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Geographically Weighted Regression Analysis of Spatial Heterogeneity for Suicide Mortality

Anwer Matti Abboosh Eeso, Zakariya Yahya Algamal*

Department of Statistics and Informatics, University of Mosul, Iraq

Abstract Suicide mortality has a great spatial variation affected by socioeconomic status such as urbanization, divorce, unemployment, domestic violence, mental illness, and dissatisfaction and should be modeled locally instead of globally. The spatial data are supplied by the application of geographically weighted regression (GWR) model which takes into account local association of variables. However, GWR model has a number of challenges that may affect its effectiveness and reliability. The bandwidth choice is one of these difficulties. When there is an inappropriate bandwidth value, it leads to either fitting the GWR model to values of noise or unexpectedly low values of output. Small bandwidth can perhaps contain excessive local variability and large bandwidth can average away significant local variations. The meta-heuristic algorithms may be explained as the optimization algorithms, which is the solution aimed at designing up the approximate solutions to problems which implies the search through the solution space in the most appropriate manner. The use of meta-heuristic algorithms in the computation of the bandwidth value in GWR model is purely new due to the application of optimization technique in computing the bandwidth value. This paper has used pelican optimization algorithm (POA) as a meta-heuristic algorithm to derive an optimal value of GWR model bandwidth based on the objective function to select the bandwidth (bandwidth minimization). According to the estimation of suicide mortality rate as a real data application, the comparison studies and assessments showed that the proposed method performed better compared to the other methods in terms of R^2 and Deviance. Based on the findings, the meta-heuristic algorithms of estimating the bandwidth value of GWR model is a prospective strategy which embraces a combination of sophisticated optimization strategies alongside with analysis of space.

Keywords Geographically weighted regression, suicide mortality rate, pelican optimization algorithm, bandwidth selectionm, kernel function

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1. Introduction

Suicide is a severe public health issue, and it takes the lives of more than 720,000 people per year across the globe and is the third leading cause of mortality in persons aged 1529. The global mortality rates are about 9-12 deaths per 100,000 population, and 73 percent of suicides are committed by the low and the middle-income countries, where preventive services are of low access. The rate in the United States has increased gradually to 14.1 per 100,000 in 2023 with a disproportionate number of male and other demographics being affected [1, 2].

Suicide numbers have decreased slowly in developed countries and continued to rise in developing ones since the 1990s and with 746,000 deaths reported in 2021 alone. These variations are due to factors like socioeconomic stressors, mental health disorders, and availability of methods with firearms and hanging taking the largest portion of completions. The burden is 8090 percent of low and middle-income countries, which is often attributed to poor psychiatric healthcare [3].

Spatial data could be defined as accumulated phenomena with locational referent of inherency or declaration. It also entails spatial data content and spatial data entities. The data that can be spatially spread is the one that is

*Correspondence to: Zakariya Y. Algamal (Email: zakariya.algamal@uomosul.edu.iq). Department of Statistics and Informatics, University of Mosul, Iraq.

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recorded and is called content data whereas the data that remain constant over space but vary with time are called change data. The content data are related with spatial data objects that are points, lines, areas, or surfaces.

Geographically modeling is a method which will give a lot of insight in the spatial analysis when the relationship between variables vary in different locations [4, 5]. This approach may be applied to approach the issue of how various phenomena vary in various geographical positions. One of the largest spheres of geographically modeling is the spatial regression models. The spatial regression models will be most appropriate to the researcher who is attempting to spatially condition a dependent variable or independent variable or attempting to model a spatial data set in which there is too much spatial autocorrelation that does not satisfy the conditions of classical regression analysis [6, 7]. Some of the disciplines and areas that use spatial regression models include economic, urban and regional planning, environmental conservation and health among others [8, 9, 10, 11, 12]. Geographically weighted regression (GWR) model is a type of spatial regression models which is used when the response variable is following the normal distribution [13, 32, 33, 34, 35].

Nonetheless, it has many aspects that may impact its effectiveness and trustworthiness, which are the issues that are typical of GWR model. One of the major challenges of the GWR model computing is the establishment of the bandwidth value to be used in the computation. Within the recent several years, many natural-inspired algorithms have been reported and applied as the techniques of the random search of the optimal solutions to a range of optimization problems.

The following paper presents the natural-inspired optimization algorithm to approximate the values of the bandwidth in GWR model. Within the framework of our approach, we will be able to choose the most optimal values having a greater predictive value. At the real data application, the effectiveness of our proposed approach is demonstrated to be better when compared to other approaches. The present study addresses the problem of selecting an optimal kernel bandwidth in geographically weighted regression (GWR) for modeling spatial heterogeneity in suicide mortality rates across Iraqi provinces. Specifically, the task is to determine a fixed bandwidth value that minimizes the GWR deviance (and thus maximizes pseudo-R2R2) over a predefined feasible interval, and to assess whether a meta-heuristic pelican optimization algorithm (POA) can outperform conventional AIC, CV, and GCV based bandwidth selection methods in this context.

2. The GWR model

GWR model was initially developed by Brunsdon, Fotheringham and Charlton [13], Brunsdon, Fotheringham and Charlton [14] and subsequently by Brunsdon, Fotheringham and Charlton [14]. GWR is a set of numerous spatial statistical models which aim to weight observations (data) according to the geographical distance to a certain point, so-called regression point. The basic conceptual idea of this model is to model the information of space in order to understand the spatial processes, based on the principle of local likelihood. Another contrast between the GWR model and global statistical models is assumed that differing regression coefficients of variables at location are not allowed,thus in a dataset with spatial heterogeneity, the spatial differences in variables can be statistically recognized.

The weighting of each observation in the data follows concept of decreasing distance whereby the weight of the observations decreases with the distance. This weighting model operates in a kernel capability which tries to down play the weight of distant observations on the point of interest [36, 37, 38, 39]. The GWR technique uses the ordinary least squares (OLS) regression model equation in its simplistic form of development. The only difference is that unlike OLS, GWR can give attention to the geographic characteristics of the collection of data because coordinates of every piece of data are inserted into the equation. To understand the development of GWR model properly, one should consider the regression establishment in general. It is written as follows:

$$y_i = \beta_0 + \sum_k \beta_k x_{ik} + \varepsilon_i \tag{1}$$

where $y_i$the dependent variable in location i, and in this study, this is the suitability of the water to be drunk (good or bad). $\beta_0$is the stationary intercept, β_k is approximated coefficients that are used to define the impact of the independent variable x_{ik} on the dependent variable and$\varepsilon_i$unobserved or unmeasured variables that influence y_i with mean 0.

When the classical assumptions of OLS are satisfied, the coefficients of the model may assume the form of a matrix as discussed below:

$$\hat{\beta} = (X'X)^{-1} X'Y \tag{2}$$

where X is a matrix of independent variables and $(X'X)^{-1}$ is the inverse of the variance–covariance matrix.

Similar to this global model, GWR model goes a step further and estimates local coefficients that are not the same in different locations [14]. Unlike global models, which assume that there is a way of fixed relationship between variables everywhere, the GWR model assumes that there is spatial non-stationarity, whereby there is a difference between the relationship between variables in one geographic point and another. This model is based on the geographic coordinates of every point (u_i, v_i) and the equation at this point is modified with the help of the introduction of spatial weights to the observation because it is located at a distance to the target point (regression point). The overall equation of the GWR can be represented in the following way:

$$y_i = \beta_0 (u_i, v_i) + \sum\nolimits_k \beta_k (u_i, v_i) x_{ik} + \varepsilon_i \tag{3}$$

where (u_i, v_i) the geographic coordinates of point i, $\beta_k (u_i, v_i)$is the local regression coefficient for the kth independent variable at location i, put differently, it is the coefficient of in dependent variable(s) at point i. The value of this parameter is usually offered and determined in a smooth surface where some points are used to show the spatial variability of the surface [14].

As indicated in equation (3), local differences in the regression coefficients $(u_i, v_i)$of the independent variables x_{ik} explain the value of the $y_i$at each geographical point (u_i, v_i). Unlike the OLS model that presumes that the coefficients are constant amongst all the locations, the GWR model provides local coefficients of each site within the study area, which includes the sites between the observation locations, thus enabling the development of smooth maps, which depict spatial variation in relationships [15, 40, 41, 42, 43]. In order to do this, GWR expands the matrix expression of OLS model by adding a spatial weight parameter which relies on the geographical closeness of points. Every single location will be assigned its weight based on the distance with the regression point using a distance decay function as proposed by [13, 15].

The model coefficients are locally estimated in a GWR model at each geographic position (u_i, v_i) with the help of a spatial weight matrix $W (u_i, v_i)$that measures the geographic closeness of observation. The local regression coefficients are written as follows:

$$\hat{\beta} (u_i, v_i) = [X^T W (u_i, v_i) X]^{-1} X^T W (u_i, v_i) Y \tag{4}$$

where $\hat{\beta} (u_i, v_i)$the vector of locally estimated coefficients at location i, $W (u_i, v_i)$ is the $n \times n$ spatial weight matrix, whose diagonal elements contain weights representing how close each point is to location i

$$W (u_i, v_i) = \begin{bmatrix} w_{i1} & 0 & \cdots & 0 \\ 0 & w_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{in} \end{bmatrix} \tag{5}$$

By studying with GWR the Eq. (4) will be written as:

$$\hat{\beta} (i) = \left(X^T W_i X \right)^{-1} X^T W_i Y \tag{6}$$

where i is the position in the matrix element, and W_i is a weighted structure, which depends on the proximity of point i to the sampling positions about i (reflecting the change of the weight W with location i).

These weights in GWR model are developed by a kernel function that characterizes the relative location of the data points. Some of the available kernel functions that can be used to weight and were borrowed to formulate the GWR model include box-car, bi-square, tri-cube, exponential, Gaussian among others [16, 17, 18, 19].

3. Bandwidth selection

GWR is used when you suspect spatial nonstationarity in relationships, especially the effect of predictors on the response changes across the study area. Typical domains include environmental processes, epidemiology, real estate, and urban planning, where local context matters, such as pollution–health links, income–education gradients, and housing prices.

In general, the GWR modeling involves estimating for each intersection a regression equation supported by the observations in other intersections. Experience has shown that observations near intersection i will contribute more to the estimation of the parameters of i than would those far from it. This impact reduces as the distance between the two places rises gradually. For estimating the smoothed geographical variations in the parameters with a distance based weighting scheme, GWR model uses a spatial kernel method [20, 30].

The weight could be pre-specified according to the distance or post-specified according to a specified number of neighbours. Similarly, the modeling results of the GWR model play important roles in the choice of the bandwidth [21]. A relatively small bandwidth that only contains a few observations can lead to instability in the fits while a conversely large value for the bandwidth can lead to bias [22].

The bandwidth parameter defines the size of the neighborhood considered by every observation during weighting. Small bandwidth can make it adapt to the local changes whereas big bandwidth may defeat the changes. The adaptive bandwidth techniques are in turn adjustable to the density of the data, this of course implies that the techniques are capable of giving a more detailed model [23, 24, 25, 44, 45, 46]. This choice may be based on either fixed selection, where a fixed value is used on all the observations, or on adaptive selection, where depends on the density of the data, and so is more flexible in picking up the variations on a local scale. Table 1 provides a summary of several functions in the kernel. Just like the version in Table 1, the kernel function gives weights to observations according to their Euclidean distance, d_{ij}, from the regression point being estimated. In addition, the bandwidth, σ, which is representing the number of neighboring data, needs to be determined.

Table 1. Kernel functions in GWR model

Kernel	Mathematical form
Gaussian	$w_{ij} = \exp\left(-\frac{1}{2}\left(\frac{d_{ij}}{\sigma}\right)^2\right)$
Exponential	$w_{ij} = \exp\left(-\frac{\lvert d_{ij}\rvert}{\sigma}\right)$
Bi-square	$w_{ij} = \begin{cases} \left(1-\left(\frac{d_{ij}}{\sigma}\right)^2\right)^2 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$
Tri-cube	$w_{ij} = \begin{cases} \left(1-\left(\frac{d_{ij}}{\sigma}\right)^3\right)^3 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$
Box-car	$w_{ij} = \begin{cases} 1 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$

This choice may be based on either fixed selection, where a fixed value is used on all the observations, or on adaptive selection, where depends on the density of the data, and so is more flexible in picking up the variations on a local scale. Table 1 provides a summary of several functions in the kernel. Just like the version in Table 1, the kernel function gives weights to observations according to their Euclidean distance, σ [24, 26, 31].

Meta-heuristic algorithm are the refined methods of optimization employed to find good solutions to problems which are hard to solve conventionally. These algorithms are widely used where the solution space is large, non-linear, or not very well defined [27]. These algorithms are planned to come out of local optimal and aimed for global optima than local search hence more accurate than local searches. Further, the nature of these algorithms is that they can give good solutions at once especially in the search spaces of high dimensions, which can be hardly solved by the traditional optimization processes [28].

From this point, our proposed idea is to use meta-heuristic algorithms for estimating the bandwidth value in GWR model which can offer a promising alternative to traditional methods. Through the application of these optimization techniques, our proposed idea is able to improve their probability of identifying optimal bandwidths that leads to accurate model representation and presentation. When the nature of spatial data analysis becomes more intricate, the implementation of complex optimization solutions to support modeling may be essential. In this paper, pelican optimization algorithm (POA) [29, 31], which is swarm-based metaheuristic algorithm inspired from the behaviors of beluga whales, is employed to tune the optimal bandwidth value in the GWR model. The following are the parameter combinations for our suggested methodology. In this work, POA is adopted as a representative recent swarm-based metaheuristic to explore the potential of optimization-based bandwidth selection in GWR. POA was chosen because of its balance between exploration and exploitation and its competitive performance reported on diverse benchmark problems, while remaining relatively easy to implement and tune.

1. The number of pelicans are 20 members and the number of iterations is $\mathrm{t}_{\max}= 1000$.
2. Every member's position is representing the bandwidth value of the kernel, σin Table 1 and it chosen at random. The members' starting positions are produced from a uniform distribution in the interval $[7, n]$where $n$represents the number of samples in the real data under the study.
3. The definition of the fitness function is considered as the deviance criterion and it is defined as

$$\text{fitness} = \min D\left(y; \hat{y}\left(\hat{\boldsymbol{\beta}}_{GWR}\right)\right) = 2\sum_{i=1}^{n}\left[y_i \log\left(\frac{y_i}{\hat{y}\left(\hat{\boldsymbol{\beta}}_{GWR}\right)}\right) - y_i + \hat{y}\left(\hat{\boldsymbol{\beta}}_{GWR}\right)\right], \tag{7}$$

4. The best bandwidth value is obtaining after updating the positions according to the of the POA algorithm until $t_{\max}$ is reached.

4. Evaluation criteria

To compare and evaluate our proposed method, POA-GWR, performance with other methods, two criteria for model evaluation were used. The first criterion is the $\mathrm{R}^2$and the second criterion is the Deviance. They are defined as, respectively,

$$\mathrm{R}^2 = 1 - \frac{D\left(y, \hat{y}\left(\hat{\boldsymbol{\beta}}_{GWR}\right)\right)}{D\left(y, \hat{y}\left(\hat{\boldsymbol{\beta}}_{0}\right)\right)} \tag{8}$$

$$\text{Deviance} = 2\sum_{i=1}^{n}\left[y_i \log\left(\frac{y_i}{\hat{y}\left(\hat{\boldsymbol{\beta}}_{GWR}\right)}\right) - y_i + \hat{y}\left(\hat{\boldsymbol{\beta}}_{GWR}\right)\right] \tag{9}$$

where $D\left(y, \hat{y}\left(\hat{\boldsymbol{\beta}}_{GWR}\right)\right)$ is the deviance of the fitted GWR model and $D\left(y, \hat{y}\left(\hat{\boldsymbol{\beta}}_{0}\right)\right)$ is the deviance of the intercept-only model. The best value of the bandwidth would be the one with the highest value of $\mathrm{R}^2$and the lowest values of the Deviance.

5. Data description

A year frame data, 2021, were collected from the 18 Iraqi provinces. The datasets for this study were obtained from Authority of Statistics and Geographic Information System, Iraq (https://cosit.gov.iq/ar/). The data included seven types of information in each individual provinces: Eq. (1) suicide mortality rate (average per (10000) persons), representing the response variable. Eq. (2) Urbanization rate (X1), Eq. (3) divorce rate (X2), Eq. (4) Unemployment rate (X3), Eq. (5) domestic violence (X4), Eq. (6) Mental illnesses (X5), and Eq. (7) Dissatisfaction index (X6). Variables X1 to X6 represent the explanatory variables. In Figure 1, the suicide mortality rate of 18 Iraqi provinces is reported. The geographical pattern of the cancer rate suggests differences between northern part and the southern part of the provinces.

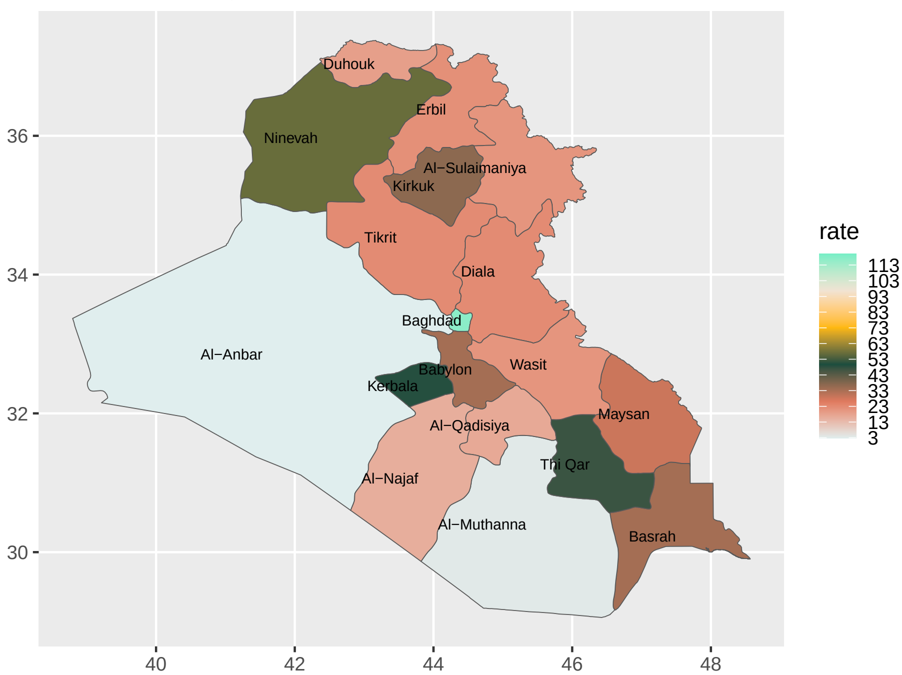

Figure 1. The spatial distribution of the suicide mortality rate of 18 Iraqi provinces under study.

6. Results and Discussion

First, the Kolmogorov Smirnov test was used in this study to test the goodness of fit of the response variable to the normal distribution. The result of the test is equal 8.351 with P-value equates 0.7447. This is pointed out by this result as an indication that normal distribution is a perfect fit for this response variable (suicide mortality rate). Table 2 provides the coefficients of OLS model (global model). According to the OLS model, Urbanization rate (X1), divorce rate (X2), Unemployment rate (X3), domestic violence (X4), and Mental illnesses (X5) had a significant effect on suicide mortality rate. Further, the association between suicide mortality rate and Urbanization rate (X1), divorce rate (X2), and Dissatisfaction index (X6) were positive. That is mean if these variables increased, the probability of suicide mortality rate increased. The association between suicide mortality rate and Unemployment rate (X3), domestic violence (X4), and Mental illnesses (X5) were negative; meaning that if these variables decreased, the probability of suicide mortality rate increased. On the other hand, Dissatisfaction index (X6) was not significantly associated with the suicide mortality rate.

Table 2. OLS model estimation

parameter	estimation	Std. error	t-value	p-value
Intercept	-34.9171	12.0279	-2.90301	**0.0231**
X1	1.1265	0.48851	2.306031	**0.0114**
X2	0.9203	0.51432	1.789422	**0.0428**
X3	-0.0395	0.00762	-5.197361	**0.0004**
X4	-8.368	1.15891	-7.220642	**0.0001**
X5	-38.2247	15.3265	-2.494026	**0.0331**
X6	2.5411	1.92412	1.3207381	0.1527

Second, the spatial heterogeneity test was done to identify the local dissimilarity in the association between the dependent variable and predictors within the Eighteen locations within the study area. The test that was conducted to determine whether the variance of the residuals is homoscedastic or heteroscedastic across the locations was the

Breusch-Pagan (BP) test. The null hypothesis is that variances are equal in the various locations with the alterative hypothesis being that there is one or more different variances in the various locations. In the event that the null hypothesis is rejected then it is said that there is spatial heterogeneity in a significant manner. Consequently, the value of the test statistic of the BP test is 15.184 and the p-value of 0.00317 that is not less than 0.05. This implies that the 18 locations within our study area have spatial diversity. To represent this spatial heterogeneity, the GWR model (local models) is taken into consideration to investigate the various spatial connections between the cancer rate and the 6 explanatory variables.

The parameter estimates of the GWR model of CV, GCV, AIC, and our proposed method, POA-GWR, of fixed bandwidth selection are summarized in Tables 2 to 5 depending on the choice of the bi-square kernel weighting function. The five statistics of the indicators are used to describe the GWR model parameters which include the minimum (Min), first quartile (Q1), median (Med), third quartile (Q3), and maximum values (Max). In addition, Table 6 gives the outcome of the assessment criteria and value of bandwidth optimum.

Two broad observations are worth noting form Tables 2 to 5 Eq. (1) With regard to the five statistics indicators (Min, Q1, Med, Q3, and Max), the direction (either positive or negative relationship) of relationships between suicide mortality rate and each of the explanatory variables of POA-GWR are similar in the respective counterparts using AIC, CV and GCV. Considering the case of Unemployment rate (X3) in the AIC, its parameters are all positive in the AIC, CV, and GCV. Eq. (2) The different parameters of each significant variables in AIC, CV, and GCV constantly lie within the range of equivalent counterparts in POA-GWR.

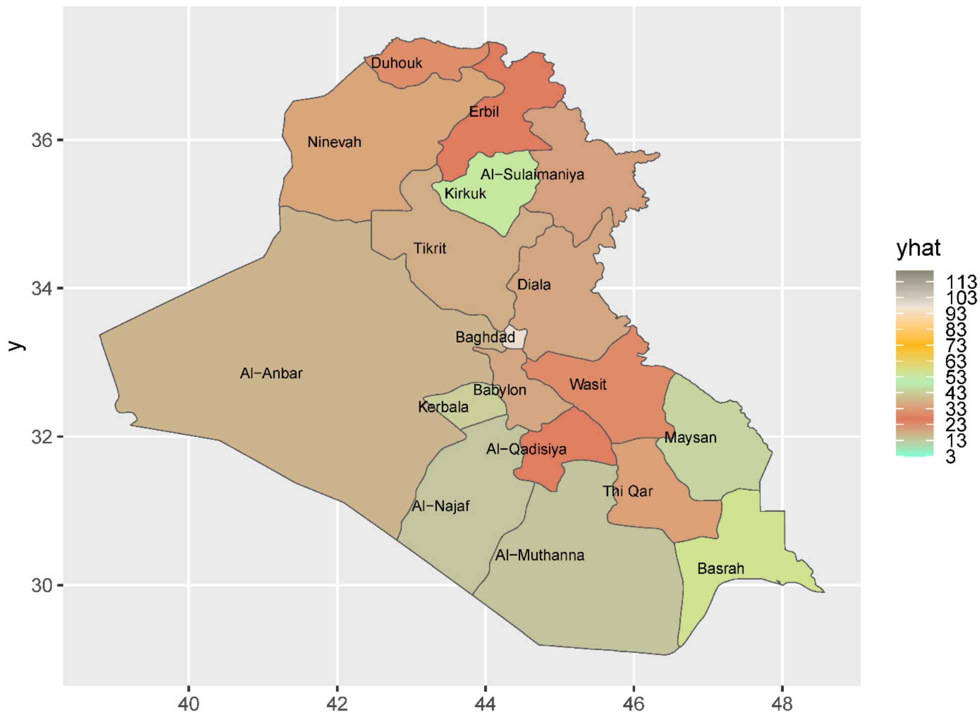

Figure 2. The suicide mortality rate prediction by AIC.

Concerning the performance of GWPR model, Table 6 reveals that R2 and Deviance attained by the proposed GWPR model, POA-GWR, are more accurate revealed that POA-GWR had a high R2 and low Deviance than AIC, CV, and GCV models. The Deviance in the POA-GWR is decreased by 48.42, 28.35, and 17.84 in comparison to the AIC, CV and GCV. The findings suggest that the GWR model based on our method, POA-GWR yields better predictions on the cancer rate in each separate Iraqi province compared to the AIC, CV, and GCV because it is able to capture the spatial heterogeneity of the data.

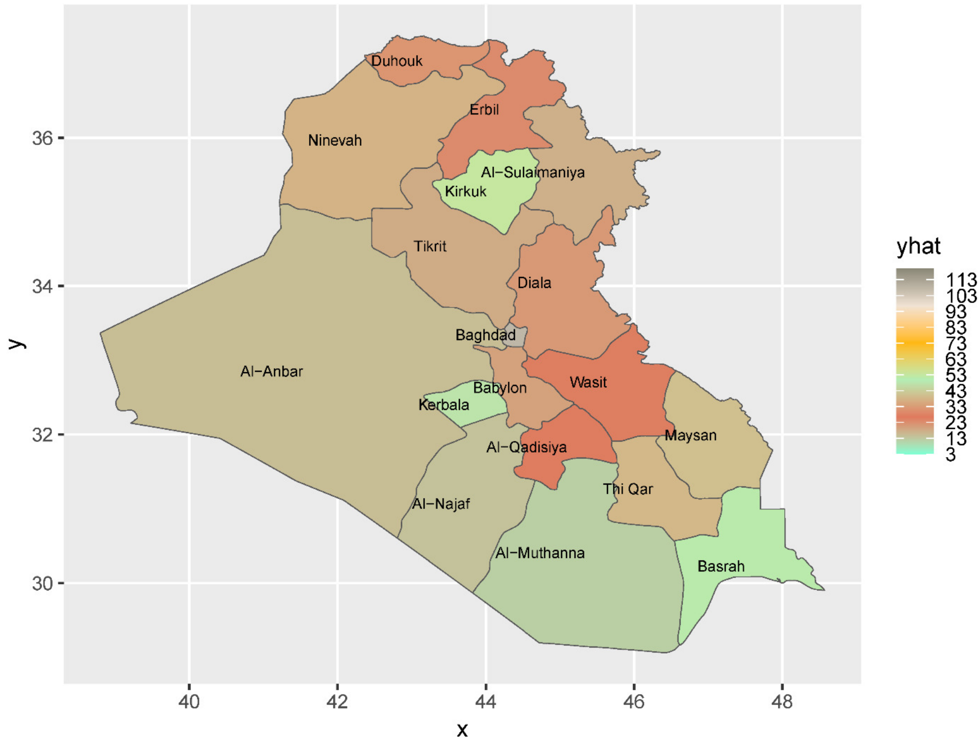

Figure 3. The suicide mortality rate prediction by CV.

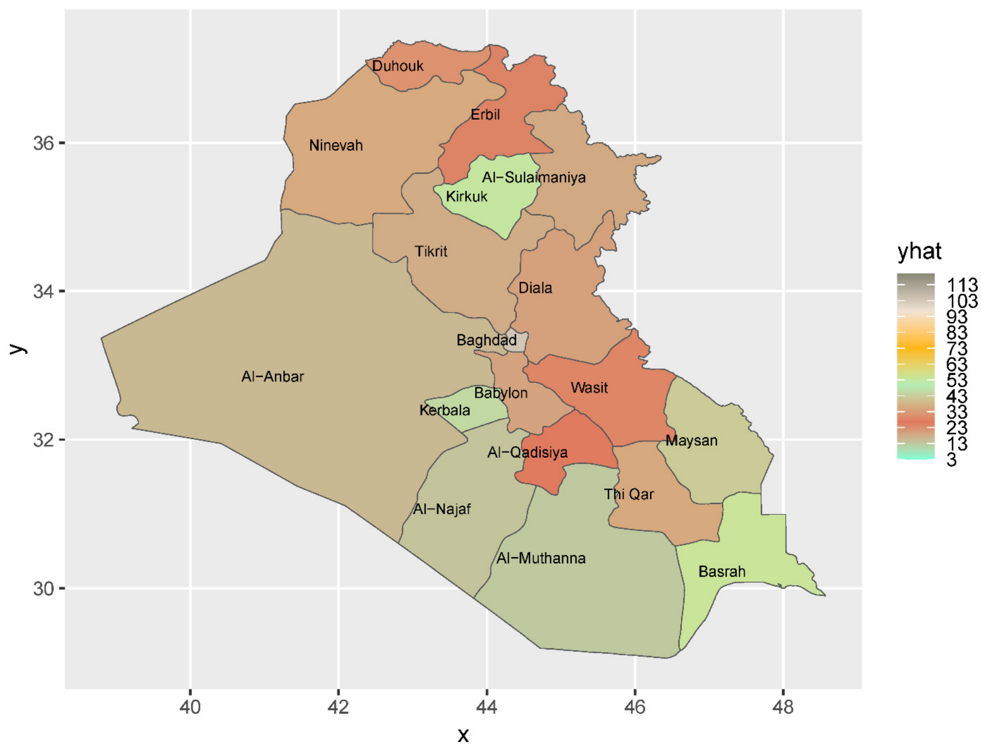

Figure 4. The suicide mortality rate prediction by GCV.

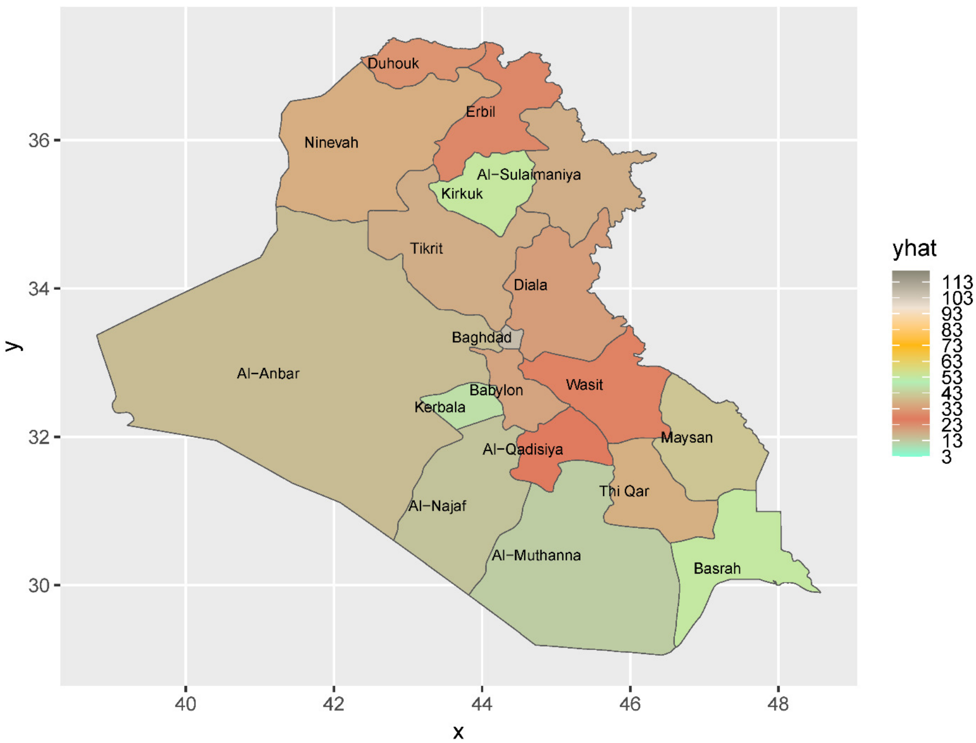

Figure 5. The suicide mortality rate prediction by POA-GWR.

Table 3. Summary of GWR parameters for AIC method

parameter	Min	Q1	Med	Q3	Max
Intercept	0.3184424	0.3184424	0.5937288	1.2241055	1.5165
X1	0.03071	0.03237	0.03701	0.03926	0.0414
X2	0.04744	0.07183	0.08408	0.09854	0.1581
X3	-0.01947	-0.01701	-0.01209	-0.008147	-0.0027
X4	-0.3498	-0.3293	-0.2633	-0.15774	-0.1231
X5	-1.1086	-0.72222	-0.09661	0.1229	0.2789
X6	-0.1452	-0.1105	-0.00196	0.1162	0.1599

Table 4. Summary of GWR parameters for CV method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.5176	2.2307	2.6719	3.3554	3.5354
X1	0.0017	0.0074	0.0087	0.0101	0.0105
X2	0.0064	0.0069	0.0074	0.0088	0.0092
X3	0.0042	0.0079	0.0171	0.0179	0.0183
X4	-4.6813	-4.0229	-3.6008	-2.8366	-0.2338
X5	-1.8386	-1.2337	0.4256	8.2163	10.0032
X6	5.5287	7.3340	10.0703	10.2899	10.6670

Table 5. Summary of GWR parameters for GCV method

parameter	Min	Q1	Med	Q3	Max
Intercept	0.01862	0.17864	0.318827	1.216624	1.6228
X1	0.02806	0.032021	0.036347	0.042945	0.0458
X2	0.04448	0.06427	0.09659	0.1263	0.2021
X3	-0.02999	-0.026836	-0.01677	-0.01376	-0.004
X4	-0.39765	-0.362104	-0.289273	-0.1262	-0.0633
X5	-1.12374	-0.48095	0.63045	1.174453	1.39
X6	-0.23109	-0.16592	0.00588	0.164148	0.2263

Table 6. Summary of GWR parameters for POA-GWR method

parameter	Min	Q1	Med	Q3	Max
Intercept	0.130789	0.30775	0.425589	1.19618	1.541
X1	0.02962	0.03153	0.036866	0.0410618	0.0432
X2	0.055638	0.078502	0.096622	0.110513	0.1865
X3	-0.02421	-0.021849	-0.014725	-0.009888	-0.0028
X4	-0.37453	-0.34858	-0.274545	-0.140201	-0.0915
X5	-0.109821	-0.609124	0.22284	0.540412	0.6558
X6	-0.18786	-0.13771	-0.000338	0.13944	0.1915

Table 7. Summary of evaluation criteria and the best bandwidth for used methods

Methods	pseudo-R^2	Deviance	best bandwidth
AIC	0.8827	7.1271	18
CV	0.9107	5.3086	17
GCV	0.9211	4.5593	16
POA-GWR	0.9626	2.7058	14

7. Conclusion

The GWR model is a particular type of statistical technique applied to the analysis of normal data that changes across space while taking into account while taking into account the spatial specificity of the relations between the variables. In addition, if the bandwidth is chosen poorly, the model fitting to the data set can be either overly complex and memorize the noise or insufficiently complex and fail to capture important patterns of the data set. This study presents an employing of meta-heuristic algorithms, which is POA for estimating the bandwidth value in GWR model which can offer a promising alternative to traditional methods. Based on suicide mortality rate estimation as a real data application, the comparison studies and evaluations demonstrated that the proposed method, POA-GWR, outperformed AIC, CV, and GCV methods regarding R^2 and Deviance. Additionally, the varying parameters of each significant variables in AIC, CV, and GCV always fall into the range of corresponding counterparts in POA-GWPR. Despite the promising performance of the proposed POA-GWR approach, this study has several limitations. First, the empirical analysis is based on suicide mortality data from only 18 Iraqi provinces for a single year (2021), which restricts the temporal dimension and may limit the generalizability of the findings to other periods or spatial contexts. Second, only a fixed bandwidth framework with a bi-square kernel was examined, while alternative kernels and adaptive bandwidth schemes might yield different or improved results.

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