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Approximate ML Estimation of Three-Parameter Exponentiated Weibull Distribution under Type-II Censoring

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Abstract In this paper, we propose an approximate maximum likelihood estimator for the three parameter exponentiated Weibull Distribution (EWD) under Type-II censoring. We also derive a percentile estimator of the scale parameter based on a Type-II censored sample. We compare both AMLE and PCE with the MLE of the scale parameter based on bias and mean square error (MSE.) through a Monte Carlo simulation study, we found that AMLE is as efficient as MLE in terms of both bias and MSE. In addition, interval estimation based on MLE and AMLE for the scale parameter is also discussed. To find more accurate values for the shape parameters, we propose a step-by-step method. This process combines a basic linear estimation technique with the standard Maximum Likelihood (ML) equation. By solving these together repeatedly, we can produce new, more precise results known as Approximate Maximum Likelihood Estimators (AMLEs). Finally, we present two data sets that include one real data set to emphasize the results of this article.

Keywords exponentiated Weibull Distribution, M L estimator, approximate M L estimator, percentile estimator, Type-II censored sample, interval estimation.

AMS 2020 subject classifications 62N02, 62N03, 62N05.

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1. Introduction

The exponentiated Weibull Distribution (EWD), first proposed by Mudholkar and Srivastava [8], represents a versatile generalization of the standard Weibull distribution through the incorporation of an additional shape parameter. This extra parameter enhances the model's flexibility, enabling it to accommodate diverse hazard rate patterns, such as monotonically increasing or decreasing rates, as well as non-monotone behaviors like bathtub-shaped and unimodal (upside-down bathtub) curves. These capabilities renders the EWD especially valuable for analyzing survival and reliability data in fields such as engineering, biology, and medicine, where real-world failure processes frequently deviate from simple monotonic hazard assumptions. The Probability density function (pdf) and the cumulative distribution function (cdf) of standard EWD are given as:

$$f(z;\alpha,\theta) = \alpha\theta z^{\alpha-1}e^{-z^{\alpha}}(1-e^{-z^{\alpha}})^{\theta-1};\ 0<z<\infty, \alpha>0 \text{ and } \theta>0. \tag{1.1}$$

$$F(z;\alpha,\theta) = (1-e^{-z^{\alpha}})^{\theta};\ 0<z<\infty, \alpha>0 \text{ and } \theta>0. \tag{1.2}$$

where α & θ are shape parameters and (1.2) is derived from (1.1).

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Since its inception, the EWD has garnered substantial interest within the statistical community. Mudholkar and Srivastava [8] originally developed maximum likelihood estimators (MLEs) for its parameters, and later works have examined a range of its properties and inference methods Later works have examined a range of EWD properties and inference methods. Mudholkar *et al.* [10], Mudholkar and Hutson [11], Nassar and Eissa [13], Nadarajah and Kotz [14], Dasari Ratna Babu *et al.* [24, 25], Jilani [18], Bhanu Prakash [21], and Pal *et al.* [15] analyzed the survival function, hazard rate, and moments of the EWD, demonstrating its advantages over the standard Weibull and Gamma distributions for fitting data with non-monotone hazard behaviors. In recent years, there has been a resurgence of interest in the EWD and related Weibull extensions, driven by their applicability in contemporary fields. Martinez *et al.* [20] illustrated the effectiveness of exponentiated Weibull models in medical survival studies, whereas Abubakar [22] introduced variational Bayesian approaches for parameter estimation in these models. Emerging flexible lifetime distributions, including the Survival Power Weibull distribution Kalt [26] and various other generalized Weibull classes Goldoust [19], underscore the continued need for versatile models that can handle intricate hazard patterns.

If a scale parameter σ is introduced in the standard EWD, we get a scaled EWD, whose PDF and CDF are given by

$$f(x;\alpha,\theta,\sigma) = \frac{\alpha\theta}{\sigma}\left(\frac{x}{\sigma}\right)^{\alpha-1} e^{-\left(\frac{x}{\sigma}\right)^{\alpha}}\left[1-e^{-\left(\frac{x}{\sigma}\right)^{\alpha}}\right]^{\theta-1};\ 0<x<\infty,\sigma>0,\alpha>0 \text{ and } \theta>0. \tag{1.3}$$

$$F(x;\alpha,\sigma,\theta) = \left(1-e^{-\left(\frac{x}{\sigma}\right)^{\alpha}}\right)^{\theta};\ 0<x<\infty,\sigma>0,\alpha>0 \text{ and } \theta>0. \tag{1.4}$$

where

- $\alpha > 0$ is the shape parameter,
- $\sigma > 0$ is the scale parameter, and
- $\theta > 0$ is the additional shape parameter.

Umesh Singh *et al.* [12] have developed generalized ML estimators, posterior mean (i.e., Bayes estimator under squared error loss function) but they are not closed form estimators. Nassar and Eissa [13] obtained a general formula for the mean residual life function. Pal *et al.* [15] have derived the survival function, failure rate, and moments of EWD using certain special formulas and also observed the failure rate of EWD compared with those of the Weibull and Gamma distributions.

Let $x_{r+1:n} < x_{r+2:n} < \ldots < x_{n-s:n}$ be a Type-II censored sample, after censoring r left most observations and s right most observations from a planned sample of size n drawn from the EWD given in equation (1.4). The log-likelihood function of the sample is

$$\log L = r\log\left(F(z_{r+1:n})\right) + s\log\left[1-F(z_{n-s:n})\right] + \sum_{i=r+1}^{n-s}\log f(z_{i:n}) \tag{1.5}$$

where $f(z), F(z)$ are as given in equations (1.1) and (1.2), and $z_{i:n} = \frac{x_{i:n}}{\sigma}$. The maximum likelihood (ML) equation of σ, obtained after simplifying equation (1.5), is given by

$$\frac{\partial \log L}{\partial \sigma} = -\frac{\alpha}{\sigma}\Bigg[(n-r-s) + r\theta G(z_{r+1:n}) - \sum_{i=r+1}^{n-s} z_{i:n}^{\alpha} + (\theta-1)\sum_{i=r+1}^{n-s} G(z_{i:n}) - s\theta H(z_{n-s:n})\Bigg] = 0. \tag{1.6}$$

$$\text{where}\quad G(z_{i:n}) = z_{i:n}^{\alpha}/[e^{z_{i:n}^{\alpha}}-1] \quad \text{and} \quad H(z_{n-s:n}) = G(z_{n-s:n})\,F(z_{n-s:n})/[1-F(z_{n-s:n})] \tag{1.7}$$

Moreover, the maximum likelihood (ML) equations in (1.6) are nonlinear, and the presence of analytically intractable terms $G(z_{r+1:n})$, $i = r+1, \ldots, n-s$, and $H(z_{n-s:n})$ prevents the derivation of closed-form solutions. Alharbi [23] highlighted the importance of developing efficient and robust estimation methods for Weibull-based models, particularly in cases where conventional ML estimators prove computationally intensive. we have to employ some iterative method, such as the Newton-Raphson method, to obtain the MLE, which requires an initial solution near the global maximum. The following sections are arranged as described below. Notwithstanding this rich body of research, estimating parameters of the exponentiated Weibull Distribution with censored data continues to pose difficulties. Specifically, under Type-II censoring—a prevalent scenario in reliability testing and life experiments—the likelihood equations are highly nonlinear, preventing a closed-form expression for the MLE of the scale parameter and necessitating iterative numerical techniques, such as Newton–Raphson, which depend on suitable starting values and can encounter convergence problems. Inspired by these obstacles, this study concentrates on proposing straightforward, closed-form, and computationally efficient estimators for the scale parameter of the exponentiated Weibull Distribution under Type-II censoring, with the shape parameters assumed known. The primary contributions of this paper are as follows:

1. A novel approximate maximum likelihood estimator (AMLE) is derived for the scale parameter through appropriate approximations of the complex terms in the likelihood equation. This estimator is explicit and has a simple structure, facilitating quick computation and enhancing its practical utility.
2. A Percentile-based Estimator (PCE) is introduced as an alternative, non-likelihood method for estimating the scale parameter from censored samples. An extensive Monte Carlo simulation investigation evaluates the proposed estimators alongside the MLE, comparing the AMLE and PCE based on bias and mean squared error (MSE). The Findings indicate that the AMLE performs nearly as efficient as the MLE while providing significant computational benefits, even with minor bias, and proves to be highly effective as an initial guess for iterative MLE algorithms. The paper proceeds as follows.

Section 2 details the derivation of the AMLE for the scale parameter under Type-II censoring. The percentile estimator is described in Section 3. Section 4 reports the results of a thorough simulation study assessing the estimators' performance, and Section 5 addresses the development of confidence intervals. Section 6 presents two illustrative examples, one involving real-world data. Concluding discussions and suggestions for future work appear in Section 7.

2. Approximate ML Estimation of Scale Parameter

In this section, we propose a simple estimator for σ by suggesting The following linear approximations to the intractable terms given in equation (1.7)

$$G(y_{i:n}) \simeq a_i + b_i y_{i:n}, \quad H(y_{n-s:n}) \simeq c_{n-s} + d_{n-s} y_{n-s:n}, \quad i = r+1, \ldots, n-s. \tag{2.1}$$

$$\text{where} \qquad G(y_{i:n}) = y_{i:n}/[e^{y_{i:n}} - 1],$$
$$H(y_{n-s:n}) \simeq [G(y_{n-s:n})\ F\left(y_{n-s:n}^{1/\alpha}\right)]/[1 - F\left(y_{n-s:n}^{1/\alpha}\right)].$$

$$\text{Here} \qquad y_{i:n} = z_{i:n}^{\alpha} \Rightarrow z_{i:n} = y_{i:n}^{1/\alpha}.$$

For evaluating the intercepts ($a_i, \quad i = r+1, \ldots, n-s.$, c_{n-s}) and the slopes ($b_i, \quad i = r+1, \ldots, n-s.$, d_{n-s}) of the above approximations, we have adopted two different linear approximation methods. The first is based on the Taylor series approximation method, as suggested by Balakrishnan [7] while studying the approximate ML estimation in the scaled Rayleigh distribution. The second method is based on the least squares (LS) approach, a new technique proposed by Vasudeva Rao *et al.* [16] while studying the linear approximate ML estimation in the scaled Type I generalized logistic distribution. Upon using the above linear approximations in equation (1.6), we

get the following approximate ML equation

$$\frac{\partial \log \tilde{L}}{\partial \sigma} = -\frac{\alpha}{\sigma}\Bigg[(n-r-s) + r\theta\left(a_{r+1} + b_{r+1}y_{r+1:n}\right) - \sum_{i=r+1}^{n-s} y_{i:n} + (\theta-1)\sum_{i=r+1}^{n-s}\left(a_i + b_i y_{i:n}\right) - s\theta\left(c_{n-s} + d_{n-s}y_{n-s:n}\right)\Bigg] = 0. \tag{2.2}$$

$$\text{Substituting back} \qquad y_{i:n} = z_{i:n}^{\alpha} = \left(\frac{x_{i:n}}{\sigma}\right)^{\alpha},$$

solving for σ, we obtain an approximate estimator of σ, which we denote by $\tilde{\sigma}$ is given by

$$\tilde{\sigma} = \left(\sum_{i=r+1}^{n-s} l_i x_{i:n}^{\alpha}\right)^{1/\alpha} \tag{2.3}$$

where

$$l_i = \begin{cases} [1-(\theta(r+1)-1)b_i]/\Delta, & \text{if } i = r+1 \\ [1-(\theta-1)b_i]/\Delta, & \text{if } r+1 < i < n-s \\ [1-(\theta-1)b_i + s\theta d_i]/\Delta, & \text{if } i = n-s \end{cases}$$

$$\Delta = (n-r-s) + r\theta a_{r+1} + (\theta-1)\sum_{i=r+1}^{n-s} a_i - s\theta c_{n-s}$$

We call the above estimator $\tilde{\sigma}$ as the approximate ML estimator (AMLE) of σ. The mean of the AMLE $\tilde{\sigma}$ is given by

$$E(\tilde{\sigma}) = \sigma \sum_{i=r+1}^{n-s} l_i \vartheta_{i:n} \tag{2.4}$$

where $\vartheta_{i:n} = E(y_{i:n})$ and $y_{i:n} = z_{i:n}^{\alpha}$, which follows a GED. Thus, to compute the mean and variance of $\tilde{\sigma}$, we need the means, variances, and covariances of the order statistics $z_{1:n} < z_{2:n} < \ldots < z_{n:n}$ of the standard EWD, those are not available in the literature. Consequently, we are unable to compute the exact variance of the AMLE. We have only simulated variances (asymptotic variances). Using these simulated variances, we can compute an approximate mean of $\tilde{\sigma}$ for the specified values of n, α, and θ. When the sample size n is large, we may use the following approximation for $\vartheta_{i:n}$.

$$\begin{aligned} \vartheta_{i:n} &= F^{-1}\big(E(U_{i:n})\big) \\ &= \Big(-\log\Big[1-[E(U_{i:n})]^{1/\theta}\Big]\Big)^{1/\alpha} \end{aligned}$$

Where $U_{i:n}$ is the i^{th} order statistic from $U(0,1)$ and it is well known that $E(U_{i:n}) = i/n+1$. Thus, we have

$$\vartheta_{i:n} \approx \left[-\log\left(1-\left(\frac{i}{n+1}\right)^{1/\theta}\right)\right]^{1/\alpha} \tag{2.5}$$

The Fisher information of $\tilde{\sigma}$ is given by

$$\begin{aligned} I &= -E\left(\frac{\partial^2 \log \tilde{L}}{\partial \sigma^2}\right) \\ &= \frac{I_{11}}{\sigma^2}, \end{aligned}$$

where

$$I_{11} = -\alpha\Bigg[n - r - s + r\theta a_{r+1} + (\theta-1)\sum_{i=r+1}^{n-s} a_i - s\theta c_{n-s} - (1+\alpha)\Bigg(\sum_{i=r+1}^{n-s}\vartheta_{i:n} - r\theta b_{r+1}\vartheta_{r+1:n} - (\theta-1)\sum_{i=r+1}^{n-s} b_i\vartheta_{i:n} + s\theta d_{n-s}\vartheta_{n-s:n}\Bigg)\Bigg]$$

The asymptotic variance of $\tilde{\sigma}$ can be obtained by inverting the above Fisher information and from which we can compute the standard error of $\tilde{\sigma}$ as

$$\text{S.E}(\tilde{\sigma}) = \frac{\tilde{\sigma}}{\sqrt{I_{11}}} \tag{2.6}$$

By applying Taylor series method to equation (2.4), we get

$$b_i = G'(\xi_{i:n}), a_i = G(\xi_i) - b_i\xi_i, \quad c_i = H'(\xi_i) \quad \text{and} \quad d_i = H(\xi_i) - c_i\xi_i, \quad i = r+1,\ldots,n-s. \tag{2.7}$$

where ξ_i is the i^{th} population quantile of EWD, given by and from the inequality (1.2), we have

$$\xi_i = F^{-1}(p_i) = \left[-\log_e\left(1-(p_i)^{\frac{1}{\theta}}\right)\right]^{\frac{1}{\alpha}}, \quad p_i = \frac{i-0.3}{n+0.4} \tag{2.8}$$

Similarly, by applying the least squares method for details of the method, see Vasudeva Rao *et al.* [17] to equation (2.3), we get

$$b_i = \frac{\left[\sum_{j=1}^{2m+1} z_{ij}G(z_{ij}) - (2m+1)\overline{z}_j\overline{G(z_j)}\right]}{\sum_{j=1}^{2m+1} z_{ij}^2 - (2m+1)(\overline{z}_j)^2}$$

$$a_i = \overline{G(z_i)} - b_i\overline{z_i}, \quad \overline{z_i} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} z_{ij}, \quad \overline{G(z_j)} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} G(z_{ij})$$

$$d_i = \frac{\left[\sum_{j=1}^{2m+1} z_{ij}H(z_{ij}) - (2m+1)\overline{z_j}\overline{H(z_j)}\right]}{\left[\sum_{j=1}^{2m+1} z_{ij}^2 - (2m+1)\overline{z_j}^2\right]}, \quad i = r+1,\ldots,n-s$$

$$\text{and} \quad c_i = \overline{H(z_i)} - d_i\overline{z_i}, \quad \overline{z_i} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} z_{ij}, \quad \overline{H(z_j)} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} H(z_{ij}) \tag{2.9}$$

$$\text{where} \quad z_{ij} = F^{-1}(p_{ij}) = \left[-\log\left(1-p_{ij}^{\frac{1}{\theta}}\right)\right]^{\frac{1}{\alpha}}.$$

For a given natural number m, we define

$$p_{ij} = p_i + \frac{j-m-1}{2m(n+0.4)}, \qquad j = 1,2,\ldots,2m+1.$$

Remark: In the above method, we may take any $m \geq 1$. However, Vasudeva Rao *et al.* [17] have suggested an optimal choice of $m = 5$, and we have taken the same choice of m.

Thus, in this section, we construct two AMLEs for σ based on Type II censored samples using two different linear approximation methods. These are based on the common assumption that the i-th standard order statistic $z_{i:n}$ lies in a narrow neighborhood of the i^{th} population quantile ξ_i. We denote the AMLEs of σ computed based on Taylor's series method and least squares method respectively by $\tilde{\sigma}_{TS}$ and $\tilde{\sigma}_{LS}$.

3. Percentiles based Estimator of Scale Parameter

Among the most easily obtained estimation methods of the parameters of the Weibull distribution are the graphical approximations to the best linear unbiased estimators. These estimates can be obtained by fitting a straight line to the theoretical points obtained from the distribution function and the percentage points of the sample. This method was originally suggested by Kao [1, 2] and also see Mann, Schafer and Singpurwalla [5] and Johnson, Kotz and Balakrishnan [9]. In case of a EWD distribution, also it is possible to use same concept to obtain the estimate of based on the percentiles because of the structure of its distribution function.
From equation (1.4), we have

$$F(x;\alpha,\theta,\sigma)=\left[1-e^{-\left(\frac{x}{\sigma}\right)^{\alpha}}\right]^{\theta} \tag{3.1}$$

which implies

$$x=\sigma\left[-\log\left(1-p^{\frac{1}{\theta}}\right)\right]^{\frac{1}{\alpha}} \tag{3.2}$$

If p denotes some estimate of $F(x;\alpha,\theta,\sigma)$, then the estimate of σ can be obtained by minimizing the error sum of squares.

$$\sum_{i=r+1}^{n-s}\left[x_{i:n}-\sigma\left(-\log\left(1-(p_i)^{\frac{1}{\theta}}\right)\right)^{\frac{1}{\alpha}}\right]^2$$

with respect to σ, which yields an approximate estimator for σ, called a percentile estimator (PCE) and denoted by σ_{PCE}, given by

$$\sigma_{\text{PCE}}=\sum_{i=r+1}^{n-s}\omega_i x_{i:n},\quad \text{where}\quad \omega_i=\frac{\left[-\log\left(1-p_i^{\frac{1}{\theta}}\right)\right]^{\frac{1}{\alpha}}}{\sum_{i=r+1}^{n-s}\left[-\log\left(1-p_i^{\frac{1}{\theta}}\right)\right]^{\frac{2}{\alpha}}} \tag{3.3}$$

It is possible to use several p_i as estimators of $F(x_{i:n})$. However, we use $p_i=\frac{i}{n+1}$ as the estimator of $F(x_{i:n})$, since $\frac{i}{n+1}$ is the expected value of $F(x_{i:n})$.

4. Comparison of AMLE and PCE with MLE

The ML equation (2.2) can be solved iteratively using the well known Newton-Raphson method, which requires an initial solution of σ near global maxima. Here, AMLE can be used as a good initial solution. We denote MLE. thus obtained, as In the case of AMLE, we cannot obtain the expected Fisher information of MLE. However, we can obtain the observed Fisher information as

$$\hat{I}=\left(-\frac{\partial^2\log L}{\partial\sigma^2}\right)_{\sigma=\hat{\sigma}} \tag{4.1}$$

where

$$\begin{aligned}\frac{\partial^2\log L}{\partial\sigma^2}=\frac{\alpha}{\sigma^2}\Bigg[&(n-r-s)+r\theta\big(G(z_{r+1:n})+z_{r+1:n}G'(z_{r+1:n})\big)-(1+\alpha)\sum_{i=r+1}^{n-s}z_{i:n}^{\alpha}\\&+(\theta-1)\sum_{i=r+1}^{n-s}\big(G(z_{i:n})+z_{i:n}G'(z_{i:n})\big)-s\theta\big(H(z_{n-s:n})+z_{n-s:n}H'(z_{n-s:n})\big)\Bigg]\end{aligned} \tag{4.2}$$

Here

$$G'(z_{i:n}) = -\frac{\alpha}{z_i}\Big[\, G(z_{i:n}) - e^{z^{\alpha}_{i:n}} G^2(z_{i:n})\,\Big],$$

$$H'(z_{n-s:n}) = \frac{f(z_{n-s:n})G(z_{n-s:n}) + F(z_{n-s:n})G'(z_{n-s:n}) + \dfrac{f(z_{n-s:n})F(z_{n-s:n})G(z_{n-s:n})}{1-F(z_{n-s:n})}}{1-F(z_{n-s:n})}.$$

From this the standard error of $\hat{\sigma}$ is obtained as

$$SE(\hat{\sigma}) = \frac{\hat{\sigma}}{\sqrt{\hat{I}}} \tag{4.3}$$

In Taylor series and LS methods discussed in the above sections, the basic principle is that the intractable terms $G(z_{i:n})$ and $H(z_{n-s:n})$ are approximated linearly in a narrow neighbourhood of the standard population quantile ξ_i. The solutions of exact and approximate log-likelihood equations tend to each other as $n \to \infty$. Hence, the exact and approximate MLEs are asymptotically identical Tiku *et al.* [3] Therefore, the asymptotic variances of exact and approximate MLEs are same Bhattacharya [6]. However, the same cannot be said in small samples. At the same time, the small sample bias and variance of MLE are not mathematically tractable. To examine the finite–sample performance of the competing estimators of the scale parameter σ, an extensive Monte Carlo simulation study was carried out. The biases and mean squared errors (MSEs) of the estimators $\hat{\sigma}$ (MLE), $\tilde{\sigma}_{TS}$, $\tilde{\sigma}_{LS}$ and σ_{PCE} were computed based on repeated samples generated from the EWD for various combinations of the shape parameters α and θ, different sample sizes n, and censoring schemes (r, s). The relative efficiencies of $\tilde{\sigma}_{TS}$, $\tilde{\sigma}_{LS}$ and σ_{PCE} with respect to the MLE were calculated as percentages and are reported in Table 6. The following conclusions are drawn.

- When $\alpha < 1$ (fixed at $\alpha = 0.5$) and the second shape parameter θ varies over 0.5, 1.5, 3.0 and 5.0, the biases of all the estimators $\hat{\sigma}$, $\tilde{\sigma}_{TS}$, $\tilde{\sigma}_{LS}$ and σ_{PCE} are positive, indicating an overall tendency to overestimate σ. Among these, the percentile estimator σ_{PCE} shows considerably larger bias than the remaining estimators.
- When $\alpha > 1$ (for example $\alpha = 1.5$) and θ varies from 0.5 to 5.0, the biases of all the estimators except σ_{PCE} are negative. Hence, the MLE, $\tilde{\sigma}_{TS}$ and $\tilde{\sigma}_{LS}$ tend to underestimate σ, whereas the percentile estimator continues to overestimate the parameter.
- In comparison with the MLE, the estimators $\tilde{\sigma}_{TS}$ and $\tilde{\sigma}_{LS}$ are generally less biased, while σ_{PCE} is more biased. Moreover, $\tilde{\sigma}_{LS}$ is slightly more biased than $\tilde{\sigma}_{TS}$ but substantially less biased than the percentile estimator.
- For all estimators, the MSE decreases as the sample size n increases, confirming their consistency. For fixed (α, θ), increases in the censoring parameters r and s lead to a mild increase in MSE due to loss of information.
- The relative efficiencies of $\tilde{\sigma}_{TS}$ and $\tilde{\sigma}_{LS}$ with respect to the MLE are approximately 100% for all considered values of α, θ, n, r and s, indicating that these estimators are almost as efficient as the MLE. The efficiencies of $\tilde{\sigma}_{TS}$ and $\tilde{\sigma}_{LS}$ are also nearly identical to each other.
- The percentile estimator σ_{PCE} is not comparable with the likelihood–based estimators in terms of both bias and MSE. Its efficiency relative to the MLE fluctuates considerably with respect to α, θ, n, r and s, although some improvement is observed for larger sample sizes.

5. Construction of Confidence Intervals

In practice, one is not only interested in getting a point estimate for a parameter, but also wishes to construct confidence interval (CI) with near nominal coverage. Since $\hat{\sigma}, \tilde{\sigma} and \sigma_{PCE}$ are asymptotically normally distributed, we have the asymptotic distributions of the quantities

$$P_1 = \frac{\hat{\sigma} - \sigma}{SE(\hat{\sigma})} \qquad P_2 = \frac{\tilde{\sigma} - \sigma}{SE(\tilde{\sigma})} \tag{5.1}$$

are standard normal $N(0,1)$. Here P_i $(i=1,2)$ are the pivotal quantities, because their distributions don't depend on the unknown scale parameter. With the aim of investigating the effect of replacing the MLE by AMLE in the coverage of CI, we conducted another simulation study. Specifically, we generated 4000 samples as described in the above section, and for each sample, we calculated the asymptotically pivotal quantities given in equation (5.1). Then, we simulated the probability coverages.

$$P(-1.645 \leq P_i \leq 1.645) \quad \text{and} \quad P(-1.96 \leq P_i \leq 1.96), \quad i=1,2. \tag{5.2}$$

which are expected to be 90% and 95%, respectively. *Table 2* display the obtained results for some selected choice of α =0.5,1.5 and 3.0; θ =0.5,1.5, 2.0 and 3.0 and for n=5,10, and 20 with various choices of r and s. Looking at this table, we observe the following. As expected, when n increases, the actual coverages become closer to nominal values.

6. Illustrations

Illustration 1:

We considered a real uncensored data set from Nichols and Padgett, 2006. The data gives 100 observations on breaking stress of carbon fibres (in Gba)

3.7 2.74 2.73 2.5 3.6 3.11 3.27 2.87 1.47 3.11 4.42 2.41 3.19 3.22 1.69 3.28 3.09 1.87 3.15 4.9 3.75 2.43 2.95 2.97 3.39 2.96 2.53 2.67 2.93 3.22 3.39 2.81 4.2 3.33 2.55 3.31 3.31 2.85 2.56 3.56 3.15 2.35 2.55 2.59 2.38 2.81 2.77 2.17 2.83 1.92 1.41 3.68 2.97 1.36 0.98 2.76 4.91 3.68 1.84 1.59 3.19 1.57 0.81 5.56 1.73 1.59 2 1.22 1.12 1.71 2.17 1.17 5.08 2.48 1.18 3.51 2.17 1.69 1.25 4.38 1.84 0.39 3.68 2.48 0.85 1.61 2.79 4.7 2.03 1.8 1.57 1.08 2.03 1.61 2.12 1.89 2.88 2.82 2.05 3.65

For the above data, we fit $EW(\sigma,\alpha,\theta)$ with known shape parameters α,θ by applying the Kolmogorov-Smirnov (KS) test. We tried different values of α,θ and found that the KS test yielded the smallest value of the D-statistic (D = 0.061935, p-value = 0.8377) with $\alpha=2.4$, $\theta=1.3$, and AMLE $\tilde{\sigma}=2.7469$, which is computed using *Eq. 2.3*. Since D is very small and equivalently the p-value is very high (≥ 0.05), it may be concluded that EW($\tilde{\sigma}$=2.689, α=2.4, θ=1.3)) is well fitted to the data, and the same is established by the following Q-Q plot and superimposed empirical and theoretical CDFs of EWD constructed based on AMLE.

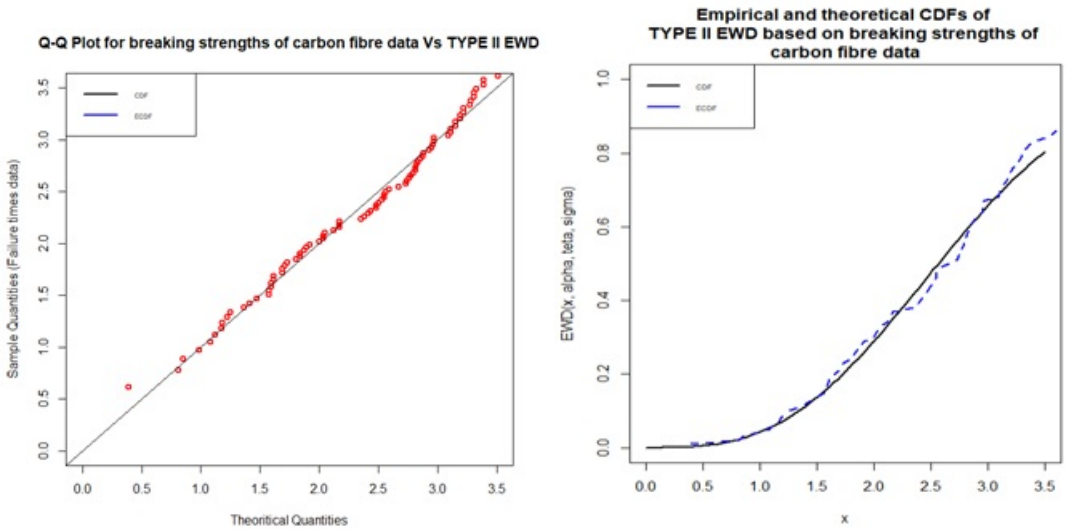


Figure 1. Q-Q plot and empirical and theoretical CDFs of $EW(\tilde{\sigma})=2.7469, \alpha=2.4, \theta=1.3$ for the breaking stress of carbon fibres (in Gba) data successive failures.

For the above data the MLE, AMLELS, AMLETS and PCE of σ are computed, using R-program with known α=2.4, θ= 1.3, for different choices of r and s. The estimates along with standard errors of σ are tabulated below.
Table 1: MLE, AMLE, PCE of σ in EW (σ, α , θ) with known α =2.4, θ = 1.3 obtained from the interval times between successive failures of the breaking stress of carbon fibres (in Gba) data with sample size n=100 with the choice of r = 0, 5, 10, 20, 30. and s= 0, 5, 10, 20, 30. The figures in brackets are the standard errors of the respective estimates.

Table 1. Estimates of σ

n	r	s	$\hat{\sigma}$ (S.E.)	$\tilde{\sigma}_{\text{LS}}$ (S.E.)	$\hat{\sigma}_{\text{TS}}$ (S.E.)	σ_{PCE}
100	0	0	2.727 (0.103)	2.689 (0.101)	2.727 (0.103)	2.266
100	0	5	2.752 (0.105)	2.717 (0.105)	2.752 (0.105)	2.392
100	0	10	2.796 (0.109)	2.768 (0.109)	2.796 (0.109)	2.583
100	0	20	3.366 (0.138)	3.349 (0.137)	3.366 (0.138)	2.876
100	0	30	2.880 (0.125)	2.868 (0.127)	2.880 (0.125)	3.138
100	5	0	2.694 (0.101)	2.650 (0.100)	2.694 (0.101)	2.232
100	5	5	2.718 (0.104)	2.677 (0.104)	2.718 (0.104)	2.353
100	5	10	2.762 (0.108)	2.727 (0.108)	2.762 (0.108)	2.539
100	5	20	3.337 (0.137)	3.317 (0.136)	3.337 (0.137)	2.822
100	5	30	2.839 (0.123)	2.819 (0.125)	2.839 (0.123)	3.069
100	10	0	2.772 (0.104)	2.660 (0.106)	2.772 (0.104)	2.195
100	10	5	2.799 (0.107)	2.689 (0.110)	2.799 (0.107)	2.311
100	10	10	2.844 (0.111)	2.742 (0.114)	2.844 (0.111)	2.493
100	10	20	3.407 (0.140)	3.352 (0.141)	3.407 (0.140)	2.766
100	10	30	2.937 (0.127)	2.844 (0.134)	2.937 (0.127)	3.001
100	20	0	2.657 (0.100)	2.555 (0.103)	2.657 (0.100)	2.091
100	20	5	2.680 (0.103)	2.580 (0.106)	2.680 (0.103)	2.193
100	20	10	2.723 (0.106)	2.631 (0.111)	2.723 (0.106)	2.365
100	20	20	3.305 (0.136)	3.264 (0.137)	3.305 (0.136)	2.611
100	20	30	2.793 (0.121)	2.708 (0.128)	2.793 (0.121)	2.809
100	30	0	2.548 (0.096)	2.458 (0.099)	2.548 (0.096)	1.983
100	30	5	2.568 (0.099)	2.480 (0.103)	2.568 (0.099)	2.071
100	30	10	2.608 (0.102)	2.528 (0.107)	2.608 (0.102)	2.234
100	30	20	3.212 (0.132)	3.184 (0.133)	3.212 (0.132)	2.456
100	30	30	2.655 (0.116)	2.581 (0.123)	2.655 (0.116)	2.617

From the above table, it may be noticed that as compared to PCE, AMLE is very much close to the MLE. AMLE has slightly less standard error compared to MLE.

Illustration 2: Here we considered another data set represents the number of revolution before failure of the 23 ball bearings in the life-test. It was originally reported in Lawless [4] it is as follows,
17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.
As in the above example, we tried KS test with different values of α, θ and found EW (σ, α, θ) is well fitted for ball bearings data with α = 1.5 , θ = 2.0 and AMLE $\tilde{\sigma}$ = 58.4128. The KS distance D= 0.13075 and the associated p-value = 0.7797. Since the p-value is high. It may be concluded that EW(α = 1.5 , θ= 2.0 , $\tilde{\alpha}$ = 58.413) is well fitted to the above data. Further the following Q-Q plot and superimposed empirical and theoretical CDFs of EWD constructed based on AMLE are ensuring the fitness of the model. Hence using R-code, we compute MLE, AMLELS, AMLETS and PCE of σ in EW(σ,α, θ) with α = 1.5, θ = 2.0 for various choices of r and s along with the standard errors. The estimates, their standard errors of σ are tabulated below.
Table 2: MLE, AMLE, PCE of σ in EW (σ, α , θ) with known α = 1.5, θ= 2.0 obtained from the interval times

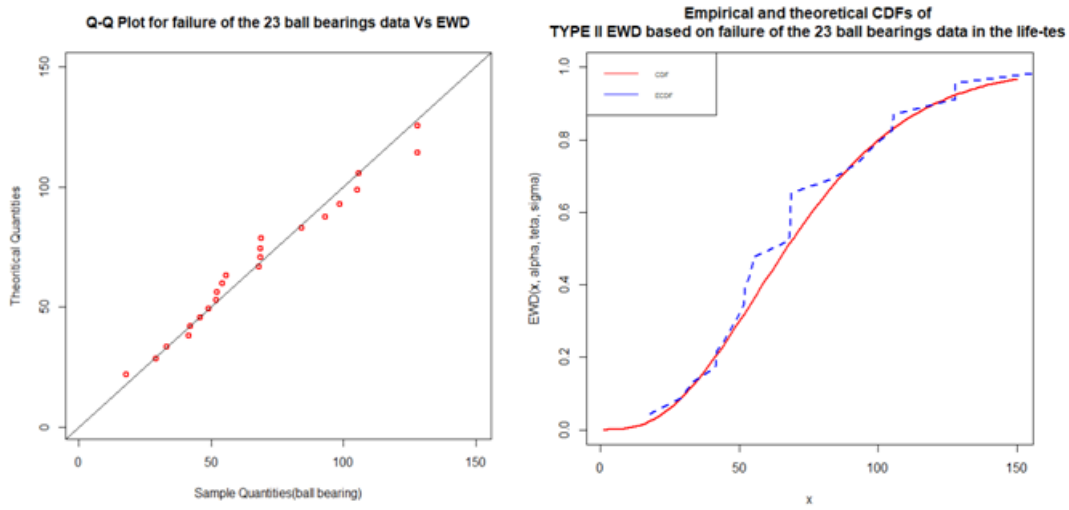


Figure 2. Q-Q plot and empirical and theoretical CDFs of $EW(\alpha = 1.5, \theta = 2.0, \tilde{\sigma} = 58.413)$ For the ball bearings data

between successive failures of the ball bearings data with sample size n= 23 with the choice of r = 0, 2, 5. and s= 0, 2, 5.

Table 2. Estimates of σ

n	r	s	$\hat{\sigma}$ (S.E.)	$\tilde{\sigma}_{\text{LS}}$ (S.E.)	$\hat{\sigma}_{\text{TS}}$ (S.E.)	σ_{PCE}
23	0	0	58.392 (6.040)	58.413 (6.210)	58.428 (6.208)	60.660
23	0	2	58.544 (6.219)	58.569 (6.335)	58.574 (6.335)	58.859
23	0	5	57.991 (6.469)	58.009 (6.556)	58.011 (6.556)	57.237
23	2	0	58.416 (6.046)	58.436 (6.220)	58.451 (6.218)	60.712
23	2	2	58.569 (6.226)	58.594 (6.346)	58.599 (6.345)	58.907
23	2	5	58.018 (6.477)	58.036 (6.568)	58.038 (6.568)	57.279
23	5	0	58.310 (6.054)	58.330 (6.233)	58.345 (6.231)	60.735
23	5	2	58.458 (6.235)	58.483 (6.359)	58.487 (6.359)	58.858
23	5	5	57.891 (6.487)	57.909 (6.582)	57.911 (6.582)	57.089

For the above table, all observations are found to be similar to the previous Illustration 1.

7. Estimating Shape and exponential Parameters using AMLE

We suggest the following iterative method, which involves solving the liner estimator (2.3) of σ and the implicit ML equation (1.6) of θ and α jointly. The resultant estimates of θ and α may be called as approximate MLEs (AMLEs). The iterative method of computation of AMLEs are described below.

Step 1. Substitute an initial estimate of $\sigma > 0$ say $\sigma^{(0)}$ in the following equation and obtain an initial estimate of α say $\alpha^{(0)}$.

$$\frac{\partial \log L}{\partial \alpha} = r\,\theta\,\frac{e^{-x_{r+1:n}}\,x_{r+1:n}\,\ln x_{r+1:n}}{1-e^{-x_{r+1:n}}} - s\theta\,\frac{F_{n-s}}{1-F_{n-s}}\,\frac{e^{-x_{n-s:n}}\,x_{n-s:n}\,\ln x_{n-s:n}}{1-e^{-x_{n-s:n}}} + \sum_{i=r+1}^{n-s}\left[\frac{1}{\alpha} + \ln z_{i:n} - x_{i:n}\ln z_{i:n} + (\theta-1)\frac{e^{-x_{i:n}}\,x_{i:n}\,\ln z_{i:n}}{1-e^{-x_{i:n}}}\right]. \tag{7.1}$$

Step 2. Substitute an initial estimate of $\sigma > 0$ say $\sigma^{(0)}$ in following equation and obtain an initial estimate of θ say $\theta^{(0)}$.

$$\frac{\partial \log L}{\partial \theta} = r\,\log\left(1-e^{-x_{r+1:n}}\right) - s\,\frac{F_{n-s}\,\log\left(1-e^{-x_{n-s:n}}\right)}{1-F_{n-s}} + \sum_{i=r+1}^{n-s}\left[\frac{1}{\theta} + \log\left(1-e^{-x_{i:n}}\right)\right]. \tag{7.2}$$

Step 3. Substitute the above estimates of α and θ ($\alpha^{(0)}$ and $\theta^{(0)}$) in the following iterative equation which was obtained by applying Newton-Raphson method to ML equations (7.1) and (7.2) to obtain the new approximate estimates of α say $\alpha^{(N)}$ and θ say $\theta^{(N)}$ with the following iterative equation

$$\begin{pmatrix}\alpha^{(i+1)}\\ \theta^{(i+1)}\end{pmatrix} = \begin{pmatrix}\alpha^{(i)}\\ \theta^{(i)}\end{pmatrix} - \left[\begin{pmatrix}\frac{\partial^2 \log L}{\partial \alpha^2} & \frac{\partial^2 \log L}{\partial \alpha\,\partial \theta}\\ \frac{\partial^2 \log L}{\partial \theta\,\partial \alpha} & \frac{\partial^2 \log L}{\partial \theta^2}\end{pmatrix}^{-1}\begin{pmatrix}\frac{\partial \log L}{\partial \alpha}\\ \frac{\partial \log L}{\partial \theta}\end{pmatrix}\right]_{\alpha=\alpha^{(i)},\,\theta=\theta^{(i)}}$$

where

$$\frac{\partial^2 \log L}{\partial \theta^2} = -s\frac{F_{n-s}^2\left[\log(1-e^{-x_{n-s}})\right]^2}{(1-F_{n-s})^2} + \sum_{i=r+1}^{n-s}\left(-\frac{1}{\theta^2}\right).$$

$$\frac{\partial^2 \log L}{\partial \alpha^2} = \sum_{i=r+1}^{n-s}\left\{-\frac{1}{\alpha^2}(\ln z_{i:n})^2 x_{i:n} + (\theta-1)\left[\frac{e^{-x_{i:n}}}{1-e^{-x_{i:n}}}(\ln z_{i:n})^2 x_{i:n}(1-x_{i:n}) + \frac{e^{-x_{i:n}}(1+e^{-x_{i:n}})}{(1-e^{-x_{i:n}})^2}(\ln z_{i:n})^2 x_{i:n}^2\right]\right\}$$

$$+ r\left[\frac{1}{F(z_{r+1:n})}\frac{\partial^2 F(z_{r+1:n})}{\partial \alpha^2} - \frac{1}{F(z_{r+1:n})^2}\left(\frac{\partial F(z_{r+1:n})}{\partial \alpha}\right)^2\right]$$

$$- s\left[\frac{1}{1-F(z_{n-s:n})}\frac{\partial^2 F(z_{n-s:n})}{\partial \alpha^2} + \frac{1}{\left(1-F(z_{n-s:n})\right)^2}\left(\frac{\partial F(z_{n-s:n})}{\partial \alpha}\right)^2\right],$$

with

$$\frac{\partial^2 F(z)}{\partial \alpha^2} = F(z)\left[\theta\frac{e^{-x}}{1-e^{-x}}(\ln z)^2 x(1-x) + \theta^2\left(\frac{e^{-x}}{1-e^{-x}}\right)^2(\ln z)^2 x^2\right].$$

$$\frac{\partial^2 F}{\partial \alpha\,\partial \theta} = \frac{\partial F}{\partial \alpha}\log\left(1-e^{-x}\right) + F\,\frac{e^{-x}}{1-e^{-x}}\frac{\partial x}{\partial \alpha}.$$

Step 4. Substitute $\alpha^{(N)}$ and $\theta^{(N)}$, obtained at step 2 and step 3, in equation (2.3) to obtain a new approximate estimate of σ say $\sigma^{(N)}$.

Step 5. Revise the old estimates $\sigma^{(0)}$, $\alpha^{(0)}$ and $\theta^{(0)}$ with the new estimates of $\sigma^{(N)}$, $\alpha^{(N)}$ and $\theta^{(N)}$ respectively and repeat Step 3 & Step 4 until the two successive estimates of α and θ do agree for a significant number of decimal places.

We denote the AMLEs of σ, α and θ, which are obtained using the above method, by $\tilde{\sigma}$, $\tilde{\alpha}$ and $\tilde{\theta}$ respectively. The standard errors of the estimates can be obtained from their variances.

8. Conclusion

Thus, our investigation of some alternative efficient linear estimators to MLE of parameter in EW distribution also it produces some new linear estimators (AMLEs). Hence, the AMLEs are alternative estimators to MLEs for the estimation of parameters in EW distribution under Type II censoring. Therefore, we recommend AMLEs to estimate parameters of EW distribution. Since, ML method does not provide explicit estimator for parameters of three parameter of EW distribution, also we have investigated some alternative simple linear estimators namely AMLEs and PCEs to σ. Based on illustrations, we have AMLEs are as efficient as MLEs though these are biased than MLEs. AMLEs can also be used as starting value to obtain MLEs using an iterative method. PCEs are not recommended as they are not as efficient as AMLEs even they are less biased.

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9. Appendix

Table 3. The bias, Mean Squared Error (MS), and Efficiencies (Eff) of $\hat{\sigma}$, $\tilde{\sigma}_{TS}$, $\tilde{\sigma}_{LS}$, $\tilde{\sigma}_{PCE}$

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
0.5	0.5	10	0	0	0.1757	0.1646	0.1699	1.1394	1.2406	1.2143(102.2)	1.2273(101.1)	7.2407(17.1)
0.5	0.5	10	0	2	0.2573	0.2550	0.2552	0.8030	1.9570	1.9489(100.4)	1.9496(100.4)	4.8338(40.5)
0.5	0.5	10	2	0	0.1941	0.1830	0.1883	1.1366	1.2200	1.1944(102.1)	1.2062(101.1)	6.7612(18.0)
0.5	0.5	10	2	2	0.2841	0.2823	0.2825	0.8271	2.0234	2.0170(100.3)	2.0177(100.3)	4.7443(42.6)
0.5	0.5	20	0	0	0.1024	0.0955	0.0975	0.7518	0.4904	0.4827(101.6)	0.4850(101.1)	2.7791(17.6)
0.5	0.5	20	0	2	0.1081	0.1052	0.1056	0.4029	0.5833	0.5798(100.6)	0.5803(100.5)	1.1489(50.8)
0.5	0.5	20	0	5	0.1453	0.1445	0.1445	0.3839	0.8464	0.8452(100.1)	0.8451(100.2)	1.3724(61.7)
0.5	0.5	20	2	0	0.0883	0.0816	0.0836	0.7125	0.4693	0.4626(101.4)	0.4646(101.0)	2.3894(19.6)
0.5	0.5	20	2	2	0.0947	0.0919	0.0923	0.3828	0.5817	0.5785(100.5)	0.5789(100.5)	1.1241(51.7)
0.5	0.5	20	2	5	0.1334	0.1327	0.1327	0.3604	0.7886	0.7874(100.2)	0.7873(100.2)	1.2561(62.8)
0.5	0.5	20	5	0	0.0804	0.0741	0.0760	0.6891	0.4719	0.4657(101.3)	0.4677(100.9)	2.3224(20.3)
0.5	0.5	20	5	2	0.1036	0.1010	0.1013	0.4016	0.6179	0.6145(100.6)	0.6149(100.5)	1.2484(49.5)
0.5	0.5	20	5	5	0.1551	0.1547	0.1547	0.3904	0.8789	0.8782(100.1)	0.8781(100.1)	1.4172(62.0)
0.5	0.5	50	0	0	0.0240	0.0209	0.0214	0.3782	0.1614	0.1602(100.7)	0.1604(100.6)	0.7428(21.7)
0.5	0.5	50	0	5	0.0380	0.0368	0.0368	0.1506	0.2057	0.2051(100.3)	0.2052(100.3)	0.2960(69.5)
0.5	0.5	50	0	10	0.0617	0.0612	0.0612	0.1523	0.2519	0.2516(100.1)	0.2516(100.1)	0.3257(77.4)
0.5	0.5	50	5	0	0.0331	0.0300	0.0305	0.4019	0.1632	0.1620(100.7)	0.1622(100.6)	0.7687(21.2)
0.5	0.5	50	5	5	0.0477	0.0466	0.0466	0.1612	0.2044	0.2039(100.3)	0.2039(100.3)	0.2899(70.5)
0.5	0.5	50	5	10	0.0499	0.0494	0.0494	0.1418	0.2407	0.2404(100.1)	0.2404(100.1)	0.3107(77.5)
0.5	0.5	50	10	0	0.0252	0.0221	0.0226	0.3871	0.1639	0.1627(100.7)	0.1629(100.6)	0.7528(21.8)
0.5	0.5	50	10	5	0.0510	0.0499	0.0500	0.1652	0.2142	0.2137(100.3)	0.2137(100.2)	0.3137(68.3)
0.5	0.5	50	10	10	0.0566	0.0561	0.0561	0.1481	0.2517	0.2515(100.1)	0.2515(100.1)	0.3295(76.4)
0.5	0.5	100	0	0	0.0170	0.0153	0.0155	0.2701	0.0781	0.0778(100.4)	0.0778(100.4)	0.3418(22.8)
0.5	0.5	100	0	10	0.0147	0.0142	0.0142	0.0701	0.0920	0.0918(100.1)	0.0918(100.1)	0.1166(78.9)
0.5	0.5	100	0	20	0.0240	0.0238	0.0238	0.0693	0.1129	0.1128(100.1)	0.1128(100.1)	0.1330(84.9)
0.5	0.5	100	0	30	0.0422	0.0422	0.0422	0.0861	0.1477	0.1477(100.0)	0.1477(100.0)	0.1717(86.0)
0.5	0.5	100	10	0	0.0063	0.0047	0.0048	0.2523	0.0789	0.0786(100.3)	0.0786(100.3)	0.3194(24.7)
0.5	0.5	100	10	10	0.0088	0.0082	0.0082	0.0643	0.0945	0.0944(100.1)	0.0944(100.1)	0.1185(79.7)
0.5	0.5	100	10	20	0.0278	0.0276	0.0276	0.0739	0.1176	0.1175(100.1)	0.1175(100.1)	0.1377(85.4)
0.5	0.5	100	10	30	0.0316	0.0315	0.0315	0.0708	0.1482	0.1482(100.0)	0.1482(100.0)	0.1690(87.7)
0.5	0.5	100	20	0	0.0178	0.0161	0.0163	0.2723	0.0800	0.0797(100.4)	0.0797(100.4)	0.3504(22.8)
0.5	0.5	100	20	10	0.0131	0.0126	0.0126	0.0667	0.0923	0.0922(100.1)	0.0922(100.1)	0.1147(80.5)
0.5	0.5	100	20	20	0.0330	0.0328	0.0328	0.0785	0.1188	0.1188(100.1)	0.1188(100.1)	0.1400(84.9)
0.5	0.5	100	20	30	0.0433	0.0432	0.0432	0.0866	0.1494	0.1494(100.0)	0.1494(100.0)	0.1725(86.6)
0.5	0.5	100	30	0	0.0148	0.0132	0.0134	0.2582	0.0763	0.0761(100.4)	0.0761(100.3)	0.3389(22.5)
0.5	0.5	100	30	10	0.0191	0.0186	0.0186	0.0749	0.0933	0.0931(100.1)	0.0931(100.1)	0.1195(78.0)
0.5	0.5	100	30	20	0.0212	0.0210	0.0210	0.0664	0.1187	0.1186(100.0)	0.1186(100.0)	0.1404(84.5)
0.5	0.5	100	30	30	0.0316	0.0316	0.0316	0.0753	0.1442	0.1442(100.0)	0.1442(100.0)	0.1663(86.7)
0.5	1.5	10	0	0	0.0875	0.0923	0.0910	0.5421	0.3404	0.3442(98.9)	0.3432(99.2)	1.4137(24.1)
0.5	1.5	10	0	2	0.0867	0.0888	0.0885	0.2615	0.4225	0.4245(99.5)	0.4242(99.6)	0.6781(62.3)
0.5	1.5	10	2	0	0.0744	0.0800	0.0787	0.5156	0.3511	0.3555(98.8)	0.3545(99.1)	1.3783(25.5)
0.5	1.5	10	2	2	0.1077	0.1109	0.1104	0.2773	0.4518	0.4550(99.3)	0.4545(99.4)	0.6920(65.3)
0.5	1.5	20	0	0	0.0528	0.0555	0.0550	0.3653	0.1584	0.1595(99.3)	0.1593(99.4)	0.6260(25.3)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
0.5	1.5	20	0	2	0.0478	0.0494	0.0493	0.1743	0.1733	0.1739(99.6)	0.1739(99.6)	0.2793(62.0)
0.5	1.5	20	0	5	0.0417	0.0426	0.0425	0.1207	0.1911	0.1916(99.8)	0.1915(99.8)	0.2452(77.9)
0.5	1.5	20	2	0	0.0504	0.0532	0.0527	0.3600	0.1649	0.1660(99.3)	0.1658(99.4)	0.6040(27.3)
0.5	1.5	20	2	2	0.0476	0.0495	0.0494	0.1790	0.1738	0.1746(99.5)	0.1745(99.6)	0.2856(60.8)
0.5	1.5	20	2	5	0.0438	0.0450	0.0449	0.1239	0.1978	0.1983(99.7)	0.1983(99.7)	0.2600(76.1)
0.5	1.5	20	5	0	0.0498	0.0531	0.0526	0.3685	0.1639	0.1652(99.2)	0.1650(99.3)	0.6381(25.7)
0.5	1.5	20	5	2	0.0441	0.0464	0.0462	0.1657	0.1799	0.1809(99.5)	0.1808(99.5)	0.2823(63.7)
0.5	1.5	20	5	5	0.0507	0.0524	0.0523	0.1254	0.2044	0.2052(99.6)	0.2052(99.6)	0.2590(78.9)
0.5	1.5	50	0	0	0.0135	0.0146	0.0145	0.2070	0.0560	0.0562(99.7)	0.0562(99.8)	0.2145(26.1)
0.5	1.5	50	0	5	0.0145	0.0152	0.0152	0.0642	0.0633	0.0634(99.8)	0.0634(99.8)	0.0846(74.8)
0.5	1.5	50	0	10	0.0100	0.0105	0.0105	0.0458	0.0669	0.0670(99.9)	0.0670(99.9)	0.0796(84.0)
0.5	1.5	50	5	0	0.0207	0.022	0.0219	0.2191	0.0615	0.0617(99.7)	0.0617(99.7)	0.2354(26.1)
0.5	1.5	50	5	5	0.0180	0.0188	0.0187	0.0713	0.0634	0.0636(99.8)	0.0636(99.8)	0.0856(74.1)
0.5	1.5	50	5	10	0.0223	0.0228	0.0228	0.0583	0.0697	0.0698(99.9)	0.0698(99.9)	0.0841(82.8)
0.5	1.5	50	10	0	0.0172	0.0185	0.0184	0.2157	0.0582	0.0584(99.7)	0.0584(99.7)	0.2369(24.6)
0.5	1.5	50	10	5	0.0097	0.0106	0.0105	0.0602	0.0640	0.0641(99.8)	0.0641(99.8)	0.0847(75.6)
0.5	1.5	50	10	10	0.0198	0.0205	0.0205	0.0555	0.0701	0.0702(99.8)	0.0702(99.8)	0.0830(84.4)
0.5	1.5	100	0	0	0.0031	0.0037	0.0037	0.1395	0.0281	0.0282(99.9)	0.0282(99.9)	0.1081(26.0)
0.5	1.5	100	0	10	0.0088	0.0091	0.0091	0.0347	0.0308	0.0308(99.9)	0.0308(99.9)	0.0393(78.3)
0.5	1.5	100	0	20	0.0146	0.0148	0.0148	0.0337	0.0338	0.0338(99.9)	0.0338(99.9)	0.0389(86.8)
0.5	1.5	100	0	30	0.0087	0.0088	0.0088	0.0230	0.0381	0.0381(100.0)	0.0381(100.0)	0.0419(90.9)
0.5	1.5	100	10	0	0.0035	0.0042	0.00410	0.1440	0.0294	0.0294(99.9)	0.0294(99.9)	0.1128(26.0)
0.5	1.5	100	10	10	0.0062	0.0066	0.0066	0.0304	0.0298	0.0299(99.9)	0.0299(99.9)	0.0376(79.3)
0.5	1.5	100	10	20	0.0114	0.0117	0.0117	0.0305	0.0358	0.0358(99.9)	0.0358(99.9)	0.0412(87.0)
0.5	1.5	100	10	30	0.0128	0.0130	0.0130	0.0272	0.0394	0.0394(99.9)	0.0394(99.9)	0.0430(91.6)
0.5	1.5	100	20	0	0.0082	0.0089	0.0089	0.1501	0.0288	0.0288(99.8)	0.0288(99.8)	0.1177(24.4)
0.5	1.5	100	20	10	0.0056	0.0060	0.0060	0.0321	0.0311	0.0311(99.9)	0.0311(99.9)	0.0385(80.8)
0.5	1.5	100	20	20	0.0104	0.0107	0.0107	0.0279	0.0346	0.0346(99.9)	0.0346(99.9)	0.0390(88.8)
0.5	1.5	100	20	30	0.0134	0.0137	0.0137	0.0271	0.0389	0.0390(99.9)	0.0390(99.9)	0.0426(91.5)
0.5	1.5	100	30	0	0.0073	0.0081	0.0081	0.1414	0.0294	0.0294(99.8)	0.0294(99.8)	0.1087(27.0)
0.5	1.5	100	30	10	0.0077	0.0082	0.0082	0.0341	0.0305	0.0306(99.9)	0.0306(99.9)	0.0384(79.4)
0.5	1.5	100	30	20	0.0079	0.0084	0.0084	0.0252	0.0342	0.0343(99.9)	0.0343(99.9)	0.0384(89.1)
0.5	1.5	100	30	30	0.0095	0.0098	0.0098	0.0225	0.0383	0.0383(99.9)	0.0383(99.9)	0.0415(92.2)
0.5	3	10	0	0	0.0647	0.0747	0.0725	0.3605	0.2023	0.2076(97.4)	0.2064(98.0)	0.6993(28.9)
0.5	3	10	0	2	0.0556	0.0613	0.0604	0.1541	0.2037	0.2066(98.6)	0.2061(98.8)	0.2880(70.7)
0.5	3	10	2	0	0.0654	0.0760	0.0738	0.3577	0.1991	0.2046(97.3)	0.2035(97.9)	0.6707(29.7)
0.5	3	10	2	2	0.0546	0.0606	0.0597	0.1457	0.2101	0.2132(98.6)	0.2128(98.8)	0.2856(73.6)
0.5	3	20	0	0	0.0252	0.0308	0.0301	0.2367	0.0857	0.0869(98.6)	0.0868(98.8)	0.2953(29.0)
0.5	3	20	0	2	0.0241	0.0282	0.0279	0.1043	0.0907	0.0916(99.0)	0.0916(99.0)	0.1377(65.9)
0.5	3	20	0	5	0.0250	0.0277	0.0275	0.0668	0.1021	0.1028(99.3)	0.1027(99.4)	0.1207(84.6)
0.5	3	20	2	0	0.0206	0.0263	0.0255	0.2418	0.0825	0.0838(98.5)	0.0836(98.7)	0.3041(27.1)
0.5	3	20	2	2	0.0303	0.0345	0.0342	0.1134	0.0947	0.0958(98.9)	0.0957(98.9)	0.1470(64.4)
0.5	3	20	2	5	0.0297	0.0326	0.0324	0.0741	0.0997	0.1005(99.2)	0.1004(99.3)	0.1218(81.9)
0.5	3	20	5	0	0.0225	0.0285	0.0277	0.2397	0.0873	0.0886(98.6)	0.0884(98.7)	0.3093(28.2)
0.5	3	20	5	2	0.0267	0.0310	0.0307	0.1057	0.0924	0.0934(98.9)	0.0933(99.0)	0.1386(66.7)
0.5	3	20	5	5	0.0248	0.0277	0.0275	0.0676	0.0973	0.0980(99.3)	0.0979(99.3)	0.1167(83.4)
0.5	3	50	0	0	0.0105	0.0129	0.0127	0.1487	0.0326	0.0328(99.4)	0.0328(99.4)	0.1168(27.9)
0.5	3	50	0	5	0.0082	0.0100	0.0099	0.0423	0.0344	0.0346(99.6)	0.0346(99.6)	0.0461(74.7)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
0.5	3	50	0	10	0.0085	0.0098	0.0098	0.0289	0.0366	0.0367(99.7)	0.0367(99.7)	0.0429(85.3)
0.5	3	50	5	0	0.0111	0.0136	0.0134	0.1472	0.03300	0.0332(99.3)	0.0332(99.4)	0.1167(28.3)
0.5	3	50	5	5	0.0118	0.0136	0.0135	0.0466	0.0355	0.0356(99.5)	0.0356(99.5)	0.0471(75.2)
0.5	3	50	5	10	0.0113	0.0127	0.0126	0.0313	0.0374	0.0375(99.6)	0.0375(99.6)	0.0434(86.1)
0.5	3	50	10	0	0.0143	0.0168	0.0166	0.1501	0.0333	0.0336(99.3)	0.0335(99.3)	0.1206(27.6)
0.5	3	50	10	5	0.0081	0.0100	0.0099	0.0426	0.0351	0.0353(99.5)	0.0353(99.5)	0.0463(75.8)
0.5	3	50	10	10	0.0125	0.0139	0.0139	0.0346	0.0373	0.0374(99.6)	0.0374(99.6)	0.0448(83.1)
0.5	3	100	0	0	0.0055	0.0068	0.0067	0.1042	0.0165	0.0165(99.7)	0.0165(99.7)	0.0597(27.5)
0.5	3	100	0	10	0.0062	0.0071	0.0071	0.0233	0.0175	0.0175(99.7)	0.0175(99.8)	0.0222(78.5)
0.5	3	100	0	20	0.0061	0.0067	0.0067	0.0164	0.0181	0.0181(99.8)	0.0181(99.8)	0.0204(88.8)
0.5	3	100	0	30	0.0060	0.0065	0.0065	0.0137	0.0200	0.0201(99.9)	0.0201(99.9)	0.0217(92.4)
0.5	3	100	10	0	0.0062	0.0074	0.0074	0.1012	0.0161	0.0162(99.7)	0.0162(99.7)	0.0575(28.1)
0.5	3	100	10	10	0.0064	0.0073	0.0073	0.0239	0.0174	0.0174(99.8)	0.0174(99.8)	0.0221(78.8)
0.5	3	100	10	20	0.0059	0.0066	0.0065	0.0157	0.0178	0.0179(99.8)	0.0179(99.8)	0.0201(88.7)
0.5	3	100	10	30	0.0078	0.0083	0.0083	0.0156	0.0202	0.0202(99.9)	0.0202(99.9)	0.0218(92.5)
0.5	3	100	20	0	0.0047	0.0060	0.0060	0.1019	0.0164	0.0164(99.6)	0.0164(99.7)	0.0598(27.4)
0.5	3	100	20	10	0.0067	0.0077	0.0077	0.0241	0.0169	0.0169(99.8)	0.0169(99.8)	0.0213(79.2)
0.5	3	100	20	20	0.0028	0.0036	0.0035	0.0132	0.0178	0.0179(99.8)	0.0179(99.8)	0.0202(88.1)
0.5	3	100	20	30	0.0056	0.0062	0.0062	0.0141	0.0190	0.0190(99.8)	0.0190(99.8)	0.0209(90.8)
0.5	3	100	30	0	0.0066	0.0079	0.0079	0.1014	0.0160	0.0161(99.6)	0.0161(99.6)	0.0580(27.7)
0.5	3	100	30	10	0.0048	0.0057	0.0057	0.0227	0.0169	0.0170(99.7)	0.0170(99.7)	0.0217(78.0)
0.5	3	100	30	20	0.0082	0.0089	0.0089	0.0194	0.0194	0.0194(99.8)	0.0194(99.8)	0.0216(89.6)
0.5	3	100	30	30	0.0050	0.0056	0.0056	0.0124	0.0200	0.0200(99.9)	0.0200(99.9)	0.0215(92.8)
0.5	5	10	0	0	0.0365	0.0485	0.0461	0.2488	0.1224	0.1263(96.9)	0.1255(97.5)	0.3624(33.8)
0.5	5	10	0	2	0.0356	0.0432	0.0420	0.0973	0.1321	0.1344(98.3)	0.1340(98.6)	0.1689(78.2)
0.5	5	10	2	0	0.0451	0.0575	0.0552	0.2686	0.1318	0.1364(96.6)	0.1355(97.2)	0.4319(30.5)
0.5	5	10	2	2	0.0394	0.0469	0.0458	0.1043	0.1384	0.1410(98.1)	0.1407(98.4)	0.1814(76.3)
0.5	5	20	0	0	0.0161	0.0230	0.0221	0.1812	0.0581	0.0592(98.1)	0.0591(98.3)	0.1936(30.0)
0.5	5	20	0	2	0.0209	0.0263	0.0258	0.0806	0.0572	0.0581(98.5)	0.0580(98.6)	0.0867(66.0)
0.5	5	20	0	5	0.0226	0.0266	0.0262	0.0512	0.0644	0.0651(98.9)	0.0651(99.0)	0.0753(85.6)
0.5	5	20	2	0	0.0221	0.0289	0.0281	0.1896	0.0587	0.0599(98.1)	0.0597(98.3)	0.1902(30.9)
0.5	5	20	2	2	0.0174	0.0226	0.0222	0.0749	0.0585	0.0593(98.6)	0.0592(98.7)	0.0845(69.2)
0.5	5	20	2	5	0.0174	0.0212	0.0209	0.0430	0.0625	0.0631(99.0)	0.0631(99.1)	0.0724(86.4)
0.5	5	20	5	0	0.0152	0.0220	0.0212	0.1812	0.0608	0.0619(98.2)	0.0618(98.4)	0.1974(30.8)
0.5	5	20	5	2	0.0274	0.0326	0.0323	0.0859	0.0650	0.0660(98.5)	0.0660(98.6)	0.0934(69.6)
0.5	5	20	5	5	0.0213	0.0248	0.0245	0.0468	0.0692	0.0698(99.1)	0.0698(99.2)	0.0774(89.4)
0.5	5	50	0	0	0.0075	0.0105	0.0103	0.1145	0.0221	0.0223(99.2)	0.0223(99.2)	0.0752(29.4)
0.5	5	50	0	5	0.0098	0.0121	0.0121	0.0355	0.0237	0.0239(99.3)	0.0239(99.4)	0.0320(74.0)
0.5	5	50	0	10	0.0070	0.0088	0.0087	0.0222	0.0239	0.0240(99.5)	0.0240(99.6)	0.0281(84.9)
0.5	5	50	5	0	0.0074	0.0103	0.0102	0.1131	0.0231	0.0233(99.2)	0.0233(99.2)	0.0775(29.9)
0.5	5	50	5	5	0.0096	0.0119	0.0118	0.0361	0.0238	0.0240(99.3)	0.0240(99.4)	0.0320(74.5)
0.5	5	50	5	10	0.0074	0.0092	0.0091	0.0217	0.0239	0.0240(99.5)	0.0240(99.5)	0.0277(86.4)
0.5	5	50	10	0	0.0096	0.0125	0.0123	0.1168	0.0225	0.0227(99.1)	0.0227(99.2)	0.0765(29.5)
0.5	5	50	10	5	0.0057	0.0078	0.0078	0.0318	0.0236	0.0237(99.5)	0.0237(99.5)	0.0311(76.0)
0.5	5	50	10	10	0.0097	0.0114	0.0114	0.0235	0.0249	0.0251(99.4)	0.0251(99.4)	0.0285(87.5)
0.5	5	100	0	0	0.0050	0.0065	0.0065	0.0790	0.0109	0.0109(99.5)	0.0109(99.5)	0.0397(27.4)
0.5	5	100	0	10	0.0049	0.0061	0.0060	0.0175	0.0114	0.0114(99.7)	0.0114(99.7)	0.0150(75.8)
0.5	5	100	0	20	0.0064	0.0074	0.0073	0.0124	0.0116	0.0116(99.7)	0.0116(99.7)	0.0132(87.6)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
0.5	5	100	0	30	0.0040	0.0048	0.0048	0.0088	0.0123	0.0124(99.8)	0.0124(99.8)	0.0132(93.7)
0.5	5	100	10	0	0.0037	0.0052	0.0051	0.0806	0.0111	0.0111(99.6)	0.0111(99.6)	0.0398(27.8)
0.5	5	100	10	10	0.0046	0.0057	0.0057	0.0179	0.0115	0.0115(99.7)	0.0115(99.7)	0.0148(77.6)
0.5	5	100	10	20	0.0023	0.0032	0.0032	0.0108	0.0121	0.0121(99.8)	0.0121(99.8)	0.0137(88.1)
0.5	5	100	10	30	0.0058	0.0066	0.0066	0.0105	0.0125	0.0125(99.8)	0.0125(99.8)	0.0136(92.1)
0.5	5	100	20	0	0.0049	0.0064	0.0064	0.0805	0.0114	0.0114(99.5)	0.0114(99.5)	0.0390(29.2)
0.5	5	100	20	10	0.0026	0.0037	0.0037	0.0145	0.0120	0.0120(99.7)	0.0120(99.7)	0.0149(80.3)
0.5	5	100	20	20	0.0053	0.0062	0.0062	0.0120	0.0125	0.0126(99.7)	0.0126(99.7)	0.0141(89.3)
0.5	5	100	20	30	0.0047	0.0054	0.0054	0.0090	0.0131	0.0131(99.8)	0.0131(99.8)	0.0141(92.8)
0.5	5	100	30	0	0.0073	0.0088	0.0087	0.0836	0.0117	0.0117(99.5)	0.0117(99.5)	0.0399(29.2)
0.5	5	100	30	10	0.0048	0.0059	0.0059	0.0180	0.0117	0.0117(99.6)	0.0117(99.6)	0.0149(78.2)
0.5	5	100	30	20	0.0053	0.0061	0.0061	0.0132	0.0126	0.0126(99.7)	0.0126(99.8)	0.0141(89.0)
0.5	5	100	30	30	0.0063	0.0069	0.0069	0.0105	0.0135	0.0136(99.8)	0.0136(99.8)	0.0146(92.9)
1.5	0.5	10	0	0	-0.0279	-0.0310	-0.0295	0.1145	0.0829	0.0826(100.4)	0.0827(100.2)	0.1216(68.2)
1.5	0.5	10	0	2	-0.0344	-0.0350	-0.0349	0.0680	0.1129	0.1128(100.1)	0.1129(100.1)	0.1436(78.6)
1.5	0.5	10	2	0	-0.0162	-0.0193	-0.0178	0.1277	0.0799	0.0796(100.4)	0.0797(100.2)	0.1216(65.7)
1.5	0.5	10	2	2	-0.0302	-0.0307	-0.0307	0.0716	0.1153	0.1152(100.1)	0.1152(100.1)	0.1493(77.2)
1.5	0.5	20	0	0	-0.0109	-0.0130	-0.0124	0.0778	0.0419	0.0418(100.3)	0.0418(100.2)	0.0562(74.5)
1.5	0.5	20	0	2	-0.0124	-0.0133	-0.0132	0.0475	0.0508	0.0508(100.1)	0.0508(100.1)	0.0603(84.3)
1.5	0.5	20	0	5	-0.0171	-0.0173	-0.0173	0.0347	0.0648	0.0648(100.0)	0.0648(100.0)	0.0770(84.2)
1.5	0.5	20	2	0	-0.0111	-0.0131	-0.0125	0.0772	0.0405	0.0404(100.3)	0.0404(100.2)	0.0544(74.5)
1.5	0.5	20	2	2	-0.0102	-0.0110	-0.0109	0.0501	0.0492	0.0491(100.1)	0.0491(100.1)	0.0594(82.8)
1.5	0.5	20	2	5	-0.0165	-0.0167	-0.0167	0.0356	0.0645	0.0645(100.0)	0.0645(100.0)	0.0771(83.8)
1.5	0.5	20	5	0	-0.0097	-0.0117	-0.0111	0.0795	0.0412	0.0411(100.3)	0.0411(100.2)	0.0558(73.8)
1.5	0.5	20	5	2	-0.0117	-0.0124	-0.0123	0.0487	0.0493	0.0492(100.1)	0.0492(100.1)	0.0586(84.1)
1.5	0.5	20	5	5	-0.0156	-0.0157	-0.0157	0.0362	0.0647	0.0647(100.0)	0.0647(100.0)	0.0767(84.4)
1.5	0.5	50	0	0	-0.0050	-0.0060	-0.0059	0.0409	0.0166	0.0166(100.1)	0.0166(100.1)	0.0201(82.5)
1.5	0.5	50	0	5	-0.0042	-0.0046	-0.0046	0.0211	0.0194	0.0194(100.1)	0.0194(100.1)	0.0218(89.1)
1.5	0.5	50	0	10	-0.0077	-0.0079	-0.0079	0.0152	0.0237	0.0237(100.0)	0.0237(100.0)	0.0272(87.3)
1.5	0.5	50	5	0	-0.0063	-0.0073	-0.0071	0.0393	0.0163	0.0163(100.1)	0.0163(100.1)	0.0196(83.3)
1.5	0.5	50	5	5	-0.008	-0.0084	-0.0084	0.0171	0.0198	0.0197(100.1)	0.0198(100.1)	0.0219(90.4)
1.5	0.5	50	5	10	-0.0047	-0.0049	-0.0049	0.0180	0.0243	0.0243(100.0)	0.0243(100.0)	0.0274(88.5)
1.5	0.5	50	10	0	-0.0034	-0.0044	-0.0042	0.0429	0.0165	0.0164(100.2)	0.0164(100.1)	0.0200(82.3)
1.5	0.5	50	10	5	-0.0078	-0.0081	-0.0081	0.0173	0.0197	0.0197(100.0)	0.0197(100.0)	0.0218(90.5)
1.5	0.5	50	10	10	-0.0068	-0.0069	-0.0069	0.0146	0.0235	0.0235(100.0)	0.0235(100.0)	0.0270(87.3)
1.5	0.5	100	0	0	-0.0035	-0.0040	-0.0039	0.02400	0.0082	0.0082(100.0)	0.0082(100.0)	0.0094(87.6)
1.5	0.5	100	0	10	-0.0032	-0.0033	-0.0033	0.0095	0.0099	0.0099(100.0)	0.0099(100.0)	0.0107(91.9)
1.5	0.5	100	0	20	-0.0048	-0.0049	-0.0049	0.0062	0.0120	0.0120(100.0)	0.0120(100.0)	0.0132(90.4)
1.5	0.5	100	0	30	-0.0033	-0.0034	-0.0034	0.0069	0.0142	0.0142(100.0)	0.0142(100.0)	0.0162(87.7)
1.5	0.5	100	10	0	-0.003	-0.0035	-0.0035	0.0243	0.0084	0.0084(100.1)	0.0084(100.1)	0.0095(87.9)
1.5	0.5	100	10	10	-0.0022	-0.0024	-0.0024	0.0110	0.0097	0.0097(100.0)	0.0097(100.0)	0.0106(91.5)
1.5	0.5	100	10	20	-0.0024	-0.0024	-0.0024	0.0087	0.0118	0.0118(100.0)	0.0118(100.0)	0.0133(88.8)
1.5	0.5	100	10	30	-0.0048	-0.0049	-0.0049	0.0064	0.0148	0.0148(100.0)	0.0148(100.0)	0.0169(87.3)
1.5	0.5	100	20	0	-0.0015	-0.0020	-0.0020	0.0259	0.0081	0.0081(100.1)	0.0081(100.1)	0.0094(86.6)
1.5	0.5	100	20	10	-0.0008	-0.0010	-0.0010	0.0121	0.0100	0.0100(100.0)	0.0100(100.0)	0.0110(90.8)
1.5	0.5	100	20	20	-0.0040	-0.0040	-0.0040	0.0069	0.0120	0.0120(100.0)	0.0120(100.0)	0.0135(88.8)
1.5	0.5	100	20	30	-0.0039	-0.0039	-0.0039	0.0070	0.0149	0.0149(100.0)	0.0149(100.0)	0.0170(87.3)
1.5	0.5	100	30	0	-0.0022	-0.0027	-0.0027	0.0250	0.0083	0.0083(100.1)	0.0083(100.0)	0.0095(87.5)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
1.5	0.5	100	30	10	-0.0012	-0.0014	-0.0014	0.0112	0.0099	0.0099(100.0)	0.0099(100.0)	0.0108(91.8)
1.5	0.5	100	30	20	-0.0034	-0.0034	-0.0034	0.0077	0.0116	0.0116(100.0)	0.0116(100.0)	0.0131(88.6)
1.5	0.5	100	30	30	-0.0019	-0.0019	-0.0019	0.0089	0.0140	0.0140(100.0)	0.0140(100.0)	0.0162(86.5)
1.5	1.5	10	0	0	-0.0045	-0.0031	-0.0035	0.0533	0.0308	0.0309(99.8)	0.0309(99.8)	0.0380(81.0)
1.5	1.5	10	0	2	-0.0101	-0.0095	-0.0096	0.0151	0.0357	0.0357(99.9)	0.0357(99.9)	0.0385(92.8)
1.5	1.5	10	2	0	-0.0053	-0.0036	-0.0040	0.0550	0.0322	0.0323(99.7)	0.0323(99.8)	0.0398(80.9)
1.5	1.5	10	2	2	-0.0051	-0.0041	-0.0043	0.0251	0.0369	0.0370(99.8)	0.0370(99.8)	0.0408(90.5)
1.5	1.5	20	0	0	-0.0041	-0.0033	-0.0034	0.0335	0.0149	0.0149(99.9)	0.0149(99.9)	0.0174(85.5)
1.5	1.5	20	0	2	-0.0017	-0.0011	-0.0012	0.0178	0.0168	0.0168(99.9)	0.0168(99.9)	0.0180(93.3)
1.5	1.5	20	0	5	-0.0067	-0.0064	-0.0064	0.0047	0.0193	0.0193(100.0)	0.0193(100.0)	0.0206(93.7)
1.5	1.5	20	2	0	-0.0030	-0.0021	-0.0023	0.0351	0.0157	0.0157(99.9)	0.0157(99.9)	0.0184(85.2)
1.5	1.5	20	2	2	-0.0051	-0.0045	-0.0046	0.0150	0.0173	0.0173(99.9)	0.0173(99.9)	0.0185(93.3)
1.5	1.5	20	2	5	-0.0049	-0.0045	-0.0045	0.0075	0.0189	0.0190(99.9)	0.0189(99.9)	0.0203(93.4)
1.5	1.5	20	5	0	-0.0008	0.0002	0.0001	0.0396	0.0157	0.0158(99.8)	0.0158(99.8)	0.0192(81.9)
1.5	1.5	20	5	2	-0.0006	0.0001	0.0001	0.0217	0.0174	0.0174(99.8)	0.0174(99.9)	0.0190(91.5)
1.5	1.5	20	5	5	-0.0038	-0.0032	-0.0033	0.0109	0.0197	0.0197(99.9)	0.0197(99.9)	0.0211(93.6)
1.5	1.5	50	0	0	-0.0003	0.0001	0.0001	0.0202	0.0062	0.0062(99.9)	0.0062(99.9)	0.0071(87.9)
1.5	1.5	50	0	5	-0.0032	-0.0030	-0.0030	0.0053	0.0068	0.0068(100.0)	0.0068(100.0)	0.0071(95.9)
1.5	1.5	50	0	10	-0.0002	-0.0001	-0.0001	0.0054	0.0074	0.0074(100.0)	0.0074(100.0)	0.0078(94.0)
1.5	1.5	50	5	0	0.0002	0.0006	0.0005	0.0211	0.0063	0.0063(99.9)	0.0063(99.9)	0.0073(87.2)
1.5	1.5	50	5	5	-0.0015	-0.0012	-0.0012	0.0073	0.0070	0.0070(100.0)	0.0070(100.0)	0.0073(95.8)
1.5	1.5	50	5	10	-0.0004	-0.0002	-0.0002	0.0061	0.0072	0.0072(100.0)	0.0072(100.0)	0.0077(93.2)
1.5	1.5	50	10	0	0.0001	0.0005	0.0005	0.0216	0.0061	0.0061(99.9)	0.0061(99.9)	0.0070(86.2)
1.5	1.5	50	10	5	-0.0014	-0.0011	-0.0011	0.0079	0.0067	0.0067(99.9)	0.0067(99.9)	0.0070(95.1)
1.5	1.5	50	10	10	0.0000	0.0002	0.0002	0.0066	0.0075	0.0075(100.0)	0.0075(100.0)	0.0080(93.7)
1.5	1.5	100	0	0	-0.0007	-0.0005	-0.0005	0.0118	0.0031	0.0031(100.0)	0.0031(100.0)	0.0034(90.9)
1.5	1.5	100	0	10	-0.0001	0.0000	0.0000	0.0042	0.0035	0.0035(100.0)	0.0035(100.0)	0.0037(95.8)
1.5	1.5	100	0	20	-0.0009	-0.0008	-0.0008	0.0020	0.0037	0.0037(100.0)	0.0037(100.0)	0.0039(94.7)
1.5	1.5	100	0	30	-0.0007	-0.0007	-0.0007	0.0008	0.0041	0.0041(100.0)	0.0041(100.0)	0.0044(92.5)
1.5	1.5	100	10	0	-0.0007	-0.0005	-0.0005	0.0120	0.0031	0.0031(100.0)	0.0031(100.0)	0.0034(90.8)
1.5	1.5	100	10	10	0.0006	0.0007	0.0007	0.0049	0.0034	0.0034(100.0)	0.0034(100.0)	0.0035(95.2)
1.5	1.5	100	10	20	-0.0013	-0.0012	-0.0012	0.0019	0.0037	0.0037(100.0)	0.0037(100.0)	0.0039(94.5)
1.5	1.5	100	10	30	-0.0017	-0.0017	-0.0017	0.0006	0.0041	0.0041(100.0)	0.0041(100.0)	0.0044(92.3)
1.5	1.5	100	20	0	-0.0006	-0.0003	-0.0004	0.0124	0.0032	0.0032(100.0)	0.0032(100.0)	0.0036(89.7)
1.5	1.5	100	20	10	-0.0001	0.0001	0.0001	0.0047	0.0034	0.0034(100.0)	0.0034(100.0)	0.0035(96.3)
1.5	1.5	100	20	20	-0.0009	-0.0008	-0.0008	0.0025	0.0037	0.0037(100.0)	0.0037(100.0)	0.0039(93.7)
1.5	1.5	100	20	30	-0.0012	-0.0011	-0.0011	0.0013	0.0042	0.0042(100.0)	0.0042(100.0)	0.0044(93.5)
1.5	1.5	100	30	0	0.0013	0.0016	0.0016	0.0145	0.0032	0.0032(99.9)	0.0032(99.9)	0.0037(87.7)
1.5	1.5	100	30	10	-0.0008	-0.0006	-0.0006	0.0039	0.0033	0.0033(100.0)	0.0033(100.0)	0.0035(94.9)
1.5	1.5	100	30	20	-0.0020	-0.0018	-0.0018	0.0015	0.0037	0.0037(100.0)	0.0037(100.0)	0.0039(94.4)
1.5	1.5	100	30	30	-0.0002	-0.0001	-0.0001	0.0025	0.0040	0.0040(100.0)	0.0040(100.0)	0.0044(92.5)
1.5	3	10	0	0	-0.0019	0.0013	0.0006	0.0323	0.0180	0.0182(99.3)	0.0181(99.5)	0.0209(86.2)
1.5	3	10	0	2	-0.0014	0.0004	0.0001	0.0075	0.0195	0.0195(99.6)	0.0195(99.7)	0.0202(96.3)
1.5	3	10	2	0	-0.0027	0.0007	0.0000	0.0364	0.0179	0.0180(99.3)	0.0180(99.5)	0.0216(83.2)
1.5	3	10	2	2	-0.0028	-0.0008	-0.0011	0.0115	0.0202	0.0203(99.6)	0.0203(99.7)	0.0212(95.2)
1.5	3	20	0	0	-0.0010	0.0008	0.0005	0.0218	0.0087	0.0087(99.7)	0.0087(99.7)	0.0099(87.2)
1.5	3	20	0	2	-0.0004	0.0009	0.0008	0.0091	0.0091	0.0091(99.8)	0.0091(99.8)	0.0095(95.7)
1.5	3	20	0	5	-0.0004	0.0005	0.0004	0.0023	0.0100	0.0100(99.8)	0.0100(99.8)	0.0103(96.6)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
1.5	3	20	2	0	-0.0017	0.0002	-0.0001	0.0230	0.0087	0.0087(99.6)	0.0087(99.7)	0.0101(85.7)
1.5	3	20	2	2	-0.0015	-0.0002	-0.0003	0.0096	0.0095	0.0095(99.8)	0.0095(99.8)	0.0099(96.1)
1.5	3	20	2	5	-0.0018	-0.0008	-0.0009	0.0027	0.0102	0.0102(99.8)	0.0102(99.8)	0.0107(95.6)
1.5	3	20	5	0	-0.0021	-0.0002	-0.0004	0.0242	0.0090	0.0091(99.7)	0.0091(99.7)	0.0108(83.7)
1.5	3	20	5	2	0.0024	0.0038	0.0037	0.0150	0.0095	0.0096(99.6)	0.0096(99.7)	0.0103(92.9)
1.5	3	20	5	5	-0.0007	0.0002	0.0002	0.0063	0.0103	0.0103(99.8)	0.0103(99.9)	0.0108(94.9)
1.5	3	50	0	0	-0.0013	-0.0005	-0.0006	0.0120	0.0035	0.0035(99.9)	0.0035(99.9)	0.0039(89.1)
1.5	3	50	0	5	-0.0013	-0.0008	-0.0008	0.0027	0.0037	0.0037(99.9)	0.0037(99.9)	0.0039(96.2)
1.5	3	50	0	10	-0.0004	0.0000	0.0000	0.0014	0.0039	0.0039(99.9)	0.0039(99.9)	0.0041(96.3)
1.5	3	50	5	0	-0.0008	0.0000	-0.0001	0.0132	0.0035	0.0035(99.8)	0.0035(99.8)	0.0040(88.3)
1.5	3	50	5	5	-0.0005	0.0001	0.0001	0.0043	0.0038	0.0038(99.9)	0.0038(99.9)	0.0040(96.3)
1.5	3	50	5	10	-0.0002	0.0002	0.0002	0.0027	0.0038	0.0038(99.9)	0.0038(99.9)	0.0040(96.2)
1.5	3	50	10	0	0.0027	0.0036	0.0035	0.0172	0.0036	0.0036(99.7)	0.0036(99.7)	0.0043(84.8)
1.5	3	50	10	5	0.0001	0.0007	0.0007	0.0053	0.0038	0.0038(99.9)	0.0038(99.9)	0.0039(96.7)
1.5	3	50	10	10	0.0012	0.0017	0.0017	0.0042	0.0041	0.0041(99.8)	0.0041(99.8)	0.0043(95.3)
1.5	3	100	0	0	0.0000	0.0004	0.0004	0.0083	0.0017	0.0017(99.9)	0.0017(99.9)	0.0019(90.3)
1.5	3	100	0	10	-0.0013	-0.0010	-0.0010	0.0008	0.0018	0.0018(100.0)	0.0018(100.0)	0.0019(98.3)
1.5	3	100	0	20	0.0003	0.0005	0.0005	0.0010	0.0020	0.0020(99.9)	0.0020(99.9)	0.0020(96.7)
1.5	3	100	0	30	0.0005	0.0007	0.0006	0.0005	0.0022	0.0022(100.0)	0.0022(100.0)	0.0023(94.0)
1.5	3	100	10	0	-0.0003	0.0001	0.0001	0.0084	0.0018	0.0018(99.9)	0.0018(99.9)	0.0020(89.4)
1.5	3	100	10	10	0.0005	0.0008	0.0008	0.0029	0.0019	0.0019(99.9)	0.0019(99.9)	0.0020(96.9)
1.5	3	100	10	20	-0.0001	0.0001	0.0001	0.0013	0.0020	0.0020(100.0)	0.0020(100.0)	0.0021(96.5)
1.5	3	100	10	30	0.0002	0.0004	0.0004	0.0009	0.0021	0.0021(99.9)	0.0021(99.9)	0.0022(95.0)
1.5	3	100	20	0	0.0002	0.0006	0.0006	0.0091	0.0018	0.0018(99.9)	0.0018(99.9)	0.0021(88.0)
1.5	3	100	20	10	0.0000	0.0003	0.0003	0.0025	0.0018	0.0018(99.9)	0.0018(99.9)	0.0019(97.1)
1.5	3	100	20	20	-0.0004	-0.0001	-0.0001	0.0011	0.0020	0.0020(99.9)	0.0020(99.9)	0.0021(94.9)
1.5	3	100	20	30	0.0006	0.0008	0.0008	0.0016	0.0021	0.0021(100.0)	0.0021(100.0)	0.0022(94.8)
1.5	3	100	30	0	0.0004	0.0009	0.0009	0.0100	0.0018	0.0018(99.9)	0.0018(99.9)	0.0021(87.2)
1.5	3	100	30	10	0.0006	0.0009	0.0009	0.0035	0.0019	0.0019(99.9)	0.0019(99.9)	0.0020(95.2)
1.5	3	100	30	20	0.0003	0.0005	0.0005	0.0022	0.0020	0.0020(99.9)	0.0020(99.9)	0.0021(95.1)
1.5	3	100	30	30	0.0000	0.0002	0.0002	0.0011	0.0022	0.0022(99.9)	0.0022(99.9)	0.0023(95.2)
1.5	5	10	0	0	0.0003	0.0042	0.0034	0.0240	0.0122	0.0123(99.1)	0.0123(99.3)	0.0140(87.3)
1.5	5	10	0	2	0.0003	0.0028	0.0024	0.0036	0.0135	0.0135(99.4)	0.0135(99.4)	0.0137(98.5)
1.5	5	10	2	0	-0.0001	0.0037	0.0029	0.029	0.0124	0.0125(99.0)	0.0125(99.2)	0.0151(82.5)
1.5	5	10	2	2	0.0021	0.0044	0.0041	0.0111	0.0135	0.0136(99.4)	0.0136(99.5)	0.0142(95.6)
1.5	5	20	0	0	0.0003	0.0025	0.0022	0.0175	0.0062	0.0062(99.4)	0.0062(99.5)	0.0071(86.9)
1.5	5	20	0	2	-0.0011	0.0007	0.0005	0.0045	0.0064	0.0065(99.6)	0.0065(99.7)	0.0066(97.7)
1.5	5	20	0	5	-0.0002	0.0011	0.0010	-0.0005	0.0066	0.0067(99.7)	0.0066(99.7)	0.0068(97.8)
1.5	5	20	2	0	0.0017	0.0039	0.0037	0.0198	0.0059	0.0060(99.3)	0.0060(99.4)	0.0070(84.5)
1.5	5	20	2	2	0.0003	0.0020	0.0019	0.0076	0.0064	0.0064(99.6)	0.0064(99.6)	0.0067(95.9)
1.5	5	20	2	5	-0.0009	0.0003	0.0002	0.0014	0.0069	0.0069(99.8)	0.0069(99.8)	0.0071(97.4)
1.5	5	20	5	0	0.0000	0.0021	0.0019	0.0213	0.0063	0.0063(99.4)	0.0063(99.5)	0.0076(82.7)
1.5	5	20	5	2	0.0012	0.0029	0.0027	0.0100	0.0064	0.0064(99.5)	0.0064(99.5)	0.0068(93.3)
1.5	5	20	5	5	-0.0002	0.0009	0.0009	0.0039	0.0070	0.0070(99.7)	0.0070(99.7)	0.0073(96.2)
1.5	5	50	0	0	-0.0003	0.0007	0.0006	0.0095	0.0025	0.0025(99.7)	0.0025(99.8)	0.0028(88.7)
1.5	5	50	0	5	0.0002	0.0009	0.0009	0.0024	0.0024	0.0024(99.8)	0.0024(99.8)	0.0025(96.1)
1.5	5	50	0	10	-0.0007	-0.0001	-0.0001	-0.0004	0.0027	0.0027(99.9)	0.0027(99.9)	0.0027(97.9)
1.5	5	50	5	0	0.0016	0.0025	0.0025	0.0123	0.0025	0.0025(99.6)	0.0025(99.6)	0.0029(84.9)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
1.5	5	50	5	5	-0.0009	-0.0002	-0.0002	0.0024	0.0024	0.0024(99.9)	0.0024(99.9)	0.0025(96.1)
1.5	5	50	5	10	0.0001	0.0007	0.0007	0.0015	0.0026	0.0026(99.9)	0.0026(99.9)	0.0027(96.9)
1.5	5	50	10	0	0.0005	0.0015	0.0014	0.0114	0.0025	0.0025(99.7)	0.0025(99.7)	0.0030(84.6)
1.5	5	50	10	5	0.0006	0.0013	0.0013	0.0045	0.0026	0.0026(99.8)	0.0026(99.8)	0.0028(94.9)
1.5	5	50	10	10	0.0010	0.0016	0.0016	0.0028	0.0027	0.0027(99.8)	0.0027(99.8)	0.0028(95.6)
1.5	5	100	0	0	0.0002	0.0007	0.0006	0.0060	0.0012	0.0012(99.8)	0.0012(99.8)	0.0013(89.4)
1.5	5	100	0	10	-0.0005	-0.0001	-0.0001	0.0007	0.0013	0.0013(100.0)	0.0013(100.0)	0.0013(97.7)
1.5	5	100	0	20	0.0001	0.0004	0.0004	0.0002	0.0013	0.0013(99.9)	0.0013(99.9)	0.0013(98.2)
1.5	5	100	0	30	-0.0006	-0.0004	-0.0004	-0.0011	0.0014	0.0014(99.9)	0.0014(99.9)	0.0015(96.0)
1.5	5	100	10	0	0.0001	0.0006	0.0006	0.0065	0.0012	0.0012(99.8)	0.0012(99.8)	0.0014(87.7)
1.5	5	100	10	10	0.0005	0.0009	0.0008	0.0022	0.0013	0.0013(99.9)	0.0013(99.9)	0.0014(97.0)
1.5	5	100	10	20	0.0004	0.0007	0.0007	0.0010	0.0013	0.0013(99.9)	0.0013(99.9)	0.0013(97.5)
1.5	5	100	10	30	0.0001	0.0003	0.0003	0.0005	0.0014	0.0014(99.9)	0.0014(99.9)	0.0014(95.1)
1.5	5	100	20	0	-0.0009	-0.0004	-0.0004	0.0059	0.0013	0.0013(99.9)	0.0013(99.9)	0.0015(87.5)
1.5	5	100	20	10	0.0001	0.0005	0.0005	0.0020	0.0013	0.0013(99.9)	0.0013(99.9)	0.0013(96.3)
1.5	5	100	20	20	-0.0013	-0.0010	-0.0011	-0.0004	0.0013	0.0013(100.0)	0.0013(100.0)	0.0014(96.3)
1.5	5	100	20	30	-0.0003	-0.0001	-0.0001	0.0001	0.0014	0.0014(100.0)	0.0014(100.0)	0.0015(95.6)
1.5	5	100	30	0	0.0000	0.0005	0.0005	0.0077	0.0012	0.0012(99.9)	0.0012(99.9)	0.0015(84.2)
1.5	5	100	30	10	0.0005	0.0009	0.0009	0.0027	0.0013	0.0013(99.9)	0.0013(99.9)	0.0014(94.0)
1.5	5	100	30	20	0.0001	0.0003	0.0003	0.0014	0.0014	0.0014(99.9)	0.0014(99.9)	0.0014(94.4)
1.5	5	100	30	30	-0.0005	-0.0002	-0.0003	0.0001	0.0014	0.0014(99.9)	0.0014(99.9)	0.0015(95.4)
3	0.5	10	0	0	-0.0226	-0.0242	-0.0235	0.0273	0.0213	0.0214(99.9)	0.0214(100.0)	0.0257(83.0)
3	0.5	10	0	2	-0.0328	-0.0331	-0.0331	0.0036	0.0326	0.0326(100.0)	0.0326(100.0)	0.0393(83.1)
3	0.5	10	2	0	-0.0262	-0.0277	-0.0270	0.0249	0.0218	0.0219(99.9)	0.0218(99.9)	0.0256(85.3)
3	0.5	10	2	2	-0.0315	-0.0318	-0.0318	0.0073	0.0308	0.0308(100.0)	0.0308(100.0)	0.0364(84.7)
3	0.5	20	0	0	-0.0114	-0.0124	-0.0121	0.0178	0.0105	0.0105(100.0)	0.0105(100.0)	0.0122(86.6)
3	0.5	20	0	2	-0.0128	-0.0132	-0.0132	0.0090	0.0128	0.0128(100.0)	0.0128(100.0)	0.0153(83.5)
3	0.5	20	0	5	-0.0172	-0.0173	-0.0173	0.0020	0.0168	0.0168(100.0)	0.0168(100.0)	0.0206(81.4)
3	0.5	20	2	0	-0.0118	-0.0128	-0.0125	0.0182	0.0102	0.0102(100.0)	0.0102(100.0)	0.0120(85.5)
3	0.5	20	2	2	-0.0124	-0.0128	-0.0128	0.0099	0.0125	0.0125(100.0)	0.0125(100.0)	0.0150(83.3)
3	0.5	20	2	5	-0.0178	-0.0179	-0.0179	0.0007	0.0167	0.0167(100.0)	0.0167(100.0)	0.0206(81.2)
3	0.5	20	5	0	-0.0107	-0.0117	-0.0114	0.0211	0.0103	0.0103(100.0)	0.0103(100.0)	0.0121(85.1)
3	0.5	20	5	2	-0.0121	-0.0125	-0.0125	0.0109	0.0129	0.0129(100.0)	0.0129(100.0)	0.0153(84.6)
3	0.5	20	5	5	-0.0180	-0.0181	-0.0181	0.0023	0.0167	0.0167(100.0)	0.0167(100.0)	0.0195(85.9)
3	0.5	50	0	0	-0.0042	-0.0047	-0.0046	0.0095	0.0042	0.0042(100.0)	0.0042(100.0)	0.0048(87.7)
3	0.5	50	0	5	-0.0054	-0.0056	-0.0056	0.0030	0.0051	0.0051(100.0)	0.0051(100.0)	0.0062(82.1)
3	0.5	50	0	10	-0.008	-0.0081	-0.0081	-0.0007	0.0062	0.0062(100.0)	0.0062(100.0)	0.0077(80.1)
3	0.5	50	5	0	-0.0035	-0.0040	-0.0039	0.0112	0.0043	0.0043(100.0)	0.0043(100.0)	0.0048(88.3)
3	0.5	50	5	5	-0.0050	-0.0052	-0.0052	0.0045	0.0051	0.0051(100.0)	0.0051(100.0)	0.0061(83.4)
3	0.5	50	5	10	-0.0081	-0.0082	-0.0082	-0.0003	0.0061	0.0061(100.0)	0.0061(100.0)	0.0073(83.0)
3	0.5	50	10	0	-0.0061	-0.0066	-0.0065	0.0084	0.0043	0.0043(99.9)	0.0043(100.0)	0.0047(91.1)
3	0.5	50	10	5	-0.0060	-0.0061	-0.0061	0.0037	0.0050	0.0050(100.0)	0.0050(100.0)	0.0058(86.4)
3	0.5	50	10	10	-0.0066	-0.0066	-0.0066	0.0018	0.0061	0.0061(100.0)	0.0061(100.0)	0.0072(84.3)
3	0.5	100	0	0	-0.0026	-0.0029	-0.0028	0.0057	0.0021	0.0021(100.0)	0.0021(100.0)	0.0023(88.4)
3	0.5	100	0	10	-0.0021	-0.0022	-0.0022	0.0026	0.0026	0.0026(100.0)	0.0026(100.0)	0.0031(83.4)
3	0.5	100	0	20	-0.0039	-0.0040	-0.0040	-0.0004	0.0031	0.0031(100.0)	0.0031(100.0)	0.0038(81.1)
3	0.5	100	0	30	-0.0035	-0.0035	-0.0035	0.0005	0.0036	0.0036(100.0)	0.0036(100.0)	0.0046(77.5)
3	0.5	100	10	0	-0.0024	-0.0026	-0.0026	0.0058	0.0021	0.0021(100.0)	0.0021(100.0)	0.0023(91.5)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
3	0.5	100	10	10	-0.0030	-0.0031	-0.0031	0.0016	0.0025	0.0025(100.0)	0.0025(100.0)	0.0030(84.3)
3	0.5	100	10	20	-0.0038	-0.0038	-0.0038	0.0007	0.0030	0.0030(100.0)	0.0030(100.0)	0.0037(82.1)
3	0.5	100	10	30	-0.0027	-0.0027	-0.0027	0.0006	0.0036	0.0036(100.0)	0.0036(100.0)	0.0045(79.8)
3	0.5	100	20	0	-0.0029	-0.0031	-0.0031	0.0057	0.0021	0.0021(100.0)	0.0021(100.0)	0.0023(90.3)
3	0.5	100	20	10	-0.0032	-0.0033	-0.0033	0.0018	0.0025	0.0025(100.0)	0.0025(100.0)	0.0029(85.8)
3	0.5	100	20	20	-0.0050	-0.0051	-0.0051	-0.0007	0.0030	0.0030(100.0)	0.0030(100.0)	0.0037(82.6)
3	0.5	100	20	30	-0.0030	-0.0030	-0.0030	0.0004	0.0037	0.0037(100.0)	0.0037(100.0)	0.0045(81.3)
3	0.5	100	30	0	-0.0023	-0.0026	-0.0025	0.0064	0.0021	0.0021(100.0)	0.0021(100.0)	0.0022(91.7)
3	0.5	100	30	10	-0.0036	-0.0037	-0.0037	0.0013	0.0025	0.0025(100.0)	0.0025(100.0)	0.0029(88.5)
3	0.5	100	30	20	-0.0041	-0.0041	-0.0041	0.0000	0.0030	0.0030(100.0)	0.0030(100.0)	0.0036(83.8)
3	0.5	100	30	30	-0.0037	-0.0037	-0.0037	0.0003	0.0038	0.0038(100.0)	0.0038(100.0)	0.0045(84.3)
3	1.5	10	0	0	-0.0053	-0.0045	-0.0047	0.0100	0.0079	0.0079(99.9)	0.0079(100.0)	0.0084(93.8)
3	1.5	10	0	2	-0.0095	-0.0092	-0.0092	-0.0055	0.0095	0.0095(100.0)	0.0095(100.0)	0.0105(90.3)
3	1.5	10	2	0	-0.0065	-0.0056	-0.0058	0.0147	0.0080	0.0080(99.9)	0.0080(100.0)	0.0085(93.4)
3	1.5	10	2	2	-0.0076	-0.0072	-0.0072	0.0027	0.0094	0.0094(100.0)	0.0094(100.0)	0.0101(93.4)
3	1.5	20	0	0	-0.0043	-0.0039	-0.0040	0.0056	0.0039	0.0039(100.0)	0.0039(100.0)	0.0041(95.8)
3	1.5	20	0	2	-0.0044	-0.0042	-0.0042	-0.0006	0.0042	0.0042(100.0)	0.0042(100.0)	0.0045(92.3)
3	1.5	20	0	5	-0.0058	-0.0056	-0.0056	-0.0060	0.0050	0.0050(100.0)	0.0050(100.0)	0.0056(88.5)
3	1.5	20	2	0	-0.0032	-0.0028	-0.0028	0.0087	0.0039	0.0039(100.0)	0.0039(100.0)	0.0041(94.5)
3	1.5	20	2	2	-0.0053	-0.0050	-0.0050	0.0011	0.0043	0.0043(100.0)	0.0043(100.0)	0.0045(93.9)
3	1.5	20	2	5	-0.0032	-0.0030	-0.0030	-0.0004	0.0050	0.0050(100.0)	0.0050(100.0)	0.0055(90.2)
3	1.5	20	5	0	-0.0020	-0.0015	-0.0015	0.0118	0.0038	0.0038(99.9)	0.0038(100.0)	0.0041(93.2)
3	1.5	20	5	2	-0.0022	-0.0019	-0.0019	0.0059	0.0042	0.0042(99.9)	0.0042(99.9)	0.0044(94.9)
3	1.5	20	5	5	-0.0031	-0.0028	-0.0028	0.0024	0.0050	0.0050(100.0)	0.0050(100.0)	0.0054(93.0)
3	1.5	50	0	0	-0.0004	-0.0002	-0.0002	0.0046	0.0015	0.0015(100.0)	0.0015(100.0)	0.0016(96.1)
3	1.5	50	0	5	-0.0016	-0.0015	-0.0015	-0.0001	0.0017	0.0017(100.0)	0.0017(100.0)	0.0018(93.1)
3	1.5	50	0	10	-0.0024	-0.0023	-0.0023	-0.0021	0.0019	0.0019(100.0)	0.0019(100.0)	0.0021(89.0)
3	1.5	50	5	0	-0.0022	-0.0020	-0.0020	0.0038	0.0015	0.0015(100.0)	0.0015(100.0)	0.0015(97.4)
3	1.5	50	5	5	-0.0015	-0.0013	-0.0013	0.0010	0.0017	0.0017(100.0)	0.0017(100.0)	0.0019(93.5)
3	1.5	50	5	10	-0.0019	-0.0019	-0.0019	-0.0002	0.0018	0.0018(100.0)	0.0018(100.0)	0.0020(90.5)
3	1.5	50	10	0	-0.0017	-0.0015	-0.0015	0.0051	0.0016	0.0016(100.0)	0.0016(100.0)	0.0016(96.2)
3	1.5	50	10	5	-0.0006	-0.0004	-0.0005	0.0026	0.0017	0.0017(100.0)	0.0017(100.0)	0.0018(94.7)
3	1.5	50	10	10	-0.0014	-0.0013	-0.0013	0.0008	0.0019	0.0019(100.0)	0.0019(100.0)	0.0020(91.4)
3	1.5	100	0	0	-0.0006	-0.0005	-0.0005	0.0024	0.0008	0.0008(100.0)	0.0008(100.0)	0.0008(96.7)
3	1.5	100	0	10	-0.0015	-0.0014	-0.0014	-0.0009	0.0008	0.0008(100.0)	0.0008(100.0)	0.0009(91.9)
3	1.5	100	0	20	-0.0009	-0.0008	-0.0008	-0.0010	0.0009	0.0009(100.0)	0.0009(100.0)	0.0011(87.8)
3	1.5	100	0	30	-0.0017	-0.0017	-0.0017	-0.0021	0.0010	0.0010(100.0)	0.0010(100.0)	0.0012(84.7)
3	1.5	100	10	0	-0.0004	-0.0003	-0.0003	0.0032	0.0008	0.0008(100.0)	0.0008(100.0)	0.0008(96.5)
3	1.5	100	10	10	-0.0011	-0.0010	-0.0010	0.0002	0.0009	0.0009(100.0)	0.0009(100.0)	0.0009(94.6)
3	1.5	100	10	20	-0.0021	-0.0020	-0.0020	-0.0013	0.0009	0.0009(100.0)	0.0009(100.0)	0.0010(91.8)
3	1.5	100	10	30	-0.0023	-0.0022	-0.0022	-0.0021	0.0010	0.0010(100.0)	0.0010(100.0)	0.0012(87.5)
3	1.5	100	20	0	-0.0010	-0.0009	-0.0009	0.0029	0.0008	0.0008(100.0)	0.0008(100.0)	0.0008(97.4)
3	1.5	100	20	10	-0.0005	-0.0004	-0.0004	0.0012	0.0008	0.0008(100.0)	0.0008(100.0)	0.0009(93.5)
3	1.5	100	20	20	-0.0013	-0.0013	-0.0013	-0.0003	0.0009	0.0009(100.0)	0.0009(100.0)	0.0010(92.3)
3	1.5	100	20	30	-0.0013	-0.0013	-0.0013	-0.0007	0.0010	0.0010(100.0)	0.0010(100.0)	0.0011(89.8)
3	1.5	100	30	0	-0.0010	-0.0009	-0.0009	0.0032	0.0008	0.0008(100.0)	0.0008(100.0)	0.0008(97.8)
3	1.5	100	30	10	-0.0013	-0.0012	-0.0012	0.0004	0.0009	0.0009(100.0)	0.0009(100.0)	0.0009(94.6)
3	1.5	100	30	20	-0.0005	-0.0004	-0.0004	0.0009	0.0009	0.0009(100.0)	0.0009(100.0)	0.0010(92.0)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
3	1.5	100	30	30	-0.0011	-0.0010	-0.0010	0.0000	0.0010	0.0010(100.0)	0.0010(100.0)	0.0011(92.0)
3	3	10	0	0	-0.0017	-0.0001	-0.0004	0.0068	0.0045	0.0045(99.8)	0.0045(99.8)	0.0046(97.1)
3	3	10	0	2	-0.0046	-0.0037	-0.0039	-0.0053	0.0050	0.0050(99.9)	0.0050(99.9)	0.0053(94.2)
3	3	10	2	0	-0.0023	-0.0006	-0.0010	0.0112	0.0045	0.0045(99.8)	0.0045(99.8)	0.0048(94.3)
3	3	10	2	2	-0.0033	-0.0023	-0.0024	0.0020	0.0052	0.0052(99.9)	0.0052(99.9)	0.0054(96.2)
3	3	20	0	0	-0.0015	-0.0006	-0.0007	0.0040	0.0023	0.0023(100.0)	0.0023(100.0)	0.0023(97.7)
3	3	20	0	2	-0.0016	-0.0009	-0.0009	-0.0008	0.0024	0.0024(99.9)	0.0024(99.9)	0.0025(97.0)
3	3	20	0	5	-0.0021	-0.0016	-0.0017	-0.004	0.0026	0.0026(100.0)	0.0026(100.0)	0.0028(93.0)
3	3	20	2	0	-0.0006	0.0003	0.0002	0.0070	0.0022	0.0022(99.9)	0.0022(99.9)	0.0023(96.1)
3	3	20	2	2	-0.0007	-0.0001	-0.0001	0.0025	0.0024	0.0024(99.9)	0.0024(99.9)	0.0025(97.0)
3	3	20	2	5	-0.0019	-0.0014	-0.0015	-0.0011	0.0026	0.0026(99.9)	0.0026(99.9)	0.0028(94.4)
3	3	20	5	0	-0.0011	-0.0002	-0.0003	0.0081	0.0023	0.0023(99.8)	0.0023(99.8)	0.0024(94.2)
3	3	20	5	2	-0.0017	-0.0010	-0.0010	0.0031	0.0024	0.0024(99.9)	0.0024(99.9)	0.0025(96.5)
3	3	20	5	5	-0.0011	-0.0006	-0.0006	0.0011	0.0025	0.0025(99.9)	0.0025(99.9)	0.0026(95.1)
3	3	50	0	0	-0.0001	0.0003	0.0003	0.0029	0.0009	0.0009(99.9)	0.0009(99.9)	0.0009(98.2)
3	3	50	0	5	-0.0004	-0.0001	-0.0001	-0.0001	0.0009	0.0009(100.0)	0.0009(100.0)	0.0010(95.3)
3	3	50	0	10	-0.0005	-0.0003	-0.0003	-0.0014	0.0010	0.0010(100.0)	0.0010(100.0)	0.0010(94.0)
3	3	50	5	0	-0.0008	-0.0004	-0.0004	0.0032	0.0009	0.0009(100.0)	0.0009(100.0)	0.0009(98.0)
3	3	50	5	5	-0.0011	-0.0008	-0.0008	0.0002	0.0009	0.0009(100.0)	0.0009(100.0)	0.0009(97.6)
3	3	50	5	10	0.0003	0.0005	0.0005	0.0009	0.0010	0.0010(99.9)	0.0010(99.9)	0.0010(94.6)
3	3	50	10	0	-0.0008	-0.0004	-0.0004	0.0038	0.0009	0.0009(99.9)	0.0009(99.9)	0.0009(96.2)
3	3	50	10	5	-0.0001	0.0002	0.0002	0.0017	0.0010	0.0010(99.9)	0.0010(99.9)	0.0010(96.8)
3	3	50	10	10	0.0004	0.0006	0.00060	0.0013	0.0010	0.0010(99.9)	0.0010(99.9)	0.0011(94.3)
3	3	100	0	0	-0.0007	-0.0004	-0.0005	0.0011	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(99.3)
3	3	100	0	10	-0.0001	0.0000	0.0000	-0.0001	0.0005	0.0005(100.0)	0.0005(100.0)	0.0005(96.3)
3	3	100	0	20	-0.0005	-0.0004	-0.0004	-0.0011	0.0005	0.0005(100.0)	0.0005(100.0)	0.0005(92.2)
3	3	100	0	30	-0.0004	-0.0003	-0.0003	-0.0016	0.0005	0.0005(100.0)	0.0005(100.0)	0.0006(88.9)
3	3	100	10	0	-0.0007	-0.0004	-0.0005	0.0017	0.0005	0.0005(100.1)	0.0005(100.0)	0.0005(98.7)
3	3	100	10	10	-0.0001	0.0000	0.0000	0.0005	0.0005	0.0005(100.0)	0.0005(100.0)	0.0005(96.1)
3	3	100	10	20	-0.0003	-0.0002	-0.0002	-0.0001	0.0005	0.0005(100.0)	0.0005(100.0)	0.0005(94.8)
3	3	100	10	30	-0.0004	-0.0003	-0.0003	-0.0004	0.0005	0.0005(100.0)	0.0005(100.0)	0.0006(93.0)
3	3	100	20	0	-0.0001	0.0002	0.0001	0.0026	0.0005	0.0005(99.9)	0.0005(99.9)	0.0005(97.1)
3	3	100	20	10	-0.0004	-0.0003	-0.0003	0.0006	0.0005	0.0005(100.0)	0.0005(100.0)	0.0005(96.8)
3	3	100	20	20	-0.0001	0.0000	0.0000	0.0005	0.0005	0.0005(100.0)	0.0005(100.0)	0.0006(94.2)
3	3	100	20	30	-0.0010	-0.0009	-0.0009	-0.0007	0.0005	0.0005(100.0)	0.0005(100.0)	0.0006(94.4)
3	3	100	30	0	-0.0002	0.0000	0.0000	0.0028	0.0004	0.0004(100.0)	0.0004(100.0)	0.0005(95.4)
3	3	100	30	10	-0.0002	-0.0001	-0.0001	0.0009	0.0005	0.0005(100.0)	0.0005(100.0)	0.0005(96.5)
3	3	100	30	20	0.0000	0.0001	0.0001	0.0007	0.0005	0.0005(100.0)	0.0005(100.0)	0.0005(94.1)
3	3	100	30	30	-0.0007	-0.0006	-0.0006	-0.0001	0.0006	0.0006(100.0)	0.0006(100.0)	0.0006(95.6)
3	5	10	0	0	0.0001	0.0021	0.0017	0.0060	0.0030	0.0031(99.3)	0.0031(99.4)	0.0031(97.7)
3	5	10	0	2	-0.0008	0.0005	0.0003	-0.003	0.0033	0.0033(99.8)	0.0033(99.8)	0.0034(97.3)
3	5	10	2	0	-0.0010	0.0009	0.0005	0.0093	0.0031	0.0031(99.6)	0.0031(99.7)	0.0033(93.9)
3	5	10	2	2	-0.0017	-0.0005	-0.0007	0.0010	0.0034	0.0034(99.8)	0.0034(99.8)	0.0034(98.1)
3	5	20	0	0	-0.0005	0.0006	0.0005	0.0035	0.0015	0.0015(99.8)	0.0015(99.8)	0.0015(98.3)
3	5	20	0	2	-0.0001	0.0008	0.0007	0.0001	0.0015	0.0016(99.8)	0.0016(99.8)	0.0016(98.4)
3	5	20	0	5	-0.0009	-0.0003	-0.0004	-0.0032	0.0016	0.0016(99.9)	0.0016(99.9)	0.0017(94.8)
3	5	20	2	0	-0.0007	0.0004	0.0003	0.0051	0.0015	0.0015(99.7)	0.0015(99.8)	0.0016(96.5)
3	5	20	2	2	-0.0009	-0.0001	-0.0001	0.0012	0.0016	0.0016(99.8)	0.0016(99.8)	0.0016(98.2)

					Bias				Mean Squared Error			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
3	5	20	2	5	-0.0014	-0.0008	-0.0008	-0.0014	0.0018	0.0018(99.9)	0.0018(100.0)	0.0018(95.7)
3	5	20	5	0	-0.0005	0.0006	0.0004	0.0068	0.0016	0.0016(99.8)	0.0016(99.9)	0.0017(93.0)
3	5	20	5	2	0.0004	0.0012	0.0011	0.0038	0.0016	0.0016(99.7)	0.0016(99.8)	0.0017(97.3)
3	5	20	5	5	-0.0002	0.0003	0.0003	0.0012	0.0017	0.0017(99.9)	0.0017(99.9)	0.0017(96.2)
3	5	50	0	0	0.0002	0.0007	0.0007	0.0024	0.0006	0.0006(99.8)	0.0006(99.8)	0.0006(98.6)
3	5	50	0	5	0.0001	0.0005	0.0004	-0.0001	0.0006	0.0006(99.8)	0.0006(99.9)	0.0006(97.9)
3	5	50	0	10	-0.0001	0.0002	0.0002	-0.0011	0.0006	0.0006(99.9)	0.0006(99.9)	0.0007(96.7)
3	5	50	5	0	-0.0006	-0.0001	-0.0001	0.0026	0.0006	0.0006(100.0)	0.0006(100.0)	0.0006(97.0)
3	5	50	5	5	-0.0002	0.0001	0.0001	0.0007	0.0006	0.0006(99.9)	0.0006(99.9)	0.0006(97.7)
3	5	50	5	10	-0.0011	-0.0008	-0.0008	-0.001	0.0007	0.0007(100.1)	0.0007(100.1)	0.0007(96.7)
3	5	50	10	0	0.0000	0.0005	0.0005	0.0038	0.0006	0.0006(99.9)	0.0006(99.9)	0.0006(93.7)
3	5	50	10	5	-0.0001	0.0003	0.0003	0.0012	0.0007	0.0007(99.9)	0.0007(99.9)	0.0007(97.1)
3	5	50	10	10	0.0000	0.0003	0.0003	0.0007	0.0007	0.0007(99.9)	0.0007(99.9)	0.0007(96.0)
3	5	100	0	0	0.0002	0.0004	0.0004	0.0015	0.0003	0.0003(99.9)	0.0003(99.9)	0.0003(98.9)
3	5	100	0	10	0.0002	0.0004	0.0004	0.0001	0.0003	0.0003(99.9)	0.0003(99.9)	0.0003(98.1)
3	5	100	0	20	0.0001	0.0003	0.0003	-0.0005	0.0003	0.0003(99.9)	0.0003(99.9)	0.0003(95.8)
3	5	100	0	30	-0.0001	0.0001	0.0001	-0.0010	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(93.1)
3	5	100	10	0	-0.0003	0.0000	0.0000	0.0016	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(97.5)
3	5	100	10	10	-0.0005	-0.0003	-0.0004	-0.0001	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(98.6)
3	5	100	10	20	-0.0001	0.0001	0.0001	0.0000	0.0003	0.0003(99.9)	0.0003(99.9)	0.0003(97.1)
3	5	100	10	30	0.0000	0.0002	0.0002	0.0000	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(94.3)
3	5	100	20	0	-0.0004	-0.0002	-0.0002	0.0018	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(95.7)
3	5	100	20	10	-0.0002	0.0000	0.0000	0.0004	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(97.4)
3	5	100	20	20	-0.0006	-0.0004	-0.0004	-0.0003	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(95.8)
3	5	100	20	30	0.0001	0.0002	0.0002	0.0001	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(94.2)
3	5	100	30	0	-0.0001	0.0001	0.0001	0.0024	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(93.5)
3	5	100	30	10	0.0003	0.0004	0.0004	0.0011	0.0003	0.0003(99.9)	0.0003(99.9)	0.0003(96.2)
3	5	100	30	20	-0.0004	-0.0003	-0.0003	0.0001	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(94.7)
3	5	100	30	30	0.0002	0.0003	0.0003	0.0005	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(94.5)
5	0.5	10	0	0	-0.0160	-0.0169	-0.0165	0.0057	0.0079	0.0079(99.8)	0.0079(99.9)	0.0100(79.2)
5	0.5	10	0	2	-0.0209	-0.0211	-0.0211	-0.0076	0.0116	0.0116(100.0)	0.0116(100.0)	0.0153(75.7)
5	0.5	10	2	0	-0.0147	-0.0157	-0.0152	0.0115	0.0080	0.0080(99.8)	0.0080(99.9)	0.0097(82.5)
5	0.5	10	2	2	-0.0225	-0.0227	-0.0227	-0.0027	0.0116	0.0116(100.0)	0.0116(99.9)	0.0141(82.6)
5	0.5	20	0	0	-0.0068	-0.0075	-0.0073	0.0043	0.0040	0.0040(99.9)	0.0040(99.9)	0.0050(78.9)
5	0.5	20	0	2	-0.0101	-0.0103	-0.0103	-0.0019	0.0048	0.0048(100.0)	0.0048(100.0)	0.0062(77.0)
5	0.5	20	0	5	-0.0107	-0.0108	-0.0108	-0.0054	0.0060	0.0060(100.0)	0.0060(100.0)	0.0083(72.2)
5	0.5	20	2	0	-0.0083	-0.0089	-0.0087	0.0044	0.0039	0.0039(99.8)	0.0039(99.9)	0.0049(80.6)
5	0.5	20	2	2	-0.0087	-0.0089	-0.0089	0.0006	0.0046	0.0046(100.0)	0.0046(100.0)	0.0059(77.7)
5	0.5	20	2	5	-0.0115	-0.0116	-0.0116	-0.0032	0.0062	0.0062(100.0)	0.0062(100.0)	0.0081(76.0)
5	0.5	20	5	0	-0.0094	-0.0100	-0.0098	0.0046	0.0039	0.0040(99.8)	0.0040(99.9)	0.0046(85.2)
5	0.5	20	5	2	-0.0103	-0.0105	-0.0105	0.0012	0.0047	0.0047(100.0)	0.0047(100.0)	0.0057(82.0)
5	0.5	20	5	5	-0.0122	-0.0122	-0.0122	-0.0018	0.0064	0.0064(100.0)	0.0064(100.0)	0.0077(83.1)
5	0.5	50	0	0	-0.0035	-0.0038	-0.0037	0.0024	0.0015	0.0015(99.9)	0.0015(100.0)	0.0019(79.6)
5	0.5	50	0	5	-0.0045	-0.0046	-0.0046	-0.0014	0.0018	0.0018(100.0)	0.0018(100.0)	0.0025(74.4)
5	0.5	50	0	10	-0.0045	-0.0045	-0.0045	-0.0021	0.0022	0.0022(100.0)	0.0022(100.0)	0.0031(71.9)
5	0.5	50	5	0	-0.0028	-0.0031	-0.0030	0.0039	0.0016	0.0016(99.9)	0.0016(100.0)	0.0020(78.3)
5	0.5	50	5	5	-0.0037	-0.0038	-0.0038	0.0001	0.0018	0.0018(100.0)	0.0018(100.0)	0.0024(76.0)
5	0.5	50	5	10	-0.0056	-0.0056	-0.0056	-0.0025	0.0021	0.0021(100.0)	0.0021(100.0)	0.0029(73.2)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
5	0.5	50	10	0	-0.0033	-0.0036	-0.0036	0.0035	0.0015	0.0015(99.9)	0.0015(99.9)	0.0018(82.7)
5	0.5	50	10	5	-0.0041	-0.0042	-0.0042	0.0006	0.0018	0.0018(100.0)	0.0018(100.0)	0.0023(78.8)
5	0.5	50	10	10	-0.0046	-0.0046	-0.0046	-0.0006	0.0022	0.0022(100.0)	0.0022(100.0)	0.0029(77.5)
5	0.5	100	0	0	-0.0012	-0.0014	-0.0014	0.0017	0.0008	0.0008(100.0)	0.0008(100.0)	0.0010(77.2)
5	0.5	100	0	10	-0.0025	-0.0026	-0.0026	-0.0012	0.0009	0.0009(100.0)	0.0009(100.0)	0.0012(72.4)
5	0.5	100	0	20	-0.0028	-0.0028	-0.0028	-0.0014	0.0011	0.0011(100.0)	0.0011(100.0)	0.0016(71.1)
5	0.5	100	0	30	-0.0015	-0.0015	-0.0015	-0.0007	0.0013	0.0013(100.0)	0.0013(100.0)	0.0019(69.8)
5	0.5	100	10	0	-0.0020	-0.0022	-0.0022	0.0014	0.0008	0.0008(99.9)	0.0008(99.9)	0.0010(80.1)
5	0.5	100	10	10	-0.0024	-0.0024	-0.0024	-0.0004	0.0009	0.0009(100.0)	0.0009(100.0)	0.0012(73.7)
5	0.5	100	10	20	-0.0022	-0.0023	-0.0023	-0.0011	0.0011	0.0011(100.0)	0.0011(100.0)	0.0015(72.7)
5	0.5	100	10	30	-0.0024	-0.0024	-0.0024	-0.0007	0.0013	0.0013(100.0)	0.0013(100.0)	0.0018(72.9)
5	0.5	100	20	0	-0.0015	-0.0016	-0.0016	0.0022	0.0007	0.0007(100.0)	0.0007(100.0)	0.0009(84.3)
5	0.5	100	20	10	-0.0019	-0.0019	-0.0019	0.0004	0.0009	0.0009(100.0)	0.0009(100.0)	0.0011(80.6)
5	0.5	100	20	20	-0.0019	-0.0019	-0.0019	0.0002	0.0011	0.0011(100.0)	0.0011(100.0)	0.0014(78.2)
5	0.5	100	20	30	-0.0027	-0.0028	-0.0028	-0.0006	0.0013	0.0013(100.0)	0.0013(100.0)	0.0017(76.1)
5	0.5	100	30	0	-0.0011	-0.0013	-0.0013	0.0031	0.0008	0.0008(100.0)	0.0008(100.0)	0.0009(85.5)
5	0.5	100	30	10	-0.0017	-0.0018	-0.0018	0.0009	0.0009	0.0009(100.0)	0.0009(100.0)	0.0011(83.3)
5	0.5	100	30	20	-0.0018	-0.0018	-0.0018	0.0006	0.0011	0.0011(100.0)	0.0011(100.0)	0.0013(80.0)
5	0.5	100	30	30	-0.0014	-0.0014	-0.0014	0.0005	0.0013	0.0013(100.0)	0.0013(100.0)	0.0016(81.6)
5	1.5	10	0	0	-0.0057	-0.0053	-0.0054	-0.0008	0.0028	0.0028(100.1)	0.0028(100.1)	0.0030(91.0)
5	1.5	10	0	2	-0.0060	-0.0058	-0.0058	-0.0078	0.0035	0.0035(100.0)	0.0035(100.0)	0.0042(85.0)
5	1.5	10	2	0	-0.0043	-0.0038	-0.0039	0.0061	0.0028	0.0028(100.0)	0.0028(100.0)	0.0029(94.9)
5	1.5	10	2	2	-0.0056	-0.0053	-0.0054	-0.0005	0.0036	0.0036(100.0)	0.0036(100.0)	0.0038(93.0)
5	1.5	20	0	0	-0.0023	-0.0021	-0.0021	0.0007	0.0015	0.0015(100.0)	0.0015(100.0)	0.0016(91.4)
5	1.5	20	0	2	-0.0028	-0.0026	-0.0026	-0.003	0.0015	0.0015(100.0)	0.0015(100.0)	0.0018(86.9)
5	1.5	20	0	5	-0.0036	-0.0035	-0.0035	-0.0061	0.0018	0.0018(100.0)	0.0018(100.0)	0.0021(82.9)
5	1.5	20	2	0	-0.0016	-0.0014	-0.0014	0.0034	0.0014	0.0014(100.0)	0.0014(100.0)	0.0015(92.8)
5	1.5	20	2	2	-0.0024	-0.0022	-0.0022	-0.0001	0.0015	0.0015(100.0)	0.0015(100.0)	0.0017(90.9)
5	1.5	20	2	5	-0.0034	-0.0033	-0.0033	-0.0027	0.0017	0.0017(100.0)	0.0017(100.0)	0.0020(85.9)
5	1.5	20	5	0	-0.0018	-0.0015	-0.0015	0.0051	0.0014	0.0014(100.0)	0.0014(100.0)	0.0015(94.0)
5	1.5	20	5	2	-0.0031	-0.0029	-0.0029	0.0013	0.0015	0.0015(100.1)	0.0015(100.1)	0.0017(93.4)
5	1.5	20	5	5	-0.0033	-0.0031	-0.0031	-0.0004	0.0018	0.0018(100.0)	0.0018(100.0)	0.0019(90.5)
5	1.5	50	0	0	-0.0012	-0.0011	-0.0011	0.0004	0.0006	0.0006(100.0)	0.0006(100.0)	0.0006(92.0)
5	1.5	50	0	5	-0.0009	-0.0008	-0.0008	-0.0013	0.0006	0.0006(100.0)	0.0006(100.0)	0.0007(86.7)
5	1.5	50	0	10	-0.0015	-0.0015	-0.0015	-0.0028	0.0007	0.0007(100.0)	0.0007(100.0)	0.0008(81.6)
5	1.5	50	5	0	-0.0007	-0.0006	-0.0006	0.0020	0.0005	0.0005(100.0)	0.0005(100.0)	0.0006(94.4)
5	1.5	50	5	5	-0.0008	-0.0007	-0.0007	0.0002	0.0006	0.0006(100.0)	0.0006(100.0)	0.0007(89.3)
5	1.5	50	5	10	-0.0010	-0.0010	-0.0010	-0.0005	0.0007	0.0007(100.0)	0.0007(100.0)	0.0008(86.4)
5	1.5	50	10	0	-0.0008	-0.0007	-0.0007	0.0024	0.0006	0.0006(100.0)	0.0006(100.0)	0.0006(95.3)
5	1.5	50	10	5	-0.0011	-0.0010	-0.0010	0.0005	0.0006	0.0006(100.0)	0.0006(100.0)	0.0007(92.3)
5	1.5	50	10	10	-0.0011	-0.0010	-0.0010	-0.0004	0.0007	0.0007(100.0)	0.0007(100.0)	0.0007(89.7)
5	1.5	100	0	0	-0.0008	-0.0007	-0.0007	0.0001	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(91.6)
5	1.5	100	0	10	-0.0004	-0.0004	-0.0004	-0.0008	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(86.6)
5	1.5	100	0	20	-0.0007	-0.0006	-0.0006	-0.0013	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(83.3)
5	1.5	100	0	30	-0.0012	-0.0012	-0.0012	-0.0022	0.0004	0.0004(100.0)	0.0004(100.0)	0.0005(78.9)
5	1.5	100	10	0	-0.0005	-0.0004	-0.0004	0.0010	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(94.9)
5	1.5	100	10	10	-0.0007	-0.0007	-0.0007	-0.0001	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(90.3)
5	1.5	100	10	20	-0.0002	-0.0002	-0.0002	0.0000	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(86.4)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
5	1.5	100	10	30	-0.0002	-0.0002	-0.0002	-0.0002	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(83.2)
5	1.5	100	20	0	-0.0009	-0.0009	-0.0009	0.0009	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(97.4)
5	1.5	100	20	10	-0.0005	-0.0005	-0.0005	0.0004	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(91.6)
5	1.5	100	20	20	-0.0003	-0.0002	-0.0002	0.0002	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(89.5)
5	1.5	100	20	30	-0.0002	-0.0002	-0.0002	0.0001	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(88.4)
5	1.5	100	30	0	-0.0007	-0.0006	-0.0006	0.0014	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(97.3)
5	1.5	100	30	10	-0.0007	-0.0006	-0.0006	0.0003	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(92.5)
5	1.5	100	30	20	-0.0004	-0.0004	-0.0004	0.0003	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(92.7)
5	1.5	100	30	30	-0.0007	-0.0007	-0.0007	-0.0002	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(92.3)
5	3	10	0	0	-0.0021	-0.0012	-0.0014	0.0005	0.0017	0.0017(99.9)	0.0017(100.0)	0.0017(97.4)
5	3	10	0	2	-0.0026	-0.0021	-0.0022	-0.0045	0.0019	0.0019(100.0)	0.0019(100.0)	0.0020(91.7)
5	3	10	2	0	-0.0016	-0.0006	-0.0008	0.0052	0.0016	0.0016(99.9)	0.0016(99.9)	0.0017(96.7)
5	3	10	2	2	-0.0016	-0.0010	-0.0011	0.0006	0.0019	0.0019(100.0)	0.0019(100.0)	0.0019(96.9)
5	3	20	0	0	-0.0015	-0.0010	-0.0010	0.0002	0.0008	0.0008(100.1)	0.0008(100.1)	0.0008(97.7)
5	3	20	0	2	-0.0009	-0.0005	-0.0005	-0.0014	0.0008	0.0008(100.0)	0.0008(100.0)	0.0009(94.6)
5	3	20	0	5	-0.0015	-0.0012	-0.0012	-0.0038	0.0010	0.0010(100.0)	0.0010(100.0)	0.0011(89.9)
5	3	20	2	0	-0.0009	-0.0004	-0.0005	0.0025	0.0008	0.0008(100.1)	0.0008(100.1)	0.0008(97.3)
5	3	20	2	2	-0.0008	-0.0004	-0.0004	0.0005	0.0008	0.0008(99.9)	0.0008(100.0)	0.0009(95.6)
5	3	20	2	5	-0.0020	-0.0017	-0.0017	-0.0020	0.0009	0.0009(100.1)	0.0009(100.1)	0.0010(94.2)
5	3	20	5	0	-0.0010	-0.0004	-0.0005	0.0036	0.0008	0.0008(100.0)	0.0008(100.0)	0.0009(96.5)
5	3	20	5	2	-0.0010	-0.0005	-0.0006	0.0016	0.0008	0.0008(100.0)	0.0008(100.0)	0.0009(95.9)
5	3	20	5	5	-0.0017	-0.0015	-0.0015	-0.0005	0.0009	0.0009(100.0)	0.0009(100.0)	0.0010(95.5)
5	3	50	0	0	-0.0002	0.0001	0.0000	0.0006	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(97.4)
5	3	50	0	5	-0.0008	-0.0006	-0.0006	-0.0012	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(94.3)
5	3	50	0	10	-0.0001	0.0001	0.0001	-0.0012	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(90.0)
5	3	50	5	0	-0.0004	-0.0002	-0.0002	0.0014	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(98.1)
5	3	50	5	5	-0.0006	-0.0004	-0.0004	0.0000	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(96.0)
5	3	50	5	10	-0.0002	-0.0001	-0.0001	-0.0001	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(93.7)
5	3	50	10	0	0.0000	0.0002	0.0002	0.0022	0.0003	0.0003(100.0)	0.0003(100.0)	0.0003(97.2)
5	3	50	10	5	-0.0007	-0.0005	-0.0005	0.0002	0.0003	0.0003(100.0)	0.0003(100.0)	0.0004(96.2)
5	3	50	10	10	-0.0004	-0.0003	-0.0003	0.0002	0.0004	0.0004(100.0)	0.0004(100.0)	0.0004(94.3)
5	3	100	0	0	-0.0004	-0.0002	-0.0002	0.0002	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(98.0)
5	3	100	0	10	-0.0003	-0.0002	-0.0002	-0.0007	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(93.7)
5	3	100	0	20	-0.0001	-0.0001	-0.0001	-0.0008	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(89.8)
5	3	100	0	30	-0.0003	-0.0002	-0.0002	-0.0013	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(84.5)
5	3	100	10	0	-0.0002	0.0000	0.0000	0.0009	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(98.4)
5	3	100	10	10	-0.0004	-0.0003	-0.0003	-0.0002	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(95.1)
5	3	100	10	20	-0.0001	0.0000	0.0000	0.0000	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(93.5)
5	3	100	10	30	-0.0005	-0.0004	-0.0004	-0.0005	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(91.0)
5	3	100	20	0	-0.0003	-0.0002	-0.0002	0.0010	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(98.5)
5	3	100	20	10	-0.0004	-0.0003	-0.0003	0.0001	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(96.3)
5	3	100	20	20	-0.0004	-0.0003	-0.0003	-0.0002	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(95.1)
5	3	100	20	30	-0.0004	-0.0004	-0.0004	-0.0003	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(93.5)
5	3	100	30	0	-0.0002	-0.0001	-0.0001	0.0013	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(97.1)
5	3	100	30	10	-0.0004	-0.0003	-0.0003	0.0003	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(95.6)
5	3	100	30	20	-0.0001	-0.0001	-0.0001	0.0003	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(94.6)
5	3	100	30	30	-0.0001	0.0000	0.0000	0.0002	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(94.9)
5	5	10	0	0	-0.0006	0.0006	0.0003	0.0014	0.0011	0.0011(99.6)	0.0011(99.7)	0.0011(99.2)

					Bias				**Mean Squared Error**			
α	θ	n	r	s	$\frac{B(\hat{\sigma})}{\sigma}$	$\frac{B(\tilde{\sigma}_{TS})}{\sigma}$	$\frac{B(\tilde{\sigma}_{LS})}{\sigma}$	$\frac{B(\sigma_{PCE})}{\sigma}$	$MS(\hat{\sigma})$	$\frac{MS(\tilde{\sigma}_{TS})}{\sigma^2}$(Eff)	$\frac{MS(\tilde{\sigma}_{LS})}{\sigma^2}$(Eff)	$\frac{MS(\sigma_{PCE})}{\sigma^2}$(Eff)
5	5	10	0	2	-0.0009	-0.0001	-0.0002	-0.0032	0.0012	0.0012(99.9)	0.0012(99.9)	0.0012(95.0)
5	5	10	2	0	-0.0008	0.0003	0.0001	0.0044	0.0011	0.0011(99.8)	0.0011(99.8)	0.0011(96.7)
5	5	10	2	2	-0.0011	-0.0004	-0.0005	0.0002	0.0012	0.0012(99.9)	0.0012(99.9)	0.0013(98.1)
5	5	20	0	0	-0.0006	0.0001	0.0000	0.0006	0.0005	0.0005(99.9)	0.0005(99.9)	0.0005(99.3)
5	5	20	0	2	-0.0001	0.0004	0.0003	-0.0010	0.0006	0.0006(99.8)	0.0006(99.8)	0.0006(97.5)
5	5	20	0	5	-0.0005	-0.0002	-0.0002	-0.0026	0.0006	0.0006(99.8)	0.0006(99.8)	0.0006(95.3)
5	5	20	2	0	-0.0003	0.0004	0.0003	0.0024	0.0006	0.0006(99.8)	0.0006(99.9)	0.0006(98.0)
5	5	20	2	2	-0.0009	-0.0004	-0.0005	-0.0001	0.0006	0.0006(99.9)	0.0006(99.9)	0.0006(98.9)
5	5	20	2	5	-0.0006	-0.0002	-0.0002	-0.0008	0.0006	0.0006(100.0)	0.0006(100.0)	0.0006(96.0)
5	5	20	5	0	-0.0005	0.0002	0.0001	0.0034	0.0006	0.0006(99.8)	0.0006(99.8)	0.0006(95.6)
5	5	20	5	2	0.0001	0.0006	0.0005	0.0019	0.0006	0.0006(99.8)	0.0006(99.8)	0.0006(97.7)
5	5	20	5	5	-0.0006	-0.0002	-0.0003	0.0001	0.0006	0.0006(100.0)	0.0006(100.0)	0.0007(96.6)
5	5	50	0	0	0.0002	0.0005	0.0005	0.0009	0.0002	0.0002(99.8)	0.0002(99.8)	0.0002(99.4)
5	5	50	0	5	-0.0005	-0.0003	-0.0003	-0.0010	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(97.1)
5	5	50	0	10	0.0003	0.0005	0.0004	-0.0007	0.0002	0.0002(99.9)	0.0002(99.9)	0.0002(92.9)
5	5	50	5	0	-0.0003	0.0000	0.0000	0.0011	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(98.9)
5	5	50	5	5	-0.0004	-0.0002	-0.0002	-0.0001	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(98.9)
5	5	50	5	10	-0.0002	0.0000	0.0000	-0.0002	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(95.8)
5	5	50	10	0	-0.0002	0.0000	0.0000	0.0016	0.0002	0.0002(100.0)	0.0002(100.0)	0.0002(97.5)
5	5	50	10	5	0.0001	0.0003	0.0003	0.0007	0.0002	0.0002(99.9)	0.0002(99.9)	0.0002(96.6)
5	5	50	10	10	-0.0003	-0.0001	-0.0001	0.0000	0.0002	0.0002(100.0)	0.0002(100.0)	0.0003(96.6)
5	5	100	0	0	0.0000	0.0001	0.0001	0.0004	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(99.3)
5	5	100	0	10	-0.0002	0.0000	0.0000	-0.0005	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(97.7)
5	5	100	0	20	-0.0003	-0.0002	-0.0002	-0.0010	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(93.6)
5	5	100	0	30	-0.0001	0.0000	0.0000	-0.0009	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(90.9)
5	5	100	10	0	-0.0001	0.0001	0.0001	0.0008	0.0001	0.0001(99.9)	0.0001(99.9)	0.0001(99.1)
5	5	100	10	10	-0.0004	-0.0002	-0.0002	-0.0002	0.0001	0.0001(100.1)	0.0001(100.1)	0.0001(98.1)
5	5	100	10	20	-0.0002	-0.0001	-0.0001	-0.0002	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(96.4)
5	5	100	10	30	-0.0003	-0.0002	-0.0002	-0.0004	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(92.4)
5	5	100	20	0	0.0001	0.0002	0.0002	0.0011	0.0001	0.0001(99.9)	0.0001(99.9)	0.0001(96.5)
5	5	100	20	10	0.0000	0.0002	0.0001	0.0003	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(97.7)
5	5	100	20	20	-0.0005	-0.0004	-0.0004	-0.0004	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(95.7)
5	5	100	20	30	-0.0003	-0.0002	-0.0002	-0.0003	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(95.0)
5	5	100	30	0	-0.0003	-0.0001	-0.0001	0.0010	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(96.2)
5	5	100	30	10	-0.0001	0.0000	0.0000	0.0004	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(96.6)
5	5	100	30	20	0.0000	0.0001	0.0001	0.0002	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(95.8)
5	5	100	30	30	-0.0003	-0.0002	-0.0002	-0.0002	0.0001	0.0001(100.0)	0.0001(100.0)	0.0001(94.6)

Here is the R code used in our analysis:

```
# THIS FUNCTION COMPUTES MLE iteratively USING N-R METHOD
MLE=function(x,n,r,s,alpha,teta,signew){
ni=0;
sigma=signew;
r1=r+1;ns=n-s;
repeat {
z=x/signew;
sigold=signew;
ni=ni+1;
y=z^alpha;
f=alpha*teta*y/z*exp(-y)*(1-exp(-y))^(teta-1);
F=(1-exp(-y))^teta;
G=y/(exp(y)-1);
Gdz=alpha*(G-exp(y)*G^2)/z;
Gd=-Gdz*z/signew;
H=G*F/(1-F);
Hdz=(f*G+F*Gdz+(f*F*G/(1-F)))/(1-F);
Hd=-Hdz*z/signew;
g=(n-r-s)+r*teta*G[r1]-s*teta*H[ns];
for (i in r1:ns) g=g-y[i]+(teta-1)*G[i];
g=-g*alpha/sigold;
gd=(n-r-s)+r*teta*(G[r1]+z[r1]*Gdz[r1])-s*teta*(H[ns]+z[ns]*Hdz[ns]);
for (i in r1:ns) gd=gd-(1+alpha)*y[i]+(teta-1)*(G[i]+z[i]*Gdz[i]);
gd=gd*alpha/sigold^2;
signew=sigold-g/gd;
if (is.nan(signew)) break;
el=abs(signew-sigold);
if (el<0.00001) break;
if (ni>200) break;
}
if (ni>200) cat("\n MLE does not converge:");
if (is.nan(signew)) cat("\n MLE does not exist:");
if (el<0.00001) MLE=signew;
#FINDING FISHER INFORMATION
FI=(n-r-s)+r*teta*(G[r1]+z[r1]*Gdz[r1])-s*teta*(H[ns]+z[ns]*Hdz[ns]);
for (i in r1:ns) FI=FI-(1+alpha)*y[i]+(teta-1)*(G[i]+z[i]*Gdz[i]);
FI=-alpha*FI/(sigma^2);
#FINDING STANDARD ERROR of MLE
SE=1/sqrt(FI);
return(c(MLE,SE));
}
# THIS FUNCTION COMPUTES PERCENTILE ESTIMATOR OF SCALE PARAMETER OF EWD
PCE=function(x,n,r,s,alpha,teta){
r1=r+1;ns=n-s;
w=array();
i=seq(1,n);
p=i/(n+1);
w=(-log(1-p^(1/teta)))^(1/alpha);
w[r1:ns]=w[r1:ns]/sum(w[r1:ns]^2);
```

```
PCE=sum(w[r1:ns]*x[r1:ns]);
return(PCE);
}
# THIS FUNCTION COMPUTES AMLE USING LEAST SQUARES METHOD
# IT ALSO COMPUTES STANDARD ERROR OF THE ESTIMATE
AMLELS=function(z,n,r,s,alpha,teta){
#COMPUTATION OF QUANTILES (ZIs)
regc=array();a=array();b=array();c=array();d=array();x=array();l=array();
r1=r+1;ns=n-s;
x=z[r1:ns];
#cat("\n Sample=",x);
for (i in 1:n){
  p=(i-0.3)/(n+0.4);
  regc=regana1(n,p,alpha,teta); #G1(Z)
  a[i]=regc[1];b[i]=regc[2];
  regc=regana2(n,p,alpha,teta); #H1(Z)
  c[i]=regc[1];d[i]=regc[2];
# cat("\n i=",i,round(a[i],4),round(b[i],4),round(c[i],4),round(d[i],4));
 }
delta=(n-r-s)+r*teta*a[r1]+(teta-1)*sum(a[r1:ns])-s*teta*c[ns];
l[r1:ns]=(1-(teta-1)*b[r1:ns])/delta;
l[r1]=(1-(r1*teta-1)*b[r1])/delta;
l[ns]=(1-(teta-1)*b[ns]+s*teta*d[ns])/delta;
#cat("\n l=",round(l[r1:ns],4));
AMLE=sum(l[r1:ns]*(x^alpha))^(1/alpha);
#FINDING FISHER INFORMATION
i=seq(1,n);
nu=-log(1-(i/(n+1))^(1/teta));
FI=(n-r-s)+r*teta*(a[r1]+(1+alpha)*b[r1]*nu[r1])-(1+alpha)*sum(nu[r1:ns]);
FI=FI+(teta-1)*(sum(a[r1:ns])+(1+alpha)*sum(b[r1:ns]*nu[r1:ns]));
FI=FI-s*teta*(c[ns]+(1+alpha)*d[ns]*nu[ns]);
FI=-alpha*FI/AMLE^2;
#Finding STANDARD ERROR of AMLE by inverting FI
SE=1/sqrt(FI);
return(c(AMLE,SE));
}
regana1=function(n,p0,alpha,teta){
i=seq(1:11);
p=p0+(i-6)/(10*n+4);
#zi=(-log(1-p^(1/teta)))^alpha;
zi=-log(1-p^(1/teta));
G1=zi/(exp(zi)-1);
slope=cov(zi,G1)/var(zi);
intercept=mean(G1)-slope*mean(zi);
return(c(intercept,slope));
}
regana2=function(n,p0,alpha,teta){
i=seq(1:11);
p=p0+(i-6)/(10*n+4);
#zi=(-log(1-p^(1/teta)))^alpha;
```

```
zi=-log(1-p^(1/teta));
G1=zi/(exp(zi)-1);
F1=(1-exp(-zi))^teta;
H1=G1*F1/(1-F1);
slope=cov(zi,H1)/var(zi);
intercept=mean(H1)-slope*mean(zi);
return(c(intercept,slope));
}
# THIS FUNCTION COMPUTES AMLE USING TAYLOR SERIES METHOD
# IT ALSO COMPUTES STANDARD ERROR OF THE ESTIMATE
AMLETS=function(z,n,r,s,alpha,teta){
a=array();b=array();c=array();d=array();x=array();l=array();
#COMPUTATION OF QUANTILES (ZIs)
r1=r+1;ns=n-s;
i=seq(1,n);
p=(i-0.3)/(n+0.4);zi=-log(1.0-p^(1.0/teta));
#COMPUTATION OF LINEAR APPROXIMATION OF G1(Z)
G1=zi/(exp(zi)-1);
G1d=(G1-exp(zi)*G1^2)/zi;
b=G1d;a=G1-b*zi;
#COMPUTATION OF LINEAR APPROXIMATION OF H1(Z)
F1=(1-exp(-zi))^teta;
f1=teta*exp(-zi)*(1-exp(-zi))^(teta-1); #derivative of F1
H1=G1*F1/(1-F1);
H1d=G1d*F1/(1-F1)+G1*f1/(1-F1)^2;
d=H1d;c=H1-d*zi;
#FINDING LINER COEFFICIENT (l) S
delta=(n-r-s)+r*teta*a[r1]+(teta-1)*sum(a[r1:ns])-s*teta*c[ns];
l[r1:ns]=(1-(teta-1)*b[r1:ns])/delta;
l[r1]=(1-(r1*teta-1)*b[r1])/delta;
l[ns]=(1-(teta-1)*b[ns]+s*teta*d[ns])/delta;
x=z[r1:ns];
AMLE=sum(l[r1:ns]*x^alpha)^(1/alpha);
#FINDING FISHER INFORMATION
i=seq(1,n);
nu=-log(1-(i/(n+1))^(1/teta));
FI=(n-r-s)+r*teta*(a[r1]+(1+alpha)*b[r1]*nu[r1])-(1+alpha)*sum(nu[r1:ns]);
FI=FI+(teta-1)*(sum(a[r1:ns])+(1+alpha)*sum(b[r1:ns]*nu[r1:ns]));
FI=FI-s*teta*(c[ns]+(1+alpha)*d[ns]*nu[ns]);
FI=-alpha*FI/AMLE^2;
#Finding STANDARD ERROR of AMLE by inverting FI
SE=1/sqrt(FI);
return(c(AMLE,SE));
}
#MAIN PROGRAM
#THIS PROGRAM COMPUTES MLE, AMLEs (both Least squares method and Taylor Series method)
# THIS PROGRAM ALSO COMPUTES PCE
# Illustration 1
#data=read.csv("carbon data.csv",header=T);
#alpha=2.4;teta=1.3;
```

```
#rs=c(0,5,10,20,30);
#sink("ILLUST1.txt");
# Illustartion 2
data=read.csv("ball_bearings data.csv",header=T);
alpha=1.5;teta=2.0;
rs=c(0,2,5);
sink("ILLUST2.txt");
y=data[,1];
n=length(y);
for (r in rs)
for (s in rs){
# Computes AMLE and it's standard error based on Least squares method
AMLE1=round(AMLELS(y,n,r,s,alpha,teta),3);
# Computes AMLE and it's standard error based on Taylor Series method
AMLE2=round(AMLETS(y,n,r,s,alpha,teta),3);
# Computes PCE
pce=round(PCE(y,n,r,s,alpha,teta),3);
# Computes MLE and it's standard error using N-R method. It uses AMLE as inititial esti
mle=round(MLE(y,n,r,s,alpha,teta,AMLE1[1]),3);
cat("\n",n,format(r,width=2),format(s,width=2)," ",format(mle[1],nsmall=3),"(",format(
");
cat(format(AMLE1[1],nsmall=3),'(',format(AMLE1[2],nsmall=3),')    ');
cat(format(AMLE2[1],nsmall=3),'(',format(AMLE2[2],nsmall=3),')
',format(pce,nsmall=3));}
sink()
cat("\n\n END OF COMPUTATION\n");
```