distribution and its generalization introduce extra shape parameters that allow for a faster rate of increase in the variance and the tails of the distribution, while allowing the conditional mean to take a regression form [19, 48, 49, 50, 51, 52]. Thus, WRM can explicitly account for situations where heterogeneity, clustering, or contagion can result in very large counts and overdispersion, which fits the overall distribution, especially the upper tail, and the standard errors and inferences better and is therefore more accurate than Poisson or negative binomial models.

In the vast majority of applications, however, the predictors in count regression models are strongly correlated with each other and cause the occurrence of multicollinearity. Multicollinearity in the WRM brings about the same unwanted effects of other regression models that produce unstable coefficients estimate and increase standard errors, but is further complicated by the high variance of the model in itself due to high level of overdispersion. Extra-Poisson dispersion is conditioned by the presence of WRM in heavy tail, and therefore covariate correlation can make maximum likelihood estimate (MLE) particularly unsteady and inaccurate. To address this problem in the WRM context, shrinkage estimators like the Waring ridge regression estimator (WRRE) [20] and the Waring Liu regression estimator (WRLE) Liu estimator [21] have been suggested, which add one or more biasing parameters to overcome a slight bias by a significant decrease in variance. Although these estimators will result in improved performance of WRM when there is multicollinearity, one can go ahead to improve them by achieving even lower reductions in the mean squared error by exploiting additional structure in the biasing mechanism [53, 54, 55].

In the linear regression model, Dawoud and Kibria [22] suggested a two-parameter biased estimator to combat multicollinearity. Consequently, the paper aims at applying the Dawoud–Kibria estimator (DKE) to WRM and obtain the theoretical aspects of it. In addition to this we also offer theory comparison of the proposed DKE with the other estimation estimators using the matrix mean squared error (MMSE) and the MSE criteria. We then carry out a Monte Carlo simulation study in different conditions such as the sample size, the number of the explanatory variables, and the degree of correlation between the explanatory variables. And the last but not the least, the productivity of the proposed method is studied, in terms of comparison with the other estimation methods, through the application of real data.

2. WRM with collinearity issue

Assume that $(y_i, u_i),\ i = 1, 2, ..., n$ is independent observed data with the predictor vector $u_i \in R^{p+1}$ and the response variable $y_i \in R$ which follows a distribution that belongs to the Waring distribution with parameters $\xi_1 > 0$ and $\xi_2 > 0$ [23]]. Then, after the parametrization of Weinberg and Gladen [24], the density function of y_i can be expressed as

$$p\left(y_i|\xi_1,\xi_2\right) = \frac{\xi_1 \Gamma\left(\xi_1+\xi_2\right)\Gamma\left(y_i+\xi_2\right)}{\Gamma\left(\xi_2\right)\Gamma\left(y_i+\xi_1+\xi_2+1\right)}, \quad y_i = 0, 1, 2, \cdots \tag{1}$$

where Γ is the gamma function. The mean, $E(y_i)$, and the variance, $var(y_i)$, of the Eq. (1) are $E(y_i) = (\xi_1/(\xi_2-1))$, $\xi_2 > 1$ and $var(y_i) = \xi_1\xi_2(\xi_1+\xi_2-1)/(\xi_2-2)(\xi_2-1)^2$, $\xi_2 > 2$. several different parameterizations of the Eq. (1) were proposed. Related to the parameterization of Rivas and Galea [16], Eq. (1) , $y_i \sim \text{Waring}(\mu_i, \chi)$, can be written as

$$p(y_i|\mu_i,\chi) = \frac{\chi_1\Gamma(\chi_1+\mu_i\chi_2)\Gamma(y_i+\mu_i\chi_2)}{\Gamma(\mu_i\chi_2)\Gamma(y_i+\chi_1+\mu_i\chi_2+1)}, \quad \mu_i > 0 \text{ and } \chi > 1 \tag{2}$$

where μ_i is the mean function of the response variable $\chi_1 = 2\chi/(\chi-1)$ and $\chi_2 = \chi + 1/\chi - 1$, and τ is the overdispersion parameter. According to Eq. (2), the mean is $E(y_i){=}\mu_i$ and the variance is $\text{var}(y_i){=}\chi\left(\mu_i^2+\mu_i\right)$.

In generalized linear model, the mean of the response variable, $\mu_i = E(y_i)$, is conditionally related to a linear function of predictors through a link function. The linear function is stated as $\eta_i = \beta_0 + \sum_{j=1}^{p} u_{ij}\beta_j = u_i^T\beta$ with $u_i^T = (1, u_{i2}, u_{i3}, ..., u_{ip})$ and $\beta = (\beta_0, \beta_1, ..., \beta_p)^T$. The link function is providing the relation of the mean and the natural parameter as $\mu_i = g^{-1}(\eta_i) = g^{-1}(u_i^T\beta)$. The WRM can be modeled by assuming $\mu_i = \exp\left(u_i^T\beta\right)$. The parameter estimation in the WRM is achieved through using the MLE based on the iteratively reweighted least-squares algorithm. The log-likelihood of Eq. (2) is defined

$$\begin{aligned}\ell(\beta,\chi) = \log\chi_1 + \log\Gamma\left(\chi_1+\exp\left(u_i^T\beta\right)\chi_2\right) + \log\Gamma\left(y_i+\exp\left(u_i^T\beta\right)\chi_2\right) - \\ \log\Gamma\left(\exp\left(u_i^T\beta\right)\chi_2\right) - \log\Gamma\left(y_i+\chi_1+\exp\left(u_i^T\beta\right)\chi_2+1\right)\end{aligned} \tag{3}$$

Then, the MLE is derived by equaling the first derivative of Eq. (3) to zero. Fisher-scoring algorithm can be used to obtain the MLE where in each iteration, the parameter is updated by

$$\beta^{(r+1)} = \beta^{(r)} + I^{-1}(\beta^{(r)})S(\beta^{(r)}), \tag{4}$$

where $I^{-1}(\beta) = \left(-E\left(\partial^2\ell(\beta,\chi)/\partial\beta\partial\beta^T\right)\right)^{-1}$. After that, the estimated coefficients of the WRM are defined as

$$\hat{\beta}_{MLE} = (\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U})^{-1}\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\hat{\mathrm{m}}, \tag{5}$$

where $\hat{\mathrm{W}} = \text{diag}\left[(\partial\mu_i/\partial\eta_i)^2/\text{var}(y_i)\right] = \text{diag}\left(\hat{\mu}_i\left(\hat{\mu}_i+1\right)\right)$ and $\hat{\mathrm{m}}$ is a vector where i^{th} element equals to $\hat{u}_i = \log\left(\hat{\mu}_i\right) + \left[\left(y_i-\hat{\mu}_i\right)/\hat{\mu}_i\left(\hat{\mu}_i+1\right)\right]$. The MLE is distributed asymptotically normal with a covariance matrix as

$$cov(\hat{\beta}_{MLE}) = \left[-E\left(\frac{\partial^2\ell(\beta,\chi)}{\partial\beta\,\partial\beta^T}\right)\right]^{-1} = \hat{\chi}(\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U})^{-1}. \tag{6}$$

where $\hat{\chi}$ is the estimated dispersion parameter. By considering $\Omega = \Upsilon^T\beta$and $\Lambda = \text{diag}(\zeta_1, \zeta_2, ..., \zeta_p) = \Omega = \Upsilon(\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U})\Upsilon^T$, where Υ is the orthogonal matrix whose columns are the eigenvectors of $\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U}$, and the $\zeta_1, \zeta_2, ..., \zeta_p \geq 0$ are the eigenvalues of $\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U}$ whereas $\Omega_j\ \forall\ j = 1, 2, ..., p$is the j^{th}element of $\Upsilon^T\beta$. The matrix mean squared error (MMSE), and the MSE of $\hat{\beta}_{\text{MLE}}$ are given respectively by

$$\text{MMSE}(\hat{\beta}_{\text{MLE}}) = \hat{\chi}\,\Upsilon\Lambda^{-1}\Upsilon^T, \tag{7}$$

and

$$\text{MSE}(\hat{\beta}_{\text{MLE}}) = \text{trace}\left(\text{MMSE}(\hat{\beta}_{\text{MLE}})\right) = \hat{\chi}\sum_{j=1}^{p}\frac{1}{\zeta_j}, \tag{8}$$

where ζ_j denotes the j^{th} eigenvalue of the $\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U}$ matrix.

In the presence of multicollinearity, the matrix $\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U}$ becomes ill-conditioned leading to high variance and instability of the MLE of the WRM parameters. Therefore, in the context of multicollinearity issue, the goal is to improve MLE performance. The accuracy of parameter estimate can be increased by using shrinkage estimators.

The ridge estimator (RE) of Hoerl and Kennard [25] in WRM, WRRE can be defined as [20]:

$$\hat{\beta}_{\text{WRRE}} = (\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U} + kI_p)^{-1}\,\mathrm{U}^{\mathrm{T}}\hat{\mathrm{W}}\mathrm{U}\,\hat{\beta}_{\text{MLE}} \tag{9}$$

where $k > 0$is a biasing factor. The bias vector and covariance matrix of the of $\hat{\beta}_{\mathrm{WRRE}}$ can be written as:

$$\mathrm{Bias}(\hat{\beta}_{\mathrm{WRRE}}) = -k\Upsilon\Lambda_{RE}^{-1}\beta, \tag{10}$$

$$Cov(\hat{\beta}_{\mathrm{WRRE}}) = \hat{\tau}\,\Upsilon\Lambda_{RE}^{-1}\Lambda\Lambda_{RE}^{-1}\Upsilon^T \tag{11}$$

Subsequently, the MMSE and MSE for the $\hat{\beta}_{\mathrm{WRRE}}$ are derived as follows:

$$\mathrm{MMSE}(\hat{\beta}_{\mathrm{WRRE}}) = \hat{\chi}\,\Upsilon\Lambda_{RE}^{-1}\Lambda\Lambda_{RE}^{-1}\Upsilon^T + \mathrm{k}^2\Upsilon\Lambda_{RE}^{-1}\beta\beta^T\Lambda_{RE}^{-1}\Upsilon^T, \tag{12}$$

$$\begin{aligned}\mathrm{MSE}(\hat{\beta}_{\mathrm{WRRE}}) &= \mathrm{trace}(\mathrm{MMSE}(\hat{\beta}_{\mathrm{WRRE}})) \\ &= \hat{\chi}\sum_{j=1}^{p}\frac{\zeta_i}{(\zeta_j+k)^2} + k^2\sum_{j=1}^{p}\frac{\gamma_j^2}{(\zeta_j+k)^2},\end{aligned} \tag{13}$$

where $\Lambda_{RE} = \mathrm{diag}(\zeta_1 + k, \zeta_2 + k, ..., \zeta_p + k)$.

On the other hand, the Liu estimator [26] in the WRM, WRLE, can be defined as [21]:

$$\hat{\beta}_{\mathrm{WRLE}} = (\mathrm{U^T\hat{W}U} + I_p)^{-1}\ (\mathrm{U^T\hat{W}U} + dI_p)\hat{\beta}_{\mathrm{MLE}}, \tag{14}$$

where $0 < d < 1$ is a biasing factor. The bias vector and covariance matrix of the of $\hat{\beta}_{\mathrm{WRLE}}$ can be written as:

$$\mathrm{Bias}(\hat{\beta}_{\mathrm{WRLE}}) = \Upsilon(d-1)\Lambda^{-1}\beta, \tag{15}$$

$$Cov(\hat{\beta}_{\mathrm{WRLE}}) = \hat{\chi}\,\Upsilon\Lambda^{-1}\Lambda_{LE}\Lambda^{-1}\Lambda_{LE}\Lambda^{-1}\Upsilon^T \tag{16}$$

Subsequently, the MMSE and MSE for the $\hat{\beta}_{\mathrm{WRLE}}$ are derived as follows:

$$\mathrm{MMSE}(\hat{\beta}_{\mathrm{WRLE}}) = \hat{\chi}\,\Upsilon\Lambda^{-1}\Lambda_{LE}\Lambda^{-1}\Lambda_{LE}\Lambda^{-1}\Upsilon^T + (d-1)^2\Upsilon\Lambda_{LE}^{-1}\beta\beta^T\Lambda_{LE}^{-1}\Upsilon^T, \tag{17}$$

$$\begin{aligned}\mathrm{MSE}(\hat{\beta}_{\mathrm{WRLE}}) &= \mathrm{trace}(\mathrm{MMSE}(\hat{\beta}_{\mathrm{WRLE}})) \\ &= \hat{\chi}\sum_{j=1}^{p}\frac{(\zeta_j+d)^2}{\zeta_j(\zeta_j+1)^2} + (d-1)^2\sum_{j=1}^{p}\frac{\gamma_j^2}{(\zeta_j+1)^2}\end{aligned} \tag{18}$$

where $\Lambda_{LE} = \mathrm{diag}(\zeta_1 + d, \zeta_2 + d, ..., \zeta_p + d)$.

3. The proposed estimator

The DKE is a modern two-parameter biased estimator introduced to stabilize inference in linear and generalized linear regression models when severe multicollinearity undermines the efficiency of traditional estimators such as ordinary least squares (OLS) and MLE. The DK estimator was proposed by Dawoud and Kibria [27]. The statistical properties of the DKE have been studied primarily under the MMSE, which accounts jointly for bias and variance. Under suitable conditions on the design matrix and the biasing parameter k and d, the DKE has been shown to dominate MLE, the classical ridge estimator, and the Liu estimator in the sense of having a smaller MSE matrix. Several papers extend DKE to GLMs, particularly Poisson and logistic regression, where multicollinearity degrades MLE performance [28, 29, 30, 31, 32].

In this paper, we employ this estimator with WRM. The proposed estimator, WRDKE, is defined as:

$$\hat{\beta}_{\mathrm{WRDKE}} = (\mathrm{Z^T\hat{W}Z} + k(1+d)I_p)^{-1}(\mathrm{Z^T\hat{W}Z} - k(1+d)I_p)\hat{\beta}_{\mathrm{MLE}}, \tag{19}$$

where $k > 0$ and $0 < d < 1$.

The MMSE and MSE of $\hat{\beta}_{\mathrm{WRDKE}}$ are defined as

$$\begin{aligned}\mathrm{MMSE}(\hat{\beta}_{\mathrm{WRDKE}}) &= \mathrm{Cov}(\hat{\beta}_{\mathrm{WRDKE}}) + Bias(\hat{\beta}_{\mathrm{WRDKE}})Bias(\hat{\beta}_{\mathrm{WRDKE}})^T \\ &= \hat{\tau}\,\Upsilon(\Lambda+k(1+d)I_p)^{-1}(\Lambda - k(1+d)I_p)\Lambda^{-1}(\Lambda - k(1+d)I_p) \\ &\quad (\Lambda + k(1+d)I_p)^{-1}\Upsilon^T + (\Lambda + k(1+d)I_p)^{-1}\beta\beta^T(\Lambda + k(1+d)I_p)^{-1},\end{aligned} \tag{20}$$

$$\begin{aligned}\mathrm{MSE}(\hat{\beta}_{\mathrm{WRDKE}}) &= \mathrm{tr}(\mathrm{MMSE}(\hat{\beta}_{\mathrm{WRDKE}})) \\ &= \hat{\tau}\sum_{j=1}^{p}\frac{(\zeta_i - k(1+d))^2}{\zeta_i(\zeta_i+k(1+d))^2} + 4k^2(1+d)^2\sum_{j=1}^{p}\frac{\gamma_j^2}{(\zeta_j+k(1+d))^2}\end{aligned} \tag{21}$$

4. Selection of k and d

The performance of the shrinkage estimators, in general, depends on the appropriate selection of the biasing factors, such as k or d. For shrinkage estimators, k or d are usually chosen by minimizing an MSE-based criterion. For our used estimators in this paper, the following formulas have been adapted from the literature.

1. The WRRE in Eq. (9) with $\hat{k} = \hat{\tau} \big/ \max\left(\hat{\gamma}_j^2\right)$.
2. WRLE in Eq. (14) with $\hat{d}$as in Amin, Noor [21].
3. WRDKE estimator in Eq. (19) with $\hat{k} = \hat{\tau} \big/ \max\left(\hat{\gamma}_j^2\right)$ and $\hat{d}$ as in Amin, Noor [21].

5. Monte Carlo Simulation study

The effectiveness of our suggested estimator, WRMRE, is examined in this section. In accordance with McDonald and Galarneau [33], predictors with varying levels of multicollinearity are produced by

$$u_{ij} = (1-\rho^2)^{1l2} b_{ij} + \rho b_{ip}, \quad i = 1, 2, ..., n, \quad j = 1, 2, ..., p, \tag{22}$$

where b_{ij}'s are produced from a standard normal distribution and ρ^2 denotes the correlation between the explanatory variables. The response variable of n observations is generated from Waring distribution with $\mu_i = \exp(u_i^T \beta)$ and the dispersion parameter τ with $\sum_{j=1}^{p} \beta_j^2 = 1$ [34, 35, 37]. Since multicollinearity can potentially affect the estimates of small samples, the small sample sizes in terms of the number of explanatory factors were used. Here, $n \in \{30, 75, 150, 500\}$. Two values of the number of explanatory variables were set to be $p = 3$ and $p = 7$. Choosing several explanatory variables is essential for capturing the complexity of relationships between predictors and the time-to-event outcome. The pairwise correlation are considered with $\rho \in \{0.85, 0.90, 0.95, 0.99\}$. The overdispersion parameter values are $\chi \in \{0.5, 1, 3\}$.The generated data is repeated 1000 times for a combination of all these factors, and the averaged mean squared errors (AMSE) are computed as

$$\text{AMSE}(\hat{\beta}) = \frac{1}{1000} \sum_{i=1}^{1000} (\hat{\beta} - \beta)^T (\hat{\beta} - \beta), \tag{23}$$

where $\hat{\beta}$ is the obtained shrinkage estimator. Tables 1–6 summarize the AMSE over 1000 replications for all competing estimators of the WRM under varying degrees of multicollinearity, censoring, sample size, and model dimension. In each configuration we compare the MLE, the WRRE, the WRLE, and the WRKLE, where the smallest AMSE in each scenario is highlighted in bold.

Several clear patterns emerge across Tables 1–6. First, for every combination of n, ρ, χ, and p, the proposed WRDKE achieves the smallest AMSE, so the bold entries always correspond to WRDKE. This indicates that incorporating the Dawoud–Kibria two-parameter shrinkage structure into the Waring regression framework yields uniformly more accurate coefficient estimates than MLE, WRRE, and WRLE in terms of AMSE. Second, as expected, AMSE increases monotonically with ρ for all estimators, reflecting the detrimental impact of stronger multicollinearity on estimation accuracy. Third, AMSE decreases with increasing sample size for all estimators, which is consistent with asymptotic theory, but the relative advantage of WRDKE over the competitors persists even at the largest sample size, $n = 500$. Finally, moving from Tables 1–3 to Tables 4–6, both higher overdispersion and larger model dimension lead to higher AMSE levels for all four estimators, yet WRDKE consistently remains the best performer.

Tables 1–3 correspond to the scenarios with the smaller number of regressors, under three increasing levels of overdispersion. In Table 1, which represents the least severe overdispersion among the three, WRDKE yields substantial reductions in AMSE relative to MLE across all settings. For instance, at $n = 30$ and $\rho = 0.99$, the AMSE of MLE is 6.074, whereas WRDKE reduces it to 2.569, cutting the error by more than half. At $n = 500$ and $\rho = 0.99$, MLE still exhibits an AMSE of 5.851, while WRDKE attains 2.357, indicating that the benefit of shrinkage does not vanish with larger samples under high multicollinearity. WRRE and WRLE also improve on

MLE, but WRDKE consistently yields the smallest AMSE; for example, at $n = 150$ and $\rho = 0.99$, the AMSEs are 5.071 of MLE, 2.664 of WRRE, 2.023 of WRLE, and 1.845 of WRDKE, confirming that WRDKE further refines the bias–variance trade-off achieved by ridge-type and Liu-type estimators.

Tables 2 and 3 increase the overdispersion parameter relative to Table 1, which systematically inflates AMSE values for all estimators. Nevertheless, the qualitative patterns remain the same: WRDKE is uniformly superior, and the performance gap relative to MLE is often slightly larger under stronger overdispersion. For example, in Table 3 with high overdispersion, at $n = 30$ and $\rho = 0.99$, MLE, WRRE, WRLE, and WRDKE have AMSEs 6.111, 3.443, 2.802, and 2.619, respectively, showing that WRDKE still offers the lowest error while both WRRE and WRLE remain distinctly worse than WRDKE. These results indicate that the proposed estimator is particularly effective in stabilizing inference for Waring regression when the variance is inflated far beyond the Poisson level.

Tables 4-6 extend the comparison to a higher-dimensional Waring regression model and higher overdispersion settings, making estimation more challenging. In Table 4, under the lowest of the three high-dimensional overdispersion levels, WRDKE again dominates its competitors in every scenario. At $n = 30$ and $\rho = 0.99$, the AMSEs are 6.177 of MLE, 3.509 of WRRE, 2.863 of WRLE, and 2.683 of WRDKE, indicating that WRDKE offers the largest reduction relative to MLE and a noticeable improvement over WRLE. As n increases, all AMSEs decline, but WRDKE continues to be the best choice; for instance, at $n = 500$ and $\rho = 0.95$, the AMSEs are 5.837 of MLE, 3.073 of WRRE, 2.443 of WRLE, and 2.249 of WRDKE.

Table 1. The AMSE values when $p = 3$ and $\chi = 0.5$

n	ρ	MLE	WRRE	WRLE	WRDKE
30	0.85	5.543	2.913	2.272	**2.089**
	0.90	5.781	3.021	2.385	**2.192**
	0.95	5.902	3.231	2.592	**2.407**
	0.99	5.206	2.789	2.155	**1.965**
75	0.85	5.459	2.834	2.189	**2.002**
	0.90	5.693	2.929	2.291	**2.105**
	0.95	5.817	3.148	2.503	**2.323**
	0.99	5.179	2.773	2.131	**1.951**
150	0.85	5.437	2.809	2.165	**1.985**
	0.90	5.677	2.911	2.275	**2.089**
	0.95	5.798	3.123	2.485	**2.299**
	0.99	5.071	2.664	2.023	**1.845**
500	0.85	5.327	2.697	2.064	**1.873**
	0.90	5.567	2.804	2.165	**1.978**
	0.95	5.688	3.019	2.377	**2.195**
	0.99	5.287	2.878	2.239	**2.052**

Table 2. The AMSE values when $p = 7$ and $\chi = 0.5$

n	ρ	MLE	WRRE	WRLE	WRDKE
30	0.85	5.449	3.038	2.401	**2.217**
	0.90	5.708	3.083	2.437	**2.255**
	0.95	5.955	3.181	2.541	**2.359**
	0.99	6.074	3.393	2.755	**2.569**
75	0.85	5.365	2.951	2.321	**2.127**
	0.90	5.621	2.992	2.351	**2.169**
	0.95	5.859	3.091	2.453	**2.267**
	0.99	5.979	3.308	2.667	**2.482**
150	0.85	5.341	2.934	2.293	**2.107**
	0.90	5.599	2.971	2.331	**2.147**
	0.95	5.837	3.072	2.435	**2.251**
	0.99	5.957	3.287	2.649	**2.463**
500	0.85	5.233	2.825	2.187	**2.001**
	0.90	5.489	2.859	2.221	**2.035**
	0.95	5.731	2.965	2.336	**2.141**
	0.99	5.851	3.181	2.542	**2.357**

Table 3. The AMSE values when $p = 3$ and $\chi = 1$

n	ρ	MLE	WRRE	WRLE	WRDKE
30	0.85	5.502	3.087	2.448	**2.261**
	0.90	5.752	3.122	2.481	**2.303**
	0.95	5.993	3.231	2.587	**2.401**
	0.99	6.111	3.443	2.802	**2.619**
75	0.85	5.412	3.001	2.361	**2.174**
	0.90	5.667	3.04	2.402	**2.211**
	0.95	5.902	3.139	2.503	**2.314**
	0.99	6.027	3.361	2.712	**2.531**
150	0.85	5.391	2.977	2.345	**2.153**
	0.90	5.647	3.021	2.377	**2.194**
	0.95	5.881	3.117	2.482	**2.293**
	0.99	6.008	3.332	2.691	**2.511**
500	0.85	5.282	2.874	2.232	**2.051**
	0.90	5.543	2.912	2.276	**2.084**
	0.95	5.779	3.013	2.371	**2.192**
	0.99	5.897	3.234	2.592	**2.404**

Table 4. The AMSE values when $p = 7$ and $\chi = 1$

n	ρ	MLE	WRRE	WRLE	WRDKE
30	0.85	5.564	3.149	2.518	**2.325**
	0.90	5.817	3.189	2.547	**2.364**
	0.95	6.053	3.289	2.651	**2.462**
	0.99	6.177	3.509	2.863	**2.683**
75	0.85	5.473	3.065	2.434	**2.241**
	0.90	5.729	3.099	2.465	**2.275**
	0.95	5.974	3.205	2.569	**2.382**
	0.99	6.092	3.417	2.781	**2.593**
150	0.85	5.455	3.041	2.407	**2.217**
	0.90	5.714	3.085	2.437	**2.261**
	0.95	5.948	3.181	2.543	**2.357**
	0.99	6.067	3.404	2.757	**2.578**
500	0.85	5.349	2.933	2.301	**2.109**
	0.90	5.602	2.973	2.329	**2.154**
	0.95	5.837	3.073	2.443	**2.249**
	0.99	5.959	3.299	2.647	**2.472**

6. Real data Application

In order to assess the quality of the suggested estimator in practice, a real data set is used. This data set include the Apprentice Migration dataset [36]. Further this data set was analyzed using WRM by Amin, Noor [21] and Noor, Amin [20]. This data set consists of 33 samples and 4 explanatory variables. The response variable represents the Apprentice. The four variables are: Eq. (1) distance (x1), Eq. (2) population (x2), Eq. (3) degree of urbanization (x3), and Eq. (4) direction from Edinburgh (x4). According to Amin, Noor [21], the WRM is more appropriate than the other count data regression model. The estimated value of χ is 2.16. In addition, the condition index is equal to 107.1079 indicating sever multicollinearity exitance. The regression coefficients which were estimated using MLE, WRRE, WRLE, and WRDKE, and were summarized in Table 7.

Across the four estimators, the estimated regression coefficients are broadly similar in sign and magnitude, but the shrinkage estimators noticeably adjust the intercept and slope estimates relative to MLE. For example, the MLE intercept is 2.5764, whereas the shrinkage estimators reduce it to about 1.71, with WRDKE producing the smallest value with 1.7109, reflecting stronger shrinkage toward a more parsimonious model. Likewise, the distance effect is slightly negative under all estimators, but its magnitude is reduced from -0.0069 by MLE to -0.0041 by

Table 5. The AMSE values when $p = 3$ and $\chi = 3$

n	ρ	MLE	WRRE	WRLE	WRDKE
30	0.85	5.561	3.152	2.511	**2.324**
	0.90	5.815	3.185	2.544	**2.361**
	0.95	6.052	3.291	2.653	**2.462**
	0.99	6.174	3.503	2.862	**2.679**
75	0.85	5.472	3.061	2.424	**2.237**
	0.90	5.734	3.103	2.461	**2.274**
	0.95	5.965	3.201	2.563	**2.377**
	0.99	6.091	3.421	2.775	**2.594**
150	0.85	5.451	3.042	2.403	**2.225**
	0.90	5.712	3.081	2.441	**2.257**
	0.95	5.944	3.181	2.542	**2.361**
	0.99	6.069	3.395	2.754	**2.571**
500	0.85	5.343	2.932	2.295	**2.113**
	0.90	5.601	2.971	2.333	**2.147**
	0.95	5.839	3.072	2.434	**2.252**
	0.99	5.965	3.291	2.651	**2.467**

Table 6. The AMSE values when $p = 7$ and $\chi = 3$

n	ρ	MLE	WRRE	WRLE	WRDKE
30	0.85	5.662	3.251	2.612	**2.421**
	0.90	5.922	3.288	2.645	**2.462**
	0.95	6.153	3.392	2.751	**2.565**
	0.99	6.275	3.604	2.963	**2.785**
75	0.85	5.573	3.162	2.525	**2.343**
	0.90	5.831	3.204	2.562	**2.375**
	0.95	6.067	3.302	2.664	**2.483**
	0.99	6.191	3.522	2.879	**2.695**
150	0.85	5.552	3.141	2.504	**2.317**
	0.90	5.811	3.182	2.541	**2.363**
	0.95	6.045	3.281	2.643	**2.457**
	0.99	6.171	3.503	2.855	**2.672**
500	0.85	5.444	3.033	2.398	**2.214**
	0.90	5.702	3.072	2.434	**2.251**
	0.95	5.943	3.173	2.535	**2.353**
	0.99	6.061	3.392	2.752	**2.574**

WRDKE, and the urbanization coefficient is shrunk from 0.0129 by MLE to 0.0112 by WRDKE. The coefficient for direction also changes substantially, increasing from 0.1327 under MLE to around 0.31 under the shrinkage estimators, with WRDKE again yielding a slightly more moderate value than WRRE and WRLE (0.3107 versus 0.3118 and 0.3111), which is consistent with its two-parameter biasing mechanism.

The most direct measure of overall fit is the model MSE reported in the last row of Table 7. Here, MLE attains an MSE of 6.1745, while WRRE and WRLE substantially reduce this to 2.5477 and 2.5136, respectively, confirming the benefit of shrinkage when the design is highly collinear. The proposed WRDKE achieves the smallest MSE, 2.4708, representing the best empirical performance among all estimators for this dataset. This improvement, though numerically modest relative to WRLE, is consistent with the simulation results, where WRDKE systematically dominated WRRE and WRLE in terms of AMSE across a wide range of scenarios. Taken together, the Apprentice Migration analysis indicates that, in a realistic setting characterized by both overdispersion and severe multicollinearity, WRDKE not only stabilizes the coefficient estimates but also yields the lowest prediction error, providing strong applied support for its use in practice.

Moreover, Figures 1-4 show the performance of the used method over several correlation values. It is clear from these Figures that the proposed method, WRDKE, exhibits very good performance compared to MLE, WRRE, and WRLE.

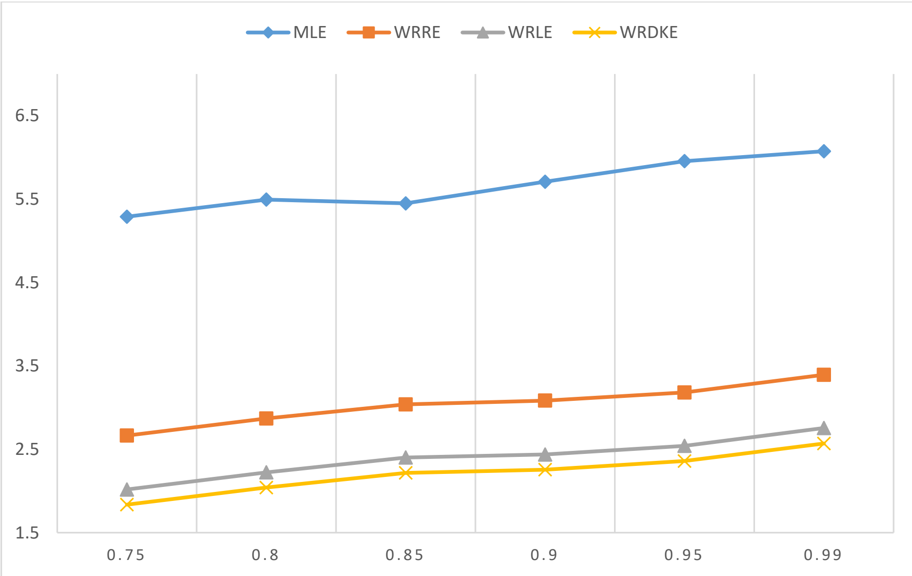

Figure 1. The performance of used methods when $n = 30$, $p = 7$, $\chi = 0.5$

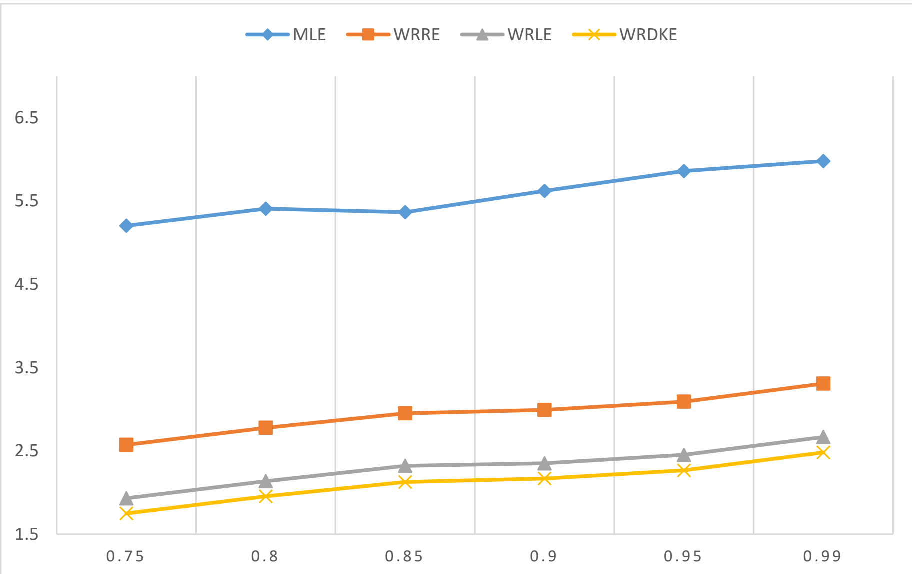

Figure 2. The performance of used methods when $n = 75$, $p = 7$, $\chi = 0.5$

7. Conclusion

The study extends the Dawoud–Kibria shrinkage idea to the WRM and shows that the proposed WRDKE provides a more efficient alternative to existing estimators when overdispersion and multicollinearity coexist. Through analytical MMSE comparisons and extensive simulations across varying sample sizes, correlation levels, and overdispersion parameters, the WRDKE consistently attains the lowest average MSE, often with

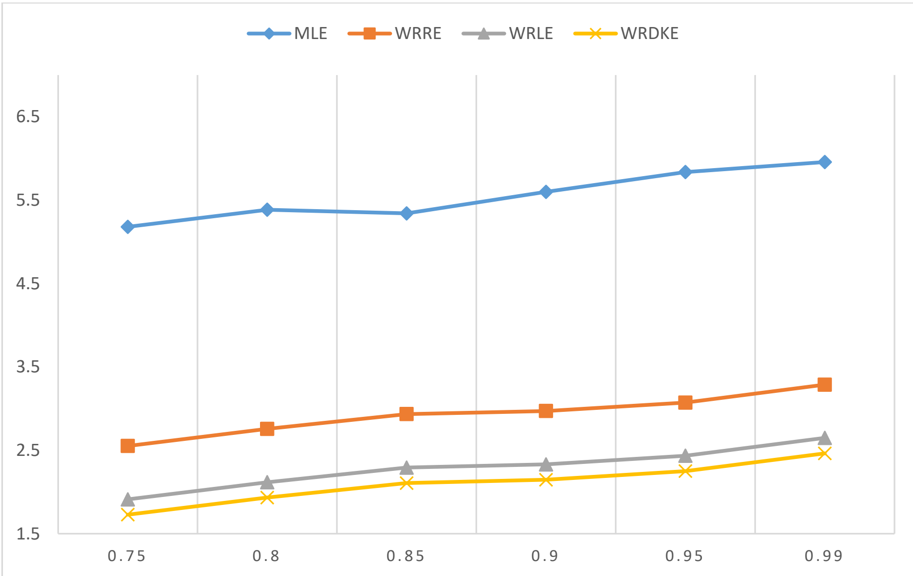

Figure 3. The performance of used methods when $n = 150$, $p = 7$, $\chi = 0.5$

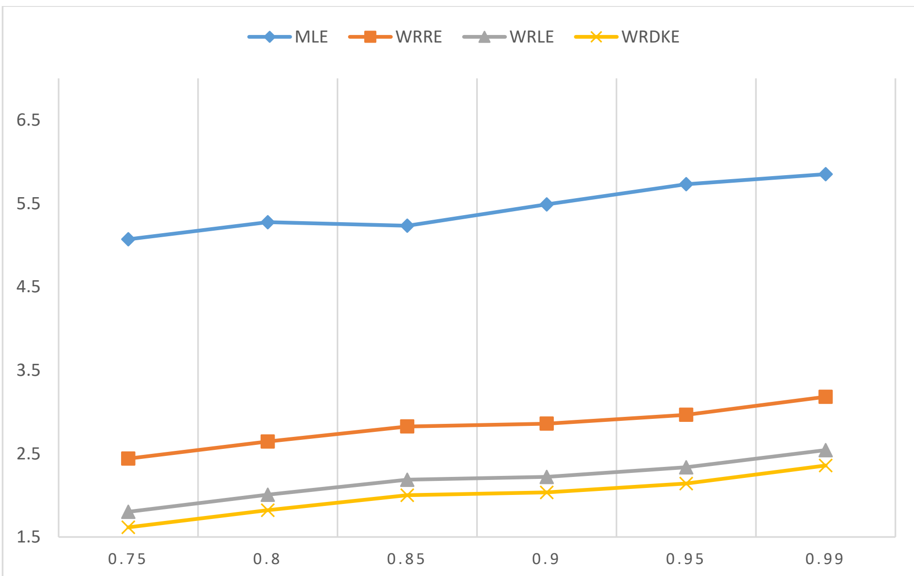

Figure 4. The performance of used methods when $n = 500$, $p = 7$, $\chi = 0.5$

substantial reductions relative to MLE, WRRE, and WRLE, especially under severe multicollinearity and strong overdispersion. These gains translate into more stable coefficient estimates and improved fit in the empirical application to overdispersed count data, confirming the practical value of the proposed method. Despite these promising findings, several limitations should be acknowledged. First, the performance assessment is restricted to specific data-generation settings, including particular ranges of sample size, correlation, and overdispersion, which may not cover all patterns encountered in practice. Second, the choice of the biasing parameters for WRDKE relies on MSE-based criteria adapted from the literature, and alternative data-driven tuning strategies have not been

Table 7. The estimated coefficients for the used estimators of Apprentice Migration dataset

	MLE	WRRE	WRLE	WRDKE
$\hat{\beta}_0$	2.5764	1.7139	1.7114	**1.7109**
$\hat{\beta}_1$	-0.0069	-0.0057	-0.0043	**-0.0041**
$\hat{\beta}_2$	0.0129	0.0113	0.0116	**0.0112**
$\hat{\beta}_3$	-0.0095	-0.0031	-0.0034	**-0.0029**
$\hat{\beta}_4$	0.1327	0.3118	0.3111	**0.3107**
MSE	6.1745	2.5477	2.5136	**2.4708**

explored systematically. Finally, the current work focuses on correctly specified WRM structures and does not investigate robustness to model misspecification or outliers. Future research could extend the WRDKE framework to more complex Waring-type models, such as hurdle or zero-inflated Waring regressions, and to high-dimensional settings where variable selection is required. Another promising direction is the development of adaptive or Bayesian versions of WRDKE with prior- or penalty-based selection of the biasing parameters. In addition, applying and validating the estimator on a wider range of real datasets from different scientific domains would help to further establish its empirical performance and practical guidelines for use.

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