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New Types of Integral Contractions in Supra Metric Space

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Abstract In the present article, we shall define the new notions of generalized $(\mathscr{S} - \psi)$ contractions of integral type A and B and prove the related fixed point theorems in the setting of supra metric space. Then, we shall deduce some new results from the proved results in the form of corollaries. Some examples will also be given to show the real existence of proved results.

Keywords fixed point, integral type A, integral type B, supra metric space

AMS 2010 subject classifications 47H10, 54H25

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1. Introduction

By seeing the utility of Banach contraction principle in metric space, many authors tried to generalize the notion of Banach contraction principle [5] and metric space. In 2002, Branciari gave the new notion of integral type contraction which was an analogue result for Banach contraction principle. Many fixed point results by various authors are given in ([1, 3, 4, 8, 13, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36]).
In 2012, Samet *et al.* [35] defined the $\mathscr{S} - \psi$-contraction by defining the $\mathscr{S}$ admissible mappings as follows:

Definition 1. *[35] Suppose $\mathfrak{N}$ be a non empty set and a map $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$. Then, a self map $\mathscr{T}$ on $\mathfrak{N}$ is said to be $\mathscr{S}$-admissible if for $a, b \in \mathfrak{N}$*

$$\mathscr{S}(a, b) \geq 1 \Rightarrow \mathscr{S}(\mathscr{T}a, \mathscr{T}b).$$

Berzig and Rus [9] gave the following definition

Definition 2. *[9] Suppose $N \in \mathbb{N}$. Then, we say that $\mathscr{S}$ is N-transitive on $\mathfrak{N}$ if*

$$a_0, a_1, ...a_{N+1} \in \mathfrak{N} : \mathscr{S}(a_i, a_{i+1}) \geq 1$$

for all $i = 0, 1, 2...N$
implies that

$$\mathscr{S}(a_0, a_{N+1}) \geq 1.$$

From Definition 2, we can conclude that

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Remark 1. *[9]*

1. *Any function* $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *is* 0*-transitive.*
2. *If* $\mathscr{S}$ *is* N*-transitive then it is* pN*-transitive for all* $p \in \mathbb{N}$*.*
3. $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *is transitive then it is* N*-transitive for all* $N \in \mathbb{N}$*.*
4. *If* $\mathscr{S}$ *is transitive then it is not necessary that it is transitive for all* $N \in \mathbb{N}$*.*

Let $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ such that

1. ψ is monotonically increasing;
2. $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for all $t > 0$.

The collection of all such type of function will be denoted by Ψ.
Define $\phi : \mathbb{R}^+ \to \mathbb{R}$ a non-negative Lebesgue integrable function such that
$\int_0^{\epsilon} \phi(t)dt > 0$ for all $\epsilon > 0$.
Collection of such function will be denoted by Φ.
In 2014, Shahi *et al.* [15] gave the new notions of $\mathscr{S} - \psi$ contractions of integral type I and II as stated below:

Definition 3. *[15] Suppose* $\mathscr{T}$ *be a self map on a metric space* $(\mathfrak{N}, \sigma)$*. We say that* $\mathscr{T}$ *is a generalized* $\mathscr{S} - \psi$*-contractive mapping of integral type I if there exists two functions* $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *and* $\psi \in \Psi$ *such that for all* $a, b \in \mathfrak{N}$*,*

$$\mathscr{S}(a, b) \int_0^{\sigma(\mathscr{T}a, \mathscr{T}b)} \phi(t)dt \leq \psi(\int_0^{M(a,b)} \phi(t)dt)$$

where $\phi \in \Phi$ *and* $M(a, b) = \max\{\sigma(a, b), \sigma(a, \mathscr{T}a), \sigma(b, \mathscr{T}b), \frac{\sigma(a, \mathscr{T}b) + \sigma(b, \mathscr{T}a)}{2}\}$*.*

Definition 4. *[15] Suppose* $\mathscr{T}$ *be a self map on a metric space* $(\mathfrak{N}, \sigma)$*. We say that* $\mathscr{T}$ *is a generalized* $\mathscr{S} - \psi$*-contractive mapping of integral type II if there exists two functions* $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *and* $\psi \in \Psi$ *such that for all* $a, b \in \mathfrak{N}$*,*

$$\mathscr{S}(a, b) \int_0^{\sigma(\mathscr{T}a, \mathscr{T}b)} \phi(t)dt \leq \psi(\int_0^{M(a,b)} \phi(t)dt)$$

where $\phi \in \Phi$ *and* $M(a, b) = \max\{\sigma(a, b), \frac{\sigma(a, \mathscr{T}b) + \sigma(a, \mathscr{T}a)}{2}, \frac{d(b, \mathscr{T}b) + d(b, \mathscr{T}a)}{2}\}$*.*

Then, they proved the following theorems.

Theorem 1. *[15] Suppose* $(\mathfrak{N}, \sigma)$ *be a complete metric space and* $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *is a transitive mapping. Suppose that* $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ *is a generalized* $\mathscr{S} - \psi$*-contractive mapping of integral type I and satisfies the following conditions:*

1. $\mathscr{T}$ *is* $\mathscr{S}$*-admissible;*
2. *there exists* $a_0 \in \mathfrak{N}$ *such that* $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$*;*
3. $\mathscr{T}$ *is continuous.*

Then, $\mathscr{T}$ *has a fixed point, that is, there exists* $a \in \mathfrak{N}$ *such that* $\mathscr{T}a = a$*.*

Theorem 2. *[15] Suppose* $(\mathfrak{N}, \sigma)$ *be a complete metric space and* $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *is a transitive mapping. Suppose that* $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ *be a a generalized* $\mathscr{S} - \psi$*-contractive mapping of integral type II and satisfies the following conditions:*

1. $\mathscr{T}$ *is* $\mathscr{S}$*-admissible;*
2. *there exists* $a_0 \in \mathfrak{N}$ *such that* $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$*;*
3. $\mathscr{T}$ *is continuous.*

Then, $\mathscr{T}$ *has a fixed point, that is, there exists* $a \in \mathfrak{N}$ *such that* $\mathscr{T}a = a$*.*

Then, many authors generalize the notion of integral contraction in metric space and other generalize spaces see ([6, 11, 15, 12, 14, 16, 18, 19, 20, 21, 22, 34, 37]).

In the spade of generalization of notion of metric space, in 2022, Berzig [7] gave the notion of supra metric space as follows:

Definition 5. *[7] Let $\mathfrak{N}$ be a non-empty set and $\sigma_{sm} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ be a map such that*

1. $\sigma_{sm}(a, b) = 0$ *iff* $a = b$;
2. $\sigma_{sm}(a, b) = \sigma_{sm}(b, a)$;
3. $\sigma_{sm}(a, b) \leq \sigma_{sm}(a, c) + \sigma_{sm}(c, b) + \rho.\sigma_{sm}(a, c)\sigma_{sm}(c, b)$;

for all $a, b, c \in \mathfrak{N}$, for some constant $\rho \in [0, \infty)$.

Then, σ_{sm} is called **supra metric** and the ordered pair $(\mathfrak{N}, \sigma_{sm})$ is called **supra metric space**.

Example 1. *[7] Suppose $(\mathfrak{N}, \sigma)$ be a metric space and α, β are positive real numbers. Then*

1. $\sigma^{\alpha}_{sm_1}(a, b) = \sigma(a, b)(\sigma(a, b) + \alpha)$ *are supra metrics with constant* $\rho = \frac{2}{\alpha}$.
2. $\sigma^{\alpha}_{sm_2}(a, b) = \beta(e^{\sigma(a,b)} - 1)$ *are supra metrics with constant* $\rho = \frac{1}{\beta}$.

But, $\sigma^{\alpha}_{sm_1}$ and $\sigma^{\beta}_{sm_2}$ are not necessary usual metrics. For example for $\alpha = 2$ and $\sigma(a, b) = |a - b|$ $\sigma^{2}_{sm_1}(4, 0) > \sigma^{2}_{sm_1}(4, 2) + \sigma^{2}_{sm_1}(2, 0)$.

Definition 6. *[7] Let $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space. The set*

$$B(a_0, r) := \{a \in \mathfrak{N} : \sigma_{sm}(a_0, a) < r\},$$

where $r > 0$ and $a_0 \in \mathfrak{N}$, is called open of radius r and center a_0. A subset $\mathfrak{M}$ of $\mathfrak{N}$ is called open if for any point $a \in \mathfrak{M}$ there exists $r > 0$ such that $B(a, r) \subset \mathfrak{M}$. The family of all open subsets of $\mathfrak{N}$ will be denoted by τ.

Proposition 1. *[7] Let $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space. Then, each open ball is an open set.*

Definition 7. *[7] Let $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space. A sequence $\{a_n\} \in \mathfrak{N}$ converges to a if for all $\epsilon > 0$ the ball $B(a, \epsilon)$ contains all but a finite numbers of terms of the sequence. So, a is the limit of the sequence and we say that*
$\lim_{n\to\infty} \sigma_{sm}(a_n, a) = 0$.

Proposition 2. *[7] Let $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space. If the sequence $\{a_n\}$ has a limit, then it is unique.*

Definition 8. *[7] Let $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space. A mapping $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is said to be continuous at a if for all $\epsilon > 0$ there exists $\delta > 0$ such that $\sigma_{sm}(\mathscr{T}b, \mathscr{T}a) < \epsilon$ whenever $\sigma_{sm}(b, a) < \delta$. If $\mathscr{T}$ is continuous at all the points of $\mathfrak{N}$ then, $\mathscr{T}$ is continuous on $\mathfrak{N}$.*

Definition 9. *[7] Let $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space. A sequence $\{a_n\} \in \mathfrak{N}$ is a Cauchy sequence if for all $\epsilon > 0$ there exists $k \in \mathbb{N}$ such that $\sigma_{sm}(a_n, a_m) < \epsilon$ for all $m, n \geq k$.*

A supra metric is complete if every Cauchy sequence is convergent.

Proposition 3. *[7] Let $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space. Then the set $\mathfrak{N}$ with respect to supra metrics in Example 1 is complete supra metric space.*

Proof
To prove the proposition, we prove the following claim first.
Claim: If $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space and $(\mathfrak{N}, \sigma)$ be a metric space. Suppose $h : \mathbb{R}^+ \to \mathbb{R}^+$ be a homeomorphism such that $\sigma_{sm} = ho\sigma$, then, a convergent sequence with respect to σ converges with respect to σ_{sm} to the same point.
Since, h is homeomorphism

1. h is continuous;
2. h^{-1} is continuous;
3. $h(0) = 0$;
4. h is strictly increasing.

Now, assume $a_n \to a$ with respect to σ, that is

$$\sigma(a_n, a) \to 0.$$

Now, apply h:

$$\sigma_{sm}(a_n, a) = h(\sigma(a_n, a)).$$

From the properties of h, we get

$$h(\sigma(a_n, a)) \to h(0) = 0.$$

Thus,

$$\sigma_{sm}(a_n, a) \to 0.$$

Therefore,
$a_n \to a$ with respect to σ_{sm}.
Hence, the claim is proved.
The 2nd claim is that the functions $h_1(t) = t(t + \alpha), h_2(t) = \beta(e^t - 1), (\alpha, \beta > 0)$ are homeomorphism of $\mathbb{R}^+$.
It is easy to verify that h_1, h_2 are continuous, bijective and their inverse is also continuous.
Thus, one can easily show that they are homeomorphism of $\mathbb{R}^+$.
Hence, the proof is complete. □

Lemma 1. [7] Every supra metric is continuous.

Remark 2. *[7] If a sequence $\{a_n\} \in \mathfrak{N}$ is a Cauchy sequence in $\mathfrak{N}$ then there exists $a \in \mathfrak{N}$ such that $\lim_{n\to\infty} \sigma_{sm}(a_n, a) = 0$ and by the property (σ_3) every subsequence converges to a.*

Then, many authors proved fixed point results in supra metric space and its other generalized spaces.
In 2025, Kumar *et al.* [17] α-admissible E-type contractions with respect to ξ and related fixed point theorems in supra metric space as stated below:

Definition 10. *[17] Suppose $\mathscr{T}$ be a self map on a supra metric space $(\mathfrak{N}, \sigma_{sm})$. If there exist $\xi \in \mathcal{Z}$ and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ such that*

$$\xi(\mathscr{S}(a, b)\sigma_{sm}(\mathscr{T}a, \mathscr{T}b), E(a, b)) \geq 0, \tag{1.1}$$

where
$E(a, b) = \sigma_{sm}(a, b) + |\sigma_{sm}(a, \mathscr{T}a) - \sigma_{sm}(b, \mathscr{T}b)|$, for all $a, b \in \mathfrak{N}$.
Then, we say that $\mathscr{T}$ is an $\mathscr{S}$-admissible E-type $\mathcal{Z}$ contraction with respect to ξ.
If $\mathscr{S}(a, b) = 1$, then $\mathscr{T}$ is said to be E-type $\mathcal{Z}$ contraction with respect to ξ.

Theorem 3. *[17] Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{T}$ is an $\mathscr{S}$-admissible E-type $\mathcal{Z}$ contraction with respect to ξ. Assume*

1. *$\mathscr{T}$ is $\mathscr{S}$-admissible;*
2. *there exists $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;*
3. *$\mathscr{T}$ is continuous.*

Then $\mathscr{T}$ has a unique fixed point.

Again in 2025, Ahmad and Albargi [2] proved the following fixed point theorem in extended b-metric space.

Theorem 4. *[2] Let $(\mathfrak{N}, \sigma)$ be a complete extended b-supra metric space with $b \geq 1$ and consider a mapping $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$. Suppose that there exists the functions $\alpha, \beta : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ and a constant $\varpi \in [0, 1)$ such that $\alpha(a, \mathscr{T}a)\alpha(b, \mathscr{T}b) \geq \beta(a, \mathscr{T}a)\beta(b, \mathscr{T}b)$ implies*

$$\sigma(\mathscr{T}a, \mathscr{T}b) \leq \frac{\varpi}{b} M(a, b)$$

where

$$M(a, b) = \max\{\sigma(a, b), \min\{\frac{\sigma(a, \mathscr{T}a)\sigma(b, \mathscr{T}b)}{1 + \sigma(a, b)}, \frac{\sigma(a, \mathscr{T}b)\sigma(b, \mathscr{T}a)}{1 + \sigma(a, b)}\}\}$$

holds for all $a, b \in \mathfrak{N}$. Assume that the following conditions hold:

1. *the mapping $\mathscr{T}$ is an α-admissible mapping with respect to β;*
2. *there exists $a_0 \in \mathfrak{N}$ such that $\alpha(a_0, \mathscr{T}a_0) \geq \beta(a_0, \mathscr{T}a_0)$;*
3. *either $\mathscr{T}$ is continuous or if $\{a_n\}$ is a sequence in $\mathfrak{N}$ such that $a_n \to a$, $\alpha(a_n, a_{n+1}) \geq \beta(a_n, a_{n+1})$ then $\alpha(a, \mathscr{T}a) \geq \beta(a, \mathscr{T}a)$.*

Then, $\mathscr{T}$ possesses a fixed point.
Moreover $\alpha(a, \mathscr{T}a)\alpha(b, \mathscr{T}b) \geq \beta(a, \mathscr{T}a)\beta(b, \mathscr{T}b)$ for all fixed points a, b. Then, the fixed point is unique.

Then, using this results they shown the existence of solution of non-linear volterra integral equation.
Again in 2025, Shammaky and Hakami [38] gave new notion of complex-valued suprametric space and gave the following common fixed point theorem.

Theorem 5. *[38] Let $(\mathfrak{N}, \mathfrak{z})$ be a complete complex valued supra metric space and $\mathscr{T}, \mathscr{S} : \mathfrak{N} \to \mathfrak{N}$. Assuming the existence of functions $k_1, k_2, k_3 : \mathfrak{N} \to [0, 1)$ such that*

1. *$k_i(\mathscr{T}a) \leq k_i(a)$ and $k_i(\mathscr{S}a) \leq k_i(a)$, for all $i = 1, 2, 3$.*
2. *$k_1(a) + 2k_2(a) + k_3(a) < 1$.*
3. $$\mathfrak{z}(\mathscr{T}a, \mathscr{S}b) \leq k_1(a)\mathfrak{z}(a, b) + k_2(a)(\mathfrak{z}(a, \mathscr{T}a) + \mathfrak{z}(b, \mathscr{S}b)) + k_3(a)\frac{\mathfrak{z}(a, \mathscr{T}a)\mathfrak{z}(b, \mathscr{S}b)}{1 + \mathfrak{z}(a, b)}$$

   for all $a, b \in \mathfrak{N}$ with $\mathfrak{z}(a, b) \neq -1$, then $\mathscr{T}, \mathscr{S}$ possesses a unique common fixed point.

Using it, they solved a Volterra-Hammerstein non-linear integral equation.
Now onward, the article is divided into two sections 2,3 respectively. In section 2, we shall define the generalized integral type contractions and proved the related fixed point theorems. Some consequences are discussed in the form of corollaries in section 3.

2. Results

Here, we shall give the notion of generalized rational $\mathscr{S} - \psi$-contractive mapping of integral type A and type B and proved related fixed point theorems.

Definition 11. *Let $(\mathfrak{N}, \sigma_{sm})$ be a supra metric space and $\mathscr{T}$ be a self map on it. We say that $\mathscr{T}$ is a generalized $\mathscr{S} - \psi$-contractive mapping of integral type A if there exist two functions $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ and $\psi \in \Psi$ such that for all $a, b \in \mathfrak{N}$,*

$$\mathscr{S}(a, b) \int_0^{\sigma(\mathscr{T}a, \mathscr{T}b)} \phi(t)dt \leq \psi(\int_0^{M(a,b)} \phi(t)dt) \tag{2.1}$$

where $\phi \in \Phi$ and
$M(a, b) = \max\{\sigma_{sm}(a, b), \sigma_{sm}(a, \mathscr{T}a), \sigma_{sm}(b, \mathscr{T}b), \frac{\sigma_{sm}(a, \mathscr{T}a) + \sigma_{sm}(b, \mathscr{T}b)}{2}, \frac{\sigma_{sm}(a, \mathscr{T}a).\sigma_{sm}(b, \mathscr{T}b)}{\sigma_{sm}(a, b)}\}$.

Theorem 6. *Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ is a transitive mapping. Suppose that $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is a generalized $\mathscr{S} - \psi$-contractive mapping of integral type A and satisfies the following conditions:*

1. *$\mathscr{T}$ is $\mathscr{S}$-admissible;*
2. *there exists $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;*
3. *$\mathscr{T}$ is continuous.*

Then, $\mathscr{T}$ has a fixed point, that is, there exists $a \in \mathfrak{N}$ such that $\mathscr{T}a = a$.

Proof
Suppose $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$. Construct a sequence $\{a_n\}$ as $a_{n+1} = \mathscr{T}a_n$.
Now, if possible assume that there exists $N \in \mathbb{N}$ such that $a_{N+1} = a_N$ then $\mathscr{T}a_N = a_N$, which implies that a_N is the fixed point of $\mathscr{T}$ and the proof is done.
So, we conclude that $a_{n+1} \neq a_n$ for all $n \in \mathbb{N}$.
Since, $\mathscr{T}$ is $\mathscr{S}$-admissible

$$\begin{aligned}
\mathscr{S}(a_0, a_1) = \mathscr{S}(a_0, \mathscr{T}a_0) \geq 1 \quad &\Rightarrow \quad \mathscr{S}(a_1, a_2) \geq 1; \\
\mathscr{S}(a_1, a_2) = \mathscr{S}(a_1, \mathscr{T}a_1) \geq 1 \quad &\Rightarrow \quad \mathscr{S}(a_2, a_3) \geq 1
\end{aligned}$$

Continuing in this way, we find that

$$\mathscr{S}(a_n, a_{n+1}) \geq 1, \tag{2.2}$$

for all $n \in \mathbb{N}$.
Using $a = a_{n-1}, b = a_n$, equation (3), equation (2) implies that

$$\begin{aligned}
\int_0^{\sigma_{sm}(a_n, a_{n+1})} \phi(t)dt &= \int_0^{\sigma_{sm}(\mathscr{T}a_{n-1}, \mathscr{T}a_n)} \phi(t)dt; \\
&\leq \mathscr{S}(a_{n-1}, a_n) \int_0^{\sigma_{sm}(\mathscr{T}a_{n-1}, \mathscr{T}a_n)} \phi(t)dt; \\
&\leq \psi(\int_0^{M(a_{n-1}, a_n)} \phi(t)dt),
\end{aligned} \tag{2.3}$$

where

$$\begin{aligned}
M(a_{n-1}, a_n) &= \max\{\sigma_{sm}(a_{n-1}, a_n), \sigma_{sm}(a_{n-1}, \mathscr{T}a_{n-1}), \sigma_{sm}(a_n, \mathscr{T}a_n) \\
&, \quad \frac{\sigma_{sm}(a_{n-1}, \mathscr{T}a_{n-1}) + \sigma_{sm}(a_n, \mathscr{T}a_n)}{2}, \frac{\sigma_{sm}(a_{n-1}, \mathscr{T}a_{n-1}).\sigma_{sm}(a_n, \mathscr{T}a_n)}{\sigma_{sm}(a_{n-1}, a_n)}\} \\
&= \max\{\sigma_{sm}(a_{n-1}, a_n), \sigma_{sm}(a_{n-1}, a_n), \sigma_{sm}(a_n, a_{n+1}) \\
&, \quad \frac{\sigma_{sm}(a_{n-1}, a_n) + \sigma_{sm}(a_n, a_{n+1})}{2}, \frac{\sigma_{sm}(a_{n-1}, a_n).\sigma_{sm}(a_n, a_{n+1})}{\sigma_{sm}(a_{n-1}, a_n)}\}, \\
&= \max\{\sigma_{sm}(a_{n-1}, a_n), \sigma_{sm}(a_n, a_{n+1}) \\
&, \quad \frac{\sigma_{sm}(a_{n-1}, a_n) + \sigma_{sm}(a_n, a_{n+1})}{2}, \sigma_{sm}(a_n, a_{n+1})\} \\
&= \max\{\sigma_{sm}(a_{n-1}, a_n), \sigma_{sm}(a_n, a_{n+1})\}
\end{aligned}$$

Now, if possible assume that $\sigma_{sm}(a_{n-1}, a_n) < \sigma_{sm}(a_n, a_{n+1})$ then

$$M(a_{n-1}, a_n) = \sigma_{sm}(a_n, a_{n+1}).$$

Using this value in equation (4), we get

$$\int_0^{\sigma_{sm}(a_n, a_{n+1})} \phi(t)dt \leq \psi(\int_0^{M(a_{n-1}, a_n)} \phi(t)dt) < \int_0^{\sigma_{sm}(a_n, a_{n+1})} \phi(t)dt,$$

By using the fact that $\psi(t) < t$ for all $t > 0$.
which is a contradiction.
Hence,

$$M(a_{n-1}, a_n) = \sigma_{sm}(a_{n-1}, a_n).$$

Thus, equation (4) becomes

$$\int_0^{\sigma_{sm}(a_n, a_{n+1})} \phi(t)dt \leq \psi(\int_0^{\sigma_{sm}(a_{n-1}, a_n)} \phi(t)dt).$$

Continuing in this way we conclude that

$$\int_0^{\sigma_{sm}(a_n, a_{n+1})} \phi(t)dt \leq \psi^n(\int_0^{\sigma_{sm}(a_0, a_1)} \phi(t)dt).$$

Applying limit as $n \to \infty$ in the above setting and using property of ψ function, we get $\int_0^{\sigma_{sm}(a_n, a_{n+1})} \phi(t)dt \to 0$ and this from property of ϕ function implies that $\sigma_{sm}(a_n, a_{n+1}) \to 0$ as $n \to \infty$.
Now, we shall prove that sequence $\{a_n\}$ is a Cauchy sequence. So, if possible assume that it is not Cauchy then there exists a $\epsilon > 0$ such that for each natural number k, there are natural numbers $m(k), n(k)$ such that $m(k) > n(k) > k$, ($m(k)$ is the smallest) with

$$\sigma_{sm}(a_{m(k)}, a_{n(k)}) \geq \epsilon, \sigma_{sm}(a_{m(k)-1}, a_{n(k)}) < \epsilon. \tag{2.4}$$

Therefore, using Definition 5 and inequality (5) for all $r \in \mathbb{N}$, we have

$$\begin{aligned} \sigma_{sm}(a_{m(k)-1}, a_{n(k)-1}) &\leq \sigma_{sm}(a_{m(k)-1}, a_{n(k)}) + \sigma_{sm}(a_{n(k)-1}, a_{n(k)}) + \rho\sigma_{sm}(a_{m(k)-1}, a_{n(k)})\sigma_{sm}(a_{n(k)-1}, a_{n(k)}) \\ &\leq \epsilon + \sigma_{sm}(a_{n(k)-1}, a_{n(k)}) + \rho\epsilon\sigma_{sm}(a_{n(k)-1}, a_{n(k)}), \end{aligned}$$

Letting $k \to \infty$ in the above inequality and using equation (5), we obtain

$$\lim_{k\to\infty} \sigma_{sm}(a_{m(k)-1}, a_{n(k)-1}) \leq \epsilon. \tag{2.5}$$

Letting $k \to \infty$ from the above inequality, we conclude that

$$\lim_{k\to\infty} \int_0^{\sigma_{sm}(a_{m(k)-1}, a_{n(k)-1})} \phi(t)dt \leq \int_0^{\epsilon} \phi(t)dt. \tag{2.6}$$

Transitive property of $\mathscr{S}$ implies that

$$\mathscr{S}(a_{m(k)-1}, a_{n(k)-1}) \geq 1. \tag{2.7}$$

Thus

$$\begin{aligned} \int_0^{\epsilon} \phi(t)dt \leq \int_0^{\sigma_{sm}(a_{m(k)}, a_{n(k)})} \phi(t)dt &= \int_0^{\sigma_{sm}(\mathscr{T}a_{m(k)-1}, \mathscr{T}a_{n(k)-1})} \phi(t)dt; \\ &\leq \mathscr{S}(a_{m(k)-1}, a_{n(k)-1}) \int_0^{\sigma_{sm}(\mathscr{T}a_{m(k)-1}, \mathscr{T}a_{n(k)-1})} \phi(t)dt; \\ &\leq \psi(\int_0^{M(a_{m(k)-1}, a_{n(k)-1})} \phi(t)dt). \end{aligned} \tag{2.8}$$

where

$$M(a_{m(k)}, a_{n(k)}) = \max\{\sigma_{sm}(a_{m(k)}, a_{n(k)}), \sigma_{sm}(a_{m(k)}, \mathscr{T}a_{m(k)}), \sigma_{sm}(a_{n(k)}, \mathscr{T}a_{n(k)})$$

$$
\begin{aligned}
, \quad & \frac{\sigma_{sm}(a_{m(k)}, \mathscr{T}a_{m(k)}) + \sigma_{sm}(a_{n(k)}, \mathscr{T}a_{n(k)}}{2}, \frac{\sigma_{sm}(a_{m(k)}, \mathscr{T}a_{m(k)}).\sigma_{sm}(a_{n(k)}, \mathscr{T}a_{n(k)})}{\sigma_{sm}(a_{m(k)}, a_{n(k)})}\} \\
= \quad & \max\{\sigma_{sm}(a_{m(k)}, a_{n(k)}), \sigma_{sm}(a_{m(k)}, a_{m(k)+1}), \sigma_{sm}(a_{n(k)}, a_{n(k)+1}) \\
, \quad & \frac{\sigma_{sm}(a_{m(k)}, a_{m(k)+1}) + \sigma_{sm}(a_{n(k)}, a_{n(k)+1}}{2}, \frac{\sigma_{sm}(a_{m(k)}, a_{m(k)+1}).\sigma_{sm}(a_{n(k)}, a_{n(k)+1})}{\sigma_{sm}(a_{m(k)}, a_{n(k)})}\}
\end{aligned}
$$

Taking limit as $k \to \infty$, we get

$$\lim_{k\to\infty} M(a_{m(k)}, a_{n(k)}) = \epsilon.$$

Thus, after applying limit $k \to \infty$, equation (9) becomes

$$\int_0^{\epsilon} \phi(t)dt \leq \int_0^{\sigma_{sm}(a_{m(k)}, a_{n(k)})} \phi(t)dt < \int_0^{\epsilon} \phi(t)dt.$$

a contradiction.

Thus, the sequence $\{a_n\}$ is a Cauchy sequence. Since, the space is complete. So, $\{a_n\}$ is a convergent sequence. This shows that there is $a \in \mathfrak{N}$ such that $a_n \to a$. It is also given that $\mathscr{T}$ is continuous. So, $\mathscr{T}a_n = a_{n+1} \to \mathscr{T}a$, and the uniqueness of the limit implies that $\mathscr{T}a = a$. □

In the next theorem, we shall remove the condition of continuity.

Theorem 7. *Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ is a transitive mapping. Suppose that $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is a generalized $\mathscr{S} - \psi$-contractive mapping of integral type A and satisfies the following conditions:*

1. *$\mathscr{T}$ is $\mathscr{S}$-admissible;*
2. *there exists $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;*
3. *If $\{a_n\}$ is a sequence in $\mathfrak{N}$ such that $\mathscr{S}(a_n, a_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $a_n \to a \in \mathfrak{N}$ as $n \to \infty$, then there exists a subsequence $\{a_{n(k)}\}$ of $\{a_n\}$ such that $\mathscr{S}(a_{n(k)}, a) \geq 1$ for all k.*

Then $\mathscr{T}$ has a fixed point, that is, there exists $a \in \mathfrak{N}$ such that $\mathscr{T}a = a$.

Proof

From the proof of above theorem, we can find a sequence $\{a_n\}$ such that $a_{n+1} = \mathscr{T}^n a_n$ which is converging to a. Now, by the condition 3, there exists a subsequence $\{a_{n(k)}\}$ of $\{a_n\}$ such that $\mathscr{S}(a_{n(k)}, a) \geq 1$ for all k.

Now, by the equation (2) and condition 3, we get

$$
\begin{aligned}
\int_0^{\sigma_{sm}(\mathscr{T}a, a_{n(k)+1})} \phi(t)dt &= \int_0^{\sigma_{sm}(\mathscr{T}a, \mathscr{T}a_n(k))} \phi(t)dt; \\
&\leq \mathscr{S}(a, a_{n(k)}) \int_0^{\sigma_{sm}(\mathscr{T}a, \mathscr{T}a_n(k))} \phi(t)dt; \\
&\leq \psi(\int_0^{M(a, a_{n(k)})} \phi(t)dt), \qquad (2.9)
\end{aligned}
$$

where

$$
\begin{aligned}
M(a, a_n(k)) \quad = \quad & \max\{\sigma_{sm}(a, a_n(k)), \sigma_{sm}(a, \mathscr{T}a), \sigma_{sm}(a_n(k), \mathscr{T}a_n(k)) \\
, \quad & \frac{\sigma_{sm}(a, \mathscr{T}a) + \sigma_{sm}(a_n(k), \mathscr{T}a_n(k))}{2}, \frac{\sigma_{sm}(a, \mathscr{T}a).\sigma_{sm}(a_n(k), \mathscr{T}a_n(k))}{\sigma_{sm}(a, a_n(k))}\}
\end{aligned}
$$

Since, $\{a_n\}$ is a convergent sequence, so by applying limit as $k \to \infty$, we get
$\lim_{k\to\infty} M(a, a_{n(k)}) = \sigma_{sm}(a, \mathscr{T}a)$.

Assume that if possible $\sigma_{sm}(a, \mathscr{T}a) > 0$ then, from the above we can say that there exist large k for which $M(a, a_{n(k)}) > 0$, which implies that

$$\int_0^{\sigma_{sm}(\mathscr{T}a, a_{n(k)+1})} \phi(t)dt \leq \psi(\int_0^{M(a,a_{n(k)})} \phi(t)dt). \tag{2.10}$$

Letting $k \to \infty$ in the above, we find that

$$\int_0^{\sigma_{sm}(\mathscr{T}a,a)} \phi(t)dt \leq \psi(\int_0^{\sigma_{sm}(\mathscr{T}a,a))} \phi(t)dt) < \int_0^{\sigma_{sm}(\mathscr{T}a,a)} \phi(t)dt.$$

a contradiction.
Thus, $\sigma_{sm}(\mathscr{T}a, a) = 0$ implies that $\mathscr{T}a = a$. □

Definition 12. *Let $(\mathfrak{N}, \sigma)$ be a supra metric space and $\mathscr{T}$ be a self map on it. We say that $\mathscr{T}$ is a generalized $\mathscr{S} - \psi$-contractive mapping of integral type B if there exist two functions $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ and $\psi \in \Psi$ such that for all $a, b \in \mathfrak{N}$,*

$$\mathscr{S}(a, b) \int_0^{\sigma_{sm}(\mathscr{T}a,\mathscr{T}b)} \phi(t)dt \leq \psi(\int_0^{M(a,b)} \phi(t)dt) \tag{2.11}$$

where $\phi \in \Phi$ and

$$\begin{aligned} M(a, b) &= \max\{\sigma_{sm}(a, b), \sigma_{sm}(a, \mathscr{T}a), \sigma_{sm}(b, \mathscr{T}b), \frac{\sigma_{sm}(a, \mathscr{T}a) + \sigma_{sm}(b, \mathscr{T}b)}{2} \\ &, \quad \frac{\sigma_{sm}(a, \mathscr{T}a).\sigma_{sm}(b, \mathscr{T}b)}{\sigma_{sm}(a, b)}, \frac{(1 + \sigma_{sm}(a, \mathscr{T}a))\sigma_{sm}(b, \mathscr{T}b)}{1 + \sigma_{sm}(a, b)}\}. \end{aligned}$$

Theorem 8. *Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ is a transitive mapping. Suppose that $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is a generalized $\mathscr{S} - \psi$-contractive mapping of integral type B and satisfies the following conditions:*

1. *$\mathscr{T}$ is $\mathscr{S}$-admissible;*
2. *there exists $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;*
3. *$\mathscr{T}$ is continuous.*

Then $\mathscr{T}$ has a fixed point, that is, there exists $a \in \mathfrak{N}$ such that $\mathscr{T}a = a$.

Proof
The proof will be the same as the proof of Theorem 7. □

Theorem 9. *Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ is a transitive mapping. Suppose that $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is a generalized $\mathscr{S} - \psi$-contractive mapping of integral type B and satisfies the following conditions:*

1. *$\mathscr{T}$ is $\mathscr{S}$-admissible;*
2. *there exists $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;*
3. *If $\{a_n\}$ is a sequence in $\mathfrak{N}$ such that $\mathscr{S}(a_n, a_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $a_n \to a \in \mathfrak{N}$ as $n \to \infty$, then there exists a subsequence $\{a_{n(k)}\}$ of $\{a_n\}$ such that $\mathscr{S}(a_{n(k)}, a) \geq 1$ for all k.*

Then $\mathscr{T}$ has a fixed point, that is, there exists $a \in \mathfrak{N}$ such that $\mathscr{T}a = a$.

Proof
Proof will be same like Theorem 8. □

3. Consequences

In this section, we shall discuss some direct consequences of our proved results.

Definition 13. *Let $(\mathfrak{N}, \sigma)$ be a supra metric space and $\mathscr{T}$ be a self map on it. We say that $\mathscr{T}$ is a $\mathscr{S} - \psi$-contractive mapping of integral type if there exist two functions $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ and $\psi \in \Psi$ such that for all $a, b \in \mathfrak{N}$,*

$$\mathscr{S}(a,b) \int_0^{\sigma_{sm}(\mathscr{T}a,\mathscr{T}b)} \phi(t)dt \leq \psi(\int_0^{\sigma_{sm}(a,b)} \phi(t)dt) \tag{3.1}$$

where $\phi \in \Phi$.

Corollary 1. Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ is a transitive mapping. Suppose that $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is a $\mathscr{S} - \psi$-contractive mapping of integral type and satisfies the following conditions:

1. $\mathscr{T}$ is $\mathscr{S}$-admissible;
2. there exists $a_0 \in X$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;
3. $\mathscr{T}$ is continuous.

Then $\mathscr{T}$ has a fixed point, that is, there exists $a \in \mathfrak{N}$ such that $\mathscr{T}a = a$.

Proof
Taking $M(a, b) = \sigma_{sm}(a, b)$ in Theorem 9. □

Corollary 2. Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ is a transitive mapping. Suppose that $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is a $\mathscr{S} - \psi$-contractive mapping of integral type and satisfies the following conditions:

1. $\mathscr{T}$ is $\mathscr{S}$-admissible;
2. there exists $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;
3. If $\{a_n\}$ is a sequence in $\mathfrak{N}$ such that $\mathscr{S}(a_n, a_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $a_n \to a \in \mathfrak{N}$ as $n \to \infty$, then there exists a subsequence $\{a_{n(k)}\}$ of $\{a_n\}$ such that $\mathscr{S}(a_{n(k)}, a) \geq 1$ for all k.

Then $\mathscr{T}$ has a fixed point, that is, there exists $a \in \mathfrak{N}$ such that $\mathscr{T}a = a$.

Proof
Taking $M(a, b) = \sigma_{sm}(a, b)$ in Theorem 10. □

Definition 14. *Let $(\mathfrak{N}, \sigma)$ be a supra metric space and $\mathscr{T}$ be a self map on it. We say that $\mathscr{T}$ is a $\mathscr{S} - \psi$-contractive mapping if there exist two functions $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ and $\psi \in \Psi$ such that for all $a, b \in \mathfrak{N}$,*

$$\mathscr{S}(a,b)\sigma_{sm}(\mathscr{T}a, \mathscr{T}b) \leq \psi(\sigma_{sm}(a,b)). \tag{3.2}$$

Corollary 3. Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ is a mapping. Suppose that $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is a $\mathscr{S} - \psi$-contractive mapping and satisfies the following conditions:

1. $\mathscr{T}$ is $\mathscr{S}$-admissible;
2. there exists $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;
3. $\mathscr{T}$ is continuous.

Then $\mathscr{T}$ has a fixed point, that is, there exists $a \in \mathfrak{N}$ such that $\mathscr{T}a = a$.

Proof
Taking $\phi(t) = 1$ in Corollary 1. □

Corollary 4. Suppose $(\mathfrak{N}, \sigma_{sm})$ be a complete supra metric space and $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ is a mapping. Suppose that $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ is a $\mathscr{S} - \psi$-contractive mapping that satisfies the following conditions:

1. $\mathscr{T}$ is $\mathscr{S}$-admissible;
2. there exists $a_0 \in \mathfrak{N}$ such that $\mathscr{S}(a_0, \mathscr{T}a_0) \geq 1$;
3. If $\{a_n\}$ is a sequence in $\mathfrak{N}$ such that $\mathscr{S}(a_n, a_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $a_n \to a \in \mathfrak{N}$ as $n \to \infty$, then there exists a subsequence $\{a_{n(k)}\}$ of $\{a_n\}$ such that $\mathscr{S}(a_{n(k)}, a) \geq 1$ for all k.

Then $\mathscr{T}$ has a fixed point, that is, there exists $a \in \mathfrak{N}$ such that $\mathscr{T}a = a$.

Proof
Taking $\phi(t) = 1$ in Corollary 2. □

Many more results from the literature can be extracted from the main results in the form of corollaries.

Example 2. *Suppose* $\mathfrak{N} = [0, \infty)$. *Define* $\sigma_{sm} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *as* $\sigma_{sm}(a, b) = |a - b|(|a - b| + \frac{1}{3})$. *From Example 1 it is clear that* $(\mathfrak{N}, \sigma_{sm})$ *is a complete supra metric space with* $\rho = 6$.
Now, consider the function $\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *as* $\mathscr{S}(a, b) = 1$ *and self map* $\mathscr{T} : \mathfrak{N} \to \mathfrak{N}$ *as*
$\mathscr{T}a = \frac{a}{3}$.
Taking $\psi(t) = \frac{t}{2}$.
Left hand side of equation (14) becomes

$$\begin{aligned} \mathscr{S}(a, b)\sigma_{sm}(\mathscr{T}a, \mathscr{T}b) &= |\frac{a}{3} - \frac{b}{3}|(|\frac{a}{3} - \frac{b}{3}| + \frac{1}{3}), \\ &= \frac{1}{9}|a - b|(|a - b| + 1). \end{aligned} \tag{3.3}$$

Now, right hand side is

$$\begin{aligned} \psi(\sigma_{sm}(a, b)) &= \frac{1}{2}(|a - b|(|a - b| + \frac{1}{3})) \\ &= \frac{1}{6}|a - b|(3|a - b| + 1). \end{aligned} \tag{3.4}$$

From equations (15) and (16), one can observe that

$$\mathscr{S}(a, b)\sigma_{sm}(\mathscr{T}a, \mathscr{T}b) \leq \psi(\sigma_{sm}(a, b)).$$

Thus, all the conditions of Corollary 3 are satisfied.
Hence, $\mathscr{T}$ *has a fixed point.*
Clearly, 0 *is the fixed point.*

Example 3. *Consider the set* $\mathfrak{N} = \{1, 2, 3\}$ *and define the* $\sigma_{sm} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *as*
$\sigma_{sm}(1, 2) = 1 = \sigma_{sm}(2, 1), \sigma_{sm}(1, 3) = 2 = \sigma_{sm}(3, 1), \sigma_{sm}(3, 2) = \sigma_{sm}(2, 3) = \frac{1}{2}, \sigma_{sm}(1, 1) = \sigma_{sm}(2, 2) = \sigma_{sm}(3, 3) = 0$ *and* $\rho = 2$.
Since,
$\sigma_{sm}(1, 3) > \sigma_{sm}(2, 3) + \sigma_{sm}(1, 2)$.
Thus, σ_{sm} *is not a metric. But, one can easily check that it is a supra metric. Hence,*
$(\mathfrak{N}, \sigma_{sm})$ *is a complete supra metric space.*
Define self map $\mathscr{T}$ *on* $\mathfrak{N}$ *as* $\mathscr{T}1 = 3, \mathscr{T}2 = 2, \mathscr{T}3 = 1$.
$\mathscr{S} : \mathfrak{N} \times \mathfrak{N} \to [0, \infty)$ *as*
$\mathscr{S}(1, 1) = \mathscr{S}(2, 2) = \mathscr{S}(3, 3) = 2, \mathscr{S}(1, 2) = \mathscr{S}(1, 3) = 0$ *and* $\mathscr{S}(2, 3) = \frac{1}{2}$.
Now, one can observe easily that
equation (2) holds trivially for all pairs except $a = 2, b = 3$. *So, we have to check only for this pair* $(2, 3)$. *For*

$a = 2, b = 3$ left hand side of equation (2) becomes

$$\begin{aligned} \mathscr{S}(a,b)\int_0^{\sigma_{sm}(\mathscr{T}a,\mathscr{T}b)}\phi(t)dt &= \frac{1}{2}\int_0^{\sigma_{sm}(\mathscr{T}2,\mathscr{T}3)}\phi(t)dt, \\ &= \frac{1}{2}\int_0^{\sigma_{sm}(2,1)}\phi(t)dt \\ &= \frac{1}{2}\int_0^{1}\phi(t)dt. \end{aligned} \tag{3.5}$$

and right hand side becomes

$$\psi(\int_0^{\sigma_{sm}(2,3)}\phi(t)dt) = \frac{1}{2}(\int_0^{\frac{1}{2}}\phi(t)dt). \tag{3.6}$$

From equations (17) and (18), we conclude that equation (2) does not hold.
Now, again equation (2) holds trivially for all the pairs except $(2,3)$. Thus,

$$\begin{aligned} M(2,3) &= \max\{\sigma_{sm}(2,3), \sigma_{sm}(2,\mathscr{T}2), \sigma_{sm}(3,\mathscr{T}3), \frac{\sigma_{sm}(2,\mathscr{T}2)+\sigma_{sm}(3,\mathscr{T}3)}{2}, \frac{\sigma_{sm}(2,\mathscr{T}2).\sigma_{sm}(3,\mathscr{T}3)}{\sigma_{sm}(2,3)}\} \\ &= \max\{\frac{1}{2}, 0, 2, 1, 0\} \\ &= 2. \end{aligned}$$

So, the right hand side becomes

$$\psi(\int_0^{M(2,3)}\phi(t)dt) = \frac{1}{2}(\int_0^{2}\phi(t)dt). \tag{3.7}$$

Hence, from equations (17) and (19) equation (2) holds.
Thus, all the conditions of Theorem 10 holds. Hence, $\mathscr{T}$ has a fixed point. Clearly, 2 is the fixed point of $\mathscr{T}$.

4. Future Scope

Here, we gave the new notion of generalized integral contractions and proved the related fixed point theorems in the complete supra metric space. Then, in the form of corollaries some consequences of proved results are also given. Then, to validate the proved result examples are given.
In future, the researcher can extend our result for two or maps in supra metric space or any other generalization of supra metric space. One can also apply our proved results for the solution of various integral, differential, and fractional calculus equations. It can be used to study the Ulam-Hyers stability of fixed point problem.

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REFERENCES

1. Ö. Acar and A.S. Özkapu, *Multivalued rational type F-contraction on orthogonal metric space*, Mathematical Foundations of Computing, vol. 6, no. 3, pp. 303–312, 2023.
2. J. Ahmad and A.H. Albargi, *Fixed point theorems in extended b-suprametric spaces with applications*, AIMS Mathematics, vol. 10, pp. 20759–20781, 2025.
3. H. Alsamir, M.S.M. Noorani, W. Shatanawi, H. Aydi, and H. Qawaqneh, *Fixed point results in metric-like spaces via $\sigma-$simulation functions*, European Journal of Pure and Applied Mathematics, vol. 12, no. 1, pp. 88–100, 2019.

4. H. Alsamir, H. Qawaqneh, G. Al-Musannef, and R. Khalil, *Common Fixed Point of Generalized Berinde Type Contraction and an Application*, European Journal of Pure and Applied Mathematics, vol. 17, no. 4, pp. 2492–2504, 2024.
5. S. Banach, *Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales*, Fundamenta Mathematicae, vol. 3, no. 1, pp. 133–181, 1922.
6. I. M. Batiha, S. A. Njadat, R. M. Batyha, A. Zraiqat, A. Dababneh, and Sh. Momani, *Design Fractional-order PID Controllers for Single-Joint Robot Arm Model*, International Journal of Advances in Soft Computing and its Applications, vol. 14, no. 2, pp. 96–114, 2022.
7. M. Berzig, *First results in suprametric spaces with applications*, Mediterranean Journal of Mathematics, vol. 19, no. 5, Article 226, 2022.
8. M. Berzig, *Nonlinear contraction in b-suprametric spaces*, The Journal of Analysis, 2024, pp. 1–14.
9. M. Berzig and M.D. Rus, *Fixed point theorems for α-contractive mappings of Meir–Keeler type and applications*, arXiv preprint arXiv:1303.5798, 2013.
10. A. Branciari, *A fixed point theorem for mappings satisfying a general contractive condition of integral type*, International Journal of Mathematics and Mathematical Sciences, vol. 29, no. 9, pp. 531–536, 2002.
11. J. Das and A. Das, *Fixed point theorems for generalized integral type weak-contraction mappings in convex modular metric spaces*, Fundamentals of Contemporary Mathematical Sciences, vol. 6, no. 2, pp. 139–153, 2025.
12. M. Elbes, T. Kanan, M. Alia, and M. Ziad, *COVD-19 Detection Platform from X-ray Images using Deep Learning*, International Journal of Advances in Soft Computing and its Applications, vol. 14, no. 1, 2022.
13. A. Hossain, F.A. Khan, and Q.H. Khan, *A relation-theoretic metrical fixed point theorem for rational type contraction mapping with an application*, Axioms, vol. 10, no. 4, Article 316, 2021.
14. A. Judeh and M. Abu Hammad, *Applications of Conformable Fractional Pareto Probability Distribution*, International Journal of Advances in Soft Computing and its Applications, vol. 14, no. 2, pp.115–124, 2022.
15. E. Karapınar, P. Shahi, and K. Tas, *Generalized $\alpha - \psi$-contractive type mappings of integral type and related fixed point theorems*, Journal of Inequalities and Applications, vol. 2014, no. 1, Article 160, 2014.
16. H. Kerim, W. Shatanawi, and A. Tallafha, *Common fixed point theorems via integral type contraction in modular metric space*, University Politehnica of Bucharest Scientific Bulletin Series A, vol. 83, pp. 125–136, 2021.
17. P. Kumar, A. Kumar, M. Kumar, V. Mor, and R. Shukla, *α-admissible E-type contractions with respect to ξ and related fixed point theorems in supra metric space*, Advances in Fixed Point Theory, vol. 15, 2025.
18. Z. Liu, X. Li, S.M. Kang, and S.Y. Cho, *Fixed point theorems for mappings satisfying contractive conditions of integral type and applications*, Fixed Point Theory and Applications, vol. 2011, no. 1, Article 64, 2011.
19. A.A.R.M. Malkawi, *Fixed point theorem in mr-metric spaces via integral type contraction*, WSEAS Transactions on Mathematics, vol. 24, pp. 295–299, 2025.
20. N. Mani, A. Sharma, R. Bhardwaj, and S.M. Bhati, *A study on metric fixed point theorems satisfying integral type contractions*, In: Advances in Applied Mathematical Analysis and Applications, pp. 229–247, River Publishers, 2022.
21. M. Meneceur, H. Qawaqneh, H. Alsamir, and G. Al-Musannef, *Multivalued Almost JS-Contractions, Related Fixed Point Results in Complete b-Metric Spaces and Applications*, European Journal of Pure and Applied Mathematics, vol. 17, no. 4, pp. 3093–3108, 2024.
22. M. Nazam, H. Aydi, M.S.M. Noorani, and H. Qawaqneh, *Existence of Fixed Points of Four Maps for a New Generalized F-Contraction and an Application*, Journal of Function Spaces, vol. 2019, ID. 5980312, 8 pages, 2019.
23. H. Qawaqneh, *Fractional analytic solutions and fixed point results with some applications*, Adv. Fixed Point Theory, vol. 14, no. 1, 2024.
24. H. Qawaqneh, H. A. Jari, A. Altalbe, and A. Bekir, *Stability analysis, modulation instability, and the analytical wave solitons to the fractional Boussinesq-Burgers system*, Phys. Scr., vol. 99, no. 125235, 2024.
25. H. Qawaqneh, M.S.Md. Noorani, and H. Aydi, *Some new characterizations and results for fuzzy contractions in fuzzy b-metric spaces and applications* , AIMS Mathematics, vol. 8, no. 3, pp. 6682–6696, 2023.
26. H. Qawaqneh, H.A. Hammad, and H. Aydi, *Exploring new geometric contraction mappings and their applications in fractional metric spaces*, AIMS Mathematics, vol. 9, no. 1, pp. 521-541, 2024.
27. H. Qawaqneh, M.S. Noorani, H. Aydi, and W. Shatanawi, *On Common Fixed Point Results for New Contractions with Applications to Graph and Integral Equations*, Mathematics, vol. 7, no. 11, ID. 1082, 2019.
28. H. Qawaqneh, and M.A. Alomair, *Mathematical and Physical Analysis of the Fractional Dynamical Model*, Fractal Fract., vol. 9, no. 453, 2025.
29. H. Qawaqneh, A. Altalbe, A. Bekir, and K.U. Tariq, *Investigation of soliton solutions to the truncated M-fractional (3+1)-dimensional Gross-Pitaevskii equation with periodic potential*, AIMS Mathematics, vol. 9, no. 9, pp. 23410–23433, 2024.
30. H. Qawaqneh, J. Manafian, A. Alsubaie, and H. Ahmad, *Investigation of exact solitons to the quartic Rosenau-Kawahara-Regularized-Long-Wave fluid model with fractional derivative and qualitative analysis*, Phys. Scr., vol. 100, no. 015270, 2025.
31. H. Qawaqneh, *New Functions For Fixed Point Results In Metric Spaces With Some Applications*, Indian J. Math., vol. 66, no. 1, 2024.
32. H. Qawaqneh, H. Aydi, *Fixed Points of Contraction Mappings Involving a Simulation Function and Applications*, Journal of Mathematical Analysis, vol. 16, no. 4, pp. 1–13, 2025.
33. I.A. Rus, *Generalized Contractions and Applications*, Cluj University Press, 2001.
34. G.S. Saluja et al., *Common fixed point theorems on S-metric spaces for integral type contractions involving rational terms and application to fractional integral equation*, Journal of Function Spaces, vol. 2024, Article 5108481, 2024.
35. B. Samet, C. Vetro, and P. Vetro, *Fixed point theorems for $\alpha - \psi$-contractive type mappings*, Nonlinear Analysis: Theory, Methods and Applications, vol. 75, no. 4, pp. 2154–2165, 2012.
36. K. Saravanan and V. Piramanantham, *New approximate fixed point results for rational contraction mappings*, Ratio Mathematica, vol. 49, 2023.

37. P. Shahi, J. Kaur, and S.S. Bhatia, *Fixed point theorems for $\alpha - \psi$-contractive type mappings of integral type with applications*, Journal of Nonlinear and Convex Analysis, vol. 16, pp. 745–760, 2015.
38. A.E. Shammaky and A.H. Hakami, *Solving Volterra–Hammerstein nonlinear integral equations via fixed point theory in complex-valued suprametric spaces*, AIMS Mathematics, vol. 10, no. 8, pp. 19974–19993, 2025.