

A Neutrosophic Three-Parameter Type I Half-Logistic Inverse Weibull Model for Survival Analysis

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Abstract In probability theory, the most important aspect is the probability distribution because it finds plenty of application in most fields that form the science. It is said to find favor on account studies of insurance and finance as well as in the medical field, in agriculture, demography and econometric studies. The contemporary research work is devoted to the introduction of a new extension, which is named the neutrosophic three-parameter type I half-logistic inverse Weibull distribution (NLIW). Numerous mathematical properties theorizing life survival have been formulated and analyzed, including survival and hazard functions, moment-generating functions and other tests of average, variance, and standard deviation, skewness and kurtosis. The effectiveness of NLIW distribution estimate has been investigated using Monte Carlo method. The findings of the simulation performed in this research reveal that task of estimating with acceptable accuracy can be achieved when the size of the sample is large. The reality of data of premature infant staying times has been adopted to elucidate the precise manner in which the expounded distribution of NLIW must be implemented. In the application aspects it has been demonstrated in the following sections that NLIW distribution is flexible since it can be used on both classical data and the data which contains uncertainties, ambiguity or imprecision.

Keywords Neutrosophic statistics, three-parameter type I half-logistic inverse Weibull distribution, survival analysis, premature infant

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1. Introduction

Neutrosophy, philosophical and mathematical formation, was created by Florentin Smarandache [?, 29, 30, 31]. As for the general themes, it refers to the interaction of opposites on the one hand and the issue of the study of indeterminacy on the other hand. Consequently, there appears a new set of approaches, neoclassical logic and set theory, which are the extensions of classical logic and set theory and aim at solving the problems of inconsistency, indeterminacy, and imperfect information.

The basis of neutrosophic statistics is that often in the data, there is information, which is vague and cannot be quantified in the classic sense and therefore cannot be properly processed in the framework of traditional statistical methods. Neutrosophic statistics is used as a way to handle and do more comprehensive analysis on such data. The use of fuzzy logic was expanded by [1] to create neutrosophy, which enables the depiction of uncertainty, ambiguity, and contradiction.

Traditional analysis often suggests that the deeper the data, the clearer it is, hence very often each of the observations gets a numerical value. However, as it has been observed, in most real life settings, information can be

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ambiguous or about which there is limited detail given. To get around these constraints, neosophic statistics offers ways of dealing with the unpredictable, scarce, and contradictory data [2, 3, 4, 32, 34, 35].

As already mentioned, neosophic statistics consider three measures that in some ways reflect the particularities of the evaluated propositions: truth membership, indeterminacy membership, and falsity membership. They all depict the extent of truth, openness, or falsehood that is correlated with a hypothesis or an observation. These degrees are represented in a manner similar to a fuzzy set by the membership functions [2, 3, 36, 37, 38].

Neutrosophic statistics are used in many different fields, including image processing, data mining, pattern recognition, and decision-making [4, 5, 6, 7]. It provides a flexible mathematical tool for the analysis and modeling of complex systems with a high level of imprecision and uncertainty.

The survival statistics are among the essential aspects of neutrosophic information that have to be examined. Basically, the idea of survival analysis, often termed as event-time analysis or time-to-event analysis, deals with the assessment of time to certain event of interest. It is commonly applied in social science, engineering, medical research and other fields where the time related results are issued. Most of the time when conducting research where the temporal order is not certain or where subjects may not have the same subsequent follow-ups survival analysis proves to be of great use. It could also be the case that occurrence of a particular event of interest; an event say of failure, a relapse or even a death, or any other event of interest [8, 39, 40, 41, 42].

There are many statistical distributions that are widely used in survival analysis to work on time-to-event data. This means that the features of the data and the assumptions made concerning the underlying survival process drives the choice of the distribution. These distributions are used for the assessment of time-to-event data in engineering, social sciences, and other medical disciplines. Different distributions may be chosen depending on the given characteristics of the data by the researchers as well as the hypotheses appropriate to the study. The literature review reveals that many articles address neutrosophic probability distribution [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

Applications for the three-parameter type I half-logistic inverse Weibull distribution can be found in many domains, including survival analysis. In this work, we extended the applications of the three-parameter type I half-logistic inverse Weibull distribution to include neutrosophical data in interval form with a degree of indeterminacy. Many qualities are investigated under the newly proposed distribution and their applications are described with the help of simulated and real data application [43, 44, 45, 46].

2. Neutrosophic three-parameter type I half-logistic inverse Weibull distribution

Probability distributions help in the portrayal of uncertainty that is prevalent in the data set through depiction of the patterns of variation. In this regard distribution summaries, the observations into a mathematical form which contain a few unknown parameters and is the best possible understanding of the basic data generating mechanism. Survival time distribution which is the probability description of the behavior of length of life is to a certain extent depends on mode of successes of the event under consideration. From the given data set, the selection of the right distribution depends with the extent of prior information regarding the physical characteristics of the process underlying the observed data [21, 22, 23, 47, 48, 49].

The three-parameter type I half-logistic inverse Weibull distribution (LIW), which was proposed by Alkarni, Afify [24], is one of the survival time distributions. Let z be continuous random variable follows a LIW distribution, then its probability density function (pdf) and the cumulative distribution function (cdf) are, respectively, as:

$$f(z; \theta, \eta, \tau) = \frac{2\theta\eta\tau (z^{-\tau-1}) \left(e^{-\eta z^{-\tau}}\right) \left(1 - e^{-\eta z^{-\tau}}\right)^{\theta-1}}{\left\{1 + \left(1 - e^{-\eta z^{-\tau}}\right)^{\theta}\right\}^2}, \quad z, \theta, \eta, \tau > 0 \quad (1)$$

$$F(z; \theta, \eta, \tau) = \frac{1 - \left(1 - e^{-\eta z^{-\tau}}\right)^{\theta}}{1 + \left(1 - e^{-\eta z^{-\tau}}\right)^{\theta}}, \quad (2)$$

The concept of neutrosophic probability as a function $NP: \rightarrow [0, 1]^3$ was originally presented by [2], where V is a neutrosophic sample space and defined the probability mapping to take the form $NP(\Omega) = (ch(\Omega), ch(neut \Omega), ch(anti \Omega)) = (\psi_1, \psi_2, \psi_3)$ with $0 \leq \psi_1, \psi_2, \psi_3 \leq 1$ and $0 \leq \psi_1 + \psi_2 + \psi_3 \leq 3$. The term Θ represents the set of sample space, R represents the set of real numbers, and ξ denotes a sample space event, Z_N and Y_N denote neutrosophic random variable.

Definition 1 Consider Z is the real-valued crisp random variable, which has the following definition: $Z: \Theta \rightarrow R$ where Θ is the event space and Z_N neutrosophic random variable as follows: $Z_N: \Theta \rightarrow R(I)$ and $Z_N = Z + I$, where I represents indeterminacy.

Theorem 1 Let $Z_N = Z + I$ be the neutrosophic random variable and the CDF and pdf of Z_N are [13], respectively

$$F_{Z_N}(z) = F_Z(z - I), \text{ and } f_{Z_N}(z) = f_Z(z - I),$$

Theorem 2 Let $Z_N = Z + I$ be the neutrosophic random variable, then the expected value and variance can be derived as follows: $E(Z_N) = E(Z) + I$ and $V(Z_N) = V(Z)$ [13].

By supposing the neutrosophic variable could be expressed as: $z_N = z_L + z_U I_N$ where $I_N \in \{I_L, I_U\}$ and z_L and $z_U I_N$ denote the determined and indeterminate parts, respectively, the neutrosophic random variable $x_N \in \{x_L, x_U\}$ which follows the LIW distribution has neutrosophic parameters, where the letters L and U are the lower values and the upper values, respectively.

Then, the neutrosophic CDF and pdf of neutrosophic LIW (NLIW) distribution is given by Eq. (3) and Eq. (4), respectively

$$f(z_N; \theta_N, \eta_N, \tau_N) = \frac{2\theta_N \eta_N \tau_N (z_N^{-\tau_N-1}) (e^{-\eta_N z_N^{-\tau_N}}) (1 - e^{-\eta_N z_N^{-\tau_N}})^{\theta_N-1}}{\left\{1 + (1 - e^{-\eta_N z_N^{-\tau_N}})^{\theta_N}\right\}^2}, \quad z_N > 0, \quad \theta_N > 0, \quad \eta_N > 0, \quad \tau_N > 0. \quad (3)$$

$$F(z_N; \theta_N, \eta_N, \tau_N) = \frac{1 - (1 - e^{-\eta_N z_N^{-\tau_N}})^{\theta_N}}{1 + (1 - e^{-\eta_N z_N^{-\tau_N}})^{\theta_N}}, \quad (4)$$

Figure 1 shows the NLIW distribution for different values of its parameters. Relating to Eq. (3) and Eq. (4), the neutrosophic survival and hazard functions of the NLIW distribution are defined in Eq. (5) and Eq. (6), respectively,

$$S(x_N) = 1 - \frac{1 - (1 - e^{-\eta_N x_N^{-\tau_N}})^{\theta_N}}{1 + (1 - e^{-\eta_N x_N^{-\tau_N}})^{\theta_N}}, \quad (5)$$

$$h(z_N; \theta_N, \eta_N, \tau_N) = \frac{\theta_N \eta_N \tau_N (z_N^{-\tau_N-1}) (e^{-\eta_N z_N^{-\tau_N}})}{(1 - e^{-\eta_N z_N^{-\tau_N}}) \left[1 + (1 - e^{-\eta_N z_N^{-\tau_N}})^{\theta_N}\right]}. \quad (6)$$

3. Parameter Estimation of NLIW Distribution

Five methods for estimating the NTPXL distribution parameters are described: (1) the maximum likelihood Method (MLE), (2) Anderson Darling method (AD), (3) Cramér-von Mises method (CVM), (4) weighted least-squares method (WLS), and (5) the maximum product spacing method (MPS).

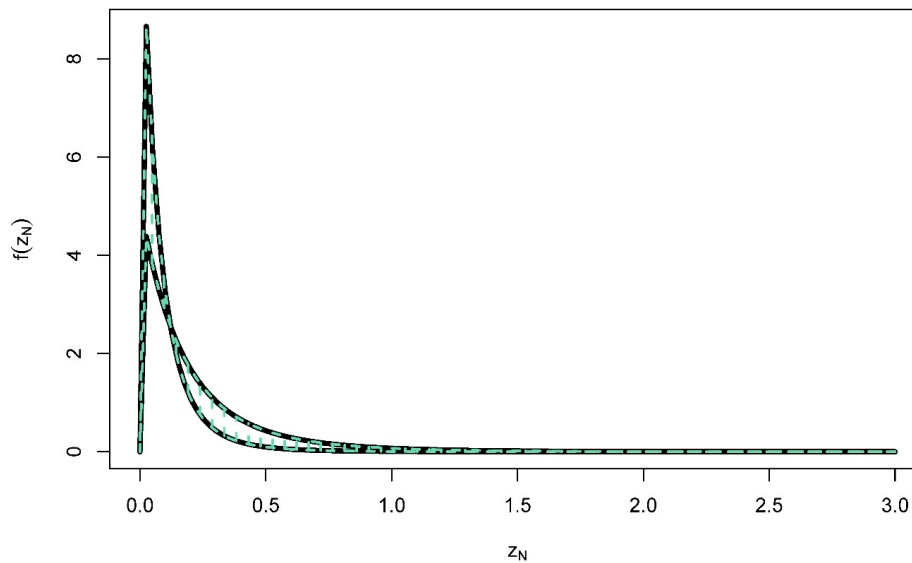


Figure 1. he pdf of NLIW when $\theta_N \in [0.5, 0.7]$, $\eta_N \in [1.5, 1.8]$ and $\tau_N \in [2, 2.3]$

3.1. MLE method

Assuming each of the random samples $z_1, z_2, \dots, z_n$ follows NLIW distribution, the log-likelihood function is given by

$$\ell = n \log (2\theta_N \eta_N \tau_N) - (\tau_N + 1) \sum_{i_N=1}^n \log z_{i_N} - \eta_N \sum_{i_N=1}^n z_{i_N}^{\tau_N} + (\theta_N + 1) \sum_{i_N=1}^n \log \left(1 - e^{-\eta_N z_{i_N}^{-\tau_N}} \right) - 2 \sum_{i_N=1}^n \log \left[1 + \left(1 - e^{-\eta_N z_{i_N}^{-\tau_N}} \right) \right] \quad (7)$$

The MLE of the parameters $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ are the solutions of the following simultaneous equations:

$$\frac{\partial \ell}{\partial \theta_N} = 0, \quad \frac{\partial \ell}{\partial \eta_N} = 0, \quad \frac{\partial \ell}{\partial \tau_N} = 0$$

$$\frac{\partial \ell}{\partial \theta_N} = \frac{n}{\theta_N} + \sum_{i_N=1}^n \log \left(1 - e^{-\eta_N z_{i_N}^{-\tau_N}} \right) \quad (8)$$

$$\frac{\partial \ell}{\partial \eta_N} = \frac{n}{\eta_N} - \sum_{i_N=1}^n z_{i_N}^{\tau_N} + (\theta_N - 1) \sum_{i_N=1}^n \frac{z_{i_N}^{-\tau_N} e^{-\eta_N z_{i_N}^{-\tau_N}}}{1 - e^{-\eta_N z_{i_N}^{-\tau_N}}} - 2 \sum_{i_N=1}^n \frac{z_{i_N}^{-\tau_N} e^{-\eta_N z_{i_N}^{-\tau_N}}}{1 + (1 - e^{-\eta_N z_{i_N}^{-\tau_N}})} \quad (9)$$

$$\begin{aligned} \frac{\partial \ell}{\partial \tau_N} = & \frac{n}{\tau_N} - \sum_{i_N=1}^n \log (z_{i_N}) - \eta_N \sum_{i_N=1}^n z_{i_N}^{\tau_N} \log (z_{i_N}) + 2 \sum_{i_N=1}^n \frac{\eta_N \log (z_{i_N}) z_{i_N}^{-\tau_N} e^{-\eta_N z_{i_N}^{-\tau_N}}}{1 + (1 - e^{-\eta_N z_{i_N}^{-\tau_N}})} \\ & + (\theta_N - 1) \sum_{i_N=1}^n \frac{\eta_N \log (z_{i_N}) z_{i_N}^{-\tau_N} (e^{-\eta_N z_{i_N}^{-\tau_N}})}{1 - e^{-\eta_N z_{i_N}^{-\tau_N}}} \end{aligned} \quad (10)$$

3.2. AD method

The AD estimates of the parameters $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ are attained by minimizing the following equation with respect to the unknown parameters:

$$AD(\theta_N, \eta_N, \tau_N) = -n - \sum_{i_N=1}^n \frac{(2i_N - 1)}{n} \left[\begin{aligned} & \log (F(Z_{i:nN}; \theta_N, \eta_N, \tau_N)) \\ & + \log (1 - F(Z_{i:nN}; \theta_N, \eta_N, \tau_N)) \end{aligned} \right] \quad (11)$$

3.3. CVM method

The CVM estimates $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ are derived by minimizing the following expression with respect to NLIW parameters as:

$$CVM(\theta_N, \eta_N, \tau_N) = \frac{1}{12n} + \sum_{i_N=1}^n \left(F(Z_{i:nN}; \theta_N, \eta_N, \tau_N) - \frac{2i_N - 1}{2n} \right)^2 \quad (12)$$

3.4. WLS method

The WLS estimates $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ are derived by minimizing Eq. (13) with respect to NLIW parameters as:

$$WLS(\theta_N, \eta_N, \tau_N) = \sum_{i_N=1}^n \frac{(1+n)^2(2+n)}{i_N(n-i_N+1)} \left[F(Z_{i:nN}; \theta_N, \eta_N, \tau_N) - \frac{i_N}{n+1} \right]^2. \quad (13)$$

3.5. MPS method

The estimation of NLIW parameters using MPS method can be obtained by maximizing Eq. (14) as:

$$MPS(\theta_N, \eta_N, \tau_N) = \frac{1}{n+1} \sum_{i_N=1}^{n+1} \log(F(Z_{i:nN}; \theta_N, \eta_N, \tau_N) - F(Z_{i-1:nN}; \theta_N, \eta_N, \tau_N)), \quad (14)$$

4. Simulation results

To investigate the efficacy of the NLIW of the neutrosophic parameters $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ of the suggested LIW, simulation research is conducted in this section. A random sample of sizes, $n = 30, 50, 150$ and 250 , is created from NLIW using different amalgams of neutrosophic parameters for the simulation. Estimated MLEs, AD, CVM, WLS, and MPS of the neutrosophic parameters for 1000 replications at different sample sizes using simulated data. Thus, for all sample sizes, the neutrosophic mean square error (NMSE) and the neutrosophic average bias (NAB) are derived. The superior neutrosophic estimator's properties are evaluated using the estimations of NAB and NMSE [25, 26, 27].

Three cases of the NLIW neutrosophic parameters are determined: Case (1): $\theta_N \in [0.5, 0.9]$, $\eta_N \in [1.5, 1.8]$, and $\tau_N \in [2, 2.5]$ Case (2): $\theta_N \in [0.7, 1.1]$, $\eta_N \in [1.8, 2.2]$, and $\tau_N \in [2.3, 2.7]$, and Case (3): $\theta_N \in [1, 1.5]$, $\eta_N \in [2, 2.5]$, and $\tau_N \in [2.6, 3]$. The results are given in Tables 1 - 6.

From Tables 1, 3, and 5, in terms of NAB, it is seen that, as predicted, the NAB for $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ decrease as sample sizes rise. It can also be deduced from Tables 1, 3, and 5 that the NAB values of the five estimators are varying as expected. The NAB values of $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ for MPS, WLS, and CVM are higher than MLE and AD estimators.

Concerning the NMSE values, and for all sample sizes, the MLE estimator of $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ has the smallest values comparing with AD, CVM, WLS, and MPS estimators. Further, it is noticed from Tables 2, 4, and 6 that when the $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$ increase regardless the values of the n , the NMSE are decreasing.

5. Real Application

From our study, we have used premature infant staying time data that we gathered from Kirkuk hospital, Iraq for about six months to apply our proposed NLIW distribution. The time corresponds to the number of days that the premature infant is alive after discharge from the hospital. The subject population in the study is 120 premature infants. However, premature infant times are not recorded accurately, the member countries need to develop clear and accurate time definitions. Therefore, defining the number of days from the birth of a premature infant with alive discharge without having explicit information is always problematic supervising unpredictable, insufficient, and incongruent data. The data related to exploratory factor analysis is given in Table 7.

Table 1. Average values of NAB for case 1

n		30	50	150	250
MLE	$\hat{\theta}_N$	[0.111,0.114]	[0.1101,0.1116]	[0.1077,0.1092]	[0.1053,0.1068]
	$\hat{\eta}_N$	[0.1226,0.1252]	[0.1202,0.1228]	[0.1178,0.1204]	[0.1154,0.118]
	$\hat{\tau}_N$	[0.1075,0.1101]	[0.1051,0.1077]	[0.1027,0.1053]	[0.1003,0.1029]
AD	$\hat{\theta}_N$	[0.1836,0.1851]	[0.1812,0.1827]	[0.1788,0.1803]	[0.1764,0.1779]
	$\hat{\eta}_N$	[0.1937,0.1963]	[0.1913,0.1939]	[0.1889,0.1915]	[0.1865,0.1891]
	$\hat{\tau}_N$	[0.1786,0.1812]	[0.1762,0.1788]	[0.1738,0.1764]	[0.1714,0.174]
CVM	$\hat{\theta}_N$	[0.2547,0.2562]	[0.2523,0.2538]	[0.2499,0.2514]	[0.2475,0.249]
	$\hat{\eta}_N$	[0.2648,0.2674]	[0.2624,0.265]	[0.26,0.2626]	[0.2425,0.2602]
	$\hat{\tau}_N$	[0.2497,0.2523]	[0.2473,0.2499]	[0.2449,0.2475]	[0.3186,0.2451]
WLS	$\hat{\theta}_N$	[0.3258,0.3273]	[0.3234,0.3249]	[0.321,0.3225]	[0.3287,0.3201]
	$\hat{\eta}_N$	[0.3359,0.3385]	[0.3335,0.3361]	[0.3311,0.3337]	[0.3136,0.3313]
	$\hat{\tau}_N$	[0.3208,0.3234]	[0.3184,0.321]	[0.316,0.3186]	[0.3897,0.3162]
MPS	$\hat{\theta}_N$	[0.3969,0.3984]	[0.3945,0.396]	[0.3921,0.3936]	[0.3998,0.3912]
	$\hat{\eta}_N$	[0.407,0.4096]	[0.4046,0.4072]	[0.4022,0.4048]	[0.3847,0.4024]
	$\hat{\tau}_N$	[0.3919,0.3945]	[0.3895,0.3921]	[0.3871,0.3897]	[0.3976,0.3873]

Table 2. Average values of NMSE for case 1

n		30	50	150	250
MLE	$\hat{\theta}_N$	[0.3182,0.3197]	[0.3158,0.3173]	[0.3134,0.3149]	[0.311,0.3125]
	$\hat{\eta}_N$	[0.3283,0.3309]	[0.3259,0.3285]	[0.3235,0.3261]	[0.3211,0.3237]
	$\hat{\tau}_N$	[0.3132,0.3158]	[0.3108,0.3134]	[0.3084,0.311]	[0.306,0.3086]
AD	$\hat{\theta}_N$	[0.3893,0.3908]	[0.3869,0.3884]	[0.3845,0.386]	[0.3821,0.3836]
	$\hat{\eta}_N$	[0.3994,0.402]	[0.397,0.3996]	[0.3946,0.3972]	[0.3922,0.3948]
	$\hat{\tau}_N$	[0.3843,0.3869]	[0.3819,0.3845]	[0.3795,0.3821]	[0.3771,0.3797]
CVM	$\hat{\theta}_N$	[0.4604,0.4619]	[0.458,0.4595]	[0.4556,0.4571]	[0.4532,0.4547]
	$\hat{\eta}_N$	[0.4705,0.4731]	[0.4681,0.4707]	[0.4657,0.4683]	[0.4633,0.4659]
	$\hat{\tau}_N$	[0.4554,0.458]	[0.453,0.4556]	[0.4506,0.4532]	[0.4482,0.4508]
WLS	$\hat{\theta}_N$	[0.5315,0.533]	[0.5291,0.5306]	[0.5267,0.5282]	[0.5243,0.5258]
	$\hat{\eta}_N$	[0.5416,0.5442]	[0.5392,0.5418]	[0.5368,0.5394]	[0.5344,0.537]
	$\hat{\tau}_N$	[0.5265,0.5291]	[0.5241,0.5267]	[0.5217,0.5243]	[0.5193,0.5219]
MPS	$\hat{\theta}_N$	[0.6026,0.6041]	[0.6002,0.6017]	[0.5978,0.5993]	[0.5954,0.5969]
	$\hat{\eta}_N$	[0.6127,0.6153]	[0.6103,0.6129]	[0.6079,0.6105]	[0.6055,0.6081]
	$\hat{\tau}_N$	[0.5976,0.6002]	[0.5952,0.5978]	[0.5928,0.5954]	[0.5904,0.593]

Table 3. Average values of NAB for case 2

n		30	50	150	250
MLE	$\hat{\theta}_N$	[0.1111,0.1126]	[0.1087,0.1102]	[0.1063,0.1078]	[0.1039,0.1054]
	$\hat{\eta}_N$	[0.1212,0.1238]	[0.1188,0.1214]	[0.1164,0.119]	[0.114,0.1166]
	$\hat{\tau}_N$	[0.1061,0.1087]	[0.1037,0.1063]	[0.1013,0.1039]	[0.0989,0.1015]
AD	$\hat{\theta}_N$	[0.1822,0.1837]	[0.1798,0.1813]	[0.1774,0.1789]	[0.175,0.1765]
	$\hat{\eta}_N$	[0.1923,0.1949]	[0.1899,0.1925]	[0.1875,0.1901]	[0.1851,0.1877]
	$\hat{\tau}_N$	[0.1772,0.1798]	[0.1748,0.1774]	[0.1724,0.175]	[0.17,0.1726]
CVM	$\hat{\theta}_N$	[0.2533,0.2548]	[0.2509,0.2524]	[0.2485,0.25]	[0.2461,0.2476]
	$\hat{\eta}_N$	[0.2634,0.266]	[0.261,0.2636]	[0.2586,0.2612]	[0.2562,0.2588]
	$\hat{\tau}_N$	[0.2483,0.2509]	[0.2459,0.2485]	[0.2435,0.2461]	[0.2411,0.2437]
WLS	$\hat{\theta}_N$	[0.3244,0.3259]	[0.322,0.3235]	[0.3196,0.3211]	[0.3172,0.3187]
	$\hat{\eta}_N$	[0.3345,0.3371]	[0.3321,0.3347]	[0.3297,0.3323]	[0.3273,0.3299]
	$\hat{\tau}_N$	[0.3194,0.322]	[0.317,0.3196]	[0.3146,0.3172]	[0.3122,0.3148]
MPS	$\hat{\theta}_N$	[0.3955,0.397]	[0.3931,0.3946]	[0.3907,0.3922]	[0.3883,0.3898]
	$\hat{\eta}_N$	[0.4056,0.4082]	[0.4032,0.4058]	[0.4008,0.4034]	[0.3984,0.401]
	$\hat{\tau}_N$	[0.3905,0.3931]	[0.3881,0.3907]	[0.3857,0.3883]	[0.3833,0.3859]

An informal graphical technique has been utilized to show that the LIW distribution is one of the plausible models for explaining the premature infant staying time data. Figure 2 displays a visual fit of the LIW distribution. Further, the χ^2 test for the goodness of fit shows that the premature infant staying time data follows LIW distribution with p-value=0.802. A descriptive assessment of the premature infant staying time data using NLIW is shown in

Table 4. Average values of NMSE for case 2

n		30	50	150	250
MLE	$\hat{\theta}_N$	[0.3168,0.3183]	[0.3144,0.3159]	[0.312,0.31535]	[0.3096,0.3111]
	$\hat{\eta}_N$	[0.3269,0.3295]	[0.3245,0.3271]	[0.3221,0.3247]	[0.3197,0.3223]
	$\hat{\tau}_N$	[0.3118,0.3144]	[0.3094,0.312]	[0.307,0.3096]	[0.3046,0.3072]
AD	$\hat{\theta}_N$	[0.3879,0.3894]	[0.3855,0.387]	[0.3831,0.3846]	[0.2807,0.3822]
	$\hat{\eta}_N$	[0.398,0.4006]	[0.3956,0.3982]	[0.3932,0.3958]	[0.3908,0.3934]
	$\hat{\tau}_N$	[0.3829,0.3855]	[0.3805,0.3831]	[0.3781,0.3807]	[0.3735,0.3783]
CVM	$\hat{\theta}_N$	[0.459,0.4605]	[0.4566,0.3581]	[0.4542,0.4557]	[0.4518,0.4533]
	$\hat{\eta}_N$	[0.4691,0.4717]	[0.4667,0.4693]	[0.4643,0.4669]	[0.4619,0.4645]
	$\hat{\tau}_N$	[0.454,0.4566]	[0.4516,0.4542]	[0.4492,0.4518]	[0.4468,0.4494]
WLS	$\hat{\theta}_N$	[0.5301,0.5316]	[0.5277,0.5292]	[0.5253,0.5268]	[0.5229,0.5244]
	$\hat{\eta}_N$	[0.5402,0.5428]	[0.5378,0.5404]	[0.5354,0.538]	[0.533,0.5356]
	$\hat{\tau}_N$	[0.5251,0.5277]	[0.5227,0.5253]	[0.5203,0.5229]	[0.5179,0.5205]
MPS	$\hat{\theta}_N$	[0.6012,0.6027]	[0.5988,0.6003]	[0.5964,0.5979]	[0.594,0.5955]
	$\hat{\eta}_N$	[0.6113,0.6139]	[0.6089,0.6115]	[0.6065,0.6091]	[0.6041,0.6067]
	$\hat{\tau}_N$	[0.5962,0.5988]	[0.5938,0.5964]	[0.5914,0.594]	[0.589,0.5916]

Table 5. Average values of NAB for case 3

n		30	50	150	250
MLE	$\hat{\theta}_N$	[0.1084,0.1099]	[0.106,0.1075]	[0.1036,0.1051]	[0.1012,0.1027]
	$\hat{\eta}_N$	[0.1185,0.1211]	[0.1161,0.1187]	[0.1137,0.1163]	[0.1113,0.1139]
	$\hat{\tau}_N$	[0.1034,0.106]	[0.101,0.1036]	[0.0986,0.1012]	[0.0962,0.2988]
AD	$\hat{\theta}_N$	[0.1795,0.181]	[0.1771,0.1786]	[0.1747,0.1762]	[0.1723,0.1738]
	$\hat{\eta}_N$	[0.1896,0.1922]	[0.1872,0.1898]	[0.1848,0.1874]	[0.1824,0.185]
	$\hat{\tau}_N$	[0.1745,0.1771]	[0.1721,0.1747]	[0.1697,0.1723]	[0.1673,0.1699]
CVM	$\hat{\theta}_N$	[0.2506,0.2521]	[0.2482,0.2497]	[0.2458,0.2473]	[0.2434,0.2449]
	$\hat{\eta}_N$	[0.2607,0.2633]	[0.2583,0.2609]	[0.2559,0.2585]	[0.2535,0.2561]
	$\hat{\tau}_N$	[0.2456,0.2482]	[0.2432,0.2458]	[0.2408,0.2434]	[0.2384,0.241]
WLS	$\hat{\theta}_N$	[0.3217,0.3232]	[0.3193,0.3208]	[0.3169,0.3184]	[0.3145,0.316]
	$\hat{\eta}_N$	[0.3318,0.3344]	[0.3294,0.332]	[0.327,0.3296]	[0.3246,0.3272]
	$\hat{\tau}_N$	[0.3167,0.3193]	[0.3143,0.3169]	[0.3119,0.3145]	[0.3095,0.3121]
MPS	$\hat{\theta}_N$	[0.3928,0.3943]	[0.3904,0.3919]	[0.388,0.3895]	[0.3856,0.3871]
	$\hat{\eta}_N$	[0.4029,0.4055]	[0.4005,0.4031]	[0.3981,0.4006]	[0.3957,0.3983]
	$\hat{\tau}_N$	[0.3878,0.3904]	[0.3854,0.388]	[0.383,0.3856]	[0.3806,0.3832]

Table 6. Average values of NMSE for case 3

n		30	50	150	250
MLE	$\hat{\theta}_N$	[0.3141,0.3156]	[0.3117,0.3132]	[0.3093,0.3108]	[0.3069,0.3084]
	$\hat{\eta}_N$	[0.3242,0.3268]	[0.3218,0.3244]	[0.3194,0.322]	[0.317,0.3196]
	$\hat{\tau}_N$	[0.3091,0.3117]	[0.3067,0.3093]	[0.3043,0.3069]	[0.3019,0.3045]
AD	$\hat{\theta}_N$	[0.3852,0.3867]	[0.3828,0.3843]	[0.3804,0.2819]	[0.378,0.3795]
	$\hat{\eta}_N$	[0.3953,0.3979]	[0.3929,0.3955]	[0.3905,0.3931]	[0.3881,0.3907]
	$\hat{\tau}_N$	[0.3802,0.3828]	[0.3778,0.3804]	[0.3754,0.378]	[0.373,0.3756]
CVM	$\hat{\theta}_N$	[0.4563,0.4578]	[0.4539,0.4554]	[0.4515,0.453]	[0.4491,0.4506]
	$\hat{\eta}_N$	[0.4664,0.469]	[0.464,0.4666]	[0.4616,0.4642]	[0.4592,0.4618]
	$\hat{\tau}_N$	[0.4513,0.4539]	[0.4489,0.4515]	[0.4465,0.4491]	[0.4441,0.4467]
WLS	$\hat{\theta}_N$	[0.5274,0.5289]	[0.525,0.5265]	[0.5226,0.5241]	[0.5202,0.5217]
	$\hat{\eta}_N$	[0.5375,0.5401]	[0.5351,0.5377]	[0.5327,0.5353]	[0.5303,0.5329]
	$\hat{\tau}_N$	[0.5224,0.525]	[0.52,0.5226]	[0.5176,0.5202]	[0.5152,0.5178]
MPS	$\hat{\theta}_N$	[0.5985,0.6]	[0.5961,0.5976]	[0.5937,0.5952]	[0.5913,0.5928]
	$\hat{\eta}_N$	[0.6086,0.6112]	[0.6062,0.6088]	[0.6038,0.6064]	[0.6014,0.604]
	$\hat{\tau}_N$	[0.5935,0.5961]	[0.5911,0.5937]	[0.5887,0.5913]	[0.5863,0.5889]

Table 8. Table 8 makes it abundantly evident that uncertainties taken into account in the observed sample are the cause of discrepancies in a number of the critical numerical statistics of the failure times data. Further, it is more clearly shown from Table 8 that there are high varies among the estimation methods in estimating the NLIW distribution parameters $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\eta}_N$.

Table 7. The staying time data

1.24,1.22, 1.23, 1.56, 1.54, 1.67, 1.49, [1.14,1.21], 1.11, 1.29, 2.03, 1.33, 1.65, 1.66, 2.81, 1.26, [1.85,2.01], 1.31,1.38, 1.13, 3.05, 1.48, 3.07, 1.27, 1.19, 1.31, [1.75,1.81], 1.62, 1.46, 1.81, [1.62,1.75], 1.62, 1.64, 1.29,1.19, 1.82, [1.66,1.74], 2.37,2.42, 1.84, 1.48,1.25, 1.48, 1.86, 2.22, [2.18,2.22], 1.96, 1.27, 1.14, 1.52, 1.64, 1.18, 2.37, 1.24, 2.23, 1.21, 1.72,1.90, 1.87, 1.74, 1.20, 2.03, 1.67, 2.03, 1.24, 1.11, 1.38, [1.33,1.41], 1.98, 2.28,1.21, 2.21,1.65,1.45,1.73,1.87,1.57,1.44,2.46,1.14,1.23,1.77,1.81,1.19,1.51,1.70,1.44,2.77, 1.74,1.37,1.23,1.36,1.18,1.12,1.37, [1.83,1.91], 2.42, 1.46, 1.57,1.16,1.68,1.41, 1.47,2.45, [1.75,1.83], 1.80,1.24, [1.27,1.31],2.09, 2.01, 1.8, 1.34, 1.28,1.54, 1.56, 1.29,1.87, 1.12, 1.79,2.01.
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In terms of survival probability, Figure 3 displays the survival curve for the five estimation methods. It can be observed that the neutrosophic survival curve using MLE methods shows higher probability than the others. This suggests that the neutrosophic MLE is better than the other four methods. Based on this observation, Figure 4 depicts the margin of the survival function between lower and upper the premature infant staying time data.

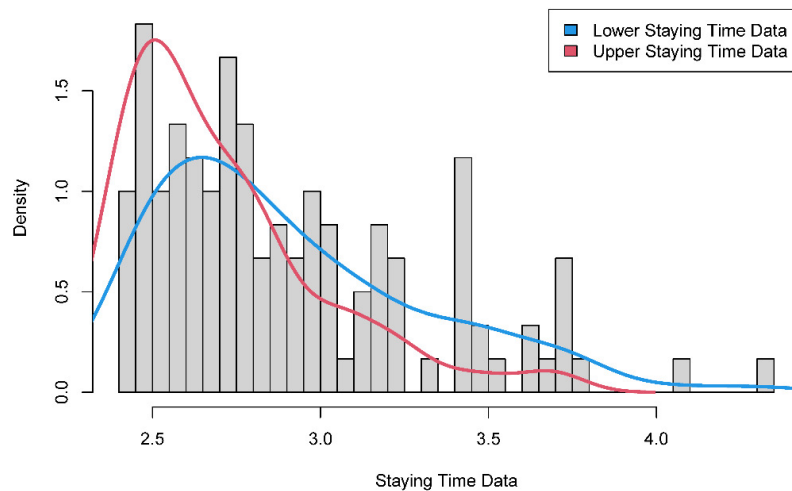


Figure 2. Fitting of LIW distribution of staying time data.

Table 8. The estimated parameters of NLIW distribution

Method	Estimated values		
	$\hat{\theta}_N$	$\hat{\eta}_N$	$\hat{\tau}_N$
MLE	[0.522,0.531]	[1.511,1.516]	[2.012,2.021]
AD	[0.535,0.538]	[1.518,1.522]	[2.025,2.031]
CVM	[0.537,0.539]	[1.523,1.529]	[2.033,2.038]
WLS	[0.541,0.544]	[1.528,1.532]	[2.041,2.047]
MPS	[0.543,0.548]	[1.530,1.539]	[2.045,2.051]

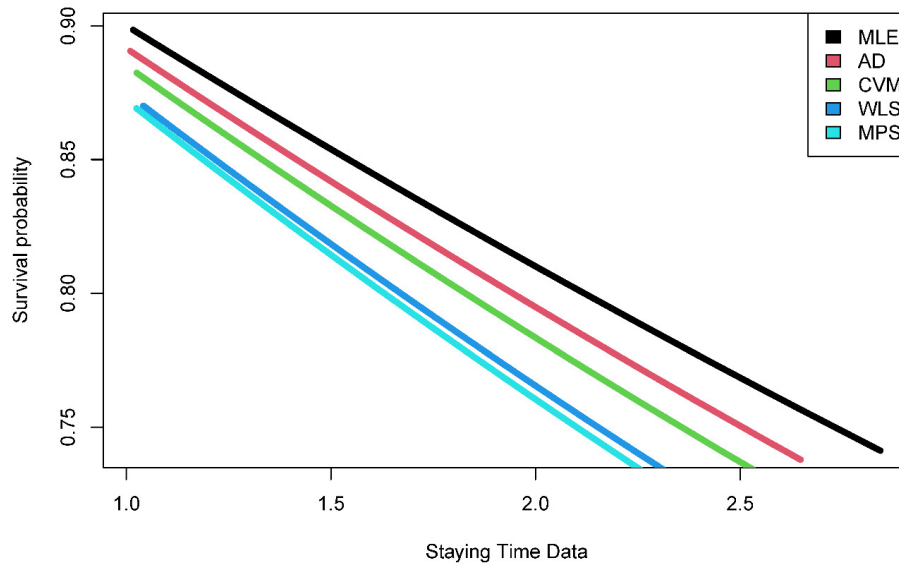


Figure 3. The survival curve plot for the NLIW distribution under several estimation methods.

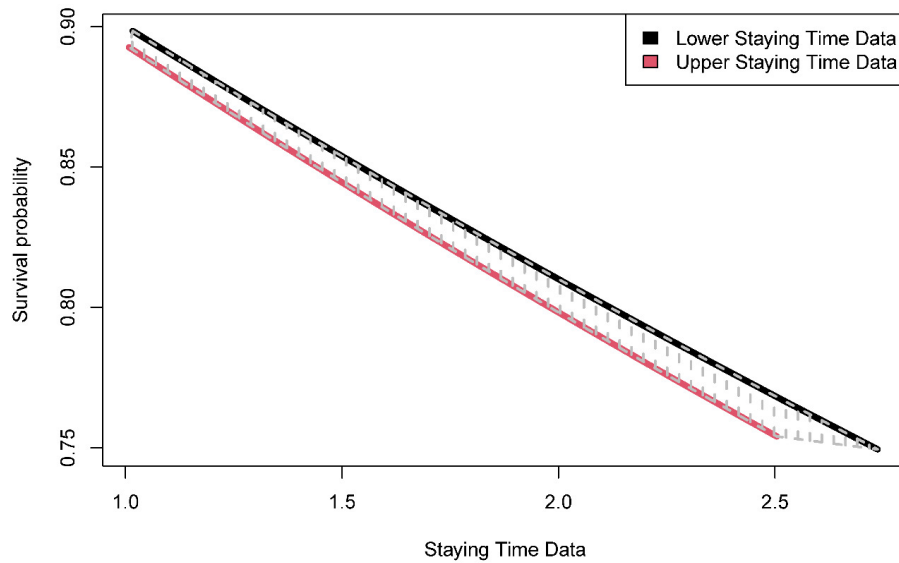


Figure 4. The survival curve plot for the NLIW distribution under MLE method.

6. Conclusions

The proposed neutrosophic three-parameter type I half-logistic inverse Weibull (NLIW) distribution offers a flexible survival model that accommodates both classical and interval-valued neutrosophic data, with its main properties, several estimation methods, and a Monte Carlo study all indicating that neutrosophic maximum likelihood estimation yields comparatively low bias and mean square error, especially for large samples, and that the model fits premature infant staying-time data well and can explicitly account for uncertainty, ambiguity, and imprecision in recorded times; however, the current assessment is limited to complete data from a single medical dataset, does not treat censoring or regression structures, and compares performance only within a small set of estimation procedures, so future work should extend the NLIW framework to censored and regression settings, evaluate it on diverse real datasets from medicine, engineering, and reliability, compare it systematically with other neutrosophic lifetime models, and investigate more deeply the computational stability, robustness, and possibly Bayesian or penalized versions of the neutrosophic estimators.

REFERENCES

1. F. Smarandache, *A unifying field in Logics: Neutrosophic Logic*, American Research Press, Philosophy Series, pp. 1–141, 1999.
2. F. Smarandache, *Introduction to neutrosophic statistics*, Infinite Study, 2014.
3. F. Smarandache, *Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version)*, International Journal of Neutrosophic Science, vol. 19, no. 1, pp. 148–165, 2022.
4. H. Guan *et al.*, *A Neutrosophic Forecasting Model for Time Series Based on First-Order State and Information Entropy of High-Order Fluctuation*, Entropy, vol. 21, no. 5, 2019.
5. X. Mao *et al.*, *A Neutrosophic-Based Approach in Data Envelopment Analysis with Undesirable Outputs*, Mathematical Problems in Engineering, vol. 2020, pp. 1–8, 2020.
6. M. Aslam, *A New Failure-Censored Reliability Test Using Neutrosophic Statistical Interval Method*, International Journal of Fuzzy Systems, vol. 21, no. 4, pp. 1214–1220, 2019.
7. F. Taş, S. Topal, and F. Smarandache, *Clustering Neutrosophic Data Sets and Neutrosophic Valued Metric Spaces*, Symmetry, vol. 10, no. 10, 2018.
8. A. A. Bibani and Z. Y. Algamal, *Survival Function Estimation for Fuzzy Gompertz Distribution with Neutrosophic Data*, International Journal of Neutrosophic Science, vol. 21, no. 3, pp. 137–142, 2023.
9. M. Ahsan-ul-Haq, *Neutrosophic Kumaraswamy distribution with engineering application*, Journal of Neutrosophic Statistics and Systems, vol. 49, pp. 269–276, 2022.
10. M. Albassam, M. Ahsan-ul-Haq, and M. Aslam, *Weibull distribution under indeterminacy with applications*, AIMS Mathematics, vol. 8, no. 5, pp. 10745–10757, 2023.
11. A. Alsoboh *et al.*, *Applications of Neutrosophic q -Poisson distribution Series for Subclass of Analytic Functions and Bi-Univalent Functions*, Mathematics, vol. 11, no. 4, 2023.
12. W.-Q. Duan *et al.*, *Neutrosophic Exponential Distribution: Modeling and Applications for Complex Data Analysis*, Complexity, vol. 2021, pp. 1–8, 2021.
13. C. J. H. Granados, *Some discrete neutrosophic distributions with neutrosophic parameters based on neutrosophic random variables*, Journal of Mathematics and Statistics, vol. 51, no. 5, pp. 1442–1457, 2022.
14. M. K. Hassan and M. Aslam, *DUS-neutrosophic multivariate inverse Weibull distribution: properties and applications*, Complex & Intelligent Systems, 2023.
15. Z. Khan *et al.*, *Generalized Pareto Model: Properties and Applications in Neutrosophic Data Modeling*, Mathematical Problems in Engineering, vol. 2022, pp. 1–11, 2022.
16. Z. Khan *et al.*, *Statistical development of the neutrosophic Lognormal model with application to environmental data*, Journal of Neutrosophic Statistics and Systems, vol. 47, no. 1, p. 1, 2021.
17. G. S. Rao, *Neutrosophic Log-Logistic Distribution Model in Complex Alloy Metal Melting Point Applications*, International Journal of Computational Intelligence Systems, vol. 16, no. 1, 2023.
18. F. Shah *et al.*, *On Neutrosophic Extension of the Maxwell Model: Properties and Applications*, Journal of Function Spaces, vol. 2022, pp. 1–9, 2022.
19. Z. Khan *et al.*, *Neutrosophic Rayleigh Model With Some Basic Characteristics and Engineering Applications*, IEEE Access, vol. 9, pp. 71277–71283, 2021.
20. M. A. Aslam, *Neutrosophic Rayleigh distribution with some basic properties and application*, In: *Neutrosophic Sets in Decision Analysis and Operations Research*, IGI Global, pp. 119–128, 2020.
21. M. Eliwa *et al.*, *A new one-parameter lifetime distribution and its regression model with applications*, PLOS ONE, vol. 16, no. 2, e0246969, 2021.
22. M. Ijaz, W. K. Mashwani, and S. B. Belhaouari, *A novel family of lifetime distribution with applications to real and simulated data*, PLOS ONE, vol. 15, no. 10, e0238746, 2020.
23. O. Kharazmi *et al.*, *A new two-sided class of lifetime distributions: Applications to complete and right censored data*, Statistics, Optimization & Information Computing, vol. 11, no. 3, pp. 595–614, 2023.
24. S. Alkarni *et al.*, *The Extended Inverse Weibull Distribution: Properties and Applications*, Complexity, vol. 2020, pp. 1–11, 2020.

25. M. Y. Mustafa and Z. Y. Algamal, *Neutrosophic inverse power Lindley distribution: A modeling and application for bladder cancer patients*, International Journal of Neutrosophic Science, vol. 21, no. 2, pp. 216–223, 2023.
26. M. M. Alanaz and Z. Y. Algamal, *Neutrosophic exponentiated inverse Rayleigh distribution: Properties and Applications*, International Journal of Neutrosophic Science, vol. 21, no. 4, pp. 36–42, 2023.
27. M. M. Alanaz, M. Y. Mustafa, and Z. Y. Algamal, *Neutrosophic Lindley distribution with application for Alloying Metal Melting Point*, International Journal of Neutrosophic Science, vol. 21, no. 4, pp. 65–71, 2023.
28. Algamal, Z. Y., & Lee, M. H. *Applying penalized binary logistic regression with correlation based elastic net for variables selection*, Journal of Modern Applied Statistical Methods, vol. 14, no.(1), p.15, 2015
29. Ewees, A. A., Algamal, Z. Y., Abualigah, L., Al-Qaness, M. A., Yousefi, D., Ghoniem, R. M., & Abd Elaziz, M. *A cox proportional-hazards model based on an improved aquila optimizer with whale optimization algorithm operators*, Mathematics, vol. 10, no.(8), p.1273, 2022
30. Algamal, Z. Y., Lee, M. H., Al-Fakih, A. M., & Aziz, M. *High-dimensional QSAR modelling using penalized linear regression model with L 1/2-norm*, SAR and QSAR in Environmental Research, vol. 27, no.(9), p.703-719, 2016
31. Algamal, Z. Y., Lukman, A. F., Abonazel, M. R., & Awwad, F. A. *Performance of the ridge and Liu estimators in the zero-inflated Bell regression model* Scientifica, vol. 1, p. 9503460, 2022.
32. Shamany, R., Alobaidi, N. N., & Algamal, Z. Y. *A new two-parameter estimator for the inverse Gaussian regression model with application in chemometrics*, Electronic Journal of Applied Statistical Analysis, vol. 12, no.(2), p.453-464, 2019
33. Algamal, Z. Y., Qasim, M. K., Lee, M. H., & Ali, H. T. M. *QSAR model for predicting neuraminidase inhibitors of influenza A viruses (H1N1) based on adaptive grasshopper optimization algorithm*, SAR and QSAR in Environmental Research, vol.31, no.11, p.803-814, 2020.
34. Al-Fakih, A. M., Algamal, Z. Y., Lee, M. H., Aziz, M., & Ali, H. T. M. *QSAR classification model for diverse series of antifungal agents based on improved binary differential search algorithm*, SAR and QSAR in Environmental Research, vol.30, no.2, p.131-143, 2019.
35. Mustafa, M. Y., & Algamal, Z. Y. *Neutrosophic inverse power Lindley distribution: A modeling and application for bladder cancer patients*, International Journal of Neutrosophic Science, 21(2), p. 216-223, 2023
36. Bibani, A. A., & Algamal, Z. Y. *Survival function estimation for fuzzy Gompertz distribution with neutrosophic data*, International Journal of Neutrosophic Science, 21(3), p.137-42, 2023
37. F. Al-Mutar & Z. Algamal, *Survival Function Estimation based on Neutrosophic two-parameter XLindley distribution* Statistics, Optimization & Information Computing, vol.14, no.3, pp. 1088–1097, 2025.
38. F. Al-Mutar & Z. Algamal, *Neutrosophic Exponentiated Power Lomax Distribution* Electronic Journal of Applied Statistical Analysis, vol.18, no.1, pp. 83–101, 2025.
39. N. M. Hammood, N. K. Rashad & Z. Y. Algamal *Neutrosophic Topp-Leone Extended Exponential distribution modeling with application for bladder cancer patients* International Journal of Neutrosophic Science, vol. 25, no. 1, pp.239-245, 2025.
40. O. E. Al-Saqal, Z. A. Hadied & Z. Y. Algamal *Modeling bladder cancer survival function based on neutrosophic inverse Gompertz distribution* International Journal of Neutrosophic Science, vol. 25, no. 1, pp.75-80, 2025.
41. M. M. Alanaz, M. Y. Mustafa & Z. Y. Algamal *Neutrosophic Lindley distribution with application for Alloying Metal Melting Point* International Journal of Neutrosophic Science, vol. 21, no. 4, pp.65-71, 2023.
42. M. M. Alanaz & Z. Y. Algamal *Neutrosophic exponentiated inverse Rayleigh distribution: Properties and Applications* International Journal of Neutrosophic Science, vol. 21, no. 4, pp.36-42, 2023.
43. Ibrahim, O.S., & Algamal, Z.Y. *Hybrid V-Support Vector Regression as Statistical Methodology for Forecasting Daily Natural Gas Prices*, Industrial Engineering and Management Systems, vol. 24, no. 4, p. 694-705, 2025.
44. Ibrahim, O.S., & Algamal, Z.Y. *Improving Nonlinear Regression Model Estimation Based on Coati Optimization Algorithm*, Statistics Optimization and Information Computing, vol. 14, no. 2, p. 923-936, 2025.
45. Ahmed, R.T., & Ibrahim, O.S. *Comparison between the FUZZY-ARFIMA Model and the Hybrid ARFIMA-FUZZY Model with Application to Agricultural Data in the City of Mosul*, Statistics Optimization and Information Computing, vol. 13, no. 2, pp. 741-758, 2025.
46. Ahmed, A. M., Jweiatee, A. F., & Algamal, Z. Y. *Integrated variable selection with hyperparameter-tuned random forest for modeling exchange rate volatility in Iraq*, Statistics, Optimization Information Computing, 17(10), p. 4822–4836, 2026
47. Alsultan, F. A., Ahmed, H. A., & Algamal, Z. Y. *Fuzzy Parameter Modeling and Survival Function Estimation for Inverse Power Maxwell Data*, Statistics, Optimization Information Computing, 16(5), p. 2795–2806, 2026
48. Ahmed, H. A., Othman, R. A., Alsultan, F. A., & Algamal, Z. Y. *A Dawoud–Kibria Shrinkage Estimator for the Waring Regression Model: Methodology, Simulation, and Application*, Statistics, Optimization Information Computing, 16(4), p. 3833-3845, 2026
49. Alharthi, A. M., Kadir, D. H., Al-Fakih, A. M., Algamal, Z. Y., Al-Thanoon, N. A., & Qasim, M. K. *Quantitative structure-property relationship modelling for predicting retention indices of essential oils based on an improved horse herd optimization algorithm*, SAR and QSAR in Environmental Research, vol. 34, no. 10, p. 831-846, 2023.