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Predicting Sunspot Numbers Based on Neutrosophic Time Series

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Abstract Neutrosophic Time Series applies Neutrosophic principles for resolving forecasting problems. It can be applied to analyze ambiguous or imprecise data that are difficult to process using conventional methods. 1D Convolutional Long Short-Term Memory Network (Conv1D-LSTM) is a hybrid of two approaches that are frequently employed to manage time series data and produce an accurate predictive model. The monthly Sunspot number data were used from the WDC-SILSO, Royal Observatory of Belgium, from January 1900 to December 2024. The dataset consists of 125 years, including 1500 months. Data normalization (Min-Max Scaling) was used to uniformly scale sunspot values to enhance model performance and stabilize training for predictive modeling. The data was divided from January 1900 to June 1987 as a training set (70%) and from July 1987 to December 2024 as a test set (30%). A hybrid method was proposed to improve the accuracy of predictions by integrating the outputs of a neutrosophic model and Conv1D-LSTM in a parallel fusion framework. We compared the proposed method with neutrosophic and Conv1D-LSTM. The evaluation metrics highlight that the proposed model outperforms other models across all performance metrics, indicating its superior forecasting ability. It is sufficiently flexible and can be readily extended to other phenomena exhibiting comparable characteristics. The neutrosophic model closely follows the Conv1D-LSTM model, indicating highly competitive results. The highest number of sunspots is predicted for June, July, September, and October 2026.

Keywords Sunspot Numbers, Neutrosophic Time Series, Conv1D-LSTM, Hybrid Neutrosophic-Conv1D-LSTM

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1. Introduction

Forecasting is a methodical framework for predicting future outcomes. It depends on valuable and logical methods, and historical data to achieve accurate predictions across different sectors to support informed decision-making [1], [2]. Technological progress aids in guiding studies through practical and accurate data forecasting to motivate researchers to develop forecasting methods that support advances in scientific knowledge and innovation to enhance progress [3], [4].

Traditional statistical methods may be confronted with challenges when the data is nonlinear, latent, or noisy [5], [6]. Therefore, researchers are increasingly using Machine Learning methods (ML) and fuzzy logic for predictions due to their crucial for enhancing forecasting accuracy, and the ability to handle massive amounts of data and identify patterns and nonlinear relationships, or when data is uncertain or contains incomplete information that may not be apparent using traditional methods [7]. Furthermore, they automate parameter optimization, minimizing the reliance on manual parameter adjustments and trial-and-error, and boost forecasting effectiveness, making it more feasible for practical implementation [8], [9].

These forecasting methods can be applied to predict the monthly average sunspot count based on historical data and astronomical parameters. The sun is a vital source of energy. Sunspots have been observed for hundreds of years and studied by numerous researchers. Sunspots are relatively lower-temperature regions with a magnetic structure

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that appears as dark areas on the solar disk [10], [11]. Each sunspot characterizes a dusky core named the umbra and a less dark halo called the penumbra [10]. Solar activity and its consequences strongly affect humans, other living organisms, the earth's climate, space navigation, air traffic on polar routes, high-frequency radio communications, radars, electric power grids, and oil pipelines, all of which are examples that many researchers have investigated [12], [13].

The number of sunspots fluctuates over time, increasing and decreasing at about 11 years, designated as the Sunspot cycle [14]. Still, it is a 22-year cycle in the solar magnetic field, as sunspots exhibit the same hemispherical magnetic polarity in alternating 11-year cycles [10]. Accurate forecasting of sunspot numbers is crucial for enhancing the ionosphere model, navigation, managing other applications, ensuring ongoing communication functionality, and understanding the sun's behaviour [15].

Researchers have been motivated to keep pace with rapid and advanced technological developments and the accompanying information revolution to meet the requirements of scientific research in developing new knowledge that enables it to explain various phenomena [16].

Predicting sunspots is a big challenge because it represents Irregular fluctuations. Successful sunspot prediction model using statistical or machine learning methods means it can be generalized to similar phenomena that have complex and irregular temporal dynamics. The development of modern algorithmic methods has improved prediction accuracy despite the presence of nonlinearity, complex patterns, random noise, large data size, and irregular trends [17]. Complementing modern methods of time series analysis and forecasting, neutrosophic can be applied to analyze ambiguous or imprecise data that are difficult to process using conventional methods [18]. The neutrosophic set can efficiently handle incomplete, inconsistent, uncertain, and indeterminate information precisely. Researchers are gaining increased attention to the Neutrosophic set because of its unique ability to solve uncertain situations in various fields [19], [20]. Guan et al., introduced neutrosophic sets to forecast the stock market [21]. They proposed an Innovative forecasting model using neutrosophic theory and fuzzy logic, dependent on the relationships among current values and historical time series datasets of the Taiwan Stock Exchange capitalization-weighted stock index and the Shanghai Stock Exchange composite index. Hashim et al. applied the neutrosophic concepts to daily challenges associated with the HOPE foundation and intended to construct a children's hospital in southwest Missouri [22]. Abdel-Basset et al. introduce the Neutrosophic TOPSIS for selecting a sugar-analyzing device for diabetes people, and it is applied to compare seven smart medical devices [23]. In 2020, Hashim et al. applied neutrosophic concepts to select the most effective treatment for some diseases [24]. Karaşan and Kahraman conducted a method based on neutrosophic concepts to choose the most appropriate renewable energy alternatives for a county municipality [25]. Sudha et al. proposed using neutrosophic concepts in decision-making circumstances on student interests to select the optimal subject combinations for the best academic performance [26]. Neutrosophic time series analysis seeks to find patterns and rules depending on time [27].Many studies have addressed Neutrosophic time series in estimating, predicting, and forecasting time series data. Cruzaty et al. proposed an approach using the Neutrosophic Statistics technique to predict taxes in Ecuador's monthly income for 2019 [28]. Salama et al. proposed a linear model-based neutrosophic time series for patient data analysis to diagnose coronavirus [29]. Veeramani et al. measure the relative importance of the financial ratios of two groups using the decision-making trial and evaluation laboratory approach under the neutrosophic environment [30]. Usmanova analyzes Uzbekistan's economic growth and fiscal policy impact on the poverty rate. The author used the neutrosophic method with the autoregressive distributed lags and vector autoregression models [31]. Edalatpanah et al. proposed a hybrid time series forecasting method based on neutrosophic logic to improve prediction accuracy and enhance performance in uncertain and complex environments [8]. Mahdi et al. proposed a hybrid neutrosophic deep learning model for better recognition of Arabic handwriting. The authors incorporate neutrosophic sets into a hybrid deep learning framework, employing a hybrid approach of CNNs and Bidirectional RNNs [32]. Abdel-Monem et al. integrated the neutrosophic set with the technique for order preference by similarity to ideal solution method (TOPSIS) and the multi-criteria decision-making model for addressing imprecise data and prioritizing ten fuel alternatives under different loads [33]. Mustapha et al. proposed a hybrid weighted similarity measure using neutrosophic sets (NS) for the analysis of symptoms and diseases in patients [34]. Paraskevas, A. and Madas, M., proposed a novel methodology incorporating single-valued neutrosophic sets with Dempster–Shafer theory to enhance uncertainty assessment and similarity in multi-criteria decision-making [35].

The emphasis on sunspot activity stems mainly from their substantial and immediate effects on space weather and scientific studies of the Sun. Sunspots exhibit periodic patterns and fluctuations, especially following an 11-year solar cycle. These patterns make the analysis valuable for further exploration. But still, they can be difficult to predict due to their nonlinear behavior, contain large fluctuations, and unseen factors. Successful sunspot analysis and improved forecasting methodologies can contribute substantially to understanding solar activity behavior, which positively impacts a wide range of similar applications. The temporal analysis of sunspots using machine learning techniques and advanced statistical models not only aids in forecasting but is also a tool for improving energy infrastructure management and understanding space operations [36].
This study aims to compare three methods, which are neutrosophic time series, Conv1DLSTM, and a hybrid neutrosophic-Conv1D-LSTM model, and predict the monthly mean sunspot number series to December 2028. The neutrosophic time series is capable of robustly handling incomplete, inconsistent, uncertain, and indeterminate information. Conversely, the sequence's local patterns and long-term dependencies can be captured by the Conv1D-LSTM hybrid model. A hybrid method was proposed to improve the accuracy of predictions by integrating the outputs of a neutrosophic model and Conv1D-LSTM in a parallel fusion framework.

2. Data Description and Preprocessing

In this study, we used the monthly Sunspot number data from the WDC-SILSO, Royal Observatory of Belgium, Brussels, DOI: https://doi.org/10.24414/qnza-ac80 from January 1900 to December 2024 (Figure 1). The dataset consists of 125 years, including 1500 months. Data normalization (Min-Max Scaling) was used to uniformly scale sunspot values to enhance model performance and stabilize training for predictive modeling. The data was divided from January 1900 to June 1987 as a training set (70%) and from July 1987 to December 2024 as a test set (30%). Table 1 shows the descriptive statistics on sunspots, which are 1,500 observations. Its range between values is 359.4, with a mean of 85.67 and a standard deviation of 70.65, indicating a large dispersion around the mean. The variance is 4991.52, and the coefficient of variation is 82.47, reflecting a high dispersion. The standard error of 1.82 demonstrates the precision of the estimate.

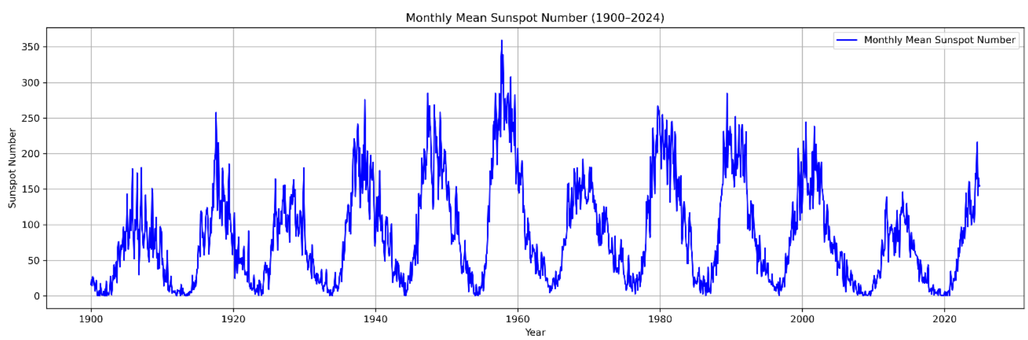

Figure 1. Time series of the monthly mean sunspot number (Jan 1900–Dec 2024).

Table 1. Descriptive Analysis of the Dataset

N	**Mean**	**SE Mean**	**StDev**	**Variance**	**CoefVar**	**Minimum**	**Maximum**
1500	85.67	1.82	70.65	4991.52	82.47	0	359.40
Q1	**Median**	**Q3**	**Range**	**Skewness**	**Kurtosis**		
24.33	70.65	131.85	359.40	0.82	-0.04		

Figure 2 represents a graph, with the data represented on the horizontal (x) axis and the probability values represented on the vertical axis. The values gradually decrease from left to right, with values eventually disappearing. The blue bars represent the actual distribution of the data, while the curve represents the expected probability distribution according to the Wakeby distribution. The graph shows an approximate fit between the data and the probability distribution. The results of the quality-of-fit test showed that the Wakeby distribution was the most suitable for representing the sunspots data, with the lowest deviation value of Kolmogorov-Smirnov being 0.0148, thus outperforming traditional probability distributions. The Wakeby is particularly suitable for describing phenomena producing heavy tails.

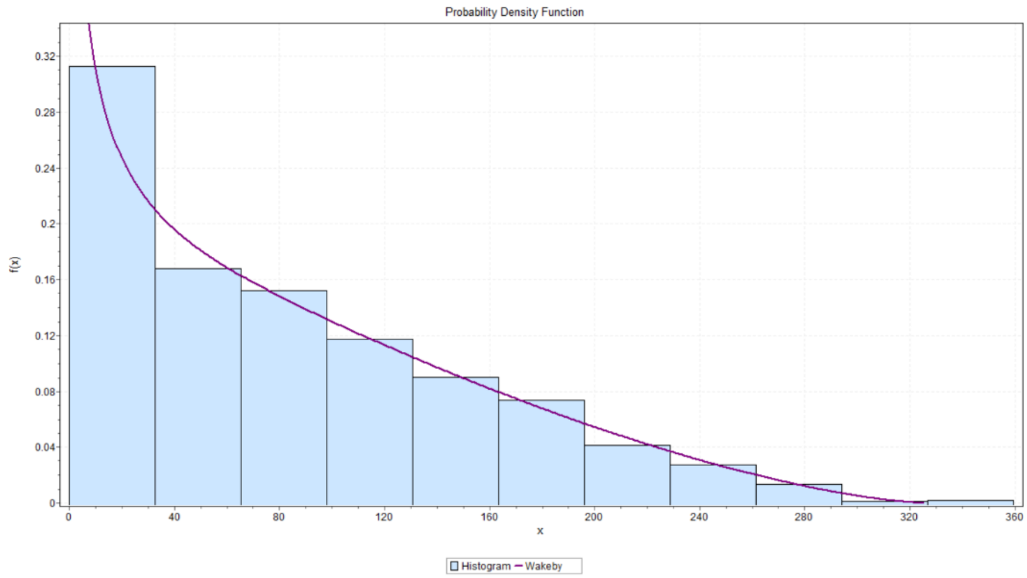

Figure 2. Histogram of the sunspots data according to a Wakeby distribution.

3. Research Methods

3.1. Fuzzy time series

Events around us are often uncertain [37]. It's an inherent and inevitable feature of real-world applications, and it's crucial to account for this when constructing models of real-world systems [38]. Although conventional techniques eliminate uncertainty effects in the real world, their applicability is limited. Therefore, Professionals in the field have extensively utilized fuzzy sets to resolve uncertainty-related problems. Fuzzy sets provide a way to deal with uncertainty, making them highly capable of tackling uncertainty-related issues by allowing for gradual membership. Fuzzy time series is a forecasting method that has been developed so far. It uses fuzzy logic, which was first introduced by Song and Chissom [39]. It has gained popularity for its ability to predict dynamic and nonlinear phenomena in a variety of areas where conventional time series models are no longer applicable [40], [41].

3.2. Neutrosophic Time Series

Classical approaches handle randomness [42], whereas fuzzy methods address vagueness. The Neutrosophic sets model explicitly defines three independent components: truth (T), indeterminacy (I), and falsity (F). Sunspot data show various sources of uncertainty, such as observational differences among observatories, errors in early record reconstruction, and cycle irregularities. Neutrosophic modeling explicitly captures measurement indeterminacy, which traditional fuzzy sets cannot fully capture, as they model only membership and non-membership. Neutrosophic Statistics is more inclusive than interval statistics (which only deal with indeterminacy that intervals can represent). It can handle all types of indeterminacies, it permits the reduction of indeterminacy, and it employs the neutrosophic probability [43]. Data in neutrosophic statistics, in contrast to classical statistics, are interval values/indeterminate values/neutrosophic numbers, which contain indeterminacy [44]. The Neutrosophic Time Series uses neutrosophic concepts for solving forecasting problems [45]. In this section, we use a neutrosophic time series to forecast sunspot numbers, depending on the method proposed by [46].

1. From all available data, select the smallest (D_s) and largest value (D_l).
2. By experts, assign in the problem domain two proper positive numbers D_1 and D_2, such that the range of the universe of discourse is lower than the designated value D_s or higher than the designated value D_l.
3. Calculate the differences between consecutive elements of the data array:

$$\text{diffs} = [d_2 - d_1,\ d_3 - d_2,\ \ldots,\ d_n - d_{n-1}]$$

4. Calculate the mean of the absolute differences between consecutive data points:

$$\text{mean\_diff} = \frac{1}{n-1}\sum_{i=1}^{n-1} |d_{i+1} - d_i|$$

5. Scale the mean difference by 2:

$$\text{avg\_diff} = \frac{\text{mean\_diff}}{2}$$

6. Determine the base value for the intervals:

$$\text{base} = 10^{\log_{10}(\text{avg\_diff})}$$

7. Calculate the length of the interval (le):

$$le = \text{round}\left(\frac{\text{avg\_diff}}{\text{base}}\right) \times \text{base}$$

8. Calculate the total range of the universe:

$$U = [D_s - D_1,\ D_l + D_2]$$

9. Calculate the number of intervals (m) by dividing the range of the universe by le:

$$m = \frac{U}{le}$$

Define the neutrosophic intervals ((low, mid, up), (T, I, F)). Each interval has two parts. The first part has three characteristics: a Lower bound, a middle value, and an upper bound, which define the range of the interval. The second part contains the truth, indeterminacy, and falsity values for the interval. For a given data point x, compute the degrees of membership based on the position of x in a triangular membership function defined by n_1, n_2, and n_3, which represent the lower, peak, and upper bounds of the triangular membership function. The degrees are scaled by the parameters truth (T), indeterminacy (I), and falsity (F), corresponding to the truth, indeterminacy, and falsity degrees, respectively. Based on a computed score derived from the truth, indeterminacy, and falsity values, assign data points to the most suitable neutrosophic interval. Captures the relationships by creating a data structure between patterns in the time series and the elements that follow them. Generate forecasted values for each point in the neutrosophic time series by leveraging the relationships (NLRGs) between subsequences (patterns) and the subsequent values.

3.3. Time Series Modeling with Conv1D-LSTM

Deep neural networks have been applied in various domains and have demonstrated outstanding performance in various applications, including image classification, audio, text, and graph applications. In recent years, the use of

1D CNN has increased for analyzing time series data, illustrating their broad applicability and ability to propel the development of deep learning. In time series data, the main principle of 1D CNN is to implement a convolution operation within local temporal windows of the input data, allowing the network to recognize temporal relationships and dependencies [47], [48]. Conv1D-LSTM is a combination of two approaches that is commonly applied for handling time series data to construct a precise predictive model. Figure 3 shows the Conv1D-LSTM architecture, which includes 1D convolution and pooling layers as the first part to extract local temporal patterns and capture short-term dependencies in the time series, followed by an activation function (e.g., ReLU, Tanh, or others) to help the network learn complex features. A max-pooling operation performs downsampling in each pooling window by choosing the maximum value to enhance computational efficiency. Flatten Layer follows the MaxPooling1D to prepare the data for LSTM and dense layers. LSTM and dense layers are utilized in the second part to handle the features, employing dropout to avoid overfitting [49].

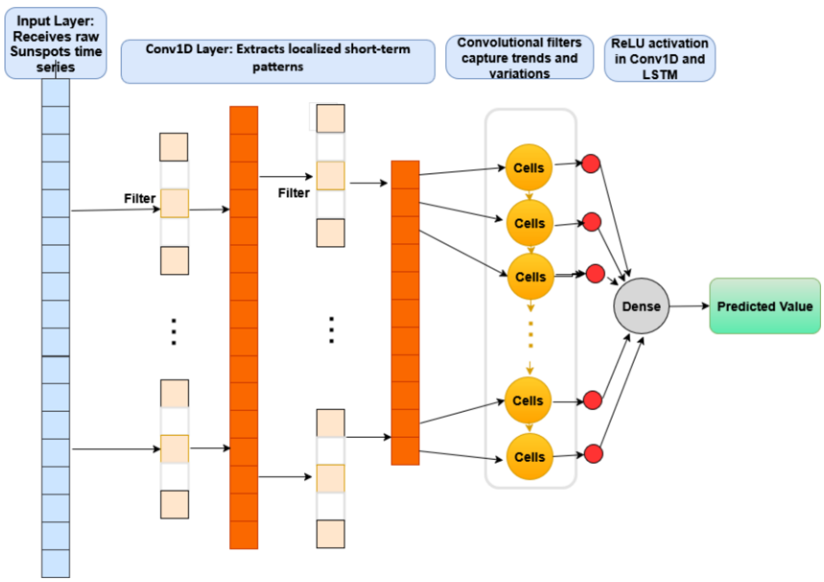

Figure 3. The hybrid Conv1D-LSTM Flowchart.

4. Results and Discussion

The neutrosophic time series model, Conv1D-LSTM model, and a hybrid neutrosophic-Conv1D-LSTM model were used to analyze and predict the time series of the monthly mean sunspot number series to December 2028. A hybrid method was developed by integrating the outputs of a neutrosophic model and Conv1D-LSTM in a parallel fusion framework. Evaluation metrics used for predictive performance of the proposed models include Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Absolute Error (MAE), and the coefficient of determination (R^2). All modeling, training, and evaluation were conducted in Python 3.12.7 using Keras 3.10.0 with TensorFlow 2.19.0 backend.

4.1. Neutrosophic Time Series Model

The data space was expanded using $D_1 = 5$ and $D_2 = 13$, and each period's triangular fuzzy intervals were computed. Several combinations of neutrosophic values (T, I, F) were tested over different ranges $(T \in$

$[0.6, 0.9]$, $I \in [0.05, 0.20]$, $F \in [0.05, 0.20]$) to evaluate their impact on result accuracy and reliability. Results indicated that the combination $T = 0.8$, $I = 0.1$, $F = 0.1$ best matched the historical sunspots data based on metrics relative to the historical mean; thus, it was adopted in the study. These values demonstrate high confidence in the measurements, with low uncertainty, reflecting the reliability of the SILSO data as the main source of solar data.

Next, the data were neutrosophicated into fuzzy intervals by maximizing

$$2 + T - I - F.$$

Prediction was implemented by creating a mapping from past sequences to the next interval index. Figures 4 and 5 show the graphs of the training and testing predictions, respectively. We can see that the two predicted results have a very precise model fit where the predicted values closely follow actual values. The model achieves an RMSE of 18.73, an MAE of 14.65, a Symmetric MAPE (SMAPE) of 27.93%, and an R^2 score of 0.93.

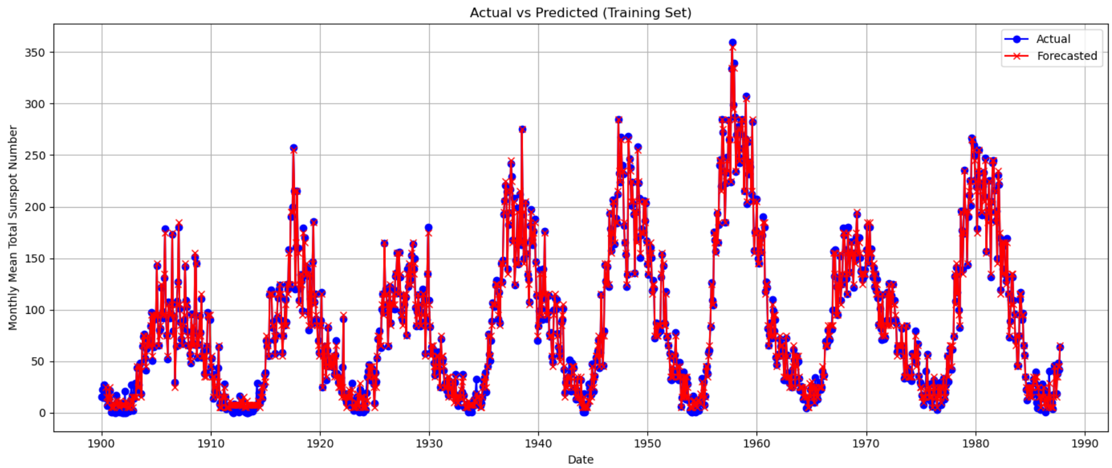

Figure 4. Actual vs. Predicted (January 1900 to June 1987) (Training Set) using Neutrosophic Time Series Model.

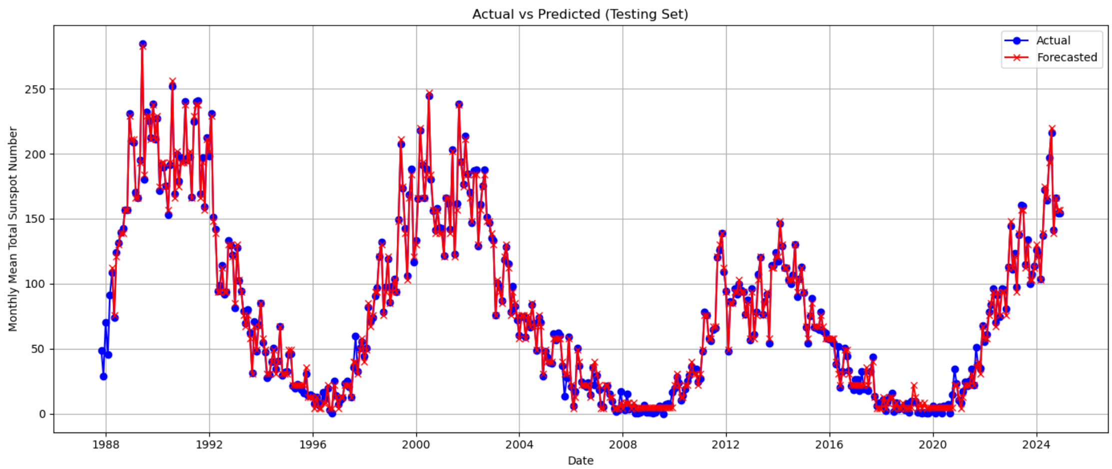

Figure 5. Actual vs. Predicted (July 1987 to Dec. 2024) (Testing Set) using Neutrosophic Time Series Model.

4.2. Hybrid Conv1D-LSTM Model

Hybrid approaches in time series modeling effectively capture the pseudo-periodic nature of sunspots, which exhibit significant fluctuations due to multiple factors. Several architectures were assessed through systematic variation of model parameters. The proposed architecture uses the Keras Sequential API, which features a single input and output branch to create the model. The Conv1D layer was implemented with 64 filters and a kernel size of 2, utilizing the ReLU activation function to perform the initial processing of the input data. To shrink the spatial dimension, the MaxPooling1D layer with a pool size of 2 was used. To recognize time-related dependencies in the data, the LSTM layer with 50 units and the activation function ReLU was applied. To generate the final prediction, the output of the LSTM layer was tied to a Dense layer with a single node. Adam was used as the optimizer, and to compile the model, the loss function was mean squared error. Three dimensions as the input sequences were used to match the requirements of the Conv1D and LSTM layers. 70% of the dataset was trained for the model using 100 epochs with a batch size of 32 samples, and the loss was monitored on the training data with a validation split used. The mean-squared error as loss during training of the model is shown in Figure 6. The figure shows that the model achieves high accuracy during training. The model achieves low RMSE, MAE, a Symmetric MAPE, and a high R^2. Consequently, the Conv1D-LSTM model aligns better with the data behavior and exhibits strong predictive performance. Figures 7 and 8 show the training and testing results, respectively, with the predicted and actual curves closely aligned. RMSE and MAE reflect the magnitude of absolute prediction errors, SMAPE provides a normalized measure of relative prediction error, and R^2 reflects how much of the data's variance is explained by the predictions. All metrics were assessed on the validation data. These metrics were selected to measure both absolute and relative prediction errors, ensuring a comprehensive evaluation of model performance Table 2.

Figure 6. Training and validation loss over 100 epochs.

4.3. Hybrid Neutrosophic-Conv1D-LSTM Time Series Model

Hybrid models enable a thorough evaluation of their performance and potential applications in similar studies due to their high adaptivity during learning, their ability to handle large and heterogeneous datasets, and their management of saturation and variability. They can capture complex relationships, as in sunspot time series. We carried out ablation and validation experiments to evaluate the effectiveness of the proposed hybrid approach. Three configurations were evaluated: (i) a Conv1D–LSTM trained on normalized sunspot data, (ii) an NTS model, and (iii) the proposed hybrid model combining NTS and Conv1D–LSTM. The hybrid model combines outputs from NTS and Conv1D-LSTM in a parallel fusion framework. Moreover, the role of neutrosophic modeling in

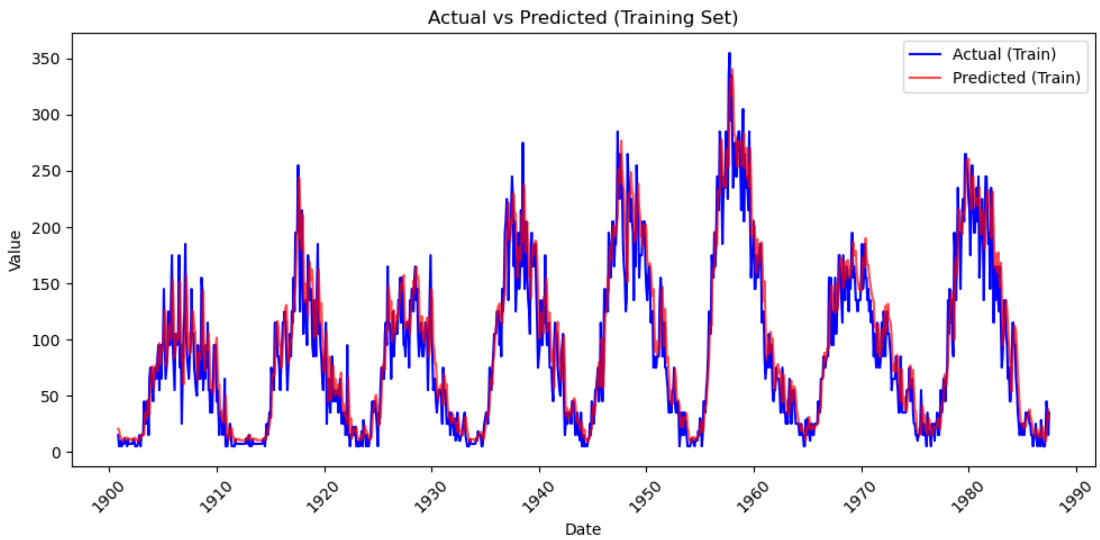

Figure 7. Actual vs. Predicted data for the training set using Hybrid Conv1D-LSTM for the period January 1900 to June 1987.

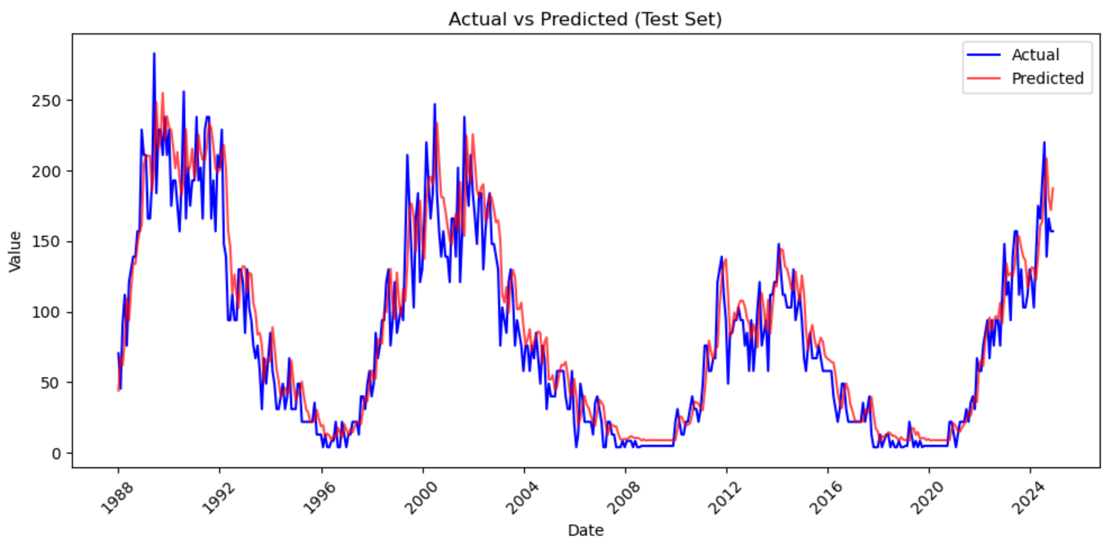

Figure 8. Actual vs. Predicted data for the testing set using Hybrid Conv1D-LSTM for the period July 1987 to Dec. 2024.

addressing uncertainty was assessed by comparing the variability of the original and neutrosophically processed series, which showed component contributes to a smoother and less noisy representation of the series within the hybrid framework. Model performance was evaluated with RMSE, MAE, SMAPE, and R^2 across multiple runs to ensure robustness, and the models' average results were reported (see Figure 9). The hybrid model consistently outperforms the individual models, confirming that the integration of neutrosophic modeling and Conv1D–LSTM contributes to capturing uncertainty and temporal patterns.

The Conv1D layer, where the filters are 64, the kernel size is 2, and the activation is ReLU, is used to identify localized trends in time series. To achieve data reduction, MaxPooling of size 2 was used. Then, an LSTM layer was used to capture long-term temporal dependencies. To predict the next time step, a dense layer is used, where the

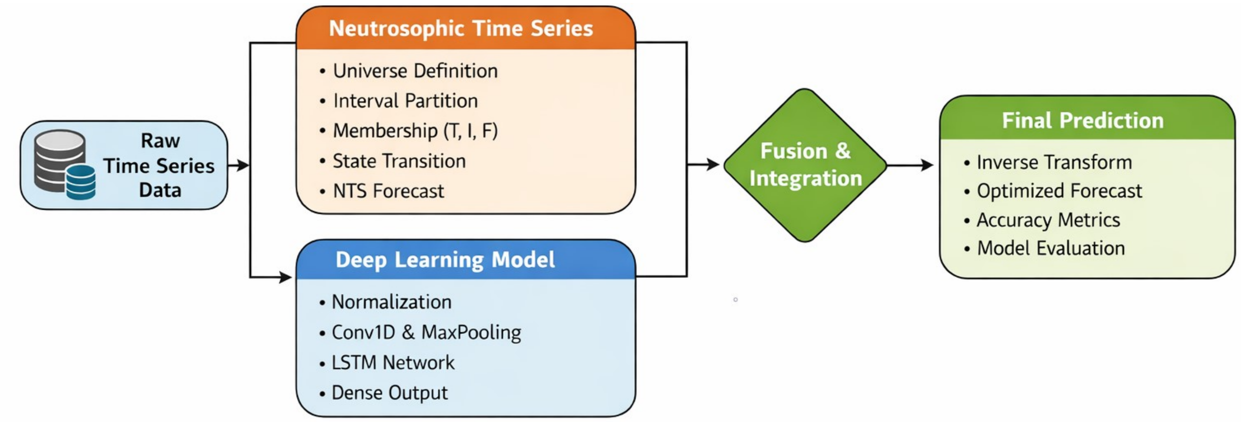

Figure 9. Parallel hybrid Neutrosophic–Conv1D-LSTM framework

number of units is 50, and the activation is relu. The model was trained for 200 epochs, with Adam as the optimizer with a learning rate of 0.001 (Figure 10). The neutrosophic Conv1D-LSTM hybrid model outperforms other models due to its ability to capture modeling uncertainty, local patterns, long-memory, nonlinear dependencies, temporal ordering, and noise resistance. The neutrosophic model is effective at capturing uncertainty, while the Conv1D-LSTM component adapts to nonlinear dependencies in the sunspot data. This integration of strengths permits the neutrosophic Conv1D-LSTM hybrid model to attain the minimum RMSE (17.60) and highest R^2 (0.94), an MAE of 13.52, and a Symmetric MAPE (SMAPE) of 25.89%, outperforming individual models, including the neutrosophic model and the Conv1D-LSTM model, that struggle to address either long-term dependencies or non-linearities. This makes the neutrosophic Conv1D-LSTM hybrid model particularly effective for forecasting tasks where these two traits coexist. Figures 11 and 12 illustrate the predicted values for the training and testing sets, respectively, obtained from the model, where the two curves are very close.

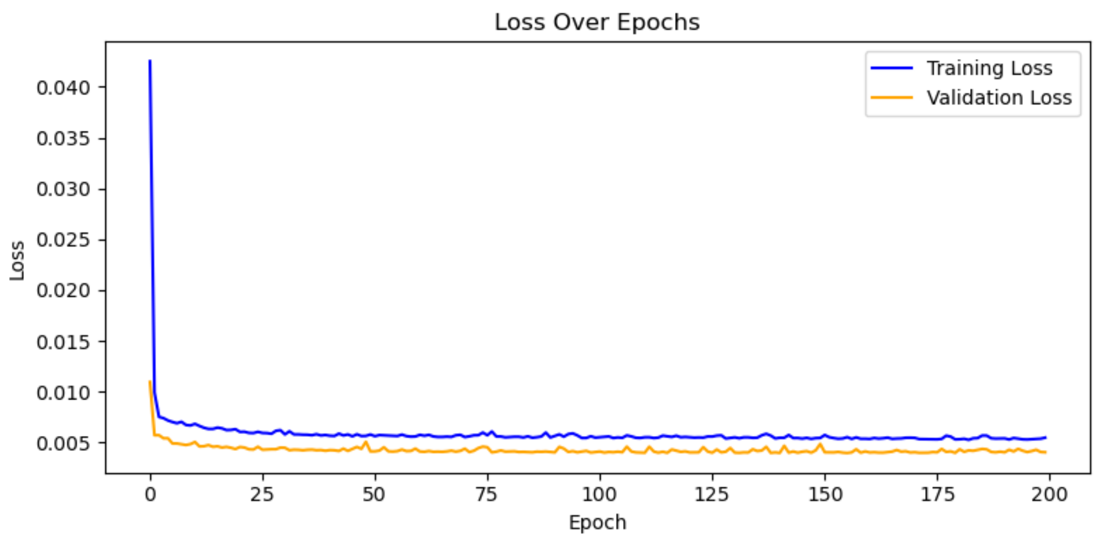

Figure 10. Training and validation loss over 200 epochs.

Table 2 shows the Hybrid Neutrosophic-Conv1D-LSTM Time Series Model, the Hybrid Conv1D-LSTM, and a neutrosophic model. The Hybrid Neutrosophic-Conv1D-LSTM performs best across all metrics, indicating its

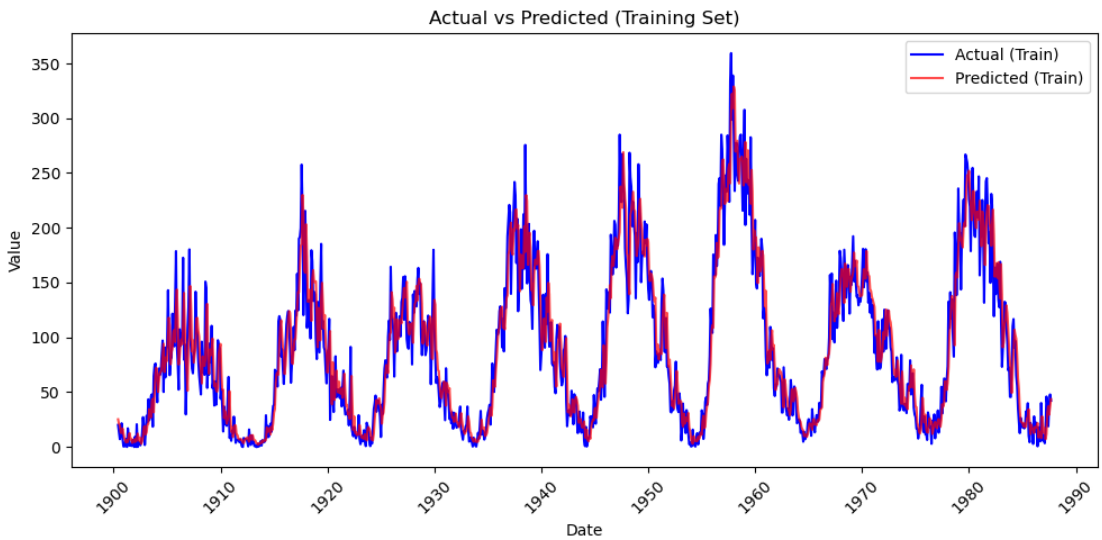

Figure 11. Actual vs. Predicted (January 1900 to June 1987) (Training Set) using Hybrid Neutrosophic-Conv1D-LSTM Mode.

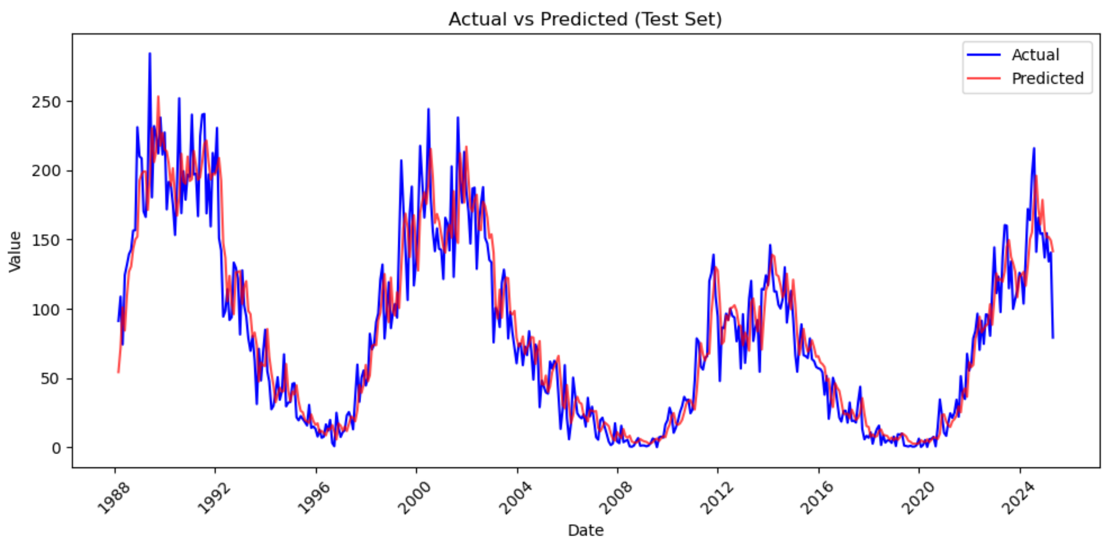

Figure 12. Actual vs. Predicted (July 1987 to Dec. 2024) (Testing Set) using Hybrid Neutrosophic-Conv1D-LSTM Mode.

superior forecasting capability. The neutrosophic model is slightly better than Conv1D-LSTM in all metrics, which means the fuzzy preprocessing adds useful structure. The RMSE and MAE values are numerically close, suggesting that large errors are not skewing the outcome. The evaluation highlights that the neutrosophic model exhibits highly competitive results, closely matching the Hybrid Neutrosophic-Conv1D-LSTM Time Series Model. Figure 13 shows the results of the future forecasting from January 2025 to December 2028. The analysis shows that the peak for sunspots among the forecasted months will be June, July, September, and October of 2026.

In this study, we proposed a hybrid neutrosophic Conv1D-LSTM model. The Neutrosophic aspects focus on the uncertainty management and data reliability analysis, while the Conv1D-LSTM focuses on improving forecasting. Therefore, the hybrid becomes a combination of improved forecasting and uncertainty management. It combines

Table 2. Evaluation Metrics Comparison of Time Series Models

Evaluation Metrics	Hybrid Neutrosophic–Conv1D–LSTM	Neutrosophic Time Series	Conv1D–LSTM
RMSE	17.60	18.73	22.45
MAE	13.52	14.65	16.09
SMAPE (%)	25.89	27.93	38.08

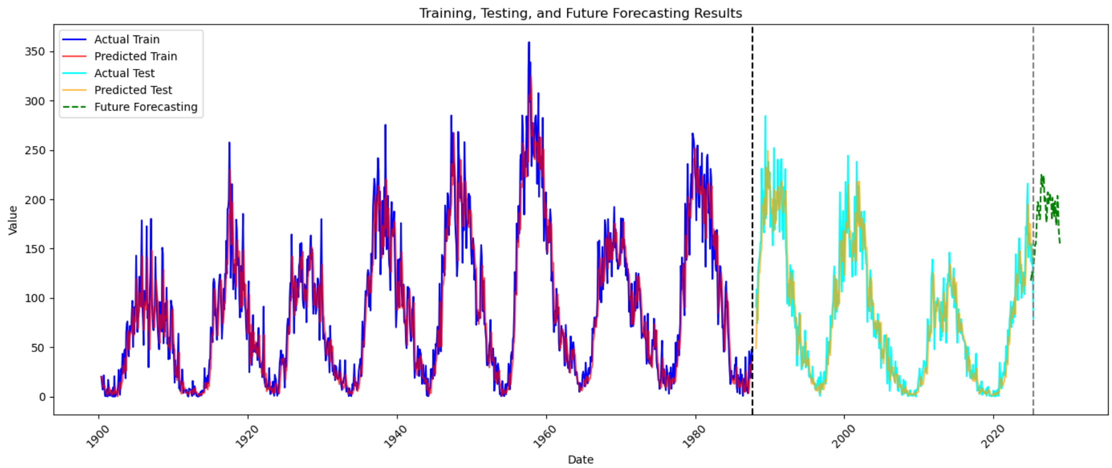

Figure 13. The Hybrid Neutrosophic-Conv1D-LSTM Model for the Training, Testing, and Future Forecasting Results to December 2028.

accuracy and reliability. The hybrid neutrosophic Conv1D-LSTM model was compared with the neutrosophic model and Conv1D-LSTM model to evaluate the performance in modelling and forecasting the mean sunspot number. The results demonstrated that the hybrid neutrosophic Conv1D-LSTM model showed the highest level of predictive performance across the methods assessed, providing higher accuracy and broadly generalizable results. This finding emphasizes the potential of hybrid neutrosophic with non-linear, data-driven methods for prediction tasks such as sunspot activity. Sunspot data are characterized by uncertainty and there is a difference in the number of sunspots between observatories. Therefore, the proposed method is the most suitable for dealing with the uncertainty and noise of the real data. It also protects the model in conditions where the data is incomplete or distorted, and this is a real reality in solar data. It is more reliable, interpretable, and stable than other methods that assume the data is correct, and it does not take into account the sources of error and uncertainty. Previous research has primarily relied on classical, fuzzy or machine learning methods. Many of these articles reported allowable error metrics, and occasionally, their RMSE, MAE, or SMAPE values may be similar to or marginally lower than those found in this analysis. However, one should interpret these comparisons with care because model performance is highly sensitive to the time periods, percentage of training dataset, and percentage of testing dataset. Importantly, the relative performance within the current dataset shows that the Hybrid Neutrosophic-Conv1D-LSTM Time Series Model consistently outperformed both the Neutrosophic model and Conv1D-LSTM Time Series Model, confirming its suitability for capturing the nonlinear interactions and complex patterns of sunspot numbers.

5. Conclusion

We compared neutrosophic, Conv1D-LSTM, and hybrid neutrosophic-Conv1D-LSTM models. The evaluation highlights that the neutrosophic-Conv1D-LSTM Hybrid model outperforms other models in terms of all performance metrics, indicating its superior forecasting capability. The neutrosophic exhibits highly competitive results, closely following the Conv1D-LSTM model. The results show that the months June, July, September, and October of 2026 will be the highest for sunspots among the predicted months.

Contributions

Muzahem, Al-Hashimi — Conceptualization, Data Analysis, Methodology, Software, Drafted the Initial Manuscript; Heyam, Hayawi— Conceptualization, Data curation, Methodology, Project Administration, Validated the Results, Writing–Review and Editing; Mohammed, Alawjar — Investigation, Supervision, Resources, Writing – review & editing.

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