STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **16**, July 2026, pp 542–566.
Published online in International Academic Press (www.IAPress.org)

Statistical Inference on Accelerated Odd Fréchet Half-Logistic Distribution under Progressive Type-II Adaptive Hybrid Censoring with Application to Dielectric Circuits Using Binomial Removals

Ehab M. Almetwally [1], Samirah Alzubaidi [2], T. S. Taher [3], Eslam Hussam [4,*]

[1] *Faculty of Business Administration, Delta University for Science and Technology, Gamasa 11152, Egypt*
[2] *Department of Mathematics, Al-Qunfudah University College, Umm Al-Qura University, Mecca, Saudi Arabia*
[3] *Department of Mathematics, Faculty of Science, Zagazig University, Zagazig 44519, Egypt*
[4] *Department of Mathematics, Faculty of Science, Capital University, Cairo, Egypt*

Abstract This research presents a statistical study, where the sample is subjected to a progressive stress-accelerated life test (PSALT) generated from the odd Fréchet half-logistic distribution (OFHLD) under adaptive progressive type-II hybrid censored (AP-II-HC) sampling. The cumulative exposure model is applied to generate incremental stress samples. Both classical and Bayesian methodologies are used to estimate the distribution's unknown parameters. Furthermore, the reliability function of the OFHLD is also calculated. The Metropolis-Hastings(MH) algorithm is applied to generate samples from the distribution. Moreover, the asymptotic and bootstrap confidence intervals (CIs) are constructed. A real data set is analyzed to illustrate the methodologies suggested in this study. Finally, some intriguing conclusions are noted and associated with future work suggestions.

Keywords Progressive-stress, Progressive type-II adaptive hybrid censoring, Maximum likelihood estimation, Bayes estimation, Simulation study

AMS 2010 subject classifications 62N01, 62N02, 62F10, 62F15

DOI: 10.19139/soic-2310-5070-3558

1. Introduction

In many conventional life testing and reliability investigations, obtaining sufficient failure times can be challenging, especially when products exhibit excellent reliability and extended lifespans. A prominent method to expedite failure occurrence is accelerated life testing (ALT), see Abd El-Raheem et al. [4], and Nelson [37], wherein products are subjected to elevated stress levels, such as humidity, temperature, pressure, voltage, see Kumar [30].

ALT subjects items or materials to heightened stress conditions that exceed standard operational thresholds, like elevated temperature, pressure, or voltage. Refer to AL-Hussaini et al. [9]. It operates in various modes, including continual stress, step stress, and PSALT, as referenced in Zhu [47].

Nelson [37] discussed these different types of ALT. The most common types of progressive stress loading include step-stress loading, where stress is increased in fixed increments after set intervals; ramp-stress loading, which applies a continuous, gradual increase in stress over time; and cyclic stress loading, see Ismail [27] where the stress fluctuates periodically but generally intensifies with each cycle, see Abd El-Raheem [3].

Many researchers worked on real data sets under different kinds of stress, for example, see [34] who worked on experimental real data that was recorded. Also see [3], who worked on a ramp-voltage experiment of miniature

*Correspondence to: Eslam Hussam (Email: eslmhussam1986@gmail.com). Department of Mathematics, Faculty of Science, Capital University, Cairo, Egypt.

ISSN 2310-5070 (online) ISSN 2311-004X (print)

light bulbs, also [23], who worked on step stress data that represents an experiment on some light bulbs with working-use stress of 2 volts. Also [4] worked on transforming data that were obtained in the laboratory in a real experiment.

The primary objective of PSALT is to expedite the occurrence of failure modes, allowing engineers and researchers to understand the product's weak points and predict its longevity in a much shorter time frame, see Abdel-Hamid [6], and Abd El-Raheem et al [1]. See Balakrishnan and Han, [17] for more information. Recent discussions on step-stress partially-accelerated life test models can be found in [43]. Competing risks in accelerated life testing with a study on step-stress models have been discussed by [25]. Tampered random variable analysis in step-stress testing with modeling, inference, and applications have been obtained by [26]. Bivariate step-stress accelerated life tests for the Kavya–Manoharan exponentiated Weibull model under progressive censoring with applications have been discussed by [14].

One of the most significant ways to reduce time and costs is censorship. Censored data are commonly utilized in reliability and life-testing studies, as noted by Dey et al. [21]. Bivariate step-stress accelerated life test for a new three-parameter model under progressive censored schemes with application in medical has been discussed by [12]. Investigators are required to gather data using censored samples to account for factors such as preserving experimental units for future use. Refer to Rastogi et al. [42] and Ng et al. [40]. Kundu and Joarder [29] introduced a progressive type-I hybrid censoring (P-I-HC), whereas Mohie El-Din et al. [34, 35] provide more flexibility by permitting the withdrawal of certain test units at different phases. Refer to Abdel-Hamid et al. [5] for additional details. Optimal test plan of step-stress model of alpha power Weibull lifetimes under progressively type-II censored samples has been introduced by [13].

An adaptive progressive type-II hybrid censoring (AP-II-HC) methodology to enhance the efficiency of statistical analysis was presented by Ng et al. [39] Refer to the works of Aggarwala et al. [15], Almetwally [8] and Cramer et al. [16].

1.1. Main contribution and novelty

The main contribution of this paper is that we performed statistical inference on the OFHLD under the AP-II-HC scheme with PSALT. We applied real experimental data, estimated the parameters and CIs, and compared our model with existing studies, demonstrating that the OFHLD provides the best fit.

The main advantage of AP-II-HC is that it saves time and money. This scheme can be summarized as follows: Suppose that n units are subjected to a life test, and $m < n$ is the desired total number of failures. At the time of the ith failure $X_{i:m:n}$, R_i units are eliminated from the test. If a failure occurs before time T (i.e., $(X_{m:m} < T)$), the experiment terminates. We have the regular progressive type-II censoring; see [38]. If, on the other hand, $X_{d:m:n} < T < X_{d+1:m:n}$, where $d + 1 < m$ and $X_{d:m:n}$ correspond to the d_{th} failure time and occur before time T, then we will not remove any living item from the experiment by placing $R_{d+1}, R_{d+2}, \ldots, R_{m-1} = 0$ and $R^*_m = n - m - \sum_{i=1}^{J} R_i$. Figure 1 presents a diagrammatic representation of the AP-II-HC strategy. See Raqab et al. [41], and Wu et al. [46].

The remainder of this paper is structured as follows. Section 2 introduces the cumulative exposure model and how it works. Section 3 presents the estimation methods and the confidence intervals. Section 4 describes the Bayesian estimation technique, the algorithms used, and the credible intervals. Section 5 discusses the simulation results and the data generation algorithm. Section 6 presents the real data application for progressive stress. Section 7 provides the conclusion drawn from the simulation and application sections.

1.2. The OFHLD

This paper introduces statistical inference for the OFHLD. The distribution was derived from the Fréchet model introduced by Bhat et al. [19], which has many applications in actuarial science, particularly in analyzing pensioner mortality data. The probability density function (PDF) and cumulative distribution function (CDF) of the OFHLD are given in equations (1) and (2), respectively. See Bhat et al. [19] for more reading about the distribution.

$$f(x) = \frac{\alpha\theta\left(2e^{-\theta x}\right)^{\alpha}}{\left(1-e^{-\theta x}\right)^{\alpha+1}} \exp\left(-\left(\frac{2e^{-\theta x}}{1-e^{-\theta x}}\right)^{\alpha}\right), \quad \alpha>0,\ \theta>0,\ x>0, \tag{1}$$

, and

$$F(x) = \exp\left(-\left(\frac{2e^{-\theta x}}{1-e^{-\theta x}}\right)^{\alpha}\right), \tag{2}$$

where $\alpha,$ is a shape parameter , and $\alpha>0,\ \theta>0,\ x>0, \theta$ is scale parameter.

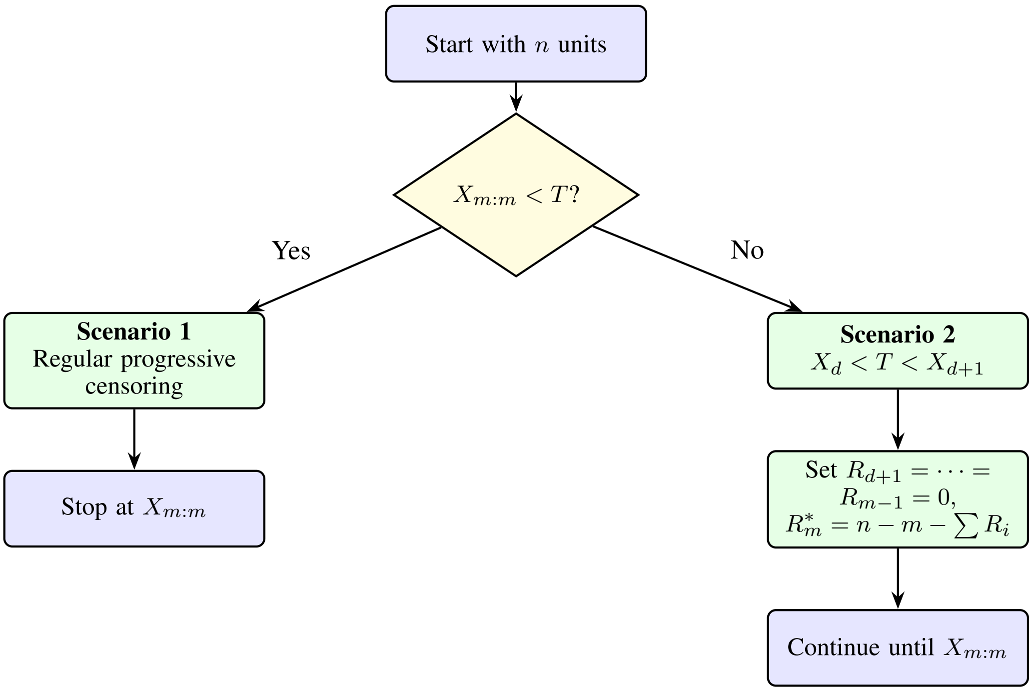

Figure 1. Diagram of the Adaptive Progressive Type-II Hybrid Censoring (AP-II-HC) strategy.

Let $x_{1:n} < \cdots < x_{d:m:n} < T < x_{d+1:m:n} < \cdots < x_{m:m:n}$ be an AP-II-HC sample from a population with a PDF $f(x)$ and CDF $F(x)$; then, the likelihood function (LF) for the observed data can be written according to Ng et al. [39] as

$$L = C\prod_{i=1}^{m} f(x_{i:m:n}) \prod_{i=1}^{J} [1-F(x_{i:m:n})]^{R_i} [1-F(x_{m:m:n})]^{R_m}, \tag{3}$$

Let $x_{1:n} < \cdots < x_{d:m:n} < T < x_{d+1:m:n} < \cdots < x_{m:m:n}$ be an AP-II-HC sample from a population with a PDF $f(x)$ and CDF $F(x)$; then, the likelihood function (LF) for the observed data can be written according to Ng et al. [39] as

$$L = C\prod_{i=1}^{m} f(x_{i:m:n}) \prod_{i=1}^{J} [1-F(x_{i:m:n})]^{R_i} [1-F(x_{m:m:n})]^{R_m}, \tag{4}$$

where C is the normalizing constant given by: $C = \prod_{j=1}^{m}\left(n-j+1-\sum_{i=1}^{j-1} R_i\right)$, which accounts for the combinatorial arrangement of removals. For further details, see Ng et al. [39].

2. Test Assumptions

Throughout this paper, we use the following notation: $j = 1, \ldots, k$ for stress levels, $i = 1, \ldots, m_j$ for failure times within level j, and θ_1 denotes the scale parameter at the baseline stress level ν_1.

In this Section, we list the assumptions that are used throughout this article in the context of progressive-stress ALT:

1. The lifetime of a unit follows the OFHLD
2. The direct proportionality between the time and the progressive-stress at level i, denoted as $\phi_i(t)$, is maintained with a fixed rate.ν_i, i.e.,
$$\phi_i(t) = \nu_i t, \quad 0 < \nu_1 < \nu_2 < \cdots < \nu_k.$$
3. The relation between θ_i and the stress loading $\phi_i(t)$, as follows:
$$\theta_i(t) = \frac{1}{a[\phi_i(t)]^\gamma}$$
Such that a and γ are greater than zero parameters. see, e.g., Nelson [37].
4. When attempting to quantify the impact of stress on failure time from one level to another, the cumulative exposure model is utilized. For more information on this topic, please refer to Nelson [37].
5. From the There is a link between life and stress at point number three, and the parameter θ_i can be represented as $\theta_i = \theta_1 \psi_i^{\gamma_i}$, where $\psi_i = \frac{\nu_1}{\nu_i}$.
6.
7. **Removal mechanism:** We assume that the number of units removed at each failure time follows a binomial distribution. This assumption is widely used in progressive censoring literature due to its mathematical convenience and flexibility. In the context of the dielectric circuit experiment, this corresponds to a scenario where each surviving unit has an independent probability p of being withdrawn after a failure occurs. We acknowledge that this is a theoretical assumption for modeling purposes; it is not derived from the physical properties of the dielectric circuits.

Based on the assumption of the cumulative exposure model, the CDF for the lifetimes that result from progressive stress $\phi_i(t)$ is

$$G_i(t) = F_i(\Delta(t)), \quad i = 1, 2, \ldots, k, \tag{5}$$

where

$$\Delta(t) = \int_0^t \theta_i(u)du = t\theta_1\psi_i^\gamma(\gamma + 1),$$

The definition of $F_i(.)$ is given in (2), where the scale parameter is set to 1. Which is

$$G_i(t) = e^{-\left(\frac{2e^{-\sigma_j t}}{1-e^{-\sigma_j t}}\right)^\alpha}, \quad t > 0,\ \gamma, \alpha, \sigma_j > 0, \theta_1 > 0, \tag{6}$$

where $\sigma_j = \theta_1 \psi_i^\gamma(\gamma + 1)$. The PDF of (6) is given by

$$g_i(t) = \frac{\alpha\sigma_j \left(2e^{-\sigma_j t}\right)^\alpha}{\left(1 - e^{-\sigma_j t}\right)^{\alpha+1}} e^{-\left(\frac{2e^{-\sigma_j t}}{1-e^{-\sigma_j t}}\right)^\alpha} \quad t > 0,\ \gamma, \alpha, \sigma_j, \theta_1 > 0. \tag{7}$$

3. Estimation Methods

This section is devoted to discussing the estimation process.

3.1. Maximum Likelihood Estimation

This section explains the procedure for estimating the maximum likelihood estimates (MLEs) of the parameters θ_1, α, and γ using AP-II-HC data within a PSALT framework. Let $t_{ij:m_i:n_i} = t_{ji}$ represent the observed lifetime values T obtained from AP-II-HC at the progressive-stress level $\phi_i(t)$, where $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, m_i$. Using the CDF in Equation (6) and the PDF in Equation (7). The likelihood function $L(\theta_1, \gamma, \alpha)$ is expressed as:

$$L(\theta_1, \gamma, \alpha) = \prod_{j=1}^{k} C_j \prod_{i=1}^{m} g_j(t_{ji}) \prod_{i=1}^{J_j} [1 - G_j(t_{ji})]^{R_j} [1 - G_j(t_{ji})]^{R_m^*}. \tag{8}$$

where

$$C_j = n_j(n_j - 1 - R_{j1})(n_j - 2 - R_{j1} - R_{j2}) \cdots \left(n_j - m_j + 1 - \sum_{i=1}^{J_j - 1} R_{ji} \right).$$

Substituting the expressions from Equations (6) and (7) into (8), we obtain:

$$\begin{aligned} L(\theta_1, \gamma, \alpha) = \prod_{j=1}^{k} C_j \prod_{i=1}^{m_j} \frac{\alpha \sigma_j \left(2e^{-\sigma_j t_{ji}}\right)^{\alpha}}{\left(1 - e^{-\sigma_j t_{ji}}\right)^{\alpha+1}} e^{-\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}} \prod_{i=1}^{J_j} \left[1 - e^{-\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}} \right]^{R_{ji}} \\ \left[1 - e^{-\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha}} \right]^{R_{jm_j}^*}, \end{aligned} \tag{9}$$

where $\sigma_j = \theta_1 \psi_j^{\gamma}(\gamma + 1)$. Therefore, the log-likelihood function is given by

$$\begin{aligned} \ell(\theta_1, \gamma, \alpha) = \sum_{j=1}^{k} \log C_j + \sum_{j=1}^{k} m_j \left(\log \alpha + \alpha \log 2 + \log \sigma_j \right) - \alpha \sum_{j=1}^{k} \sum_{i=1}^{m_j} \sigma_j t_{ji} - \\ (\alpha + 1) \sum_{j=1}^{k} \sum_{i=1}^{m_j} \log\left(1 - e^{-\sigma_j t_{ji}}\right) - \sum_{j=1}^{k} \sum_{i=1}^{m_j} \left(\frac{2e^{-\sigma_j t_{ji}}}{1 - e^{-\sigma_j t_{ji}}} \right)^{\alpha} + \\ \sum_{j=1}^{k} \sum_{i=1}^{J_j} R_{ji} \log \left[1 - e^{-\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}} \right] + \sum_{j=1}^{k} R_{jm_j}^* \log \left[1 - e^{-\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha}} \right]. \end{aligned} \tag{10}$$

where the likelihood equations of α, θ, and γ are, respectively:

$$\frac{\partial \ell(\alpha,\theta_1,\gamma)}{\partial \alpha} = \sum_{j=1}^{k}\frac{m_j}{\alpha} + \log 2\sum_{j=1}^{k} m_j - \sum_{j=1}^{k}\sum_{i=1}^{m_j}\sigma_j t_{ji} - \sum_{j=1}^{k}\sum_{i=1}^{m_j}\log\left(1-e^{-\sigma_j t_{ji}}\right) - \sum_{j=1}^{k}\sum_{i=1}^{m_j}\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}\log\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right) + \sum_{j=1}^{k}\sum_{i=1}^{J_j} R_{ji}\frac{\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}\log\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)e^{-\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}}}{1-e^{-\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}}} + \sum_{j=1}^{k} R^*_{jm_j}\frac{\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha}\log\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)e^{-\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha}}}{1-e^{-\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha}}}, \tag{11}$$

$$\frac{\partial \ell(\theta_1,\gamma,\alpha)}{(\gamma+1)\partial \theta_1} = \sum_{j=1}^{k}\frac{m_j}{\sigma_j}\psi_j^{\gamma} - \alpha\sum_{j=1}^{k}\sum_{i=1}^{m_j}\psi_j^{\gamma}t_{ji} + 2\alpha\sum_{j=1}^{k}\sum_{i=1}^{m_j}\frac{\psi_j^{\gamma}t_{ji}e^{-\sigma_j t_{ji}}}{(1-e^{-\sigma_j t_{ji}})^2}\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha-1} - (\alpha+1)\sum_{j=1}^{k}\sum_{i=1}^{m_j}\frac{\psi_j^{\gamma}t_{ji}e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}} - 2\alpha\sum_{j=1}^{k}\sum_{i=1}^{J_j}R_{ji}\frac{\psi_j^{\gamma}t_{ji}e^{-\sigma_j t_{ji}}}{(1-e^{-\sigma_j t_{ji}})^2}\frac{\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha-1}}{e^{\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}}-1} - 2\alpha\sum_{j=1}^{k}R^*_{jm_j}\frac{\psi_j^{\gamma}t_{jm_j}e^{-\sigma_j t_{jm_j}}}{(1-e^{-\sigma_j t_{jm_j}})^2}\frac{\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha-1}}{e^{\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha}}-1}, \tag{12}$$

$$\frac{\partial \ell(\alpha,\theta_1,\gamma)}{\partial \gamma} = \sum_{j=1}^{k}\frac{m_j}{\sigma_j}A_j - \alpha\sum_{j=1}^{k}\sum_{i=1}^{m_j}A_j t_{ji} + 2\alpha\sum_{j=1}^{k}\sum_{i=1}^{m_j}\frac{A_j t_{ji}e^{-\sigma_j t_{ji}}}{(1-e^{-\sigma_j t_{ji}})^2}\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha-1} - (\alpha+1)\sum_{j=1}^{k}\sum_{i=1}^{m_j}A_j\frac{t_{ji}e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}} + 2\alpha\sum_{j=1}^{k}\sum_{i=1}^{J_j}R_{ji}\frac{A_j t_{ji}e^{-\sigma_j t_{ji}}}{(1-e^{-\sigma_j t_{ji}})^2}\frac{\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha-1}}{e^{\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}}-1} + 2\alpha\sum_{j=1}^{k}R^*_{jm_j}\frac{A_j t_{jm_j}e^{-\sigma_j t_{jm_j}}}{(1-e^{-\sigma_j t_{jm_j}})^2}\frac{\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha-1}}{e^{\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha}}-1}, \tag{13}$$

where $A_j = \left((\gamma+1)\theta_1\psi_j^{\gamma}\log\psi_j + \theta_1\psi_j^{\gamma}\right)$.

We now have a system of three nonlinear equations with three unknowns: α, θ_1, and γ. Finding a closed-form solution is challenging. Hence, an iterative method like the Newton-Raphson algorithm can be employed to numerically solve the nonlinear system represented by equations (11), (12), and (13).

3.2. Asymptotic Confidence Intervals

In this subsection, approximate CIs for the parameters are derived using the asymptotic distributions of the maximum likelihood estimators (MLEs) of the elements in the vector of unknown parameters, denoted as $\Theta = (\alpha, \theta_1, \gamma)$, the asymptotic distribution of the MLEs of Θ is expressed as:

$$((\hat{\alpha}-\alpha),(\hat{\theta}_1-\theta_1),(\hat{\gamma}-\gamma)) \sim N(0,\Sigma),$$

where $\Sigma=(\sigma_{ij})$ for $i,j=1,2,3$ is the variance-covariance matrix of the unknown parameters $\Theta=(\alpha,\theta_1,\gamma)$. The approximate $100(1-\beta)\%$ two-sided CIs for each parameter Θ_i are given by:

$$(\hat{\Theta}_{iL},\hat{\Theta}_{iU})=\hat{\Theta}_i \pm Z_{1-\beta/2}\sqrt{\sigma_{ii}}, \quad i=1,2,3,$$

where $\hat{\Theta}_1=\hat{\alpha}$, $\hat{\Theta}_2=\hat{\theta}_1$, $\hat{\Theta}_3=\hat{\gamma}$, and Z_q represents the $100q$-th percentile of a standard normal distribution.

3.3. Bootstrap confidence intervals

Bootstrap CIs are resampling techniques used to estimate the precision of a statistic, such as a mean or median, by repeatedly sampling from the observed data. Unlike traditional methods, bootstrap CIs don't rely on strict distributional assumptions, making them particularly useful when dealing with complex or non-normal data. By drawing numerous random samples with replacements from the dataset, bootstrap CIs generate an empirical distribution of the statistic. This allows for calculating percentiles that form the bounds of the CI, offering a robust and flexible approach to interval estimation. The Fisher information matrix (FIM) is obtained as the negative of the expected Hessian matrix of the log-likelihood function. For further details on the FIM and other information measures, see [24, 18].

3.4. Bayes Estimation

Bayesian estimation is a statistical approach that combines prior knowledge with observed data to estimate unknown parameters. This method is based on Bayes' theorem, which updates the probability of a hypothesis as more evidence or information becomes available. In Bayesian estimation, a prior distribution reflects initial beliefs about the parameter, which is then updated using the likelihood of observed data to produce a posterior distribution, see Chen [20].

The posterior distribution is used to estimate parameter values for point estimates and interval estimates. Bayesian estimation is flexible and particularly useful when prior information is available or when working with complex models. See Hleil [11] for more reading.

Using the square error (SE) loss function and the linear exponential loss function (LELF), Bayesian estimates (BEs) for the model parameters α, θ_1, and γ are derived under type-II adaptive progressive hybrid censoring, see [5]. We adopt independent gamma priors for the parameters α, θ_1, and γ. The gamma distribution is chosen for the following reasons: (i) it has positive support, matching the parameter spaces; (ii) it is flexible, allowing both informative and non-informative specifications; (iii) its density is simple to evaluate, which facilitates the Metropolis-Hastings algorithm. However, due to the complexity of the likelihood function, the resulting posterior distributions are not of standard forms. Therefore, we do not claim conjugacy, and we rely on MCMC methods for posterior inference. Gamma priors have been effectively used in recent reliability studies [7, 36, 44]. Taking into consideration the fact that the model parameters α, θ_1, and γ adhere to a gamma prior, we progress in the following manner:

$$\pi_1(\alpha) \propto \alpha^{\mu_1-1}e^{-\frac{\alpha}{\lambda_1}}, \quad \alpha>0,\, \mu_1>0,\, \lambda_1>0$$
$$\pi_1(\theta_1) \propto \theta_1^{\mu_2-1}e^{-\frac{\theta_1}{\lambda_2}}, \quad \theta_1>0,\, \mu_2>0,\, \lambda_2>0$$
$$\pi_1(\gamma) \propto \gamma^{\mu_3-1}e^{-\frac{\gamma}{\lambda_3}}, \quad \gamma>0,\, \mu_3>0,\, \lambda_3>0$$

Before performing Bayesian estimation with gamma priors, the hyperparameter values must be established. In this study, we adopt an empirical approach based on the data, using the MLEs to elicit informative priors. For a general parameter δ_l, the hyperparameters a_l and b_l are derived by matching the mean and variance of the MLEs (see Dey et al. [22]):

$$a_l=\frac{(\bar{\hat{\delta}}_l)^2}{s_l^2}, \quad b_l=\frac{\bar{\hat{\delta}}_l}{s_l^2},$$

where $\bar{\hat{\delta}}_l = \frac{1}{L}\sum_{j=1}^{L}\hat{\delta}_l^j$ and $s_l^2 = \frac{1}{L-1}\sum_{j=1}^{L}(\hat{\delta}_l^j - \bar{\hat{\delta}}_l)^2$ are the mean and variance of the MLEs over L bootstrap samples. This data-driven approach ensures that the priors reflect the information in the observed data. Assuming that the model parameters α, θ_1, and γ are independent, the joint prior PDF of α, θ_1, and γ can be expressed as:

$$\pi_1(\alpha,\theta_1,\gamma) \propto \alpha^{\mu_1-1}e^{-\frac{\alpha}{\lambda_1}}\theta_1^{\mu_2-1}e^{-\frac{\theta_1}{\lambda_2}}\gamma^{\mu_3-1}e^{-\frac{\gamma}{\lambda_3}}. \tag{14}$$

The joint posterior probability density function of the parameters α, θ_1, and γ can be derived using equations (9) and (14) are given as follows:

$$\begin{aligned}\Pi(\alpha,\theta_1,\gamma|data) = &\prod_{j=1}^{k}\prod_{i=1}^{m_j}\frac{\sigma_j\left(2e^{-\sigma_j t_{ji}}\right)^{\alpha}}{\left(1-e^{-\sigma_j t_{ji}}\right)^{\alpha+1}}e^{-\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}}\prod_{i=1}^{J_j}\left[1-e^{-\left(\frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}\right)^{\alpha}}\right]^{R_{ji}} \\ &\alpha^{m_j+\mu_1-1}e^{-\frac{\alpha}{\lambda_1}}\theta_1^{\mu_2-1}e^{-\frac{\theta_1}{\lambda_2}}\gamma^{\mu_3-1}e^{-\frac{\gamma}{\lambda_3}}\left[1-e^{-\left(\frac{2e^{-\sigma_j t_{jm_j}}}{1-e^{-\sigma_j t_{jm_j}}}\right)^{\alpha}}\right]^{R^*_{jm_j}},\end{aligned} \tag{15}$$

Using the SE loss function, the Bayes estimator for the function of the parameters $U = U(\Theta)$, where $\Theta = (\alpha,\theta_1,\gamma)$, is expressed as:

$$\hat{U}_{\text{SE}} = \int_{\Theta} U\,\pi^*(\Theta)\,d\Theta, \tag{16}$$

Here, $\pi^*(\Theta)$ is defined in Equation (15). Using the LINEX loss function (LLF), the Bayes estimator for $U = U(\Theta)$ is given as:

$$\hat{U}_{\text{LINEX}} = -\frac{1}{c}\log\left[\int_{\Theta} e^{-cU}\pi^*(\Theta)\,d\Theta\right], \tag{17}$$

Here, $c \neq 0$ represents the shape parameter of the LINEX loss function. It is evident that the integrals in Equations (16) and (17) cannot be solved analytically. Therefore, the Markov chain Monte Carlo (MCMC) method is used to approximate these integrals. For more reading, see Abd El-Raheem et al. [2]

In this section, the MCMC method is used to generate samples from the posterior distribution, enabling the computation of the BEs for α, θ_1, and γ under progressive-stress ALT. Based on the joint posterior density function in (15), the conditional posterior distributions of α, θ_1, and γ are provided as follows for more reading see Hleil [10]: For notational simplicity, define:

$$A_{ji} = \frac{2e^{-\sigma_j t_{ji}}}{1-e^{-\sigma_j t_{ji}}}, \quad B_{ji} = 1 - e^{-A_{ji}^{\alpha}}$$

Then, the conditional posterior distributions become:

$$\Pi(\alpha|\theta_1,\gamma,data) \propto \prod_{j=1}^{k}\prod_{i=1}^{m_j}\frac{A_{ji}^{\alpha}}{(1-e^{-\sigma_j t_{ji}})}e^{-A_{ji}^{\alpha}}\prod_{i=1}^{J_j}B_{ji}^{R_{ji}}\alpha^{m_j+\mu_1-1}e^{-\frac{\alpha}{\lambda_1}}B_{jm_j}^{R^*_{jm_j}} \tag{18}$$

$$\Pi(\theta_1|\alpha,\gamma,data) \propto \prod_{j=1}^{k}\prod_{i=1}^{m_j}\frac{\sigma_j e^{-\alpha\sigma_j t_{ji}}}{(1-e^{-\sigma_j t_{ji}})^{\alpha+1}}e^{-A_{ji}^{\alpha}}\prod_{i=1}^{J_j}B_{ji}^{R_{ji}}\theta_1^{\mu_2-1}e^{-\frac{\theta_1}{\lambda_2}}\gamma^{\mu_3-1}e^{-\frac{\gamma}{\lambda_3}}B_{jm_j}^{R^*_{jm_j}} \tag{19}$$

$$\Pi(\gamma|\alpha,\theta_1,data) \propto \prod_{j=1}^{k}\prod_{i=1}^{m_j}\frac{\sigma_j e^{-\alpha\sigma_j t_{ji}}}{(1-e^{-\sigma_j t_{ji}})^{\alpha+1}}e^{-A_{ji}^{\alpha}}\prod_{i=1}^{J_j}B_{ji}^{R_{ji}}\gamma^{\mu_3-1}e^{-\frac{\gamma}{\lambda_3}}B_{jm_j}^{R^*_{jm_j}} \tag{20}$$

The conditional posterior distributions of α, θ_1, and γ in equations (18), (19), and (20) cannot be simplified analytically into standard distributions. As a result, the Metropolis-Hastings algorithm is employed to generate random samples from these distributions, as discussed by Upadhyay and Gupta [45]. The following algorithm is proposed to compute the Bayes estimators of $U = U(\alpha,\theta_1,\gamma)$ under the SE and LINEX loss functions, see Metropolis [32].

3.5. Credible Confidence Intervals

A Bayesian credible interval (or posterior interval) at a confidence level of $100(1-\beta)\%$ for a random variable η is defined as the range of values within which the posterior probability of η lying in the interval is $1-\beta$. This is expressed as:

$$P(L \leq \eta \leq U) = \int_L^U \eta^*(\eta|t)d\eta = 1-\beta.$$

There are several types of credible intervals. A common type is the central credible interval, which includes the range of values corresponding to the $\beta/2$ and $(1-\beta/2)$ percentiles of the posterior distribution.

3.6. Credible Confidence Intervals

A Bayesian credible interval (or posterior interval) at a confidence level of $100(1-\beta)\%$ for a random variable η is defined as the range of values within which the posterior probability of η lying in the interval is $1-\beta$. This is expressed as:

$$P(L \leq \eta \leq U) = \int_L^U \eta^*(\eta|t)d\eta = 1-\beta.$$

There are several types of credible intervals. A common type is the central credible interval, which includes the range of values corresponding to the $\beta/2$ and $(1-\beta/2)$ percentiles of the posterior distribution.

The following steps are used to compute credible confidence intervals for α, θ_1, and γ:

- Obtain MCMC samples $\{\alpha^{(i)}, \theta_1^{(i)}, \gamma^{(i)}\}_{i=1}^N$ after burn-in.
- Sort the samples in ascending order for each parameter.
- For a $100(1-\beta)\%$ credible interval, take the $\beta/2$ and $1-\beta/2$ percentiles:

$$(\alpha_{(\beta/2)}, \alpha_{(1-\beta/2)}), \quad (\theta_{1(\beta/2)}, \theta_{1(1-\beta/2)}), \quad (\gamma_{(\beta/2)}, \gamma_{(1-\beta/2)})$$

- This provides the central credible interval for each parameter.

4. Simulation Work

Monte Carlo simulation is conducted to investigate the performance of the Maximum Likelihood Estimators (MLEs) and Bayesian Estimators (BEs) under both the Square Error (SE) loss function and the LINEX loss function. The performance is evaluated in terms of their Mean Squared Errors (MSEs), Relative Absolute Biases (RABs), Length of Asymptotic Confidence Intervals (LACI), Coverage Probability (CP), Length of Bootstrap Percentile (LBP), Length of Bootstrap-t (LBT), and Length of Credible Confidence Intervals (LCCI) for various sample sizes $(n_i, m_i, i = 1, 2, \ldots, k)$ and censoring schemes (CSs) $(R_{ij}, j = 1, 2, \ldots, m_i)$, see [28] which contains many previous works on MCMC algorithms.

All simulation and application studies were conducted using R.

For the Maximum Likelihood Estimation (MLE), we used the maxlike function from the maxlike package.

For the Bayesian estimation, we implemented the Metropolis-Hastings algorithm via the coda package to perform MCMC sampling.

In both the simulation study and the real data analysis, we generated 12,000 MCMC samples, with a burn-in period of 2,000 samples, which were discarded to ensure convergence. The choice of burn-in was based on visual inspection of trace plots and preliminary convergence diagnostics. For Bayesian estimation, we employed the Square Error Loss Function (SELF) as well as two LINEX loss functions: LLFI with asymmetry parameter $c = 0.5$ (which penalizes overestimation more heavily) and LLFII with $c = -0.5$ (which penalizes underestimation more heavily).

Each simulation experiment was repeated for 10,000 iterations to ensure statistical reliability of the results.

1. **First Design:** The simple ramp-stress test consists of two stress levels $(k = 2)$ with ramp rates $\phi_1 = 40$ and $\phi_2 = 60$.
2. **Second Design:** The multiple ramp-stress test consists of four stress levels $(k = 4)$ with ramp rates $\phi_1 = 40$, $\phi_2 = 60$, $\phi_3 = 80$, and $\phi_4 = 100$.
3. The actual parameters have been selected as $(\alpha = 0.4; \theta_1 = 0.4; \gamma = 0.3)$, $(\alpha = 0.4; \theta_1 = 1.2; \gamma = 1.1)$, and $(\alpha = 2; \theta_1 = 1.2; \gamma = 1.1)$.
4. The sample sizes have been selected as:
   For a simple ramp: Sample size I: $n_1 = 20, n_2 = 10$.
   Sample size II: $n_1 = 40, n_2 = 30$.
   For Multi ramp: Sample size I: $n_1 = 12, n_2 = 10, n_3 = 8, n_4 = 5$.
   Sample size II: $n_1 = 25, n_2 = 22, n_3 = 18, n_4 = 15$.
5. The censored sample has been obtained by $r = \frac{m}{n}$ is 0.7, and 0.9.
6. The censored time of AP-II-HC has been obtained by quantile Q is 0.6 and 0.85, which means that after this percentage of failure times, the test is terminated.
7. Assuming that each test unit extracted from the life test is independent of the others and has a constant probability P, the number of units removed at each failure time follows a binomial distribution. This can be expressed as:

$$P(R_1 = r_1) = \binom{n-m}{r_1} p^{r_1} (1-p)^{n-m-r_1}$$

   where $0 \le r_1 \le n - m$, and

$$P(R_i = r_i \mid R_{i-1} = r_{i-1}, \dots, R_1 = r_1) = \binom{n - m - \sum_{l=1}^{i-1} r_l}{r_i} p^{r_i} (1-p)^{n-m-\sum_{l=1}^{i} r_l}$$

   with the condition $0 \le r_i \le n - m - \sum_{l=1}^{i-1} r_l$, for $i = 2, \dots, m-1$. If there are any remaining units, they are all removed during the m-th failure with a probability of 1.
   Further, if we assume that R_i is independent of X_i for all i, we can express the total probability as:

$$P(R, p) = P(R_m = r_m \mid R_{m-1} = r_{m-1}, \dots, R_1 = r_1) \cdots P(R_2 = r_2 \mid R_1 = r_1) P(R_1 = r_1)$$

   Thus, the overall probability is:

$$P(R, p) = \frac{(n-m)!}{\prod_{i=1}^{m-1} r_i! (n - m - \sum_{i=1}^{m-1} r_i)!} p^{\sum_{i=1}^{m-1} r_i} (1-p)^{(m-1)(n-m) - \sum_{i=1}^{m-1} (m-i) r_i}$$

   We select different values of parameters p as 0.3 and 0.8.
8. Lengths and coverage probabilities of 95% approximate and credible confidence intervals for the model parameters. Also, we estimated the reliability function $S1, S2, S3, S4$ under different true values.

Algorithm 1: Simulation Algorithm (With Parameters α, θ_1, and γ)

1. Begin with an initial guess for $(\alpha, \theta_1, \gamma)$, denoted as $(\alpha^{(0)}, \theta_1^{(0)}, \gamma^{(0)})$.
2. Set $i = 1$.
3. Compute $\alpha^{(i)}$, $\theta_1^{(i)}$, and $\gamma^{(i)}$ using Equations (18), (19), and (20), respectively and then use the MH algorithm see [4].
4. Increment i by 1 $(i = i + 1)$.
5. Repeat steps 3 and 4 for N iterations.
6. The approximate mean values of U and e^{-cU} can be calculated as follows:

$$E(U) = \frac{1}{N-M} \sum_{i=M+1}^{N} U(\alpha^{(i)}, \theta_1^{(i)}, \gamma^{(i)})$$

$$E(e^{-cU}) = \frac{1}{N-M} \sum_{i=M+1}^{N} \exp\{-cU(\alpha^{(i)}, \theta_1^{(i)}, \gamma^{(i)})\}$$

Here, M represents the burn-in period, which is the initial phase of the iterations excluded from the final calculation to allow the algorithm to stabilize.

$$\tilde{R} = \int_0^\infty \int_0^\infty \int_0^\infty S(t)\pi^*(\alpha, \theta_1, b \mid data)\, d\alpha\, d\theta_1\, db. \tag{21}$$

Also, we must estimate the reliability function of the model, so the integral given in Eq. 21 is obviously impossible to calculate analytically. As a result, the Bayesian estimator of R, specifically the Gibbs sampling methods, is obtained using this approach. Gibbs sampling is one of the Markov Chain Monte Carlo techniques used to obtain samples from the posterior distribution when direct analytical calculation is impossible, and these samples are then used to approximate Bayesian estimators.

4.1. Algorithm 2: Bootstrap Algorithm

- Perform steps 1 through 6 of Algorithm 1.
- Repeat step 1 K times, and arrange the estimates in ascending order as $\{\hat{\Theta}_i^{[SE1]}, \hat{\Theta}_i^{[SE2]}, \ldots, \hat{\Theta}_i^{[SEK]}\}$, where $i = 1, 2, 3$, with $\hat{\Theta}_1^{SE} \equiv \hat{\alpha}^{SE}$, $\hat{\Theta}_2^{SE} \equiv \hat{\theta}_1^{SE}$, and $\hat{\Theta}_3^{SE} \equiv \hat{\gamma}^{SE}$.
- The $100(1-\beta)\%$ credible interval for Θ_i is then given by:

$$(\hat{\Theta}_i^{[SE\lfloor \beta K/2 \rfloor]}, \hat{\Theta}_i^{[SE\lfloor (1-\beta/2)K \rfloor]}), \quad i = 1, 2, 3. \tag{22}$$

The Bootstap algorithm is discussed clearly in Abd El-Raheem, A. M. [4]

4.2. Remarks from the Simulation Tables

Based on the simulation results presented in Tables 1-8, we draw the following conclusions:

- **Bayesian vs. MLE:** BEs yield lower MSE and Bias than MLEs across all scenarios.
- **Consistency:** As sample size n increases, Bias, MSE, and CI lengths decrease for all estimators.
- **Effect of removal probability p:** For fixed n, increasing p reduces Bias, MSE, and CI lengths.
- **Loss functions:** LINEX-based BEs (especially LLFI) outperform SELF-based BEs in terms of MSE and Bias.
- **Ramp designs:** Multiple ramp-stress designs yield smaller MSE than simple ramp designs for the same sample size.
- **Confidence intervals:** Bootstrap CIs (particularly bootstrap-t) are shorter than asymptotic CIs, indicating higher precision.
- **LINEX parameter:** LLFI ($c = 0.5$) consistently outperforms LLFII ($c = -0.5$), producing lower MSE and Bias.

Table 1. The MLE values and Bayesian estimate values for simple ramp with sample size I, where $\alpha = 0.4; \theta_1 = 0.4; \gamma = 0.3$

Size I				MLE		SELF		LLFI		LLFII		CI by MLE				CI by SELF		LLFI	LLFII
p	Q	r		RAB	MSE	RAB	MSE	RAB	MSE	RAB	MSE	LACI	CP	LBP	LBT	LCCI	CP	LCCI	LCCI
0.3	0.6	0.7	α	0.1707	0.5019	0.0210	0.0048	0.0230	0.0052	0.0190	0.0042	0.4745	100.00%	0.0153	0.0153	0.0738	98.30%	0.2808	0.2536
			θ_1	0.2353	0.1694	0.0385	0.0102	0.0295	0.0106	0.0474	0.0099	1.5716	99.80%	0.0522	0.0520	0.3880	98.50%	0.4015	0.3827
			γ	0.1508	0.6819	0.0244	0.0511	0.0645	0.0557	0.0147	0.0471	3.2337	100.00%	0.1036	0.1036	0.7558	99.70%	0.9227	0.8508
			S1	0.1112	0.0915	0.0500	0.0065	0.0354	0.0065	0.0644	0.0065	0.4433	96.00%	0.0142	0.0143	0.3053	97.40%	0.3116	0.3006
			S2	0.1261	0.0142	0.0527	0.0062	0.0421	0.0061	0.0632	0.0062	0.4231	96.40%	0.0134	0.0137	0.2933	97.50%	0.3004	0.2930
		0.9	α	0.1248	0.2118	0.0177	0.0028	0.0192	0.0032	0.0162	0.0024	0.3776	100.00%	0.0122	0.0122	0.0815	98.80%	0.2208	0.1900
			θ_1	0.2194	0.1535	0.0212	0.0078	0.0254	0.0080	0.0170	0.0077	1.4474	99.90%	0.0447	0.0447	0.3302	98.70%	0.3475	0.3426
			γ	0.0854	0.4518	0.0225	0.0243	0.0628	0.0245	0.0916	0.0241	2.8210	99.90%	0.0903	0.0898	0.5077	99.70%	0.6080	0.5994
			S1	0.0113	0.0877	0.0117	0.0050	0.0061	0.0050	0.0174	0.0049	0.3670	95.10%	0.0115	0.0115	0.2686	96.20%	0.2770	0.2745
			S2	0.0142	0.0129	0.0383	0.0045	0.0087	0.0045	0.0085	0.0045	0.3617	97.00%	0.0115	0.0112	0.2629	97.60%	0.2636	0.2617
	0.85	0.7	α	0.1160	0.0174	0.0122	0.0004	0.0139	0.0004	0.0106	0.0004	0.4522	100.00%	0.0150	0.0150	0.0664	99.40%	0.0803	0.0777
			θ_1	0.1109	0.1265	0.0848	0.0099	0.0771	0.0101	0.0924	0.0098	1.3839	99.90%	0.0449	0.0444	0.3317	98.60%	0.3743	0.3597
			γ	0.2914	0.6092	0.0078	0.0496	0.0390	0.0541	0.0365	0.0456	3.2447	99.90%	0.1051	0.1038	0.7253	99.80%	0.9111	0.8360
			S1	0.4169	0.0141	0.2779	0.0065	0.2444	0.0061	0.3115	0.0069	0.3952	98.80%	0.0119	0.0119	0.2696	98.50%	0.2702	0.2691
			S2	0.4736	0.0133	0.2738	0.0060	0.2490	0.0057	0.2988	0.0063	0.3768	98.30%	0.0123	0.0120	0.2637	98.20%	0.2568	0.2576
		0.9	α	0.1029	0.0125	0.0113	0.0004	0.0115	0.0003	0.0103	0.0004	0.3888	100.00%	0.0122	0.0124	0.0896	98.70%	0.0934	0.0922
			θ_1	0.0933	0.1044	0.0176	0.0073	0.0218	0.0074	0.0134	0.0072	1.2978	100.00%	0.0437	0.0438	0.3278	98.80%	0.3366	0.3319
			γ	0.1987	0.3961	0.0068	0.0232	0.0371	0.0233	0.0290	0.0231	3.0418	100.00%	0.0958	0.0979	0.5068	99.30%	0.5933	0.5862
			S1	0.0083	0.0057	0.0690	0.0038	0.0555	0.0038	0.0825	0.0039	0.2961	99.70%	0.0096	0.0096	0.2284	98.30%	0.2385	0.2402
			S2	0.0465	0.0061	0.0465	0.0034	0.0352	0.0033	0.0579	0.0034	0.3044	99.90%	0.0098	0.0099	0.2179	97.70%	0.2246	0.2262
0.8	0.6	0.7	α	0.2589	0.0318	0.0313	0.0009	0.0333	0.0009	0.0294	0.0008	0.5691	100.00%	0.0189	0.0187	0.0868	99.10%	0.1072	0.1028
			θ_1	0.3619	0.2044	0.0133	0.0102	0.0228	0.0107	0.0040	0.0098	1.6797	99.80%	0.0537	0.0535	0.3705	99.20%	0.4040	0.3880
			γ	0.1773	0.7500	0.0753	0.0584	0.1177	0.0643	0.0340	0.0532	3.3902	99.90%	0.1123	0.1118	0.7850	99.70%	0.9850	0.9040
			S1	0.0236	0.0142	0.0093	0.0061	0.0247	0.0064	0.0059	0.0058	0.4655	95.60%	0.0145	0.0146	0.2949	96.00%	0.3113	0.2998
			S2	0.0427	0.0128	0.0011	0.0051	0.0122	0.0052	0.0099	0.0050	0.4388	96.70%	0.0130	0.0131	0.2814	97.40%	0.2830	0.2766
		0.9	α	0.1433	0.0145	0.0129	0.0005	0.0141	0.0005	0.0117	0.0005	0.4162	100.00%	0.0131	0.0131	0.0832	98.50%	0.0887	0.0877
			θ_1	0.3635	0.1681	0.0130	0.0078	0.0203	0.0080	0.0025	0.0077	1.5033	99.90%	0.0487	0.0486	0.3376	98.30%	0.3458	0.3410
			γ	0.0452	0.5132	0.0580	0.0234	0.0484	0.0235	0.0307	0.0232	2.8091	99.90%	0.0901	0.0891	0.5025	99.80%	0.5991	0.5924
			S1	0.0167	0.0099	0.0014	0.0048	0.0044	0.0048	0.0047	0.0047	0.3895	96.10%	0.0119	0.0121	0.2708	96.50%	0.2716	0.2694
			S2	0.0111	0.0094	0.0009	0.0043	0.0088	0.0043	0.0008	0.0042	0.3791	96.20%	0.0112	0.0113	0.2551	96.30%	0.2570	0.2552
	0.85	0.7	α	0.2390	0.0277	0.0301	0.0006	0.0320	0.0016	0.0282	0.0015	0.5349	100.00%	0.0172	0.0172	0.0879	99.00%	0.1495	0.1446
			θ_1	0.3831	0.1921	0.0191	0.0091	0.0294	0.0136	0.0090	0.0122	1.6869	99.70%	0.0551	0.0552	0.4313	99.40%	0.4552	0.4329
			γ	0.1243	0.7062	0.1350	0.0424	0.1797	0.0818	0.0913	0.0674	3.4213	99.90%	0.1101	0.1087	0.8813	99.60%	1.1015	1.0121
			S1	0.0500	0.0085	0.0102	0.0045	0.0237	0.0045	0.0445	0.0046	0.3595	100.00%	0.0117	0.0117	0.2511	98.80%	0.2635	0.2650
			S2	0.0623	0.0080	0.0344	0.0041	0.0091	0.0041	0.0600	0.0042	0.3496	100.00%	0.0111	0.0112	0.2523	98.40%	0.2502	0.2524
		0.9	α	0.1197	0.0117	0.0116	0.0005	0.0128	0.0005	0.0104	0.0005	0.3802	100.00%	0.0121	0.0121	0.0798	98.20%	0.0882	0.0869
			θ_1	0.3736	0.1510	0.0104	0.0068	0.0205	0.0087	0.0084	0.0083	1.5329	99.90%	0.0478	0.0469	0.3511	98.50%	0.3579	0.3511
			γ	0.1090	0.4792	0.0953	0.0227	0.0858	0.0229	0.0810	0.0227	2.9765	99.90%	0.0954	0.0939	0.4914	99.60%	0.5843	0.5773
			S1	0.0434	0.0055	0.0101	0.0036	0.0203	0.0036	0.0417	0.0037	0.2888	100.00%	0.0091	0.0091	0.2248	98.70%	0.2340	0.2357
			S2	0.0042	0.0057	0.0087	0.0032	0.0024	0.0032	0.0199	0.0032	0.2957	99.90%	0.0098	0.0098	0.2197	99.10%	0.2209	0.2224

Table 2. The MLE values and Bayesian estimate values for simple ramp with sample size II $\alpha = 0.4; \theta_1 = 0.4; \gamma = 0.3$

Size II				MLE		SELF		LLFI		LLFII		CI by MLE							
p	Q	r		RAB	MSE	RAB	MSE	RAB	MSE	RAB	MSE	LACI	CP	LBP	LBT	LCCI	CP	LCCI	LCCI
0.3	0.6	0.7	α	0.1287	0.0076	0.0167	0.0002	0.0174	0.0002	0.0160	0.0002	0.2773	99.80%	0.0090	0.0089	0.0450	98.70%	0.0496	0.0488
			θ_1	0.1138	0.0630	0.0351	0.0052	0.0050	0.0053	0.0057	0.0050	0.9682	100.00%	0.0294	0.0297	0.2823	98.40%	0.2852	0.2782
			γ	0.1917	0.4228	0.0268	0.0501	0.0639	0.0547	0.0094	0.0460	2.5403	100.00%	0.0807	0.0815	0.7077	99.80%	0.9140	0.8408
			S1	0.0264	0.0061	0.0075	0.0033	0.0035	0.0035	0.0184	0.0032	0.3043	96.10%	0.0101	0.0101	0.2177	97.30%	0.2311	0.2217
			S2	0.0462	0.0061	0.0107	0.0027	0.0034	0.0027	0.0179	0.0027	0.2980	97.20%	0.0095	0.0094	0.2069	96.90%	0.2039	0.2003
		0.9	α	0.0672	0.0038	0.0084	0.0001	0.0090	0.0001	0.0079	0.0001	0.2158	98.90%	0.0067	0.0066	0.0437	99.00%	0.0457	0.0453
			θ_1	0.1095	0.0072	0.0267	0.0043	0.0043	0.0044	0.0042	0.0042	0.9007	100.00%	0.0310	0.0312	0.2469	98.30%	0.2553	0.2522
			γ	0.1331	0.3594	0.0207	0.0214	0.0604	0.0215	0.0083	0.0213	2.3459	100.00%	0.0765	0.0770	0.4860	99.20%	0.5701	0.5639
			S1	0.0167	0.0044	0.0014	0.0025	0.0029	0.0025	0.0058	0.0025	0.2591	96.60%	0.0080	0.0080	0.1938	97.10%	0.1980	0.1965
			S2	0.0030	0.0042	0.0058	0.0022	0.0029	0.0022	0.0023	0.0022	0.2556	97.30%	0.0080	0.0078	0.1806	96.30%	0.1850	0.1837
	0.85	0.7	α	0.1208	0.0070	0.0141	0.0002	0.0147	0.0002	0.0134	0.0002	0.2680	100.00%	0.0086	0.0086	0.0436	99.60%	0.0462	0.0456
			θ_1	0.0570	0.0564	0.0154	0.0052	0.0103	0.0053	0.0205	0.0051	0.9272	100.00%	0.0273	0.0276	0.2847	98.70%	0.2864	0.2787
			γ	0.4554	0.4155	0.0438	0.0500	0.0801	0.0541	0.0082	0.0464	2.6970	100.00%	0.0862	0.0862	0.7501	99.60%	0.9071	0.8450
			S1	0.0593	0.0044	0.0654	0.0025	0.0410	0.0024	0.0899	0.0025	0.2569	98.90%	0.0085	0.0083	0.1885	97.80%	0.1911	0.1897
			S2	0.1637	0.0051	0.0719	0.0021	0.0554	0.0020	0.0886	0.0022	0.2623	98.80%	0.0087	0.0087	0.1802	97.90%	0.1739	0.1744
		0.9	α	0.0723	0.0035	0.0090	0.0001	0.0095	0.0002	0.0084	0.0002	0.2381	99.60%	0.0078	0.0078	0.0438	98.20%	0.0486	0.0481
			θ_1	0.0392	0.0060	0.0140	0.0042	0.0103	0.0043	0.0199	0.0041	0.9187	100.00%	0.0290	0.0293	0.2438	98.40%	0.2518	0.2489
			γ	0.1789	0.3191	0.0388	0.0202	0.0791	0.0220	0.0078	0.0217	2.4424	100.00%	0.0809	0.0802	0.4895	99.20%	0.5737	0.5668
			S1	0.0260	0.0034	0.0261	0.0019	0.0159	0.0019	0.0363	0.0019	0.2276	98.70%	0.0074	0.0074	0.1709	97.50%	0.1695	0.1701
			S2	0.0081	0.0031	0.0029	0.0016	0.0051	0.0016	0.0109	0.0016	0.2166	99.50%	0.0070	0.0070	0.1578	97.50%	0.1568	0.1573
0.8	0.6	0.7	α	0.1349	0.0078	0.0182	0.0002	0.0189	0.0002	0.0175	0.0002	0.2743	99.80%	0.0086	0.0083	0.0446	99.80%	0.0504	0.0497
			θ_1	0.1119	0.0583	0.0169	0.0050	0.0224	0.0052	0.0114	0.0049	0.9307	100.00%	0.0302	0.0305	0.2746	98.20%	0.2805	0.2726
			γ	0.3348	0.4656	0.0407	0.0481	0.0772	0.0524	0.0050	0.0443	2.6471	100.00%	0.0820	0.0816	0.7166	99.80%	0.8928	0.8259
			S1	0.0259	0.0039	0.0039	0.0023	0.0279	0.0023	0.0203	0.0023	0.2444	99.00%	0.0074	0.0074	0.1838	97.70%	0.1874	0.1857
			S2	0.0480	0.0042	0.0005	0.0018	0.0171	0.0018	0.0162	0.0018	0.2519	99.10%	0.0078	0.0078	0.1681	97.50%	0.1659	0.1663
		0.9	α	0.0725	0.0045	0.0089	0.0002	0.0094	0.0002	0.0084	0.0002	0.2376	99.70%	0.0074	0.0074	0.0450	97.90%	0.0467	0.0463
			θ_1	0.1023	0.0481	0.0110	0.0043	0.0203	0.0044	0.0103	0.0043	0.9051	100.00%	0.0331	0.0329	0.2524	98.50%	0.2553	0.2522
			γ	0.0282	0.3818	0.0407	0.0213	0.0453	0.0215	0.0046	0.0211	2.4232	100.00%	0.0767	0.0777	0.4980	99.50%	0.5727	0.5650
			S1	0.0175	0.0034	0.0031	0.0019	0.0049	0.0019	0.0155	0.0019	0.2289	98.50%	0.0074	0.0074	0.1671	97.70%	0.1712	0.1718
			S2	0.0177	0.0031	0.0004	0.0017	0.0159	0.0017	0.0018	0.0017	0.2177	99.50%	0.0071	0.0071	0.1577	97.60%	0.1592	0.1597
	0.85	0.7	α	0.1348	0.0078	0.0180	0.0002	0.0187	0.0002	0.0174	0.0002	0.2752	99.70%	0.0087	0.0087	0.0455	99.60%	0.0500	0.0493
			θ_1	0.1080	0.0580	0.0198	0.0045	0.0254	0.0054	0.0143	0.0051	0.9293	100.00%	0.0307	0.0296	0.2773	98.00%	0.2861	0.2779
			γ	0.3493	0.4570	0.0480	0.0482	0.0851	0.0525	0.0116	0.0443	2.6192	100.00%	0.0798	0.0801	0.7229	99.80%	0.8928	0.8253
			S1	0.0088	0.0021	0.0146	0.0022	0.0258	0.0037	0.0036	0.0033	0.3115	96.60%	0.0100	0.0100	0.2273	96.80%	0.2352	0.2254
			S2	0.0260	0.0036	0.0093	0.0017	0.0166	0.0027	0.0019	0.0026	0.3040	97.30%	0.0095	0.0094	0.2011	97.10%	0.2031	0.1991
		0.9	α	0.0725	0.0045	0.0089	0.0002	0.0094	0.0002	0.0084	0.0002	0.2376	99.70%	0.0074	0.0074	0.0450	97.90%	0.0467	0.0463
			θ_1	0.0923	0.0481	0.0110	0.0043	0.0213	0.0044	0.0128	0.0043	0.9051	100.00%	0.0331	0.0329	0.2524	98.50%	0.2553	0.2522
			γ	0.0282	0.3818	0.0467	0.0213	0.0453	0.0215	0.0096	0.0211	2.4232	100.00%	0.0767	0.0777	0.4980	99.50%	0.5727	0.5650
			S1	0.0071	0.0020	0.0061	0.0018	0.0105	0.0027	0.0017	0.0026	0.2855	96.00%	0.0090	0.0091	0.1947	97.50%	0.2014	0.1997
			S2	0.0100	0.0028	0.0081	0.0015	0.0148	0.0023	0.0009	0.0023	0.2630	96.80%	0.0085	0.0085	0.1876	96.60%	0.1875	0.1861

Table 3. The MLE values and Bayesian estimate values for simple ramp with sample size I $\alpha = 0.4; \theta_1 = 0.4; \gamma = 0.3$

Size I				MLE		SELF		LLFI		LLFII		CI by MLE				CI by SELF		LLFI	LLFII
p	Q	r		RAB	MSE	RAB	MSE	RAB	MSE	RAB	MSE	LACI	CP	LBP	LBT	LCCI	CP	LCCI	LCCI
0.3	0.6	0.7	α	0.1094	0.0080	0.0160	0.0029	0.0069	0.0019	0.0051	0.0018	0.3174	99.90%	0.0105	0.0106	0.0482	99.40%	0.0531	0.0521
			θ_1	0.1045	0.0634	0.0606	0.0094	0.0562	0.0042	0.0650	0.0042	0.7192	100.00%	0.0232	0.0225	0.2363	98.00%	0.2373	0.2312
			γ	0.4295	0.2987	0.0949	0.0542	0.0834	0.0587	0.0611	0.0500	2.1434	99.10%	0.0674	0.0669	0.7436	99.90%	0.9455	0.8772
			S1	0.5082	0.0159	0.2666	0.0051	0.2426	0.0049	0.2906	0.0054	0.4115	96.30%	0.0127	0.0125	0.2564	97.80%	0.2654	0.2552
			S2	0.4710	0.0101	0.2546	0.0039	0.2383	0.0038	0.2709	0.0041	0.2810	97.10%	0.0088	0.0087	0.2123	96.40%	0.2040	0.2004
			S3	0.4753	0.0120	0.2521	0.0039	0.2419	0.0038	0.2624	0.0040	0.3381	97.00%	0.0109	0.0109	0.2138	97.20%	0.2005	0.1967
			S4	0.4935	0.0168	0.2526	0.0044	0.2474	0.0044	0.2580	0.0044	0.4406	96.80%	0.0139	0.0138	0.2220	97.20%	0.2214	0.2148
		0.9	α	0.0706	0.0055	0.0147	0.0021	0.0054	0.0011	0.0040	0.0012	0.2686	99.80%	0.0089	0.0087	0.0519	99.30%	0.0564	0.0558
			θ_1	0.0962	0.0211	0.0291	0.0063	0.0058	0.0037	0.0050	0.0036	0.5691	100.00%	0.0188	0.0187	0.2373	97.70%	0.2378	0.2347
			γ	0.4184	0.2773	0.0648	0.0236	0.0386	0.0226	0.0580	0.0222	2.0057	98.10%	0.0633	0.0640	0.5060	99.80%	0.5874	0.5800
			S1	0.0319	0.0083	0.0777	0.0041	0.0672	0.0029	0.0883	0.0029	0.3564	96.80%	0.0121	0.0120	0.2050	97.60%	0.2095	0.2076
			S2	0.1170	0.0040	0.0657	0.0036	0.0573	0.0022	0.0741	0.0022	0.2464	96.50%	0.0077	0.0078	0.1839	97.20%	0.1825	0.1811
			S3	0.2373	0.0071	0.0605	0.0030	0.0538	0.0022	0.0672	0.0021	0.3201	97.60%	0.0100	0.0101	0.1855	97.30%	0.1821	0.1806
			S4	0.3466	0.0126	0.0582	0.0029	0.0528	0.0024	0.0636	0.0024	0.4256	97.20%	0.0148	0.0148	0.1962	97.00%	0.1929	0.1911
	0.85	0.7	α	0.0796	0.0067	0.0080	0.0015	0.0042	0.0009	0.0026	0.0008	0.2969	99.60%	0.0095	0.0097	0.0463	99.10%	0.0481	0.0473
			θ_1	0.0648	0.0371	0.0602	0.0047	0.0285	0.0041	0.0609	0.0040	0.7484	100.00%	0.0238	0.0235	0.2351	98.10%	0.2317	0.2257
			γ	0.0729	0.2830	0.0312	0.0479	0.0228	0.0512	0.0453	0.0439	2.1610	99.30%	0.0682	0.0688	0.6966	99.80%	0.8868	0.8202
			S1	0.1285	0.0137	0.0559	0.0049	0.0457	0.0049	0.0660	0.0049	0.3934	98.20%	0.0126	0.0124	0.2378	98.90%	0.2348	0.2324
			S2	0.1344	0.0080	0.0620	0.0033	0.0554	0.0032	0.0685	0.0034	0.2792	97.60%	0.0090	0.0089	0.2079	97.80%	0.1957	0.1958
			S3	0.1332	0.0103	0.0653	0.0032	0.0612	0.0032	0.0694	0.0033	0.3251	98.40%	0.0099	0.0099	0.2013	98.10%	0.1956	0.1944
			S4	0.1293	0.0153	0.0672	0.0038	0.0650	0.0039	0.0694	0.0038	0.4173	99.60%	0.0134	0.0134	0.2315	99.30%	0.2169	0.2119
		0.9	α	0.0572	0.0053	0.0028	0.0012	0.0035	0.0002	0.0021	0.0002	0.2698	99.60%	0.0085	0.0083	0.0513	98.10%	0.0557	0.0552
			θ_1	0.0216	0.0193	0.0091	0.0031	0.0056	0.0031	0.0041	0.0031	0.5443	100.00%	0.0180	0.0178	0.2135	98.30%	0.2187	0.2158
			γ	0.0618	0.2638	0.0283	0.0233	0.0186	0.0204	0.0379	0.0213	1.9690	96.80%	0.0601	0.0599	0.5190	99.40%	0.6024	0.5933
			S1	0.0054	0.0054	0.0162	0.0024	0.0118	0.0023	0.0207	0.0024	0.2892	99.60%	0.0093	0.0093	0.1842	98.00%	0.1848	0.1853
			S2	0.0248	0.0035	0.0120	0.0017	0.0085	0.0017	0.0155	0.0017	0.2233	98.60%	0.0070	0.0070	0.1558	97.30%	0.1560	0.1565
			S3	0.0522	0.0067	0.0086	0.0016	0.0058	0.0016	0.0115	0.0016	0.2896	99.20%	0.0095	0.0097	0.1523	98.40%	0.1527	0.1528
			S4	0.0708	0.0122	0.0576	0.0018	0.0035	0.0018	0.0081	0.0018	0.3825	99.90%	0.0115	0.0115	0.1634	97.50%	0.1615	0.1611
0.8	0.6	0.7	α	0.1005	0.0072	0.0072	0.0025	0.0168	0.0003	0.0150	0.0002	0.3359	99.90%	0.0109	0.0109	0.0530	98.40%	0.0574	0.0563
			θ_1	0.1003	0.0560	0.0012	0.0042	0.0066	0.0044	0.0041	0.0041	0.9145	100.00%	0.0295	0.0292	0.2521	97.20%	0.2590	0.2509
			γ	0.1431	0.2828	0.0919	0.0525	0.1318	0.0576	0.0529	0.0479	2.0787	99.40%	0.0671	0.0676	0.7502	99.60%	0.9282	0.8557
			S1	0.1174	0.0121	0.0200	0.0042	0.0203	0.0044	0.0443	0.0040	0.4213	96.70%	0.0133	0.0132	0.2437	95.90%	0.2587	0.2480
			S2	0.0919	0.0055	0.0272	0.0026	0.0106	0.0026	0.0439	0.0025	0.2840	96.80%	0.0092	0.0090	0.1970	96.00%	0.2011	0.1969
			S3	0.1269	0.0077	0.0404	0.0026	0.0300	0.0026	0.0509	0.0025	0.3401	96.70%	0.0106	0.0104	0.1998	97.40%	0.2012	0.1963
			S4	0.1812	0.0129	0.0542	0.0032	0.0490	0.0033	0.0596	0.0031	0.4449	96.80%	0.0140	0.0139	0.2256	96.90%	0.2244	0.2163
		0.9	α	0.0593	0.0046	0.0070	0.0023	0.0103	0.0002	0.0088	0.0002	0.2697	99.50%	0.0083	0.0082	0.0543	98.10%	0.0581	0.0576
			θ_1	0.0945	0.0202	0.0010	0.0038	0.0062	0.0038	0.0032	0.0037	0.6882	100.00%	0.0216	0.0218	0.2223	99.40%	0.2403	0.2362
			γ	0.1404	0.2490	0.0511	0.0226	0.0415	0.0228	0.0461	0.0224	1.9497	97.90%	0.0612	0.0612	0.5028	99.70%	0.5906	0.5830
			S1	0.0251	0.0081	0.0050	0.0031	0.0052	0.0031	0.0425	0.0030	0.3680	96.20%	0.0116	0.0117	0.2139	96.10%	0.2183	0.2160
			S2	0.0078	0.0040	0.0015	0.0023	0.0049	0.0023	0.0238	0.0023	0.2490	96.40%	0.0075	0.0078	0.1827	96.30%	0.1889	0.1870
			S3	0.0024	0.0063	0.0079	0.0022	0.0031	0.0023	0.0141	0.0022	0.3111	96.50%	0.0103	0.0103	0.1794	96.50%	0.1863	0.1842
			S4	0.0087	0.0111	0.0115	0.0025	0.0103	0.0025	0.0090	0.0024	0.4127	96.50%	0.0135	0.0134	0.1914	97.30%	0.1954	0.1931
	0.85	0.7	α	0.0652	0.0041	0.0051	0.0023	0.0160	0.0002	0.0141	0.0002	0.3319	100.00%	0.0107	0.0107	0.0519	99.70%	0.0551	0.0541
			θ_1	0.0626	0.0245	0.0012	0.0037	0.0008	0.0038	0.0039	0.0036	0.8253	100.00%	0.0269	0.0270	0.2233	99.00%	0.2432	0.2347
			γ	0.0196	0.2730	0.0084	0.0478	0.1215	0.0520	0.0463	0.0441	2.2253	99.70%	0.0683	0.0684	0.7245	99.80%	0.8826	0.8218
			S1	0.0567	0.0079	0.0088	0.0027	0.0042	0.0027	0.0026	0.0027	0.3406	100.00%	0.0109	0.0109	0.1978	98.10%	0.2039	0.2019
			S2	0.0437	0.0039	0.0016	0.0017	0.0059	0.0017	0.0090	0.0017	0.2402	98.70%	0.0075	0.0074	0.1623	98.10%	0.1608	0.1608
			S3	0.0316	0.0061	0.0081	0.0016	0.0035	0.0016	0.0128	0.0016	0.2982	99.60%	0.0096	0.0096	0.1641	97.30%	0.1577	0.1564
			S4	0.0208	0.0108	0.0126	0.0021	0.0107	0.0021	0.0151	0.0020	0.3942	100.00%	0.0127	0.0126	0.1781	98.20%	0.1787	0.1738
		0.9	α	0.0419	0.0036	0.0024	0.0013	0.0102	0.0001	0.0071	0.0001	0.2782	99.50%	0.0087	0.0088	0.0584	99.40%	0.0629	0.0622
			θ_1	0.0139	0.0182	0.0008	0.0029	0.0006	0.0035	0.0027	0.0032	0.5799	100.00%	0.0183	0.0182	0.2400	98.00%	0.2446	0.2416
			γ	0.0137	0.2370	0.0074	0.0213	0.0372	0.0203	0.0357	0.0203	1.9410	97.90%	0.0607	0.0615	0.5319	99.90%	0.5989	0.5888
			S1	0.0046	0.0051	0.0042	0.0023	0.0032	0.0023	0.0024	0.0023	0.2907	100.00%	0.0095	0.0093	0.1842	97.80%	0.1874	0.1879
			S2	0.0049	0.0029	0.0013	0.0015	0.0047	0.0012	0.0012	0.0015	0.2110	98.00%	0.0065	0.0065	0.1593	97.50%	0.1631	0.1636
			S3	0.0017	0.0047	0.0073	0.0012	0.0005	0.0013	0.0054	0.0013	0.2640	98.80%	0.0089	0.0089	0.1587	98.40%	0.1611	0.1613
			S4	0.0052	0.0085	0.0003	0.0011	0.0025	0.0019	0.0084	0.0019	0.3511	100.00%	0.0110	0.0108	0.1680	97.50%	0.1697	0.1694

Table 4. The MLE values and Bayesian estimate values for simple ramp with sample size II $\alpha = 0.4; \theta_1 = 0.4; \gamma = 0.3$

Size II				MLE		SELF		LLFI		LLFII		CI by MLE				CI by SELF		LLFI	LLFII
p	Q	r		RAB	MSE	RAB	MSE	RAB	MSE	RAB	MSE	LACI	CP	LBP	LBT	LCCI	CP	LCCI	LCCI
0.3	0.6	0.7	α	0.0922	0.0054	0.0611	0.0009	0.0112	0.0001	0.0104	0.0001	0.1970	99.00%	0.0060	0.0060	0.0319	98.30%	0.0326	0.0324
			θ_1	0.0837	0.0206	0.0311	0.0019	0.0377	0.0018	0.0417	0.0015	0.3877	100.00%	0.0121	0.0121	0.1362	98.10%	0.1309	0.1289
			γ	0.3584	0.1605	0.0938	0.0342	0.1044	0.0365	0.0418	0.0403	1.5411	97.90%	0.0478	0.0479	0.6260	99.90%	0.7449	0.7038
			S1	0.1026	0.0065	0.0789	0.0023	0.0620	0.0023	0.0958	0.0022	0.2942	97.30%	0.0090	0.0092	0.1747	95.80%	0.1865	0.1796
			S2	0.1666	0.0027	0.0860	0.0010	0.0758	0.0010	0.0962	0.0011	0.1804	96.40%	0.0057	0.0056	0.1195	97.00%	0.1201	0.1182
			S3	0.2382	0.0044	0.0976	0.0012	0.0929	0.0019	0.1025	0.0011	0.2200	96.50%	0.0069	0.0069	0.1140	97.10%	0.1105	0.1089
			S4	0.3059	0.0077	0.1096	0.0012	0.1092	0.0016	0.1101	0.0015	0.3002	96.80%	0.0097	0.0097	0.1346	97.20%	0.1308	0.1269
		0.9	α	0.0476	0.0021	0.0062	0.0007	0.0057	0.0001	0.0051	0.0001	0.1617	99.20%	0.0051	0.0051	0.0316	98.60%	0.0316	0.0314
			θ_1	0.0513	0.0094	0.0190	0.0017	0.0167	0.0017	0.0133	0.0014	0.3607	100.00%	0.0112	0.0112	0.1537	98.50%	0.1593	0.1577
			γ	0.0265	0.1206	0.0606	0.0171	0.0524	0.0172	0.0406	0.0197	1.3523	97.40%	0.0422	0.0421	0.4477	98.90%	0.5112	0.5060
			S1	0.0361	0.0042	0.0281	0.0015	0.0209	0.0015	0.0354	0.0015	0.2553	97.00%	0.0082	0.0082	0.1438	97.80%	0.1513	0.1501
			S2	0.0113	0.0016	0.0107	0.0010	0.0054	0.0010	0.0161	0.0010	0.1588	97.00%	0.0051	0.0051	0.1207	97.00%	0.1229	0.1221
			S3	0.0129	0.0025	0.0064	0.0010	0.0081	0.0008	0.0053	0.0009	0.1946	96.40%	0.0064	0.0064	0.1230	97.00%	0.1235	0.1225
			S4	0.0247	0.0046	0.0100	0.0012	0.0112	0.0012	0.0088	0.0012	0.2659	96.40%	0.0084	0.0085	0.1306	96.80%	0.1362	0.1350
	0.85	0.7	α	0.0824	0.0034	0.0083	0.0007	0.0087	0.0001	0.0080	0.0001	0.1887	99.20%	0.0057	0.0057	0.0288	99.10%	0.0311	0.0308
			θ_1	0.0733	0.0143	0.0310	0.0015	0.0289	0.0015	0.0332	0.0015	0.4505	100.00%	0.0142	0.0141	0.1390	96.50%	0.1384	0.1365
			γ	0.2609	0.1581	0.0729	0.0340	0.0664	0.0345	0.0091	0.0387	1.6138	96.70%	0.0513	0.0521	0.6694	99.70%	0.8218	0.7698
			S1	0.0334	0.0046	0.0220	0.0020	0.0149	0.0020	0.0290	0.0020	0.2586	99.10%	0.0085	0.0085	0.1661	98.40%	0.1701	0.1667
			S2	0.0625	0.0026	0.0267	0.0010	0.0226	0.0010	0.0309	0.0011	0.1746	98.40%	0.0056	0.0056	0.1139	97.50%	0.1151	0.1146
			S3	0.0805	0.0042	0.0295	0.0010	0.0274	0.0010	0.0316	0.0010	0.2180	97.60%	0.0069	0.0070	0.1058	97.20%	0.1084	0.1072
			S4	0.0927	0.0072	0.0312	0.0010	0.0307	0.0014	0.0318	0.0014	0.2944	98.20%	0.0091	0.0091	0.1381	98.50%	0.1338	0.1289
		0.9	α	0.0452	0.0020	0.0054	0.0006	0.0055	0.0001	0.0049	0.0001	0.1633	98.90%	0.0051	0.0050	0.0309	98.50%	0.0310	0.0308
			θ_1	0.0345	0.0086	0.0098	0.0013	0.0115	0.0014	0.0081	0.0013	0.3766	100.00%	0.0117	0.0116	0.1517	98.70%	0.1535	0.1521
			γ	0.0051	0.1189	0.0551	0.0169	0.0469	0.0169	0.0069	0.0191	1.3614	97.10%	0.0439	0.0435	0.4660	99.60%	0.5426	0.5362
			S1	0.0195	0.0031	0.0035	0.0013	0.0004	0.0012	0.0066	0.0013	0.2175	98.90%	0.0069	0.0068	0.1347	96.60%	0.1376	0.1378
			S2	0.0015	0.0013	0.0021	0.0007	0.0044	0.0007	0.0001	0.0008	0.1401	98.80%	0.0044	0.0045	0.1079	97.30%	0.1071	0.1072
			S3	0.0014	0.0019	0.0013	0.0007	0.0026	0.0007	0.0048	0.0007	0.1689	98.50%	0.0053	0.0054	0.1042	96.80%	0.1042	0.1040
			S4	0.0032	0.0033	0.0043	0.0009	0.0072	0.0009	0.0015	0.0008	0.2265	98.70%	0.0070	0.0070	0.1101	97.60%	0.1150	0.1143
0.8	0.6	0.7	α	0.0910	0.0042	0.0132	0.0005	0.0135	0.0001	0.0128	0.0001	0.1978	99.40%	0.0066	0.0066	0.0307	97.80%	0.0326	0.0323
			θ_1	0.0085	0.0135	0.0046	0.0016	0.0022	0.0017	0.0069	0.0016	0.4553	100.00%	0.0143	0.0137	0.1564	98.50%	0.1601	0.1577
			γ	0.1056	0.1571	0.0734	0.0377	0.1050	0.0409	0.0423	0.0349	1.5498	96.80%	0.0476	0.0471	0.6472	99.20%	0.7832	0.7314
			S1	0.0465	0.0060	0.0244	0.0025	0.0143	0.0026	0.0407	0.0024	0.3023	96.30%	0.0094	0.0097	0.1900	96.20%	0.1995	0.1915
			S2	0.0450	0.0026	0.0217	0.0012	0.0115	0.0012	0.0320	0.0012	0.1962	96.50%	0.0060	0.0061	0.1356	97.10%	0.1359	0.1339
			S3	0.0692	0.0039	0.0253	0.0011	0.0199	0.0011	0.0307	0.0011	0.2400	96.60%	0.0076	0.0076	0.1263	97.40%	0.1304	0.1284
			S4	0.1012	0.0069	0.0307	0.0015	0.0293	0.0015	0.0323	0.0014	0.3223	96.80%	0.0104	0.0103	0.1494	96.80%	0.1524	0.1475
		0.9	α	0.0457	0.0019	0.0055	0.0004	0.0058	0.0007	0.0052	0.0001	0.1622	99.20%	0.0056	0.0055	0.0297	98.80%	0.0308	0.0307
			θ_1	0.0084	0.0092	0.0032	0.0015	0.0021	0.0011	0.0062	0.0014	0.3755	100.00%	0.0113	0.0114	0.1560	99.10%	0.1615	0.1598
			γ	0.0098	0.1124	0.0607	0.0190	0.0584	0.0191	0.0375	0.0189	1.4206	97.50%	0.0442	0.0456	0.4844	99.80%	0.5375	0.5317
			S1	0.0082	0.0040	0.0018	0.0016	0.0036	0.0016	0.0180	0.0016	0.2625	97.00%	0.0082	0.0083	0.1530	97.20%	0.1568	0.1555
			S2	0.0081	0.0016	0.0016	0.0010	0.0105	0.0010	0.0044	0.0010	0.1589	96.50%	0.0049	0.0049	0.1181	96.30%	0.1247	0.1239
			S3	0.0069	0.0024	0.0059	0.0010	0.0183	0.0010	0.0175	0.0010	0.1989	97.20%	0.0065	0.0064	0.1231	96.10%	0.1243	0.1232
			S4	0.0027	0.0041	0.0069	0.0012	0.0232	0.0013	0.0261	0.0012	0.2759	97.10%	0.0083	0.0083	0.1381	97.40%	0.1375	0.1361
	0.85	0.7	α	0.0697	0.0033	0.0071	0.0005	0.0128	0.0001	0.0120	0.0001	0.1920	99.20%	0.0062	0.0062	0.0291	98.30%	0.0316	0.0314
			θ_1	0.0072	0.0114	0.0037	0.0016	0.0019	0.0016	0.0059	0.0016	0.4182	100.00%	0.0137	0.0139	0.1557	98.60%	0.1574	0.1549
			γ	0.0999	0.1573	0.0254	0.0328	0.0543	0.0351	0.0030	0.0308	1.5511	97.80%	0.0487	0.0484	0.6204	99.60%	0.7316	0.6888
			S1	0.0108	0.0041	0.0066	0.0017	0.0080	0.0017	0.0009	0.0017	0.2508	98.90%	0.0079	0.0079	0.1553	98.80%	0.1607	0.1585
			S2	0.0238	0.0018	0.0017	0.0009	0.0028	0.0009	0.0061	0.0009	0.1638	98.10%	0.0054	0.0054	0.1120	98.20%	0.1153	0.1152
			S3	0.0308	0.0027	0.0070	0.0008	0.0048	0.0008	0.0092	0.0008	0.1983	98.30%	0.0061	0.0061	0.1107	97.40%	0.1092	0.1085
			S4	0.0350	0.0049	0.0106	0.0010	0.0102	0.0011	0.0111	0.0010	0.2674	99.20%	0.0083	0.0084	0.1293	97.20%	0.1274	0.1240
		0.9	α	0.0436	0.0018	0.0053	0.0004	0.0056	0.0001	0.0050	0.0001	0.1749	99.30%	0.0054	0.0055	0.0309	98.40%	0.0320	0.0318
			θ_1	0.0073	0.0079	0.0023	0.0015	0.0016	0.0009	0.0052	0.0015	0.3683	100.00%	0.0118	0.0118	0.1548	97.90%	0.1598	0.1580
			γ	0.0042	0.1031	0.0247	0.0182	0.0462	0.0183	0.0028	0.0182	1.3800	97.80%	0.0419	0.0417	0.4577	99.20%	0.5259	0.5206
			S1	0.0071	0.0031	0.0011	0.0012	0.0012	0.0012	0.0009	0.0012	0.2184	99.00%	0.0072	0.0072	0.1297	97.00%	0.1356	0.1357
			S2	0.0080	0.0012	0.0014	0.0008	0.0027	0.0008	0.0021	0.0008	0.1361	98.30%	0.0042	0.0042	0.1062	96.80%	0.1074	0.1074
			S3	0.0012	0.0018	0.0012	0.0007	0.0021	0.0007	0.0075	0.0007	0.1645	98.30%	0.0052	0.0053	0.1032	97.40%	0.1050	0.1048
			S4	0.0023	0.0033	0.0029	0.0009	0.0051	0.0009	0.0102	0.0009	0.2243	99.10%	0.0068	0.0069	0.1126	97.40%	0.1150	0.1145

Table 5. MLE and Bayesian: multi ramp with sample size I $\alpha = 0.4; \theta_1 = 1.2; \gamma = 1.1$

Size I				MLE		SELF		LLFI		LLFII		CI by MLE				CI by SELF		LLFI	LLFII
p	Q	r		RAB	MSE	RAB	MSE	RAB	MSE	RAB	MSE	LACI	CP	LBP	LBT	LCCI	CP	LCCI	LCCI
0.3	0.6	0.7	α	0.0985	0.0081	0.0066	0.0019	0.0066	0.0020	0.0063	0.0019	0.3181	99.90%	0.0102	0.0104	0.0513	98.70%	0.0544	0.0535
			θ_1	0.1279	0.0959	0.0745	0.0279	0.0682	0.0279	0.0957	0.0282	1.0550	100.00%	0.0339	0.0339	0.6065	98.50%	0.6231	0.6000
			γ	0.1284	0.3565	0.0322	0.1281	0.0151	0.1320	0.0492	0.1210	2.3410	97.00%	0.0745	0.0734	1.2473	98.40%	1.3688	1.3484
			S1	0.1381	0.0153	0.2586	0.0065	0.2255	0.0051	0.2921	0.0056	0.4345	96.30%	0.0132	0.0132	0.2541	96.30%	0.2660	0.2632
			S2	0.1385	0.0080	0.2143	0.0025	0.1962	0.0024	0.2327	0.0026	0.2764	96.40%	0.0091	0.0092	0.1741	98.00%	0.1736	0.1708
			S3	0.1327	0.0104	0.1971	0.0029	0.1904	0.0029	0.2040	0.0029	0.3421	96.80%	0.0112	0.0112	0.1987	95.90%	0.1987	0.1931
			S4	0.1254	0.0164	0.1914	0.0046	0.1934	0.0047	0.1896	0.0045	0.4614	95.70%	0.0143	0.0143	0.2583	96.80%	0.2603	0.2534
		0.9	α	0.0826	0.0060	0.0058	0.0002	0.0065	0.0002	0.0050	0.0002	0.2758	100.00%	0.0083	0.0084	0.0567	98.70%	0.0599	0.0593
			θ_1	0.0366	0.0806	0.0141	0.0175	0.0119	0.0154	0.0163	0.0155	1.0100	99.90%	0.0337	0.0337	0.4723	97.30%	0.4831	0.4829
			γ	0.1043	0.2822	0.0064	0.0266	0.0039	0.0255	0.0079	0.0257	2.0344	97.30%	0.0664	0.0670	0.6087	97.80%	0.6268	0.6278
			S1	0.0976	0.0084	0.0513	0.0018	0.0425	0.0017	0.0602	0.0018	0.3601	96.50%	0.0115	0.0116	0.1633	97.60%	0.1620	0.1624
			S2	0.0397	0.0044	0.0474	0.0012	0.0414	0.0012	0.0535	0.0012	0.2531	96.80%	0.0081	0.0082	0.1321	97.40%	0.1344	0.1348
			S3	0.0653	0.0079	0.0485	0.0014	0.0445	0.0014	0.0525	0.0014	0.3335	96.60%	0.0107	0.0106	0.1452	98.00%	0.1436	0.1440
			S4	0.0830	0.0137	0.0516	0.0018	0.0491	0.0018	0.0540	0.0018	0.4408	96.30%	0.0138	0.0139	0.1677	97.90%	0.1654	0.1658
	0.85	0.7	α	0.0973	0.0076	0.0027	0.0003	0.0041	0.0002	0.0017	0.0002	0.3308	99.90%	0.0102	0.0103	0.0483	97.90%	0.0509	0.0500
			θ_1	0.1198	0.0912	0.0503	0.0268	0.0430	0.0261	0.0808	0.0277	1.0126	100.00%	0.0307	0.0315	0.5035	98.90%	0.5465	0.5311
			γ	0.0157	0.3247	0.0312	0.1246	0.0140	0.1265	0.0484	0.1236	2.2337	97.20%	0.0703	0.0688	1.3083	99.20%	1.3932	1.3626
			S1	0.1241	0.0147	0.0676	0.0054	0.0538	0.0050	0.0813	0.0059	0.3990	98.50%	0.0123	0.0123	0.2316	96.80%	0.2420	0.2470
			S2	0.1151	0.0071	0.0591	0.0025	0.0514	0.0023	0.0668	0.0027	0.2794	97.10%	0.0090	0.0092	0.1433	97.40%	0.1485	0.1499
			S3	0.1053	0.0091	0.0504	0.0028	0.0471	0.0028	0.0539	0.0029	0.3260	97.60%	0.0107	0.0107	0.1753	96.90%	0.1762	0.1721
			S4	0.1156	0.0159	0.0423	0.0045	0.0423	0.0047	0.0424	0.0044	0.4202	98.40%	0.0132	0.0132	0.2330	97.80%	0.2422	0.2338
		0.9	α	0.0651	0.0053	0.0023	0.0002	0.0031	0.0002	0.0016	0.0002	0.2664	99.90%	0.0084	0.0083	0.0574	98.40%	0.0590	0.0584
			θ_1	0.0305	0.0670	0.0118	0.0172	0.0097	0.0147	0.0140	0.0147	0.9874	99.40%	0.0334	0.0321	0.5075	97.70%	0.5123	0.5109
			γ	0.0092	0.2592	0.0047	0.0266	0.0015	0.0247	0.0026	0.0266	2.0062	97.50%	0.0620	0.0622	0.6363	96.90%	0.6403	0.6397
			S1	0.0926	0.0057	0.0165	0.0017	0.0129	0.0016	0.0201	0.0017	0.2946	99.70%	0.0092	0.0092	0.1551	98.20%	0.1566	0.1586
			S2	0.0277	0.0038	0.0152	0.0012	0.0128	0.0011	0.0176	0.0012	0.2313	98.10%	0.0076	0.0076	0.1297	98.10%	0.1300	0.1313
			S3	0.0592	0.0070	0.0138	0.0012	0.0122	0.0012	0.0154	0.0013	0.2959	98.30%	0.0095	0.0094	0.1310	98.50%	0.1352	0.1360
			S4	0.0809	0.0126	0.0124	0.0016	0.0115	0.0016	0.0133	0.0016	0.3878	99.50%	0.0116	0.0116	0.1506	98.90%	0.1524	0.1527
0.8	0.6	0.7	α	0.1603	0.0109	0.0172	0.0004	0.0181	0.0003	0.0162	0.0002	0.3223	99.80%	0.0101	0.0101	0.0493	98.70%	0.0551	0.0542
			θ_1	0.0556	0.0987	0.0096	0.0286	0.0020	0.0299	0.0132	0.0275	1.2043	99.90%	0.0396	0.0394	0.6372	98.90%	0.6776	0.6478
			γ	0.0413	0.3368	0.0363	0.1176	0.0191	0.1190	0.0536	0.1171	2.2693	98.00%	0.0697	0.0702	1.2571	98.20%	1.3507	1.3219
			S1	0.0630	0.0132	0.0863	0.0045	0.0537	0.0045	0.0411	0.0046	0.4397	95.90%	0.0138	0.0141	0.2472	97.90%	0.2614	0.2567
			S2	0.0502	0.0058	0.0606	0.0020	0.0422	0.0020	0.0793	0.0020	0.2878	96.50%	0.0093	0.0092	0.1745	95.90%	0.1751	0.1710
			S3	0.0379	0.0085	0.0602	0.0027	0.0529	0.0028	0.0678	0.0026	0.3576	96.20%	0.0115	0.0115	0.2094	96.10%	0.2067	0.1999
			S4	0.0268	0.0149	0.0692	0.0046	0.0705	0.0048	0.0682	0.0045	0.4763	96.60%	0.0153	0.0152	0.2733	97.80%	0.2709	0.2624
		0.9	α	0.1042	0.0073	0.0130	0.0003	0.0137	0.0002	0.0122	0.0002	0.2932	99.70%	0.0091	0.0090	0.0574	98.70%	0.0610	0.0603
			θ_1	0.0351	0.0770	0.0035	0.0166	0.0013	0.0166	0.0057	0.0166	1.0885	99.90%	0.0354	0.0346	0.5045	97.90%	0.5060	0.5041
			γ	0.0407	0.2648	0.0168	0.0308	0.0135	0.0306	0.0201	0.0311	2.0077	97.70%	0.0655	0.0652	0.6833	96.80%	0.6833	0.6859
			S1	0.0223	0.0091	0.0336	0.0019	0.0247	0.0019	0.0304	0.0019	0.3723	96.20%	0.0118	0.0118	0.1633	97.30%	0.1687	0.1694
			S2	0.0452	0.0042	0.0223	0.0012	0.0162	0.0012	0.0285	0.0012	0.2539	96.40%	0.0081	0.0081	0.1344	97.60%	0.1378	0.1380
			S3	0.0108	0.0070	0.0186	0.0014	0.0145	0.0015	0.0227	0.0014	0.3267	96.20%	0.0108	0.0107	0.1449	96.90%	0.1493	0.1492
			S4	0.0192	0.0123	0.0181	0.0020	0.0155	0.0020	0.0207	0.0020	0.4339	95.90%	0.0135	0.0135	0.1739	97.90%	0.1750	0.1748
	0.85	0.7	α	0.1500	0.0108	0.0147	0.0002	0.0156	0.0002	0.0137	0.0002	0.3336	99.50%	0.0108	0.0107	0.0547	98.50%	0.0563	0.0553
			θ_1	0.0530	0.0949	0.0081	0.0253	0.0015	0.0261	0.0105	0.0248	1.1714	99.90%	0.0358	0.0353	0.6257	98.70%	0.6331	0.6097
			γ	0.0058	0.3269	0.0086	0.1025	0.0087	0.1028	0.0258	0.1023	2.3833	97.50%	0.0725	0.0742	1.3192	98.90%	1.4035	1.3722
			S1	0.0517	0.0097	0.0271	0.0037	0.0131	0.0035	0.0312	0.0039	0.3736	100.00%	0.0119	0.0120	0.2167	98.00%	0.2291	0.2344
			S2	0.0491	0.0044	0.0158	0.0015	0.0080	0.0015	0.0237	0.0016	0.2510	98.50%	0.0084	0.0084	0.1453	97.40%	0.1484	0.1491
			S3	0.0301	0.0064	0.0056	0.0021	0.0023	0.0021	0.0090	0.0021	0.3022	99.20%	0.0097	0.0098	0.1762	97.70%	0.1782	0.1741
			S4	0.0219	0.0112	0.0036	0.0038	0.0035	0.0039	0.0036	0.0037	0.3995	100.00%	0.0130	0.0125	0.2322	98.20%	0.2424	0.2343
		0.9	α	0.0945	0.0067	0.0104	0.0002	0.0111	0.0002	0.0097	0.0002	0.2848	99.60%	0.0089	0.0088	0.0577	98.10%	0.0604	0.0598
			θ_1	0.0253	0.0689	0.0034	0.0165	0.0011	0.0165	0.0046	0.0164	1.0291	99.40%	0.0318	0.0317	0.4965	96.90%	0.5039	0.5014
			γ	0.0040	0.2481	0.0082	0.0287	0.0050	0.0287	0.0114	0.0287	2.0664	97.70%	0.0665	0.0666	0.6578	97.40%	0.6637	0.6631
			S1	0.0157	0.0057	0.0100	0.0015	0.0064	0.0015	0.0136	0.0016	0.2969	99.90%	0.0098	0.0096	0.1469	97.80%	0.1517	0.1536
			S2	0.0129	0.0029	0.0044	0.0010	0.0020	0.0010	0.0068	0.0011	0.2114	99.00%	0.0064	0.0067	0.1237	97.90%	0.1252	0.1264
			S3	0.0067	0.0049	0.0021	0.0012	0.0018	0.0012	0.0014	0.0012	0.2729	98.90%	0.0085	0.0086	0.1305	98.20%	0.1333	0.1340
			S4	0.0131	0.0091	0.0034	0.0015	0.0034	0.0015	0.0032	0.0015	0.3655	100.00%	0.0118	0.0119	0.1491	98.00%	0.1536	0.1537

Table 6. The MLE values and Bayesian estimate values for simple ramp with sample size II $\alpha = 0.4; \theta_1 = 1.2; \gamma = 1.1$

Size II				MLE		SELF		LLFI		LLFII		CI by MLE				CI by SELF		LLFI	LLFII
p	Q	r		RAB	MSE	RAB	MSE	RAB	MSE	RAB	MSE	LACI	CP	LBP	LBT	LCCI	CP	LCCI	LCCI
0.3	0.6	0.7	α	0.0852	0.0036	0.0089	0.0001	0.0093	0.0001	0.0085	0.0001	0.1921	99.40%	0.0061	0.0060	0.0313	98.50%	0.0319	0.0316
			θ_1	0.1076	0.0544	0.0379	0.0108	0.0197	0.0108	0.0279	0.0108	0.7119	99.70%	0.0230	0.0230	0.3705	98.70%	0.3971	0.3866
			γ	0.1060	0.1866	0.0156	0.0667	0.0279	0.0685	0.0334	0.0653	1.5659	97.10%	0.0502	0.0494	0.9507	98.60%	1.0196	1.0019
			S1	0.0974	0.0063	0.0786	0.0023	0.0579	0.0023	0.0996	0.0024	0.3073	96.40%	0.0095	0.0096	0.1807	97.50%	0.1894	0.1872
			S2	0.1688	0.0031	0.0795	0.0009	0.0694	0.0009	0.0897	0.0009	0.1989	96.50%	0.0066	0.0066	0.1101	96.40%	0.1119	0.1102
			S3	0.2442	0.0046	0.0895	0.0013	0.0874	0.0013	0.0918	0.0012	0.2365	96.50%	0.0077	0.0076	0.1351	96.40%	0.1348	0.1312
			S4	0.3133	0.0078	0.1021	0.0023	0.1062	0.0024	0.0982	0.0023	0.3149	96.80%	0.0101	0.0101	0.1821	98.00%	0.1864	0.1817
		0.9	α	0.0462	0.0021	0.0058	0.0001	0.0061	0.0001	0.0055	0.0001	0.1625	99.00%	0.0051	0.0051	0.0305	98.20%	0.0316	0.0314
			θ_1	0.0964	0.0267	0.0043	0.0099	0.0025	0.0099	0.0060	0.0098	0.6404	98.90%	0.0203	0.0205	0.3791	97.20%	0.3903	0.3881
			γ	0.0495	0.1164	0.0134	0.0231	0.0106	0.0230	0.0163	0.0233	1.3377	97.70%	0.0459	0.0444	0.5888	97.20%	0.5932	0.5939
			S1	0.0272	0.0046	0.0303	0.0013	0.0074	0.0013	0.0369	0.0013	0.2648	96.40%	0.0082	0.0082	0.1367	97.00%	0.1404	0.1404
			S2	0.0624	0.0017	0.0185	0.0008	0.0041	0.0007	0.0227	0.0008	0.1626	97.10%	0.0053	0.0054	0.1061	96.60%	0.1072	0.1070
			S3	0.1062	0.0023	0.0135	0.0009	0.0120	0.0009	0.0159	0.0009	0.1885	97.00%	0.0059	0.0060	0.1130	97.20%	0.1150	0.1146
			S4	0.2488	0.0042	0.0115	0.0012	0.0127	0.0012	0.0126	0.0012	0.2551	96.80%	0.0082	0.0082	0.1359	97.30%	0.1375	0.1371
	0.85	0.7	α	0.0800	0.0036	0.0074	0.0001	0.0078	0.0001	0.0070	0.0001	0.1974	99.60%	0.0061	0.0062	0.0303	99.00%	0.0321	0.0319
			θ_1	0.0879	0.0458	0.0343	0.0098	0.0163	0.0097	0.0238	0.0100	0.6697	99.00%	0.0211	0.0209	0.3372	98.80%	0.3582	0.3491
			γ	0.0758	0.1710	0.0121	0.0660	0.0245	0.0676	0.0128	0.0649	1.5562	98.40%	0.0503	0.0505	0.9578	98.20%	1.0144	0.9991
			S1	0.0291	0.0043	0.0155	0.0018	0.0097	0.0018	0.0243	0.0020	0.2497	98.20%	0.0077	0.0076	0.1554	97.40%	0.1609	0.1635
			S2	0.0601	0.0027	0.0225	0.0007	0.0182	0.0007	0.0268	0.0007	0.1760	97.50%	0.0055	0.0054	0.0945	98.10%	0.0931	0.0931
			S3	0.0795	0.0044	0.0262	0.0011	0.0250	0.0011	0.0273	0.0010	0.2186	98.20%	0.0069	0.0069	0.1156	97.50%	0.1179	0.1149
			S4	0.0927	0.0078	0.0282	0.0020	0.0295	0.0021	0.0269	0.0020	0.2926	98.90%	0.0096	0.0096	0.1656	98.90%	0.1694	0.1643
		0.9	α	0.0431	0.0020	0.0045	0.0001	0.0048	0.0001	0.0042	0.0001	0.1636	99.10%	0.0053	0.0053	0.0303	98.50%	0.0317	0.0316
			θ_1	0.0420	0.0218	0.0042	0.0091	0.0024	0.0091	0.0059	0.0091	0.6521	98.80%	0.0207	0.0213	0.3704	98.10%	0.3738	0.3723
			γ	0.0391	0.1021	0.0097	0.0228	0.0069	0.0227	0.0124	0.0229	1.3662	97.50%	0.0408	0.0412	0.5858	96.80%	0.5905	0.5905
			S1	0.0043	0.0033	0.0103	0.0010	0.0058	0.0010	0.0131	0.0010	0.2239	98.90%	0.0074	0.0073	0.1207	98.40%	0.1203	0.1214
			S2	0.0402	0.0014	0.0058	0.0005	0.0021	0.0005	0.0076	0.0006	0.1443	97.90%	0.0045	0.0044	0.0901	98.30%	0.0909	0.0913
			S3	0.0092	0.0019	0.0122	0.0006	0.0110	0.0006	0.0033	0.0006	0.1707	98.60%	0.0051	0.0051	0.0992	98.00%	0.0985	0.0986
			S4	0.0530	0.0034	0.0103	0.0009	0.0104	0.0009	0.0033	0.0009	0.2275	99.10%	0.0070	0.0070	0.1183	98.70%	0.1189	0.1188
0.8	0.6	0.7	α	0.1086	0.0046	0.0144	0.0001	0.0148	0.0001	0.0141	0.0001	0.2054	99.30%	0.0065	0.0066	0.0324	98.90%	0.0344	0.0342
			θ_1	0.0545	0.0359	0.0083	0.0098	0.0041	0.0100	0.0124	0.0097	0.6971	98.90%	0.0213	0.0213	0.3656	98.60%	0.3912	0.3811
			γ	0.0287	0.1598	0.0079	0.0640	0.0201	0.0659	0.0043	0.0627	1.5629	97.80%	0.0473	0.0475	0.9674	98.30%	1.0030	0.9817
			S1	0.0092	0.0058	0.0030	0.0021	0.0058	0.0021	0.0118	0.0021	0.2994	96.80%	0.0097	0.0096	0.1746	96.30%	0.1804	0.1779
			S2	0.0228	0.0023	0.0070	0.0007	0.0027	0.0007	0.0114	0.0007	0.1836	96.90%	0.0062	0.0062	0.0984	96.40%	0.1062	0.1046
			S3	0.0301	0.0035	0.0087	0.0012	0.0076	0.0012	0.0099	0.0011	0.2281	96.70%	0.0075	0.0076	0.1351	97.30%	0.1345	0.1307
			S4	0.0343	0.0066	0.0093	0.0022	0.0106	0.0023	0.0080	0.0022	0.3133	97.10%	0.0105	0.0104	0.1898	96.30%	0.1881	0.1828
		0.9	α	0.0465	0.0022	0.0052	0.0001	0.0055	0.0001	0.0049	0.0001	0.1688	99.20%	0.0055	0.0055	0.0315	98.10%	0.0330	0.0329
			θ_1	0.0061	0.0266	0.0004	0.0103	0.0013	0.0104	0.0021	0.0102	0.6393	98.60%	0.0198	0.0200	0.3754	98.40%	0.3991	0.3968
			γ	0.0058	0.1196	0.0056	0.0229	0.0028	0.0229	0.0084	0.0230	1.3562	97.50%	0.0418	0.0407	0.5703	97.60%	0.5928	0.5933
			S1	0.0048	0.0043	0.0042	0.0013	0.0014	0.0013	0.0069	0.0013	0.2583	96.60%	0.0088	0.0087	0.1391	96.60%	0.1434	0.1433
			S2	0.0103	0.0016	0.0026	0.0008	0.0008	0.0008	0.0043	0.0008	0.1574	97.70%	0.0050	0.0051	0.1034	96.90%	0.1082	0.1080
			S3	0.0246	0.0024	0.0010	0.0008	0.0000	0.0008	0.0021	0.0008	0.1909	97.50%	0.0061	0.0061	0.1134	96.80%	0.1141	0.1138
			S4	0.0167	0.0045	0.0004	0.0012	0.0009	0.0012	0.0000	0.0012	0.2617	96.90%	0.0084	0.0084	0.1356	98.20%	0.1356	0.1353
	0.85	0.7	α	0.1049	0.0043	0.0138	0.0001	0.0142	0.0001	0.0134	0.0001	0.1979	99.30%	0.0064	0.0065	0.0315	97.40%	0.0331	0.0329
			θ_1	0.0513	0.0391	0.0054	0.0107	0.0012	0.0110	0.0096	0.0105	0.7369	99.00%	0.0237	0.0237	0.3714	99.70%	0.4105	0.3991
			γ	0.0441	0.1574	0.0006	0.0617	0.0112	0.0626	0.0124	0.0611	1.5441	97.20%	0.0495	0.0488	0.9618	97.30%	0.9804	0.9680
			S1	0.0043	0.0038	0.0247	0.0017	0.0045	0.0017	0.0451	0.0018	0.2427	99.20%	0.0077	0.0076	0.1573	98.30%	0.1611	0.1633
			S2	0.0192	0.0017	0.0141	0.0006	0.0038	0.0006	0.0245	0.0006	0.1623	97.40%	0.0051	0.0052	0.0934	96.80%	0.0954	0.0953
			S3	0.0557	0.0027	0.0159	0.0008	0.0129	0.0008	0.0190	0.0008	0.2008	97.90%	0.0061	0.0061	0.1119	96.70%	0.1133	0.1110
			S4	0.0974	0.0050	0.0222	0.0016	0.0250	0.0017	0.0196	0.0016	0.2703	99.10%	0.0081	0.0081	0.1524	97.60%	0.1593	0.1553
		0.9	α	0.0452	0.0021	0.0054	0.0001	0.0057	0.0001	0.0051	0.0001	0.1670	99.20%	0.0054	0.0054	0.0310	98.80%	0.0322	0.0320
			θ_1	0.0041	0.0258	0.0016	0.0107	0.0001	0.0107	0.0033	0.0106	0.6291	98.80%	0.0212	0.0210	0.3846	98.50%	0.4063	0.4044
			γ	0.0080	0.1290	0.0055	0.0230	0.0028	0.0229	0.0083	0.0231	1.4083	97.20%	0.0419	0.0413	0.5786	97.60%	0.5939	0.5946
			S1	0.0014	0.0031	0.0216	0.0010	0.0150	0.0010	0.0283	0.0011	0.2188	99.10%	0.0072	0.0073	0.1237	97.60%	0.1249	0.1261
			S2	0.0161	0.0012	0.0131	0.0006	0.0089	0.0006	0.0173	0.0006	0.1330	98.80%	0.0042	0.0042	0.0953	97.00%	0.0967	0.0972
			S3	0.0056	0.0017	0.0105	0.0007	0.0079	0.0007	0.0130	0.0007	0.1627	98.90%	0.0051	0.0051	0.1025	97.60%	0.1037	0.1038
			S4	0.0140	0.0033	0.0104	0.0010	0.0092	0.0010	0.0117	0.0010	0.2243	99.10%	0.0072	0.0071	0.1208	98.00%	0.1232	0.1231

Table 7. The MLE values and Bayesian estimate values for simple ramp with sample sizeI $\alpha = 2; \theta_1 = 1.2; \gamma = 1.1$

Size I				MLE		SELF		LLFI		LLFII		CI by MLE				CI by SELF		LLFI	LLFII
p	Q	r		RAB	MSE	RAB	MSE	RAB	MSE	RAB	MSE	LACI	CP	LBP	LBT	LCCI	CP	LCCI	LCCI
0.3	0.6	0.7	α	0.0403	0.1288	0.0064	0.0170	0.0051	0.0107	0.0095	0.0099	1.4072	99.60%	0.0467	0.0458	0.2979	98.70%	0.3347	0.3226
			θ_1	0.0563	0.0196	0.0319	0.0032	0.0318	0.0031	0.0299	0.0018	0.2778	98.20%	0.0088	0.0089	0.1382	98.20%	0.1379	0.1372
			γ	0.0305	0.0323	0.0184	0.0132	0.0094	0.0133	0.0138	0.0112	0.7030	97.00%	0.0234	0.0228	0.4155	98.50%	0.4161	0.4111
			S1	0.2017	0.0194	0.0767	0.0051	0.0647	0.0048	0.0887	0.0054	0.4457	96.90%	0.0131	0.0131	0.2431	97.70%	0.2511	0.2518
			S2	0.1829	0.0119	0.0628	0.0019	0.0575	0.0018	0.0681	0.0020	0.3167	97.40%	0.0102	0.0104	0.1396	98.30%	0.1397	0.1394
			S3	0.1761	0.0151	0.0561	0.0025	0.0557	0.0026	0.0566	0.0025	0.3953	97.40%	0.0118	0.0119	0.1797	96.90%	0.1786	0.1749
			S4	0.1745	0.0220	0.0527	0.0047	0.0561	0.0048	0.0494	0.0045	0.5132	97.40%	0.0164	0.0166	0.2517	97.60%	0.2580	0.2515
		0.9	α	0.0371	0.0952	0.0034	0.0094	0.0022	0.0095	0.0045	0.0094	1.1743	99.10%	0.0377	0.0381	0.3773	97.40%	0.3809	0.3787
			θ_1	0.0211	0.0052	0.0094	0.0016	0.0043	0.0016	0.0048	0.0016	0.2660	97.60%	0.0089	0.0089	0.1511	97.90%	0.1578	0.1572
			γ	0.0241	0.0280	0.0085	0.0126	0.0076	0.0127	0.0031	0.0103	0.6479	98.00%	0.0206	0.0204	0.4377	98.60%	0.4411	0.4391
			S1	0.0318	0.0111	0.0167	0.0045	0.0111	0.0045	0.0223	0.0046	0.4107	97.90%	0.0135	0.0132	0.2573	97.40%	0.2628	0.2636
			S2	0.0553	0.0061	0.0194	0.0016	0.0164	0.0016	0.0223	0.0016	0.2936	98.20%	0.0091	0.0090	0.1502	98.70%	0.1534	0.1534
			S3	0.0811	0.0097	0.0252	0.0025	0.0242	0.0026	0.0261	0.0019	0.3649	98.10%	0.0118	0.0119	0.1943	97.00%	0.1955	0.1942
			S4	0.1051	0.0168	0.0317	0.0025	0.0323	0.0045	0.0312	0.0041	0.4803	98.50%	0.0155	0.0156	0.2709	98.10%	0.2773	0.2747
	0.85	0.7	α	0.0335	0.1059	0.0052	0.0061	0.0024	0.0062	0.0080	0.0060	1.2489	99.60%	0.0390	0.0388	0.2804	99.30%	0.3085	0.2971
			θ_1	0.0476	0.0131	0.0289	0.0024	0.0280	0.0023	0.0298	0.0012	0.2677	98.20%	0.0088	0.0087	0.1347	97.70%	0.1338	0.1332
			γ	0.0073	0.0312	0.0103	0.0117	0.0049	0.0117	0.0116	0.0102	0.7238	97.70%	0.0228	0.0233	0.4171	99.40%	0.4242	0.4199
			S1	0.5448	0.0154	0.1851	0.0025	0.1637	0.0023	0.2066	0.0028	0.3669	99.70%	0.0124	0.0124	0.1600	96.80%	0.1629	0.1663
			S2	0.4917	0.0103	0.1487	0.0010	0.1379	0.0010	0.1595	0.0011	0.2742	98.40%	0.0085	0.0083	0.0853	97.80%	0.0898	0.0903
			S3	0.4915	0.0113	0.1367	0.0011	0.1335	0.0011	0.1400	0.0011	0.3014	98.60%	0.0092	0.0092	0.0972	99.00%	0.1029	0.1011
			S4	0.5122	0.0148	0.1353	0.0018	0.1380	0.0019	0.1328	0.0018	0.3704	99.60%	0.0115	0.0117	0.1327	98.90%	0.1495	0.1455
		0.9	α	0.0301	0.0818	0.0023	0.0060	0.0012	0.0060	0.0034	0.0050	1.0970	99.20%	0.0360	0.0358	0.3760	98.70%	0.3875	0.3868
			θ_1	0.0207	0.0051	0.0054	0.0016	0.0041	0.0016	0.0036	0.0010	0.2656	97.70%	0.0085	0.0085	0.1558	97.20%	0.1565	0.1558
			γ	0.0062	0.0256	0.0077	0.0114	0.0040	0.0114	0.0026	0.0101	0.6470	98.40%	0.0215	0.0215	0.4135	96.80%	0.4175	0.4159
			S1	0.0564	0.0045	0.0405	0.0015	0.0318	0.0015	0.0492	0.0016	0.2604	99.80%	0.0087	0.0086	0.1498	97.40%	0.1518	0.1534
			S2	0.1100	0.0031	0.0403	0.0006	0.0352	0.0006	0.0455	0.0007	0.2072	98.70%	0.0066	0.0066	0.0949	97.60%	0.0968	0.0972
			S3	0.1829	0.0046	0.0533	0.0009	0.0508	0.0009	0.0559	0.0009	0.2434	98.90%	0.0075	0.0076	0.1109	98.30%	0.1123	0.1122
			S4	0.2599	0.0079	0.0710	0.0016	0.0705	0.0016	0.0715	0.0016	0.3129	99.90%	0.0096	0.0096	0.1481	97.50%	0.1527	0.1517
0.8	0.6	0.7	α	0.0947	0.1779	0.0126	0.0108	0.0159	0.0115	0.0093	0.0102	1.4779	99.90%	0.0477	0.0476	0.3815	97.80%	0.4019	0.3895
			θ_1	0.0353	0.0069	0.0121	0.0014	0.0113	0.0014	0.0130	0.0014	0.2790	97.90%	0.0088	0.0089	0.1327	97.40%	0.1341	0.1332
			γ	0.0197	0.0325	0.0116	0.0097	0.0067	0.0097	0.0164	0.0098	0.7021	98.00%	0.0228	0.0227	0.3863	98.10%	0.3857	0.3808
			S1	0.1245	0.0184	0.0546	0.0045	0.0433	0.0043	0.0659	0.0047	0.4948	97.70%	0.0156	0.0159	0.2389	98.10%	0.2478	0.2478
			S2	0.0891	0.0083	0.0366	0.0014	0.0314	0.0014	0.0418	0.0015	0.3299	97.30%	0.0112	0.0112	0.1347	98.00%	0.1356	0.1353
			S3	0.0744	0.0106	0.0269	0.0017	0.0260	0.0018	0.0277	0.0017	0.3873	97.80%	0.0127	0.0127	0.1606	96.20%	0.1601	0.1571
			S4	0.0690	0.0173	0.0210	0.0035	0.0236	0.0036	0.0185	0.0034	0.5046	98.50%	0.0165	0.0167	0.2304	99.30%	0.2310	0.2253
		0.9	α	0.0726	0.1150	0.0013	0.0101	0.0025	0.0112	0.0002	0.0091	1.2020	99.20%	0.0368	0.0376	0.4101	97.40%	0.4151	0.4136
			θ_1	0.0125	0.0049	0.0005	0.0012	0.0017	0.0013	0.0012	0.0011	0.2668	97.80%	0.0085	0.0084	0.1507	98.20%	0.1546	0.1539
			γ	0.0163	0.0288	0.0026	0.0090	0.0047	0.0081	0.0004	0.0090	0.6613	97.70%	0.0218	0.0216	0.4212	97.80%	0.4168	0.4154
			S1	0.0048	0.0108	0.0075	0.0040	0.0019	0.0040	0.0130	0.0041	0.4082	98.00%	0.0123	0.0122	0.2516	97.10%	0.2483	0.2489
			S2	0.0169	0.0054	0.0072	0.0010	0.0043	0.0011	0.0102	0.0015	0.2880	97.30%	0.0094	0.0094	0.1483	97.60%	0.1489	0.1487
			S3	0.0365	0.0093	0.0109	0.0014	0.0098	0.0016	0.0119	0.0012	0.3728	98.60%	0.0120	0.0121	0.1897	98.80%	0.1905	0.1894
			S4	0.0568	0.0165	0.0156	0.0035	0.0160	0.0035	0.0153	0.0030	0.4963	98.80%	0.0176	0.0177	0.2644	98.10%	0.2685	0.2665
	0.85	0.7	α	0.0766	0.1317	0.0075	0.0079	0.0107	0.0085	0.0042	0.0075	1.2901	99.30%	0.0421	0.0413	0.3300	97.80%	0.3508	0.3391
			θ_1	0.0341	0.0068	0.0111	0.0013	0.0104	0.0013	0.0126	0.0012	0.2848	98.40%	0.0089	0.0090	0.1374	97.20%	0.1381	0.1374
			γ	0.0052	0.0333	0.0050	0.0097	0.0007	0.0091	0.0100	0.0097	0.7150	97.30%	0.0227	0.0228	0.3628	97.90%	0.3881	0.3836
			S1	0.1870	0.0082	0.0881	0.0016	0.0682	0.0016	0.1081	0.0018	0.3370	100.00%	0.0111	0.0112	0.1456	98.80%	0.1491	0.1521
			S2	0.1338	0.0043	0.0650	0.0006	0.0543	0.0006	0.0757	0.0006	0.2443	99.00%	0.0082	0.0082	0.0835	97.90%	0.0878	0.0883
			S3	0.1376	0.0051	0.0600	0.0007	0.0559	0.0007	0.0642	0.0007	0.2673	99.40%	0.0086	0.0086	0.0923	97.90%	0.0988	0.0972
			S4	0.1638	0.0077	0.0627	0.0013	0.0639	0.0013	0.0617	0.0013	0.3303	99.90%	0.0103	0.0108	0.1260	98.20%	0.1378	0.1342
		0.9	α	0.0671	0.1104	0.0011	0.0071	0.0004	0.0071	0.0001	0.0061	1.1920	99.10%	0.0370	0.0379	0.4122	98.00%	0.4203	0.4187
			θ_1	0.0118	0.0035	0.0003	0.0011	0.0012	0.0011	0.0010	0.0009	0.2756	97.80%	0.0087	0.0088	0.1514	97.50%	0.1567	0.1561
			γ	0.0042	0.0210	0.0023	0.0061	0.0005	0.0069	0.0003	0.0081	0.6705	97.00%	0.0199	0.0200	0.4431	97.50%	0.4436	0.4422
			S1	0.0130	0.0046	0.0366	0.0016	0.0275	0.0015	0.0457	0.0016	0.2664	100.00%	0.0084	0.0084	0.1471	98.00%	0.1532	0.1552
			S2	0.0262	0.0029	0.0303	0.0005	0.0250	0.0005	0.0356	0.0005	0.2092	99.60%	0.0066	0.0065	0.0963	97.20%	0.0979	0.0985
			S3	0.0724	0.0040	0.0405	0.0006	0.0379	0.0006	0.0431	0.0006	0.2457	99.30%	0.0079	0.0078	0.1187	97.10%	0.1183	0.1181
			S4	0.1292	0.0067	0.0568	0.0011	0.0564	0.0012	0.0573	0.0011	0.3113	100.00%	0.0099	0.0099	0.1566	98.70%	0.1627	0.1614

Table 8. The MLE values and Bayesian estimate values for simple ramp with sample size II $\alpha = 2; \theta_1 = 1.2; \gamma = 1.1$

Size II				MLE		SELF		LLFI		LLFII		CI by MLE				CI by SELF		LLFI	LLFII
p	Q	r		RAB	MSE	RAB	MSE	RAB	MSE	RAB	MSE	LACI	CP	LBP	LBT	LCCI	CP	LCCI	LCCI
0.3	0.6	0.7	α	0.0345	0.0529	0.0055	0.0043	0.0070	0.0040	0.0040	0.0045	0.8672	99.10%	0.0265	0.0261	0.1831	99.20%	0.1983	0.1943
			θ_1	0.0503	0.0071	0.0271	0.0019	0.0217	0.0019	0.0218	0.0020	0.1910	97.30%	0.0060	0.0059	0.0898	96.90%	0.0902	0.0900
			γ	0.0409	0.0167	0.0085	0.0049	0.0211	0.0053	0.0070	0.0051	0.4669	97.70%	0.0142	0.0146	0.2522	97.60%	0.2525	0.2509
			S1	0.0846	0.0080	0.0371	0.0019	0.0315	0.0019	0.0427	0.0020	0.3256	97.80%	0.0101	0.0103	0.1616	97.20%	0.1614	0.1616
			S2	0.1075	0.0051	0.0464	0.0009	0.0440	0.0009	0.0489	0.0009	0.2242	97.30%	0.0072	0.0072	0.0932	97.10%	0.0913	0.0912
			S3	0.1281	0.0071	0.0544	0.0012	0.0542	0.0012	0.0545	0.0012	0.2636	97.80%	0.0084	0.0082	0.1083	98.60%	0.1095	0.1084
			S4	0.1460	0.0111	0.0612	0.0022	0.0629	0.0022	0.0596	0.0021	0.3440	98.10%	0.0103	0.0103	0.1552	97.40%	0.1563	0.1543
		0.9	α	0.0319	0.0406	0.0018	0.0039	0.0026	0.0039	0.0010	0.0039	0.7492	99.20%	0.0230	0.0234	0.2478	97.60%	0.2454	0.2443
			θ_1	0.0104	0.0052	0.0028	0.0006	0.0025	0.0007	0.0032	0.0010	0.1685	96.90%	0.0054	0.0055	0.0911	98.40%	0.0929	0.0927
			γ	0.0016	0.0158	0.0012	0.0048	0.0026	0.0048	0.0042	0.0048	0.4081	97.70%	0.0129	0.0129	0.2626	98.00%	0.2721	0.2714
			S1	0.0275	0.0050	0.0099	0.0018	0.0065	0.0018	0.0132	0.0019	0.2731	97.50%	0.0089	0.0089	0.1674	97.60%	0.1676	0.1676
			S2	0.0223	0.0022	0.0095	0.0006	0.0079	0.0006	0.0112	0.0006	0.1792	97.40%	0.0058	0.0057	0.0906	97.20%	0.0917	0.0916
			S3	0.0224	0.0030	0.0110	0.0008	0.0106	0.0008	0.0113	0.0008	0.2125	98.10%	0.0063	0.0063	0.1134	97.70%	0.1124	0.1119
			S4	0.0247	0.0053	0.0130	0.0017	0.0135	0.0017	0.0124	0.0017	0.2819	98.00%	0.0087	0.0087	0.1614	97.40%	0.1626	0.1617
	0.85	0.7	α	0.0187	0.0453	0.0025	0.0022	0.0040	0.0023	0.0010	0.0022	0.8217	98.80%	0.0259	0.0259	0.1746	99.80%	0.1860	0.1819
			θ_1	0.0474	0.0061	0.0203	0.0012	0.0199	0.0012	0.0207	0.0012	0.2079	97.70%	0.0066	0.0066	0.0939	97.80%	0.0962	0.0960
			γ	0.0239	0.0155	0.0080	0.0044	0.0115	0.0045	0.0065	0.0043	0.4766	97.80%	0.0146	0.0147	0.2563	97.90%	0.2584	0.2566
			S1	0.1726	0.0038	0.0716	0.0007	0.0620	0.0007	0.0812	0.0007	0.2196	99.60%	0.0070	0.0070	0.0900	98.70%	0.0945	0.0954
			S2	0.2044	0.0030	0.0829	0.0004	0.0779	0.0004	0.0878	0.0004	0.1797	98.70%	0.0059	0.0058	0.0564	97.80%	0.0582	0.0584
			S3	0.2460	0.0041	0.0961	0.0005	0.0945	0.0005	0.0977	0.0005	0.2038	98.80%	0.0066	0.0066	0.0703	97.40%	0.0692	0.0686
			S4	0.2892	0.0059	0.1094	0.0009	0.1105	0.0009	0.1083	0.0008	0.2496	99.10%	0.0080	0.0079	0.0952	98.50%	0.0963	0.0949
		0.9	α	0.0278	0.0405	0.0006	0.0039	0.0014	0.0039	0.0002	0.0039	0.7583	98.50%	0.0262	0.0257	0.2447	98.10%	0.2454	0.2441
			θ_1	0.0101	0.0021	0.0026	0.0005	0.0023	0.0006	0.0030	0.0006	0.1745	97.70%	0.0056	0.0057	0.0945	99.00%	0.0939	0.0937
			γ	0.0012	0.0115	0.0011	0.0046	0.0014	0.0046	0.0014	0.0046	0.4215	97.00%	0.0132	0.0132	0.2651	98.00%	0.2670	0.2665
			S1	0.0337	0.0020	0.0218	0.0006	0.0162	0.0006	0.0274	0.0006	0.1745	98.60%	0.0058	0.0057	0.0938	98.10%	0.0943	0.0951
			S2	0.0231	0.0012	0.0160	0.0002	0.0128	0.0002	0.0191	0.0002	0.1374	98.50%	0.0045	0.0045	0.0572	97.00%	0.0572	0.0574
			S3	0.0295	0.0016	0.0172	0.0003	0.0159	0.0003	0.0186	0.0003	0.1550	98.80%	0.0053	0.0054	0.0633	98.40%	0.0664	0.0663
			S4	0.0425	0.0024	0.0213	0.0006	0.0214	0.0006	0.0212	0.0005	0.1898	99.20%	0.0062	0.0060	0.0901	98.10%	0.0916	0.0911
0.8	0.6	0.7	α	0.0603	0.0618	0.0099	0.0031	0.0114	0.0033	0.0083	0.0029	0.8525	99.30%	0.0276	0.0272	0.1967	98.80%	0.2065	0.2012
			θ_1	0.0277	0.0036	0.0123	0.0008	0.0119	0.0008	0.0127	0.0008	0.1973	97.10%	0.0064	0.0064	0.0961	98.20%	0.0948	0.0945
			γ	0.0088	0.0152	0.0034	0.0039	0.0059	0.0040	0.0010	0.0039	0.4828	97.60%	0.0157	0.0160	0.2462	97.90%	0.2469	0.2452
			S1	0.0541	0.0077	0.0289	0.0018	0.0233	0.0018	0.0344	0.0019	0.3340	98.20%	0.0108	0.0106	0.1589	97.40%	0.1608	0.1609
			S2	0.0604	0.0039	0.0317	0.0007	0.0293	0.0007	0.0342	0.0008	0.2253	98.30%	0.0070	0.0070	0.0943	98.10%	0.0939	0.0938
			S3	0.0699	0.0054	0.0350	0.0010	0.0348	0.0010	0.0353	0.0010	0.2664	97.80%	0.0084	0.0085	0.1101	97.50%	0.1101	0.1091
			S4	0.0801	0.0090	0.0383	0.0018	0.0398	0.0018	0.0369	0.0017	0.3496	98.20%	0.0111	0.0112	0.1507	97.70%	0.1540	0.1521
		0.9	α	0.0359	0.0395	0.0021	0.0031	0.0028	0.0028	0.0013	0.0027	0.7268	99.10%	0.0230	0.0230	0.2470	97.60%	0.2410	0.2396
			θ_1	0.0054	0.0019	0.0003	0.0006	0.0057	0.0006	0.0007	0.0006	0.1699	98.40%	0.0052	0.0052	0.0888	97.60%	0.0927	0.0924
			γ	0.0051	0.0122	0.0010	0.0035	0.0004	0.0040	0.0008	0.0025	0.4324	97.20%	0.0138	0.0139	0.2817	98.40%	0.2772	0.2765
			S1	0.0176	0.0054	0.0061	0.0017	0.0027	0.0015	0.0095	0.0011	0.2874	98.20%	0.0090	0.0090	0.1749	98.40%	0.1733	0.1734
			S2	0.0064	0.0023	0.0030	0.0006	0.0013	0.0006	0.0046	0.0006	0.1879	97.40%	0.0059	0.0059	0.0904	96.90%	0.0933	0.0932
			S3	0.0029	0.0033	0.0024	0.0008	0.0020	0.0008	0.0028	0.0008	0.2242	98.20%	0.0073	0.0073	0.1113	97.20%	0.1118	0.1113
			S4	0.0027	0.0058	0.0029	0.0017	0.0035	0.0017	0.0024	0.0017	0.2977	98.40%	0.0090	0.0091	0.1598	97.10%	0.1623	0.1614
	0.85	0.7	α	0.0536	0.0562	0.0081	0.0031	0.0118	0.0053	0.0086	0.0029	0.8422	99.10%	0.0269	0.0266	0.1932	98.80%	0.2057	0.2010
			θ_1	0.0270	0.0031	0.0119	0.0007	0.0115	0.0008	0.0123	0.0008	0.2080	97.50%	0.0067	0.0068	0.0993	98.60%	0.0977	0.0975
			γ	0.0078	0.0159	0.0029	0.0042	0.0111	0.0054	0.0062	0.0042	0.4892	97.40%	0.0153	0.0153	0.2462	97.50%	0.2536	0.2517
			S1	0.0271	0.0029	0.0245	0.0006	0.0151	0.0006	0.0339	0.0006	0.2124	99.70%	0.0070	0.0070	0.0878	98.40%	0.0923	0.0932
			S2	0.0416	0.0018	0.0348	0.0003	0.0298	0.0002	0.0398	0.0003	0.1659	99.30%	0.0055	0.0054	0.0561	98.60%	0.0585	0.0586
			S3	0.0719	0.0023	0.0471	0.0004	0.0453	0.0004	0.0490	0.0004	0.1851	98.80%	0.0059	0.0059	0.0655	98.00%	0.0685	0.0679
			S4	0.1070	0.0036	0.0596	0.0006	0.0603	0.0007	0.0590	0.0006	0.2276	98.90%	0.0070	0.0072	0.0890	99.10%	0.0935	0.0921
		0.9	α	0.0334	0.0377	0.0013	0.0030	0.0020	0.0041	0.0005	0.0024	0.7150	99.20%	0.0224	0.0218	0.2482	98.70%	0.2500	0.2490
			θ_1	0.0051	0.0016	0.0002	0.0005	0.0006	0.0005	0.0013	0.0005	0.1714	97.90%	0.0053	0.0053	0.0884	97.80%	0.0911	0.0909
			γ	0.0024	0.0118	0.0001	0.0041	0.0003	0.0050	0.0017	0.0039	0.4263	97.50%	0.0139	0.0140	0.2751	98.30%	0.2760	0.2753
			S1	0.0071	0.0020	0.0079	0.0001	0.0023	0.0005	0.0135	0.0006	0.1737	98.90%	0.0051	0.0052	0.0942	98.40%	0.0943	0.0949
			S2	0.0032	0.0011	0.0071	0.0002	0.0040	0.0002	0.0102	0.0002	0.1282	98.10%	0.0041	0.0041	0.0566	96.10%	0.0556	0.0557
			S3	0.0035	0.0013	0.0123	0.0003	0.0110	0.0003	0.0136	0.0003	0.1427	98.20%	0.0048	0.0047	0.0652	97.80%	0.0660	0.0658
			S4	0.0168	0.0021	0.0197	0.0006	0.0198	0.0006	0.0195	0.0006	0.1774	99.40%	0.0058	0.0057	0.0908	98.50%	0.0928	0.0923

5. Real Data Analysis and Applications on circuits

To prove the applicability of the work in the paper, we applied the model to real data that was collected in a real-time experiment. Real data description: To demonstrate the proposed methodologies, we analyze an accelerated life test data set from Mahto et al. [31], which represents failure times for the time-dependent dielectric breakdown of metal-oxide-semiconductor integrated circuits. The test was conducted under three different elevated temperatures: 170°C, 200°C, and 250°C. However, our study considers only data obtained at 170°C and 200°C. To assess the suitability of the data for fitting to a log-logistic distribution, we calculate the Kolmogorov-Smirnov (KS) distance and the associated p-values at each stress level. These results and the data sets are reported in Table 9. The scheme used is in the same table, the data at each level, and p.

Table 9. Data under AP-II-HC

Q	m	p	Stress	Data	Scheme
2	9,12	0.3	170	164 164 218 230 263 639 1148 1678 1678	2 1 2 1 0 0 0 0 0
			200	76 82 210 385 412 491 504 522 884 1446 1827 2385	2 3 1 0 2 0 0 0 0 0 0 0
		0.8	170	164 164 218 230 467 538 917 1678 1678	1 4 0 0 1 0 0 0 0
			200	76 82 210 315 412 504 1131 1446 1824 1827 2385 3077	1 6 1 0 0 0 0 0 0 0 0 0
3	13,15	0.3	170	164 164 218 230 263 467 538 639 917 1148 1678 1678 1678	1 0 1 0 0 0 0 0 0 0 0 0 0
			200	76 82 210 315 385 504 522 646 678 775 884 1131 1827 2248 3077	0 1 2 1 0 1 0 0 0 0 0 0 0 0 0
		0.8	170	164 164 218 263 467 538 639 669 1148 1678 1678 1678 1678	0 1 1 0 0 0 0 0 0 0 0 0 0
			200	76 82 210 315 385 522 646 678 884 1131 1446 1824 2248 2385 3077	1 4 0 0 0 0 0 0 0 0 0 0 0 0 0

Table 10. MLE for different models under stress level with different statistical measures

	Stress		estimates	SE	AIC	BIC	KSD	PVKS
OFHLD	170	α	0.6797	0.0190	233.5849	235.7090	0.1627	0.8280
		θ_1	0.0019	0.0001				
		γ	0.0424	1.0865				
	200	α	0.5407	0.1656	320.7256	323.7128	0.0966	0.9830
		θ_1	0.0011	0.0049				
		γ	0.1146	0.7344				
ExEx	170	α	0.00000901	0.00000000024	235.2229	237.3470	0.1630	0.8203
		θ_1	0.6000	0.0577				
		γ	0.4989	0.0148				
	200	α	0.0000	0.0000	321.4052	323.5293	0.1041	0.9661
		θ_1	0.5327	0.0353				
		γ	0.4509	0.0375				
LogEx	170	α	0.9643	0.0483	234.7455	236.8696	0.1652	0.8078
		θ_1	0.00001652	0.00000000027				
		γ	0.3794	0.0072				
	200	α	0.9439	0.0018	321.2075	323.3316	0.1193	0.9067
		θ_1	0.00005045	0.00000002247				
		γ	0.2552	0.0660				

Interpretation of the "Scheme" column: Each number represents the number of units removed after the corresponding failure. For example, "2 1 2 1 0 0 0 0 0" means: after the 1st failure, 2 units were removed; after the 2nd failure, 1 unit; after the 3rd failure, 2 units; after the 4th failure, 1 unit; and no further removals.

The tabulated values show acceptable outcomes for both the KS distance (KSD) and the p-values in Table 10. Also, MLE and different computing models under stress levels by using an extension of the exponential (ExEx) distribution [33], and logistic exponential (LogEx) distribution [30] with different statistical measures, an Akaike information criterion (AIC), Bayesian Information Criterion (BIC), KSD, and P-value KS (PVKS). For a detailed

Table 11. Comparison with competing models by MLE

		estimates	SE	AIC	BIC
OFHLD	α	0.5617	0.0060	549.4835	554.1496
	θ_1	0.0012	0.0000		
	γ	0.1134	0.3557		
ExEx	α	0.0000	0.0000	551.0718	555.7378
	θ_1	0.5821	0.0487		
	γ	0.4720	0.0018		
LogEx	α	0.9736	0.0194	551.1106	555.7766
	θ_1	0.0001	0.0000		
	γ	0.2570	0.0068		

study of the OFHLD properties and its comparison with other well-known distributions, see Bhat et al. [19]. The present paper focuses on statistical inference under progressive-stress ALT with AP-II-HC.

From Table 10, the best model is the one with the lowest AIC, BIC, and Kolmogorov-Smirnov distance (KSD) values and the highest p-values, indicating a better fit to the OFHLD. Table 11 presents the AIC and BIC for each data set at different stress levels, assessing the fit of the data to an OFHLD. The AIC and BIC values are relatively low, indicating a good fit for the OFHLD at both 170°C and 200°C. These results demonstrate that the chosen distribution adequately describes the failure-time data for the tested conditions under the progressive-stress model.

Table 12. MLE and Bayesian estimation with different cases

				MLE			Bayesian		
Q	m			estimates	SE	L.CI	estimates	SE	L.CI
2	9,12	0.3	α	0.6012	0.0887	0.1100	0.6017	0.0374	0.1466
			θ_1	0.0014	0.0026	0.0032	0.0014	0.0004	0.0014
			γ	0.1017	0.0873	0.1082	0.1078	0.0341	0.1253
		0.8	α	0.5617	0.0795	0.0985	0.5588	0.0337	0.1305
			θ_1	0.0012	0.0499	0.0619	0.0012	0.0006	0.0021
			γ	0.1134	0.3093	0.3834	0.1253	0.1058	0.3183
3	13,15	0.3	α	0.5978	0.0815	0.3193	0.5881	0.0868	0.3372
			θ_1	0.0015	0.0186	0.0730	0.0016	0.0009	0.0030
			γ	0.0764	0.1763	0.6911	0.1154	0.1166	0.3661
		0.8	α	0.5617	0.0774	0.3034	0.5542	0.0766	0.2976
			θ_1	0.0012	0.0715	0.2804	0.0015	0.0006	0.0023
			γ	0.1134	0.3301	1.2941	0.0786	0.0754	0.2347

Table 12 presents MLE and Bayesian estimation results for different parameter values across various cases. For each case, it compares the estimated parameters (α, θ_1, and γ) along with their standard errors (SE) and lower confidence intervals (L.CI). The results demonstrate consistency between MLE and Bayesian methods, with slight differences in parameter estimates and associated uncertainties. Notably, Bayesian estimation generally yields lower standard errors and narrower confidence intervals, suggesting higher precision. The variability in estimates and uncertainties appears to be influenced by the parameter m and the grouping Q, reflecting their roles in the estimation process.

Figure 2 describes the existence and uniqueness of the likelihood function, which proves that the solution exists and is unique. Figure 3 shows the MCMC plots for the generated solution during the iteration, which proves the stability of the solution after 10,000 iterations. Also, this figure shows that the solution tends to be normally distributed. Convergence of the MCMC chains was assessed visually using trace plots (Figure 3), which show stable behavior after discarding 2,000 iterations as burn-in.

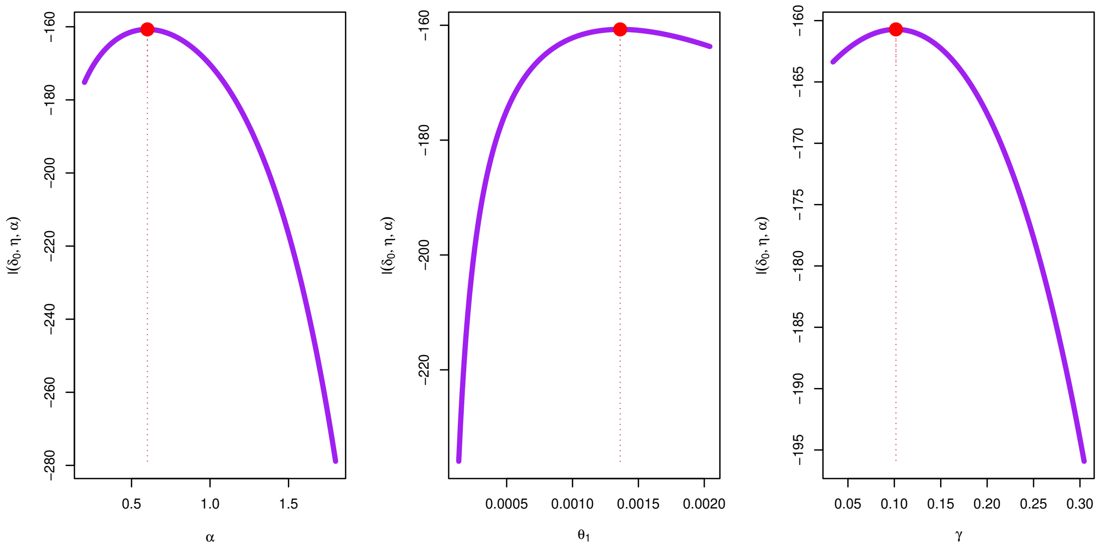

Figure 2. Profile likelihood with P=0.3 and Q=2

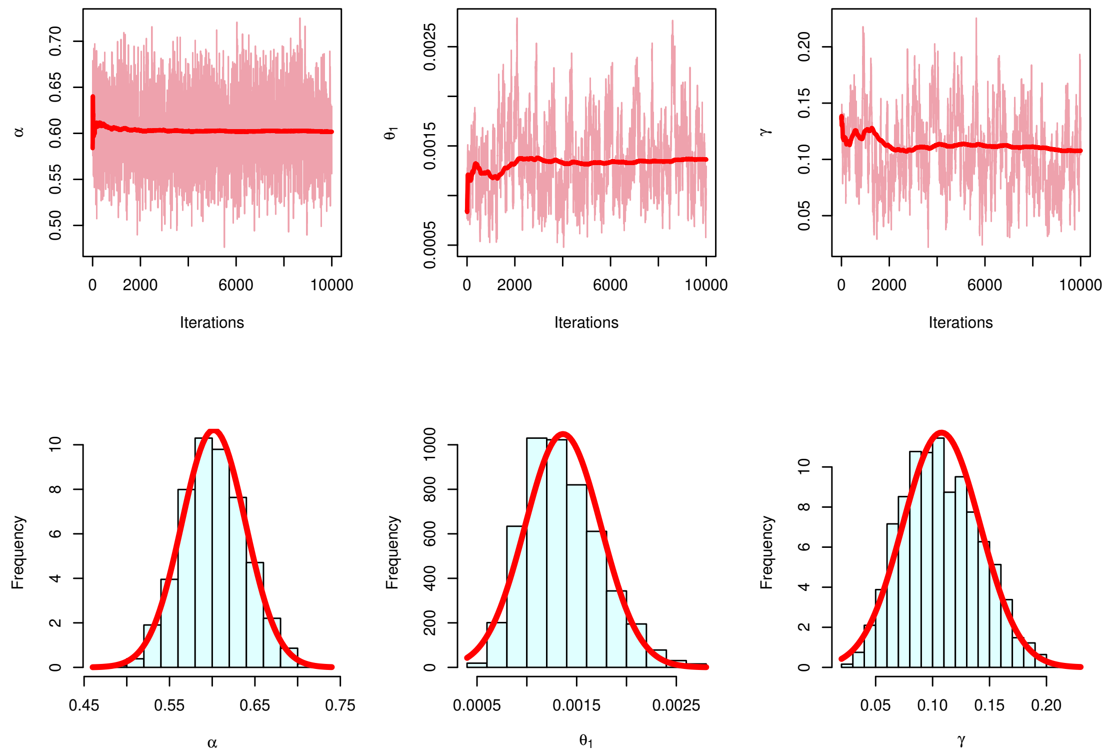

Figure 3. MCMC plots for the estimated values of the parameters from the MH algorithm

6. Conclusion

Under the PSALT conditions, we performed statistical inferences for the OFHLD using data that had been progressively type-II censored. Estimations of the unknown parameters of the distribution were done. These estimates were derived by the use of a variety of methods, including Bayesian and conventional research methodologies. After discovering that Bayesian estimation is superior to the classical estimation method, we concluded that the bootstrap confidence interval is superior to the asymptotic ones in terms of length. We used a real-world data example that was created in a practical manner, and we compared our distribution to new ones under normal and PSALT conditions. By using the AIC and BIC results in Tables 11 and 12, we can conclude that the OFHLD is the best model to fit the data under consideration.

6.1. Future Work

For future work, we will explore several new avenues to enhance the model's applicability and robustness. One potential direction is to develop Bayesian inference methods, which could incorporate prior knowledge and offer more flexibility, particularly in handling complex data structures. Additionally, researchers could investigate alternative estimation techniques, such as machine learning-based methods, to improve parameter estimation accuracy and computational efficiency.

Acknowledgements

The authors would like to thank the anonymous reviewers and the editor for their valuable comments and suggestions on this paper, which lead to great improvements of the presentation. The authors also gratefully acknowledge Umm Al-Qura University, Saudi Arabia, for supporting this research through grant number 26UQU4340101GSSR02.

Funding Statement

This research work was funded by Umm Al-Qura University, Saudi Arabia under grant number: 26UQU4340101GSSR02.

REFERENCES

1. A. M. Abd El-Raheem, *Optimal plans and estimation of constant-stress accelerated life tests for the extension of the exponential distribution under type-I censoring*, Journal of Testing and Evaluation, vol. 47, pp. 3781–3821, 2019.
2. A. M. Abd El-Raheem, *Optimal design of multiple accelerated life testing for generalized half-normal distribution under type-I censoring*, Journal of Computational and Applied Mathematics, vol. 368, article 112539, 2019.
3. A. M. Abd El-Raheem, M. H. Abu-Moussa, M. M. Mohie El-Din, and E. H. Hafez, *Accelerated Life Tests under Pareto-IV Lifetime Distribution: Real Data Application and Simulation Study*, Mathematics, vol. 8, article 1786, 2020.
4. A. M. Abd El-Raheem, E. M. Almetwally, M. S. Mohamed, and E. H. Hafez, *Accelerated life tests for modified Kies exponential lifetime distribution: binomial removal, transformers turn insulation application and numerical results*, AIMS Mathematics, vol. 6, no. 5, pp. 5222–5255, 2021.
5. A. H. Abdel-Hamid, and E. K. AL-Hussaini, *Progressive stress accelerated life tests under finite mixture models*, Metrika, vol. 66, pp. 213–231, 2007.
6. A. H. Abdel-Hamid, and T. A. Abushal, *Inference on progressive-stress model for the exponentiated exponential distribution under type-II progressive hybrid censoring*, Journal of Statistical Computation and Simulation, vol. 85, pp. 1165–1186, 2015.
7. E. M. Almetwally, *Uncertainty in lifetime data quantified with Bayesian adaptive Hamiltonian methods*, Measurement, article 121455, 2026.
8. E. M. Almetwally, *Statistical inference for a one-parameter lifetime model under unified progressive hybrid censoring with binomial random removals*, AIMS Mathematics, vol. 11, no. 1, pp. 211–242, 2026.
9. E. K. AL-Hussaini, A. H. Abdel-Hamid, and A. F. Hashem, *One-sample Bayesian prediction intervals based on progressively type-II censored data from the half-logistic distribution under progressive stress model*, Metrika, vol. 78, pp. 771–783, 2015.

10. H. Alrweili, *Analysis of recent decade rainfall data with new exponential-exponential distribution: Inference and applications*, Alexandria Engineering Journal, vol. 95, pp. 306–320, 2024.
11. H. Alrweili, and E. S. Alotaibi, *Bayesian and non-bayesian estimation of Marshall-Olkin XLindley distribution in presence of censoring, cure fraction, and application on medical data*, Alexandria Engineering Journal, vol. 112, pp. 633–646, 2025.
12. N. Alotaibil, A. S. Al-Moisheer, I. Elbatal, and S. A. Alyami, *Bivariate step-stress accelerated life test for a new three-parameter model under progressive censored schemes with application in medical*, AIMS Mathematics, vol. 9, no. 2, pp. 3521–3558, 2024.
13. R. Alotaibi, E. M. Almetwally, D. Kumar, and H. Rezk, *Optimal test plan of step-stress model of alpha power Weibull lifetimes under progressively type-II censored samples*, Symmetry, vol. 14, no. 9, article 1801, 2022.
14. N. Alotaibi, I. Elbatal, E. M. Almetwally, S. A. Alyami, A. S. Al-Moisheer, and M. Elgarhy, *Bivariate step-stress accelerated life tests for the Kavya–Manoharan exponentiated Weibull model under progressive censoring with applications*, Symmetry, vol. 14, no. 9, article 1791, 2022.
15. N. Balakrishnan, and R. Aggarwala, *Progressive censoring: theory, methods, and applications*, Springer, 2000.
16. N. Balakrishnan, and E. Cramer, *The art of progressive censoring*, Springer, 2014.
17. N. Balakrishnan, and D. Han, *Exact inference for a simple step stress model with competing risks for failure from exponential distribution under type-II censoring*, Journal of Statistical Planning and Inference, vol. 138, pp. 4172–4186, 2008.
18. H. M. Barakat, M. A. Alawady, T. S. Taher, and I. A. Husseiny, *Second-order concomitants of order statistics from bivariate Cambanis family: some information measures – estimation*, Communications in Statistics — Simulation and Computation, vol. 54, no. 8, pp. 3195–3221, 2024.
19. A. A. Bhat, S. P. Ahmad, A. M. Gemeay, A. H. Muse, M. E. Bakr, and O. S. Balogun, *A novel extension of half-logistic distribution with statistical inference, estimation and applications*, Scientific Reports, vol. 14, no. 1, article 4326, 2024.
20. M.-H. Chen, and Q.-M. Shao, *Monte Carlo Estimation of Bayesian Credible and HPD Intervals*, Journal of Computational and Graphical Statistics, vol. 8, no. 1, pp. 69–92, 1999.
21. S. Dey, M. Nassar, and D. Kumar, *Moments and estimation of reduced Kies distribution based on progressive type-II right censored order statistics*, Hacettepe Journal of Mathematics and Statistics, vol. 48, no. 1, pp. 332–350, 2019.
22. S. Dey, S. Singh, Y. M. Tripathi, and A. Asgharzadeh, *Estimation and prediction for a progressively censored generalized inverted exponential distribution*, Statistical Methodology, vol. 32, pp. 185–202, 2016.
23. E. H. Hafez, et al., *Study on Lindley distribution accelerated life tests: Application and numerical simulation*, Symmetry, vol. 12, no. 12, article 2080, 2020.
24. I. A. Husseiny, H. M. Barakat, T. S. Taher, and M. A. Alawady, *Fisher information in order statistics and their concomitants for Cambanis bivariate distribution*, Mathematica Slovaca, vol. 74, no. 2, pp. 501–520, 2024.
25. H. Haj Ahmad, E. M. Almetwally, and D. A. Ramadan, *Competing Risks in Accelerated Life Testing: A Study on Step-Stress Models with Tampered Random Variables*, Axioms, vol. 14, no. 1, article 32, 2025.
26. H. H. Ahmad, D. A. Ramadan, and E. M. Almetwally, *Tampered random variable analysis in step-stress testing: Modeling, inference, and applications*, Mathematics, vol. 12, no. 8, article 1248, 2024.
27. A. A. Ismail, *Inference for a step-stress partially accelerated life test model with an adaptive Type-II progressively hybrid censored data from Weibull distribution*, Journal of Computational and Applied Mathematics, vol. 260, pp. 533–542, 2014.
28. Z. F. Jaheen, H. M. Moustafa, and G. H. Abd El-Monem, *Bayes inference in constant partially accelerated life tests for the generalized exponential distribution with progressive censoring*, Communications in Statistics — Theory and Methods, vol. 43, pp. 2973–2988, 2014.
29. D. Kundu, and A. Joarder, *Analysis of Type-II progressively hybrid censored data*, Computational Statistics & Data Analysis, vol. 50, no. 10, pp. 2509–2528, 2006.
30. A. Kumar Mahto, S. Dey, and Y. Mani Tripathi, *Statistical inference on progressive-stress accelerated life testing for the logistic exponential distribution under progressive type-II censoring*, Quality and Reliability Engineering International, vol. 36, no. 1, pp. 112–124, 2020.
31. A. K. Mahto, Y. M. Tripathi, S. Dey, B. S. O. Alsaedi, M. H. Alhelali, F. M. Alghamdi, A. Alrumayh, and E. Alshawarbeh, *Bayesian estimation and prediction under progressive-stress accelerated life test for a log-logistic model*, Alexandria Engineering Journal, vol. 101, pp. 330–342, 2024.
32. N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, et al., *Equations of state calculations by fast computing machines*, Journal of Chemical Physics, vol. 21, pp. 1087–1092, 1953.
33. M. M. MohieElDin, S. E. Abu-Youssef, N. S. Ali, and A. M. Abd El-Raheem, *Classical and Bayesian inference on progressive-stress accelerated life testing for the extension of the exponential distribution under progressive type-II censoring*, Quality and Reliability Engineering International, vol. 33, no. 8, pp. 2483–2496, 2017.
34. M. M. Mohie El-Din, S. E. Abu-Youssef, N. S. A. Ali, and A. M. Abd El-Raheem, *Optimal plans of constant-stress accelerated life tests for the Lindley distribution*, Journal of Testing and Evaluation, vol. 45, pp. 1463–1475, 2017.
35. M. M. Mohie El-Din, M. M. Amein, A. M. Abd El-Raheem, E. H. Hafez, and F. H. Riad, *Bayesian inference on progressive-stress accelerated life testing for the exponentiated Weibull distribution under progressive type-II censoring*, Journal of Statistical Applications and Probability Letters, vol. 7, pp. 109–126, 2020.
36. S. G. Nassr, T. S. Taher, T. Alballa, and N. M. Elharoun, *Reliability analysis of the Lindley distribution via unified hybrid censoring with applications in medical survival and biological lifetime data*, AIMS Mathematics, vol. 10, no. 6, pp. 14943–14974, 2025.
37. W. B. Nelson, *Accelerated testing: statistical models, test plans, and data analysis*, John Wiley & Sons, 2009.
38. H. K. T. Ng, D. Kundu, and P. S. Chan, *Statistical analysis of exponential lifetimes under an adaptive Type-II progressive censoring scheme*, Naval Research Logistics, vol. 56, no. 8, pp. 687–698, 2009.
39. H. K. T. Ng, P. S. Chan, and N. Balakrishnan, *Optimal progressive censoring plans for the Weibull distribution*, Technometrics, vol. 46, no. 4, pp. 470–481, 2004.
40. H. K. T. Ng, L. Luo, Y. Hu, and F. Duan, *Parameter estimation of three parameter Weibull distribution based on progressively type II censored samples*, Journal of Statistical Computation and Simulation, vol. 82, no. 11, pp. 1661–1678, 2012.

41. M. Z. Raqab, and M. T. Madi, *Inference for the generalized Rayleigh distribution based on progressively censored data*, Journal of Statistical Planning and Inference, vol. 141, no. 10, pp. 3313–3322, 2011.
42. M. K. Rastogi, and Y. M. Tripathi, *Estimating the parameters of a Burr distribution under progressive type II censoring*, Statistical Methodology, vol. 9, no. 3, pp. 381–391, 2012.
43. T. S. Taher, *Comment on "Statistical inference for a step-stress partially-accelerated life test model with an adaptive Type-I progressively hybrid censored data from Weibull distribution"*, Statistical Papers, vol. 67, no. 3, pp. 53, 2026.
44. T. S. Taher, and E. Hussam, *Statistical Inference and Reliability Evaluation of the Perks Distribution under Unified Hybrid Censoring: Applications to Carbon-Fiber Gauge Data*, Computational Journal of Mathematical and Statistical Sciences (Accepted), 2026.
45. S. K. Upadhyay, and A. Gupta, *A Bayes analysis of modified Weibull distribution via Markov chain Monte Carlo simulation*, Journal of Statistical Computation and Simulation, vol. 80, no. 3, pp. 241–254, 2010.
46. S. F. Wu, C. C. Wu, C. H. Chou, and H. M. Lin, *Statistical inferences of a two-parameter distribution with the bathtub shape based on progressive censored sample*, Journal of Statistical Computation and Simulation, vol. 81, no. 3, pp. 315–329, 2011.
47. Y. Zhu, *Optimal design and equivalency of accelerated life testing plans*, Doctoral dissertation, Rutgers The State University of New Jersey, 2010.