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Computer Network Access Codes with Various Types of Network Design by Using Dominant Local Resolving Set Theory

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Abstract Computer networks spanning multiple buildings face increasing challenges in efficient access control, optimal server placement, and rapid fault identification. As network structures grow more complex, there is a strong motivation to develop a mathematical framework that minimizes resource usage while ensuring full network coverage and unique node identification. This study addresses these challenges by applying dominant local resolving set theory to computer network design. The methodology adopts an axiomatic–deductive approach, modeling computer networks as connected graphs where devices are represented by vertices and communication links by edges. The analysis focuses on dominant local metric dimensions of standard graph families and vertex amalgamation products, which effectively model heterogeneous multi-building network topologies. A structured framework is developed to determine dominant local bases and corresponding access codes. The most significant results establish exact formulas for the dominant local metric dimension of vertex amalgamation graphs. The results show that this dimension is determined by the dominant local metric dimensions of the constituent graphs and is strongly influenced by the choice of linkage vertices, with reductions occurring under specific modular conditions. These findings demonstrate the broader significance of dominant local resolving set theory in designing cost-effective and scalable computer networks. The proposed framework provides practical guidance for minimizing the number of service providers, improving fault diagnosis, and optimizing access code assignment in complex distributed network environments.
Keywords: network code, network design, local resolving set, dominant local metric dimension, vertex amalgamation

AMS 2010 subject classifications 05C12

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1. Introduction

The basic concept of a computer network is a series of systems that are combined from several basic subsystems consisting of a communication system and communication protocols [1]. The system forms a channel that is related and supports each other for the purpose of sharing data or sharing devices [2]. The communication system is a combination of several systems. The system is formed from signal transmission, signal carrier or what is usually called a carrier and communication protocol. Signal transmission is usually in analog or digital form [3]. Signal transmission requires a medium as a signal transmitter (carrier), which can be in the form of cables, radio waves (RF), light, and others. To be able to convey data, communication systems also need rules (rules/protocols). A communication protocol is a rule that is used to control data exchange between nodes [4]. These settings include the initialization process, verification, how to communicate, and how to end communication. A computer network can be defined as two or more computers that can be connected using a communication system [5]. Conceptually, a computer network is a computer-based work network consisting of nodes connected to each other, either by wires

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or can be wireless. Each node/point will act as a workstation which functions as a server service provider to manage certain functions from other nodes or points [20].

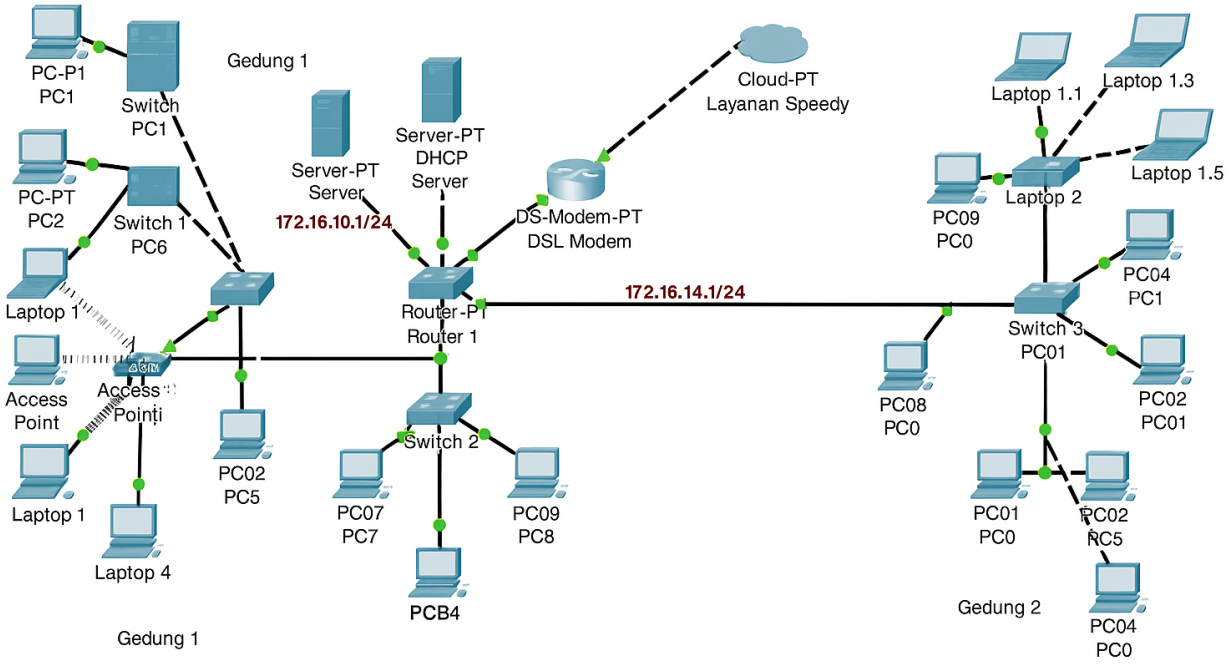


Figure 1. Example Network topology connecting using routers, switches, and access points of two building.

Computer networks can be modeled as graphs, and the mathematical study of these graph structures is called graph theory. Within the theory of graphs is the important concept of dominating set, which refers to the set of vertices from the same graph in which every other vertex in the graph is adjacent to at least one vertex in this set. Since the dominating set of a graph is not unique, it is usually more interesting to study the dominating set with the smallest possible size, known as the minimum dominating set [14].

The concept of metric dimensions was introduced by Harary and Melter [15]. Harary defines a resolving (differentiating) set in a graph as a set of vertices uniquely required to represent or identify all other nodes based on shortest path distances. This representation of a vertex in the set of differences is presented as k-ordered pairs whose elements are the distance between the vertices on the graph and the vertices in the set of differences. The resolving set that has smallest cardinality is called a basis. While the metric dimensions is obtained from distinct representations for all vertices in the graph, the local metric dimensions is such that each pair of adjacent vertices has a distinct representation [11].

From this definition, several new studies have been explored, such as local metric dimensions in non-bipartite graphs and rooted product graph operations [12] as well as in graphs formed from join operations. In addition, Susilowati et al. studied the similarity between metric dimensions and local metric dimensions in graphs resulting from certain operations [15], [17]. Furthermore, they examined the commutative characterization of graphs obtained from the amalgamation and join operations based on their metric dimensions [13].

As a substructure of metric dimensions and dominating set, the concept of a resolving dominating set was introduced by Brigham et al., satisfying the definition of both a dominating set as well as a distinguishing (resolving) set [18]. The minimum cardinality of such a set is called the resolving dominating number. In a later paper, the concept of locating dominating set and metric dimensions was integrated to form the term metric locating dominating set. These two concepts are essentially the same idea.

Recent developments in graph theory have increasingly focused on the applicability of resolving sets to complex systems. For instance, Ali et al. (2024) explored dominant metric dimensions in zero-divisor graphs, highlighting the theoretical bounds of resolving sets in algebraic structures [6]. Similarly, Prabhu et al. (2025) investigated the metric dimension of star fan graphs, providing new insights into the resolvability of hybrid network structures [7]. Nadeem et al. (2025) further demonstrated the utility of localized metric resolvability for optimizing resource placement [8]. Additionally, Batiha et al. (2024) and Sardar et al. (2024) have expanded these concepts to secure metric dimensions and claw-free cubic graphs, respectively [9], [10]. However, few have explicitly addressed the dominant local variant in the context of vertex amalgamation products.

In 2021 and 2022 Umilasari et al. expanded on the idea of the metric locating dominating set to introduce the concept of local dominance metric dimensions in graphs, which has relevance to computer network design [14]. In particular, Umilasari et al.'s idea is relevant in determining how to design computer networks if the computers to be connected are located in several buildings, as well as how to identify and trace faulty servers in areas experiencing problems.

Such cases align closely with the principles in local dominance differentiating sets. Figure 1 illustrates a practical network topology connecting two distinct buildings (building 1 and building 2) via a central router and distributed switches. In this schema, the router acts as the central hub or cut-vertex that amalgamates the local networks of each building. By modeling this physical infrastructure as a graph, where routers and switches represent vertices and cables represent edges, we can apply graph theory to analyze the flow of information. This visual representation serves as the baseline for our vertex amalgamation model, demonstrating how complex multi-building networks can be decomposed into simpler fundamental graph structures for analysis.

As a consequence, this study explores different approaches for determining computer network codes across various network designs, and extends the theoretical framework by investigating graph structures resulting from amalgamation products operation, with a special focus on heterogeneous graphs.

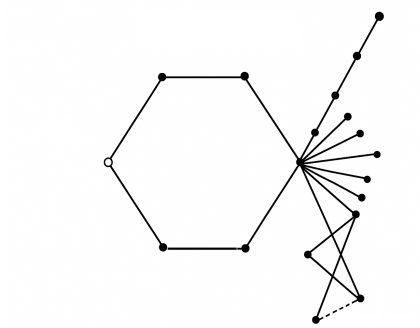


Figure 2 Examples of Vertex Amalgamation of Four Different Graphs

Let $G_1, G_2, \ldots, G_t$ be subgraphs of G and let $v \in V$. If $G = \bigcup_{i=1}^{t} G_i$ and $\bigcap_{i=1}^{t} G_i = v_1$, then G is a vertex amalgamation of $G_1, G_2, \ldots, G_t$ on vertex v, denoted by $G = \bigvee_{v}^{1}$.

If G_i has n_i vertices for $i = 1, 2, \ldots, t$, then the graph $\bigvee_{v}^{1} G_1, G_2, \ldots, G_t$ each has $n_1, n_2, \ldots, n_t$ vertices, and $\bigvee_{v}^{1} G_1, G_2, \ldots, G_t$ $\sum_{i=1}^{t} n_i - t + 1$ vertices, respectively [21]. In this paper, the dimensions of the local dominance metric of vertex amalgamation graphs are determined. Several special graphs that were studied include path graph, cycle graph, complete graphs, complete bipartite graphs and star graphs [22, 23]. The developed theory is then applied to designing computer network nodes for network distributed across multiple buildings, each with different structural layouts. Therefore, the general problems and specific objectives to be achieved from this study are outlined below.

1. To develop a graph-theoretic framework for modeling computer network designs using dominant local resolving set theory.

2. To determine the dominant local metric dimension of various standard network topologies, including path, cycle, star, complete, and amalgamated graphs.
3. To analyze the effect of vertex amalgamation and linkage vertex selection on the dominant local metric dimension of heterogeneous network structures.
4. To provide a mathematical basis for designing cost-efficient computer networks by minimizing the number of service providers while ensuring full network coverage and unique node identification.

2. Method

This research employs an axiomatic-deductive approach implemented through a structured framework. We propose a "Dominant Local Resolving Set Framework" (DLRSF) that processes network topologies to output optimal server placements. The methodology is divided into four key subsections.

1. Dataset Description
   The "datasets" used in this study are theoretical graph models representing standard computer network topologies. These include:
   - Path Graphs (P_n): Representing Bus topology networks.
   - Cycle Graphs (C_n): Representing Ring topology networks.
   - Star Graphs (S_n): Representing Centralized Hub networks.
   - Vertex Amalgamation Graphs ($Amal(G)$): Representing multi-building networks connected via a gateway (linkage vertex).

   All graphs are simple, connected, and undirected, reflecting standard wired network constraints.
2. Proposed Model Architecture
   The proposed framework operates in four phases, as illustrated in the workflow description below:
   - Topology Modeling: Convert physical network layout into a graph $G(V, E)$.
   - Dominance Check: Identify set S such that every node $v \notin S$ is adjacent to at least one node in S.
   - Resolving Assessment: Calculate distance vectors $r(v|W)$ for all adjacent pairs to ensure uniqueness.
   - Optimization: Determine the minimum cardinality set W_l that satisfies both conditions (2) and (3).
3. Network Mapping Process
   To bridge graph theory and network engineering, we apply the following mapping rules:
   - Vertex (v) $\rightarrow$ Workstation/Device: Each node represents a PC, printer, or router.
   - Edge (e) $\rightarrow$ Communication Link: Cables or wireless channels connecting devices.
   - Dominant Set (W_l) $\rightarrow$ Service Providers: The minimum set of servers required to manage the network.
   - Metric Representation ($r(v|W_l)$) $\rightarrow$ Access Code: The unique coordinate vector assigned to each device, serving as a logical address (similar to an IP subnet mask).
4. Assessment Method Recommendation
   The recommended assessment method for determining the dominant local basis involves three decision rules:
   - Full-Matching: A set is rejected if any two adjacent vertices u, v have identical metric representations ($r(u|W_l) = r(v|W_l)$).
   - Semi-Matching: A set is accepted only if it is a dominating set; i.e., every node is within distance 1 of the set.
   - Minimality: Among all valid sets, the one with the smallest cardinality $|W_l|$ is selected as the Dominant Local Metric Dimension ($Ddim_l$).

   To theoretically validate the minimality condition of our assessment framework, we rely on the following fundamental property established by Umilasari et al.. This lemma confirms that if a valid solution does not exist at a certain cardinality, no smaller subset can satisfy the requirements.
   Lemma 1. Let G be a connected graph. If there is no local dominant resolving set with cardinality p, then for every $S \subseteq V(G)$ with $|S| < p$, S is not a local dominant resolving set.

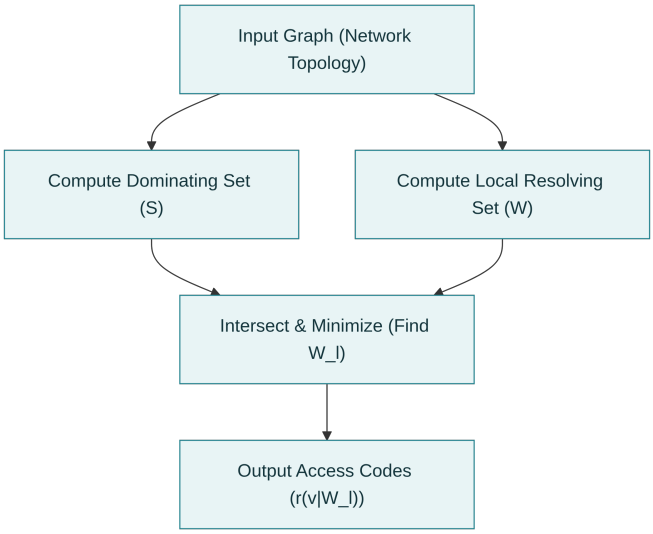


Figure 3 Dominant Local Resolving Set Framework

To formalize the assessment, we define the criteria mathematically. Let $G = (V, E)$ be a connected graph. An ordered set $W_l = \{w_1, w_2, \ldots, w_k\} \subseteq V$ is called a *local resolving set* if for every pair of adjacent vertices $u, v \in E(G)$,

$$r(u \mid W_l) \neq r(v \mid W_l),$$

where the representation of a vertex u with respect to W_l is defined as

$$r(u \mid W_l) = (d(u, w_1), d(u, w_2), \ldots, d(u, w_k)).$$

Furthermore, W_l is called a *dominant set* if for every vertex $v \in V \setminus W_l$, there exists a vertex $w \in W_l$ such that

$$d(v, w) = 1.$$

The *dominant local metric dimension*, denoted by $Ddim_l(G)$, is defined as

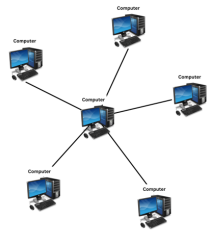
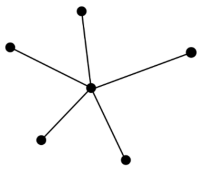
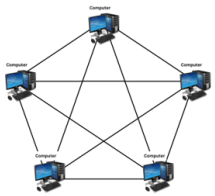
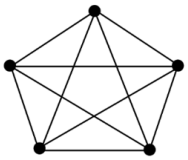
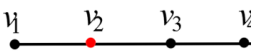
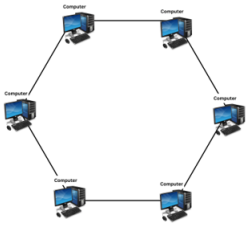
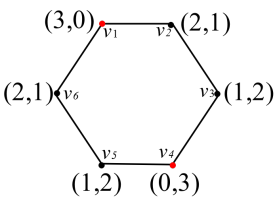
$$Ddim_l(G) = \min\{|W_l| \ : \ W_l \text{ is a dominant local resolving set of } G\}.$$

3. Results and Discussion

Conceptually, a computer network is a system of interconnected nodes (vertices) either to each other through the use of wires or as wireless networks. Each node or point functions as a workstation and can also work as a server for managing some certain tasks functions from other nodes within the network.

Below are several observations of several network types and the number of service providers required as well as examples of network access codes that can be given to each computer. Examples of selected networks include networks in the form of star graphs, complete graphs, path graphs and cycle graphs.

Table 1. Examples of network types and the number of service providers

Network Type Image	Representation in Graphs	Information
		The graph formed from this network representation is a star graph with 6 nodes, so that with just 1 provider, all five computers can be reached. Therefore, the access code for each computer can be given the same code, for example: {1} because each computer is not connected to each other.
		The graph formed from the representation of this network form is a Complete graph with 6 vertices, so it takes 5 providers to secure each computer. because each computer is connected to each other. The providers needed are quite a lot because each computer has a network connection with every other computer. The codes that can be given are for example: computer 1: {1, 1, 1, 1, 1, 1} computer 2: {0, 1, 1, 1, 1, 1} computer 3: {1, 0, 1, 1, 1, 1} computer 4: {1, 1, 0, 1, 1, 1} computer 5: {1, 1, 1, 0, 1, 1} computer 6: {1, 1, 1, 1, 0, 1}
		The graph formed from the representation of this network form is a Path graph with 4 vertices, so 2 providers are needed, then all four computers can be reached. Therefore, the access code for each computer is for example: computer 1: {1, 2} computer 2: {0, 1} computer 3: {1, 0} computer 4: {2, 1}
		The graph formed from the representation of this network form is a Cycle graph with 5 vertices, so 3 providers are needed, then all four computers can be reached. Therefore, the access code for each computer is for example: computer 1: {3, 0} computer 2: {2, 1} computer 3: {1, 2} computer 4: {0, 3} computer 5: {1, 2} computer 6: {2, 1}

Furthermore, in several theorems below, the results of the dominant local metric dimension in graphs are presented. The results of vertex amalgamation product with several special graphs are also presented including for cycle graphs, path graphs, star graphs, and complete bipartite graphs.

3.1. Dominant Local Resolving Set of Vertex Amalgamation Product Graph

Theorem 1. Let $amal(G) = \vee_v^1\{P_n, C_m\}$ be a vertex amalgamation of a path and cycle graph for $m, n \geq 3$, and v is a pendant of the path attached to any vertex of the cycle, then:

$$Ddim_l(amal(G)) = \begin{cases} Ddim_l(P_n) + Ddim_l(C_m), m = n; m, n \equiv 0 \pmod 3 \\ Ddim_l(P_n) + Ddim_l(C_m) - 1, m = n; m, n \not\equiv 0 \pmod 3 \end{cases}$$

Proof.

a. Case 1. $m, n \equiv 0 \pmod 3$

Let the vertex set of $amal(G) = \vee_v^1\{P_n, C_m\}$ is $V(G) = \{v, v_i, u_j \mid i = 1, 2, \ldots, n-1;\ j = 1, 2, 3 \ldots, m-1\}$ and the edge set is $E(G) = \{E(P_n) \cup E(C_m)\}$. Select

$$W_l = \{v_1, v_4, v_7, v_{10} \ldots, v_{3i-2}\} \cup \{u_2, u_5, u_8, u_{11} \ldots, u_{3i-1}\}$$

which is a combination of dominant local resolving set of path and cycle as dominant local basis of G for every $i = 1, 2, \ldots, n-1; j = 1, 2, 3 \ldots, m-1$, $|W_l| = Ddim_l(P_n) + Ddim_l(C_m)$. It can be seen that the representation every two vertex of $V(G)$ is definitely different. Next, because the element of W_l is chosen by combining dominant local basis of path (P_n) and cycle (C_m) then every vertex of $V(G)$ can be dominated by elements of W_l. So, it can be said that W_l dominant local resolving set of $amal(G) = \vee_v^1\{P_n, C_m\}$. Choose any $S \subseteq V(amal(G)) = \vee_v^1\{P_n, C_m\}$ with $|S| < |W_l|$. Without loss of generality, suppose $|S| = |W_l| - 1$ with $|W_l| = Ddim_l(P_n) + Ddim_l(C_m) - 1$, so there must be at least 1 vertex t that is not adjacent to S. Therefore S is not a dominant local resolving set of $amal(S_n, v, k)$. **Based on Lemma 1**, any set $|T|$ with $|S|$ is not a dominant local resolving set of G Then, $W_l = Ddim_l(P_n) + Ddim_l(C_m)$, is a dominant local basis of $amal(G) = \vee_v^1\{P_n, C_m\}$. Therefore, it was proven that $Ddim_l = Ddim_l(P_n) + Ddim_l(C_m)$. □

b. Case 2. $m, n \not\equiv 0 \pmod 3$

Let the vertex set of $amal(G) = \vee_v^1\{P_n, C_m\}$ be $V(G) = \{v, v_i, u_j \mid i = 1, 2, \ldots, n-1;\ j = 1, 2, \ldots, m-1\}$ and the edge set $E(G) = E(P_n) \cup E(C_m)$. Choose

$$W_l = \{v\} \cup \{v_2, v_5, v_8, \ldots, v_{3i-1}\} \cup \{u_1, u_4, u_7, \ldots, u_{3i-2} = 1 + Ddim_l(P_n) + Ddim_l(C_m) - 1$$

which is a combination of the local dominating sets of the path graph and the cycle graph as dominant local basis of G for every $i = 1, 2, \ldots, n-1; j = 1, 2, 3 \ldots, m-1$, and the linkage vertex of path and cycle are element of W_l, then it can be written $|W_l| = Ddim_l(P_n) + Ddim_l(C_m) - 1$. The result show that the representation of every two vertices on $V(G)$ certainly different.

Next, because the element of W_l of W_l are taken by merger the the local dominant basis of P_n and C_m then all the vertices of $V(G)$ can be dominated by W_l, or it can be said that W_l is dominant local resolving set of $amal(G) = \vee_v^1\{P_n, C_m\}$. Take any $S \subseteq V(amal(G)) = \vee_v^1\{P_n, C_m\}$ with $|S| < |W_l|$. Without reduce the generality of proof, assume that $|S| = |W_l| - 1$ with $|W_l| = Ddim_l(P_n) + Ddim_l(C_m) - 1 - 1$, then there exist one vertex which is not neighbors with S. Therefore, S is not a dominant local resolving set of $amal(S_n, v, k)$. **By Lemma 1**, any set $|T|$ with $|S|$ is not a dominant local resolving set of G Then, $W_l = Ddim_l(P_n) + Ddim_l(C_m) - 1$, is a local basis of $amal(G) = \vee_v^1\{P_n, C_m\}$. So, it was proven that $Ddim_l = Ddim_l(P_n) + Ddim_l(C_m) - 1$. □

Observation 2. Let $amal(G) = \vee_v^1\{P_n, C_m, S_k\}$ be the vertex amalgamation of a path, cycle, and star graph for $m, n, k \geq 3$, where v is a leaf of the path which is attached to any vertex of the cycle and the center vertex of the star. Then:

$$Ddim_l(amal(G)) = \begin{cases} Ddim_l(P_n) + Ddim_l(C_m), m = n; m, n \equiv 0 \pmod 3, \\ Ddim_l(P_n) + Ddim_l(C_m) - 1, m = n; m, n \not\equiv 0 \pmod 3. \end{cases}$$

a. Case 1. $m, n \equiv 0 \pmod 3$

By attaching the center point of Star graph to $amal(G) = \bigvee_v^1\{P_n, C_m\}$ and the proof of case $m, n \equiv 0 \pmod 3$ in Theorem 1 with $v \in W_l$, then it clearly that adding the star graph is not change for choosing dominant local basis of $amal(G) = \bigvee_v^1\{P_n, C_m, S_k\}$. All the pendant of star graph can be dominated by W_l because $v \in W_l$. □

b. Case 2. $m, n \not\equiv 0 \pmod 3$

In line with the case in point a., we have $Ddim_l(amal(G)) = Ddim_l(P_n) + Ddim_l(C_m) - 1$, where $m = n$ and $m, n \not\equiv 0 \pmod 3$. □

Observation 3. Let $amal(G) = \bigvee_v^1\{P_n, C_m, S_k, K_{x,y}\}$ be the vertex amalgamation of path, cycle, star, and complete bipartite graph for $m, n, k \geq 3$, and v is the leaf of path graph which attache to any vertex of cycle and the center point of star, then:

$$Ddim_l(amal(G)) = \begin{cases} Ddim_l(P_n) + Ddim_l(C_m) + 1, m = n; m, n \equiv 0 \pmod 3, \\ Ddim_l(P_n) + Ddim_l(C_m), m = n; m, n \not\equiv 0 \pmod 3. \end{cases}$$

a. **Case 1.** $m, n \equiv 0 \pmod 3$
   In line with the case in Observation 2 point a and complete bipartite graph, $Ddim_l(K_{x,y}) = 2$, then it is necessary to add one vertex element of W_l from one of the complete Bipartite graph partitions other than v. Then,
   $Ddim_l(amal(G) = \bigvee_v^1\{P_n, C_m, S_k, K_{x,y}\}) = Ddim_l(amal(G) = \bigvee_v^1\{P_n, C_m, S_k\}) + 1 = Ddim_l(P_n) + Ddim_l(C_m) + 1, m = n; m, n \equiv 0 \pmod 3$
b. **Case 2.** $m, n \not\equiv 0 \pmod 3$
   Likewise for the case $m, n \not\equiv 0 \pmod 3$, then
   $Ddim_l(amal(G) = \bigvee_v^1\{P_n, C_m, S_k, K_{x,y}\}) = Ddim_l(amal(G) = \bigvee_v^1\{P_n, C_m, S_k\}) - 1 + 1 = Ddim_l(P_n) + Ddim_l(C_m), m = n; m, n \not\equiv 0 \pmod 3$

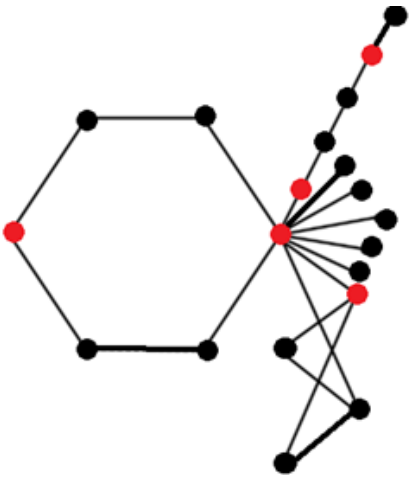


Figure 3. Graf $amal(G) = \bigvee_v^1\{P_n, C_m, S_k, K_{(x,y)}\}$

3.2. The Local Resolving Set of Network Design Results Graph for a Building

Next, in this section, several examples of network types are shown. The first network can be seen in Figure 4 The network design has the same form as the graph $amal(G) = \bigvee_v^1\{P_4, C_4\}$, so the graphic representation can be seen in Figure 5 With the theory of dominant local resolving set, 3 computer devices can be selected that can be used as the main network service provider devices. The code for each device can be made as follows:

For example, the network service providers = $w_1 = \{v_2, v, u_1\}$, then the network code that can be created is:

$$\begin{aligned} r(v_1|W_1) &= (1,3,4);\\ r(v_2|W_1) &= (0,2,3);\\ r(v_3|W_1) &= (1,1,2);\\ r(v|W_1) &= (2,0,1);\\ r(u_1|W_1) &= (3,1,0);\\ r(u_2|W_1) &= (4,2,1);\\ r(u_3|W_1) &= (3,1,2); \end{aligned}$$

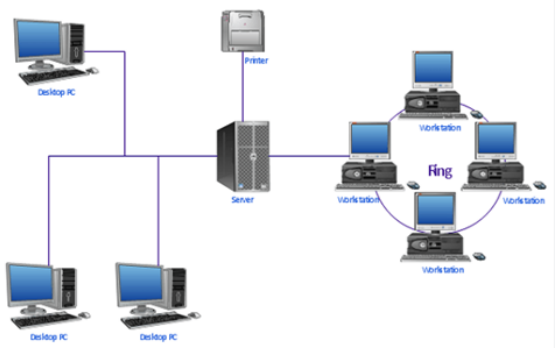


Figure 4 First Example of Computer Network

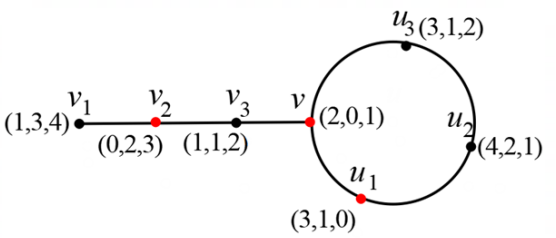


Figure 5 First Computer Network Representation

The second network example can be seen in Figure 6, so its graph representation can be seen in Figure 7. With the theory of dominant local resolving set, 7 computer devices can be selected that can be used as the main network service provider devices. The code for each device can be made as follows:

For example, the network service providers = $w_2 = d, e, h, k, l, n, r$, then the network code that can be created is:

$$\begin{aligned} r(a|W_2) &= (1,2,4,4,5,5,5)\\ r(b|W_2) &= (1,2,4,4,5,5,5)\\ r(c|W_2) &= (1,2,4,4,5,5,5)\\ r(d|W_2) &= (0,1,3,3,4,4,4) \end{aligned}$$

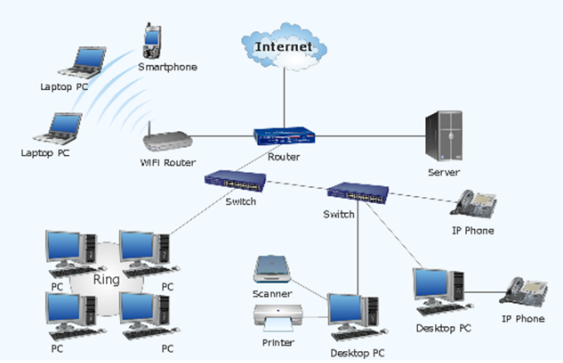


Figure 6 Second Example of Computer Network

$$
\begin{aligned}
r(e|W_2) &= (1,0,2,2,3,3,3)\\
r(f|W_2) &= (2,1,3,3,4,4,4)\\
r(g|W_2) &= (2,1,1,1,2,2,2)\\
r(h|W_2) &= (3,2,0,2,3,1,1)\\
r(i|W_2) &= (4,3,1,3,4,2,2)\\
r(j|W_2) &= (4,3,3,1,2,4,4)\\
r(k|W_2) &= (3,2,2,0,1,3,3)\\
r(l|W_2) &= (4,3,3,1,0,4,4)\\
r(m|W_2) &= (5,4,4,2,1,5,5)\\
r(n|W_2) &= (4,3,1,3,4,0,2)\\
r(o|W_2) &= (5,4,2,4,5,1,3)\\
r(p|W_2) &= (5,4,2,4,5,1,3)\\
r(q|W_2) &= (5,4,2,4,5,3,1)\\
r(r|W_2) &= (4,3,1,3,4,2,0)
\end{aligned}
$$

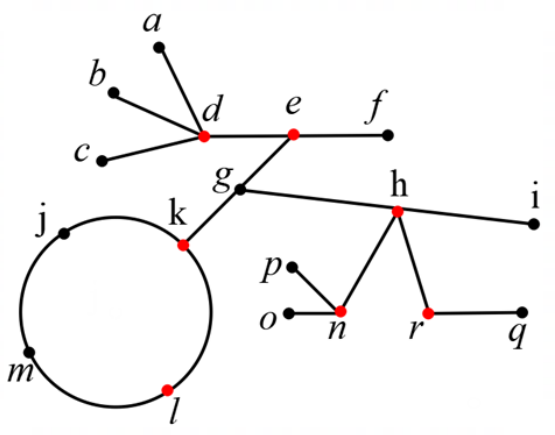


Figure 3.5 Second Computer Network Representation

First computer network with 7 devices requires at least 3 devices as the main network service provider, while the second computer network with 18 devices requires at least 7 devices as the main service provider. Thus, determining the number of devices as the main network service provider depends on the network pattern/design.

4. Conclusion

This study successfully applied the theory of dominant local resolving sets to optimize computer network design, specifically within vertex amalgamation structures. We concluded that the dominant local metric dimension of an amalgamated network is directly determined by the sum of the dimensions of its constituent graphs, conditioned by the modularity of their cycle lengths and the choice of the linkage vertex. Our comparative analysis demonstrates that Star topology networks offer the highest efficiency, requiring the fewest service providers, whereas fully connected meshes require the most. These findings provide a theoretical basis for network architects to design cost-efficient, identifiable, and secure multi-building network infrastructures. Future work will extend this framework to dynamic wireless sensor networks and investigate the fault-tolerance of these resolving sets under node failure conditions.

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