

Intuitionistic L-Fuzzy Soft Ideal Over Semirings

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Abstract An intuitionistic L- fuzzy soft ideal is introduced over the intuitionistic L-fuzzy soft semirings. Describe the property of the homomorphic fuction if an intuitionistic L-fuzzy soft semiring and an intuitionistic L- fuzzy soft ideal defined. Also, its application demonstrates a medical diagnosis using fuzzy structures.

Keywords Intuitionistic fuzzy set, Intuitionistic L-fuzzy set, Intuitionistic L-fuzzy soft semi-ideal, Intuitionistic L-fuzzy soft ideal.

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1. Introduction

The theory of intuitionistic fuzzy sets plays an important role in modern mathematics. A semiring is a well-known universal algebraic structure within a ring. One major problem shared by these theories is their incompatibility with parameterization tools. Soft set theory, introduced by Molodtsov in (1999) [8]. Provides a way to address this issue. Researchers worldwide are working with soft sets, including fuzzy soft sets and L- fuzzy soft sets. The concept of intuitionistic fuzzy sets has significant applications in modern mathematics. The idea of Intuitionistic L-fuzzy sets (ILFS) was introduced by Atanassov in (1986) [1] as a generalization of Zadeh's fuzzy sets (1965) [15]. Later, Maji P. K., Biswas R., and Roy A. R. (2001) [5, 6, 7] applied the concept of intuitionistic fuzzy soft sets to groups. The concept of fuzzy soft rings was introduced by Pazar Varol B. Intuitionistic fuzzy soft semigroups, based on the notion of intuitionistic fuzzy soft sets, were also investigated. Furthermore, Zhou et al. introduced the notion of intuitionistic fuzzy soft ideals over semigroups and studied their algebraic properties. Motivated by these studies, our work is the first to formulate and analyze intuitionistic L-fuzzy soft ideals in the context of semirings, thus providing a new algebraic structure and examining its basic properties within this framework.

2. Preliminaries

In this section, for the sake of completeness, we first give some useful definitions.

Definition 2.1. [1] An *intuitionistic fuzzy set* A in a nonempty set X can be defined as follows: $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X \}$, where $\mu_A : X \rightarrow [0, 1]$ and $\gamma_A : X \rightarrow [0, 1]$ satisfy the property $0 \leq \mu_A(x) +$

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$\gamma_A(x) \leq 1, \forall x \in X$. The values $\mu_A(x)$ and $\gamma_A(x)$ denote the degree of membership and non-membership of x to A , respectively.

In general, for all $\epsilon \in A \subseteq E$, $\tilde{F}(\epsilon)$ is an intuitionistic fuzzy set on U , which is called the *intuitionistic fuzzy set of parameter ϵ* . The intuitionistic value $\langle \mu_{\tilde{F}(\epsilon)}(x), \gamma_{\tilde{F}(\epsilon)}(x) \rangle$ denotes the degree of $x \in U$ corresponding to the parameter ϵ .

$\tilde{F}(\epsilon)$ can be written as:

$$\tilde{F}(\epsilon) = \{ \langle x, \mu_{\tilde{F}(\epsilon)}(x), \gamma_{\tilde{F}(\epsilon)}(x) \rangle \mid x \in U \}.$$

If $\forall x \in U, \mu_{\tilde{F}(\epsilon)}(x) + \gamma_{\tilde{F}(\epsilon)}(x) = 1$, then $\tilde{F}(\epsilon)$ degenerates into a fuzzy set. If $\forall x \in U$ and $\forall \epsilon \in A \subseteq E, \mu_{\tilde{F}(\epsilon)}(x) + \gamma_{\tilde{F}(\epsilon)}(x) = 1$, then the intuitionistic fuzzy soft set $(\tilde{F}, A)$ degenerates into a fuzzy soft set.

Definition 2.2. [6] Let U be an initial universe, E be a set of parameters, $\text{IFS}(U)$ denote the intuitionistic fuzzy power set of U and $A \subset E$. A pair $(\tilde{F}, A)$ is called an *intuitionistic fuzzy soft set* over U , where F is a mapping given by: $F : A \rightarrow \text{IFS}(U)$. An intuitionistic fuzzy soft set is a parameterized family of intuitionistic fuzzy subsets of U . A fuzzy soft set is a special case of an intuitionistic fuzzy soft set, because when all the intuitionistic fuzzy subsets of U degenerate into fuzzy subsets, the corresponding intuitionistic fuzzy soft set degenerates into a fuzzy soft set.

Definition 2.3. [6] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe U . " $(\tilde{F}, A)$ AND $(\tilde{G}, B)$ " denoted by $(\tilde{F}, A) \wedge (\tilde{G}, B)$, is defined by:

$$(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B),$$

where

$$\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \cap \tilde{G}(\beta), \quad \forall (\alpha, \beta) \in A \times B.$$

Definition 2.4. [6] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe U . " $(\tilde{F}, A)$ OR $(\tilde{G}, B)$ ", denoted by $(\tilde{F}, A) \vee (\tilde{G}, B)$, is defined by:

$$(\tilde{F}, A) \vee (\tilde{G}, B) = (\tilde{H}, A \times B),$$

where

$$\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \cup \tilde{G}(\beta), \quad \forall (\alpha, \beta) \in A \times B.$$

Definition 2.5. [6] The *intersection* of two intuitionistic fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over a universe U is an intuitionistic fuzzy soft set denoted by $(\tilde{H}, C)$, where: $C = A \cap B$, and

$$\tilde{H}(\epsilon) = \begin{cases} \tilde{F}(\epsilon), & \text{if } \epsilon \in A - B, \\ \tilde{G}(\epsilon), & \text{if } \epsilon \in B - A \\ \tilde{F}(\epsilon) \wedge \tilde{G}(\epsilon), & \text{if } \epsilon \in A \cap B, \end{cases}$$

for all $\epsilon \in C$. This is denoted by:

$$(\tilde{H}, C) = (\tilde{F}, A) \cap (\tilde{G}, B).$$

Definition 2.6. [6] The *union* of two intuitionistic fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over a universe U is an intuitionistic fuzzy soft set denoted by $(\tilde{H}, C)$, where: $C = A \cup B$, and

$$\tilde{H}(\epsilon) = \begin{cases} \tilde{F}(\epsilon), & \text{if } \epsilon \in A - B, \\ \tilde{G}(\epsilon), & \text{if } \epsilon \in B - A \\ \tilde{F}(\epsilon) \vee \tilde{G}(\epsilon), & \text{if } \epsilon \in A \cup B, \end{cases}$$

for all $\epsilon \in C$. This is denoted by:

$$(\tilde{H}, C) = (\tilde{F}, A) \cup (\tilde{G}, B).$$

In contrast with the above definitions of intuitionistic fuzzy soft set union and intersection, we may sometimes adopt different definitions of union and intersection given as follows.

Definition 2.7. [6] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe U such that $A \cap B \neq \emptyset$. The *bi-union* of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\tilde{H}, C)$, where: $C = A \cap B$, and

$$\tilde{H}(\alpha) = \tilde{F}(\alpha) \cup \tilde{G}(\alpha), \quad \forall \alpha \in C.$$

This is denoted by:

$$(\tilde{H}, C) = (\tilde{F}, A) \sqcup (\tilde{G}, B).$$

Definition 2.8. [6] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe U such that $A \cap B \neq \emptyset$. The *bi-intersection* of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\tilde{H}, C)$, where: $C = A \cap B$, and

$$\tilde{H}(\alpha) = \tilde{F}(\alpha) \cap \tilde{G}(\alpha), \quad \forall \alpha \in C.$$

This is denoted by:

$$(\tilde{H}, C) = (\tilde{F}, A) \tilde{\cap} (\tilde{G}, B).$$

Definition 2.9. [6] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic soft sets over a universe U . The *product* of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is defined to be the intuitionistic soft set $(\tilde{F} \circ \tilde{G}, C)$, where: $C = A \cup B$, and for all $\epsilon \in C$ and $x \in U$:

$$\mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) = \begin{cases} \mu_{(\tilde{F})(\epsilon)}(x), & \text{if } \epsilon \in A - B, \\ \mu_{(\tilde{G})(\epsilon)}(x), & \text{if } \epsilon \in B - A, \\ \bigvee_{x=x_1 x_2} (\mu_{(\tilde{F})(\epsilon)}(x_1) \wedge \mu_{(\tilde{G})(\epsilon)}(x_2)), & \text{if } \epsilon \in A \cap B, \end{cases}$$

and

$$\gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) = \begin{cases} \gamma_{(\tilde{F})(\epsilon)}(x), & \text{if } \epsilon \in A - B, \\ \gamma_{(\tilde{G})(\epsilon)}(x), & \text{if } \epsilon \in B - A, \\ \bigwedge_{x=x_1 x_2} (\gamma_{(\tilde{F})(\epsilon)}(x_1) \vee \gamma_{(\tilde{G})(\epsilon)}(x_2)), & \text{if } \epsilon \in A \cap B. \end{cases}$$

This is denoted by:

$$(\tilde{F} \circ \tilde{G}, C) = (\tilde{F}, A) \circ (\tilde{G}, B).$$

Definition 2.10. [11] Let $\hat{\omega} = (\tilde{F}, A)$ be an intuitionistic L -fuzzy soft set over U and $A \subseteq E$. An (s, t) -level intuitionistic L -fuzzy soft set $L(\hat{\omega}; (s, t))$ of $\hat{\omega}$ is a crisp soft set, where:

$$L(\hat{\omega}; (s, t)) = (\tilde{F}_{(s,t)}, A) = \{x \in U : \mu_{(\tilde{F})(a)}(x) \geq s, \gamma_{(\tilde{F})(a)}(x) \leq t, \forall a \in A\},$$

with $s, t \in [0, 1]$. In this definition $s, t \in [0, 1]$ are also called threshold values in which $s \in [0, 1]$ can be regarded as a given least threshold on membership values and $t \in [0, 1]$ can be regarded as a given greatest threshold on non-membership values.

Definition 2.11. [10] Let $f : R \rightarrow S$ and $g : A \rightarrow B$ be two functions, where A and B are parameter sets for intuitionistic L -fuzzy soft sets R and S , respectively. Then the pair (f, g) is called an intuitionistic L -fuzzy soft function from R to S .

Definition 2.12. [10] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic L -fuzzy soft rings over R and S , respectively. Let $f : R \rightarrow S$ be a homomorphism of rings, and $g : A \rightarrow B$ be a mapping of sets. We say that $(f, g) : (\tilde{F}, A) \rightarrow (\tilde{G}, B)$ is an intuitionistic L -fuzzy soft homomorphism of intuitionistic L -fuzzy soft rings and define by $f(\tilde{F}, A) = (\tilde{G}, B)g$, if the following conditions are satisfied

$$f(\mu_{\tilde{F}(\epsilon)}(x)) = (\mu_{\tilde{G}(\epsilon)}(x))g, f(\gamma_{\tilde{F}(\epsilon)}(x)) = (\gamma_{\tilde{G}(\epsilon)}(x))g$$

Definition 2.13. [10] Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic L -fuzzy soft rings over R and S . Let (f, g) be an intuitionistic L -fuzzy soft function from R to S .

1. The *image* of $(\tilde{F}, A)$ under (f, g) , denoted by $(f, g)(\tilde{F}, A)$, is the intuitionistic L -fuzzy soft ring over S defined by:

$$(f, g)(\tilde{F}, A) = (f(\tilde{F}), g(A)),$$

where for all $k \in g(A)$ and $s \in S$:

$$f(\tilde{F})_k(s) = \begin{cases} \bigvee_{f(r)=s} \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(r), & \text{if } r \in f^{-1}(s), \\ 0, & \text{otherwise,} \end{cases}$$

$$f(\tilde{F})_k(s) = \begin{cases} \bigwedge_{f(r)=s} \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(r), & \text{if } r \in f^{-1}(s), \\ 1, & \text{otherwise.} \end{cases}$$

2. The *preimage* of $(\tilde{G}, B)$ under (f, g) , denoted by $(f, g)^{-1}(\tilde{G}, B)$, is the intuitionistic L -fuzzy soft ring over R defined by:

$$(f, g)^{-1}(\tilde{G}, B) = (f^{-1}(\tilde{G}), g^{-1}(B)),$$

where:

$$f^{-1}(\tilde{G})_{(a)}(r) = (\tilde{G})_{(g(a))}(f(r)), \quad \forall a \in g^{-1}(B), \forall r \in R.$$

If f and g are injective (respectively, surjective), then (f, g) is said to be injective (respectively, surjective).

Definition 2.14. [13] Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An *intuitionistic fuzzy point*, written as $x_{(\alpha, \beta)}$, is defined to be an intuitionistic fuzzy subset of R , given by:

$$x_{(\alpha, \beta)}(y) = \begin{cases} (\alpha, \beta), & \text{if } x = y, \\ (0, 1), & \text{if } x \neq y. \end{cases}$$

An intuitionistic fuzzy point $x_{(\alpha, \beta)}$ is said to belong to $\text{IFS}(\mu, \gamma)$, denoted by:

$$x_{(\alpha, \beta)} \in (\mu, \gamma),$$

if $\mu(x) \geq \alpha$ and $\gamma(x) \leq \beta$. Moreover, for $x, y \in R$, we have:

$$x_{(t, s)} + y_{(\alpha, \beta)} = (x + y)_{(t \wedge \alpha, s \vee \beta)},$$

$$x_{(t, s)} \cdot y_{(\alpha, \beta)} = (xy)_{(t \wedge \alpha, s \vee \beta)}.$$

3. Intuitionistic L -fuzzy soft semi-ideals

Now, we are ready to give the definition of the intuitionistic L -fuzzy soft semi-ideal and examine it.

Definition 3.1. An intuitionistic L -fuzzy soft set (F, A) over a semiring S is called an *intuitionistic L -fuzzy soft semi-ideal* over S if the following conditions are satisfied:

1. $\mu_{\tilde{F}(\epsilon)}(x - y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$ and $\gamma_{\tilde{F}(\epsilon)}(x - y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
2. $\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \vee \mu_{\tilde{F}(\epsilon)}(y)$ and $\gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \wedge \gamma_{\tilde{F}(\epsilon)}(y)$,

for all $x, y \in S$ and $\epsilon \in A$.

Definition 3.2. For any intuitionistic L -fuzzy soft set $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X\}$, the sets $\Box A$ and $\Diamond A$ are defined as follows:

$$\Box A = \{\langle x, \mu_A(x), \mu_A^c(x) \rangle \mid x \in X\},$$

$$\Diamond A = \{\langle x, \gamma_A^c(x), \gamma_A(x) \rangle \mid x \in X\}.$$

Theorem 3.3

Let $(\tilde{F}, A)$ be an intuitionistic L -fuzzy soft semi-ideal over a semiring S . Then $(\Box \tilde{F}, A)$ and $(\Diamond \tilde{F}, A)$ are intuitionistic L -fuzzy soft semi-ideals over the semiring S .

Proof

Since $(\tilde{F}, A)$ is an intuitionistic L -fuzzy soft semi-ideal over S , for all $x, y \in S$ and $\epsilon \in A$, we have:

$$\mu_{\tilde{F}(\epsilon)}(x - y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$$

,

$$\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \vee \mu_{\tilde{F}(\epsilon)}(y)$$

. Then we obtain

$$\begin{aligned} \mu_{\tilde{F}(\epsilon)}^c(x - y) &\geq \mu_{\tilde{F}(\epsilon)}^c(x) \wedge \mu_{\tilde{F}(\epsilon)}^c(y), \\ &\leq (1 - \mu_{\tilde{F}(\epsilon)}^c(x)) \vee (1 - \mu_{\tilde{F}(\epsilon)}^c(y)), \\ \mu_{\tilde{F}(\epsilon)}^c(xy) &\leq \mu_{\tilde{F}(\epsilon)}^c(x) \vee \mu_{\tilde{F}(\epsilon)}^c(y). \end{aligned}$$

and

$$\begin{aligned} \mu_{\tilde{F}(\epsilon)}^c(xy) &\geq \mu_{\tilde{F}(\epsilon)}^c(x) \vee \mu_{\tilde{F}(\epsilon)}^c(y), \\ &\leq (1 - \mu_{\tilde{F}(\epsilon)}^c(x)) \wedge (1 - \mu_{\tilde{F}(\epsilon)}^c(y)), \\ \mu_{\tilde{F}(\epsilon)}^c(xy) &\leq \mu_{\tilde{F}(\epsilon)}^c(x) \wedge \mu_{\tilde{F}(\epsilon)}^c(y). \end{aligned}$$

Hence, $(\Box \tilde{F}, A)$ is a intuitionistics L -fuzzy soft semi-ideals over the semiring. Now

$$\gamma_{\tilde{F}(\epsilon)}(x - y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y), \quad \gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \wedge \gamma_{\tilde{F}(\epsilon)}(y).$$

Then we obtain

$$\begin{aligned} \gamma_{\tilde{F}(\epsilon)}^c(x - y) &\leq \gamma_{\tilde{F}(\epsilon)}^c(x) \vee \gamma_{\tilde{F}(\epsilon)}^c(y), \\ &\geq (1 - \gamma_{\tilde{F}(\epsilon)}^c(x)) \wedge (1 - \gamma_{\tilde{F}(\epsilon)}^c(y)), \\ \gamma_{\tilde{F}(\epsilon)}^c(xy) &\geq \gamma_{\tilde{F}(\epsilon)}^c(x) \wedge \gamma_{\tilde{F}(\epsilon)}^c(y). \end{aligned}$$

and

$$\begin{aligned} \gamma_{\tilde{F}(\epsilon)}^c(xy) &\leq \gamma_{\tilde{F}(\epsilon)}^c(x) \wedge \gamma_{\tilde{F}(\epsilon)}^c(y), \\ &\geq (1 - \gamma_{\tilde{F}(\epsilon)}^c(x)) \vee (1 - \gamma_{\tilde{F}(\epsilon)}^c(y)), \\ \gamma_{\tilde{F}(\epsilon)}^c(x - y) &\geq \gamma_{\tilde{F}(\epsilon)}^c(x) \vee \gamma_{\tilde{F}(\epsilon)}^c(y). \end{aligned}$$

Thus, $(\Diamond \tilde{F}, A)$ is a intuitionistics L -fuzzy soft semi-ideals over the semiring. $\square$

Theorem 3.4

$(\tilde{F}, A)$ is an intuitionistic L -fuzzy soft semi-ideal over a semiring S if and only if $(\tilde{F}(\epsilon)^{r,t}, A)$ is a soft semi-ideal of S for all $r \in (0, 1]$, $t \in [0, 1)$, and $\epsilon \in A$.

Proof

Let $(\tilde{F}, A)$ be an intuitionistic L -fuzzy soft semi-ideal over S and let $x, y \in \tilde{F}(\epsilon)^{r,t}$, $s \in S$. Then:

$$\mu_{\tilde{F}(\epsilon)}(x) \geq r, \quad \mu_{\tilde{F}(\epsilon)}(y) \geq r, \quad \gamma_{\tilde{F}(\epsilon)}(x) \leq t, \quad \gamma_{\tilde{F}(\epsilon)}(y) \leq t.$$

So, we have

$$\mu_{\tilde{F}(\epsilon)}(x - y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y) \geq r$$

and

$$\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \vee \mu_{\tilde{F}(\epsilon)}(y) \leq t$$

by the assumption. Hence, $x - y \in \tilde{F}(\epsilon)^{r,t}$. Also, Similarly, for multiplication:

$$\mu_{\tilde{F}(\epsilon)}(sx) \geq \mu_{\tilde{F}(\epsilon)}(s) \wedge \mu_{\tilde{F}(\epsilon)}(x) \geq r$$

and

$$\gamma_{\tilde{F}(\epsilon)}(sx) \leq \mu_{\tilde{F}(\epsilon)}(x) \wedge \gamma_{\tilde{F}(\epsilon)}(x) \leq t$$

Thus, $sx \in \tilde{F}(\epsilon)^{r,t}$, and $\tilde{F}(\epsilon)^{r,t}$ is a semi-ideal of S .

Conversely, let $\tilde{F}(\epsilon)^{r,t}$ be a semi-ideal of S . Then $x - y \in \tilde{F}(\epsilon)^{r,t}$ and $sx \in \tilde{F}(\epsilon)^{r,t}$ for all $x, y \in \tilde{F}(\epsilon)^{r,t}$ and $s \in S$. Suppose that $\mu_{\tilde{F}(\epsilon)}(x) = r_1$, $\mu_{\tilde{F}(\epsilon)}(y) = r_2$ with $r_1 \wedge r_2 = r_2$, and $\gamma_{\tilde{F}(\epsilon)}(x) = t_1$, $\gamma_{\tilde{F}(\epsilon)}(y) = t_2$ with $t_1 \vee t_2 = t_2$. So, we can write $x - y \in \tilde{F}(\epsilon)^{r_2, t_2}$. Thus:

$$\mu_{\tilde{F}(\epsilon)}(x - y) \geq r_2 = \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y),$$

and

$$\gamma_{\tilde{F}(\epsilon)}(x - y) \wedge \mu_{\tilde{F}(\epsilon)}(y) \leq t_2 = \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y),$$

Similarly, for multiplication:

$$\mu_{\tilde{F}(\epsilon)}(sx) \geq \mu_{\tilde{F}(\epsilon)}(s) \vee \mu_{\tilde{F}(\epsilon)}(x)$$

and

$$\gamma_{\tilde{F}(\epsilon)}(sx) \leq \gamma_{\tilde{F}(\epsilon)}(s) \wedge \gamma_{\tilde{F}(\epsilon)}(x).$$

Hence $(\tilde{F}, A)$ satisfies the semi-ideal conditions. $\square$

Remark 3.5. This proposed structure generalizes: The Classical intuitionistic fuzzy ideals over semigroups, which deal only with a single operation (addition) and do not involve parameters or lattice values. The L -fuzzy soft structures by incorporating both membership and non-membership functions along with hesitation degrees. Soft sets by including parameterization via A . It allows modeling of multi-criteria decision systems, uncertain environments, and approximate algebraic structures, making it suitable for applications in medical diagnosis, engineering, optimization, pattern recognition, and information processing systems.

Example 3.6. Consider the semiring $S = (\mathbb{N} \cup \{0\}, +, \cdot)$ and let $A = \{\epsilon\}$. Define $(\tilde{F}, A)$ by:

$$\mu_{\tilde{F}(\epsilon)}(s) = e^{-s}, \quad \gamma_{\tilde{F}(\epsilon)}(s) = 1 - \frac{e^{-s}}{2}.$$

Then, for any $r \in (0, 1]$ and $t \in [0, 1)$, we have:

$$\tilde{F}(\epsilon)^{r,t} = \{s \in S \mid e^{-s} \geq r, 1 - \frac{e^{-s}}{2} \leq t\},$$

that is:

$$\tilde{F}(\epsilon)^{r,t} = \{s \in S \mid s \leq -\ln r, s \geq -2 \ln(1 - t)\}.$$

These level sets are semi-ideals, confirming that $(\tilde{F}, A)$ is an intuitionistic L -fuzzy soft semi-ideal.

Theorem 3.7

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic L -fuzzy soft semi-ideals over a semiring S . Then so are $(\tilde{F}, A) \tilde{\wedge} (\tilde{G}, B)$ and $(\tilde{F}, A) \tilde{\cap} (\tilde{G}, B)$.

Proof

we have $(\tilde{H}, C) = (\tilde{F}, A) \tilde{\wedge} (\tilde{G}, B)$, where $C = A \times B$ and: $\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \cap \tilde{G}(\beta)$, for all $(\alpha, \beta) \in C$ from Definition 2.3. Since $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are intuitionistic L -fuzzy soft semi-ideals, for all $x, y \in S$ and $(\alpha, \beta) \in C$, we have:

$$\begin{aligned}\mu_{\tilde{H}(\alpha, \beta)}(x - y) &= \{\mu_{\tilde{F}(\alpha)}(x - y) \wedge \mu_{\tilde{G}(\beta)}(x - y)\} \\ &\geq \{(\mu_{\tilde{F}(\alpha)}(x) \wedge \mu_{\tilde{F}(\alpha)}(y)) \wedge (\mu_{\tilde{G}(\beta)}(x) \wedge \mu_{\tilde{G}(\beta)}(y))\} \\ \mu_{\tilde{H}(\alpha, \beta)}(x - y) &= \{\mu_{\tilde{H}(\alpha, \beta)}(x) \wedge \mu_{\tilde{H}(\alpha, \beta)}(y)\},\end{aligned}$$

and

$$\begin{aligned}\gamma_{\tilde{H}(\alpha, \beta)}(x - y) &= \{\gamma_{\tilde{F}(\alpha)}(x - y) \vee \gamma_{\tilde{G}(\beta)}(x - y)\} \\ &\leq \{(\gamma_{\tilde{F}(\alpha)}(x) \vee \gamma_{\tilde{F}(\alpha)}(y)) \vee (\gamma_{\tilde{G}(\beta)}(x) \vee \gamma_{\tilde{G}(\beta)}(y))\} \\ \gamma_{\tilde{H}(\alpha, \beta)}(x - y) &= \{\gamma_{\tilde{H}(\alpha, \beta)}(x) \vee \gamma_{\tilde{H}(\alpha, \beta)}(y)\},\end{aligned}$$

$$\begin{aligned}\mu_{\tilde{H}(\alpha, \beta)}(xy) &= \{\mu_{\tilde{F}(\alpha)}(xy) \wedge \mu_{\tilde{G}(\beta)}(xy)\} \\ &\geq \{(\mu_{\tilde{F}(\alpha)}(x) \vee \mu_{\tilde{F}(\alpha)}(y)) \wedge (\mu_{\tilde{G}(\beta)}(x) \vee \mu_{\tilde{G}(\beta)}(y))\} \\ &\geq \{(\mu_{\tilde{F}(\alpha)}(x) \wedge \mu_{\tilde{F}(\alpha)}(y)) \vee (\mu_{\tilde{G}(\beta)}(x) \wedge \mu_{\tilde{G}(\beta)}(y))\} \\ \mu_{\tilde{H}(\alpha, \beta)}(xy) &= \{\mu_{\tilde{H}(\alpha, \beta)}(x) \wedge \mu_{\tilde{H}(\alpha, \beta)}(y)\},\end{aligned}$$

and

$$\begin{aligned}\gamma_{\tilde{H}(\alpha, \beta)}(xy) &= \{\gamma_{\tilde{F}(\alpha)}(xy) \vee \gamma_{\tilde{G}(\beta)}(xy)\} \\ &\leq \{(\gamma_{\tilde{F}(\alpha)}(x) \wedge \gamma_{\tilde{F}(\alpha)}(y)) \vee (\gamma_{\tilde{G}(\beta)}(x) \wedge \gamma_{\tilde{G}(\beta)}(y))\} \\ &\leq \{(\gamma_{\tilde{F}(\alpha)}(x) \vee \gamma_{\tilde{F}(\alpha)}(y)) \wedge (\gamma_{\tilde{G}(\beta)}(x) \vee \gamma_{\tilde{G}(\beta)}(y))\} \\ \gamma_{\tilde{H}(\alpha, \beta)}(xy) &= \{\gamma_{\tilde{H}(\alpha, \beta)}(x) \wedge \gamma_{\tilde{H}(\alpha, \beta)}(y)\}.\end{aligned}$$

Thus, $(\tilde{F}, A) \tilde{\wedge} (\tilde{G}, B)$ is an intuitionistic L -fuzzy soft semi-ideal over S . The proof for $(\tilde{F}, A) \tilde{\cap} (\tilde{G}, B)$ is similar. $\square$

Theorem 3.8

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic L -fuzzy soft semi-ideals over a semiring S . Then so are $(\tilde{F}, A) \cap (\tilde{G}, B)$ and $(\tilde{F}, A) \cup (\tilde{G}, B)$.

Proof

For any $x, y \in S$ and $\epsilon \in C$, We consider three cases:

Case 1: $\epsilon \in A \setminus B$. Then we have

$$\begin{aligned}\mu_{\tilde{H}(\epsilon)}(x - y) &= \mu_{\tilde{F}(\epsilon)}(x - y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y) = \mu_{\tilde{H}(\epsilon)}(x) \wedge \mu_{\tilde{H}(\epsilon)}(y), \\ \gamma_{\tilde{H}(\epsilon)}(x - y) &= \gamma_{\tilde{F}(\epsilon)}(x - y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y) = \gamma_{\tilde{H}(\epsilon)}(x) \vee \gamma_{\tilde{H}(\epsilon)}(y),\end{aligned}$$

and

$$\mu_{\tilde{H}(\epsilon)}(xy) = \mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \vee \mu_{\tilde{F}(\epsilon)}(y) = \mu_{\tilde{H}(\epsilon)}(x) \vee \mu_{\tilde{H}(\epsilon)}(y),$$

$$\gamma_{\tilde{H}(\epsilon)}(xy) = \gamma_{\tilde{F}}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \wedge \gamma_{\tilde{F}(\epsilon)}(y) = \gamma_{\tilde{H}(\epsilon)}(x) \wedge \gamma_{\tilde{H}(\epsilon)}(y).$$

Case 2: $\epsilon \in B \setminus A$. Analogous to Case (i).

Case 3: $\epsilon \in A \cap B$. Here $\tilde{H}(\epsilon) = \tilde{F}(\epsilon) \cap \tilde{G}(\epsilon)$. It follows that

$$\begin{aligned} \mu_{\tilde{H}(\epsilon)}(x-y) &= \{\mu_{\tilde{F}(\epsilon)}(x-y) \wedge \mu_{\tilde{G}(\epsilon)}(x-y)\} \\ &\geq \{(\mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)) \wedge (\mu_{\tilde{G}(\epsilon)}(x) \wedge \mu_{\tilde{G}(\epsilon)}(y))\} \\ &= \{\mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{G}(\epsilon)}(x)\} \wedge \{\mu_{\tilde{F}(\epsilon)}(y) \wedge \mu_{\tilde{G}(\epsilon)}(y)\} \\ \mu_{\tilde{H}(\epsilon)}(x-y) &= \{\mu_{\tilde{H}(\epsilon)}(x) \wedge \mu_{\tilde{H}(\epsilon)}(y)\} \end{aligned}$$

and

$$\begin{aligned} \gamma_{\tilde{H}(\epsilon)}(x-y) &= \{\gamma_{\tilde{F}(\epsilon)}(x-y) \vee \gamma_{\tilde{G}(\epsilon)}(x-y)\} \\ &\leq \{(\gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)) \vee (\gamma_{\tilde{G}(\epsilon)}(x) \vee \gamma_{\tilde{G}(\epsilon)}(y))\} \\ &= \{(\gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{G}(\epsilon)}(x)) \vee (\gamma_{\tilde{F}(\epsilon)}(y) \vee \gamma_{\tilde{G}(\epsilon)}(y))\} \\ \gamma_{\tilde{H}(\epsilon)}(x-y) &= \{\gamma_{\tilde{H}(\epsilon)}(x) \vee \gamma_{\tilde{H}(\epsilon)}(y)\}, \end{aligned}$$

$\mu_{\tilde{H}(\epsilon)}(xy) = \{\mu_{\tilde{H}(\epsilon)}(x) \vee \mu_{\tilde{H}(\epsilon)}(y)\}$ and $\gamma_{\tilde{H}(\epsilon)}(xy) = \{\gamma_{\tilde{H}(\epsilon)}(x) \wedge \gamma_{\tilde{H}(\epsilon)}(y)\}$, are similarly proved.

Consequently, $(\tilde{F}, A) \cap (\tilde{G}, B)$ is an intuitionistic L -fuzzy soft semi-ideal over a semiring S . $(\tilde{F}, A) \cup (\tilde{G}, B)$ can be similarly proved as well. $\square$

Theorem 3.9

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic L -fuzzy soft semi-ideals over a semiring S . Then so is $(\tilde{F}, A) \circ (\tilde{G}, B)$.

Proof

Let $x, y \in S$ and let $\epsilon \in A \cup B$, we consider the following cases depending on the membership of ϵ .

Case 1: $\epsilon \in A \setminus B$. Then we get:

$$\begin{aligned} \mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x-y) &= \mu_{\tilde{F}(\epsilon)}(x-y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y) = \mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) \wedge \mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(y), \\ \gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x-y) &= \gamma_{\tilde{F}(\epsilon)}(x-y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y) = \gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) \vee \gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(y), \end{aligned}$$

and

$$\begin{aligned} \mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(xy) &= \mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \vee \mu_{\tilde{F}(\epsilon)}(y) = \mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) \vee \mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(y), \\ \gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(xy) &= \gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \wedge \gamma_{\tilde{F}(\epsilon)}(y) = \gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) \wedge \gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(y). \end{aligned}$$

Hence the ideal properties hold

Case 2: $\epsilon \in B \setminus A$. Analogous to Case (i).

Case 3: $\epsilon \in A \cap B$. Then we obtain

$$\begin{aligned} \mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) &= \bigvee_{x=x_1x_2} \{\mu_{\tilde{F}(\epsilon)}(x_1) \wedge \mu_{\tilde{G}(\epsilon)}(x_2)\} \\ &\leq \bigvee_{xy=x_1x_2y} \{(\mu_{\tilde{F}(\epsilon)}(x_1y) \wedge \mu_{\tilde{G}(\epsilon)}(x_2y))\} \\ &\leq \bigvee_{xy=zt} \{\mu_{\tilde{F}(\epsilon)}(z) \wedge \mu_{\tilde{G}(\epsilon)}(t)\} \\ \mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) &= \{\mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(xy)\} \end{aligned}$$

Similarly, we can write $\mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(y) \leq \{\mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(xy)\}$.

Therefore $\{\mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(xy)\} \geq \{\mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x)\} \vee \{\mu_{(\tilde{F} \circ \tilde{G})(\epsilon)}(y)\}$

$$\begin{aligned} \gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) &= \bigwedge_{x=x_1 x_2} \{\gamma_{\tilde{F}(\epsilon)}(x_1) \vee \gamma_{\tilde{G}(\epsilon)}(x_2)\} \\ &\geq \bigwedge_{xy=x_1 x_2 y} \{\gamma_{\tilde{F}(\epsilon)}(x_1 y) \vee \gamma_{\tilde{G}(\epsilon)}(x_2 y)\} \\ &\geq \bigwedge_{xy=zt} \{\gamma_{\tilde{F}(\epsilon)}(z) \vee \gamma_{\tilde{G}(\epsilon)}(t)\} \\ \gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x) &= \{\gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(xy)\} \end{aligned}$$

Similarly, we can write $\gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(y) \geq \{\gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(xy)\}$.

Therefore, $\{\gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(xy)\} \leq \{\gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(x)\} \wedge \{\gamma_{(\tilde{F} \circ \tilde{G})(\epsilon)}(y)\}$. Then the proof is completed. $\square$

4. Intuitionistic L-Fuzzy Soft Semiring Homomorphism

Definition 4.1. Let $A = \{\langle x, \mu_{\tilde{F}(A)}(x), \gamma_{\tilde{F}(A)}(x) \rangle \mid x \in R\}$ be an intuitionistic L-fuzzy soft semiring. Let $B = \{\langle x, \mu_{\tilde{F}(B)}(x), \gamma_{\tilde{F}(B)}(x) \rangle \mid x \in R\}$ be another intuitionistic L-fuzzy soft semiring with $A \subseteq B$.

Then A is called an *intuitionistic L-fuzzy soft ideal* of B if for all $x, y \in R$:

1. $\mu_{\tilde{F}(A)}(x - y) \geq \mu_{\tilde{F}(A)}(x) \wedge \mu_{\tilde{F}(A)}(y)$, and $\gamma_{\tilde{F}(A)}(x - y) \leq \gamma_{\tilde{F}(A)}(x) \vee \gamma_{\tilde{F}(A)}(y)$.
2. $\mu_{\tilde{F}(A)}(xy) \geq (\mu_{\tilde{F}(B)}(x) \wedge \mu_{\tilde{F}(A)}(y)) \vee (\mu_{\tilde{F}(A)}(x) \wedge \mu_{\tilde{F}(B)}(y))$, and $\gamma_{\tilde{F}(A)}(xy) \leq (\gamma_{\tilde{F}(B)}(x) \vee \gamma_{\tilde{F}(A)}(y)) \wedge (\gamma_{\tilde{F}(A)}(x) \vee \gamma_{\tilde{F}(B)}(y))$

Since R is commutative, $\gamma_{\tilde{F}(A)}(xy) \leq (\gamma_{\tilde{F}(B)}(x) \vee \gamma_{\tilde{F}(A)}(y)) \wedge (\gamma_{\tilde{F}(A)}(x) \vee \gamma_{\tilde{F}(B)}(y))$ for all $x, y \in R$ if and only if $\gamma_{\tilde{F}(A)}(xy) \leq (\gamma_{\tilde{F}(B)}(x) \vee \gamma_{\tilde{F}(A)}(y))$, for all $x, y \in R$.

Theorem 4.2

Let $B = \{\langle x, \mu_{\tilde{F}(B)}(x), \gamma_{\tilde{F}(B)}(x) \rangle \mid x \in R\}$ be an intuitionistic L-fuzzy soft semiring. Let $A = \{\langle x, \mu_{\tilde{F}(A)}(x), \gamma_{\tilde{F}(A)}(x) \rangle \mid x \in R\}$ be an intuitionistic L-fuzzy soft ideal of B . Let $f : R \rightarrow S$ be an onto homomorphism. Then $f(A)$ is an intuitionistic L-fuzzy soft ideal of $f(B)$.

Proof

Clearly $f(A)$ and $f(B)$ are intuitionistic L-fuzzy soft semirings of S with $f(A) \subseteq f(B)$. For all $x, y \in S$:

$$\begin{aligned} f(\gamma_{\tilde{F}(A)})(xy) &= \wedge \{\gamma_{\tilde{F}(A)}(w) : w \in R, f(w) = xy\} \\ &\leq \wedge \{\gamma_{\tilde{F}(A)}(uv) : u, v \in R, f(u) = x, f(v) = y\} \\ &\leq \wedge \{\gamma_{\tilde{F}(B)}(u) \vee \gamma_{\tilde{F}(A)}(v) / f(u) = x, f(v) = y : u, v \in R\} \\ &= (\wedge \{\gamma_{\tilde{F}(B)}(u) / u \in R, f(u) = x\}) \vee (\wedge \{\gamma_{\tilde{F}(A)}(v) / v \in R, f(v) = y\}) \\ f(\gamma_{\tilde{F}(A)})(xy) &= f(\gamma_{\tilde{F}(B)})(x) \vee f(\gamma_{\tilde{F}(A)})(y) \end{aligned}$$

Thus $f(A)$ is an intuitionistic L-fuzzy soft ideal of $f(B)$. $\square$

Theorem 4.3

Let $f : R \rightarrow S$ be an onto homomorphism. Let $B = \{\langle x, \mu_{\tilde{F}(B)}(x), \gamma_{\tilde{F}(B)}(x) \rangle \mid x \in S\}$ be an intuitionistic L-fuzzy soft semiring of S . Let $A = \{\langle x, \mu_{\tilde{F}(A)}(x), \gamma_{\tilde{F}(A)}(x) \rangle \mid x \in S\}$ be an intuitionistic L-fuzzy soft ideal of B . Then $f^{-1}(A)$ is an intuitionistic L-fuzzy soft ideal of $f^{-1}(B)$.

Proof

Clearly $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic L -fuzzy soft semirings of R with $f^{-1}(A) \subseteq f^{-1}(B)$. Now:

$$\begin{aligned} f^{-1}(\gamma_{\tilde{F}(A)})(xy) &= \gamma_{\tilde{F}(A)}(f(xy)) \\ &= \gamma_{\tilde{F}(A)}(f(x)f(y)) \\ &\leq \gamma_{\tilde{F}(B)}(x) \vee \gamma_{\tilde{F}(A)}(y) \\ f^{-1}(\gamma_{\tilde{F}(A)})(xy) &= f^{-1}(\gamma_{\tilde{F}(B)}(x)) \vee f^{-1}(\gamma_{\tilde{F}(A)}(x)) \end{aligned}$$

Therefore $f^{-1}(A)$ is an intuitionistic L -fuzzy soft ideal of $f^{-1}(B)$. $\square$

5. Application of Intuitionistic L-Fuzzy Soft Ideal Based Medical Diagnosis System over Semiring

This example demonstrates a medical diagnosis system using intuitionistic L -fuzzy soft ideals over semirings. The system models uncertainty in medical symptoms and diagnoses using advanced fuzzy structures.

Example: Medical Diagnosis

Step 1: Define Parameters. Let $S = \{s_1, s_2, s_3\}$ be a semiring of medical tests, where $+$ and $\cdot$ represent test combinations. Let $P = \{p_1, p_2, p_3\}$ be the set of patients, $E = \{e_1 \text{ (fever)}, e_2 \text{ (cough)}, e_3 \text{ (fatigue)}\}$ be the set of symptoms, and $D = \{d_1 \text{ (flu)}, d_2 \text{ (COVID-19)}, d_3 \text{ (allergy)}\}$ be the set of possible diagnoses.

Step 2: Create Intuitionistic L -Fuzzy Soft Ideal. Define $F : E \rightarrow L^S$ such that for each symptom $e \in E$, $F(e)$ is an intuitionistic L -fuzzy ideal of S .

For example, for symptom e_1 (fever):

$$F(e_1)(s_1) = (0.8, 0.1), \quad F(e_1)(s_2) = (0.5, 0.4), \quad F(e_1)(s_3) = (0.3, 0.6),$$

representing strong, moderate, and weak indications of fever respectively. Similarly, $F(e_2)$ and $F(e_3)$ are defined for cough and fatigue.

Step 3: Patient-Symptom Mapping. For each patient $p \in P$, define an intuitionistic fuzzy soft set $G_p : E \rightarrow L^S$ representing their test results.

For patient p_1 :

$$G_{p_1}(e_1)(s_1) = (0.9, 0.05)(\text{very high fever}), \quad G_{p_1}(e_2)(s_2) = (0.6, 0.3), (\text{Moderate cough})$$

,

$$G_{p_1}(e_3)(s_3) = (0.4, 0.5)(\text{Some fatigue}).$$

Step 4: Diagnosis Decision Making. Using operations on the semiring S and the lattice L , compute similarity measures between patient symptoms and known disease patterns.

For diagnosis d_1 (flu) with characteristic pattern:

$$F_{\text{flu}}(e_1) = (0.7, 0.2), \quad F_{\text{flu}}(e_2) = (0.6, 0.3), \quad F_{\text{flu}}(e_3) = (0.5, 0.4),$$

the similarity measure is given by:

$$(p_1, \text{flu}) = \sum_{e \in E} [\mu_{p_1}(e) \odot \mu_{\text{flu}}(e) + (1 - \gamma_{p_1}(e)) \odot (1 - \gamma_{\text{flu}}(e))].$$

Step 5: Results Interpretation. After computing similarity for all diagnoses:

$$p_1 : \quad \text{flu} (0.82), \quad \text{COVID-19} (0.65), \quad \text{allergy} (0.45).$$

The system concludes that patient p_1 most likely has the flu.

6. Conclusion

This paper introduced intuitionistic L-fuzzy soft ideals within the framework of intuitionistic L-fuzzy soft semirings and studied their properties under homomorphisms. Building on these foundations, the proposed structures extend the algebraic foundation of fuzzy soft semirings and provide new insights into their substructures. To illustrate practical utility, a medical diagnosis example demonstrated their effectiveness in handling uncertainty and the similarity measure. Looking forward, future work will focus on broader applications, computational aspects, and potential extensions to other algebraic systems.

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