



# A Two-step Fixed-point Iterative Scheme for Solving Generalized Absolute Value Matrix Equations

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**Abstract** This study introduces a numerical approach for solving the Generalized Absolute Value Matrix Equation. The motivation of this work lies in the fact that such equations arise in various applied mathematical and engineering problems, where the presence of the absolute value term makes the system strongly nonlinear and difficult to solve using standard linear algebra techniques. The elementwise absolute value introduces a nonlinearity, which makes standard linear solvers unsuitable. A tailored two-step fixed-point iteration is developed and tested on problems of varying sizes. Numerical experiments demonstrate that, with a suitable relaxation parameter, the method achieves reliable convergence, maintaining robustness and accuracy even for moderately sized problems.

**Keywords** Generalized Absolute Value Matrix Equation; Fixed Point Iteration; Nonlinear Matrix Equation.

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## 1. Introduction

Nonlinear matrix equations involving absolute value operators constitute a distinctive class of problems that have gained increasing attention in both theoretical and applied mathematics. Their importance stems from the fact that such formulations appear naturally in numerous applications, including optimization, numerical linear algebra, control theory, computational mechanics, and various branches of engineering and applied sciences [12, 16, 25]. These problems are often characterized by inherent nonlinearity and nonsmoothness, which poses significant challenges for the design of fast and reliable numerical algorithms. Among the wide variety of such problems, the Generalized Absolute Value Matrix Equation (GAVME) plays a central role. It can be stated in the form

$$AX + B|X| = C, \quad (1)$$

where  $A, B, C \in \mathbb{R}^{n \times n}$  are given coefficient matrices, and  $X \in \mathbb{R}^{n \times n}$  is the matrix to be determined. The notation  $|X|$  denotes the entrywise absolute value of  $X$ , i.e.,  $(|X|)_{ij} = |x_{ij}|$ . The GAVME generalizes the well-known *Absolute Value Equation* (AVE) from the vector setting to the matrix setting, thereby extending its modeling capabilities to a significantly broader range of applications. Problems of this type are encountered, for example, in:

- piecewise linear and nonsmooth optimization,
- linear and nonlinear complementarity problems,
- modeling of contact phenomena with friction and unilateral constraints in structural and mechanical systems,

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- image deblurring, restoration, and noise suppression in signal processing,
- simulation of nonsmooth or hybrid dynamical systems.

The vector-based AVE [19],

$$Ax + B|x| = b,$$

emerges as a special case when the matrices and unknowns reduce to their vector analogues. While the AVE already presents notable computational difficulties, the matrix generalization introduces additional challenges arising from the interaction between matrix structure and the nonsmooth absolute value operation. These factors make the development of efficient solution strategies for GAVMEs a topic of continuing research interest, particularly when the problem size becomes large.

The aim of the present work is to investigate robust and computationally efficient iterative algorithms for solving GAVMEs in both moderate-scale settings. Our focus is on the *Two-Step Fixed-Point Iteration*, an approach designed to leverage contraction properties under suitable conditions to achieve global convergence. We analyze its theoretical properties, establish sufficient conditions ensuring convergence, and validate the method through extensive numerical experiments covering a range of problem sizes.

**Notation.** Throughout this paper,  $\langle x, y \rangle$  denotes the standard inner product in  $\mathbb{R}^n$ , and  $\|x\|$  refers to the Euclidean norm  $\|x\| = \sqrt{x^T x}$ . For a matrix  $X \in \mathbb{R}^{n \times n}$ :

- $|X|$  is the matrix obtained by taking the absolute value of each entry:  $|X| = (|x_{ij}|)$ ,
- the **Frobenius norm** is given by  $\|X\|_F = \sqrt{\sum_{i,j} x_{ij}^2}$ ,
- the sign function  $\text{sign}(X)$  is defined entrywise as

$$(\text{sign}(X))_{ij} = \begin{cases} 1 & \text{if } x_{ij} > 0, \\ 0 & \text{if } x_{ij} = 0, \\ -1 & \text{if } x_{ij} < 0, \end{cases}$$

- for a vector  $x \in \mathbb{R}^n$ ,  $\text{diag}(\text{sign}(x))$  denotes the diagonal matrix whose diagonal entries are the components of  $\text{sign}(x)$ .
- Let  $\mathbf{0}_{n \times n}$  denote the zero matrix of dimension  $n \times n$ .
- Let  $\mathbf{I}_n$  denote the identity matrix of size  $n \times n$ .
- Let

$$\mathbb{S}_+^n := \{X \in \mathbb{R}^{n \times n} \mid X = X^T, x^T X x \geq 0 \text{ for all } x \in \mathbb{R}^n\}.$$

**Structure of the paper.** The remainder of this manuscript is organized as follows. Section 2 introduces the problem formulation, establishes notation, and reviews relevant background on AVEs and their matrix extensions. Section 3 presents the proposed Two-Step Fixed-Point Iteration for the GAVME. Section 4 provides a convergence analysis of the method. Section 5 reports numerical results that illustrate the efficiency, scalability, and accuracy of the proposed approach. Finally, Section 7 summarizes the main contributions and outlines directions for future research.

## 2. Problem Formulation and Preliminaries

The Absolute Value Equation (AVE) in its standard vector form can be expressed as

$$Ax + |x| = b, \quad x, b \in \mathbb{R}^n, \quad A \in \mathbb{R}^{n \times n}, \tag{2}$$

where the absolute value is applied componentwise, i.e.,

$$(|x|)_i = |x_i|, \quad i = 1, \dots, n.$$

This equation is both nonlinear and nonsmooth, with the nonlinearity arising from the absolute value operation. A natural extension of the AVE to the matrix setting leads to the *Generalized Absolute Value Matrix Equation* (GAVME), formulated as

$$AX + B|X| = C, \quad A, B, C, X \in \mathbb{R}^{n \times n},$$

where  $|X|$  denotes the matrix obtained by taking the absolute value of each entry of  $X$ :

$$|X| = (|x_{ij}|).$$

A particular case of the GAVME is obtained when  $B = I_n$ :

$$AX + |X| = C.$$

The GAVME inherits the nonlinear and nonsmooth characteristics of the AVE while introducing additional complexity due to the matrix structure. The problem consists of determining the unknown matrix  $X$  that satisfies the given equation.

### 3. Two-Step Fixed-Point Iteration for GAVME

We reformulate the GAVME (1) as a fixed-point problem:

$$X = -A^{-1}B|X| + A^{-1}C,$$

defining the map  $F(X) := -A^{-1}B|X| + A^{-1}C$ .

#### 3.1. Algorithm: Two-Step Fixed-Point Method for GAVME

Given an initial matrix  $X_0 \in \mathbb{R}^{n \times n}$ , a tolerance  $\varepsilon > 0$ , and a relaxation parameter  $\alpha_k \in (0, 1]$ , for  $k = 0, 1, 2, \dots$ , perform:

**Step 1: Prediction step**

$$Y_{k+1} = -A^{-1}B|X_k| + A^{-1}C.$$

**Step 2: Correction step**

$$X_{k+1}^{\text{corr}} = -A^{-1}B|Y_{k+1}| + A^{-1}C.$$

**Step 3: Relaxation (optional)**

$$X_{k+1} = (1 - \alpha_k)X_k + \alpha_k X_{k+1}^{\text{corr}}.$$

**Step 4: Stopping criterion**

$$\text{Stop if } \|X_{k+1} - X_k\|_F < \varepsilon.$$

### 4. Convergence Analysis

#### Proposition 4.1

Let  $A, B \in \mathbb{R}^{n \times n}$ , and suppose that  $A$  is invertible. Assume that the operator norm of the matrix product  $A^{-1}B$  satisfies

$$\|A^{-1}B\| < 1,$$

for some consistent matrix norm (e.g., spectral).

#### Remark 4.2

In this work, the convergence of the two-step fixed-point iteration is guaranteed under the condition  $\|A^{-1}B\| < 1$ , which also ensures that the solution of the GAVME is unique. Nonetheless, uniqueness can hold under weaker conditions, as indicated in the literature [5, 18, 20]. Moreover, the two-step fixed-point iteration reduces the effective spectral radius of the iteration matrix compared to the single-step method, which contributes to faster convergence and improved stability.

*Lemma 4.3*

Let the mapping  $F : \mathbb{R}^{n \times n} \rightarrow \mathbb{R}^{n \times n}$  be defined by

$$F(X) = -A^{-1}B|X| + A^{-1}C. \tag{3}$$

Under Proposition 4.1, the fixed-point iteration

$$X_{k+1} = F(X_k)$$

converges globally and linearly to the unique fixed point  $X^* \in \mathbb{R}^{n \times n}$ , which is the solution of the Generalized Absolute Value Matrix Equation (GAVME)

$$AX + B|X| = C.$$

*Proof*

We begin by observing that if  $X^*$  satisfies  $F(X^*) = X^*$ , then:

$$X^* = -A^{-1}B|X^*| + A^{-1}C \Rightarrow AX^* + B|X^*| = C,$$

i.e.,  $X^*$  solves the GAVME.

Next, we consider the difference between two iterates:

$$\begin{aligned} \|F(X) - F(Y)\| &= \| -A^{-1}B(|X| - |Y|)\| \leq \|A^{-1}B\| \| |X| - |Y| \| \\ &\leq \| -A^{-1}B(|X| - |Y|)\| \leq \|A^{-1}B\| \|X - Y\|, \end{aligned}$$

By Proposition 4.1, the constant  $\|A^{-1}B\|$  is strictly less than 1. Therefore,  $F$  is a contraction mapping on  $\mathbb{R}^{n \times n}$  endowed with the chosen norm.

The Banach Fixed-Point Theorem then guarantees that the sequence  $(X_k)$  defined by  $X_{k+1} = F(X_k)$  converges linearly to the unique fixed point  $X^*$ , independently of the initial guess  $X_0$ .  $\square$

Now, we present the two-step Fixed-Point algorithm for solving the GAVME given in equation (1), as follows.

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**Two-step Fixed-Point method for solving GAVME**

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**Step 0 (Initialization):**  
 Matrices  $A, B, C \in \mathbb{R}^{n \times n}$ ;  
 an accuracy parameter  $\varepsilon > 0$  ( $\varepsilon = 10^{-6}$ );  
 an initial point  $X^0$  and a relaxation parameter  $\alpha \in (0, 1]$  ;

**Step 1 (Test of convergence):** while  $\|X_{k+1} - X_k\|_F > \varepsilon$  do

**Step 2 (Compute )**  
 $Y_{k+1} \leftarrow -A^{-1}B|X_k| + A^{-1}C$ ;  
 $X_{k+1} \leftarrow -A^{-1}B|Y_{k+1}| + A^{-1}C$ ;  
 $k \leftarrow k + 1$ ;

**Step 3 (Update iterate):** Set  $X_{k+1} \leftarrow (1 - \alpha)X_k + \alpha X_{k+1}$  and go to **Step 1**.

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Figure 1. Algorithm 1

**4.1. On the Selection of the Relaxation Parameter  $\alpha_k$**

To improve the numerical stability and convergence behavior of the fixed-point iteration applied to the Generalized Absolute Value Matrix Equation (GAVME), we introduce a relaxation parameter  $\alpha_k \in (0, 1]$ . The general form of the relaxed iteration is given by

$$X_{k+1} = (1 - \alpha_k)X_k + \alpha_k F(X_k),$$

where the operator  $F$  is defined in (3). The convergence of this iterative scheme is influenced by the spectral characteristics of the matrix product  $A^{-1}B$ . When  $\|A^{-1}B\| < 1$ , the operator  $F$  is contractive under appropriate norms, ensuring global convergence of the iteration.

From a theoretical standpoint, one can derive a suitable value for the relaxation parameter based on the Lipschitz constant  $L$  of the mapping  $F$ . A commonly used and justifiable choice is:

$$\alpha_{Theo} = \frac{1}{1+L}, \quad \text{with } L = \|A^{-1}B\|.$$

This value balances the current approximation  $X_k$  and the computed image  $F(X_k)$ , thus stabilizing the iterative process and avoiding overshooting in cases where the operator is only weakly contractive.

In the particular case where  $\|A^{-1}B\|$  is significantly less than 1, the iteration remains stable without relaxation, and setting  $\alpha_k = 1$  is both theoretically and computationally advantageous.

For practical purposes, the norm  $\|A^{-1}B\|$  can be computed or estimated numerically, and  $\alpha_k$  can be chosen accordingly. A conservative and effective strategy is to use

$$\alpha_{Theo} = \min\left(1, \frac{1}{1+\|A^{-1}B\|}\right),$$

which guarantees convergence while preserving numerical robustness.

## 5. Numerical Experiments

In this section, we present some preliminary numerical results to compare the performance of the fixed-point iteration applied to the Generalized Absolute Value Matrix Equation (GAVME). The implementations are performed by using MatlabR2025a. "Iter" and "CPU" denote the number of iterations and the CPU running time in seconds for this algorithm, we consider the following examples to evaluate and analyze the efficiency and accuracy of the method.

**Problem 1.** Matrices:

$$A = - \begin{bmatrix} 6 & -1 & 0 & 0 & 0 \\ -0.1 & 6 & -1 & 0 & 0 \\ 0 & -0.1 & 6 & -1 & 0 \\ 0 & 0 & -0.1 & 6 & -1 \\ 0 & 0 & 0 & -0.1 & 6 \end{bmatrix}, \quad B = - \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -0.4 & 1 & 0 & 0 & 0 \\ -0.4 & -0.4 & 1 & 0 & 0 \\ -0.4 & 0 & -0.4 & 1 & 0 \\ -0.4 & 0 & 0 & -0.4 & 1 \end{bmatrix} \quad \text{and } C = I_5.$$

Frobenius norm:

$$\|A^{-1}B\|_F = 0.4046 < 1$$

The unique solution  $X^* \in \mathbb{S}_+^5$  is given by:

$$X^* = \begin{bmatrix} 0.1445 & 0.0209 & 0.0030 & 0.0004 & 0.0001 \\ 0.0118 & 0.1461 & 0.0211 & 0.0030 & 0.0004 \\ 0.0106 & 0.0119 & 0.1461 & 0.0211 & 0.0030 \\ 0.0103 & 0.0022 & 0.0107 & 0.1459 & 0.0208 \\ 0.0090 & 0.0014 & 0.0009 & 0.0104 & 0.1443 \end{bmatrix}$$

The numerical results corresponding to this example, using a feasible initial point  $X^0 = I_5$ , are summarized below (Table 1):

Table 1. Computational results for Problem 1 with different sizes of  $\alpha$  and  $\alpha_{Theo}$

$\alpha$	Iter	CPU
$\alpha = 0.3$	40	0.0148
$\alpha = 0.4$	29	0.0096
$\alpha = 0.5$	22	0.0074
$\alpha = \alpha_{Theo}$	11	0.0612

**Problem 2.** Define:

$$A(i, j) = \begin{cases} n & i = j \\ 0.5 & |i - j| = 1 \\ 0 & \text{otherwise} \end{cases}, \quad B(i, j) = \begin{cases} 1 & i = j \\ 0.5 & |i - j| = 2 \\ 0 & \text{otherwise} \end{cases} \text{ and } C = I_n.$$

The unique solution in the case  $n = 5$ :

$$X^* = \begin{bmatrix} 0.1667 & -0.0417 & 0.0000 & 0.0000 & 0.0000 \\ -0.0417 & 0.1667 & -0.0417 & 0.0000 & 0.0000 \\ 0.0000 & -0.0417 & 0.1667 & -0.0417 & 0.0000 \\ 0.0000 & 0.0000 & -0.0417 & 0.1667 & -0.0417 \\ 0.0000 & 0.0000 & 0.0000 & -0.0417 & 0.1667 \end{bmatrix}$$

The numerical results corresponding to this example, using a feasible initial point  $X^0 = I_n$ , are summarized below (Table 2):

Table 2. Computational results for Problem 2 with  $\alpha = 0.5$  and  $\alpha_{Theo}$

Dim	$\alpha = 0.5$		$\alpha = \alpha_{Theo}$	
	Iter	CPU	Iter	CPU
5	22	0.0093	12	0.0219
10	22	0.0167	9	0.0103
50	23	0.0697	6	0.0302
100	24	0.1060	6	0.0649
500	25	2.5543	5	0.5814

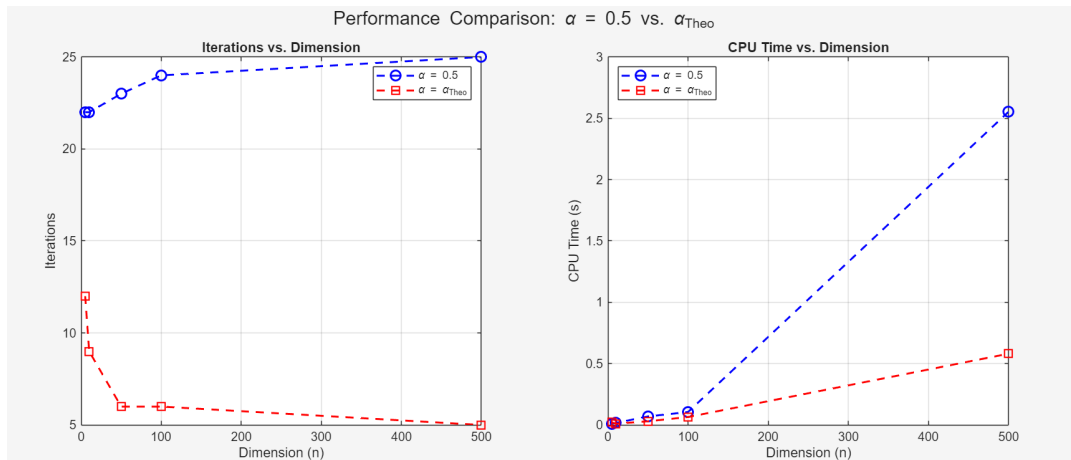


Figure 2. Comparison of the different value of  $\alpha$  for Problem 2.

**Problem 3.** Define:

$$A = \begin{bmatrix} n & 0.5 & 0 & 0 & \cdots & 0 \\ 0.5 & n & 0.5 & 0 & \cdots & 0 \\ 0 & 0.5 & n & 0.5 & \cdots & 0 \\ 0 & 0 & 0.5 & n & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & n \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0 & 0.2 & 0 & \cdots & 0 \\ 0 & 0.5 & 0 & 0.2 & \cdots & 0 \\ 0.2 & 0 & 0.5 & 0 & \cdots & 0 \\ 0 & 0.2 & 0 & 0.5 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0.5 \end{bmatrix}, \text{ and } C = A + B.$$

The numerical results corresponding to this example, using a feasible initial point  $X^0 = \mathbf{0}_{n \times n}$ , are summarized below (Table 3), with the unique solution  $X^* = I_n$ :

Table 3. Computational results for Problem 3 with  $\alpha = 0.5$  and  $\alpha_{Theo}$

Dim	$\alpha = 0.5$		$\alpha = \alpha_{Theo}$	
	Iter	CPU	Iter	CPU
10	22	0.0176	8	0.0110
20	23	0.0264	6	0.0174
60	23	0.0436	5	0.0209
500	25	2.8183	4	0.4537
1000	25	9.1425	4	1.9650

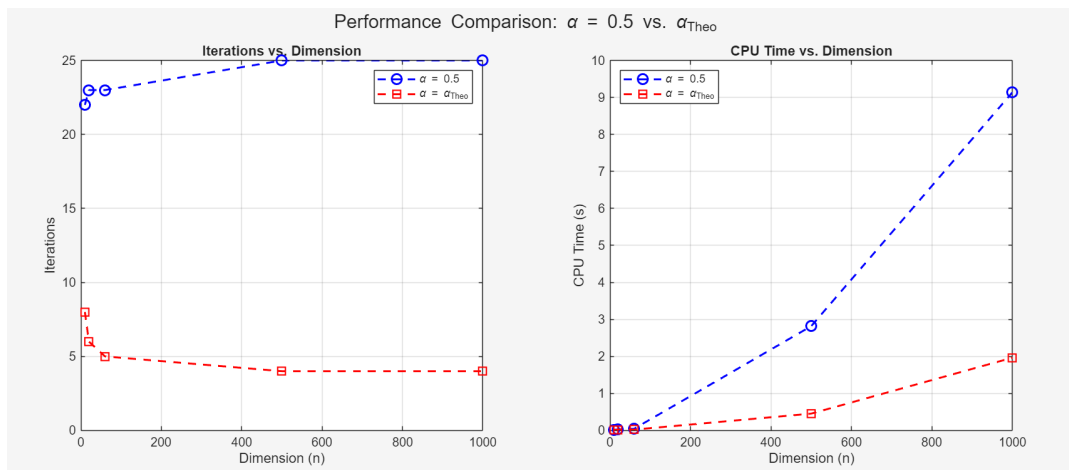


Figure 3. Comparison of the different value of  $\alpha$  for Problem 3.

### 6. Comparison of single-step and two-step fixed-point iterations

In this section, we compare the performance of the single-step relaxed Picard iteration and the two-step fixed-point iterative scheme for solving the Generalized Absolute Value Matrix Equation (GAVME):

$$AX + B|X| = C, \quad A, B, C \in \mathbb{R}^{n \times n}.$$

**6.1. Algorithms**

**Single-step relaxed picard:**

$$X_{k+1} = (1 - \alpha)X_k + \alpha(-A^{-1}B|X_k| + A^{-1}C),$$

with a relaxation parameter  $\alpha \in (0, 1]$ .

**Two-step fixed-point iteration:**

$$\begin{cases} Y_{k+1} = -A^{-1}B|X_k| + A^{-1}C, \\ X_{k+1} = -A^{-1}B|Y_{k+1}| + A^{-1}C, \end{cases}$$

optionally with a relaxation parameter  $\alpha$ :

$$X_{k+1} \leftarrow (1 - \alpha)X_k + \alpha X_{k+1}.$$

**6.2. Numerical Comparison**

Examples 2 and 3 are used to compare the performance of the single-step and two-step fixed-point iterative methods.

Table 4. Iteration counts for single-step and two-step methods for different problem sizes (Examples 2 and 3)

Problem	$n$	Single-Step Picard		Two-Step ( $\alpha = \alpha_{\text{Theo}}$ )	
		Iter	CPU	Iter	CPU
Example 2	5	29	0.0262	12	0.0219
	10	29	0.0291	9	0.0103
	50	30	0.0457	6	0.0302
	100	30	0.0719	6	0.0649
	500	31	1.1355	5	0.5814
Example 3	10	24	0.0289	8	0.0110
	20	25	0.0216	6	0.0174
	60	26	0.0937	5	0.0209
	500	26	1.1020	4	0.4537
	1000	26	4.2003	4	1.9650

**Discussion**

From Table 4, we observe the following:

- The single-step relaxed Picard method generally requires a higher number of iterations to converge compared to the two-step fixed-point method with the theoretical relaxation parameter  $\alpha_{\text{Theo}}$ .
- The two-step method reduces the iteration count significantly, especially for moderate-scale problems (e.g.,  $n = 500$  and  $n = 1000$ ), illustrating the efficiency of the additional intermediate step  $Y_{k+1}$ .

This comparison confirms that the two-step fixed-point iteration is generally superior to the single-step relaxed Picard iteration in both robustness and computational efficiency, particularly for high-dimensional GAVMEs.

*Remark 6.1*

From the computational results in Tables 2,3 and Figures 2,3, it is evident that the choice of the relaxation parameter  $\alpha$  has a significant impact on both the number of iterations and the total CPU time. For all tested problem dimensions, the theoretical parameter  $\alpha_{\text{Theo}}$  consistently reduces the iteration count compared to the fixed value  $\alpha = 0.5$ . In particular, for moderately sized problems, the iteration count decreases from 25 to only 5 in Problem 2 and 4 in Problem 3 when  $\alpha_{\text{Theo}}$  is applied.

## 7. Conclusion

In this work, we addressed the solution of the Generalized Absolute Value Matrix Equation

$$AX + B|X| = C$$

by employing a two-step fixed-point iterative scheme. Numerical experiments demonstrate that the proposed method performs efficiently, particularly for high-dimensional matrices, when an appropriately chosen theoretical relaxation parameter  $\alpha_{\text{Theo}}$  is employed, leading to improved convergence behavior and numerical stability. Future work will focus on enhancing the convergence properties of the method and extending its application to additional classes of matrices, including real-world and randomly generated instances.

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