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Vertex Antimagic Total Labeling of Modified Fan Graphs

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Abstract Graph labeling is the process of assigning labels to the vertices and edges of a graph according to specific rules or conditions. A vertex antimagic total labeling (VATL) is a distinct assignment of positive integers to all vertices and edges of a graph such that the total weight obtained by adding the label of each vertex and the labels of all edges incident with it, is unique for every vertex in that graph. This paper investigates and proves the vertex antimagic total labeling of a modified fan graph, which is obtained by merging the isolated vertices K_1 from n copies of the disjoint union $K_1 \bigcup \overline{(K_2)} \bigcup \overline{(K_m)}$ into a single apex vertex. This study extends it by proposing an algorithm and providing a computational complexity of O(nm) as evidence for verification of this labeling.

Keywords graph labeling, vertex antimagic total labeling, modified fan graph, vertex weights, computational verification.

AMS 2010 subject classifications 05C78

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1. Introduction

Let a graph G = (V, E) be a simple, finite, undirected graph with $|V|$ = n vertices and $|E|$= m edges. Graph labeling was first introduced in the late 1960s. Many studies in graph labeling refer to Rosa's research (1967). An antimagic labeling of a graph G = (V, E) is a one-to-one mapping from E to $\{1, 2, ..., m\}$ such that distinct vertices receive different label sums from the edges incident to them. G is called antimagic if it admits an antimagic labeling.
Vertex Antimagic Total Labeling $(VATL)$: A vertex antimagic total labeling of G is a bijection f : V$(G) \bigcup$ E(G) $\rightarrow$ $\{1, 2, 3, ..., n + m\}$ such that the vertex weight w(v) of each vertex v $\in$ V(G) defined by w(v) = [f(v) + $\sum$ f(uv)] $\in$ E(G), $(uv) \in$ E(G), is distinct for all vertices of G. If such a labeling exists, then the graph G is said to admit a vertex antimagic total labeling.
The antimagic labeling of graphs was introduced by [1] (also in [2]), who verified the antimagicness of paths, 2-regular graphs, and complete graphs. An extensive survey of contributions to graceful labeling of a variety of graphs was given by [3]. The notation and terminology used in this paper are taken from [3]. Regular graphs with an odd degree are antimagic, as proved in [4]. Antimagic labeling of complete multipartite graphs constructed in [5]. In[6], graceful and antimagic labelings were presented, and in the same year [7] provided vertex antimagic total labelings. Ladders and fangraphs are cycle-antimagic and were discussed by [8]. Every connected graph other than K_2 was shown to be local antimagic in [9]. A labeling concept called "local antimagic labeling," defined as a labeling in which the adjacent vertices have distinct weights is discussed in [9]. Antimagic labeling of regular graphs was studied in [10]. Dense graphs were shown antimagic in [11]. Antimagic labeling of triangular book graphs and double fan graphs was discussed in [12]. A proof of a local antimagic conjecture was given in [13]. Fans were shown cycle-antimagic in [14]. Results on even and odd antimagic total labeling are provided in [15]. Investigation of super lehmer–3 mean labeling in theta related-graphs was discussed in [16]. A study on neutrosophic cordial labeling

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graphs with an algorithm shown in [17]. The above contributions motivated us to provide a vertex antimagic total labeling of a special type of fan graph called a modified fan graph denoted by $F^n_{(2,m)}$ for all integers $n \geq 2$, $m \geq 2$. This graph is constructed by merging the isolated vertices K_1 from n copies of the disjoint union $K_1 \bigcup \overline{(K_2)} \bigcup \overline{(K_m)}$ into a single apex vertex. The resulting structure features a central vertex connected to supporting vertices, which in turn connect to incidental vertices only, forming an edgeless subgraph. The total number of edges and vertices in $F^n_{((2,m))}$ is precisely $n(4+3m)+1$. This article proves that $F^n_{(2,m)}$ admits a vertex antimagic total labeling for all valid values of m and n, thereby contributing a new family of graphs to the growing interest in vertex antimagic total structures.

2. Main Result

Theorem: A modified Fan graph $F^n_{(2,m)}$ for all integers $n \geq 2$, $m \geq 2$ satisfies vertex antimagic total labeling (VATL).
Proof:
The assignment of edge values in the proposed graph structure follows a systematic vertex-edge labeling scheme. Let n denote the number of copies of the fan graph and m represent the number of internal tip vertices within each blade B_i. For each $i \in \{1, 2, ..., n\}$, the edge labels are defined as follows.

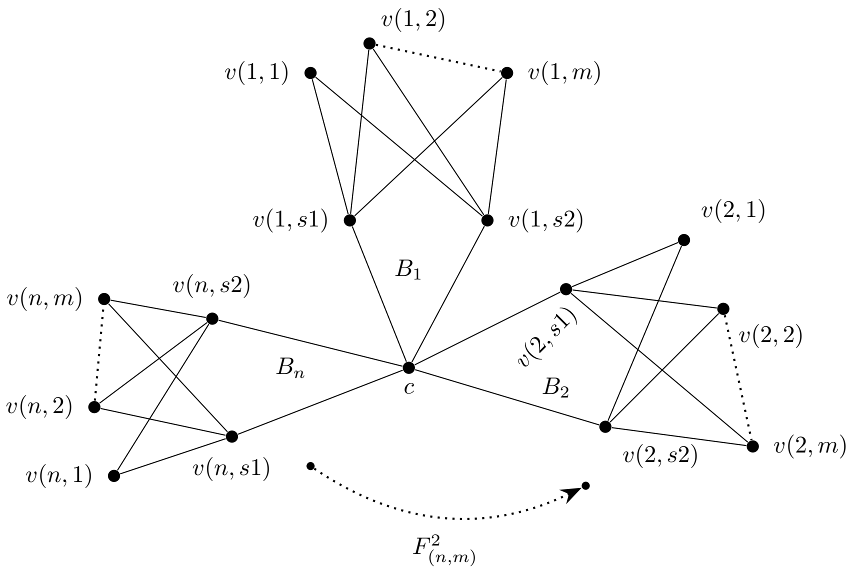

Figure 1. Modified Fan Graph

The central vertex **c** is connected to two supporting vertices, **s1** and **s2**.
If **m is either odd or m ($\geq$ 4) is even,** and n can be either odd or even, the labels for the edges connecting the central vertex to the supporting vertices are assigned as follows.
$f(c) = n(4+3m)+1$.
$f(cv(i,s1)) = (n-i)(3m+4) + 2(m+2)$, i = 1, 2,... n.
$f(cv(i,s2)) = (n-i)(3m+4) + 2m + 3$, i = 1, 2,... n.
These assignments ensure that each edge receives a unique label, with values decreasing as the index i increases.
For each internal vertex $v(i,j)$, where $j \in \{1, 2, ..., m\}$ the edges connecting $v(i,s1)$ to $v(i,j)$ are assigned the labels:
$f(v(i,s1)v(i,j)) = (n-i)(3m+4) + 1 + 2(j-1)$, i = 1, 2, ... n, j = 1, 2... m.
similarly, the connecting $v(i,s2)$ to $v(i,j)$ is assigned with the label

f$(v(i,j)v(i,s2))$ = $(n-i)$ $(3m+4)$ + 2 + 2$(j-1)$, i = 1, 2,... n, j = 1, 2,... m.
This pattern ensures a consistent interleaving of even and odd labels within each blade B$_i$.
The labels for the vertices given in the following expressions:
f $(v(i,s1))$ = $(n-i)(3m+4)$ + 2m + 1, i = 1, 2,..., n.
f$(v(i,s2))$ = $(n-i)(3m+4)$ + 2$(m+1)$, i = 1, 2,..., n.
f$(v(i,j))$ = $(3m+4)(n-i+1)$ + j - m, i = 1, 2,..., n, j = 1,2,..., m.
Special case: for m = 2, and for any n (odd or even), the labeling is given below.
f(c) = 10n + 1. From the earlier labeling calculations, the changes occur only between the edges of central and supporting vertices through their exchange of labeling values. i.e f $(cv(i,s1)) \to$ f $(cv(i,s2))$ and f $(cv(i,s2)) \to$ f $(cv(i,s1))$. f $(cv(i,s1))$ = 10$(n-i)$ + 7, i = 1,2, ..., n. f $(cv(i,s2))$ = 10 $(n-i)$ + 8, i = 1,2, ..., n.
Vertex weights:
The resultant vertex weights, obtained by summing each vertex values with its incident edge values as follows.
w(c) = 3mn^2 + 4n^2 + 4mn + 7n + 1.
w$(v(i,s1))$ = $(3n-3i+1)$ m^2 + $(10n-10i+4)$ m + $(8n-8i+5)$, for i = 1, 2, ..., n.
w$(v(i,s2))$ = $(3n-3i+1)$ m^2 + $(10n-10i+5)$ m + $(8n-8i+5)$, for i = 1, 2, ..., n.
w$(v(i,j))$ = $(9n-9i+2)$m + $(12n-12i+3+5j)$, for each i = 1, 2, ..., n, for j = 1, 2, ..., m.
For the special case m = 2 the change happens only on w$(v(i,s1))$ = 40$(n-i)$ + 16, for i = 1, 2, ..., n. w$(v(i,s2))$ = 40$(n-i)$ + 20, for i = 1, 2, ..., n . The remaining outcomes are same.
This labeling ensures that all edge and vertex labelled values are distinct and that the corresponding vertex weights (denoted within a bracket) are unique, satisfying the conditions of a vertex antimagic total labeling for a modified fan graph.

2.1. Algorithmic view

Algorithm: Vertex and Edge label assignment: odd m or m (≥ 4) even

Input:
Positive integers n and m.
Output:
A labeling function f assigning distinct labels to every vertex and edge of the graph.

Step 1: Initialize

Compute the base value
B = 3m + 4
f(c) = nB + 1.

Step 2: For i = 1, 2, ...,n, do:

(a) Labels of connection edges cv$(i,s1)$ and cv$(i,s2)$
f $(cv(i,s1))$ = $(n-i)$ B + 2$(m+2)$
f $(cv(i,s2))$ = $(n-i)$ B + 2m + 3
(b) Labels of supporting vertices v$(i,s1)$, v$(i,s2)$
f $(v(i,s1))$ = $(n-i)$ B + 2m + 1
f$(v(i,s2))$ = $(n-i)$ B + 2$(m+1)$

(c) For j = 1, 2, ..., m do (1) Label of edge from v$(i,s1)$ to tip vertex v(i,j):

f $(v(i,s1)v(i,j))$ = $(n-i)$ B + 1 + 2$(j-1)$
(2) Label of edge from v$(i,s2)$ to tip vertex v(i,j):
f$(v(i,j)v(i,s2))$ = $(n-i)$ B + 2 + 2$(j-1)$

(d) Label of the terminal vertex v(i, j)

Assign
f$(v(i,j))$ = B $(n - i + 1)$ + j - m
End For
End For
Return f.

2.1.1. *For m = 2*

Algorithm: Vertex and Edge label assignment: for m = 2(only change happens in step 2 (a) part)

Step 1: Initialize
Compute the base value
B = 10
f(c) = 10n + 1.

Step 2: For i = 1, 2, ...,n, do:

(a) Labels of connection edges cv$(i, s1)$ and cv$(i, s2)$
f $(cv(i, s1))$ = 10 $(n - i)$ + 7
f $(cv(i, s2))$ = 10 $(n - i)$ + 8
remaining parts are same as earlier.

An algorithm is said to run in polynomial time if its complexity can be expressed as O$\left(n^k\right)$ for some constant k. For this problem the nested loop (outer loop n times and inner loop m times) runs over n . m times result time complexity is O(nm) polynomial time.
To prove the uniqueness in the labeling, consider edge difference and supporting vertex differences. In step 2, (a) and (b) part differences are 2m + 1, 2m + 2, 2m + 3 and 2m + 4. In the (c) part differences between the edges are consecutive integers differing by 1. The (d) part value is different from (a), (b) and (c). Hence all difference values are distinct, ensuring uniqueness in labeling.
Similarly consider vetex weight differences: w(c) is different from all other values. Difference between | w$(v(i, s1))$ – w$(v(i, s2))$ | = m for all i. Internally, $|w\ (v(i, j+1)) - w\ (v(i, j))\,|$ = 5, $|w\ (v(i+1, j)) - w\ (v(i, j))\,|$ = 9m + 12 and | w$(v(i+1, s1))$ – w$(v(i, s1))$ | = (m + 2)(3m + 4) = | w$(v(i+1, s2))$ – w$(v(i, s2))$ | for supporting vertices w$(v(i, s1))$. For m = 2, difference between | w$(v(i, s1))$ – w$(v(i, s2))$ | = 2m and $|w\ (v(i, j+1)) - w\ (v(i, j))\,|$ = 5 internally. This result the vertex weights are pairwise different and not same with earlier values. Hence all vertex weights (denoted within a bracket) are distinct, results uniqueness in labeling outcome.

2.2. *Example*

The resultant vertex weights for for n = 3, m = 3(shown in bracket) given below for understanding.
central vertex weight: 175. The supporting vertices weights are w(v $(1, s1)$): 156, w(v $(1, s2)$): 159, w(v$(2, s1)$): 91, w(v$(2, s2)$): 94, w(v$(3, s1)$): 26, w(v$(3, s2)$): 29.
The internal vertex weights are w(v$(1, 1)$) : 92, w(v$(1, 2)$) : 97, w(v$(1, 3)$) : 102, w(v$(2, 1)$) : 53, w(v$(2, 2)$) : 58, w(v$(2, 3)$) : 63, w(v$(3, 1)$) : 14, w(v$(3, 2)$) : 19, w(v$(3, 3)$) : 24.
All 16 vertex weights are distinct for n = 3, m = 3, the expressions yield a valid vertex-antimagic total outcome (no two vertex weights coincide).

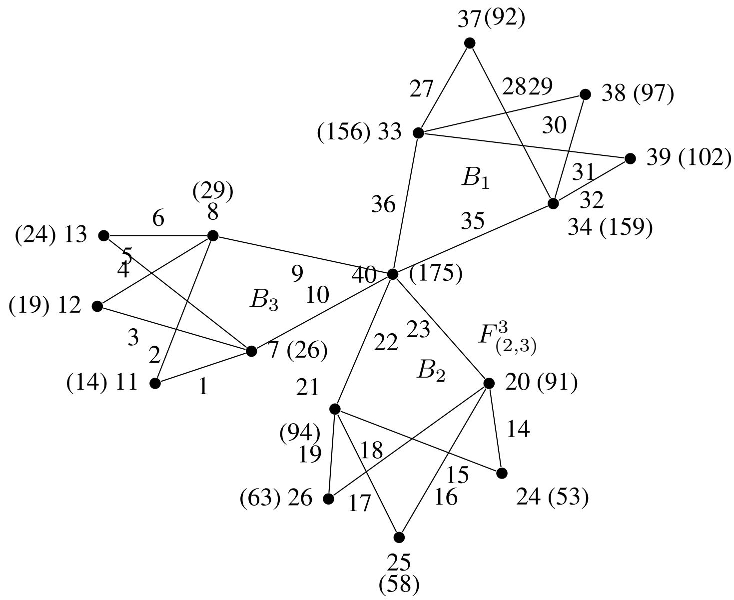

Figure 2. modified $F^3_{(2,3)}$ Graph

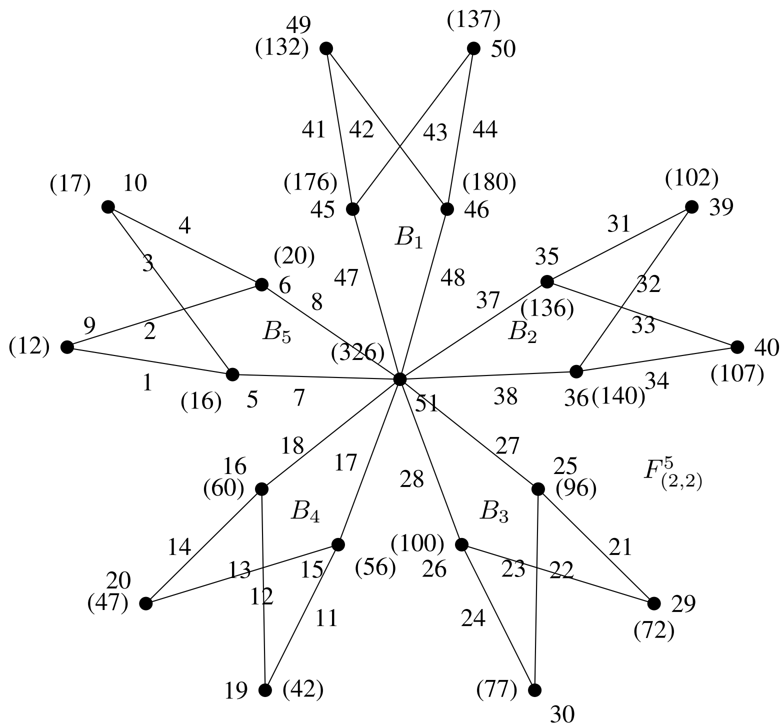

Figure 3. modified $F^5_{(2,2)}$ Graph

3. Conclusion

This work confirms the vertex antimagic total labeling of modified fan graphs, thereby extending the study of graph labeling in fan graph structures. By assigning distinct integers to edges and ensuring distinct pairwise vertex weights (denoted within a bracket), the results demonstrate the feasibility of vertex antimagic total labeling in non-trivial graph families. Moreover, the algorithmic approach enables automated verification for larger graphs, opening new directions for computational study in labeling schemes.

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