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A generalized bivariate Fibonacci-like polynomial and a new subclass of bi-univalent functions with Neutrosophic Poisson distribution

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Abstract A new subclass of bi-univalent functions is introduced in this paper through the use of the Neutrosophic Poisson Distribution and the creation of generalized bivariate Fibonacci-like polynomials. These polynomials include Horadam and Chebyshev. The first Taylor-Maclaurin coefficients of these functions are identified, and bounds for those coefficients are established. Additionally, the well-known Fekete-Szegő problem and its implications for these subclasses are investigated in this paper. Furthermore, several observations are taken for particular values of the variables, which allow for a more in-depth understanding of the characteristics of this newly created subclass.

Keywords Generalized bivariate Fibonacci-like Polynomial, bi-univalent functions, regular functions, Fekete-Szegö problem.

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1. INTRODUCTION

Legendre initially presented orthogonal polynomials in 1784 [9]. Under particular model restrictions, they are frequently used to solve ordinary differential equations. Furthermore, orthogonal polynomials are essential to approximation theory [3, 5].

Many mathematicians have been greatly impacted by the application of specific special polynomials in geometric function theory, including Chebyshev, Faber, Horadam, Lucas, and Fibonacci. Consequently, the literature has presented several subclasses of univalent and bi-univalent functions that are holomorphic in the unit disc.

For these new subclasses of holomorphic functions, well-known problems like initial coefficient estimates, the Fekete-Szegö problem, and the Hankel determinant problem have been investigated using the well-known concept of subordination and some basic geometric function theory concepts in complex analysis.

This study presents a novel subfamily of holomorphic and bi-univalent functions, inspired by the relationships between classes of holomorphic functions and special polynomials. Next, we get upper limits for these subclasses' functions' first two coefficients. For the specified function classes, we also tackle the Fekete-Szegö issue. Lastly, depending on the primary findings, we offer a number of corollaries and observations.

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Let $\mathcal{A}$ be the class of regular functions $f\ (z)$ of the form

$$f(z) = z + \rho_2 z^2 + \rho_3 z^3 + \cdots = z + \sum_{n=2}^{\infty} \rho_n z^n. \tag{1.1}$$

are univalent in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and satisfy the conditions $f(0) = 0$ and $f'(0) = 1$ which are normalized conditions.

Let S denote the subclass of $\mathcal{A}$ consisting functions of the form (1.1).

Due to the well-known Koebe one quarter theorem [14] is be said that $f \in S$ has an inverse f^{-1} defined by $f^{-1}(f(z)) = z$, $(z \in \mathbb{U})$ and $f(f^{-1}(w)) = w$, $(|w| < r_0(f), r_0(f) \geq \frac{1}{4})$, where

$$f^{-1}(w) = w - \rho_2 w^2 + \left(2\rho_2^2 - \rho_3\right) w^3 - \left(5\rho_2^3 - 5\rho_2\rho_3 + \rho_4\right) w^4 + \cdots = g(w). \tag{1.2}$$

It is well-known if a function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ then the both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ stands for the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). In fact, very recently Srivastava et al. [11]. Have work actually others are reviving the study of holomorphic and bi-univalent functions in recent years, see for example, [2, 4, 7, 12, 17, 18].

Generalized Bivariate Fibonacci-like Polynomial. Chebyshev, Faber, Horadam, Lucas, and Fibonacci polynomials, and their generalizations hold significant importance in applied sciences, including physics, engineering, and related fields. Among these, the Fibonacci polynomial is one of the most prominent special polynomials. Due to its extensive applications in the applied sciences, several generalizations of the Fibonacci polynomial have been introduced in the literature (see [22]).

The following recurrence relation defines the Fibonacci numbers:

$$F_n = F_{n-1} + F_{n-2}; F_0 = 0; F_1 = 1,$$

for $n \geq 2$, readers can find a concise history and comprehensive information about the generalized bivariate Fibonacci polynomial in [21] and its references. Additionally, the authors in [21] introduced a new generalization of the Fibonacci polynomial, referred to as the generalized bivariate Fibonacci-like polynomial.

Let p, q be positive integers and x, y be real numbers. For $n \geq 2$, the generalized bivariate Fibonacci-like polynomials are defined by the following recurrence relation:

$$H_n(x, y) = pxH_{n-1}(x, y) + qyH_{n-2}(x, y), \tag{1.3}$$

where $H_0(x, y) = a$; $H_1(x, y) = b$ and $px,\ qy \neq 0$, $p^2x^2\ +4qy \neq 0$. The generating functions of generalized bivariate Fibonacci-like polynomials is (see [21])

$$H^{(x,y)}(z) = \sum_{n=0}^{\infty} H_0(x, y) z^n = \frac{a + (b - apx)z}{1 - pxz - qyz^2}, \tag{1.4}$$

he first three generalized bivariate Fibonacci-like polynomials are expressed as follows:

$$H_0(x, y) = a,\ H_1(x, y) = b,\ H_2(x, y) = pbx + aqy. \tag{1.5}$$

By selecting different values for p, q, a, b, and y, various polynomial sequences can be derived using the recurrence relation. These polynomial sequences are summarized in Table 1 below:

(p,q)	(a,b)	(x,y)	$H_n(x,y)$
$(1,1)$	$(0,1)$	(x,y)	Bivariate Fibonacci, $F_n(x,y)$
$(1,1)$	$(0,1)$	$(x,1)$	Fibonacci, $F_n(x)$
$(2,1)$	$(0,1)$	$(x,1)$	Pell, $P_n(x)$
$(1,1)$	$(2,x)$	(x,y)	Bivariate Lucas, $L_n(x,y)$
$(2,1)$	$(1,2t)$	$(t,-1)$	Chebyshev of the second kind, $U_n(x)$
(p,q)	(a,bx)	$(x,1)$	Horadam, $H_{n+1}(x)$

Table 1: Special cases of the generalized bivariate Fibonacci-like polynomials

The Poisson, Pascal, Borel, Mittag-Leffler-type Poisson distributions, among others, have been extensively used in recent studies to explore key aspects of geometric function theory (see, [20, 19, 30, 28, 27, 26, 24, 25, 31, 32, 33]). These studies include topics such as coefficient estimates, inclusion relations, and criteria for membership in specific function classes.

In 1995, Neutrosophic theory was introduced by Smarandache as a new branch of philosophy, serving as a generalization of both fuzzy logic and intrinsic fuzzy logic (see [23]). While the neutrosophic Poisson distribution of a discrete variable X is fundamentally a classical Poisson distribution, its parameter m is not fixed. Instead, m can be specified as a range or interval, encompassing two or more elements. This type of distribution is most commonly encountered when m is represented as an interval. Let

$$NP\left(X=d\right)=\frac{(m)^{d}}{d!}e^{-m},\ \ d=0,1,2,..., \tag{1.6}$$

where the expected value and the variance, denoted by the distribution parameter m

$$NL(X)=NV(X)=m\ ,$$

a neutrosophic statistical number is $N=O+I$; for more information, see [23] and the sources therein. We are going to provide a new power series, and the coefficients of this series will be the probabilities of the Neutrosophic Poisson Distribution

$$\mathbb{B}(m,z)=z+\sum_{n=2}^{\infty}\frac{(m)^{n-1}e^{-m}}{(n-1)!}z^{n},\qquad z\in\mathbb{U}. \tag{1.7}$$

Consider the linear operator $\mathbb{P}_m:\mathcal{A}\to\mathcal{A}$ defined by the convolution

$$\mathbb{P}_{l}f(z)=\mathbb{B}(m,z)*f(z)=z+\sum_{n=2}^{\infty}\frac{(m)^{n-1}e^{-m}}{(n-1)!}\rho_{n}z^{n},\qquad z\in\mathbb{U}. \tag{1.8}$$

Remark. Although the Neutrosophic Poisson distribution is introduced in a general form where the parameter m may belong to an interval, in this work we consider a fixed representative value of m for the purpose of constructing a well-defined analytic operator and obtaining explicit coefficient estimates.

This assumption is standard in the literature of geometric function theory to ensure mathematical tractability while preserving the structural features of the underlying probabilistic model. Therefore, throughout this paper, m is treated as a positive constant and the operator $\mathbb{P}_m$ is used in its classical sense.

Nowadays, many researchers have been studying bi-univalent functions associated with orthogonal polynomials, few to mention [1, 13, 8, 16, 15, 19, 20, 29].

Nevertheless, there are very few results about the generalised bivariate Fibonacci-like polynomials in this context, despite the large body of work on bi-univalent functions linked with special polynomials. Specifically, subclasses defined by subordination to these polynomials in conjunction with probabilistic operators like the neutrosophic Poisson distribution have not been thoroughly studied.The primary goal of this study is to bring together two significant areas of geometric function theory: (i) probabilistic operators originating from neutrosophic distributions and (ii) special polynomial structures, especially generalised bivariate Fibonacci-like polynomials. This combination enables us to obtain more precise coefficient estimates and functional inequalities, as well as a novel analytical framework for building and researching subclasses of bi-univalent functions.This paper's primary contributions may be summed up as follows:A linear operator related to the neutrosophic Poisson distribution and generalised bivariate Fibonacci-like polynomials are used to establish a new subclass of bi-univalent functions. We derive estimates of the coefficients for $|\rho_2|$ and $|\rho_3|$. For the suggested class, the Fekete–Szeg"o functional issue is examined. By selecting suitable parameter values, a number of special situations and corollaries are produced.

Lastly, we investigate novel subclasses of bi-univalent functions and provide coefficient bounds for the Fekete-Szeg"o issue, $|\rho_2|$, and $|\rho_3|$.

2. Definition and Examples

The following new subclasses of bi-univalent functions are presented in this section:

Definition 2.1. Suppose that $ß \geq 0,\ -\pi < \varphi \leq \pi$, and $0 \leq Y \leq 1$: It is claimed that a function $f \in \Sigma$ of the type (1.1) belongs to the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$ if the subordinations listed below are true:

$$\left(1+ße^{i\varphi}\right)\left\{(1-Y)\frac{\mathbb{P}_l f(z)}{z}+Y\left(\mathbb{P}_l f(z)\right)'\right\}-ße^{i\varphi}\prec(h(z))$$

and

$$\left(1+ße^{i\varphi}\right)\left\{(1-Y)\frac{\mathbb{P}_l f(w)}{w}+Y\left(\mathbb{P}_l f(w)\right)'\right\}-ße^{i\varphi}\prec(h(w)),$$

where the function g has the form (1.2), $z;w \in \mathbb{U}$, and $p^2x^2+4qy>0$.

Example 2.2
Let $Y=0$, a function $f \in \Sigma$ is said to be in the family $\mathfrak{G}_{ß,\varphi,n,\Sigma,0}(p,q,x,y)(h(z))$ if it satisfies the subordinations:

$$\left(1+ße^{i\varphi}\right)\left\{\frac{\mathbb{P}_l f(z)}{z}\right\}-ße^{i\varphi}\prec(h(z))$$

and

$$\left(1+ße^{i\varphi}\right)\left\{\frac{\mathbb{P}_l f(w)}{w}\right\}-ße^{i\varphi}\prec(h(w)),$$

where the function g has the form (1.2), $z;w \in \mathbb{U}$, and $p^2x^2+4qy>0$.

Example 2.3
Let $Y=1$, a function $f \in \Sigma$ is said to be in the family $\mathfrak{G}_{ß,\varphi,n,\Sigma,1}(p,q,x,y)(h(z))$ if it satisfies the subordinations:

$$\left(1+ße^{i\varphi}\right)\left\{\left(\mathbb{P}_l f(z)\right)'\right\}-ße^{i\varphi}\prec(h(z))$$

and

$$\left(1+ße^{i\varphi}\right)\left\{\left(\mathbb{P}_l f(w)\right)'\right\}-ße^{i\varphi}\prec(h(w)),$$

where the function g has the form (1.2), $z;w \in \mathbb{U}$, and $p^2x^2+4qy>0$.

Example 2.4
Let $ß=0$, a function $f \in \Sigma$ is said to be in the family $\mathfrak{G}_{0,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$ if it satisfies the subordinations:

$$(1-Y)\frac{\mathbb{P}_l f(z)}{z}+Y\left(\mathbb{P}_l f(z)\right)'\prec(h(z))$$

and

$$(1-Y)\frac{\mathbb{P}_l f(w)}{w} + Y\left(\mathbb{P}_l f(w)\right)' \prec (h(w)),$$

where the function g has the form (1.2), $z; w \in \mathbb{U}$, and $p^2x^2 + 4qy > 0$.

Example 2.5
Let $Y = ß = 0$, a function $f \in \Sigma$ is said to be in the family $\mathfrak{G}_{0,\varphi,n,\Sigma,0}(p,q,x,y)(h(z))$ if it satisfies the subordinations:

$$\frac{\mathbb{P}_l f(z)}{z} \prec (h(z))$$

and

$$\frac{\mathbb{P}_l f(w)}{w} \prec (h(w)),$$

where the function g has the form (1.2), $z; w \in \mathbb{U}$, and $p^2x^2 + 4qy > 0$.

Example 2.6
Let $Y = 1$ and $ß = 0$, a function $f \in \Sigma$ is said to be in the family $\mathfrak{G}_{0,\varphi,n,\Sigma,1}(p,q,x,y)(h(z))$ if it satisfies the subordinations:

$$\left(\mathbb{P}_l f(z)\right)' \prec (h(z))$$

and

$$\left(\mathbb{P}_l f(w)\right)' \prec (h(w)),$$

where the function g has the form (1.2), $z; w \in \mathbb{U}$, and $p^2x^2 + 4qy > 0$.

3. Coefficient bounds of the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$ and special case

First, we give the coefficient estimates for the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$ given in Definition 2.1.

Theorem 3.1
let $f \in \mathcal{A}$ by given (1.1) and $Y \in \mathbb{R}$ then $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$ and

$$|\rho_2| \leq \frac{|b|\sqrt{|2b|}}{\sqrt{\left|\left[\left(1+ße^{i\varphi}\right)(1+2Y)b^2 - 2(pbx+aqy)\left(1+ße^{i\varphi}\right)^2(1+Y)^2e^{-m}\right]m^2e^{-m}\right|}}.$$

and

$$|\rho_3| \leq \frac{b}{\left(1+ße^{i\varphi}\right)(1+2Y)m^2e^{-m}} + \frac{b^2}{\left(1+ße^{i\varphi}\right)^2(1+Y)^2m^2e^{-2m}}.$$

Proof
Let $f \in \mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$. Then there are two analytic functions ϕ, ϱ given by

$$\phi(z) = c_1z + c_2z^2 + c_3z^3 + \cdots \quad (z \in \mathbb{U}) \tag{3.1}$$

and

$$\varrho(w) = d_1w + d_2w^2 + d_3w^3 + \cdots \quad (w \in \mathbb{U}), \tag{3.2}$$

with $\phi(0) = \varrho(0) = 0$, $|\phi(z)| < 1$, $|\varrho(w)| < 1$, $z, w \in \mathbb{U}$ such that

$$\left(1+ße^{i\varphi}\right)\left\{(1-Y)\frac{\mathbb{P}_l f(z)}{z} + Y\left(\mathbb{P}_l f(z)\right)'\right\} - ße^{i\varphi} = h(x,\phi(z)) \tag{3.3}$$

and

$$\left(1+ße^{i\varphi}\right)\left\{(1-Y)\frac{\mathbb{P}_l f(w)}{w}+Y\left(\mathbb{P}_l f(w)\right)'\right\}-ße^{i\varphi}=h(x,\varrho(w)). \tag{3.4}$$

Combining (3.1), (3.2) and (3.3) yields

$$\begin{aligned}&\left(1+ße^{i\varphi}\right)\left\{(1-Y)\frac{\mathbb{P}_l f(z)}{z}+Y\left(\mathbb{P}_l f(z)\right)'\right\}-ße^{i\varphi}\\&=1+H_1(x,y)c_1z+\left[H_1(x,y)c_2+H_2(x,y)c_1^2\right]z^2+\cdots\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}&\left(1+ße^{i\varphi}\right)\left\{(1-Y)\frac{\mathbb{P}_l f(w)}{w}+Y\left(\mathbb{P}_l f(w)\right)'\right\}-ße^{i\varphi}\\&=1+H_1(x,y)d_1w+\left[H_1(x,y)d_2+H_2(x,y)d_1^2\right])w^2+\cdots.\end{aligned} \tag{3.6}$$

It is quite well-known that if $|\phi(z)|<1$ and $|\varrho(w)|<1$, $z,w\in\mathbb{U}$, then

$$|c_i|\leq 1 \quad\text{and}\quad |d_i|\leq 1 \text{ for all } i\in\mathbb{N}. \tag{3.7}$$

Comparing the corresponding coefficients in (3.5) and (3.6), after simplifying, we have

$$\left(1+ße^{i\varphi}\right)(1+Y)me^{-m}\rho_2=H_1(x,y)c_1, \tag{3.8}$$

$$\frac{1}{2}\left(1+ße^{i\varphi}\right)(1+2Y)m^2e^{-m}\rho_3=H_1(x,y)c_2+H_2(x,y)c_1^2, \tag{3.9}$$

$$-\left(1+ße^{i\varphi}\right)(1+Y)me^{-m}\rho_2=H_1(x,y)d_1, \tag{3.10}$$

and

$$\frac{1}{2}\left(1+ße^{i\varphi}\right)(1+2Y)m^2e^{-m}\left[2\rho_2^2-\rho_3\right]=H_1(x,y)d_2+H_2(x,y)d_1^2. \tag{3.11}$$

It follows from (3.8) and (3.10) that

$$c_1=-d_1 \tag{3.12}$$

and

$$2\left(1+ße^{i\varphi}\right)^2(1+Y)^2m^2e^{-2m}\rho_2^2=\left[H_1(x,y)\right]^2\left(c_1^2+d_1^2\right). \tag{3.13}$$

If we add (3.9) to (3.11), we find that

$$\left(1+ße^{i\varphi}\right)(1+2Y)m^2e^{-m}\rho_2^2=H_1(x,y)\left(c_2+d_2\right)+H_2(x,y)\left(c_1^2+d_1^2\right). \tag{3.14}$$

Substituting the value of $c_1^2+d_1^2$ from (3.13) in the right hand side of (3.14), we deduce that

$$\rho_2^2=\frac{\left[H_1(x,y)\right]^3(c_2+d_2)}{\left[\left(H_1(x,y)\right)^2\left(1+ße^{i\varphi}\right)(1+2Y)-2H_2(x,y)\left(1+ße^{i\varphi}\right)^2(1+Y)^2e^{-m}\right]m^2e^{-m}}. \tag{3.15}$$

Further computations using (1.5), (3.7) and (3.15), we obtain

$$|\rho_2|\leq\frac{|b|\sqrt{|2b|}}{\sqrt{\left|\left[\left(1+ße^{i\varphi}\right)(1+2Y)b^2-2(pbx+aqy)\left(1+ße^{i\varphi}\right)^2(1+Y)^2e^{-m}\right]m^2e^{-m}\right|}}.$$

Next, if we subtract (3.11) from (3.9), we can easily see that

$$\left(1+ße^{i\varphi}\right)(1+2Y)m^2e^{-m}(\rho_3-\rho_2^2)=H_1(x,y)(c_2-d_2)+H_2(x,y)(c_1^2-d_1^2). \tag{3.16}$$

In view of (3.12) and (3.13), we get from (3.16)

$$\rho_3 = \frac{H_1(x,y)(c_2-d_2)}{(1+ßе^{i\varphi})\,(1+2Y)m^2e^{-m}} + \frac{[H_1(x,y)]^2\,(c_1^2+d_1^2)}{2\,(1+ße^{i\varphi})^2\,(1+Y)^2m^2e^{-2m}}.$$

Thus applying (1.5), we obtain

$$|\rho_3| \leq \frac{b}{(1+ße^{i\varphi})\,(1+2Y)m^2e^{-m}} + \frac{b^2}{(1+ße^{i\varphi})^2\,(1+Y)^2m^2e^{-2m}}.$$

This completes the proof of Theorem 3.1. $\square$

Remark 3.2. By giving different values to the parameters in Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the generalized bivariate Fibonacci-like polynomials, respectively.

$i)$ Assume that $Y=0$, let (1.1) be in the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,0}(p,q,x,y)(h(z))$.Then

$$|\rho_2| \leq \frac{|b|\,\sqrt{|2b|}}{\sqrt{\left|\left[(1+ße^{i\varphi})\,b^2 - 2\,(1+ße^{i\varphi})^2\,e^{-m}(pbx+aqy)\right]\,m^2e^{-m}\right|}},$$

and

$$|\rho_3| \leq \frac{|2b|}{(1+ße^{i\varphi})\,m^2e^{-m}} + \frac{b^2}{(1+ße^{i\varphi})^2\,m^2e^{-2m}}.$$

$ii)$ Assume that $Y=1$, let (1.1) be in the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,1}(p,q,x,y)(h(z))$.Then

$$|\rho_2| \leq \frac{|b|\,\sqrt{|2b|}}{\sqrt{\left|\left[3\,(1+ße^{i\varphi})\,b^2 - 8\,(1+ße^{i\varphi})^2\,e^{-m}(pbx+aqy)\right]\,m^2e^{-m}\right|}},$$

and

$$|\rho_3| \leq \frac{|2b|}{3\,(1+ße^{i\varphi})\,m^2e^{-m}} + \frac{b^2}{4\,(1+ße^{i\varphi})^2\,m^2e^{-2m}}.$$

Remark 3.3. By giving $b=bx$, $y=1$ in class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$ of Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the Horadam polynomials, respectively.

$i)$ Assume that $Y=0$, let (1.1) be in the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,0}(p,q,x,1)(h(z))$.Then

$$|\rho_2| \leq \frac{|bx|\,\sqrt{|2bx|}}{\sqrt{\left|\left[(1+ße^{i\varphi})\,(bx)^2 - 2\,(1+ße^{i\varphi})^2\,e^{-m}(pbx^2+aq)\right]\,m^2e^{-m}\right|}},$$

and

$$|\rho_3| \leq \frac{|2bx|}{(1+ße^{i\varphi})\,m^2e^{-m}} + \frac{(bx)^2}{(1+ße^{i\varphi})^2\,m^2e^{-2m}}.$$

$ii)$ Assume that $Y=1$, let (1.1) be in the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,1}(p,q,x,1)(h(z))$.Then

$$|\rho_2| \leq \frac{|bx|\,\sqrt{|2bx|}}{\sqrt{\left|\left[3\,(1+ße^{i\varphi})\,(bx)^2 - 8\,(1+ße^{i\varphi})^2\,e^{-m}(pbx^2+aq)\right]\,m^2e^{-m}\right|}},$$

and

$$|\rho_3| \leq \frac{|2bx|}{3\left(1+ße^{i\varphi}\right)m^2e^{-m}} + \frac{(bx)^2}{4\left(1+ße^{i\varphi}\right)^2 m^2e^{-2m}}.$$

Remark 3.4. By giving $p=q=1, a=0, b=1, y=1$ in class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$ of Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the Fibonacci polynomials, respectively.

$i)$ Assume that $Y=0$, let (1.1) be in the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,0}(1,1,x,1)(h(z))$.Then

$$|\rho_2| \leq \frac{\sqrt{2}}{\sqrt{\left|\left[\left(1+ße^{i\varphi}\right)-2\left(1+ße^{i\varphi}\right)^2 e^{-m}x\right]m^2e^{-m}\right|}},$$

and

$$|\rho_3| \leq \frac{2}{\left(1+ße^{i\varphi}\right)m^2e^{-m}} + \frac{1}{\left(1+ße^{i\varphi}\right)^2 m^2e^{-2m}}.$$

$ii)$ Assume that $Y=1$, let (1.1) be in the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,1}(1,1,x,1)(h(z))$.Then

$$|\rho_2| \leq \frac{\sqrt{2}}{\sqrt{\left|\left[3\left(1+ße^{i\varphi}\right)-8\left(1+ße^{i\varphi}\right)^2 e^{-m}x\right]m^2e^{-m}\right|}},$$

and

$$|\rho_3| \leq \frac{2}{3\left(1+ße^{i\varphi}\right)m^2e^{-m}} + \frac{1}{4\left(1+ße^{i\varphi}\right)^2 m^2e^{-2m}}.$$

Remark 3.5. By giving different values to the parameters in Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the generalized bivariate Fibonacci-like polynomials, respectively.

$i)$ Assume that $ß=0$, let (1.1) be in the class $\mathfrak{G}_{0,\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$.Then

$$|\rho_2| \leq \frac{|b|\sqrt{|2b|}}{\sqrt{|[(1+2Y)b^2-2(pbx+aqy)(1+Y)^2e^{-m}]\,m^2e^{-m}|}}.$$

and

$$|\rho_3| \leq \frac{b}{(1+2Y)m^2e^{-m}} + \frac{b^2}{(1+Y)^2m^2e^{-2m}}.$$

Remark 3.6. By giving different values to the parameters in Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the generalized bivariate Fibonacci-like polynomials, respectively.

$i)$ Assume that $Y=ß=0$, let (1.1) be in the class $\mathfrak{G}_{0,\varphi,n,\Sigma,0}(p,q,x,y)(h(z))$.Then

$$|\rho_2| \leq \frac{|b|\sqrt{|2b|}}{\sqrt{|[b^2-2e^{-m}(pbx+aqy)]\,m^2e^{-m}|}},$$

and

$$|\rho_3| \leq \frac{|2b|}{m^2e^{-m}} + \frac{b^2}{m^2e^{-2m}}.$$

$ii)$ Assume that $Y=1$ and $ß=0$, let (1.1) be in the class $\mathfrak{G}_{0,\varphi,n,\Sigma,1}(p,q,x,y)(h(z))$.Then

$$|\rho_2| \leq \frac{|b|\sqrt{|2b|}}{\sqrt{|[3b^2 - 8e^{-m}(pbx + aqy)]\, m^2 e^{-m}|}},$$

and

$$|\rho_3| \leq \frac{|2b|}{3m^2 e^{-m}} + \frac{b^2}{4m^2 e^{-2m}}.$$

Remark 3.7. By giving $b = bx, y = 1$ in class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p, q, x, y)(h(z))$ of Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the Horadam polynomials, respectively.

$i)$ Assume that $Y = ß = 0$, let (1.1) be in the class $\mathfrak{G}_{0,\varphi,n,\Sigma,0}(p, q, x, 1)(h(z))$.Then

$$|\rho_2| \leq \frac{|bx|\sqrt{|2bx|}}{\sqrt{|[(bx)^2 - 2e^{-m}(pbx^2 + aq)]\, m^2 e^{-m}|}},$$

and

$$|\rho_3| \leq \frac{|2bx|}{m^2 e^{-m}} + \frac{(bx)^2}{m^2 e^{-2m}}.$$

$ii)$ Assume that $Y = 1$ and $ß = 0$, let (1.1) be in the class $\mathfrak{G}_{0,\varphi,n,\Sigma,1}(p, q, x, 1)(h(z))$.Then

$$|\rho_2| \leq \frac{|bx|\sqrt{|2bx|}}{\sqrt{|[3(bx)^2 - 8e^{-m}(pbx^2 + aq)]\, m^2 e^{-m}|}},$$

and

$$|\rho_3| \leq \frac{|2bx|}{3m^2 e^{-m}} + \frac{(bx)^2}{4m^2 e^{-2m}}.$$

Remark 3.8. By giving $p = q = 1, a = 0, b = 1, y = 1$ in class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p, q, x, y)(h(z))$ of Theorem 3.1, we obtain some bounds on the coefficients $|\rho_2|$ and $|\rho_3|$ of bi-starlike and bi-convex functions defined by the Fibonacci polynomials, respectively.

$i)$ Assume that $Y = ß = 0$, let (1.1) be in the class $\mathfrak{G}_{0,\varphi,n,\Sigma,0}(1, 1, x, 1)(h(z))$.Then

$$|\rho_2| \leq \frac{\sqrt{2}}{\sqrt{|[1 - 2e^{-m}x]\, m^2 e^{-m}|}},$$

and

$$|\rho_3| \leq \frac{2}{m^2 e^{-m}} + \frac{1}{m^2 e^{-2m}}.$$

$ii)$ Assume that $Y = 1$ and $ß = 0$, let (1.1) be in the class $\mathfrak{G}_{0,\varphi,n,\Sigma,1}(1, 1, x, 1)(h(z))$.Then

$$|\rho_2| \leq \frac{\sqrt{2}}{\sqrt{|[3 - 8e^{-m}x]\, m^2 e^{-m}|}},$$

and

$$|\rho_3| \leq \frac{2}{3m^2 e^{-m}} + \frac{1}{4m^2 e^{-2m}}.$$

4. Fekete–Szegö inequality for the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p, q, x, y)(h(z))$

Fekete-Szegö inequality is one of the famous problems related to coefficients of univalent analytic functions [6]. In this section, our aim is to study Fekete-Szegö type inequality for the function in the class $\mathfrak{G}_{ß,\varphi,n,\Sigma,Y}(p, q, x, y)(h(z))$.

Theorem 4.1
let $f \in \mathcal{A}$ by given (1.1) and $Y \in \mathbb{R}$ then $f \in \mathfrak{G}_{\text{ß},\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$ and

$$\left|\rho_3-\eta\rho_2^2\right| \leq \begin{cases} \frac{|2b|}{(1+\text{ß}e^{i\varphi})(1+2Y)m^2e^{-m}}, & |\eta-1|\leq \ell \\ \frac{2|b|^3|1-\eta|}{\left|\left[(1+\text{ß}e^{i\varphi})(1+2Y)b^2-2(pbx+aqy)(1+\text{ß}e^{i\varphi})^2(1+Y)^2e^{-m}\right]m^2e^{-m}\right|}, & |\eta-1|\geq \ell, \end{cases}$$

where

$$\ell = \left|1-\frac{\left[2\left(1+\text{ß}e^{i\varphi}\right)(1+Y)^2(pbx+aqy)\right]e^{-m}}{(1+2Y)\,b^2}\right|$$

Proof
It follows from (3.15) and (3.16) that

$$\begin{aligned}
&\rho_3-\eta\rho_2^2\\
&=\frac{H_1(x,y)(c_2-d_2)}{(1+\text{ß}e^{i\varphi})(1+2Y)\,m^2e^{-m}}+(1-\eta)\,\rho_2^2\\
&=\frac{H_1(x,y)(c_2-d_2)}{(1+\text{ß}e^{i\varphi})(1+2Y)\,m^2e^{-m}}\\
&+\frac{\left[H_1(x,y)\right]^3(c_2+d_2)(1-\eta)}{\left[(H_1(x,y))^2(1+\text{ß}e^{i\varphi})(1+2Y)-2H_2(x,y)(1+Y)^2\left(1+\text{ß}e^{i\varphi}\right)^2e^{-m}\right]m^2e^{-m}}\\
&=H_1(x,y)\left[\left(\top(n)+\frac{1}{(1+2Y)(1+\text{ß}e^{i\varphi})\,m^2e^{-m}}\right)c_2+\left(\top(n)-\frac{1}{(1+2Y)(1+\text{ß}e^{i\varphi})\,m^2e^{-m}}\right)d_2\right],
\end{aligned}$$

where

$$\top(n)=\frac{\left[H_1(x,y)\right]^2(1-\eta)}{\left(\left[H_1(x,y)\right]^2(1+\text{ß}e^{i\varphi})(1+2Y)-2H_2(x,y)\left(1+\text{ß}e^{i\varphi}\right)^2(1+Y)^2e^{-m}\right)m^2e^{-m}}.$$

According to (1.5), we find that

$$\left|\rho_3-\eta\rho_2^2\right|\leq\begin{cases}\frac{2|H_1(x,y)|}{(1+\text{ß}e^{i\varphi})(1+2Y)m^2e^{-m}} & |\top(n)|\leq\frac{1}{|(1+\text{ß}e^{i\varphi})(1+2Y)m^2e^{-m}|},\\ 2\left|H_1(x,y)\right|\left|\top(n)\right| & |\top(n)|\geq\frac{1}{|(1+\text{ß}e^{i\varphi})(1+2Y)m^2e^{-m}|}.\end{cases}$$

This completes the proof of Theorem 4.1. □

Remark 4.2. Let (3.1) be in the class $\mathfrak{G}_{\text{ß},\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$. By taking some special values in the parameters in Theorem (4.1) we obtain the followings:
$i)$ Assume that $\eta=1$,

$$\left|\rho_3-\rho_2^2\right|\leq\frac{2b}{(1+\text{ß}e^{i\varphi})(1+2Y)m^2e^{-m}}.$$

$ii)$ Assume that $Y=0$,

$$\left|\rho_3-\eta\rho_2^2\right|\leq\begin{cases}\frac{|2b|}{(1+\text{ß}e^{i\varphi})m^2e^{-m}}, & |\eta-1|\leq\left|1-\frac{\left[2\left(1+\text{ß}e^{i\varphi}\right)(pbx+aqy)\right]e^{-m}}{b^2}\right|,\\ \frac{2|b|^3|1-\eta|}{\left|\left[(1+\text{ß}e^{i\varphi})b^2-2(1+\text{ß}e^{i\varphi})^2(pbx+aqy)e^{-m}\right]m^2e^{-m}\right|}, & |\eta-1|\geq\left|1-\frac{\left[2\left(1+\text{ß}e^{i\varphi}\right)(pbx+aqy)\right]e^{-m}}{b^2}\right|.\end{cases}$$

$iii)$ Assume that $Y=1$,

$$\left|\rho_3-\eta\rho_2^2\right| \leq \begin{cases} \frac{|2b|}{3(1+\text{ß}e^{i\varphi})m^2e^{-m}}, & |\eta-1| \leq \left|1-\frac{\left[8\left(1+\text{ß}e^{i\varphi}\right)(pbx+aqy)\right]e^{-m}}{3b^2}\right|, \\ \frac{2|b|^3|1-\eta|}{\left|\left[3(1+\text{ß}e^{i\varphi})b^2-8(1+\text{ß}e^{i\varphi})^2(pbx+aqy)e^{-m}\right]m^2e^{-m}\right|}, & |\eta-1| \geq \left|1-\frac{\left[8\left(1+\text{ß}e^{i\varphi}\right)(pbx+aqy)\right]e^{-m}}{3b^2}\right|. \end{cases}$$

Remark 4.3. Let (3.1) be in the class $\mathfrak{G}_{\text{ß},\varphi,n,\Sigma,Y}(p,q,x,y)(h(z))$. By taking some special values in the parameters in Theorem (4.1) we obtain the followings:

$i)$ Assume that $\eta=1$ and $\text{ß}=0$,

$$\left|\rho_3-\rho_2^2\right| \leq \frac{2b}{(1+2Y)m^2e^{-m}}.$$

$ii)$ Assume that $Y=\text{ß}=0$,

$$\left|\rho_3-\eta\rho_2^2\right| \leq \begin{cases} \frac{|2b|}{m^2e^{-m}}, & |\eta-1| \leq \left|1-\frac{[2(pbx+aqy)]e^{-m}}{b^2}\right|, \\ \frac{2|b|^3|1-\eta|}{|[b^2-2(pbx+aqy)e^{-m}]m^2e^{-m}|}, & |\eta-1| \geq \left|1-\frac{[2(pbx+aqy)]e^{-m}}{b^2}\right|. \end{cases}$$

$iii)$ Assume that $Y=1$ and $\text{ß}=0$,

$$\left|\rho_3-\eta\rho_2^2\right| \leq \begin{cases} \frac{|2b|}{3m^2e^{-m}}, & |\eta-1| \leq \left|1-\frac{[8(pbx+aqy)]e^{-m}}{3b^2}\right|, \\ \frac{2|b|^3|1-\eta|}{|[3b^2-8(pbx+aqy)e^{-m}]m^2e^{-m}|}, & |\eta-1| \geq \left|1-\frac{[8(pbx+aqy)]e^{-m}}{3b^2}\right|. \end{cases}$$

5. Concluding Remarks

In this work, we present and investigate a novel subclass of analytic bi-univalent functions that are subservient to the generating function of the generalized bivariate Fibonacci-like polynomials (GBFP). We provide estimates for the initial Maclaurin coefficients ρ_2 and ρ_3 for functions in this subclass, and we also study the Fekete–Szegö functional of the type $\left|\rho_3-\eta\rho_2^2\right|$. If the related parameters are chosen appropriately, these results can be further extended to derivative-based operator classes. Specifically, the resulting findings provide novel coefficient bounds for hitherto unstudied subclasses and generalize some known results in the literature as special instances when the parameters p, q, x, y, a, and b are chosen appropriately. To highlight the significance and originality of the suggested subclass, the connection between the current findings and previous contributions has been made clear. Lastly, this subclass might be studied in the context of higher-order Hankel and Toeplitz determinant issues, including fourth and higher-order situations, as a potential avenue for future study.

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