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Portfolio Selection on the Jakarta Islamic Index using the UTASTAR Method and Compromise Programming

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Abstract This paper proposes a data-driven multicriteria decision analysis framework for portfolio selection by integrating the Utility Additive STAR (UTASTAR) method with compromise programming. The proposed framework seeks a Pareto-efficient compromise portfolio by simultaneously maximizing expected return, minimizing portfolio risk, and maximizing the global utility estimated by the UTASTAR model. Continuing the conventional UTASTAR approach, which relies on preference information elicited directly from decision-makers, the proposed framework constructs data-driven reference information from historical market data. The Entropy Weight Method is first employed to derive pseudo-utility values from multiple financial criteria, which are subsequently incorporated as supplementary reference information in a modified UTASTAR formulation while preserving ordinal preference consistency through ranking constraints. The estimated utility function is then integrated with expected return and portfolio risk within the Compromise Programming model to determine the portfolio allocation. The Euclidean distance metric is adopted because it provides a balanced trade-off among the conflicting objectives. Experimental results on stocks listed in the Jakarta Islamic Index 70 demonstrate that the proposed framework successfully constructs a compromise portfolio while preserving ordinal preference relations with low utility reconstruction errors. Comparisons with the classical Markowitz mean–variance model and the Equal-Weighted portfolio indicate that the proposed framework provides a competitive multicriteria alternative by incorporating utility information derived from multiple financial indicators. Furthermore, sensitivity analysis using various norms shows that different compromise metrics produce different return–risk–utility trade-offs while maintaining relatively stable portfolio allocations.

Keywords Data Driven, Multi-Criteria Decision Analysis, Entropy Weight Method, Pseudo Score, Markowitz.

AMS 2020 subject classifications 90C29, 90B50, 91G10, 90C90.

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1. Introduction

Retail investors are individual investors who invest using their personal funds and generally execute transactions on a smaller scale than institutional investors [1]. The Indonesian Financial Services Authority reported that the share of retail investor transactions in the stock market increased significantly to approximately 50% throughout 2025, with more than 70% of Indonesian retail investors belonging to the younger generations (Generation Y and Generation Z) [2]. Retail investors are also commonly classified as non-professional investors who often face limitations in information, financial expertise, and investment experience, making them more susceptible to behavioral biases in investment decision-making [3].

Indonesia is home to the world's largest Muslim population, with more than 85% of its citizens adhering to Islam and approximately 13% of the global Muslim population residing in the country [4, 5]. Consequently, the implementation of Islamic financial principles plays a significant role in shaping investment decisions, particularly in relation to Sharia-compliant stocks, which operate in accordance with Islamic principles and are not involved

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in prohibited sectors such as gambling, interest-based financial services, conventional insurance, or the production and distribution of prohibited goods and services [4, 5]. In Indonesia, Sharia-compliant stocks are represented by the Jakarta Islamic Index (JII), which was introduced on July 3, 2000 and initially consisted of 30 highly liquid Sharia-compliant stocks listed on the Indonesia Stock Exchange (IDX), with its constituents reviewed semi-annually. To provide broader market representation, the Jakarta Islamic Index 70 (JII70) was launched on May 17, 2018, comprising 70 Sharia-compliant stocks that satisfy both Sharia compliance and liquidity criteria. Owing to its broader market coverage, JII70 provides a more comprehensive investment universe for evaluating portfolio optimization models [4, 6, 7]. The rapid growth of Indonesia's capital market, together with the benefits of global portfolio diversification, has also attracted increasing interest from foreign investors [5]. In addition to domestic investors, international investors have shown growing interest in Sharia-compliant stocks because of their adherence to Islamic investment principles and their potential to provide greater portfolio stability, as reported in previous studies [4, 5, 8]. Despite this growing interest, studies on the Indonesian stock market, particularly those focusing on Sharia-compliant stocks, remain relatively limited [5]. Accordingly, this study focuses on stocks included in the JII70.

Portfolio selection was first formulated by Markowitz (1952) as a bi-objective optimization problem that balances expected return and portfolio risk. This framework became the foundation of Modern Portfolio Theory and has been widely adopted in the development of portfolio optimization models [9, 10]. Nevertheless, the mean–variance approach has several limitations, as it considers only two decision dimensions and does not fully capture the complexity of asset characteristics or investor preferences under dynamic market conditions [9]. To address these limitations, utility-based approaches have been widely incorporated into portfolio optimization to represent investor preferences within the investment decision-making process. Various utility functions have been proposed in the literature, including linear, quadratic, exponential, power, and logarithmic utility functions, each providing a different mathematical representation of investor preferences and attitudes toward risk [11]. In contrast, this study employs the UTASTAR method, which constructs an additive utility function from multiple financial criteria within a Multiple Criteria Decision Analysis (MCDA) framework.

Utility Additive STAR (UTASTAR) is a method for estimating additive utility functions within the framework of MCDA. It is used to construct a global utility function based on the evaluation of a set of alternatives across multiple criteria [12, 13, 14]. The method extends the original Utility Additives (UTA) approach by employing a linear programming formulation in which differences in marginal utilities between successive criterion levels are treated as decision variables. It also accommodates preference inconsistencies through the introduction of two error variables, resulting in more stable and accurate estimations of utility functions and alternative rankings [15, 16, 17]. These advantages enable UTASTAR to generate a single global utility value for each alternative while preserving the consistency of the preference ordering implied by the evaluation criteria [18]. In this study, Portfolio evaluation involves multiple fundamental criteria, including profitability, valuation, firm size, and liquidity-related indicators, which jointly characterize the attractiveness of investment alternatives. This complexity requires a decision-making framework capable of systematically handling multiple conflicting objectives while constructing an interpretable additive utility function from these financial criteria. To address this, the UTASTAR method is employed to derive an additive utility function based on selected firm-specific financial indicators. The resulting utility function systematically represents the relationship between these indicators and the relative attractiveness of the evaluated assets. As a MCDA preference disaggregation method, UTASTAR is applied to portfolio selection problems involving multiple assets, multiple evaluation criteria, and conflicting objectives, such as maximizing expected return, minimizing risk, and maximizing utility.

As research on UTASTAR has advanced, several variants have been developed to accommodate different data characteristics and decision-making problems. For example, WUTASTAR utilizes interval-valued data to address uncertainty in supplier evaluation [15], Fuzzy UTASTAR integrates fuzzy logic to support decision-making in the transportation domain [12], and IM-UTASTAR combines quantitative and qualitative data represented by intuitionistic multiplicative numbers for low-carbon tourism destination selection[14]. However, these variants are not adopted in the present study because all input data consist of objective numerical values derived from companies' fundamental indicators. Consequently, interval-based uncertainty representation, fuzzy logic, and linguistic information are unnecessary in this context. Instead, this study extends the conventional UTASTAR framework by modifying the objective function to not only minimize the fitting error but also penalize violations of the preference ordering (ranking violations). This approach is consistent with recent developments in regularization-based preference disaggregation methods, which balance multiple objective function components through penalty parameters [19].

Furthermore, this study differs from several previous applications of UTASTAR that consider only adjacent preference pairs, such as warehouse location selection [18], and the determination of decision criteria weights for renewable energy cooperation between the European Union and North Africa [13]. In contrast, the proposed approach incorporates all pairs of alternatives that satisfy the preference relation, thereby exploiting more comprehensive preference information during the estimation of the utility function rather than restricting the estimation process to adjacent ranking pairs. Furthermore, based on the literature review, the application of UTASTAR in stock investment remains relatively limited. One relevant study applied UTASTAR to the Iranian Stock Exchange, where the preference model was constructed based on expert evaluations of risk, return, and liquidity [20] In contrast, the present study performs preference disaggregation using a data-driven preference proxy generated through the Entropy Weight Method (EWM) based on four corporate fundamental indicators. The resulting additive utility function is then integrated into the Compromise Programming (CP) model as one of the objective functions for portfolio optimization.

The EWM is used to assess the variation in data values among several alternatives to avoid relying on subjective judgments [21]. In this study, EWM is employed to calculate objective weights for four corporate fundamental indicators and to generate pseudo-utility values, which are subsequently converted into a pseudo-ranking as a data-driven proxy for preferences. This pseudo-ranking is then used as reference information in the preference disaggregation process to estimate the global utility function using UTASTAR. To reduce look-ahead bias and provide a more realistic evaluation, the data are divided chronologically into a training dataset for utility function estimation and a testing dataset used exclusively for out-of-sample portfolio evaluation. The estimated utility function is subsequently incorporated into CP, a multi-criteria optimization method introduced by Zeleny (1973) that identifies compromise solutions by minimizing the distance to the ideal point [22]. An extension of CP is Nadir Compromising Programming (NCP), which uses the nadir (anti-ideal) point as a reference point when information regarding the ideal point is unavailable or less relevant [23]. Empirical studies show that the L_2-norm (Euclidean) can represent proportional geometric distances across all criterion dimensions, produce stable and more efficient solutions compared to other norms [24]. Accordingly, the L_2-norm is adopted in this study because previous studies have shown that it provides a balanced compromise among multiple objectives while preserving proportional distances across criteria.

Most UTASTAR applications employ expert-provided preference information and terminate at the ranking stage without integrating the estimated utility function into a subsequent portfolio optimization model. This study addresses these limitations by proposing a data-driven portfolio optimization framework in which the utility function estimated from selected financial criteria is integrated into CP to determine a compromise solution that simultaneously considers the trade-offs among return, risk, and utility. The effectiveness of the proposed framework is evaluated by comparing its portfolio performance with the classical Markowitz Mean–Variance model and an Equal-Weighted portfolio. Accordingly, the proposed framework offers an interpretable decision-support tool for retail investors by translating multiple financial criteria into a portfolio that balances return, risk, and utility. Therefore, this study contributes to the development of interpretable multi-criteria portfolio selection frameworks, particularly for applications in the Indonesian Islamic stock market.

2. Methodology

In this study, a data-driven multi-criteria portfolio selection framework integrating objective weighting, preference disaggregation, and multi-objective optimization is proposed. To evaluate the generalization capability of the proposed framework, the stock alternatives are partitioned into separate training and testing subsets. The training set is used to construct the UTASTAR utility model, whereas the testing set is reserved for out-of-sample evaluation.

The proposed framework consists of three stages: (i) objective weighting using the Entropy Weight Method (EWM) to generate pseudo-rankings from four fundamental criteria, namely Return on Equity (ROE), Stock Turnover Ratio (STR), Market Capitalization (MarCap), and Price-to-Book Value (PBV); (ii) preference disaggregation using UTASTAR to estimate an additive utility function from the training data; and (iii) portfolio optimization using CP, where expected return, portfolio risk (variance), and the estimated utility are simultaneously optimized. By separating the evaluation criteria used in UTASTAR from the optimization objectives, the proposed framework avoids redundant information and prevents the overemphasis of return and risk during the utility estimation process. The data format and evaluation criteria are adapted to the classical UTASTAR framework.

2.1. Assumptions and Dataset

In order to preserve methodological rigor and analytical coherence, a set of fundamental premises was created prior to the formalization of the suggested optimization model, see assumption 2.1 below:

Assumption 2.1 (Performance Matrix [12, 14, 15, 16, 17, 18, 25]). Let $A = \{a_1, a_2, \ldots, a_n\}$ denote a finite set of stock alternative consisting of n alternatives, where $a_i \in A,\ \forall i = 1, 2, \ldots, n$ with $C = \{c_1, c_2, \ldots, c_m\}$ denote the set of evaluation criteria consisting of m criteria, where $c_j \in C,\ \forall j = 1, 2, \ldots, m$. The performance value of stock alternative a_i under of criterion c_j is denoted by $g_j(a_i) = g_{i,j}$, with the normalized performance value is denoted by $g^N_{i,j} = z_{i,j}$ so the normalized decision matrix is expressed as:

$$\mathbf{Z} = [z_{i,j}] \in \mathbb{R}^{n\times m} \tag{1}$$

The empirical analysis is based on 35 Sharia-compliant stocks that remained continuously listed in the JII70 for five consecutive years (November 2020–November 2025), as reported by IDX Islamic [27]. Raw financial and market data were collected from company annual reports, Indonesian Stock Exchange (IDX), and `investing.com`. This sampling criterion provides a stable sample and facilitates data comparability throughout the observation period.

Inspired by the data-splitting principle proposed in [28], the dataset is defined as follows: Let the complete set of reference alternatives be denoted by $A = \{a_1, a_2, \ldots, a_n\}$ where n is the total number of alternatives. The dataset is partitioned into a training set $A_{\text{tr}} \subset A$, where $A_{\text{tr}} = a_1, a_2, \ldots, a_{n_{\text{tr}}}$ with $n_{\text{tr}} < n$, and the testing set be denoted by $A_{\text{te}} = A \setminus A_{\text{tr}}$, where $n_{\text{te}} + n_{\text{tr}} = n$. The training set is used to estimate the parameters of the UTASTAR model, whereas the testing set is employed to evaluate the ranking consistency and generalization capability of the estimated utility function on unseen stock alternatives.

Given the limited number of stock alternatives, the training–testing split ratio requires careful consideration. Common split ratios include 80:20, 70:30, 60:40, and 50:50 [28, 29], with 80:20 being the most widely adopted. However, because this study involves only 35 stock alternatives, a 70:30 split was selected. An 80:20 split would allocate only seven stocks to the testing set, which may provide a less representative basis for out-of-sample evaluation. The 70:30 split therefore offers a more balanced compromise between model estimation and evaluation.

Table 1 summarizes the financial evaluation criteria used to construct the utility model, together with their classifications and descriptions.

Table 1. Evaluation Criteria to Building UTASTAR Model's

Description	Notation	Financial Characteristic	Type
ROE	c_1	Profitability	Benefit
STR	c_2	Stock Liquidity	Benefit
MarCap	c_3	Firm Size	Benefit
PBV	c_4	Valuation	Cost

Benefit criteria are those for which higher values are preferred, whereas cost criteria are those for which lower values are desirable. The selected criteria capture key aspects of firm fundamentals, namely profitability, stock

liquidity, firm size, and valuation, thereby providing objective financial information for constructing the additive utility model.
ROE measures the profitability generated from shareholders' equity and reflects the return earned on shareholders' investments [30, 31].

$$ROE = \frac{\textit{Net Income}}{\textit{Shareholders' Equity}} \tag{2}$$

MarCap represents the total market value of a company, calculated as the product of the share price and the number of outstanding shares [32, 33, 34].

$$MarCap = \textit{Share Price} \times \textit{Shares Outstanding} \tag{3}$$

To reduce scale differences among firms and mitigate the influence of extremely large market capitalization values, the MarCap values are transformed using the natural logarithm (ln) [35, 36]. Accordingly, the transformed market capitalization is expressed as:

$$\ln(MarCap)$$

STR measures stock liquidity by comparing trading volume with the number of outstanding shares, where a higher STR indicates greater liquidity and stronger investor interest [37, 38, 39].

$$STR = \frac{\textit{Trading Volume}}{\textit{Shares Outstanding}} \tag{4}$$

PBV compares a company's market price per share with its book value per share to indicate whether a stock is relatively undervalued or overvalued [34], [40].

$$PBV = \frac{\textit{Price per Share}}{\textit{Book Value per Share}} \tag{5}$$

The book value per share is calculated from total equity divided by the number of outstanding shares [41].

$$BV = \frac{\textit{Total Equity}}{\textit{Shares Outstanding}} \tag{6}$$

Moreover, The construction of the portfolio optimization model relies on several financial measures. Return on assets is defined based on Definition 2.1:

Definition 2.1 (Asset Return [42]). Let $P_{i,t}$ denote the closing price of asset a_i at time t. The simple return (excluding dividends) of asset a_i at time t, denoted by $R_{i,t}$, is defined as:

$$R_{i,(t-1,t)} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}. \tag{7}$$

Accordingly, the expected (average) daily return of asset a_i over T trading days is calculated as follows [42, 43]:

$$\mathbb{E}(R_i) = \mu_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t} \tag{8}$$

This study employs the variance of returns as the risk measure due to its simplicity and its compatibility with the mean–variance optimization framework. The variance of the return of alternative a_i denoted by σ_i^2, [43] is defined as:

$$\text{Var}(R_i) = \sigma_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_{i,t} - \mathbb{E}(R_i))^2 \tag{9}$$

and Covariance between alternative Return a_α and a_β is defined as [43]:

$$\text{Cov}(R_\alpha, R_\beta) = \sigma_{\alpha,\beta} = \frac{1}{T-1}\sum_{t=1}^{T}(R_{\alpha,t} - \mathbb{E}(R_\alpha))(R_{\beta,t} - \mathbb{E}(R_\alpha)), \quad \alpha, \beta = 1, \ldots, n. \tag{10}$$

2.2. *Portfolio Optimization*

Let w_i represents the proportion of the total investment allocated to asset a_i and $\mathbf{w}$ denote the vector of portfolio weights, defined as:

$$\mathbf{w} = (w_1, w_2, \ldots, w_n)^\top \tag{11}$$

The portfolio is assumed to be fully invested, satisfying the constraint [43]

$$\sum_{i=1}^{n} w_i = 1 \tag{12}$$

The Portfolio Expected Return is defined as:

$$\mathbb{E}(R_{port}) = \sum_{i=1}^{n} w_i \mathbb{E}(R_i) \tag{13}$$

and The Portfolio Return in matrix form can be expressed as:

$$\boldsymbol{\mu}_{port} = \mathbf{w}^\top \boldsymbol{\mu}, \tag{14}$$

The Portfolio Risk (Variance) is defined as [42, 43]:

$$\sigma^2_{port} = \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} w_\alpha w_\beta \text{Cov}(R_\alpha, R_\beta) \tag{15}$$

and the portfolio risk in matrix form can be expressed as:

$$\boldsymbol{\sigma}^2_{port} = \mathbf{w}^\top \boldsymbol{\Sigma} \mathbf{w}, \tag{16}$$

where, $\boldsymbol{\Sigma}$ is covariance matrix, and

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nm} \end{bmatrix}. \tag{17}$$

The mean–variance portfolio optimization proposed by Markowitz is one of the most widely used approaches in modern portfolio theory. The model is based on the fundamental trade-off between expected return and risk, where investors seek to maximize the expected return for a given level of risk or, equivalently, minimize the portfolio risk for a desired level of expected return. These two equivalent optimization formulations are expressed [44, 45]:

(i) Maximum Return Portfolio

$$\begin{aligned} \max_{\mathbf{w}} \quad & \mathbf{w}^T \boldsymbol{\mu} \\ \text{s.t.} \quad & \mathbf{w}^T \mathbf{e} = 1, \\ & \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \leq R^*, \\ & \mathbf{w} \geq 0. \end{aligned} \tag{18}$$

(ii) Minimum Variance (Risk) Portfolio

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^T \mathbf{e} = 1, \\ & \mathbf{w}^T \boldsymbol{\mu} \geq R_*, \\ & \mathbf{w} \geq 0. \end{aligned} \tag{19}$$

where,

$\mathbf{w}$: the portfolio weight vector
$\boldsymbol{\mu}$: the expected return vector
$\boldsymbol{\Sigma}$: the covariance matrix
$\mathbf{e}$: a vector whose elements are all equal to 1 ($\mathbf{e}^\top = \begin{bmatrix}1 & 1 & \cdots & 1\end{bmatrix}$)
R^* : the maximum allowable portfolio risk (risk budget)
R_* : the minimum required expected portfolio return

To ensure that both portfolio return and risk are expressed on the same annual time horizon, the daily expected return vector and covariance matrix are annualized. Let $\mathcal{D}$ denote the number of trading days in one year. The annualized quantities are given by [43]:

$$\boldsymbol{\mu}^{annual} = \mathcal{D} \times \boldsymbol{\mu}, \tag{20}$$

and annualized Covariance matrix is:

$$\boldsymbol{\Sigma}^{annual} = \mathcal{D} \times \boldsymbol{\Sigma}. \tag{21}$$

2.3. Data Preprocessing and Objective Information Extraction

Objective information extraction and cross-criterion comparability were ensured by normalizing the raw performance data prior to entropy-based weighting.

2.3.1. Min-Max Normalization Normalization is a preliminary step to ensure that values across different criteria are directly comparable while preserving the proportional differences among alternatives, thus preparing objective information for entropy-based weight calculation, follow as [46, 47, 48]:

$$\mathrm{z}_{i,j} = \begin{cases} \dfrac{g_{i,j} - g_j^{min}}{g_j^{\max} - g_j^{\min}}, & \text{if } c_j \text{ is benefit type,} \\ \dfrac{g_j^{\max} - g_{i,j}}{g_j^{\max} - g_j^{\min}}, & \text{if } c_j \text{ is cost type.} \end{cases} \tag{22}$$

where $z_{i,j}$ denotes the normalized performance value of alternative a_i under criterion c_j. This transformation maps all criteria onto the unit interval [0,1], thereby ensuring cross-criterion comparability and non-negativity of the normalized values, which are required conditions in entropy-based weighting schemes [46, 47]. Moreover, min–max normalization preserves the proportional differences among alternatives (stocks), a property that contributes to the stability and interpretability of entropy-based multi-criteria evaluations [48].

2.3.2. Entropy Weight Method (EWM) The EWM is used to obtain objective criterion weights based on the variability of market data. The normalized performance values are first transformed into proportional measures that capture each alternative's relative contribution to each criterion. Let p_{ij} denote the proportion of alternative a_i with respect to criterion c_j defined as [49]:

$$p_{i,j} = \frac{z_{i,j}}{\sum_{i=1}^{n} z_{i,j}}, \quad \forall i = 1, \dots, n, \ \forall j = 1, \dots, m. \tag{23}$$

where n denotes the number of alternatives and m denotes the number of criteria. e_j denotes the entropy value of criterion c_j, which measures the degree of dispersion of the criterion values across alternatives. A lower entropy value indicates a higher information content and stronger discriminating ability among alternatives. The entropy value is computed as [21, 48, 49]:

$$e_j = -\lambda \sum_{i=1}^{n} p_{i,j} \ln(p_{i,j}), \quad \lambda = \frac{1}{\ln(n)} \tag{24}$$

where λ is a normalization constant used to ensure that the entropy value satisfies $0 \leq e_j \leq 1$, where a lower entropy indicates that the corresponding criterion contains more informative content for the weighting process. d_j

denotes the divergence degree of criterion c_j, measuring its discriminating power among alternatives [48]:

$$d_j = 1 - e_j \tag{25}$$

consider ω_j to represent the entropy-based weight, which is defined as [49]:

$$\omega_j = \frac{d_j}{\sum_{j=1}^{m} d_j}, \text{ where } \sum_{j=1}^{m} \omega_j = 1 \tag{26}$$

2.3.3. Pseudo-Ranking Construction The criterion weights obtained from EWM are subsequently used to calculate a pseudo-score for each individual stock, which forms the initial pseudo-ranking of asset alternatives. This ranking serves as an initial objective reference representing investor preferences based on market data prior to utility modelling and portfolio optimization. Consider $Ps(a_i)$ denotes the pseudo-score from asset a_i, so a composite pseudo-score for each asset is computed as [48]:

$$Ps(a_i) = \sum_{j=1}^{m} \omega_j z_{i,j} \tag{27}$$

consider $a_{(k)}$ to represent the alternative ranked k-th based on the pseudo-scores, which induces a complete ordinal ranking of all alternatives as follows:

$$a_{(1)} \succ a_{(2)} \succ \cdots \succ a_{(K)}, \quad \forall k = 1, \dots, K \tag{28}$$

where $\succ$ indicates which is more desirable.

The pseudo-ranking results are used as a data-driven proxy for market preferences and serve as the primary ordinal reference in the UTASTAR preference disaggregation process. The pseudo-utility values ($Ps(a_i) = U_i^{Ps}$) in the training data are still utilized as complementary cardinal reference information to support the estimation process. However, the primary objective of the model is to maintain the consistency of ordinal preferences, while conformity to the pseudo-utility (cardinal) values serves only as supporting information and is not the primary optimization objective. Therefore, the model does not aim to directly reproduce pseudo-utility values, but rather to construct a utility function that maintains the ordinal preference structure while still utilizing cardinal information as a complementary reference.

2.4. Preference Disaggregation via UTASTAR

The pseudo-ranking serves as an ordinal reference in the UTASTAR disaggregation process, providing an objective approximation of investor preferences without explicitly using EWM weights.

2.4.1. Utilities Function A utility function is used in multi-criteria decision making to map the performance of an alternative to a numerical value representing investor preferences. Consider the following utility function:

Definition 2.2. [50] The utility function $u(x)$ is defined as a function $u : X \to \mathbb{R}$, where X is the set of consumption goods. For $x, y \in X$, the statement $u(x) > u(y)$ means that the consumer prefers x to y, while $u(x) = u(y)$ indicates that the consumer is indifferent between the two. Util is the unit of the utility function.

In economic and investment utility theory, utility functions are typically assumed to be monotonically increasing and concave, reflecting preferences for higher performance and risk aversion [50]. However, in the UTASTAR framework, the utility function is inferred from the ranking of reference alternatives through a preference disaggregation procedure rather than being specified a priori [20, 12, 51]. UTASTAR was utilized as a preference calibration stage prior to portfolio optimization, producing marginal utility functions for each criterion and a global utility function by aggregating them. The estimated marginal and global utility functions are required to be monotonically non-decreasing [12, 20, 13, 51], without imposing a specific functional shape such as concavity or convexity

2.4.2. Marginal Utility For each marginal utility function $u_j(\cdot)$ is assumed to be continuous and piecewise-linear. To calculate the marginal value of each asset, the ranges for each criterion must first be established. Let z_j represent the general range of the criteria, with $[z_j^{min},\ z_j^{\max}]$ denoting the worst and best observed normalized values for criterion c_j, respectively (Figure 1) [12, 14].

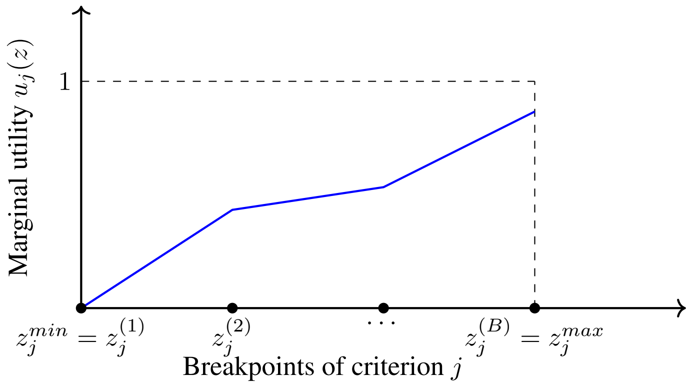


Figure 1. Piecewise linear marginal utility function.

Since the normalized criterion values satisfy $z_{i,j} \in [z_j^{\min},\ z_j^{\max}] = [0,\ 1]$ the interval is uniformly partitioned into $(B-1)$ equal subintervals, producing B cutting points (breakpoints) which $z_j^{(\ell)}$ with $\ell = 1, 2, \ldots, B$. The interval is uniformly partitioned into $B-1$ equal subintervals with step size:

$$\Delta = \frac{z_j^{\max} - z_j^{\min}}{B-1} \tag{29}$$

Accordingly, the ℓ-th breakpoint is given by:

$$z_j^{(\ell)} = z_j^{\min} + (\ell - 1)\Delta \tag{30}$$

subtitution Δ value:

$$z_j^{(\ell)} = z_j^{\min} + (\ell - 1)\frac{z_j^{\max} - z_j^{\min}}{B-1} \tag{31}$$

obtained that:

$$z_j{}^{(\ell)} = z_j{}^{\min} + \frac{\ell - 1}{B-1}(z_j{}^{\max} - z_j{}^{\min}) \qquad \forall \ell = 1, 2, \ldots, B \tag{32}$$

Thus, value of ℓ breakpoint with $z_j{}^{\min} = 0$ and $(z_j{}^{\max} - z_j{}^{\min}) = 1$ is given by:

$$z_j{}^{(\ell)} = \frac{\ell - 1}{B-1}, \qquad \forall \ell = 1, 2, \ldots, B \tag{33}$$

For any alternative a_i whose normalized performance satisfies:

$$z_j{}^{(\ell)} \leq z_{i,j} \leq z_j{}^{(\ell+1)} \tag{34}$$

The Marginal Utility is computed using linear interpolation as follows:

$$u_j(z_{i,j}) = u_j\left(z_j^{(\ell)}\right) + \frac{z_{i,j} - z_j^{(\ell)}}{z_j^{(\ell+1)} - z_j^{(\ell)}}\left[u_j\left(z_j^{(\ell+1)}\right) - u_j\left(z_j^{(\ell)}\right)\right], \qquad \forall \ell = 1, 2, \ldots, B-1. \tag{35}$$

the piecewise linear marginal utility function for criterion c_j can be expressed as:

$$u_j(z) = \begin{cases} u_j\left(z_j^{(1)}\right) + \dfrac{z - z_j^{(1)}}{z_j^{(2)} - z_j^{(1)}} \left[u_j\left(z_j^{(2)}\right) - u_j\left(z_j^{(1)}\right)\right], & z \in \left[z_j^{(1)}, z_j^{(2)}\right], \\ u_j\left(z_j^{(2)}\right) + \dfrac{z - z_j^{(2)}}{z_j^{(3)} - z_j^{(2)}} \left[u_j\left(z_j^{(3)}\right) - u_j\left(z_j^{(2)}\right)\right], & z \in \left[z_j^{(2)}, z_j^{(3)}\right], \\ \vdots & \vdots \\ u_j\left(z_j^{(B-1)}\right) + \dfrac{z - z_j^{(B-1)}}{z_j^{(B)} - z_j^{(B-1)}} \left[u_j\left(z_j^{(B)}\right) - u_j\left(z_j^{(B-1)}\right)\right], & z \in \left[z_j^{(B-1)}, z_j^{(B)}\right]. \end{cases} \tag{36}$$

Generalizing the marginal utility function for criterion c_j given by:

$$u_j(z) = u_j\left(z_j^{(\ell)}\right) + \frac{z - z_j^{(\ell)}}{z_j^{(\ell+1)} - z_j^{(\ell)}} \left[u_j\left(z_j^{(\ell+1)}\right) - u_j\left(z_j^{(\ell)}\right)\right], \qquad \forall z \in [{z_j}^{(\ell)},\, {z_j}^{(\ell+1)}] \tag{37}$$

where,

ℓ	:	the interval index between consecutive breakpoints, $\ell = 1, 2, \ldots, B-1$
j	:	the criterion index, $j = 1, 2, \ldots, m$
B	:	the total number of breakpoints
z	:	the value of an alternative on criterion j
$u_j(z)$	:	the marginal utility value of criterion j at value z
$z_j^{(\ell)}$	:	the ℓ-th breakpoints under criterion j
$u_j(z_j^{(\ell)})$	:	the corresponding marginal utility values at (ℓ)-th breakpoints
$z_j^{(\ell+1)}$	:	$(\ell+1)$-th breakpoint under criterion j
$u_j(z_j^{(\ell+1)})$	:	the corresponding marginal utility values at $(\ell+1)$-th breakpoints

2.4.3. Global utility Function The global utility of alternative a_i, denoted by $U(a_i) = U_i$, is computed using an additive aggregation model as follows [12, 15, 18]:

$$U_i = \sum_{j=1}^{m} u_j(z_{i,j}), \qquad i = 1, 2, \ldots, n_{\text{tr}}, \tag{38}$$

where,

U_i	:	the global utility of the i-th alternative estimated using the UTASTAR model
$u_j(z_{i,j})$	:	the marginal utility function associated with criterion j
U_i^P	:	the pseudo utility of the i-th alternative used as the reference information
n_{tr}	:	the number of alternatives in the training dataset
m	:	the number of evaluation criteria

UTASTAR is a preference disaggregation method that estimates an additive utility function from the ranking of reference alternatives by solving a linear optimization problem. In the original UTASTAR formulation, discrepancies between the estimated additive utility function and the reference preferences are represented by overestimation and underestimation deviation variables, commonly denoted as σ^+ and σ^-, which are incorporated directly into the utility estimation model [12, 14, 15, 16, 17, 18, 25].

In contrast, the proposed approach preserves the classical additive representation of the UTASTAR global utility function while introducing two distinct groups of deviation variables. The first group consists of the fitting deviation variables, ζ^+ and ζ^-, which measure the discrepancy between the estimated utility values and the pseudo-utility values generated by the Entropy Weight Method (EWM). The second group consists of the ranking deviation variables, ξ^+ and ξ^-, which enforce the consistency of the estimated utility function with the preference ordering of the reference alternatives. By separating these two types of deviations, the proposed formulation independently controls utility fitting errors and violations of the preference ordering within the optimization model.

Accordingly, the proposed model can be regarded as a modified UTASTAR formulation. Although it retains the classical additive utility structure, the deviation variables used in the original formulation are replaced by separate fitting and ranking deviation variables that explicitly distinguish utility estimation errors from preference-ordering violations. Consequently, the original variables σ^+ and σ^- are not introduced explicitly, thereby avoiding redundant representations of the same sources of deviation while preserving the fundamental principles of the UTASTAR methodology.

2.5. Constraints of UTASTAR Disaggregation

Since the reference pseudo utility serves as a data-driven proxy for market preference rather than an exact target to be reproduced, the UTASTAR estimation procedure is formulated with a set of constraints that preserve consistency with the reference information while ensuring a valid additive utility model. These constraints include model fit, ranking consistency, marginal utility monotonicity, utility scale normalization, and non-negativity of the deviation variables.

2.5.1. Model's fit Constraints. In the UTASTAR estimation stage, the reference pseudo utility is used as a data-driven proxy for market preference to guide the estimation of the global utility function. The proposed formulation exploits both the cardinal information contained in the pseudo utility values and the ordinal information represented by the corresponding preference ranking. The cardinal pseudo utility provides a reference for estimating the utility scale through controlled fitting constraints, whereas the preference ranking serves as the primary reference for preserving the relative ordering of alternatives. Therefore, the model does not seek to exactly reproduce the pseudo utility values but instead estimates an additive utility function that remains numerically close to the reference while prioritizing ranking consistency.

$$U_i \approx U_i^{Ps}, \quad i = 1, 2, \dots, n_{\mathrm{tr}}. \tag{39}$$

The deviation between the estimated global utility and the reference pseudo utility is represented by two non-negative deviation variables:

$$U_i - U_i^{Ps} = \zeta_i^+ - \zeta_i^-, \quad i = 1, 2, \dots, n_{\mathrm{tr}}, \tag{40}$$

the deviation variables satisfy by:

$$\zeta_i^+ \geq 0, \qquad \zeta_i^- \geq 0, \qquad \text{and} \qquad i = 1, 2, \dots, n_{tr}$$

where,

ζ_i^+ : Non-negative variable representing positive deviations

ζ_i^- : Non-negative variable representing negative deviations

Although the deviation variables quantify the difference between the estimated global utility and the reference pseudo utility, the proposed model does not enforce an exact fit. Instead, a fitting tolerance parameter ε_{fit} is introduced to allow small deviations that do not compromise the consistency with the reference information. The fitting requirement is therefore expressed as To avoid overfitting, a fitting tolerance parameter $\varepsilon_{\text{fit}} \geq 0$ is introduced.

$$|U_i - U_i^{Ps}| \leq \varepsilon_{\text{fit}}$$

the transformation to linear form defined as

$$-\varepsilon_{\text{fit}} \leq U_i - U_i^{Ps} \leq \varepsilon_{\text{fit}}$$

To allow controlled violations while maintaining feasibility, the lower bound is relaxed by the non-negative deviation variable ζ_i^-, whereas the upper bound is relaxed by the non-negative deviation variable ζ_i^+.

Lower bound constraint

$$-\varepsilon_{\text{fit}} - \zeta_i^- \leq U_i - U_i^{Ps}$$

Upper bound constraint

$$U_i - U_i^{Ps} \leq \varepsilon_{\text{fit}} + \zeta_i^+$$

Therefore, small deviations within the tolerance range are allowed without being penalized. The fitting constraint is formulated as

$$-\varepsilon_{\text{fit}} - \zeta_i^- \leq U_i - U_i^{Ps} \leq \varepsilon_{\text{fit}} + \zeta_i^+, \quad i = 1, 2, \ldots, n_{\text{tr}}. \tag{41}$$

where, ε_{fit} : Fitting tolerance. This formulation preserves the linear programming structure of the UTASTAR model while allowing both overestimation and underestimation of the estimated global utility relative to the reference pseudo utility. The fitting constraints allow the estimated global utility to utilize the cardinal information contained in the reference pseudo utility while preventing overfitting through the tolerance parameter ε_{fit}. Consequently, the estimated utility is encouraged to remain close to the reference utility values without requiring exact numerical replication. Although the cardinal pseudo utility contributes additional information to the estimation process, the primary objective of the proposed model is to preserve the ordinal preference structure represented by the reference ranking. Therefore, numerical differences between the estimated utility and the reference pseudo utility are acceptable provided that the resulting utility function maintains the intended ordering of alternatives.

2.5.2. Ranking Consistency Constraints. Let $a_{(k)}$ denote the alternative ranked in the k-th position according to the pseudo-ranking derived from the ordinal reference data, where $k = 1, 2, \ldots, n_{tr}$, the preference ordering is expressed as:

$$a_{(1)} \succeq a_{(2)} \succeq a_{(3)} \succ \cdots \succeq a_{(n_{\text{tr}})} \tag{42}$$

where, $\succeq$ denotes a weak preference relationship, which includes both strict preference ($\succ$) and indifference ($\sim$). Rather than considering only adjacent alternatives [13, 18],the proposed model leverages complete ordinal information by constructing all pairwise preference relations among the training alternatives.
The preference pairs that occur

$$\begin{gathered}(a_1, a_2), (a_1, a_3), (a_1, a_4), \ldots, (a_1, a_{n_{\text{tr}}}) \\ (a_2, a_3), (a_2, a_4), \ldots, (a_2, a_{n_{\text{tr}}}) \\ \vdots \\ (a_{n_{\text{tr}}-1}, a_{n_{\text{tr}}})\end{gathered}$$

Therefore, the set of preference pairs is defined

$$\mathcal{C} = \{(a_\alpha, a_\beta) \mid 1 \leq \alpha < \beta \leq n_{\text{tr}}\}$$

where, $\mathcal{C}$ denotes the combination of the set of all pairwise preference constraints [26].

$$\mathcal{C}_{(n_{\text{tr}},2)} = \binom{n_{\text{tr}}}{2} = \frac{(n_{\text{tr}})!}{2 \times (n_{\text{tr}} - 2)!} = \frac{(n_{\text{tr}}) \times (n_{\text{tr}} - 1) \times (n_{\text{tr}} - 2)!}{2 \times (n_{\text{tr}} - 2)!}$$

Consequently, the total number of pairwise comparisons is

$$|\mathcal{C}| = \binom{n_{\text{tr}}}{2} = \frac{n_{\text{tr}}(n_{\text{tr}} - 1)}{2}. \tag{43}$$

By combining all pairwise comparisons, the proposed formulation allows the estimated utility function to preserve not only the local ordering between adjacent alternatives but also the global ordinal structure across the training set. For a pair of alternatives satisfying

$$a_\alpha \succ a_\beta$$

the estimated global utility must satisfy the following.

$$U(a_\alpha) > U(a_\beta)$$

Given that $U(a_i) = U_i$, the above relation can be written as

$$U_\alpha > U_\beta \quad \forall(\alpha,\beta) \in \mathcal{K}^{\succ}, \quad \text{with} \quad \mathcal{K}^{\succ} \subset \mathcal{K}$$

where $\mathcal{K}^{\succ}$ denotes the subset of strictly preferred pairs. To ensure minimum discrimination between alternatives, a non-negative discrimination threshold $varepsilon_{rank}$ is introduced, which produces

$$U_\alpha - U_\beta \geq \varepsilon_{\text{rank}}$$

The global utility function is estimated from pseudo-rank, therefore a perfect consistency cannot always be achieved. To preserve model feasibility, two non-negative deviation variables $\xi^+_{\alpha,\beta}, \xi^-_{\alpha,\beta}$, are introduced (see Fig.2. The resulting ranking consistency constraint is formulated as [18].

$$U_\alpha - U_\beta \geq \varepsilon_{\text{rank}} - \xi^+_{\alpha,\beta} + \xi^-_{\alpha,\beta} \quad if \quad a_\alpha \succ a_\beta \tag{44}$$

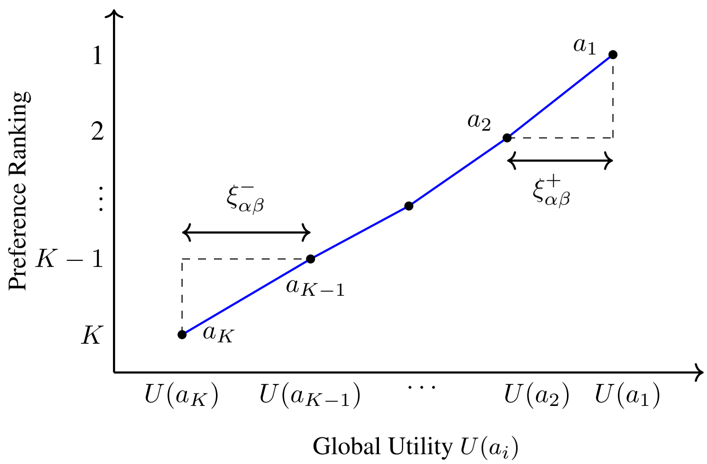


Figure 2. Illustration of the relationship between global utility and preference ranking in the UTASTAR model.

Furthermore, if two alternatives have the same pseudo-ranking $(a_\alpha \sim a_\beta)$, the estimated global utility values must satisfy

$$U_\alpha = U_\beta, \quad if \quad a_\alpha \sim a_\beta \tag{45}$$

where,

$\xi^+_{\alpha,\beta}$	:	Overestimation error associated with the preference constraint between alternatives a_α and a_β
$\xi^-_{\alpha,\beta}$	:	Underestimation error associated with the preference constraint between alternatives a_α and a_β
$\varepsilon_{\text{rank}}$	:	Positive threshold parameter specifying the minimum utility separation between two alternatives with a strict preference relation.

2.5.3. Monotonicity Constraints. Consider Equation (42), where the reference rankings yield the following strict preference ordering:

$$a_{(1)} \succ a_{(2)} \succ \cdots \succ a_{(K)}$$

For every pair of neighboring alternatives $(a_{(k)}, a_{(k+1)})$, the difference in their global utility is defined as:

$$\Delta(a_{(k)}, a_{(k+1)}) = U(a_{(k)}) - U(a_{(k+1)})$$

Since $a_{(k)} \succ a_{(k+1)}$, it follows that:

$$\Delta(a_{(k)}, a_{(k+1)}) \geq 0.$$

Using the additive form of the global utility function, and obtain:

$$\Delta(a_{(k)}, a_{(k+1)}) = \sum_{j=1}^{m} \big(u_j[z_j(a_{(k)})] - u_j[z_j(a_{(k+1)})]\big)$$

Because the marginal utility functions are piecewise linear, the change in utility for criterion c_j is represented by the utility increments between breakpoints:

$$u_j(z_j{}^{(\ell+1)} - u_j(z_j{}^{(\ell)})$$

Therefore, to ensure that every increase in performance does not decrease the utility, the utility increments between breakpoints must be non-negative, so that the following condition holds:

$$u_j(z_j^{(\ell+1)}) - u_j(z_j^{(\ell)}) \geq 0, \tag{46}$$

This constraint guarantees that the marginal utility functions are monotonically non-decreasing, thereby ensuring consistency with the reference rankings.

2.5.4. Normalization and Feasibility Constraints. The marginal utility functions are defined over a set of breakpoints for each criterion. Let $z_j^{(\ell)}, \quad \forall j = 1, 2, \ldots, m$ and $\ell = 1, 2, \ldots, B$ denote the ℓ-th breakpoint of criterion j, given by:

$$z_j^{(1)}, z_j^{(2)}, \ldots, z_j^{(B)}$$

Then, the utility values at the breakpoints are defined as:

$$u_j(z_j^{(1)}), u_j(z_j^{(2)}), \ldots, u_j(z_j^{(B)})$$

Subsequently, the utility increment between two consecutive breakpoints is defined as:

$$\Delta u_j^{(1)} = u_j(z_j^{(2)}) - u_j(z_j^{(1)})$$

$$\Delta u_j^{(2)} = u_j(z_j^{(3)}) - u_j(z_j^{(2)})$$

$$\vdots$$

$$\Delta u_j^{(B-1)} = u_j(z_j^{(B)}) - u_j(z_j^{(B-1)})$$

Hence, the utility increment for each segment can be generally expressed as follows:

$$\Delta u_j^{(\ell)} = u_j(z_j^{(\ell+1)}) - u_j(z_j^{(\ell)})$$

Therefore, the cumulative utility increment satisfies the following:

$$\sum_{\ell=1}^{B-1} \Delta u_j^{(\ell)} = [u_j(z_j^{(2)}) - u_j(z_j^{(1)})] + [u_j(z_j^{(3)}) - u_j(z_j^{(2)})] + \cdots + [u_j(z_j^{(B-1)}) - u_j(z_j^{(B)})]$$

$$\sum_{\ell=1}^{B-1} \Delta u_j^{(\ell)} = -u_j(z_j^{(1)}) + u_j(z_j^{(B)})$$

The cumulative utility increment can be expressed as

$$\sum_{\ell=1}^{B-1} \Delta u_j^{(\ell)} = u_j(z_j^{(B)}) - u_j(z_j^{(1)})$$

$$\sum_{\ell=1}^{B-1} \Delta u_j^{(\ell)} = u_j(z_j^{(B)}) - u_j(z_j^{(1)})$$

The marginal utility at the first breakpoint is normalized to zero and is defined as follows:

$$u_j(z_j^{(1)}) = 0 \tag{47}$$

it follows that

$$\sum_{\ell=1}^{B-1} \Delta u_j^{(\ell)} = u_j(z_j^{(B)})$$

equivalent to

$$\sum_{\ell=1}^{B-1} [u_j(z_j^{(\ell+1)}) - u_j(z_j^{(\ell)})] = u_j(z_j^{(B)})$$

where $z_j^{(B)} = z_j^{(best)}$ corresponds to the best performance level can be written as:

$$u_j(z_j{}^{best}) = \sum_{\ell=1}^{B-1} \left[u_j(z_j{}^{(\ell+1)}) - u_j(z_j{}^{(\ell)}) \right], \quad \forall j$$

Furthermore, because additive utility functions are unique only up to positive affine transformations, a normalization constraint is required to obtain a unique solution.

$$U^{max} = \sum_{j=1}^{m} u_j(z_j{}^{best})$$

Without loss of generality, the utility scale is set as:

$$U^{max} = 1$$

Thus, the UTASTAR normalization constraint:

$$\sum_{j=1}^{m} u_j(z_j{}^{best}) = 1 \tag{48}$$

which is equivalent to:

$$\sum_{j=1}^{m} \sum_{\ell=1}^{B-1} [u_j(z_j{}^{(\ell+1)}) - u_j(z_j{}^{(\ell)})] = 1$$

This normalization defines the global utility on the interval [0,1], thereby providing a unique utility scale and ensuring comparability across alternatives.

2.5.5. Linear Programming Formulation. Following the basic UTASTAR formulation proposed by [12, 14, 15, 16, 17, 18, 25], and adopting a less complex structure compared with the formulation presented by Zhao et al. [19], this study extends the objective function by introducing the weighting parameter Ω. While the conventional UTASTAR formulation relies solely on ordinal preference information, the proposed formulation additionally incorporates pseudo-cardinal utility values generated by the Entropy Weight Method (EWM) as supplementary reference information during model estimation. The modified linear programming formulation is expressed as follows:

$$\min F = \frac{1}{\Omega}\sum_{i=1}^{n_{\text{tr}}}(\zeta_i^+ + \zeta_i^-) + \Omega\sum_{(\alpha,\beta)\in\mathcal{K}}(\xi_{\alpha\beta}^+ + \xi_{\alpha\beta}^-), \qquad \text{with } \Omega > 1. \tag{49}$$

subject to:

$$\begin{cases} \text{Model fit constraints} & \\ -\varepsilon_{\text{fit}} - \zeta_i^- \le U_i - U_i^P \le \varepsilon_{\text{fit}} + \zeta_i^+, & \forall i = 1,2,\dots,n_{tr}, \\ \text{Ranking consistency constraints} & \\ U_\alpha - U_\beta \ge \varepsilon_{\text{rank}} - \xi_{\alpha\beta}^+ + \xi_{\alpha\beta}^-, & \text{if } a_\alpha \succ a_\beta, \\ U_\alpha = U_\beta, & \text{if } a_\alpha \sim a_\beta, \\ \text{Monotonicity constraints} & \\ u_j(z_j{}^{(\ell+1)}) - u_j(z_j^{(\ell)}) \ge 0, & \forall \ell = 1,2,\dots,B-1, \\ \text{Normalization constraint} & \\ u_j(z_j{}^{(1)}) = 0 \quad \text{and} \quad \displaystyle\sum_{j=1}^{m} u_j(z_j{}^{best}) = 1, & \forall j, \\ \text{Non-negativity constraints} & \\ \zeta_i^+ \ge 0 \quad \text{and} \quad \zeta_i^- \ge 0, & \forall a_i, i = 1,2,\dots,n_{tr} \\ \xi_{(\alpha,\beta)}^+ \ge 0 \quad \text{and} \quad \xi_{(\alpha,\beta)}^- \ge 0, & \forall (\alpha,\beta) \in \mathcal{K}. \end{cases}$$

where:

n_{tr} : Number of training alternatives used to estimate the marginal utility functions.
ζ_i^+ : Overestimation for training alternative i.
ζ_i^- : Underestimation for training alternative i.
K : Set of ordered pairs of alternatives (a_α, a_β) for which a strict preference relation is specified in the reference ranking.
$\xi_{\alpha,\beta}^+$: Overestimation error associated with the preference constraint between alternatives a_α and a_β.
$\xi_{\alpha,\beta}^-$: Underestimation error associated with the preference constraint between alternatives a_α and a_β.
Ω : Penalty coefficient assigned to ranking errors, where $\Omega > 1$, ensuring that violations of the preference ordering receive a larger penalty than fitting errors.
ε_{fit} : Small positive tolerance parameter controlling the maximum allowable fitting error in the estimated utility function.
$\varepsilon_{\text{rank}}$: Positive threshold parameter specifying the minimum utility separation between two alternatives with a strict preference relation.

The pseudo-cardinal utility serves only as auxiliary numerical guidance for estimating the global utility function and does not replace the ordinal preference structure. The weighting parameter Ω controls the trade-off between utility fitting errors and ranking inconsistency errors. Since $\Omega > 1$, the fitting error term is down-weighted by $1/\Omega$, whereas the ranking inconsistency term is emphasized by Ω. This design follows the preference disaggregation philosophy of UTASTAR, where preserving the ordinal preference ordering is prioritized over minimizing numerical deviations between the estimated and pseudo-cardinal utility values. Consequently, the estimated global utility function is guided by the pseudo-cardinal information while remaining primarily determined by the ordinal preference relations.

The variables $\xi^{+}_{\alpha,\beta}$ and $\xi^{-}_{\alpha,\beta}$ are auxiliary slack variables associated with the pairwise ranking constraints for each preference pair $(\alpha, \beta) \in K$, whereas ζ^{+}_{i} and ζ^{-}_{i} are auxiliary slack variables associated with the utility fitting constraints. These variables are used only during the estimation of the global utility function. After solving the linear programming model, the estimated global utility function is used as an input to the subsequent Compromise Programming portfolio optimization.

2.6. Compromise Programming (CP) Formulation

The objective of Compromise Programming (CP) is to determine the optimal portfolio allocation vector $\mathbf{w}$, defined in Equation (11), where $\mathbf{w} \in D \subset \mathbb{R}^n$ [23, 52].

$$\mathbf{w} = (w_1, \dots, w_n)^\top \in D \subset \mathbb{R}^n$$

where $\mathbf{w}$ denotes the portfolio allocation vector and w_i denotes the proportion of capital invested in asset a_i with $i = 1, 2, \dots, n$. The portfolio is assumed to be fully invested and long-only, satisfying the following constraints:

$$\sum_{i=1}^{n} w_i = 1, \qquad w_i \geq 0, \quad i = 1, \dots, n.$$

Here, D denotes the feasible set of portfolios defined as:

$$D = \left\{ \mathbf{w} \in \mathbb{R}^n \;\middle|\; \sum_{i=1}^{n} w_i = 1,\ w_i \geq 0,\ i = 1, \dots, n \right\}$$

2.6.1. Multi Objective Function. Let $f_h : D \to \mathbb{R}, \quad h = 1, 2, \dots, H$ denote the objective function. The multi-objective vector function is defined:

$$f(\mathbf{w}) = (f_1(\mathbf{w}), f_2(\mathbf{w}), \dots, f_H(\mathbf{w}))^\top$$

To obtain an optimal portfolio, investors allocate their wealth among risky assets while pursuing three decision-making objectives: maximizing expected returns, minimizing portfolio risk, and maximizing expected utility. The first objective, maximizing expected returns, represents the potential for investment gains. Although previous studies have reported a positive relationship between return and risk [53], investment decision makers are generally assumed to exhibit loss-averse behavior, implying that they tend to avoid potential losses when making investment decisions [54]. Consequently, the second objective is to minimize portfolio risk in order to reduce investment uncertainty. Since risk cannot be completely eliminated, this study measures risk using portfolio variance. The third objective is to maximize the utility value derived from the UTASTAR model, where the utility function provides a mathematical representation of data-driven preference structures constructed from selected financial indicators.

$$f_1(\mathbf{w}) : \text{portfolio Risk}, \quad f_2(\mathbf{w}) : \text{portfolio Return}, \quad f_3(\mathbf{w}) : \text{UTASTAR-based utility}.$$

These objectives are formulated as follows:

$$f(w) \begin{cases} f_1(\mathbf{w}) = \text{Risk}(\mathbf{w}) = \mathbf{w}^T \Sigma \mathbf{w} & \text{(minimization)} \\ f_2(\mathbf{w}) = \text{Return}(\mathbf{x}) = \mathbf{w}\mu & \text{(maximization)} \\ f_3(\mathbf{w}) = \text{UTASTAR Utility}(\mathbf{w}) = \mathbf{w}\mathbf{U} & \text{(maximization)} \end{cases} \tag{50}$$

To formulate a consistent multi-objective minimization problem, the maximization objectives are transformed by

$$\widetilde{f}_1(\mathbf{w}) = f_1(\mathbf{w}), \quad \widetilde{f}_2(\mathbf{w}) = -f_2(\mathbf{w}), \quad \widetilde{f}_3(\mathbf{w}) = -f_3(\mathbf{w})$$

Thus, the transformed objective vector becomes:

$$\widetilde{f}(\mathbf{w}) = (\widetilde{f}_1(\mathbf{w}), \widetilde{f}_2(\mathbf{w}), \widetilde{f}_3(\mathbf{w}))^\top$$

Furthermore, a multi-objective optimization problem can be written as:

$$\min_{w \in D} \widetilde{f}(\mathbf{w}) = \min_{w \in D} \left(\widetilde{f}_1(\mathbf{w}), \widetilde{f}_2(\mathbf{w}), \widetilde{f}_3(\mathbf{w})\right)$$

Hence, the feasible set D is defined by the following constraints:

$$\sum_{i=1}^{n} w_i = 1, \qquad w_i \geq 0, \qquad i = 1, \ldots, n$$

2.6.2. Pareto-Optimality. The existence of conflicting objectives means that not all can be optimized simultaneously without sacrificing certain criteria. This creates a trade-off between risk, return, and utility, resulting in a set of efficient solutions called Pareto-optimal.

Definition 2.3 (Pareto-Optimal Solution [52])**.** Let $D \subseteq \mathbb{R}^n$ denote the feasible set and $f_h : D \to \mathbb{R}$, where $h = 1, \ldots, H$, is the objective function. Then, a feasible solution $\mathbf{w}^* \in D$ is said to be Pareto-optimal if it belongs to the Pareto set Λ_P, which contains all non-dominated feasible solutions and is defined as:

$$\Lambda^P = \{\mathbf{w}^* \in D \mid \nexists \mathbf{w} \in D \text{ such that } \mathbf{w} \prec \mathbf{w}^*\}$$

where a solution $\mathbf{w}$ is said to dominate $\mathbf{w}^*$ (denoted by $\mathbf{w} \prec \mathbf{w}^*$) if $f_h(\mathbf{w}) \leq f_h(\mathbf{w}^*), \quad \forall h = 1, 2, \ldots, H$ and $f_h(\mathbf{w}) < f_h(\mathbf{w}^*)$ for at least one h.

Consequently, any feasible solution that is dominated by another solution cannot be Pareto-optimal.

2.6.3. Ideal and Nadir Points. To select a representative solution from the Pareto set, CP introduces reference points that represent the best and worst values for each objective. The ideal point $f^{Ideal} = f^{\mathcal{I}}$, represents the best achievable value for each objective. The ideal point vector [23, 52]:

$$\mathbf{f}^{\mathcal{I}} = \left(f_1^{\mathcal{I}}, f_2^{\mathcal{I}}, \ldots, f_H^{\mathcal{I}}\right)^\top \in \mathbb{R}^H$$

Formally, for h-objectives, is defined as:

$$f_h^{\mathcal{I}} = \min_{w \in D} \widetilde{f}_h(\mathbf{w}), \quad h = 1, \ldots, H$$

Specifically, for the portfolio objectives:
$f_1^{\mathcal{I}} = \min_{w \in D} f_1(\mathbf{w})$,
$f_2^{\mathcal{I}} = \min_{w \in D} \widetilde{f}_2(\mathbf{w}) = \min_{w \in D} (-f_2(\mathbf{w})) = -\max_{w \in D} f_2(\mathbf{w})$,
$f_3^{\mathcal{I}} = \min_{w \in D} \widetilde{f}_3(\mathbf{w}) = \min_{w \in D} (-f_3(\mathbf{w})) = -\max_{w \in D} f_3(\mathbf{w})$.
Similarly, the nadir point $f^{\mathcal{N}adir} = f^{\mathcal{N}}$ represents the worst value over the Pareto set.

$$f_h^{\mathcal{N}} = \max_{w \in \Lambda^P} \widetilde{f}_h(\mathbf{w}), \quad h = 1, \ldots, H$$

For portfolio objectives:

$$f_1^{\mathcal{N}} = \max_{w \in \Lambda^P} f_1(\mathbf{w})$$

$$f_2^{\mathcal{N}} = \max_{w \in \Lambda^P} \widetilde{f}_2(\mathbf{w}) = \max_{w \in \Lambda^P} (-f_2(\mathbf{w})) = -\min_{w \in \Lambda^P} f_2(\mathbf{w})$$

$$f_3^{\mathcal{N}} = \max_{w \in \Lambda^P} \widetilde{f}_3(\mathbf{w}) = \max_{w \in \Lambda^P} (-f_3(\mathbf{w})) = -\min_{w \in \Lambda^P} f_3(\mathbf{w})$$

Hence, the nadir point vector is expressed as

$$\mathbf{f}^{\mathcal{N}} = \left(f_1^{\mathcal{N}}, f_2^{\mathcal{N}}, \ldots, f_H^{\mathcal{N}}\right)^\top \in \mathbb{R}^H$$

The points were subsequently used to normalize the deviations. This normalization ensures that all objectives are dimensionless and directly comparable within the CP framework.

2.6.4. L_γ Norm Metric. The normalized deviation of each objective function from the ideal point is calculated as follows [52]:

$$d_h(\mathbf{w}) = \frac{\widetilde{f}_h(\mathbf{w}) - f_h^{\mathcal{I}}}{f_h^{\mathcal{N}} - f_h^{\mathcal{I}}}, \quad h = 1, \ldots, H \tag{51}$$

The vector of deviation:

$$\mathbf{d}(\mathbf{w}) = \left(d_1(\mathbf{w}), d_2(\mathbf{w}), \ldots, d_H(\mathbf{w})\right)^\top \in \mathbb{R}^H$$

The weighted L_γ-metric of the deviation vector is defined as:

$$\| \mathbf{d}(\mathbf{w}) \|_{\gamma,\phi} = \left(\sum_{h=1}^{H} \phi_h \left|d_h(\mathbf{w})\right|^\gamma\right)^{1/\gamma}, \quad \gamma \geq 1$$

where ϕ_h denotes the relative importance (weight) of h-th objective function.
Thus, the L_γ-metric can be expressed as:

$$\min L_\gamma(\mathbf{w}) = \| \mathbf{d}(\mathbf{w}) \|_{\gamma,\phi} = \left(\sum_{h=1}^{H} \phi_h \left|d_h(\mathbf{w})\right|^\gamma\right)^{\frac{1}{\gamma}}$$

The CP optimization problem based on the L_γ-metric is formulated as follows [23, 52]:

$$\min L_p(\mathbf{w}) = \left(\sum_{h=1}^{H} \phi_h \left|\frac{\widetilde{f}_h(\mathbf{w}) - f_h^{\mathcal{I}}}{f_h^{\mathcal{N}} - f_h^{\mathcal{I}}}\right|^\gamma\right)^{\frac{1}{\gamma}}, \qquad \text{where} \quad \gamma = 1, 2 \quad \text{or} \quad \infty. \tag{52}$$

In this study, the importance weights among the objectives (ϕ_h) in the CP method are assumed to be equal, that is, $\phi_1 = \phi_2 = \phi_3 = 1/3$. This assumption reflects a neutral approach in which risk, return, and utility are given equal weight in portfolio optimization.

Norm L_γ for $\gamma = 1$ such as:

$$L_1(\mathbf{w}) = \| \mathbf{d}(\mathbf{w}) \|_{1,\phi} = \sum_{h=1}^{H} \phi_h \left|d_h(\mathbf{w})\right| = \sum_{h=1}^{H} \phi_h \left|\frac{\widetilde{f}_h(\mathbf{w}) - f_h^{\mathcal{I}}}{f_h^{\mathcal{N}} - f_h^{\mathcal{I}}}\right| \tag{53}$$

The distance metric reduces to the Manhattan norm (L_1), which captures the total absolute deviation from the ideal solution by applying a linear penalty to the deviations in each criterion :

Norm L_γ for $\gamma = 2$ such as:

$$L_2(\mathbf{w}) = \| \mathbf{d}(\mathbf{w}) \|_{2,\phi} = \left(\sum_{h=1}^{H} \phi_h \left|d_h(\mathbf{w})\right|^2\right)^{\frac{1}{2}} = \left(\sum_{h=1}^{H} \phi_h \left|\frac{\widetilde{f}_h(\mathbf{w}) - f_h^{\mathcal{I}}}{f_h^{\mathcal{N}} - f_h^{\mathcal{I}}}\right|^2\right)^{\frac{1}{2}} \tag{54}$$

The distance measure reduces to the Euclidean norm (L_2), which applies quadratic penalization. As a result, larger normalized deviations receive greater penalties, providing a smooth measure of the compromise distance from the ideal solution.

For $\gamma \to \infty$, the distance measure reduces to the Chebyshev norm (L_∞):

$$L_\infty(\mathbf{w}) = \| \mathbf{d}(\mathbf{w}) \|_{\infty,\phi} = \lim_{\gamma \to \infty} \left(\sum_{h=1}^{H} |d_h(\mathbf{w})|^\gamma \right)^{\frac{1}{\gamma}} = \max_{h=1,\ldots,H} |d_h(\mathbf{w})| \tag{55}$$

Emphasizes the maximum deviation (minimax), which accounts for the object with the greatest divergence from the ideal.

2.6.5. Portfolio Optimization Problem. Considering the multi-objective nature of portfolio selection, Compromise Programming (CP) is adopted to identify the portfolio solution that provides the closest distance to the ideal solution by balancing conflicting objectives, as formulated in previous studies [22, 23, 52]. Furthermore, to mitigate concentration risk and improve portfolio diversification, a maximum weight constraint is incorporated into the Compromise Programming framework. This constraint prevents excessive allocation to individual assets, thereby promoting a more balanced portfolio structure, consistent with the role of diversification in reducing portfolio-specific risk and improving risk management. The ultimate diversified portfolio optimization model is formulated as follows [55, 56]:

$$\begin{aligned} \min_{w \in D} \quad & L_\gamma(\mathbf{w}) \\ \text{s.t.} \quad & f_h(\mathbf{w}) \le f_h^{\mathcal{N}}, \quad h = 1, \ldots, H, \\ & \sum_{i=1}^{n} w_i = 1, \\ & 0 \le w_i \le 0.30, \quad i = 1, \ldots, n, \end{aligned} \tag{56}$$

where the normalized deviation is defined as:

$$d_h(\mathbf{w}) = \frac{\tilde{f}_h(\mathbf{w}) - f_h^{\mathcal{I}}}{f_h^{\mathcal{N}} - f_h^{\mathcal{I}}}, \qquad h = 1, \ldots, H.$$

The CP method employs the L_γ metric with $\gamma = 1, 2$, or ∞ to align with the investor's preferences in balancing multiple objectives. The constraint $f_h(w) \le f_h^{\mathcal{N}}$ guarantess that the resulting solution meets or exceeds the Nadir point, as defined by the Compromise Solution Problem.

The optimal portfolio is defined by:

$$\mathbf{w}^* = (w_1^*, w_2^*, \ldots, w_n^*)^\top \in D$$

where,

D	:	feasible set of portfolio allocations satisfying all portfolio constraints
$\mathbf{w}$	:	portfolio weight vector ($\mathbf{w} = (w_1, w_2, \ldots, w_n)$)
$\mathbf{w}^*$	:	portfolio allocation that achieves the best trade-off among risk, expected return, and UTASTAR utility
w_i	:	proportion of the total investment allocated to stock i
n	:	number of stock alternatives
H	:	number of objective functions
h	:	index of the objective function, $h = 1, \ldots, H$
$L_\gamma(\mathbf{w})$	:	L_γ metric measuring the normalized distance of portfolio $\mathbf{w}$ from the ideal solution
γ	:	Minkowski metric parameter controlling the compromise distance
$f_h(\mathbf{w})$	:	value of the h-th objective function for portfolio $\mathbf{w}$
$\tilde{f}_h(\mathbf{w})$	:	transformed value of the h-th objective function used for normalization
f_h^I	:	ideal value of the h-th objective function
f_h^N	:	nadir value of the h-th objective function
$d_h(\mathbf{w})$	:	normalized deviation of portfolio $\mathbf{w}$ from the ideal value for objective h

The optimal portfolio is obtained by minimizing the weighted normalized distance from the Ideal point while considering the Nadir point as the upper bound for normalization. Consequently, the resulting allocation represents a compromise solution that balances the conflicting objectives of minimizing portfolio risk while simultaneously maximizing expected return and UTASTAR utility.

3. Result

3.1. Preference Proxy and UTASTAR Utility Estimation

The proposed UTASTAR model was implemented using a fixed set of parameters determined through preliminary experiments. The fitting tolerance was set to $\varepsilon_{\mathrm{fit}} = 1 \times 10^{-3}$, while the ranking tolerance was fixed at $\varepsilon_{\mathrm{rank}} = 1 \times 10^{-4}$. The number of breakpoints was set to B=4, which provided a feasible solution while maintaining the desired properties of the piecewise linear marginal utility functions. The same parameter configuration was applied throughout all experiments to ensure consistency and comparability of the results.

The UTASTAR model was constructed using a preference-based training and validation strategy. From the 35 stock alternatives, 70% of the alternatives (n_{tr}) were selected as reference alternatives for estimating the utility functions, while the remaining 30% were reserved for validation. The reference alternatives were determined based on the market data-based preference proxy generated using EWM. The resulting pseudo utility values provide supplementary cardinal information, while the corresponding pseudo-ranking provides ordinal preference information used as reference knowledge in the UTASTAR formulation.

The objective function of the proposed UTASTAR formulation incorporates the parameter Ω as an additional preference-preservation factor to balance numerical fitting errors and ranking inconsistencies. In this study, Ω was set to 10 ($\Omega > 1$), indicating that ranking deviations are penalized more strongly than deviations from pseudo-utility values. This configuration is designed to prioritize the preservation of ordinal preference information while maintaining limited flexibility in estimating the cardinal utility representation.

In contrast with a direct numerical approximation approach, UTASTAR does not aim to reproduce the pseudo-utility values exactly; rather, it derives a constrained cardinal utility representation that is consistent with the available preference information. The fitting constraints with $\varepsilon_{\mathrm{fit}}$ tolerance prevent the estimated utility function from becoming a direct replication of the pseudo utility generated by EWM. Instead, UTASTAR estimates a cardinal utility function under structural requirements, including monotonicity, normalization, and preference-ranking consistency. Meanwhile, the ranking constraints with $\varepsilon_{\mathrm{rank}}$ ensure that the ordinal preference relationships among the reference alternatives are maintained during the estimation process.

After estimating the marginal utility functions using the training alternatives, the obtained UTASTAR model was applied to both training and validation alternatives to calculate their global utility values. Table 2 presents the comparison between the market data-based preference proxy (pseudo utility) and the UTASTAR-estimated global utility values for all stock alternatives.

The results indicate that the estimated global utility values exhibit a close numerical correspondence with the pseudo utility values, although small deviations are observed. These differences are expected because UTASTAR estimates the utility function under multiple structural constraints rather than directly minimizing the difference from pseudo-utility values alone. Therefore, the obtained global utility values represent a constrained cardinal approximation of the market-implied preference structure rather than an exact numerical replication of the EWM-based preference proxy. The consistency between the pseudo-ranking and the UTASTAR ranking was further evaluated using ranking differences, Spearman's rank correlation coefficient, and Kendall's rank correlation coefficient. The results show that the estimated utility function successfully preserves the ordinal preference structure. The Spearman correlation coefficient and Kendall correlation coefficient achieved values of $\rho = 1.000$ and $\tau = 1.000$, respectively, with p-values below 0.001, indicating statistically significant perfect agreement between the pseudo-ranking and the UTASTAR ranking.

For the validation alternatives, the same evaluation procedure was conducted after applying the estimated utility function obtained from the training alternatives. The consistent ranking relationship observed in the validation set demonstrates that the estimated utility function can maintain the reference preference structure beyond the

Table 2. Comparison of Preference Proxy and UTASTAR Global Utility

Stock	**Pseudo Utility**	**UTASTAR Phase**	**UTASTAR Global Utility**	**Pseudo Rank**	**UTASTAR Rank**
PTPP	0.637807	Training	0.637953	1	1
ANTM	0.617669	Training	0.616669	2	2
ACES	0.549949	Training	0.549609	3	3
ADRO	0.492262	Training	0.491262	4	4
ERAA	0.468491	Training	0.468932	5	5
MDKA	0.451958	Training	0.450958	6	6
MNCN	0.437852	Training	0.438852	7	7
SMRA	0.433523	Training	0.433675	8	8
PGAS	0.395709	Training	0.394819	9	9
BRPT	0.395619	Training	0.394719	10	10
SMGR	0.393880	Training	0.392985	11	11
TLKM	0.386040	Training	0.385040	12	12
JPFA	0.369195	Training	0.368458	13	13
UNTR	0.367703	Training	0.367737	14	14
PTBA	0.360194	Training	0.359194	15	15
ITMG	0.349278	Training	0.349103	16	16
UNVR	0.347502	Training	0.348502	17	17
INCO	0.343547	Training	0.343638	18	18
EXCL	0.337684	Training	0.337617	19	19
AKRA	0.337608	Training	0.337517	20	20
MAPI	0.331707	Training	0.331641	21	21
BTPS	0.317755	Training	0.318079	22	22
KLBF	0.313871	Training	0.314558	23	23
INDF	0.311984	Training	0.312984	24	24
CTRA	0.303627	Testing	0.304031	25	25
BRIS	0.302490	Testing	0.303644	26	26
TPIA	0.289574	Testing	0.288549	27	27
ICBP	0.286440	Testing	0.287870	28	28
ISAT	0.281714	Testing	0.283215	29	29
SIDO	0.270933	Testing	0.267316	30	30
INTP	0.263688	Testing	0.264996	31	31
MIKA	0.263165	Testing	0.264078	32	32
CPIN	0.258794	Testing	0.260631	33	33
PWON	0.252140	Testing	0.253521	34	34
MYOR	0.237506	Testing	0.239481	35	35

Notes: Pseudo Utility denotes the market data-based preference proxy used as the reference for UTASTAR estimation. The UTASTAR model perfectly reproduced the ordinal ranking of all 35 stocks. The rank agreement between the Preference Proxy and UTASTAR was confirmed by Spearman's rank correlation coefficient ($\rho = 1.000$, $p < 0.001$) and Kendall's rank correlation coefficient ($\tau = 1.000$, $p < 0.001$).

alternatives used during calibration. This result indicates that the UTASTAR model provides a generalized preference representation rather than merely fitting the selected reference alternatives. It should be emphasized that the main output of UTASTAR is the estimated cardinal utility function rather than the ranking itself. The ranking comparison using Spearman and Kendall correlation is performed only to verify that the transformation from ordinal preference information into cardinal utility does not distort the original preference structure. The resulting utility values are subsequently incorporated as the utility objective in the Compromise Programming portfolio optimization model.

This study employed a 70:30 ratio to split the dataset into training and testing sets. With a total of 35 stock alternatives, the theoretical split yields 24.5 stock alternatives for the training set and 10.5 stock alternatives for the testing set. Since the number of stock alternatives must be an integer, Python allocates 24 stock alternatives to the training set and 11 stock alternatives to the testing set. This allocation preserves the total number of stock alternatives while providing the closest possible approximation to the intended 70:30 ratio.

3.2. Marginal Utility Function

After the UTASTAR model was successfully estimated, the marginal utility functions for each criterion were obtained. These marginal utility functions represent the contribution of each criterion achievement level to the overall utility learned by the model. The utility values were calculated at each normalized breakpoint, allowing the model to capture the gradual changes in preferences across the range of criterion values. The normalized criterion interval was partitioned into $B = 4$ equally spaced breakpoints, generating four cutting points ($\ell = 1, 2, 3, 4$). The estimated marginal utility values corresponding to these breakpoints are presented in Table 3.

Table 3. Marginal Utility Values at Each Breakpoint

ℓ	$z^{(\ell)}$	**ROE**	**STR**	**MarCap**	**PBV**
1	0.000	0.000000	0.000000	0.000000	0.000000
2	0.333	0.052618	0.188310	0.065441	0.000000
3	0.667	0.085975	0.380431	0.133264	0.045468
4	1.000	0.162763	0.570450	0.199070	0.067717

To provide a more intuitive interpretation of the utility changes across different segments, the marginal utility functions for each criterion are visualized as curves, as shown in Figure 3.

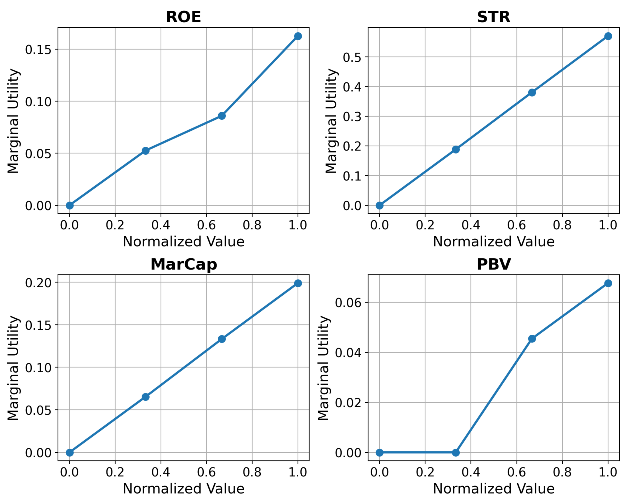


Figure 3. Marginal utility functions obtained using the UTASTAR method.

Based on Table 3, all marginal utility functions satisfy the monotonicity constraint, meaning that the utility values do not decrease as the criterion values increase. This indicates that improvements in each criterion contribute positively to the overall utility generated by the UTASTAR model.

Among the four criteria, STR exhibits the largest increase in marginal utility, rising from 0 to 0.570450 at the final breakpoint. This result indicates that changes in STR contribute the most to the improvement of the overall utility compared with the other criteria. In contrast, PBV shows a relatively small increase in marginal utility, with the utility remaining zero until the second breakpoint, indicating that increases in PBV within the lower value range do not provide a significant contribution to the overall utility. Meanwhile, ROE and MarCap exhibit a more gradual increase in marginal utility across all breakpoints.

Figure 3 complements the numerical results in Table 3 by illustrating the shapes of the estimated marginal utility functions. Although all criteria satisfy the monotonicity constraint, the curves reveal different utility growth patterns. STR exhibits the steepest and most linear increase, indicating that improvements in this criterion consistently generate the largest utility gains. ROE and MarCap show gradual and nearly linear increases, reflecting stable contributions to the overall utility. In contrast, PBV presents a nonlinear pattern with an initially flat curve followed by a sharp increase after the second breakpoint, suggesting that improvements in PBV become influential only after exceeding a certain threshold.

3.3. Compromise Programming

The covariance matrix was constructed for 35 stocks, resulting in a 35×35 matrix. The minimum eigenvalue of the covariance matrix was 0.0322, confirming that the covariance matrix is positive definite. This indicates that the estimated covariance matrix satisfies the mathematical properties required for portfolio risk modeling and subsequent optimization. The ideal and nadir values required in CP were then obtained by optimizing each objective individually while recording the corresponding values of the remaining objectives. The resulting payoff table is presented in Table 4.

Table 4. Payoff Tables with and without the Upper Bound Constraint

Objective	**Without Upper Bound**			**With Upper Bound**		
	Return	**Risk**	**Utility**	**Return**	**Risk**	**Utility**
Max Return	0.674242	0.388243	0.394719	0.469637	0.117983	0.345883
Min Risk	0.015832	0.016091	0.322442	0.015832	0.016001	0.322444
Max Utility	-0.041642	0.297757	0.637953	0.047237	0.078831	0.590395

Based on the research results, Table 4 clear trade-offs can be observed among the three objectives. Without the upper bound constraint, the maximum expected return objective produced the highest expected return of 0.6742, but it was also associated with the highest portfolio risk of 0.3882 and a utility score of 0.3947. Conversely, the minimum risk objective achieved the lowest portfolio risk of 0.0161, although the expected return decreased substantially to 0.0158, with a utility score of 0.3224. Meanwhile, the maximum utility objective yielded the highest utility score of 0.6380 with a moderate portfolio risk of 0.2978, but the corresponding expected return became -0.0416, indicating that maximizing investor preference alone may lead to an unfavorable financial outcome.

Thus, with the upper bound constraint, the trade-off pattern remained similar, although the feasible solution space became more restricted. The maximum expected return decreased from 0.6742 to 0.4696, while the associated portfolio risk was reduced considerably from 0.3882 to 0.1180. Similarly, the maximum utility score declined from 0.6380 to 0.5904, accompanied by a reduction in portfolio risk from 0.2978 to 0.0788 and an improvement in expected return from -0.0416 to 0.0472. These results indicate that imposing an upper bound constraint limits extreme portfolio allocations, resulting in more balanced portfolios with substantially lower risk, although at the expense of reduced maximum return and utility. The observed conflicts among the three objectives justify the application of Compromise Programming to identify a balanced portfolio solution.

Furthermore, the individually optimized objective values were used to determine the ideal and nadir points required for objective normalization in the Compromise Programming model.

Table 5 reports the ideal and nadir values of the three objective functions. These values were obtained by individually optimizing each objective and were subsequently used to normalize the objective deviations in the Compromise Programming model. Without the upper bound constraint, the ideal point consists of an expected return of 0.6742, a minimum portfolio risk of 0.0161, and a utility score of 0.6380, whereas the nadir point comprises an expected return of -0.0416, a portfolio risk of 0.3882, and a utility score of 0.3224. With the upper bound constraint, the ideal point changes to an expected return of 0.4696, a minimum portfolio risk of 0.0160, and a utility score of 0.5904, while the corresponding nadir point consists of an expected return of 0.0158, a portfolio risk of 0.1180, and a utility score of 0.3224. The negative nadir value of the expected return in the unconstrained case

Table 5. Ideal and Nadir Points with and without the Upper Bound Constraint

Objective	Without Upper Bound		With Upper Bound	
	Ideal	Nadir	Ideal	Nadir
Expected Return	0.674242	-0.041642	0.469637	0.015832
Portfolio Risk	0.016001	0.388243	0.016001	0.117983
Utility Score	0.637953	0.322442	0.590395	0.322444

indicates that the worst attainable portfolio under conflicting objectives is expected to generate a negative return, whereas the constrained model maintains a positive nadir return due to the restriction on portfolio concentration. These ideal and nadir values serve solely as reference points for normalizing the objectives and do not represent the final optimal portfolio obtained through Compromise Programming.

Using the identified ideal and nadir points, the Compromise Programming model was solved using the L_2 norm to obtain the optimal portfolio allocation. To examine the effect of the upper bound constraint on portfolio diversification, two optimization scenarios were considered: one without the upper bound constraint and another with a maximum asset weight of 30%. The resulting portfolio allocations, together with their corresponding expected return, portfolio risk, and global utility, are summarized in Table 6.

Table 6. Effect of the Upper Bound Constraint on Optimal Portfolio Weights

Stock	**Without Upper Bound (%)**	**With Upper Bound (%)**
ACES	–	13.78
ANTM	67.43	30.00
BRPT	32.57	16.64
ERAA	–	1.09
EXCL	–	9.30
JPFA	–	17.28
PTPP	–	6.84
TPIA	–	5.07
Expected Return	0.376110	0.279796
Portfolio Risk	0.123454	0.052628
Global Utility	0.544379	0.484861
Number of Selected Stocks	2	8
Maximum Weight (%)	67.43	30.00

Table 6 compares the optimal portfolios obtained using the L_2 norm with and without the upper bound constraint. Without the upper bound, the optimization allocated 67.43% of the portfolio to ANTM and the remaining 32.57% to BRPT, resulting in a highly concentrated portfolio consisting of only two assets. Such concentration exposes the portfolio to substantial idiosyncratic risk because its performance depends heavily on a small number of stocks.

After imposing a 30% upper bound on individual asset weights (see Figure 4), the portfolio allocation became substantially more diversified, with investments distributed across eight different stocks. The maximum allocation to any single stock was successfully limited to 30%, thereby preventing excessive concentration while maintaining feasible portfolio weights. The increased diversification was accompanied by a reduction in expected return from 0.376110 to 0.279796 and a slight decrease in the global utility from 0.544379 to 0.484861. However, the portfolio risk decreased considerably from 0.123454 to 0.052628, indicating that the diversification achieved through the upper bound constraint effectively reduced the overall portfolio risk.

These results demonstrate the trade-off introduced by the upper bound constraint. Although restricting the maximum allocation to each asset slightly reduces the expected return and utility, it produces a more balanced portfolio with significantly lower concentration risk. Therefore, the upper bound constraint enhances the practical

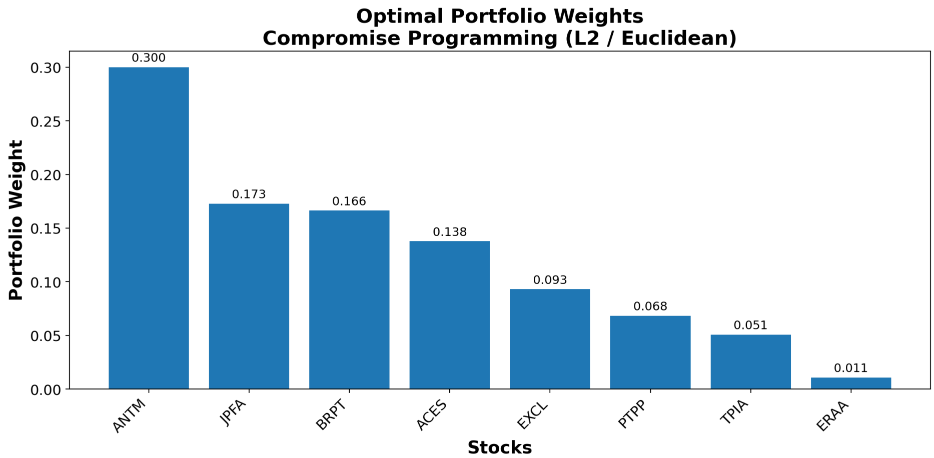


Figure 4. Marginal utility functions obtained using the UTASTAR method.

applicability of the proposed optimization model by generating portfolios that are more diversified and consistent with prudent portfolio management practices.

4. Sensitivity Analysis

In the context of MCDA, sensitivity analysis has become a core approach for validating the resilience and reliability of decision-making outcomes [57]. Sensitivity analysis is an important phase in model development and communication, providing deeper insight into model behavior, internal structure, and the responsiveness of the resulting solution to changes in input parameters. Systematic sensitivity analysis examines solution stability, parameter influence, and decision consistency, thereby supporting the development of robust decision models [58]. In multi-objective optimization, sensitivity analysis is commonly employed to evaluate the impact of parameter changes on the compromise solution. It also provides insight into how variations in input data and preference parameters affect the trade-offs among conflicting objectives and the stability of the resulting solution [59].

4.1. Sensitivity Analysis of the UTASTAR Model

The number of breakpoints was initially fixed at $B = 4$ before conducting the sensitivity analysis of the parameters ε_{fit} and $\varepsilon_{\text{rank}}$. The choice of $B = 4$ instead of $B = 3$ was based on the consideration that four breakpoints produce a greater number of piecewise linear segments, thereby providing greater flexibility for the utility function to represent preferences without introducing excessive model complexity. This fixed setting serves as a baseline configuration, allowing the effect of each parameter to be examined independently without being confounded by changes in the number of breakpoints.

The sensitivity analysis began with the parameter ε_{fit}, while fixing $\varepsilon_{\text{rank}} = 1 \times 10^{-4}$. This procedure was adopted because ε_{fit} controls the tolerance level for the goodness-of-fit between the estimated UTASTAR utility (U) and the pseudo-utility (U^{Ps}). By keeping $\varepsilon_{\text{rank}}$ constant, the effect of varying ε_{fit} on the fitting performance can be evaluated directly without being influenced by changes in the ranking discrimination tolerance. Once the value of ε_{fit} yielding the best performance was identified, a second sensitivity analysis was conducted for $\varepsilon_{\text{rank}}$ using the selected optimal value of ε_{fit}. This sequential approach enables each parameter to be evaluated independently, allowing its individual influence on the UTASTAR modeling results to be identified more clearly. Finally, after determining the optimal combination of ε_{fit} and $\varepsilon_{\text{rank}}$, a sensitivity analysis of the number of breakpoints (B) was performed to evaluate the effect of the utility function's flexibility on the UTASTAR modeling results.

Table 7. Sensitivity Analysis Results for the ε_{fit} Parameter

ε_{fit}	Total Error	Total ζ	Utility Value	Ranking	Spearman's ρ	Kendall's τ
0	1.4098×10^{-5}	1.4098×10^{-4}	Different	Unchanged	1	1
1×10^{-5}	1.8357×10^{-6}	1.8357×10^{-5}	Different	Unchanged	1	1
1×10^{-4}	0	0	Different	Unchanged	1	1
1×10^{-3}	0	0	Different	Unchanged	1	1
1×10^{-2}	0	0	Different	Changed	9.1818×10^{-1}	7.8182×10^{-1}
1×10^{-1}	0	0	Different	Changed	9.6364×10^{-1}	8.9091×10^{-1}
1	0	0	Different	Changed	9.6364×10^{-1}	8.9091×10^{-1}

As shown in Table 7, the total value of the deviation variable ξ was zero for all tested values of ε_{fit}. This indicates that the preference-order constraints were satisfied without activating the ξ variables in any experiment. Consequently, variations in ε_{fit} only affected the allowable fitting tolerance and the size of the feasible solution space while leaving the ξ component unchanged. For ε_{fit} between 0 and 10^{-3}, the ranking of the alternatives remained unchanged, as confirmed by Spearman's ρ and Kendall's τ values of 1.

Furthermore, $\varepsilon_{\text{fit}} = 10^{-2}$ exhibited lower rank correlation than those obtained at 10^{-1} and 1, although all three cases achieved zero fitting deviations. This result suggests that increasing the fitting tolerance beyond the point where all fitting deviations become zero does not necessarily preserve the same ranking performance on unseen data. Since the UTASTAR model optimizes its objective function rather than directly maximizing rank-correlation measures, different optimal or near-optimal utility functions may produce different rankings on the testing set. The identical results obtained for $\varepsilon_{\text{fit}} = 10^{-1}$ and 1 indicate that, for the present dataset, further increasing the fitting tolerance did not change the solution returned by the solver.

Based on these results, $\varepsilon_{\text{fit}} = 10^{-3}$ was selected as the parameter for the subsequent analysis. Although the total error reached zero starting from $\varepsilon_{\text{fit}} = 10^{-4}$, the value $\varepsilon_{\text{fit}} = 10^{-3}$ represents the largest tolerance that still preserves perfect ranking consistency (Spearman's $\rho = 1$ and Kendall's $\tau = 1$).

Next, after fixing ε_{fit} at 1×10^{-3}, the sensitivity analysis focused on varying $\varepsilon_{\text{rank}}$ to examine its effect on the UTASTAR modeling results. This analysis evaluates the trade-off between reproducing the pseudo-utility values (U^{Ps}) and satisfying the ranking constraints.

Table 8. Sensitivity analysis of the optimization results with respect to the $\varepsilon_{\text{rank}}$ parameter.

$\varepsilon_{\text{rank}}$	Total Error	Total ζ	Total ξ	Objective Value	Ranking
0	0	0	0	Different	Unchanged
1×10^{-5}	0	0	0	Different	Unchanged
1×10^{-4}	0	0	0	Different	Unchanged
1×10^{-3}	2.1090×10^{-4}	2.1090×10^{-3}	0	Different	Changed
1×10^{-2}	5.8640×10^{-1}	3.8262×10^{-1}	5.4814×10^{-2}	Different	Changed
1×10^{-1}	62.5951	1.6080	6.2434	Different	Changed

Table 9. Rank correlation coefficients for different values of the $\varepsilon_{\text{rank}}$ parameter.

$\varepsilon_{\text{rank}}$	Spearman's ρ	Kendall's τ
0	1	1
1×10^{-5}	1	1
1×10^{-4}	1	1
1×10^{-3}	9.9091×10^{-1}	9.6364×10^{-1}
1×10^{-2}	9.7273×10^{-1}	9.2727×10^{-1}
1×10^{-1}	9.6364×10^{-1}	8.9091×10^{-1}

The results (Tables 8 and 9) indicate that when $\varepsilon_{\text{rank}} = 0$, 1×10^{-5} and 1×10^{-4}, the model produced both total ζ and total ξ equal to zero, resulting in a total error of zero. This indicates that both the utility fitting constraints and the ranking constraints were satisfied perfectly. Although the estimated utility values U differed from the pseudo-utility values U^P (due to the existence of multiple optimal solutions), the ordering of alternatives remained identical to the pseudo-ranking. Consequently, both the Spearman and Kendall rank correlation coefficients were equal to 1.

Thus, when $\varepsilon_{\text{rank}}$ was increased to 1×10^{-3}, a trade-off between the two objectives began to emerge. The total error increased to 2.1090×10^{-4}, entirely due to the utility fitting deviation ($\zeta = 2.1090 \times 10^{-3}$), while the total ranking violation remained zero ($\xi = 0$). This indicates that the model was still able to satisfy all ranking constraints without violation but at the expense of a reduced agreement between the estimated utilities and the pseudo-utility values. As a result, the ranking changed slightly, with the Spearman and Kendall coefficients decreasing to 0.9909 and 0.9636, respectively. Following this, for $\varepsilon_{\text{rank}} = 1 \times 10^{-2}$, the total error increased substantially to 0.5864. Besides the increase in utility fitting deviation ($\zeta = 0.38262$), violations of the ranking constraints also appeared ($\xi = 0.054814$). Nevertheless, the ordinal relationship remained relatively strong, with Spearman's coefficient equal to 0.9727 and Kendall's coefficient equal to 0.9273. Furthermore, $\varepsilon_{\text{rank}}$ was further increased to 1×10^{-1}, the total error rose dramatically to 62.5951. The total utility fitting deviation increased to $\zeta = 1.6080$, while the total ranking violation reached $\xi = 6.2434$, indicating that the model could no longer preserve both utility fitting and ranking consistency simultaneously. This deterioration was reflected by the decline in the Spearman and Kendall coefficients to 0.9636 and 0.8909, respectively.

Overall, increasing $\varepsilon_{\text{rank}}$ provides greater flexibility in enforcing the ranking constraints. However, this flexibility comes at the cost of larger deviations from the pseudo-utility values and, eventually, violations of the ranking constraints, resulting in a deterioration of the overall solution quality. In this study, $\varepsilon_{\text{rank}}$ was set to 1×10^{-4} as the minimum separation margin between the utility values of alternatives with different preference ranks. This value was selected because it is sufficiently small to preserve the original preference structure while being large enough to guarantee a strict separation between alternatives according to the pseudo-ranking. A smaller value, such as $\varepsilon_{\text{rank}} = 1 \times 10^{-5}$, was not adopted because it provides an extremely narrow separation margin, making the differences between utility values almost negligible relative to the numerical tolerance of the optimization solver. Consequently, such a value may produce nearly identical utility values (near ties) without providing any meaningful improvement in the model quality.

Similarly, $\varepsilon_{\text{rank}} = 0$ was not selected because the ranking constraint reduces to $U_\alpha \geq U_\beta$, allowing two alternatives with different preference ranks to receive identical utility values ($U_\alpha = U_\beta$). This situation is inconsistent with the pseudo-ranking adopted as the reference preference information in this study, where all alternatives have unique ranks and no ties are present. Therefore, a strictly positive value of $\varepsilon_{\text{rank}}$ was employed to ensure that every pair of alternatives with different preference orders also exhibits a minimum utility difference, thereby preserving the consistency of the ordinal preference representation within the UTASTAR model.

Based on the sensitivity analysis, the tolerance values were fixed at $\varepsilon_{\text{fit}} = 1 \times 10^{-3}$ and $\varepsilon_{\text{rank}} = 1 \times 10^{-4}$. Using these parameter settings, the subsequent analysis focuses on evaluating the effect of the number of breakpoints on the UTASTAR model.

Table 10. Effect of parameter B on ranking consistency

B	**Total Error**	**Value**	**Rank**	**Spearman's ρ**	**Kendall's τ**
3	0	Different	Same	1.0000	1.0000
4	0	Different	Same	1.0000	1.0000
5	0	Different	Changed	0.9909	0.9636
7	0	Different	Changed	0.9454	0.8545
11	0	Different	Changed	0.5909	0.4909

The result (see Table 10) of all breakpoint configurations produced a total error of zero, indicating that all constraints in the model are satisfied without requiring penalty variables. Therefore, both the pseudo-utility fitting constraints and the ranking consistency constraints are fully satisfied by the obtained solutions. Nevertheless, the resulting ranking is not always identical to the pseudo-ranking, particularly for $B \geq 5$. This occurs even though the total ranking error remains zero. The reason is that the ranking error variables (ξ_i^+ and ξ_i^-) become positive only when the ranking constraints defined in the model are violated. As long as every constrained pair of alternatives satisfies $U_\alpha - U_\beta \geq \varepsilon_{\text{rank}}$, the ranking error remains zero. However, small changes in utility values caused by the optimization process may still alter the relative order of alternatives whose utility values are very close to each other.

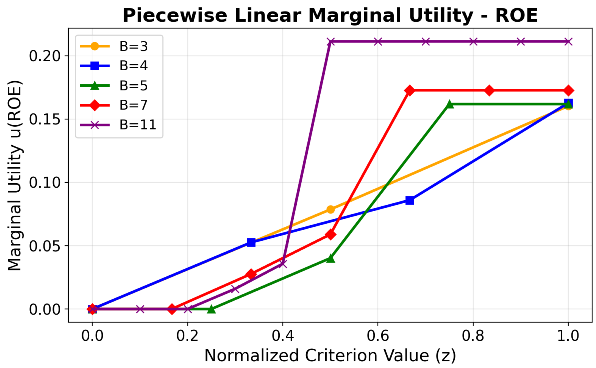
Consequently, the ranking changes observed through the Spearman and Kendall correlation coefficients reflect changes in the global ordering of alternatives rather than violations of the ranking constraints that would generate penalties in the objective function. Furthermore, all utility values estimated by UTASTAR (U) differ from pseudo-utility values (U^P). This is consistent with the fitting constraint formulation, $-\varepsilon_{\text{fit}} - \zeta_i^- \leq U_i - U_i^{Ps} \leq \varepsilon_{\text{fit}} + \zeta_i^+$, which does not enforce $U_i = U_i^{Ps}$, but only restricts the deviation to remain within the tolerance level of $\varepsilon_{\text{fit}} = 10^{-3}$. The influence of the number of breakpoints is reflected in the ranking consistency. For $B = 3$ and $B = 4$, both the Spearman and Kendall correlation coefficients equal 1, indicating that the UTASTAR ranking is identical to the pseudo-ranking. When the number of breakpoints is increased to $B = 5$, the ranking correlation remains very high (Spearman = 0.9909 and Kendall = 0.9636), indicating only minor changes in the ordering of alternatives. However, for $B = 7$ and $B = 11$, both correlation coefficients decrease further, suggesting that increasing the number of breakpoints provides greater flexibility for the utility function, resulting in a ranking that gradually deviates from the pseudo-ranking.

However, the ranking correlation alone is insufficient to determine the most appropriate number of breakpoints. The selection of breakpoints should also consider the shape of the resulting marginal utility functions. Although $B = 3$ achieves perfect ranking consistency, the small number of breakpoints produces an overly simplified utility function, causing the marginal utility curve to closely resemble a straight line. This indicates that the model has limited flexibility to represent variations in preferences across each criterion. In other words, the model becomes overly rigid and is therefore less capable of capturing the nonlinear relationship between criterion values and utility. Consequently, the final breakpoint selection is further evaluated through a visual analysis of the marginal utility curves for each breakpoint configuration. This analysis aims to identify the number of breakpoints that not only preserves ranking consistency but also produces marginal utility functions that are sufficiently representative of the underlying preference structure.

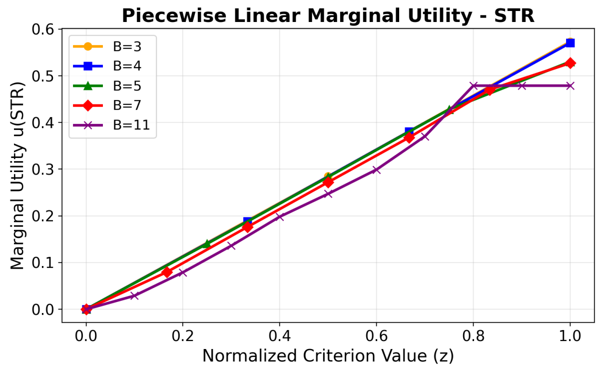
Figure 5 further illustrates the effect of the breakpoint configuration on the estimated marginal utility functions. For ROE and PBV, using only three breakpoints results in overly simplified utility curves that are close to linear, limiting the model's ability to represent changes in marginal preference across the criterion range. Increasing the number of breakpoints to four introduces sufficient flexibility to capture moderate nonlinearities while maintaining smooth and interpretable utility functions. In contrast, configurations with five or more breakpoints produce increasingly irregular shapes, including abrupt jumps and extended plateau regions, particularly for ROE and PBV. These patterns indicate that the model becomes highly sensitive to local variations in the training data without providing a corresponding improvement in ranking consistency.

A different pattern is observed for STR and Market Capitalization, where the marginal utility functions remain nearly identical across all breakpoint configurations. This indicates that increasing the number of breakpoints contributes little to representing the underlying preference structure for these criteria. Therefore, the additional model complexity associated with larger breakpoint values is not justified. Considering both the perfect agreement with the pseudo-ranking (Spearman's $\rho = 1$ and Kendall's $\tau = 1$) and the visual characteristics of the estimated utility functions, $B = 4$ provides the most appropriate balance between model flexibility and simplicity. It preserves ranking consistency while avoiding both the oversimplification observed for $B = 3$ and the excessive flexibility exhibited by larger breakpoint configurations. Based on the sensitivity analysis, the parameter combination of $B = 4$, $\varepsilon_{\text{fit}} = 10^{-3}$ and $\varepsilon_{\text{rank}} = 10^{-4}$ was selected for the subsequent analysis. This configuration maintains the consistency between the UTASTAR ranking and the market-derived pseudo-ranking while avoiding unnecessary model complexity. The resulting utility values are then used as a data-driven proxy for market preferences and incorporated as the utility objective in the subsequent Compromise Programming model.

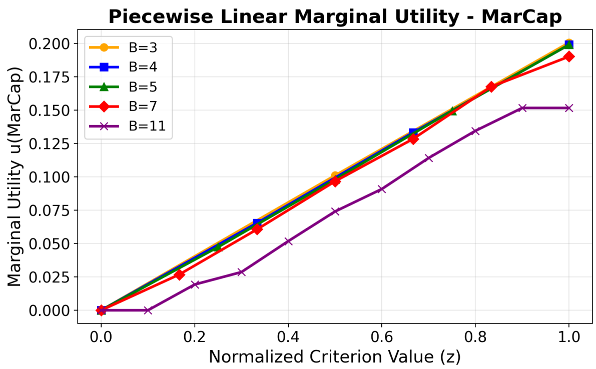
Furthermore, Figure 5 illustrates the piecewise linear marginal utility functions obtained for the four evaluation criteria under different breakpoint configurations. Overall, all marginal utility functions satisfy the monotonicity



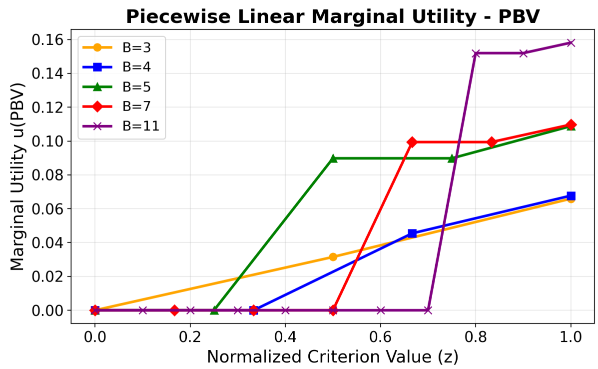
(a) ROE



(b) STR



(c) MarCap



(d) PBV

Figure 5. Piecewise linear marginal utility functions for the four evaluation criteria.

assumption of the UTASTAR method, indicating that the utility value never decreases as the normalized criterion value increases. However, the shapes of the utility functions differ across criteria, reflecting different levels of sensitivity to changes in criterion values. For ROE (Figure 5 a), the marginal utility function is monotonically increasing for all breakpoint configurations, indicating that firms with higher profitability consistently receive greater utility values. This behavior is consistent with financial theory, as a higher return on equity reflects a more efficient use of shareholders' capital to generate earnings. Nevertheless, the increase is nonlinear. For larger numbers of breakpoints ($B = 7$ and $B = 11$), the utility rises sharply within the middle range of ROE values and subsequently reaches a plateau, suggesting that further improvements in profitability beyond a certain level provide little additional utility.

Thus, the marginal utility function for STR (Figure 5 b) also exhibits a monotonically increasing pattern across all breakpoint configurations. Compared with the other criteria, the function is nearly linear and remains relatively stable as the number of breakpoints increases, indicating a consistent preference structure. A slight plateau appears only for $B = 11$ near the upper end of the criterion range, whereas the remaining configurations continue to increase until the maximum normalized value. For Market Capitalization (Figure 5 c), the marginal utility function increases monotonically throughout the entire criterion range. This result suggests that companies with larger market capitalization are consistently assigned higher utility, reflecting the common perception that larger firms are generally more stable and liquid. Similar to STR, the utility function is almost linear for most breakpoint configurations. A plateau is observed only for $B = 11$ at high criterion values, while the remaining configurations continue to increase steadily.

Finally, the marginal utility function for PBV (Figure 5 d) also satisfies the monotonicity requirement but exhibits a different shape from the other criteria. Since PBV is treated as a cost criterion, the increasing utility function represents a preference for lower PBV values after the transformation and normalization procedures in UTASTAR. Therefore, the results remain consistent with value investing principles, where firms with lower price-to-book ratios are generally considered more attractive. Several breakpoint configurations ($B = 5$, $B = 7$, and $B = 11$) exhibit plateau regions, indicating that improvements beyond a certain valuation level no longer produce substantial gains in utility.

Overall, the marginal utility functions preserve the monotonicity property required by the UTASTAR method while capturing different preference sensitivities across the evaluation criteria. ROE and PBV exhibit stronger nonlinear behavior with noticeable plateau regions, whereas STR and Market Capitalization display nearly linear utility functions over most of the criterion range. These findings suggest that the proposed model not only produces utility functions consistent with financial theory but also captures nonlinear relationships between criterion values and investor preferences.

4.2. Sensitivity Analysis of Maximum Weight Constraint

To evaluate the effect of different maximum weight constraints under the selected L_2-norm Compromise Programming model, a sensitivity analysis was conducted, and the results are presented in Table 11.

Table 11. Portfolio Performance under Different Maximum Weight Constraints

Maximum Weight	Return	Risk	Utility	Number of Stocks
30%	0.279796	0.052628	0.484861	8
35%	0.297538	0.061366	0.497347	7
40%	0.314026	0.070923	0.508105	7
50%	0.350495	0.094713	0.524184	5
None	0.376110	0.123454	0.544379	2

As the upper bound increases from 30% to no constraint, the expected return rises from 0.279796 to 0.376110, accompanied by an increase in utility from 0.484861 to 0.544379. However, these improvements are obtained at the expense of substantially higher portfolio risk, which increases from 0.052628 to 0.123454. Moreover, relaxing the upper bound reduces the number of selected stocks from eight to only two, indicating that the portfolio becomes increasingly concentrated. This pattern is also reflected in the maximum portfolio weight, which consistently reaches the specified upper bound (30%, 35%, 40%, and 50%), while in the absence of a constraint the optimization allocates 67.43% of the investment to a single stock. These results indicate that the optimizer tends to exploit the maximum allowable allocation to improve portfolio performance.

Although higher upper bounds produce greater expected returns and utility, they also lead to a disproportionate increase in portfolio risk and a significant reduction in diversification. The 30% upper bound provides the most balanced solution by maintaining the lowest portfolio risk while still achieving competitive return and utility values. In addition, limiting the maximum allocation to approximately one-third of the total portfolio prevents excessive concentration in a single asset, thereby preserving the diversification principle. Therefore, the 30% upper bound is adopted in this study as a realistic diversification constraint that achieves a better compromise between return, risk, utility, and portfolio diversification.

Moreover, Table 12 presents the resulting portfolio compositions obtained under different maximum weight constraints.

The results (Table 12) show that imposing a maximum weight constraint significantly affects the degree of portfolio diversification. Without a maximum weight constraint, the optimal portfolio is highly concentrated in two assets, namely ANTM (67.43%) and BRPT (32.57%). This indicates that the optimization model tends to allocate a large proportion of wealth to assets with the most favorable trade-off between return, risk, and preference utility when no diversification restriction is applied.

Furthermore, when the maximum weight constraint is imposed, the allocation to ANTM is limited by the specified upper bound, causing the remaining investment proportion to be redistributed among the other selected assets. For instance, under the 30% upper bound, ANTM reaches the imposed limit, while the portfolio expands

Table 12. Portfolio Composition under Different Maximum Weight Constraints

Upper Bound	Portfolio Composition
30%	ANTM (0.3000), JPFA (0.1728), BRPT (0.1664), ACES (0.1378), EXCL (0.0930), PTPP (0.0684), TPIA (0.0507), ERAA (0.0109)
35%	ANTM (0.3500), BRPT (0.2054), JPFA (0.1612), ACES (0.1285), PTPP (0.0673), EXCL (0.0454), TPIA (0.0423)
40%	ANTM (0.4000), BRPT (0.2377), JPFA (0.1491), ACES (0.1139), PTPP (0.0609), TPIA (0.0355), EXCL (0.0030)
50%	ANTM (0.5000), BRPT (0.3165), JPFA (0.0766), ACES (0.0624), PTPP (0.0445)
None	ANTM (0.6743), BRPT (0.3257)

to include eight stocks with smaller allocations. As the upper bound is gradually increased from 30% to 50% he allocation to ANTM increases, whereas the number of selected assets decreases, indicating that a less restrictive constraint allows the model to move closer to the unconstrained optimal solution.

Thus, the results also demonstrate that BRPT and JPFA remain among the selected assets across all upper-bound levels, suggesting that these stocks consistently exhibit favorable characteristics within the compromise solution. However, their portfolio weights vary as the maximum weight constraint becomes less restrictive due to the reallocation mechanism. At the 50% upper bound, the portfolio becomes more concentrated, consisting of only five assets, whereas the unconstrained portfolio contains only two assets and exhibits the highest level of concentration.

These findings indicate that the maximum weight constraint plays an important role in controlling portfolio concentration without altering the fundamental preference structure obtained from the proposed model. Therefore, applying the maximum weight constraint through different upper-bound levels provides a practical mechanism for improving portfolio diversification and reducing excessive exposure to individual assets while preserving the compromise between expected return, portfolio risk, and investor preference.

4.3. Sensitivity Analysis of the Compromise Programming Norm

A sensitivity analysis was conducted to evaluate the effect of norm selection in Compromise Programming by comparing three distance measures: the L_1, L_2, and L_∞ norms. Since each norm employs a different distance metric to measure the proximity between the feasible and ideal solutions, different trade-offs among expected return, portfolio risk, and global utility may arise. The optimization results under the maximum weight constraint ($w_i \leq 0.30$) indicate that the choice of L_γ norm influences the trade-off between return, risk, and utility in the resulting portfolio, as shown in Table 13.

Table 13. Comparison of Portfolio Performance under Different L_γ Norms

Norm	Upper Constraint	Expected Return	Risk	Utility
L_1	$w_i \leq 0.30$	0.266933	0.050078	0.486311
L_2	$w_i \leq 0.30$	0.279796	0.052628	0.484861
L_∞	$w_i \leq 0.30$	0.291438	0.056047	0.485177

Table 13 show that the L_1 norm produces the lowest expected return (0.266933) and the lowest risk (0.050078), indicating a more conservative portfolio allocation that prioritizes minimizing the overall deviation from the ideal solution across objectives. In contrast, the L_∞ norm provides the highest expected return (0.291438) accompanied by a higher risk level (0.056047). This occurs because the Chebyshev distance minimizes the maximum normalized deviation among objectives, resulting in a solution that controls the largest deviation from the ideal solution.

Meanwhile, the L_2 norm results in an intermediate trade-off, with an expected return of 0.279796 and risk of 0.052628. This intermediate position indicates that the L_2 norm provides a balance between return improvement and risk control compared with the more conservative L_1 and more aggressive L_∞ solutions. However, the obtained utility values are relatively close across all norms, ranging from 0.484861 to 0.486311, suggesting that different distance metrics lead to comparable overall preference outcomes despite producing different risk-return combinations.

These findings demonstrate that the selection of the L_γ norm affects the balance between return maximization and risk control rather than consistently improving the utility value. Therefore, the L_2 norm is selected as the primary distance measure because it provides a balanced compromise between the conservative L_1 solution, and the return-oriented L_∞ solution while maintaining a moderate trade-off between expected return and portfolio risk.

Table 14. Comparison of Normalized Distances under Different L_γ Norms

Norm	**Upper Constraint**	d_{Return}	d_{Risk}	$d_{Utility}$
L_1	$w_i \leq 0.30$	0.446677	0.334144	0.388446
L_2	$w_i \leq 0.30$	0.418332	0.359151	0.393857
L_∞	$w_i \leq 0.30$	0.392677	0.392677	0.392677

This behavior is also reflected in the normalized deviations presented in Table 14. The Chebyshev (L_∞) norm produces identical normalized deviations across the three objectives, $[0.392677,\ 0.392677,\ 0.392677]$, reflecting its minimax characteristic of minimizing the maximum deviation from the ideal solution. In contrast, the L_1 and L_2 norms produce different distributions of normalized deviations, corresponding to alternative compromise strategies that place different emphasis on return, risk, and utility. Therefore, the L_2 norm is adopted as the final distance measure in the Compromise Programming model.

Table 15. Optimal Portfolio Weights under Different L_γ Norms

Stock	L_1 **(%)**	L_2 **(%)**	L_∞ **(%)**
ANTM	30.00	30.00	30.00
JPFA	16.99	17.28	17.40
BRPT	14.40	16.64	19.09
ACES	14.40	13.78	13.45
EXCL	10.43	9.30	7.68
PTPP	7.10	6.84	6.90
TPIA	5.05	5.07	4.92
ERAA	1.65	1.09	0.56

Finally, Table 15 presents the optimal portfolio weights obtained under the three L_γ norms. The results show that the same eight stocks are selected under all optimization scenarios, namely ANTM, JPFA, BRPT, ACES, EXCL, PTPP, TPIA, and ERAA. This consistency indicates that the portfolio composition remains unchanged despite variations in the norm parameter. Although the selected assets are identical, several allocation weights vary across the three norms. ANTM consistently reaches the maximum allocation limit of 30%, indicating that it is the most preferred asset under all scenarios. The allocation of BRPT increases from 14.40% under the L_1 norm to 19.09% under the L_∞ norm, whereas the weights of EXCL and ERAA decrease accordingly. In contrast, the allocations of JPFA, ACES, PTPP, and TPIA remain relatively stable across all optimization scenarios.

Overall, the choice of norm affects only the distribution of portfolio weights while preserving the portfolio composition. Combined with the results in Tables 13, 14 and 15, these findings suggest that the proposed Compromise Programming model exhibits limited sensitivity to the choice of norm, with only modest differences in allocation weights. Among the evaluated norms, the Euclidean (L_2) norm provides the most balanced trade-off between return, risk, and utility.

4.4. Benchmark Comparison

Table 16 compares the performance of the proposed UTASTAR-CP model with the Markowitz mean–variance model and the equal-weighted portfolio. The evaluation considers expected return, portfolio risk, and utility value where applicable.

Table 16. Portfolio Performance Comparison

Method	**Return**	**Risk**	**Utility**
Markowitz	0.350859	0.050000	–
Equal-Weighted Portfolio	0.030102	0.157171	–
UTASTAR-CP	0.279796	0.052628	0.484861

The utility value is reported only for the proposed UTASTAR-CP model because it is derived from the UTASTAR utility function. In contrast, the Markowitz and equal-weighted portfolio models optimize portfolio allocation without incorporating a utility representation based on multiple financial criteria.

As shown in Table 16 the Markowitz model achieved the highest expected return (0.3509) and the lowest portfolio risk (0.0500) in this experiment. The proposed UTASTAR-CP model generated an expected return of 0.2798 with a portfolio risk of 0.0526, while the equal-weighted portfolio produced the lowest expected return (0.0301) and the highest portfolio risk (0.1572). These results indicate that the equal-weighted strategy is less effective than optimization-based approaches for the dataset considered.

In contrast, the equal-weighted portfolio serves as a non-optimized benchmark that allocates identical weights across all selected assets without considering differences in expected return, volatility, correlation structure, or multi-criteria preferences. The relatively low return and higher risk obtained in this experiment suggest that equal allocation may not effectively exploit the heterogeneous characteristics of the selected stocks. This result indicates that incorporating optimization mechanisms can improve the ability to balance portfolio performance compared with a simple diversification strategy.

The differences between the Markowitz and UTASTAR-CP results are consistent with their distinct optimization objectives. The Markowitz model is designed exclusively to optimize the trade-off between expected return and portfolio risk. In contrast, the proposed UTASTAR-CP model simultaneously considers three objectives: maximizing expected return, minimizing portfolio risk, and maximizing the UTASTAR global utility, which summarizes information from multiple financial criteria. Consequently, the proposed model does not aim to maximize return or minimize risk individually, but rather to identify a Pareto-efficient compromise portfolio that balances these conflicting objectives.

Finally, although the UTASTAR-CP portfolio does not achieve the highest return or the lowest risk individually, it provides a more comprehensive decision-support framework by integrating multiple financial criteria into the portfolio optimization process. Therefore, the benchmark comparison should be interpreted as illustrating the trade-offs between different optimization philosophies rather than demonstrating the absolute superiority of one model over another. While the Markowitz model remains effective for optimizing the return–risk trade-off, the proposed UTASTAR-CP model offers an alternative multicriteria perspective by incorporating a utility-based representation of financial criteria to identify a balanced compromise solution.

5. Conclusion

This study developed a portfolio selection framework that integrates the Entropy Weight Method (EWM), UTASTAR, and Compromise Programming to construct portfolios by simultaneously considering expected return, portfolio risk, and utility information derived from multiple financial criteria. Within the proposed framework, the EWM is employed to generate pseudo utility values and pseudo rankings from historical market data, which are subsequently incorporated as reference information during the UTASTAR estimation process to infer a global utility function. The inferred utility function is then used as an additional optimization criterion within the Compromise Programming model alongside expected return and portfolio risk. In this way, the proposed framework complements the conventional mean–variance perspective by incorporating a utility criterion derived from multiple financial indicators into the portfolio optimization process.

The empirical results indicate that the proposed UTASTAR–CP framework is capable of generating a compromise portfolio by simultaneously considering expected return, portfolio risk, and utility. Comparisons with the Markowitz Mean–Variance and Equal-Weighted benchmark portfolios show that the proposed framework provides a competitive alternative for incorporating utility information into portfolio optimization. In addition, the sensitivity analysis across the L_1, L_2, and L_∞ norms demonstrates that different compromise metrics lead to different return–risk–utility trade-offs while maintaining a relatively stable portfolio composition..

The findings suggest that the proposed framework may serve as a decision-support tool for investors during the portfolio screening and selection process. For retail investors, the model may provide a systematic approach to evaluating stock alternatives by considering multiple financial characteristics rather than relying solely on expected return and portfolio risk. Nevertheless, the resulting portfolio should not be interpreted as a fixed investment recommendation, as investment decisions remain dependent on individual risk preferences, investment objectives, prevailing market conditions, and other practical considerations.

This study has several limitations. First, the proposed framework relies on historical financial data collected over a specific observation period; consequently, the resulting portfolio reflects the prevailing market conditions during that period and may differ under alternative market environments. Second, the estimates of expected returns, the covariance matrix, and the utility representation inferred through UTASTAR depend on historical information and may not fully capture future changes in market conditions, macroeconomic conditions, or evolving market dynamics. Moreover, the proposed framework treats the covariance matrix and the selected fundamental financial indicators as deterministic inputs and does not explicitly account for estimation uncertainty or non-stationarity. Third, the pseudo-cardinal utility values and pseudo-rankings generated by the EWM should be interpreted as objective, data-driven reference information rather than as representations of the intrinsic or psychological preferences of individual investors. Because historical financial data may also reflect herding behavior, speculative episodes, market regime shifts, and other forms of market noise, the estimated utility function should be interpreted as a utility representation inferred from objective data-driven reference information rather than as a behavioral model of investor decision-making. Fourth, although ranking consistency was evaluated using Spearman's rho and Kendall's tau, the proposed framework does not quantify the uncertainty of the estimated marginal utility functions through confidence intervals or other statistical uncertainty measures. Finally, the proposed framework is intended to support portfolio selection under historical market conditions rather than to predict future stock price movements. Therefore, the resulting portfolio should be interpreted as an optimization outcome under the observed historical conditions rather than as a permanent investment recommendation.

Furthermore, this study does not incorporate risk-adjusted performance measures, such as the Sharpe Ratio, because its primary objective is the development of an integrated portfolio selection framework combining UTASTAR and Compromise Programming. Future research may extend the proposed framework by incorporating risk-adjusted performance measures, transaction costs, portfolio turnover constraints, liquidity considerations, and dynamic portfolio rebalancing to improve its practical applicability. Future studies may also consider chronological rolling-window (walk-forward) validation, explicit modeling of estimation uncertainty and non-stationarity in the covariance matrix and financial indicators, confidence intervals for the estimated marginal utility functions, and additional financial or sustainability-related criteria when sufficiently consistent historical data are available. Alternative multi-criteria decision-making methods and preference inference approaches may also be investigated to further improve the flexibility and generalizability of the proposed framework.

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Author Contributions

Eka Kristya Rahayu: Conceptualization, methodology, data simulation, software, formal analysis, visualization, and writing–original draft, writing–review and editing. Farikhin: Conceptualization, methodology, formal analysis, writing–review and editing, supervision. Moch. Fandi Ansori: Conceptualization, methodology, validation, writing–review and editing, supervision. All the authors have read and approved the final version of the manuscript.

Conflict of Interest

The authors declare no conflicts of interest.

Data Availability

The data used in this study are publicly available. Historical daily stock prices were obtained from `investing.com`, while company financial indicators were derived from the annual reports of the respective companies downloaded from the Indonesia Stock Exchange (IDX). The processed datasets and Python code developed for the analysis are available from the corresponding author upon reasonable request.

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