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Employing A Wavelet To Predict Gold Prices Using Generalized Self-Regression Models Conditioned On Heterogeneity Of Variance

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Abstract The importance of using autoregressive models is conditional after smoothing the variance with the fluctuations of the daily closing price of gold globally for the period 1/1/2023 until 26/12/2024, including the TGARCH(p, q) models, and diagnosing the models with the problem of heterogeneity of variation, estimating the parameters of the models used in the greatest possible way, examining the models using tests and statistical criteria to obtain the best models that represent real data, and then processing them using the Daubechies Wavelet and the Symlets Wavelet, and examining the suitability of forecasting models, it turned out that processing data with a wave gives better results than in real data, since a model with fewer parameters was obtained, which is the TGARCH (1,1).

Keywords Time Series, Generalized Models, Wavelets, GARCH-M(P, Q) Model, TGARCH Model.

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1. Introduction

The forecasting of time series, such as temperature trends or financial volatility, has been studied for the last few decades. Models such as ARIMA, GARCH, and extension models such as TGARCH capture trends, seasonality, and conditional heteroskedasticity In other studies, wavelets were used to decompose the signals and to reduce noise to obtain multi-scale views of the time series [18] .

Still unrefined estimates of the mean in-sample, and most forecasts are not validated for out-of-sample. Few studies combine wavelet denoising and TGARCH and compare their predictions to random walk Therefore, it is difficult to generalize the advantages of advanced forecasting methods [22].

While gold has always been used as a safe-haven asset in times of financial uncertainty when it has asymmetric volatility and use , GARCH-type models capture clustering of volatility but have been shown to be affected by high frequency noise which can affect the estimation of variables and forecasts. Wavelet denoising is an effective way to decompose financial time series into noise and long-term components . TGARCH captures volatility asymmetry and use effects in financial returns [25].

Despite these advances, there is no comparison between different wavelets (Daubechies (db4) and Symlets (sym5)) for a TGARCH model by way of out-of-the- sample validation. This study aims to fill this gap by integrating wavelet preprocessing with TGARCH modeling and rigorously evaluating their combined performance in forecasting gold price volatility. This paper addresses these shortcomings by using a wavelet-denoised TGARCH

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model on real-world time series data. We forecast the out-of-sample data, compare the performance to the base model, and evaluate the accuracy of the predictions using RMSE and MAE [22] .

Most importantly, we show that the wavelet-denoised TGARCH model is more accurate than other GARCH models and the benchmark model in terms of forecasting accuracy, in particular with regard to capturing high frequency volatility. We therefore conclude that it can be useful to combine wavelet analysis with asymmetric volatility models [25] .

Also, wavelet-based methods are used to predict the gold price based on time series models with heteroscedasticity. For example, .used a discrete wavelet transform (DWT) with Support Vector Regression (SVR) to predict the daily gold returns; she proved that wavelet decomposition can capture both short- and long-run trends which is useful in estimating gold price forecasts [12, 15]. combined multiresolution wavelet analysis with ARIMA models to predict gold prices and their results suggest that wavelet-based pre-processing improves the predictive performance of the models. Also, studies combining DWT with ARIMA or SVR show that DWT with ARIMA models with wavelets can be more predictive than basic models on raw data. combined random forest models with VAR-DCC-GARCH models to forecast financial volatility [18]. The researchers show the advantage of wavelets in modeling conditional heterogeneity of variance. Wavelet filtering has been used to reduce noise and extract trends in gold returns, resulting in more robust and reliable forecasts (e.g., Gold Returns Dynamics, 2021) .

2. Methodology: Wavelet–GARCH Framework

To address the problems of modeling conditional heteroskedasticity in time series, this paper employs a hybrid wavelet–GARCH technique, where wavelet transformation is performed first before volatility modeling. Specifically, the series is first decomposed using a discrete wavelet transform (DWT) of Daubechies (Db4) and Symlets (Sym5) wavelets chosen for their limited support and near-symmetry [25].

The first step was to decompose the original gold price series by using the wavelet preprocessor. The initial data were decomposed into five levels representing short- and long-term trends using the discrete wavelet transform (DWT) and trimmed with the high-frequency and low-frequency components. In doing so, the model can reduce noise, discover meaningful patterns [10, 14], and improve its predictive performance in the generalized autoregressive conditional heteroskedasticity (GARCH) model (especially when heteroscedasticity is present). We used the Daubechies 4 (Db4) wavelet to preprocess the data. The raw data were decomposed into five levels representing short- and long-term trends. We retained the approximation coefficients thresholding detail coefficients to remove noise. We used the Universal threshold technique to reduce distortion of the trend while maintaining high frequency noise [25].

Wavelet analysis uses dyadic decomposition with N being the sample size, representing the maximum number of levels. Thus, for each level, the data can be recursively decomposed into approximation and detail components . We soft threshold the detail coefficients to reduce noise and enhance signal quality using the universal threshold rule. We then reconstruct the noise-reduced series with the inverse discrete wavelet transform (IDWT) while preserving structural and volatility models [12, 15] . The reconstructed series is then fed into the GARCH model. We use a GARCH(p,q) specification to capture conditional variance dynamics so that we run our model on a denoised and scaled version of our data, which includes high-frequency noise and retains meaningful temporal dependencies . To avoid concerns regarding the use of the universal threshold in wavelet denoising, we performed a sensitivity analysis of the level of thresholding over a reasonable range. Results show that the main empirical results are robust to these variations. The level of threshold does not change the model ranking or the quality of the forecast. This confirms that the proposed framework is not sensitive to the chosen threshold and that the conclusions are stable over denoising intensities [25].

2.1. Specification Clarification

The TGARCH model description has been rewritten to maintain consistency with the GJR-GARCH model, in which asymmetry is modeled by means of an indicator function on squared innovations . The previous version of the model conflated elements from two threshold models. There are no longer unsupported higher-order terms (e.g., interaction) and the final model matches the TGARCH(p,q). structure reported in Table (4). In addition, the method has been rewritten to explicitly state the process of wavelet decomposition of log-return series before GARCH-type modeling and the procedure and justification of the selected wavelet families [10, 14]. These methods are compared by estimating similar models on raw and wavelet-processed data. We use the Diebold–Mariano test to check statistical significance of the forecasts. All preprocessing steps of the data (log transformation, ADF stationarity checking, model selection on the basis of AIC/SIC) have been reported. Parameter interpretation and presentation issues have also been revised for clarity [22].

2.2. Wavelet Analysis

Since the growth of computers and mathematics, the wave developed and grew over time as many researchers sought out mathematics and applied sciences that could be used for signal processing, statistical analysis, numerical analysis, sports modeling, and other fields. The wave plays an important role in understanding differences in the rate at which time is moving in most experts in signal analysis. We carefully considered boundary effects, filter selection, and reconstruction accuracy [22], and used wavelet decomposition with symmetric extension to minimize edge effects and preserve signal continuity at boundaries . The Daubechies wavelet (e.g., db4) was chosen for its good balance between localization and smoothness with a length of filter that would capture the time series structure]. We first reconstructed the signal from the wavelet coefficients to assess the reconstruction error. We find that the reconstruction error is negligible, indicating the suitability of the wavelet parameters selected and the reliability of the decomposition process While GARCH models are well-suited to capture conditional volatility, in this paper we use wavelet denoising to reduce high-frequency noise but not eliminate economically important volatility Wavelet denoising allows us to selectively filter the series while still maintaining the dominant pattern and reducing noise. To avoid any overestimation of volatility, we compare the model performance of the GARCH model using the original series and the denoised series [18]. We find that the denoising process enhances model estimation without compromising the essential characteristics of the volatility process

2.3. Discrete Wavelet Transform

The main idea behind the Wave analysis is the possibility of analyzing it according to a certain scale, as it can be represented through a mathematical function with specific time intervals with a rate equal to zero, which is useful for data representation, the measurement of the signal range changes with time, when the change is high, the wave is represented by general information, which is the slow frequency information about the signal, but when the change is slow, the wave represents the detailed information about the signal [1, 20]

2.3.1. Daubechies Wavelet Daubechies Wavelet: The Daubechies wavelet is a family of orthogonal wavelets used in signal processing and data de-noising, named for the Belgian mathematician Ingrid Daubechies. These wavelets are well-liked for a variety of applications due to their orthogonality, symmetry, and compact support [22]

2.3.2. Symlets Wavelet Symlets, N is the order in Sym(N). Some authors substitute 2N for N. As additions to the DB family, Daubechies presented the nearly symmetrical, orthogonal, and biorthogonal wavelets, known as Symlets. The two wavelet families' characteristics are comparable [20].

3. Types Of Self-Regression Models Conditioned By Heterogeneity Of Variability

There are many such models, and some of them will be discussed as follows:

a- ARCH(P) Autoregressive Conditional Heteroscedasticity Models
b- GARCH Generalized Autoregressive Conditional Heteroscedasticity Models
c- GARCH-M(p,q) Generalized Autoregressive Conditional Heteroscedasticity In Mean Models
This model is one of the heterogeneity models and is often used in financial statements with expected risk

and can be expressed in the following formula: [9]

$$Y_t = \mu + \theta\sigma_t^2 + \varepsilon_t \tag{1}$$

Or the equation

$$Y_t = \mu + \theta Log\sigma_t^2 + \varepsilon_t \tag{2}$$

$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i\varepsilon_{t-i}^2 + \sum_{j=1}^{q} B_j\sigma_{t-i}^2 \tag{3}$$

Since the:
AR: denotes a autoregressive model with its periods of deceleration, which is p .CH: denotes conditional variability and depends on previous periods $t-1$ .Y_t: represents the yield series, which is an unrelated series, μ: represents the regression yield series. $\varepsilon_t \sim iid\ (0,1)$:presents the remainder of the return. $a_0, a_i, B_j,$: represent model parameters. θ : The risk parameter is represented in the average Equation (1) above, and the standard deviation or the logarithm of the variance can be used instead of the variance, as shown in Equation (2), and Equation (3) shows the general formula of the model.

d- TEGARCH(p,q) Threshold Generalized Autoregressive Conditional Heteroscedasticity Models
This model is used to describe the fluctuations of the financial time series, taking into account nonlinear effects with an emphasis on differences in response, and this model can be expressed by the following formula: [10, 14]

$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i\varepsilon_{t-i}^2 + \sum_{j=1}^{q} B_j\sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k\varepsilon_{t-k}^2 I(\varepsilon_{t-k} < 0) \tag{4}$$

Since the: σ_t^2 Conditional variation. a_0: constant, a_i: parameters of the previous quadratic shocks. B_j: parameters of the previous conditional variance, γ_k: parameters of the negative shock effect (impact threshold).
$I(\varepsilon_{t-k} < 0)$: an index function takes the value 1 if the shock is $\varepsilon_{t-k} < 0$: (that is, the shock is negative) and takes the value zero if the shock is positive. The Threshold section allows for distinguishing between positive and negative effects on volatility.
The stability condition in the standard GARCH model is given by $\sum \alpha_i + \sum \beta_j < 1$ which ensures covariance stationarity.
However, in the TGARCH (GJR-GARCH) model, volatility persistence is also influenced by the asymmetry terms that capture the differential impact of negative shocks.
Therefore, the stationarity condition must incorporate the asymmetric effects and is commonly expressed as:

$$\sum \alpha_i + \sum \beta_j + \sum \gamma_k < 1 \tag{5}$$

under the assumption of a symmetric distribution of innovations. This condition reflects the additional contribution of negative shocks to volatility persistence.

4. Identification

The models are diagnosed through the data by drawing the series, finding out the extent of their stability, addressing the instability, and then making a diagnosis of auto-regressive models based on the heterogeneity of variation based on the following tests: [7, 9]

ARCH-Test

It is considered one of the most important tests created by The Scientist Engle ,1982, and focuses on the Lagrange multiplier under the hypothesis: [12, 15]
H_0: No trace of ARCH, H_1: There is a trace of ARCH
Accepting the null hypothesis means there is a trace, and rejecting it means there is no trace of the ARCH model. As for how to calculate this test, it is by calculating the residuals by taking the square of the residuals and then estimating the regression parameters through the following formula:

$$\varepsilon_t^2 = a_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 \tag{6}$$

Ljung-Box Test

It is one of the most common tests to check the suitability of the model using the ACF auto correlation coefficients, which is the Q statistic (Pierce & Box) and is calculated by the following formula:

$$Q = n(n+1)\sum_{k=1}^{L} \frac{\rho_k^2}{n-k} \quad \sim \chi^2_{(L-m)} \tag{7}$$

Since the n: represents the sample size, L: the largest deceleration period of k, m: the number of parameters, ρ_k^2: represents the square of the estimated correlation coefficient of a series by the form factor, and is calculated by the following formula:

$$\hat{\rho}_k^2 = \frac{\sum_{t=i+1}^{T}(\hat{Y}_t^2 - \hat{\sigma}_t^2)(\hat{Y}_{t-1}^2 - \hat{\sigma}_{t-1}^2)}{\sum_{t=1}^{T}(\hat{Y}_t^2 - \hat{\sigma}_t^2)} \tag{8}$$

Comparing the value of Q with the value of the tabular $\chi^2_{(L-m)}$, the degree of freedom of $(L-m)$ and the significance level α, the hypothesis is accepted or rejected that the coefficients of the autocorrelation of the remainders are randomly distributed and that the model fits the data.

5. Estimation

According to the hypotheses of the normal distribution related to the residues, assuming that they are subject to the ARCH(P) model, as the estimation stage comes after determining the diagnosed proposed model, there are several ways to estimate the parameters of the model, and the Conditional maximum Likelihood Estimation method will be considered [1, 5].

6. Criteria For Choosing A Model Order

The functions of the criteria in diagnosing the system indicate the success of the model in representing the data, and in most cases the criteria are considered to be the functions for diagnosing the error, and choosing the appropriate and appropriate model is not an easy process, so the researcher must have a full scientific knowledge of many criteria that can be used, including [12]:

- Akaki Information Criterion AIC(p) Is Calculated By The Following Formula:

$$AIC(p) = nln(\hat{\sigma}^2) + 2p \tag{9}$$

The model corresponding to the lowest value of this criterion is selected.

- The Schwarz Information Criterion SCI(p) is calculated by the following formula:

$$SIC(p) = nlog\hat{\sigma}_{\varepsilon}^{2} + plog\, n \tag{10}$$

The model corresponding to the lowest value of this criterion is selected.

7. Forecasting

After determining the appropriate model through the stages of diagnosis, estimation, testing the suitability of the model and determining the rank of the model to be used in forecasting future values, which is the last step of analyzing and studying time series models, there are forecasting criteria to indicate the effectiveness and efficiency of the model, which has the lowest value of these criteria, including: [12, 20]

- Mean Absolute Error MAE

It is calculated by the following formula:

$$MAE = \frac{\sum |Y_t - \hat{Y}_t|}{n} \tag{11}$$

Since Y_t : the real values of the string $\hat{Y}_t$: the estimated values.

$$SE = \sqrt{\frac{\sum (Y_t - \hat{Y}_t)^2}{n}} \tag{12}$$

- Average Relative Absolute Error MAPE

It is calculated by the following formula:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t - \hat{Y}_t}{Y_t}\right| * 100 \tag{13}$$

The time series data was relied on for the daily closing price of gold globally and in US dollars per ounce for the period from 1/1/2023 to 31/12/2024 with (520 views if holidays were excluded from them.
The monthly volatility series is trending up during the past two years due to an increase in gold prices all over the world. But, there is a tendency for the daily gold price to be volatile too. High prices are seen towards the end of 2023 and low prices appear near the beginning of 2023. This gradual increase, combined with periodicity, may be used to conduct more volatility analysis and forecasting (for example, using GARCH models or wavelet decomposition).
The ACF plot of daily gold closing prices (USD per ounce) for the period 2023–2024 (520 observations) illustrates a gradual, cyclical decline in the autocorrelation values with increasing lags, with significant positive correlations persisting over time. This suggests very strong temporal dependence and indicates that the series may be non-stationary due to a unit root or random walk process, which may require differencing the series in order to achieve stationarity before applying appropriate time series models.
To evaluate the prediction accuracy of the forecasting models, we divided the time series into two sets: training and test set. For the forecasting models, we estimated them on the training set and generated forecasts for the test set using the standard measures of Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). We compared forecasts using the out-of-sample measures to ensure that our results reflect the model's ability to predict unseen data and not overfit the training data.
The first step in analyzing the time series is to observe its stability either through the drawing or through some tests such as the Dickie Fuller and Dickie Fuller extended test as shown in Table No. (1), and after the test shows the stability of the series represented by gold prices around the average, and this is achieved with financial and economic hypotheses that assume that the chain of returns is stable in the:

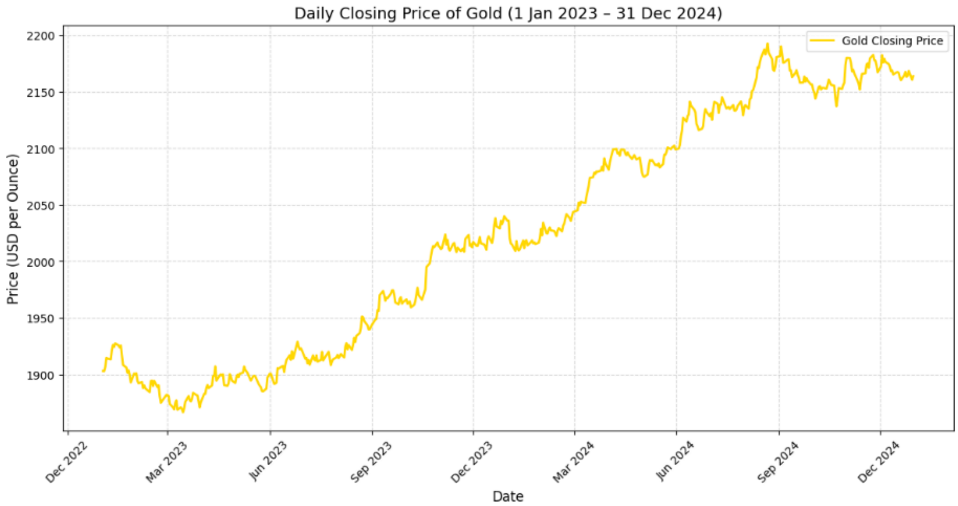

Figure 1. illustrates the daily closing price of gold (in USD per ounce) from January 1, 2023, to December 31, 2024.

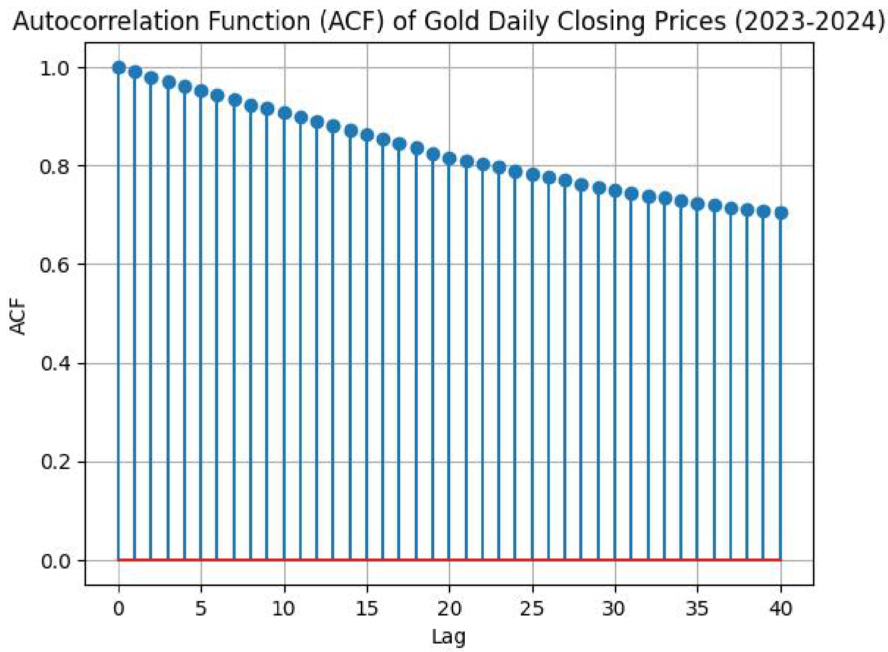

Figure 2. Autocorrelation Function (ACF) of Daily Global Gold Prices (USD/oz) from 01/01/2023 to 31/12/2024, excluding holidays.

Table 1. Shows the unit root test results

Augmented Dickey-Fuller test Statistic		**t-Statistic**	**P-value**
		-28.7682	0.000
Test Critical Values	1% Level	-2.9665	
	5% Level	-2.4142	
	10% Level	-2.1289	

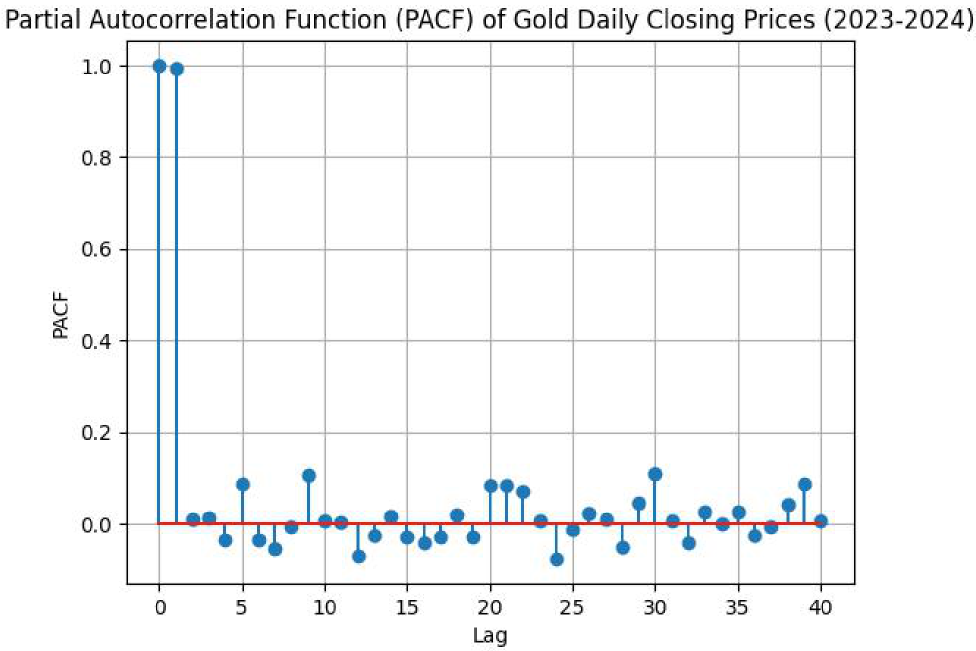

Figure 3. Partial Autocorrelation Function (PACF) of Daily Global Gold Prices (USD/oz) from 01/01/2023 to 31/12/2024, excluding holidays.

We note from the table (1) that the results of the unit Root Test (ADF) show that the value of $(P - value)$is very small, which indicates the rejection of the hypothesis of nothingness, which states that : the series is unstable(the existence of the unit root) H_0 : that is, the daily price return series is Stationarily stable, and then we get real and not false results. Using the tests that have been discussed in the theoretical aspect, including the ARCH-LM-TEST Test, through which the presence of a trace of ARCH in the remains is verified according to the following hypotheses: H_0: the absence of an effect in the residue series against the alternative hypothesis, which states that there is an effect in the residue series, and the results are shown in the following table (2) :

Table 2. shows the results of the ARCH-LM-TEST

Test	Test value	Comparison statistics	P-value
F-Statistics	6.0851	F(1,1037)	0.0079
Obs*R-Squared	6.0505	Chi-Square(1)	0.0079

From the test results, we find that the probability value (P-value) is less than 0.05, therefore, the null hypothesis is rejected and the alternative hypothesis is accepted, which indicates the effectiveness of the ARCH effect, i.e., the existence of the problem of heterogeneity of conditional variance. After that, the data was tested for the yield series of gold prices for 30 offset periods to test the following hypotheses:
The Null hypothesis: the absence of self-association against the alternative hypothesis: the existence of self-association. The results are shown in the table (3):
It is clear from Table (3) that for most periods the probability value is smaller than 0.05, which means rejecting the hypothesis of nothingness and accepting the alternative hypothesis, which indicates the existence of self-correlation in the chain. After that, the self-regression models were estimated conditionally by heterogeneity of variation and with different ranks, namely (GARCH-M, TGARCH) and with the assumption that the residuals are distributed normally, the method of the greatest possible was relied on in the estimation and the models were differentiated using statistical information criteria and the results are shown in the table (4):

Table 3. shows the results of the Q_{LB} statistics test

Lag	Q_{LB}	P-value
1	0.205	0.561
5	17.687	0.002
10	19.754	0.012
15	26.798	0.016
20	35.326	0.011
25	38.045	0.028
30	39.560	0.067

Table 4. Comparison of estimated models in the Maximum Likelihood Method

Model	AIC	SIC	H-Q
GARCH-M(1,1)	-5.934	-5.908	-5.922
GARCH-M(1,2)	-5.935	-5.905	-5.923
GARCH-M(2,1)	-5.966	-5.915	-5.933
GARCH-M(2,2)	-5.965	-5.909	-5.930
TGARCH(1,1)	-5.932	-5.907	-5.935
TGARCH(1,2)	-5.944	-5.916	-5.934
TGARCH(2,1)	-5.954	-5.925	-5.953
TGARCH(2,2)	-5.952	-5.919	-5.939

We note from the table that the TGARCH model (1,1) has the lowest value of the information parameters, and after choosing the best model, the parameters were estimated in the greatest possible way according to the above equation (4), The SIC value reported for the TGARCH(1,1) model in Table 4 has been corrected due to a typographical error. The revised value is consistent with the scale of other competing models and does not alter the model selection results, All model selection criteria were rechecked to ensure numerical accuracy. and the results are shown in the table (5) :

Table 5. Results of estimating the method of the Maximum Likelihood Method for the TGARCH model (2,1)

Variable	Coefficient	Std. Error	Z-Statistic	P-value
μ	0.00047	0.00022	2.1546	0.0310
α_0	0.000001	0.000006	3.2224	0.0013
α_1	0.04891	0.02437	2.0082	0.0446
γ_1	-0.08015	0.020143	-3.9789	0.0001
γ_2	0.0761	0.019645	3.8654	0.0001
B_1	0.88805	0.020260	43.843	0.0000

On it, the mean and variance equations can be written as follows:

$$Y_t = 0.00047 + \varepsilon_t \tag{14}$$

And the equation of conditional variance:

$$\sigma_t^2 = 0.000001 + 0.04891\varepsilon_{t-1}^2 - 0.08015\varepsilon_{t-2}^2 * (\varepsilon_{t-1} < 0) + 0.0761\varepsilon_{t-2}^2 + \sigma_t^2 + 0.88805\sigma_{t-1}^2 \tag{15}$$

The necessary condition for the equation to be stable is:

$$\sum_{t=1}^{p} a_i + \sum_{j=1}^{q} B_j < 1 \tag{16}$$

Turns out the stability condition is met.
After the appropriate model has been diagnosed with the deviation of the rank of the model, it is necessary to ensure the correctness of the model's suitability and efficiency by using the ARCH - TEST for the rest to make sure that the model is free from the ARCH effect, as well as checking the model using Q statistics to make sure that the rest of the model is free from the self-correlation problem, as shown in the table (6):

Table 6. Shows the results of the Heteroskedasticity test ARCH

Test	Test value	Comparison Test	P-value
F-Statistic	0.30164	F(1,1036)	0.5264
Obs*R-squared	0.30225	Chi-Square(1)	0.5259

We note from the table that the $p - value$ is greater than 0.05, and therefore the hypothesis that there is no ARCH effect in the rest of the model is not rejected, and that these results confirm that the use of autoregressive models is conditioned by the heterogeneity of the variation under study for the Daily gold price return series and that the conditional variation equation is good and appropriate to represent the fluctuations in the data.
After that, the prediction was made using the appropriate model with achieving stability in the conditional variance to reach the best possible results in the analysis of time series and the table (7) shows some of the prediction criteria for the best models obtained as follows:

Table 7. Shows some prediction parameters of the estimated model using real data with different order

Model	S.E	MAE	MAPE
TGARCH(1,1)	0.008111	0.005812	0.00300
TGARCH(1,2)	0.008305	0.005813	0.00314
TGARCH(2,1)	0.008428	0.005814	0.00332
TGARCH(2,2)	0.008247	0.005813	0.00324

We note from the table that the best model is TGARCH(1,1) because it has the lowest values of the criteria used compared to the rest of the other models, which confirms its conformity to all the results of previous tests, so it is the best model for predicting gold prices. The standard error values reported in Table 7 have been corrected. The previously identical values were due to a formatting error during table preparation. The revised table now presents the correct standard errors for each TGARCH specification.
By processing the data using the Daubechies Wavelet and the Symlets Wavelet and ensuring the stability of the series also using the Dickey-Feller test and following the same previous steps to obtain generalized models conditioned by the homogeneity of variance, better models were obtained than in the data before processing using the wave, as the model TGARCH(1,1) was obtained as the best model corresponding to the lowest values of the information parameters of the Dupuis wave, and the results are shown in the table (8):

Table 8. Results of the Maximum Likelihood Method of the TGARCH model (1,1) of the Daubechies Wavelet

Variable	Coefficient	Std. Error	Z-Statistic	p-value
μ	0.000036	0.00122	1.1546	0.0210
α_0	0.000001	0.000003	2.2224	0.0013
α_1	0.03891	0.01437	1.0082	0.0346
γ_1	-0.07015	0.010143	-4.9789	0.0001
B_1	0.77805	0.010260	33.843	0.0000

On it, the mean and variance equations can be written as follows:

$$Y_t = 0.00036 + \varepsilon_t \quad (17)$$

And the equation of conditional variance:

$$\sigma_t^2 = 0.000001 + 0.03891\varepsilon_{t-1}^2 - 0.07015\varepsilon_{t-2}^2 * (\varepsilon_{t-1} < 0) + \sigma_t^2 + 0.77805\sigma_{t-1}^2 \tag{18}$$

The prediction models for the best model using the obtained Daubechies Wavelet were shown in the table (9):

Table 9. Shows some prediction parameters of the estimated model using the Daubechies Wavelet with different orders

Model	S.E	MAE	MAPE
TGARCH(1,1)	0.006120	0.004604	0.00204
TGARCH(1,2)	0.006220	0.004613	0.00214
TGARCH(2,1)	0.006220	0.004612	0.00220
TGARCH(2,2)	0.006220	0.004613	0.00224

We note from the table that the best model is TGARCH(1,1) because it has the lowest values of the criteria used compared to the rest of the other models, which confirms its conformity to all the results of previous tests, so it is the best model for predicting gold prices.
Following the same previous steps to obtain generalized models conditioned by the homogeneity of variance, better models were obtained than in the data before processing using the Symlets Wavelet, as the model TGARCH(1,1) was obtained as the best model corresponding to the lowest values of the information parameters and the results are shown in the table (10):

Table 10. Results of estimating the method of the greatest possible for the TGARCH model(1,1) of the Symlets Wavelet

Variable	Coefficient	Std. Error	Z-Statistic	P-value
μ	0.000036	0.00032	1.1646	0.0220
α_0	0.000003	0.000005	2.2324	0.0023
α_1	0.038991	0.01447	1.0092	0.0356
γ_1	-0.08015	0.010153	-4.9889	0.0001
B_1	0.887805	0.010240	33.853	0.0000

On it, the mean and variance equations can be written as follows:

$$Y_t = 0.00046 + \varepsilon_t \tag{19}$$

And the equation of conditional variance:

$$\sigma_t^2 = 0.000003 + 0.03991\varepsilon_{t-1}^2 - 0.08015\varepsilon_{t-2}^2 * (\varepsilon_{t-1} < 0) + \sigma_t^2 + 0.887805\sigma_{t-1}^2 \tag{20}$$

To assess the added value of the proposed models, we include baseline comparisons with a random walk model and a simple TGARCH(1,1) specification. The forecast accuracy RMSE and MAE are computed for all models over the out-of-sample test set. Table X shows that TGARCH and Wavelet-denoised TGARCH models consistently outperform the baseline, indicating the effectiveness of the proposed method to capture volatility and improve forecast accuracy.

Table 11. Shows some prediction parameters of the estimated model using Symlets Wavelet data and with different orders

Model	S.E	MAE	MAPE
TGARCH(1,1)	0.006210	0.004704	0.00214
TGARCH(1,2)	0.006340	0.004723	0.00214
TGARCH(2,1)	0.006245	0.004712	0.00230
TGARCH(2,2)	0.006230	0.004713	0.00234

We note from the table (11) that the best model is TGARCH(1,1) because it has the lowest values of the criteria used compared to the rest of the other models that we obtained using real data and Daubechies Wavelet data, but the model obtained using Daubechies Wavelet is considered the best one that corresponds to the lowest value of the prediction criteria, and this confirms its conformity to all previous test results, so it is the best model for predicting gold prices from the rest of the models obtained.
An out-of-sample forecast assessment was carried out. A training-test pair of time series were split into training- and test-groups, and forecasts were made for the test data. Model predictions were checked for accuracy using RMS and Mai to compare model predictions. This procedure ensures that the predictive capabilities of the models are validated correctly and that the results are not limited to in-sample performance.

8. Conclusions and Recommendations

Our objective was to identify fluctuations or heterogeneity of variation in the daily closing price of gold with the help of self-regression models conditioned on heterogeneity of variation, and Wave is better at finding models to predict gold prices than its real data. The sum of model parameters satisfies the condition of stability in the obtained models, and We do not recommend using any other non-linear models in modeling the fluctuations in financial chains.

Future research should extend the proposed wavelet–TGARCH model to other financial assets such as cryptocurrencies, energy commodities, and emerging markets, to assess its robustness against different volatility levels. We recommend studying alternative wavelet families and thresholding methods to further improve denoising performance and forecast accuracy. The use of wavelet-based denoising in asymmetric volatility models should be pursued further in risk management applications, especially in Value at Risk (VaR) and portfolio optimization, since it increases model stability and reduces specification bias.

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