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The new integral transform, DPR Transform and its Application

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Abstract In this paper, a new integral DPR transform is developed and introduced. It is a superior transform, exhibiting dualities with more than eighty other integral transforms. Many researchers have been introduced different integral transform with various kernels. This transform is generalization of all such more than eighty integral transforms. Instead of studying eighty different transforms this single transform can solve the various problem solved by these different integral transform. A connection is established between this recent transformation and the previously recognized advantageous transforms, thereby extending the theoretical framework.We further discuss fundamental properties of the DPR transform which include linearity property, scaling property, domain shifting property, and its operation on derivatives are rigorously demonstrated. The transform is applied to a variety of equations,which include ordinary differential equations, partial differential equations, integro-differential equations, fractional differential equations, difference equations, and differential–difference equations. In addition, the practical utility of the DPR integral transform is illustrated through applications in forensic science, where it provides outstanding results without requiring extensive computational effort.

Keywords Integral transform, forensic science, boundary value problems, DPR transform.

AMS 2010 subject classifications : 35A22, 65R10

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1. Introduction

Now a days, integral transforms have become one of the most widely used mathematical techniques for solving the advanced problems in space science, medicine, forensic science, technology, engineering, commerce, and economics. The DPR transform has an important feature that they have ability to give exact solutions to problems without lengthy calculations.

The general form of an integral transform as follows

$$T[F(t)][s] = \int_0^{\infty} K(s,t)f(t)dt$$

where $K(s,t)$ is the kernel, $f(t)$ is the input function, and $T[F(t)][s]$ is the output function of the integral transform. Several transforms are commonly named after the mathematicians who introduced them, such as the Laplace transform [1], the Fourier transform (Joseph Fourier) [2], and the Mellin transform (Mellin) [3]. Integral transforms are used for solving inital value problem and boundary value problems

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which involved ordinary differential equations and partial differential equations.They further used to solved other problems in mathematics, science, engineering and technology.

In recent years, many researchers have been engaged in developing new integral transforms for solving the problems in ordinary differential equations, partial differential equations, integral equations, and integro-differential equations. These transforms have also been applied to medical and forensic problems. In 2020, Hossein Jafari developed an integral transform which encompasses many classes of integral transforms within the Laplace transform framework, including those developed during the last two decades, such as the Sumudu transform (1993) [4], the Natural transform (2008) [5], the Elzaki transform (2011) [6], the Aboodh transform (2013) [7], the Tarig transform (2013) [8], the Kamal transform (2016) [9], the Maghoub transform (2016) [10], the Mohand transform (2017) [11], the Shahu transform (2019) [12], and the Sawi transform (2019) [13].

In 2021, Patil, together with Kushare and Takate, introduced the Kushare transform [14]. Subsequently, Khakale and Patil proposed the Soham transform [15], while Derle, Rahane, and Patil developed the double Rangaig transform [16]. Jafri introduced another general integral transform in 2021 [17]. More recently, Rachid El Aitouni and Dinkar P. Patil introduced a Taki transform [18]. Building upon these developments, this work proposes a more general integral transform which is named as DPR Transform. The DPR transform can be further expanded to handle fuzzy members that represent imprecise or ambiguous functional data. [19] [20].

2. PRELIMINARY

Result 1
For any positive real number $a > 0$ and real number x,

$$a^x = e^{x \ln a} \tag{1}$$

Result 2
For any $n > 0$ and $a > 0$,

$$\int_0^\infty x^{n-1} e^{-ax} dx = \frac{\Gamma(n)}{a^n} \tag{2}$$

Result 3

$$\int uv dx = u \int v dx - \int u' (\int v dx) dx \tag{3}$$

where u and v are differentiable functions.

Result 4
We consider functions of exponential order in the set A, defined as

$$A = \Big\{ f(t) : |f(t)| < M e^{|t| k_i} \text{ if } t \in (-1)^i \times [0, \infty], \text{where } \ i = 1, 2 \text{and } \ M, k_1, k_2 > 0$$

In the above set A, the constant M must be a finite.

3. MAIN RESULTS

In this section, we introduce a new integral transform called the DPR transform and its inverse, defined as follows

Definition: 1
The DPR integral transform of order n of a continuous function $f(t)$ in the interval $(0,\infty)$ is defined as

$$DPR_n[f(t)](u,q(s)) = DPR(n,u,q(s)) \\ = p(s)\int_0^\infty t^{n-1}k^{-q(s)t}f(ut)dt, \quad s>0, u>0$$

In this if, we replace ut by t, then

$$DPR_n[f(t)](u,q(s)) = DPR(n,u,q(s)) \\ = p(s)\int_0^\infty \left(\frac{t}{u}\right)^{n-1} k^{\frac{-q(s)}{u}t}f(t)\frac{dt}{u}, s>0, u>0$$

The DPR transform can be expressed as follows:

$$DPR_n[f(t)](u,q(s)) = DPR(n,u,q(s)) \\ = \frac{p(s)}{u^n}\int_0^\infty t^{n-1}\cdot k^{\frac{-q(s)}{u}t}f(t)dt$$

Where

- $p(s)$ is a finite, non-zero function for all $s>0$.
- $q(s)$ is a real-valued function such that $q(s)>0$ for all $s>0$.
- $k>0$, $k\neq 1$, is a fixed constant.
- $n\in\mathbb{N}$ represents the order of the transform.

Definition: 2
The inverse of the DPR transform is defined as

$$f(t) = DPR_{n+1}^{-1}\left[DPR_{n+1}f(t)\right]$$

Theorem: 1
Existence theorem of DPR transform:
For the existence of the DPR transform the sufficient condition can be stated as follows:
Let the function $f(t)$ be piecewise continuous on every finite interval $[0,a]$ further satisfies

$$\mid t^{n-1}f(t)\mid < Mk^{\alpha t}, \quad M,k,\alpha>0............(*)$$

then the DPR transform of f(t) exists for all $s>\alpha$

Proof:
By definition of the DPR transform,

$$DPR_n[f(t)](u,q(s)) = p(s)\int_0^\infty t^{n-1}k^{-q(s)t}f(ut)dt$$

Replace ut with t then, we obtain

$$DPR_n[f(t)](u,q(s)) = p(s)\int_0^\infty \left(\frac{t}{u}\right)^{n-1} k^{-q(s)\left(\frac{t}{u}\right)}f(t)\frac{dt}{u}$$

$$= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{\frac{-q(s)}{u}t}f(t)dt$$

$$= \frac{p(s)}{u^n}\int_0^a t^{n-1}k^{\frac{-q(s)}{u}t}f(t)dt + \frac{p(s)}{u^n}\int_a^\infty (t)^{n-1}k^{\frac{-q(s)}{u}t}f(t)dt$$

As $f(t)$ is piecewise continuous function, the first integral exists on the interval $0 \leq t \leq a$
For the second integral,

$$\begin{aligned}
&\left|\frac{p(s)}{u^n}\int_a^\infty t^{n-1}k^{-q(s)\left(\frac{t}{4}\right)}f(t)dt\right| \\
&\leqslant \frac{p(s)}{u^n}\int_a^\infty k^{\frac{-q(s)}{u}t}\left|(t)^{n-1}f(t)\right|dt \\
&\leqslant \frac{p(s)}{u^n}\int_a^\infty k^{\frac{q(s)}{u}t}Mk^{\alpha t}dt\ldots \text{ from (*)} \\
&\leqslant \frac{p(s)M}{u^n}\int_a^\infty k^{-\left(\frac{q(s)}{u}-\alpha\right)t}dt \\
&= \frac{p(s)M}{u^n}\left[\frac{k^{-\left(\frac{q(s)}{u}-\alpha\right)t}}{lnk\left(\frac{q(s)}{u}-\alpha\right)}\right]_a^\infty \\
&= \frac{p(s)M}{u^n}\lim_{b\to\infty}\left[\frac{k^{-\left(\frac{q(s)}{u}-\alpha\right)t}}{lnk\left(\frac{q(s)}{u}-\alpha\right)}\right]_a^b \\
&= \frac{p(s)M}{u^{n-1}lnk\left(q(s)-\alpha u\right)}k^{-a\left(\frac{q(s)-\alpha u}{u}\right)}
\end{aligned}$$

This integral converges whenever $\frac{q(s)}{u}-\alpha > 0$. Hence, the DPR transform exists for $\frac{q(s)}{u}-\alpha > 0$.

4. DPR TRANSFORM OF SOME FUNCTIONS

The DPR transform is must satisfy the condition $t > 0$, and it can be written in the following form:

$$\begin{aligned}
DPR_n[f(t)](u,q(s)) &= p(s)\int_0^\infty f(ut)k^{-q(s)t}t^{n-1}dt \\
&= \frac{p(s)}{u^n}\int_0^\infty f(t)k^{-\frac{q(s)}{u}t}t^{n-1}dt
\end{aligned}$$

1. Let $f(t) = 1$

$$\begin{aligned}
DPR_n(1)(u,q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u})t}(1)dt \\
&= \frac{p(s)}{u^n}\left[\int_0^\infty t^{n-1}e^{-\frac{q(s)}{u}t\cdot\ln k}dt\right]\ldots \qquad \text{from (1)} \\
&= \frac{p(s)}{u^n}\left[\frac{\Gamma(n)}{\left(\frac{q(s)\ln k}{u}\right)^n}\right]\ldots.. \qquad \text{from (2)} \\
&= \frac{p(s)}{u^n}\left[\frac{\Gamma(n)\cdot u^n}{(q(s)\ln k)^n}\right] \\
&= \frac{p(s)\Gamma(n)}{(q(s)\ln k)^n}
\end{aligned}$$

2. Let $f(t) = t$

$$
\begin{aligned}
DPR_n\left[t\right](u,q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}t \quad dt \\
&= \frac{p(s)}{u^n}\int_0^\infty t^{n}k^{-\frac{q(s)}{u}}dt \\
&= \frac{p(s)}{u^n}\frac{\Gamma(n+1)}{\left(\frac{q(s)}{u}\ln k\right)^{n+1}} \cdots \text{ from (2)} \\
&= \frac{up(s)\Gamma(n+1)}{\left(q(s)\ln(k)\right)^{n+1}}
\end{aligned}
$$

3. Let $f(t)=t^m$

$$
\begin{aligned}
DPR_n\left[t^m\right](u,q(s)) &= \frac{p(s)}{u^n}\int^\infty t^{(n-1}k^{-\frac{q(s)}{u^n}t}\cdot t^m \quad dt \\
&= \frac{p(s)}{u^n}\int_0^\infty t^{(m+n)-1}k^{-\frac{q(s)}{u}t}dt \\
&= \frac{p(s)}{u^n}\int_0^\infty t^{(n+m)-1}e^{\frac{-q(S)}{u}t\cdot\ln k}d \\
&= \frac{p(s)}{u^n}\frac{\Gamma(m+n)}{\left(\frac{q(s)\ln k)^{n+m}}{u}\right)} \cdots \text{ from (2)} \\
&= \frac{p(s)u^m\Gamma(m+n)}{(q(s)\ln k)^{m+n}}
\end{aligned}
$$

4. Let $f(t)=e^{at}$

$$
\begin{aligned}
DPR_n\left[e^{at}\right](u,q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}\cdot k^{-\frac{q(s)}{u}t}\cdot e^{at}dt \\
&= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}e^{\frac{-q(s)t}{u}t\ln k}\cdot e^{at}dt \cdots \quad \text{from(1)} \\
&= \frac{p(s)}{u^n}\frac{\Gamma(n)}{\left(\frac{q(s)\ln k}{u}-a\right)^n} \\
&= \frac{p(s)u^n\Gamma(n)}{u^n(q(s)\ln k-au)^n} \\
&= \frac{p(s)\Gamma(n)}{\left(q(s)\ln k-au\right)^n}
\end{aligned}
$$

5. Let $DPRf(t) = e^{-at}$.

$$\begin{aligned} DPR_n\left[e^{-at}\right](u, q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}\cdot k^{-\frac{q(s)}{u}t}\cdot e^{-at}dt \\ &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}e^{\frac{-q(s)t}{u}t\ln k}\cdot e^{-at}dt \\ &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}.e^{-\left(\frac{q(s)}{u}\ln k+a\right)t}dt \\ &= \frac{p(s)}{u^n}\frac{\Gamma(n)}{\left(\frac{q(s)\ln k}{u}+a\right)^n} \qquad \cdots \text{from}(2) \\ &= \frac{p(s)u^n\Gamma(n)}{u^n(q(s)\ln k+a)^n} \\ &= \frac{p(s)\Gamma(n)}{(q(s)\ln k+au)^n} \end{aligned}$$

6. Let $\quad f(t) = t^m e^{at}$

$$\begin{aligned} &DPR_n\left[t^m e^{at}\right](u, q(s)) = \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}\left(t^m e^{at}\right)dt \\ &= \frac{p(s)}{u^n}\int_0^\infty t^{(m+n)-1}e^{-\frac{q(s)}{u}(t)\ln k}e^{at}dt \\ &= \frac{p(s)}{u^n}\int_0^\infty t^{(m+n)-1}e^{-\left(\frac{q(s)}{u}\ln k-a\right)t}dt \\ &= \frac{p(s)}{u^n}\frac{\Gamma(m+n)}{\left(\frac{q(s)}{u}\ln k-a\right)^{m+n}}\cdots \text{ from } (2) \\ &= \frac{p(s)}{u^n}\frac{u^{m+n}\Gamma(m+n)}{(q(s)\ln k-au)^{m+n}} \\ &= \frac{p(s)u^m\Gamma(m+n)}{(q(s)\ln k-au)^{m+n}} \end{aligned}$$

7. $f(t) = \sin at$

$$\begin{aligned} DPR_n[\sin at]\,(u_1 q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}\sin at\ \ dt \\ &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}\cdot e^{-\frac{q(s)}{u}t\ln k}\left[\frac{e^{iat}-e^{-iat}}{2i}\right]dt \qquad \cdots \text{from}(I) \\ &= \frac{p(s)}{2iu^n}\left\{\int_0^\infty t^{n-1}e^{-\left(\frac{q(s)}{u}\ln k-ia\right)t}dt-\int_0^\infty t^{n-1}\cdot e^{-\left(\frac{q(s)}{u}\ln k+ia\right)t}dt\right\} \\ &= \frac{p(s)}{2iu^n}\left\{\frac{\Gamma(n)}{\left(\frac{q(s)\ln k}{u}-ia\right)^n}-\frac{\Gamma(n)}{\left(\frac{q(s)\ln k}{u}+ia\right)^n}\right\} \end{aligned}$$

$$
\begin{aligned}
&= \frac{p(s)u^n}{2iu^n}\left\{\frac{\Gamma(n)}{(q(u)\ln k - iua)^n} - \frac{\Gamma(n)}{(q(i)\ln k + iqu)^n}\right. \\
&= \frac{p(s)\Gamma(n)}{2i}\left\{\frac{(q(s)\ln k + iau)^n - (q(s)\ln k - iau)^2}{(q(s)\ln k - iau)^n.(q(s)\ln k + iau)^n}\right\} \\
&= \frac{p(s)\Gamma(n)}{2i}\left\{\frac{(q(s)\ln k + iua)^n - (q(s)\ln k - iau)^n}{(q(s)\ln k)^2)^n + ((au)^2)^n}\right\}
\end{aligned}
$$

8. $f(t) = \cos at$

$$
\begin{aligned}
DPR_n[\cos at]\,(u, q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}\cos at \quad dt \\
&= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}\cdot e^{-\frac{q(s)}{u}t\ln k}\left[\frac{e^{iat}+e^{-iat}}{2}\right] \quad dt \\
&= \frac{p(s)}{2u^n}\left\{\int_0^\infty t^{n-1}e^{-\left(\frac{q(s)}{u}\ln k - ia\right)t}dt + \int_0^\infty t^{n-1}\cdot e^{-\left(\frac{q(s)}{u}\ln k + ia\right)t}dt\right\} \\
&= \frac{p(s)}{2u^n}\left\{\frac{\Gamma(n)}{\left(\frac{q(s)\ln k}{u} - ia\right)^n} + \frac{\Gamma(n)}{\left(\frac{q(s)\ln k}{u} + ia\right)^n}\right\} \\
&= \frac{p(s)u^n}{2u^n}\left\{\frac{\Gamma(n)}{(q(s)\ln k - iua)^n} + \frac{\Gamma(n)}{(q(s)\ln k + iau)^n}\right. \\
&= \frac{p(s)\Gamma(n)}{2}\left\{\frac{(q(s)\ln k + i\ln k)^n + (q(s)\ln k - iau)^2}{(q(s)\ln k - iau)^n.(q(s)\ln k + iau)^n}\right\} \\
&= \frac{p(s)\Gamma(n)}{2}\left\{\frac{(q(s)\ln k + iua)^n + (q(s)\ln k - iau)^n}{(q(s)\ln k)^2)^n + ((au)^2)^n}\right\}
\end{aligned}
$$

9. Let $f(t) = \sinh at$

$$
\begin{aligned}
DPR_n[\sinh at](u, q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}\sinh at dt \\
&= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}\cdot\left(\frac{e^{at}-e^{-at}}{2}\right)dt \\
&= \frac{p(s)}{u^n}\int_0^{t_0} t^{n-1}e^{-\frac{q(s)}{u}t}\left(\frac{e^{at}-e^{-at}}{u}\right)dt\cdots\text{from (1)} \\
&= \frac{p(s)}{2u^n}\left\{\int_0^\infty t^{n-1}\left(e^{-\left(\left(\frac{q(s)}{u}\right)\ln k - a\right)t} - e^{-\left(\left(\frac{q(s)}{u}\right)\ln k + a\right)t}\right)dt\right\} \\
&= \frac{p(s)}{2u^n}\left\{\frac{\Gamma(n)u^n}{(q(s)\ln k - au)^n} - \frac{\Gamma(n)u^n}{(q(s)\ln k + au)^n}\right\} \\
&= \frac{p(s)[n}{2}\left\{\frac{1}{(q(s)\ln k - au)^n} - \frac{1}{(q(s)\ln k + au)^n}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{p(s)\Gamma(n)}{2}\left\{\frac{(q(s)\ln k + au)^n - (q(s)\ln k - au)^n}{(q(s)\ln k - au)^n(q(s)\ln k + au)^n}\right\} \\
&= \frac{p(s)\Gamma(n)}{2}\left\{\frac{(q(s)\ln k + au)^n - (q(s)\ln k - au)^n}{((q(s)\ln k)^2)^n - ((au)^2)^n}\right\}
\end{aligned}
$$

10. Let $f(t) = \cosh at$

$$
\begin{aligned}
DPR_n \cosh at](u, q(s)) &= \frac{p(s)}{u^n} \int_0^\infty t^{n-1} \cdot k^{-\frac{q(s)t}{u}} \cdot \cosh at dt \\
&= \frac{p(s)}{u^n} \int_0^\infty t^{n-1} \cdot e^{-\frac{q(s)}{u} t \ln k} \cdot \left(\frac{e^{at} + e^{-at}}{2} \right) dt \quad \text{from (1)} \\
&= \frac{p(s)}{u^n} \left\{ \int_0^\infty t^{n-1} e^{-\left(\frac{q(s)\ln k}{u} - a\right)t} dt + \int_0^\infty t^{n-1} e^{-\left(\frac{q(s)\ln k}{u} + a\right)t} dt \right\} \\
&= \frac{p(s)}{2u^n} \left\{ \frac{\Gamma(n)}{\left(\frac{q \sin k - a}{u}\right)^n} + \frac{\Gamma(n)}{\left(\frac{q(s)\ln k + a}{u}\right)^n} \right\} \qquad \text{from (2)} \\
&= \frac{p(s)\Gamma(n)}{2} \left\{ \frac{1}{(q(s)\ln k - au)^n} + \frac{1}{(q(s)\ln k + au)^n} \right\} \\
&= \frac{p(s)\Gamma(n)}{2} \left\{ \frac{(q(s)\ln k + au)^n + (q(s)\ln k - au)^n}{(q(s)\ln k - au)^n (q(s)\ln k + au)^n} \right\} \\
&= \frac{p(s)\Gamma(n)}{2} \left\{ \frac{(q(s)\ln k + au)^n + (q(s)\ln k - au)^n}{((q(s)\ln k)^2)^n - ((au)^2)^n} \right\}
\end{aligned}
$$

5. PROPERTIES OF DPR TRANSFORM

Theorem: 2

Linearity Property: Let $f(t)$ and $g(t)$ be two functions for which a DPR transform exists and α and β are constants, then

$$DPR_n[\alpha f(t) + \beta g(t)] = \alpha DPR_n[f(t)] + \beta DPR_n[g(t)]$$

Proof: Consider

$$
\begin{aligned}
DPR_n[\alpha f(t) + \beta g(t)](u, q(s)) &= \frac{p(s)}{u^n} \int_0^\infty t^{n-1} \cdot k^{\frac{-q(s)}{u}t} (\alpha f(t) + \beta g(t)) dt \\
&= \frac{\alpha p(s)}{u^n} \int_0^\infty t^{n-1} k^{\frac{-q(s)}{u}t} f(t) + \beta \frac{p(s)}{u^n} \int_0^\infty t^{n-1} k^{\frac{-q(s)}{u}t} g(t) dt \\
&= \alpha DPR_n[f(t)] + \beta DPR_n[g(t)]
\end{aligned}
$$

Theorem: 3

Change of Scale Property:

$$DPR_n[f(at)](u, q(s)) = \frac{1}{a^{n-1}} DPR\left(n, u, \frac{q(s)}{a}\right)$$

Proof: $DPR_n[f(at)](u, q(s)) = \frac{p(s)}{u^n} \int_0^\infty t^{n-1} k^{-\frac{q(s)}{u}t} f(at) dt$

put $at = v$

$$
\begin{aligned}
DPR_n[f(at)](u, q(s)) &= \frac{p(s)}{u^n} \int_0^\infty \left(\frac{v}{a}\right)^{n-1} k^{-\frac{q(s)}{u}\left(\frac{v}{a}\right)} f(v) \frac{dv}{a} \\
&= \frac{p(s)}{u^n} \frac{1}{a^n} \int_0^\infty (v)^{n-1} k^{-\frac{q(s)}{u} \cdot \left(\frac{v}{a}\right)} f(v) dv \\
&= \frac{1}{a^{n-1}} \frac{p(s)}{u^n} \frac{1}{a} \int_0^\infty (v)^{n-1} k^{-\frac{1}{a}\frac{q(s)}{u}v} f(v) dv. \\
&= \frac{1}{a^{n-1}} DPR_n[f(t)](u, q(s))
\end{aligned}
$$

Theorem: 4
Shifting in s domain:

$$DPR_n\left[e^{-ct}f(t)\right](u,q(s)) = DPR_n[f(t)]\left(u, q(s)+\frac{cu}{\ln k}\right)$$

Proof:

$$DPR_n\left[e^{-ct}f(t)\right](u,q(s)) = \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}e^{-ct}f(t)dt$$
$$Putk^{-ct} = e^{-ct\ln k}$$
$$\left(k^{-ct}\right)^{\frac{1}{\ln k}} = e^{-ct}$$
$$k^{-\frac{ct}{\ln k}} = e^{-ct}$$

$$\begin{aligned}DPR_n\left[e^{-ct}f(t)\right](u,q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-q(s)}tk^{-\frac{ct}{\ln k}}f(t)dt\\ &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{\left(\frac{q(s)}{u}+\frac{c}{\ln k}\right)t}f(t)dt\\ &= DPR_n[F(t)]\left(u, q(s)+\frac{cu}{\ln k}\right)\end{aligned}$$

Theorem: 5
Shifting in N domain:

$$DPR_n\left[t^m f(t)\right](u,q(s) = DPR_{m+n}[f(t)](u,q(s))$$

Proof:

$$\begin{aligned}\text{DPR: } \left[t^m f(t)\right](u,q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)t}{u}}\cdot t^m f(t)dt\\ &= \frac{p(s)}{u^n}\int_0^\infty t^{m+n-1}k^{-\frac{q(s)}{u}t}f(t)dt\\ &= u^m DPR_{m+m}[f(t)](u,q(s))\\ \text{Also, } DPR_n\left[\frac{f(t)}{t^m}\right](u,q(s)) &= DPR_{n-m}[f(t)](u,q(s))\\ DPR_n\left[\frac{f(t)}{t^m}\right](u,q(s)) &= \frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}\frac{f(t)}{t^m}dt\\ &= \frac{p(s)}{u^n}\int_0^\infty t^{(n-m)-1}k^{\frac{-q(s)}{u}t}f(t)dt\\ &= u^{-m}\cdot DPR_{n-m}[f(t)](u,q(s))\end{aligned}$$

6. DERIVATIVES OF DPR TRANSFORM:

This section provide the DPR transform of derivatives of the function.

Theorem: 6
Let $f(t)$ and $f'(t)$ be piecewise continuous functions on $[0,\infty)$ and are exponential order so that the DPR transform exists, then

$DPR_n\left[f'(t)\right](u,q(s)) = -\frac{(n-1)}{u}DPR_{n-1}[f(t)](u,q(s)) + \frac{q(s)}{u}\ln k DPR_n[F(t)](u,q(s))$

Proof:

$$DPR_n\left[f'(t)\right](u,q(s))$$
$$=\frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{\frac{-q(s)}{u}t}f'(t)dt$$
$$=\frac{p(s)}{u^n}\left[\left(t^{n-1}k^{\frac{-q(s)}{u}t}f(t)\right)_0^\infty-\int_0^\infty\left((n-1)t^{n-2}\cdot k^{\frac{-q(s)}{u}t}f(t)dt+t^{n-1}k^{\frac{-q(s)}{u}t}\left(\frac{-q(s)}{u}\right)\ln k\right)f(t)dt\right]$$
$$=\frac{p(s)}{u^n}\left[-(n-1)\int_0^\infty t^{n-2}k^{-\frac{q(s)}{u}t}f(t)dt+\frac{q(s)}{u}\ln k\int_0^\infty t^{n-1}k^{-q(s)}tf(t)dt\right]$$
$$=\frac{-(n-1)}{u}DPR_{n-1}[f(t)](u,q(s))+\frac{q(s)}{u}\ln kDPR_n[f(t)](u,q(s))$$

DPR transform for derivative when $n=1$

$$DPR_1\left[f'(t)\right](u,q(s))=-\frac{p(s)}{u}f(0)+\frac{q(s)}{u}\ln kDPR_1f(t)(u,q(s))$$

Proof:

$$DPR,\{f'(t)\}=\frac{p(s)}{u}\int_0^\infty t^{n-1}f'(t)k^{-\frac{q(s)}{u}t}dt$$
$$=\frac{p(s)}{u}\int_0^\infty f'(t)k^{-\frac{q(s)}{u}t}dt$$
$$=\frac{p(s)}{u}\left\{\left[k^{-\frac{q(s)}{u}t}\cdot f(t)\right]_0^\infty-\int_0^\infty k^{-\frac{q(s)t}{u}}\left(\frac{-q(s)}{u}\right)\cdot\ln kf(t)dt\right\}$$

$$=\frac{p(s)}{u}\left[0-k^0f(0)\right]-\frac{p(s)}{u}\ln k\left(\frac{-q(s)}{u}\right)\int_0^\infty k^{-\frac{q(s)}{u}t}f(t)dt$$
$$=\frac{p(s)}{u}f(0)+\frac{q(s)}{u}\ln k\left[\frac{p(s)}{u}\int_0^\infty k^{-\frac{q(s)}{u}t}f(t)dt\right]$$
$$=-\frac{p(s)}{u}f(0)+\frac{q(s)}{u}\ln kDPR,f(t)$$

$$DPR_1f'(t)=-\frac{p(s)}{u}f(0)+\frac{q(s)}{u}\ln kDPR_1f(t)(u,q(s))$$

Theorem: 7

Let $f(t)$ and $f'(t)$ and $f''(t)$ be piecewise continuous functions on $[0,\infty)$ and are of exponential order so that the DPR transform exists, then

$$DPR_n\left\{f''(t)\right\}(u,q(s))=\frac{(n-1)}{u}DPR_{n-1}\left[f'(t)+\frac{q(s)}{u}\ln kf(t)\right]+\left(\frac{q(s)}{u}\ln k\right)^2DPR_nf(t)$$

Poof:

$$DPRR_n\left\{f''(t)\right\}(u,q(s))=\frac{p(s)}{u^n}\int_0^\infty t^{n-1}k^{-\frac{q(s)}{u}t}f''(t)dt$$
$$=\frac{p(s)}{u^n}\left[t^{n-1}k^{-\frac{q(s)}{u}t}\int_0^\infty f''(t)dt-\int_0^\infty\left\{(n-1)t^{n-2}k^{\frac{q(s)}{u}t}+t^{n-1}k^{-\frac{q(s)}{u}t}\left(-\frac{q(s)}{u}\right)\ln k\int f''(t)dt\right\}\right]$$
$$=\frac{p(s)}{u^n}\left[\left(t^{n-1}k^{\frac{-q(s)}{u}t}f'(t)\right)_0^\infty-(n-1)\int_0^\infty t^{n-2}k\frac{-q(s)t}{u}f'(t)dt-\int_0^\infty t^{n-1}k\frac{-q(s)}{u}t\left(\frac{-q(s)}{u}\right)\ln kf'(t)dt\right]$$

$$= \frac{-(n-1)}{u} DPR_{n-1} f'(t) + \frac{q(s)}{u} \ln k DPR_n f'(t)$$
$$= -\frac{(n-1)}{u} DPR_{n-1} f'(t) + \frac{q(s)}{u} \ln k \left[-\frac{(n-1)}{u} DPR_{n-1} f(t) + \frac{q(s) \ln k DPR_n f(t)}{u} \right]$$
$$= \frac{-(n-1)}{u} DPR_{n-1} \left[f'(t) + \frac{q(s)}{u} \ln k f(t) \right] + \left(\frac{q(s)}{u} \ln k \right)^2 DPR_n f(t)$$

Theorem: 8

Let $f(t)$, $f'(t)$,$f''(t)$ and $f'''(t)$ be piecewise continuous functions on $[0, \infty)$ and are of exponential order, so that the DPR transform exists, then

$$DPR_n \{f'''(t)\}(u, q(s)) = \frac{-(n-1)}{u} DPR_{n-1} \left[f''(t) + \frac{q(s) \ln k}{u} f'(t) + \left(\frac{q(s) \ln k}{u} \right)^2 f(t) \right] + \left(\frac{q(s)}{u} \ln k \right)^3 DPR_n f(t)$$

Proof:

$$DPR_n \{f'''(t)\}(u, q(s)) = \frac{p(s)}{u^n} \int_0^\infty t^{n-1} \cdot k^{-\frac{q(s)}{u} t} f'''(t) dt$$
$$= \frac{p(s)}{u^n} \left(t^{n-1} k^{\frac{-q(s)}{u} t} \int_0^\infty f'''(t) dt \right) - \int_0^\infty \Big\{ (n-1) t^{n-2} k^{\frac{-q(s)}{u} t} + t^{n-1} k^{\frac{-q(s)}{u} t} \left(\frac{-q(s)}{u} \right) \ln k \int f'''(t) dt \Big\}$$
$$= \frac{p(s)}{u^n} \left[\left(t^{n-1} k^{\frac{-q(s)}{u} t} f''(t) \right)_0^\infty - (n-1) \int_0^\infty t^{n-2} k^{\frac{-q(s)}{u} t} f''(t) dt + \frac{q(s)}{u} \ln k \int_0^\infty t^{n-1} k^{\frac{-q(s)}{u} t} f''(t) dt \right]$$
$$= \frac{-(n-1)}{u} DPR_{n-1} f''(t) + \frac{q(s)}{u} \ln k \quad DPR_n f''(t)$$
$$= \frac{-(n-1)}{u} DPR_{n-1} f''(t) + \frac{q(s)}{u} \ln k \left[\frac{-(n-1)}{u} DPR_{n-1} f'(t) + \frac{q(s)}{u} \ln k f(t) + \left(\frac{q(s)}{u} \ln k \right)^2 DPR_n f(t) \right]$$

$$DPR_n \{f'''(t)\}(u, q(s)) = -\frac{(n-1)}{u} DPR_{n-1} \left[f''(t) + \frac{q(s)}{u} \ln k f'(t) + \left(\frac{q(s) \ln k}{u} \right)^2 f(t) \right] + \left(\frac{q(s)}{u} \ln k \right)^3 DPR_n f(t).$$

Theorem: 9

Let $f(t)$ ..... $f^n(t)$ be piecewise continuous function on $[0, \infty)$ and are of exponential order so that the DPR transform exists, then

$$DPR_n \{f^n(t)\}(u, q(s)) = -\frac{(n-1)}{u} DPR_{n-1} \left[f^{(n-1)}(t) + \left(\frac{q(s)}{u} \ln k \right) f^{(n-2)} \cdots\cdots + \left(\frac{q(s) \ln k}{u} \right)^{n-1} f(t) \right] + \left(\frac{q(s)}{u} \ln k \right)^n DPR_n f(t)$$

7. RELATION BETWEEN DPR TRANSFORM AND OTHER USEFUL INTEGRAL TRANSFORM:

In this section, we establish the relationship of the DPR transform with other integral transforms. By assigning particular values to the parameters k, u, n, $p(s)$, $q(s)$ in the definition of the DPR transform, we obtain several well-known transforms as special cases. We have formulated a table that illustrates these substitutions and the corresponding transforms. In a similar manner, by choosing different parameter values, many other transforms can be derived. Thus, we conclude that the DPR transform represents a more general form of integral transform.

Sr.No.	k	p(s)	q(s)	n	u	Formula	Name of transform
1	e	s	s^2	1	1	$s\int_0^\infty e^{-s^2t}f(t)dt$	H Y transform[21]
2	e	$\frac{1}{s}$	$\frac{1}{s^2}$	1	1	$\frac{1}{s}\int_0^\infty e^{-\frac{t}{s^2}}f(t)dt$	Kashuri and Fundo trans.[22]
3	e	λ_1	λ_2	1	λ_3	$\lambda_1\int_0^\infty e^{-\lambda_2 t}f(\lambda_3 t)\,dt$	Upadhyaya transform [23]
4	e	1	s	1	1	$\int_0^\infty e^{-st}f(t)dt$	Laplace transform [24]
5	e	s	s	1	1	$s\int_0^\infty e^{-st}f(t)dt$	Laplace-Carson transform [25]
6	e	$\frac{1}{s}$	$\frac{1}{s}$	1	1	$\frac{1}{s}\int_0^\infty e^{-\frac{1}{s}t}f(t)dt$	Sumudu transform
7	e	1	s	1	u	$\int_0^\infty e^{-st}f(ut)dt$	Natural transform [26]
8	e	s	$\frac{1}{s}$	1	1	$s\int_0^\infty e^{-\frac{1}{s}t}f(t)dt$	Elzaki transform
9	e	$\frac{1}{s}$	s	1	1	$\frac{1}{s}\int_0^\infty e^{-st}f(t)dt$	Aboodh transform
10	e	s	s	1	u	$s\int_0^\infty e^{-st}f(ut)dt$	ZZ transform [27]
11	e	1	s	1	u	$\int_0^\infty e^{-st}f(ut)dt$	Ramdhan transform [28]
12	e	s	s	1	1	$s\int_0^\infty e^{-st}f(t)dt$	Mangoub transform
13	e	1	$\frac{1}{s}$	1	1	$\int_0^\infty e^{-\frac{1}{s}t}f(t)dt$	Kamal transform
14	e	s^2	s	1	1	$s^2\int_0^\infty e^{-\frac{1}{s}t}f(t)dt$	Mohand transform
15	e	$\frac{1}{s}$	$\frac{1}{s^2}$	1	1	$\frac{1}{s}\int_0^\infty e^{-s^2t}f(t)dt$	Tarig transform
16	e	$\frac{1}{s^\beta}$	s^α	1	1	$\frac{1}{s^\beta}\int_0^\infty e^{-s^\alpha t}f(t)dt$	Sadik transform [29]
17	e	1	$\frac{s}{v}$	1	1	$\int_0^\infty e^{-\frac{1}{s}t}f(t)dt$	Yang transform [30]
18	e	1	$\frac{1}{s}$	1	1	$\int_0^\infty e^{-\frac{s}{v}t}f(t)dt$	Shehu transform [31]
19	e	$\frac{1}{s^2}$	$\frac{1}{s}$	1	1	$\frac{1}{s^2}\int_0^\infty e^{-\frac{1}{s}t}f(t)dt$	Sawi transform
20	e	1	s	$n+1$	1	$\int_0^\infty e^{-st}t^n f(t)dt$	Atangana Kilicman transform [32]
21	e	1	0	2	1	$\int_0^\infty tJ_v(st)f(t)dt$	Hankel transform [33]
22	e	s^3	$\frac{1}{s^2}$	1	1	$s^3\int_0^\infty e^{-\frac{1}{s^2}t}f(t)dt$	Kharrat Toma transform [34]
23	e	$m(s)$	$\frac{1}{n(s)}$	1	1	$m(s)\int_0^\infty e^{-\frac{1}{n(s)}}f(t)dt$	Alenzi transform [35]
24	e	s^a	s^b	1	1	$s^a\int_0^\infty e^{-st}f(t)dt$	W transform [36]
25	e	s	s	1	1	$s\int_0^\infty e^{-st}f(t)dt$	Pourreza transform [37]
26	e	s^a	$\frac{1}{s}$	1	1	$s^a\int_0^\infty e^{-\frac{1}{s}t}f(t)dt$	G transform [1]
27	e	s	s	1	u	$s\int_0^\infty e^{-st}f(ut)dt$	Formable transform [38]
28	e	s	s^a	1	1	$s\int_0^\infty e^{-s^a t}f(t)dt$	Kushare transform
29	e	$\frac{1}{s}$	s^a	1	1	$s^a\int_0^\infty e^{-s^a t}f(t)dt$	Soham transform
30	e	s	s	n	1	$s\int_0^\infty t^{n-1}e^{-st}f(t)dt$	ARA transform [39]
31	e	s^2	$\frac{1}{s}$	1	1	$s^2\int_0^\infty e^{-\frac{1}{s}t}f(t)dt, \quad s>0$	Anuj transform [40]
32	e	$\frac{1}{s^2}$	s	1	1	$\frac{1}{s^2}\int_0^\infty e^{-st}f(t)dt, \quad s>0$	Emad-Sara transform [41]
33	e	$\frac{1}{s}$	s^2	1	1	$\frac{1}{s}\int_0^\infty e^{-s^2t}f(t)dt, \quad s>0$	Emad-Fail transform [42]
34	e	$p(s)$	$q(s)$	1	1	$p(s)\int_0^\infty e^{-q(s)t}f(t)dt, \quad s>0$	General Integral transform [43]
35	e	$\frac{1}{s^2}$	$\frac{1}{s^2}$	1	1	$\frac{1}{s^2}\int_0^\infty e^{-\frac{1}{s^2}t}f(t)dt$	AR- transform [44]
36	e	1	s^2	1	1	$\int_0^\infty e^{-s^2t}f(t)dt$	Fareena transform [45]
37	e	$\frac{1}{s}$	s	1	u	$\frac{1}{s^u}\int_0^\infty e^{-\frac{st}{u}}f(t)dt$	NE transform [46]
38	e	p^5	p	1	1	$p^5\int_0^\infty e^{-pt}f(t)dt$	DVT transform [47]

39	e	s^3	$-\frac{1}{s}$	1	1	$\int_0^\infty s^3 e^{\frac{t}{s}} f(t)dt$	GF1 integral transform [48]
40	e	s	is	1	u	$\frac{s}{u}\int_0^\infty e^{-i(\frac{s}{u})t} f(t)dt$	Sea transform [49]
41	e	$\frac{1}{\beta}$	β	1	α	$\frac{-1}{\beta}\int_0^\infty e^{\beta x} f(\alpha x)dx$	KKAT transform [50]
42	e	$p(s)$	$iqt(s)$	1	1	$p(s)\int_0^\infty e^{iqt(s)t} f(t)dt$	SEJI transform [51]
43	e	$\frac{1}{s}$	$\frac{1}{s}$	1	1	$\frac{1}{s}\int_0^\infty e^{-\frac{t}{s}} f(t)dt$	ZMA transform [52]
44	e	s^3	s	1	1	$s^3\int_0^\infty e^{-st} f(t)dt$	Rohit transform [53]
45	e	$\frac{1}{s^2}$	s^2	1	1	$\frac{1}{s^2}\int_0^\infty e^{-s^2 t} f(t)dt$	IMAN transform [54]
46	e	$\frac{1}{s^3}$	s	1	1	$\frac{1}{s^3}\int_0^\infty e^{-st} f(t)dt$	Gupta transform [55]
47	e	1	$\frac{1}{s^\alpha}$	1	1	$\int_0^\infty e^{-\frac{t}{s^\alpha}} f(t)dt$	α integral Laplace transform [43]
48	e	s^α	s^β	1	u	$s^\alpha\int_0^\infty e^{-s^\beta t} f(t)dt$	Taki transform
49	e	$\frac{1}{s^n}$	is	1	1	$\frac{1}{s^n}\int_0^\infty e^{-ist} f(t)dt$	Complex SEE transform [56]
50	e	$\frac{1}{s^n}$	1	1	$\frac{1}{u}$	$\frac{1}{s^n}\int_0^\infty e^{-t} f(\frac{t}{u})dt$	KAJ transform [57]
51	e	s	s	1	1	$s\int_0^\infty e^{-st} f(t)dt$	RAJ transform [58]
52	e	$p(s)$	$(q(s))^{\frac{1}{n}}$	1	1	$p(s)\int_0^\infty e^{-(q(s))^{\frac{1}{n}} t} f(t)dt$	INEM transform [59]
53	e	1	$\frac{1}{s^2}$	1	1	$\int_0^\infty e^{-\frac{1}{s^2}t} f(t)dt$	Mhase transform [60]
54	e	$\frac{1}{s^n}$	s	1	1	$\frac{1}{s^n}\int_0^\infty e^{-st} f(t)dt$	SEE transform [61]
55	e	$\frac{1}{s}$	$\frac{s}{v}$	1	1	$\frac{1}{s}\int_0^\infty e^{-\frac{s}{v}t} f(t)dt$	J- transform [62]
56	e	$\frac{s}{\epsilon}$	$\frac{\epsilon}{s}$	1	1	$\frac{s}{\epsilon}\int_0^\infty e^{-\frac{\epsilon}{s}t} f(t)dt$	Rishi transform [63]
57	e	1	is^n	1	1	$\int_0^\infty e^{-is^n t} f(t)dt$	Complex EE transform [64]
58	e	1	0	s	1	$\int_0^\infty t^{s-1} f(t)dt$	Mellin transform [65]
59	e	s^2	$\frac{v}{s}$	1	1	$s^2\int_0^\infty e^{-\frac{v}{s}t} f(t)dt$	RAHMOH transform [66]
60	e	$\frac{1}{\sqrt{s}}$	s	1	1	$\frac{1}{\sqrt{s}}\int_0^\infty e^{-st} f(t)dt$	HK- transform [67]
61	e	$\frac{s}{\lambda_n}$	$\frac{s}{\lambda_n}$	1	1	$\frac{s}{\lambda_n}\int_0^\infty e^{-\frac{s}{\lambda_n}t} f(t)dt$	Khalouta transform [68]
62	e	s^β	s^α	1	1	$s^\beta\int_0^\infty e^{-s^\beta t} f(t)dt$	Hunaiber transform [69]
64	e	1	$\frac{1}{s}$	1	u	$\int_0^\infty e^{-\frac{1}{s}t} f(ut)dt$	Abaub Shkheam transform [22]
64	e	s	s	1	$\frac{1}{s}$	$s\int_0^\infty e^{-st} f(\frac{t}{s})dt$	AMJ Shkheam transform [67]
65	e	s^n	s^k	1	u	$s^n\int_0^\infty e^{-s^k t} f(t)dt$	g- transform [67]
66	e	pq	$\frac{p}{q}$	1	1	$pq\int_0^\infty e^{-\frac{p}{q}t} f(t)dt$	Quideen transform [67]
67	e	s^2	$\frac{1}{s^2}$	1	1	$s^2\int_0^\infty e^{-\frac{1}{s^2}t} f(t)dt$	Bayawa transform [70]
68	e	$p(s)$	$q(s)$	1	1	$p(s)\int_0^\infty e^{-q(s)t} f(t)dt$ $= p(s)\int_1^\infty e^{-(q(s)+1)t} f(\ln t)dt$	Abdalla transform [71]
69	e	$h(v)$	$\sigma(v)$	1	1	$h(v)\int_0^\infty e^{-\sigma(v)t} f(\psi(v)t)dt$	Generalization of transform [72]
70	e	v^α	$\frac{1}{v}$	1	1	$v^\alpha\int_0^\infty e^{-\frac{1}{v}t} f(t)dt$	G_α transform [73]
71	e	s	s	η	γ	$\frac{s}{\gamma^n}\int_0^\infty \eta^{n-1} e^{-\frac{s}{\gamma}t} f(\eta)d\eta$ here $\eta, \gamma > 0$	GB transform [74]
72	e	$p(s)$	s^n	1	1	$p(s)\int_0^\infty e^{-s^n t} f(t)dt$	He- transform [74]
73	e	$\frac{1}{s}$	s	1	-1	$\frac{1}{s}\int_{-\infty}^0 e^{st} f(t)dt$	Rangaig- transform
74	e	$p(s)$	$q(s)$	1	-1	$p(s)\int_{-\infty}^0 e^{p(s)t} f(t)dt$	Emad-Salam transform.
75	e	$\frac{1}{v^\beta}$	$v^{i\alpha}$	1	1	$\frac{1}{v^\beta}\int_0^\infty e^{-v^{i\alpha} t} f(t)dt$	Mayan Integral transform [75]
76	e	1	$q(v)$	1	1	$\lim_{p\to\infty}\int_{t=0}^p e^{-v^{iq(v)} t} f(t)dt$	Complex EFG transform [76]
77	a	$\frac{1}{s^r}$	s	1	1	$\frac{1}{s^r}\int_0^\infty a^{-st} f(t)dt$	SUM transform [77]
78	e	$\frac{1}{q^2}$	$\frac{1}{q^3}$	1	1	$\frac{1}{q^2}\int_0^\infty e^{-\frac{1}{q^3}t} w(t)dt$	Dukari transform [78]
79	e	$\frac{1}{v}$	iv	1	1	$\frac{1}{v}\int_0^\infty e^{-ivt} h(t)dt$	SEL transform [?]
80	e	1	is	1	1	$\int_0^\infty e^{-ist} f(t)dt$	Fourier transform [79]
81	e	$\frac{1}{m^\delta}$	jm^ω	1	1	$\frac{1}{m^\delta}\int_0^\infty e^{-jm^\omega t} f(t)dt$	CSI Copmlex Sadik transform

Table 1. Relation between DPR Transform and other useful transform

8. APPLICATION OF DPR TRANSFORM IN FORENSIC SCIENCE:

We solve boundary value problems on Newton's law of cooling by applying the DPR transform.

Example: 1
One morning a murder victim is discovers by a detective at $9am$. Temperature of the body is 80°F one hour later at $10am$, the body has cooled to 75°F. The room was kept at a constant temperature of 70°F. Assumed that, victim had a normal temperature of 98.6°F at the time of death. find the time the murder took place by using differential equation.

Solution: we have $T(t)$ = Temperature of the body at time t.
$T_{\text{room}} = 70°\text{F}$ K = positive constant.
t = time since death in hours
$T_0 = 98° \cdot 6°\text{F}$ (initial body temperature at time of death)
Temperature of body at 9.00 Am is

$$T\left(t_1\right) = 80°\text{F}$$

An hour later, the temperature of body is

$$T\left(t_1 + 1\right) = 75°\text{F}.$$

To find the time when tempearture is 98.6°F
By Newton's law of cooling, the differential equation is

$$\frac{dT}{dt} = -K(T - 70)$$

Now, by DPR transform, we get

$$DPR\left(\frac{dT}{dt}\right) = -kDPR(T) + 70 \times kDPR(1)$$

$$\frac{q(s)\ln k}{u}T(u, q(s)) - \frac{p(s)}{u}T(0) = -KT(u, q(s)) + k \times 70 \times \frac{p(s)}{q(s)\ln k}$$

$$\therefore \left(\frac{q(s)\ln k}{u} + k\right)T(u, q(s)) = k \times 70 \times \frac{p(s)}{q(s)\ln k} + \frac{p(s)T(0)}{u}$$

$$T(u, q(s)) = \frac{k \times 70 \times u \times p(s)}{(q(s)\ln k)(q\ln k + ku)} + \frac{p(s)T(0)}{(q(s)\ln k + ku)}$$

$$T(u, q(s)) = \frac{k \times 70 \times u \times p(s)}{(q(s)\ln k)(q\ln k + ku)} + \frac{p(s)T(0)}{(q(s)\ln k + ku)}.$$

$$T(u, q(s)) = \frac{70 \times p(s)}{(q(s)\ln k)} - \frac{70 \times p(s)}{(q(s)\ln k + ku)} + \frac{p(s)T(0)}{(q(s)\ln k + ku)}.$$

By applying inverse DPR

$$\begin{aligned}
T(t) =& 70DPR^{-1}\left(\frac{p(s)}{q(s)\ln k}\right) - 70DPP^{-1}\left(\frac{p(s)}{q(s)\ln k + ku}\right) + T(0)DPR^{-1}\left(\frac{p(s)}{q(s)\ln k + ku}\right) \\
T(t) =& 70(1) - 70\left(e^{-kt}\right) + T(0)e^{-kt} \\
\therefore T(t) =& 70 + T(0)e^{-kt} - 70e^{-kt} \\
\therefore T(t) =& 70 + (T(0) - 70)e^{-kt}.
\end{aligned}$$

Now, we have

$$T(0) = 80, \quad T(1) = 75 \implies k = \ln 2$$

Let τ is the time before $9:00am$ the person died.

$$\begin{aligned} T(t) &= 98.6 = 70 + 10e^{k\tau} \\ 98.6 &= 70 + 10e^{(\ln 2)\tau} \\ 28.6 &= 10.2^{\tau} \\ 2^{\tau}F &= 2.86 \\ \tau &= \log_2(2.86) \\ \tau &= \frac{\ln(2.86)}{\ln(2)} \\ &= \frac{1.05}{0.693} \\ &= 1.52 \text{ hours} \end{aligned}$$

$\therefore$ Time of death $= 9.00\text{am} - 1.52\text{hr} \approx 7:29\text{am}$.

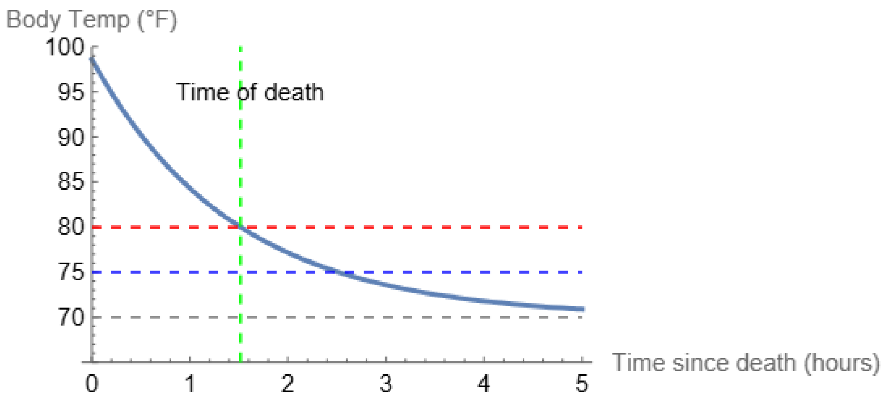

Figure 1. Death Rate

9. Conclusion

In this paper, we have defined the DPR transform, a new and more standardized integral transform, together with some of its fundamental properties and theorems. We have also demonstrated its application to solving the death-time estimation problem in forensic science. From these results, we conclude that the DPR transform serves as a generalization of several existing integral transforms. In particular, the Laplace, HY, and Elzaki transforms emerge as special cases under suitable parameter substitutions. Hence, the DPR transform provides a powerful and versatile tool for solving integro-differential equations, ordinary differential equations, partial differential equations, and a wide range of problems encountered in applied mathematics and everyday life.

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The authors don't have any conflict of interest among them. The authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest (such as personal or professional relationships, affiliations, knowledge, or beliefs) in the subject matter or materials discussed in this manuscript.

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