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Analyzing the Impact of Adaptive Type-II Progressive Hybrid Censoring on Overlap Measures with Gompertz Data

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Abstract This paper examines statistical inference regarding the overlap between two Gompertz distributions with varying parameters, employing the adaptive Type-II progressive hybrid censoring scheme developed by Ng et al. (2009). We focus on key metrics to assess overlap and derive the asymptotic bias and variance of the estimators. Additionally, we construct confidence intervals using bootstrap resampling and Taylor series approximation. The effectiveness of the method is demonstrated through an analysis of customer waiting times at the bank until service is provided.

Keywords Bootstrap method; Matusita's measure; Morisita's measure; Weitzman's measure; adaptive type-II progressive hybrid censoring.

AMS 2010 subject classifications 62E10, 62N01, 62N02, 62G30

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1. Introduction

Life-testing experiments face a fundamental trade-off: researchers need to gather enough failure data for accurate analysis, but they must also manage limited time and resources. Censoring techniques help resolve this by allowing an experiment to end before all units have failed. A beneficial method is progressive censoring, where specific units are removed at various points during the test, either on a fixed schedule or at random times. This realistically models situations where units are lost or must be removed from the study early.

Progressive censoring models are generally divided into two types. Progressive Type-I censoring fixes the end time of the experiment in advance, while Progressive Type-II censoring fixes the number of failures to be observed. Both methods allow for the removal of units at stages other than the very end, adding flexibility.

However, each method has a weakness. With Type-I, if the scheduled end time is too short, there may be too few failures to allow for meaningful analysis, especially for long-lasting products. Type-II avoids this problem but can lead to an unacceptably long experiment if the units are highly durable and failures occur slowly.

To address these limitations, [21] introduced hybrid censoring schemes. These methods set both a maximum time limit (T) and a minimum number of failures (m). This ensures the experiment will end by a specific time, but can also stop early if a sufficient number of failures occur quickly. One scheme (Type-I Hybrid) may yield fewer than m failures, while the other (Type-II Hybrid) may run slightly past time T to ensure the target number of failures is met.

Recognizing that real-world experiments are dynamic, [30] developed an even more flexible model called adaptive Type-II progressive hybrid censoring. This approach allows the researcher to change the censoring plan

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mid-experiment if the initial plan proves impractical. A threshold time T acts as a trigger: if the target number of failures has not been reached by T, the remaining units are subjected to a modified, more aggressive censoring scheme to accelerate the test's completion. This model, as shown in Figure 1, adds a crucial layer of practicality to life testing.

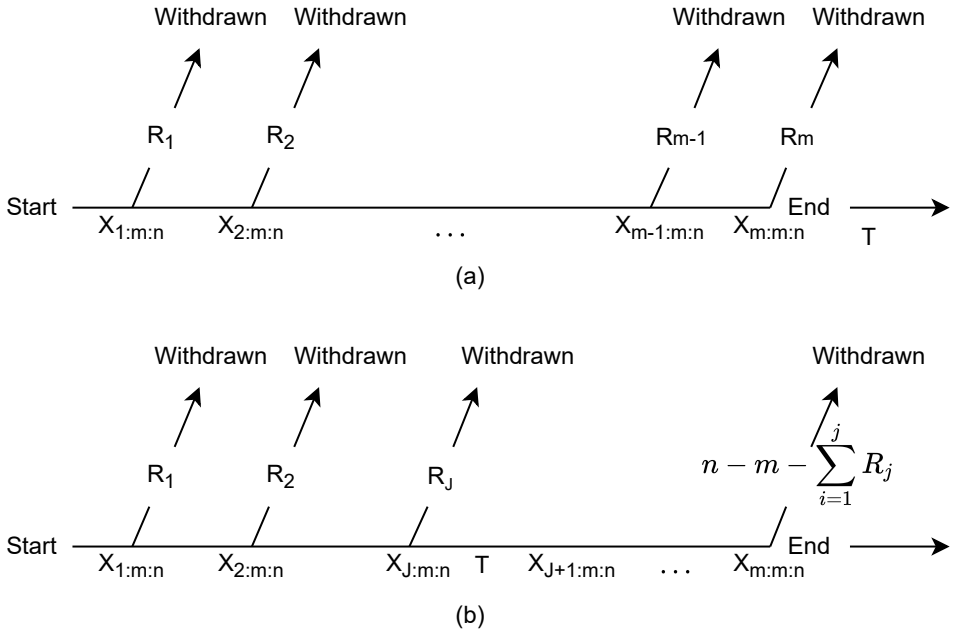

Figure 1. (a) Scenario when m-th failure occurs before time T; (b) Scenario when time T is reached before m failures are observed

Consider a life-testing experiment that begins with n units. The goal is to observe a predetermined number of failures, m (where $m < n$), according to an initial censoring plan $(K_1, K_2, \ldots, K_m)$. This plan is adaptable. The experimenter also sets an ideal maximum duration for the test, denoted T. The procedure adapts based on when failures occur relative to time T, leading to two distinct scenarios:

Scenario 1: The m-th failure occurs before time T. In this case (Figure 1(a)), the experiment follows the original type-II progressive censoring plan exactly. It concludes at the time of the m-th failure, X_m, with the pre-set removals $(K_1, K_2, \ldots, K_m)$.

Scenario 2: Time T is reached before m failures are observed. Suppose only j failures have occurred by time T. At this point, the censoring scheme is modified to accelerate the test. No further units are removed until the m-th failure occurs. This is done by setting $K_{j+1} = K_{j+2} = \cdots = K_{m-1} = 0$. The remaining surviving units are all censored at the m-th failure, so:

$$K_m = n - m - \sum_{i=1}^{j} K_i$$

This adjustment (Figure 1(b)) ensures that the target of m failures is met without letting the test duration extend too far beyond the ideal time T.

Special Cases

This adaptive model includes two well-known methods as special cases:

- If $T = 0$, the method simplifies to a conventional type-II censoring scheme.
- If $T \to \infty$ (i.e., the time limit is effectively removed), the process becomes a standard progressive type-II censoring technique.

The Adaptive-IIPH censoring scheme has significant practical utility, as demonstrated by its diverse applications in recent research. For instance, [1] created novel estimators for the stress-strength reliability of the Inverse Weibull distribution using this adaptive approach. In a different domain, [9] applied Adaptive-IIPH censoring to accelerated life testing, studying the persistence of virus-containing microdroplets over time to simulate cough events. The method has also been applied in engineering reliability; [8] utilized it to model the lifetime of sodium-sulfur batteries using the XLindley distribution. Furthermore, its flexibility is shown in environmental science, where [20] used Adaptive-IIPH censoring to analyze radar-measured rainfall data from cumulus clouds. More recently, [7] have also employed adaptive progressive censoring to estimate fundamental entropy measures, with an application to breast cancer data.

In reliability engineering, a key problem is estimating the proportion of components from different populations that share similar failure time characteristics. This is often quantified using overlap coefficients (OVL), which measure the common area between two probability density functions. Standard measures include Matusita's ρ, Morisita's λ, and Weitzman's Δ. As illustrated in Figure2, the OVL corresponds to the shaded area common to both density curves, intuitively representing the probability mass shared by the two distributions.

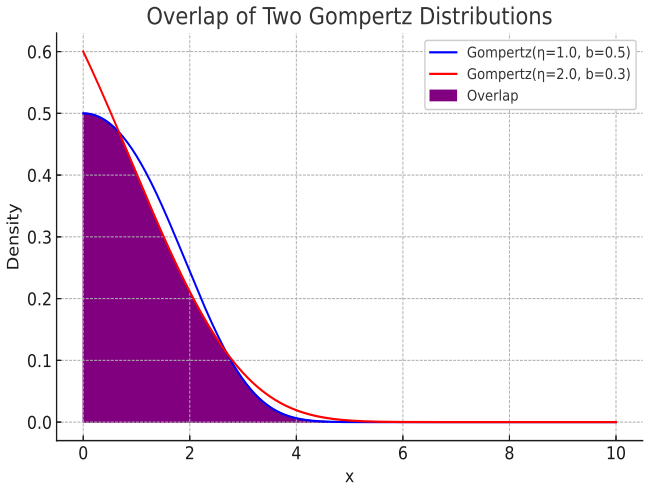

Figure 2. Overlap of two densities.

The utility of overlap coefficients (OVL) extends across a broad spectrum of disciplines, from quantitative ecology [15] and electromyographic assessment ([17] to clinical trial analysis [26], with comprehensive reviews available in [34] and [23]. Parallel to these applications, significant methodological progress has been made in the statistical inference of OVL. The sampling behavior of nonparametric estimators was analyzed by [19], and the comparative advantages of OVL over the Youden index were established by [31]. Most recently, [2] and [4] have advanced the field by applying the Adaptive-IIPH censoring scheme to parametric models. In a seminal work, they developed inference procedures for the OVL of both Rayleigh and Pareto distributions, deriving asymptotic properties and employing Monte Carlo simulations with bootstrap and Taylor approximation methods to construct robust confidence intervals for small samples, each illustrated with practical data examples.

In this article, our primary focus lies in making inferences regarding the measure of overlap (OVL) while utilizing Adaptive-IIPH censoring data from two independent Gompertz distributions with different scale parameters.

The OVL measures can be formally defined as follows: Consider two samples drawn from continuous distributions with probability density functions $f_1(x)$ and $f_2(x)$. The corresponding overlap coefficients are given by:

$$\text{Matusita's Measure (1955): } \rho = \int \sqrt{f_1(x)f_2(x)}dx,$$
$$\text{Morisita's Measure (1959): } \lambda = \frac{2\int f_1(x)f_2(x)dx}{\int [f_1(x)]^2dx + \int [f_2(x)]^2dx},$$
$$\text{Weitzman's Measure (1970): } \Delta = \int \min(f_1(x), f_2(x))dx.$$

These measures can be adapted for discrete distributions by replacing integrals with summations and can also be generalized to multivariate settings. OVL values are bounded between 0 and 1. A value near 0 indicates minimal similarity (significant disagreement) between the two density functions, while a value of 1 denotes perfect agreement.

The mathematical formulation of these coefficients is complex, and the exact sampling distributions of their estimators are generally unknown. Previous research includes [32] work on discrete Weitzman's measure, [25] investigation of sampling properties under homogeneity, and [28] simulation studies for normal densities. [22] examined sampling variability, [13] applied bootstrap and jackknife methods, and [29] tackled related inference problems.

[19] analyzed the sampling distribution of a nonparametric OVL estimator, [5]. Furthermore, [31] conducted a comparative study between the overlap coefficient (OVL) and the Youden index (J), demonstrating the relative advantages of using OVL.

This article focuses on statistical inference for the overlap coefficient (OVL) when the data are obtained from two independent Gompertz distributions with different scale parameters under an Adaptive-IIPH censoring scheme. The paper is structured as follows: Section 2 introduces the Gompertz distribution and derives the OVL measures. Section 3 presents the estimators, examines their approximate biases, and constructs confidence intervals using the delta method and bootstrap techniques. Section 4 details the simulation results and provides a discussion. Finally, Section 5 demonstrates the methodology with a practical example using real-life data.

2. The model

A random variable U is said to have a Gompertz distribution with probability density function (pdf)

$$f(u) = \frac{1}{\theta}\exp\left[\beta u - \frac{1}{\theta\beta}\left(e^{\beta u} - 1\right)\right],\ u > 0 \tag{1}$$

where $\theta > 0$ is the shape parameter and $\beta > 0$ is the scale parameter. The cumulative distribution function (cdf) corresponding to (1) for $u > 0$, is

$$F(u) = 1 - e^{-\frac{1}{\theta\beta}\left(e^{\beta u}-1\right)}. \tag{2}$$

The Gompertz distribution is a suitable model for describing human mortality trends, accurately characterizing the exponential growth in mortality rates over time. In practice, estimating the shape parameter is much more important than the scale parameter in the Gompertz distribution.

For known β, the distribution of $X = \frac{1}{\beta}\left(e^{\beta U} - 1\right)$ is a one-parameter exponential distribution $(Exp(\theta))$, with pdf and cdf as follows:

$$f(x) = \frac{1}{\theta}e^{-x/\theta}, \quad \text{for } x > 0, \theta > 0 \tag{3}$$

and

$$F(x) = 1 - e^{-\frac{x}{\theta}}, \quad \text{for } x > 0, \theta > 0 \tag{4}$$

Because the transformation $X = \frac{1}{\beta}\left(e^{\beta u} - 1\right)$ is a one-to-one function that strictly increases, both the original data set U and the transformed data set X have an equivalent impact on overlap measures. Furthermore, transforming the data to X simplifies the computation of essential properties.

Let $R = \frac{\theta_1}{\theta_2}$, as in [19], the continuous version of the three proposed overlap measures can be expressed as a function of R as follows:

$$\rho = \frac{2\sqrt{R}}{1+R}, \tag{5}$$

$$\lambda = \frac{4R}{(1+R)^2}, \tag{6}$$

and

$$\Delta = 1 - R^{\frac{1}{1-R}}\left|1 - \frac{1}{R}\right|, R \neq 1. \tag{7}$$

According to [29], ρ, λ, and Δ are not monotone for all $R > 0$. However, they exhibit certain properties, such as symmetry in R, meaning that $OVL(R) = OVL(\frac{1}{R})$. They also remain invariant under linear transformations, $Y = aX + b$, $a \neq 0$ and attain the maximum value of 1 at $R = 1$.

2.1. Maximum likelihood estimates

The joint pdf of the Adaptive-IIPH censored sample coming from an absolutely continuous distribution with pdf $f(.)$ and cdf $F(.)$ (see [10] can be written as

$$L(\theta_1|\mathbf{U}) = C[1 - F(u_m)]^{K^*}\prod_{i=1}^{m} f(u_i)\prod_{i=1}^{J}[1 - F(u_i)]^{K_i} \tag{8}$$

where,

$$C = n(n - K_1 - 1)(n - K_1 - K_2 - 2)...(n - K_1 - K_2 - ... - K_{m-1} - m + 1),$$

is the normalizing constant. Let $\mathbf{U} = \{U_{1:m_1:n_1} < U_{2:m_1:n_1} < ... < U_{m_1:m_1:n_1}\}$ be an Adaptive-IIPH censoring sample from Gompertz (θ_1, β) distribution under the censoring scheme $\{n_1, m_1, K_1, ..., K_{J_1}, 0, ..., 0, K^* = n_1 - m_1 - \sum_{i=1}^{J_1} K_i\}$ such that $U_{J_1:m_1:n_1} < T_1 < U_{J_1+1:m_1:n_1}$. For simplicity, let $U_i = U_{i:m_1:n_1}$. Since the lifetime of product U follows Gompertz(θ_1, β) with known (given) scale parameter β, then by substituting Eqs. (1) and (2) into Eq. 8, the log-likelihood function reads

$$\begin{aligned} L(\theta_1|u_1, u_2, ..., u_{m_1}) \propto \frac{1}{\theta_1^{m_1}} e^{-\frac{1}{\theta_1\beta}K^*(e^{\beta u_{m_1}}-1)} e^{-\frac{1}{\theta_1\beta}\sum_{i=1}^{m_1}(e^{\beta u_i}-1)} \\ \times e^{-\frac{1}{\theta_1\beta}\sum_{i=1}^{J_1}K_i(e^{\beta u_i}-1)} \end{aligned} \tag{9}$$

Using the transformation $X = \frac{1}{\beta}\left(e^{\beta U} - 1\right)$, the order statistic $\{X_1 < X_2 < ... < X_{m_1}\}$ will be the corresponding Adaptive-IIPH from the one parameter exponential distribution with mean θ_1. Hence, the associated log-likelihood function of the observed transformed data becomes

$$l_x \propto -m_1 \log\theta_1 - \frac{\left(K^* x_{m_1} + \sum_{i=1}^{m_1} x_i + \sum_{i=1}^{J_1} K_i x_i\right)}{\theta_1} \tag{10}$$

Similarly for the second data set, $\mathbf{V} = \{V_1 < V_2 < ... < V_{m_2}\}$ is an Adaptive-IIPH censoring sample under the scheme $\{n_2, m_2, L_1, ..., L_{J_2}, 0, ..., 0, L^* = n_2 - m_2 - \sum_{i=1}^{J_2} L_i\}$ such that $V_{J_2} < T_2 < V_{J_2+1}$ from Gompertz(θ_2, β).
Using the transformation $Y = \frac{1}{\beta}\left(e^{\beta V} - 1\right)$, the order statistic $\{Y_1 < Y_2 < ... < Y_{m_2}\}$ will be the corresponding Adaptive-IIPH from the one parameter exponential distribution with mean θ_2. Hence, the joint associated log-likelihood function of the observed transformed data becomes

$$l_y \propto -m_2 \log \theta_2 - \frac{\left(L^* y_{m_2} + \sum_{i=1}^{m_2} y_i + \sum_{i=1}^{J_2} L_i y_i\right)}{\theta_2} \tag{11}$$

The MLEs of the parameters θ_1 and θ_2 can be obtained by taking the first derivative of Eqs. 10 and 11 with respect to θ_1 and θ_2, respectively, and equating the normal equations to 0 to get

$$\hat{\theta}_1 = \frac{K^* x_{m_1} + \sum_{i=1}^{m_1} x_i + \sum_{i=1}^{J_1} K_i x_i}{m_1}, \tag{12}$$

$$\hat{\theta}_2 = \frac{\left(L^* y_{m_2} + \sum_{i=1}^{m_2} y_i + \sum_{i=1}^{J_2} L_i y_i\right)}{m_2}. \tag{13}$$

[33] established that for an exponential distribution with mean θ, the maximum likelihood estimator (MLE) based on a progressive Type-II censored sample is given by $\hat{\theta} = \frac{\sum_{i=1}^{m}(K_i+1)w_i}{m}$. They further showed that $\hat{\theta}$ follows a Gamma distribution with parameters m and θ/m, denoted $\hat{\theta} \sim$ Gamma$(m, \frac{\theta}{m})$.

[11] (Theorems 5 and 7) demonstrated that the MLE derived from an Adaptive-IIPH censored sample is identical to the MLE from a deterministic progressive Type-II scheme. This key result implies that the sampling distribution of $\hat{\theta}$ remains unchanged regardless of whether the censoring is pre-determined or adapts during the experiment. Therefore, for two independent samples, we have $\hat{\theta}_i \sim$ Gamma$(m_i, \frac{\theta_i}{m_i})$ for $i = 1, 2$. As a direct consequence, the means and variances of the MLEs in equations (12) and (13) are as follows:

$$E(\hat{\theta}_1) = \theta_1, \qquad E(\hat{\theta}_2) = \theta_2, \tag{14}$$

$$Var(\hat{\theta}_1) = \frac{\theta_1^2}{m_1}, \qquad Var(\hat{\theta}_2) = \frac{\theta_2^2}{m_2}. \tag{15}$$

By the invariant property, the MLE of R is $\hat{R} = \frac{\hat{\theta}_1}{\hat{\theta}_2}$. Hence, $\frac{\theta_2}{\theta_1}\hat{R}$ has F-distribution with $2m_1$ and $2m_2$ degrees of freedom ($F_{2m_1,2m_2}$). Thus, by using the delta method of approximation, the variance of $\hat{R}$ can be approximated by:

$$Var(\hat{R}) = \frac{m_2^2(m_1 + m_2 - 1)}{m_1(m_2 - 1)^2(m_2 - 2)} R^2. \tag{16}$$

Clearly, an unbiased estimator of R is given by $\hat{R}^* = \frac{(m_2-1)}{m_2}\hat{R}$ with variance $Var(\hat{R}^*) = \frac{(m_1+m_2-1)}{m_1(m_2-2)}R^2$ and hence $Var(\hat{R}^*) < Var(\hat{R})$. Since the OVL measures are functions of R, therefore, based on the MLE estimate of R, the OVL measures can be estimated by

$$\hat{\rho} = \frac{2\sqrt{\hat{R}^*}}{1 + \hat{R}^*}, \tag{17}$$

$$\hat{\lambda} = \frac{4\hat{R}^*}{\left(1 + \hat{R}^*\right)^2}, \tag{18}$$

and,

$$\hat{\Delta} = 1 - \hat{R}^{*\frac{1}{1-\hat{R}^*}} \left|1 - \frac{1}{\hat{R}^*}\right|, \hat{R}^* \neq 1. \tag{19}$$

3. Asymptotic properties of OVL

Using the delta method, the asymptotic variance and bias for OVL measures are as follows: Let $OVL = g(\hat{R}^*)$, then the asymptotic variances are given by

$$Var(\hat{\rho}) = \sigma^2_{\hat{\rho}} \cong \frac{(m_1+m_2-1)}{m_1(m_2-2)} \frac{R(1-R)^2}{(1+R)^4}, \tag{20}$$

$$Var(\hat{\lambda}) = \sigma^2_{\hat{\lambda}} \cong \frac{16(m_1+m_2-1)}{m_1(m_2-2)} \frac{R^2(1-R)^2}{(1+R)^6}, \tag{21}$$

$$Var(\hat{\Delta}) = \sigma^2_{\hat{\Delta}} \cong \frac{(m_1+m_2-1)}{m_1(m_2-2)} \frac{R^{\frac{2}{1-R}}(\log R)^2}{(1-R)^2}. \tag{22}$$

with the asymptotic bias

$$Bias(\hat{\rho}) \cong \frac{(m_1+m_2-1)}{m_1(m_2-2)} \times \frac{\sqrt{R}(3R^2-6R-1)}{2(1+R)^3}, \tag{23}$$

$$Bias(\hat{\lambda}) \cong \frac{(m_1+m_2-1)}{m_1(m_2-2)} \times \frac{4R^2(R-2)}{(1+R)^4}, \tag{24}$$

and,

$$Bias(\hat{\Delta}) \cong \left\{ \begin{array}{ll} H(R)\frac{(m_1+m_2-1)}{m_1(m_2-2)}, & R>1 \\ -H(R)\frac{(m_1+m_2-1)}{m_1(m_2-2)}, & R<1 \end{array} \right\}, \tag{25}$$

where, $H(R) = R^2\left[\frac{R^{\frac{2R-1}{1-R}}R\{2R-\log R-2\}\log R-(R-1)^2}{(R-1)^3}\right]$.

Consistent estimators for the above variances and biases can be obtained by substituting R by $\hat{R}^*$ in the above formulas.

3.1. Interval estimation

Two types of interval estimation for the OVL measure are considered, namely the asymptotic confidence interval and the bootstrap confidence interval that were introduced by [14]. For a large sample, normal approximation to the sampling distribution using the delta-method works fairly well. Therefore, the asymptotic $100(1-\alpha)\%$ confidence interval for the OVL measures is given by: $\left\{\widehat{OVL} \mp \hat{\sigma}_{\widehat{OVL}} Z_{\alpha/2}\right\}$, where $Z_{\alpha/2}$ is the $\frac{\alpha}{2}$ upper quantile of the standard normal distribution.

There is an obvious bias involved in all OVL measure estimates, however, for large samples, they work fairly well. Thus, the bias corrected interval can be computed as follows:

$$\left(\widehat{OVL} - Bias(\widehat{OVL})\right) \pm \hat{\sigma}_{\widehat{OVL}} Z_{\alpha/2}. \tag{26}$$

However, uniform bootstrap resampling approach for estimating bootstrap confidence intervals as described by [14], is designed for one sample case. For a two-sample case, the uniform resampling rules will apply to each sample separately and independently (see [19]).

Let $\mathbf{X} = (X_1, X_2, \cdots, X_{m_1})$ and $\mathbf{Y} = (Y_1, Y_2, \cdots, Y_{m_2})$ be two independent Adaptive-IIPH samples drawn from $f_1(x)$ and $f_2(y)$ respectively. Assume that the parameter of interest is the OVL coefficient. Let S be an estimate of OVL based on the mentioned two random samples. For B uniform re-samples, say $(X^*_{i1}, X^*_{i2}, ..., X^*_{im_1})$ and $(Y^*_{i1}, Y^*_{i2}, ..., Y^*_{im_2}), i = 1, 2, ..., B$, let $S^*_1, S^*_2, ..., S^*_B$ be the re-sampling realization of S. Then, the uniform re-sampling approximation to the $100(1-\alpha)\%$ bootstrap confidence limits can be obtained as follows: Let $S^*_{(1)}, S^*_{(2)}, ..., S^*_{(B)}$ be the order statistics of $S^*_1, S^*_2, ..., S^*_B$. Define $\omega_1 = \text{integer}(B(\alpha))$ and $\omega_2 = \text{integer}(B(1-\alpha))$. Then the uniform re-sampling approximation of the $100(1-\alpha)\%$ confidence interval is $\left(\frac{S^*_{(\omega_1)}+S^*_{(\omega_1+1)}}{2}, \frac{S^*_{(\omega_2)}+S^*_{(\omega_2+1)}}{2}\right)$.

4. Real life data

This section presents an illustrative example to demonstrate the practical application of the proposed methodology. Since real-life datasets that naturally arise from an Adaptive-IIPH censoring scheme for the Gompertz distribution are scarce, we employ a common validation technique: we take a complete dataset and artificially impose censoring. This approach serves two key purposes: 1) it provides a concrete "proof of concept" for the computational procedure outlined in previous sections, and 2) it allows us to compare the estimates obtained from the censored samples against the known results from the full data, thus offering a check on the method's performance.

The dataset consists of the waiting times of customers from two banks, A and B, measured in minutes until they are served. According to [12], the Gompertz distribution is appropriate for analyzing these data sets. Table 1 presents the complete waiting times.

The legitimacy of the Gompertz model for Bank A and Bank B is assessed using Kolmogorov-Smirnov (K-S), Anderson-Darling (A-D), and chi-square tests with $\beta = 1$, $\theta_1 = 1.679$, and $\theta_2 = 2.067$, respectively. Table 2 shows that the Gompertz model fits both data sets well with a significance level of 0.05.

Table 1. Waiting time before customer service in Bank A and B

Bank A	1.1	1.2	1.8	1.9	2.0	2.2	2.3	2.3	2.3	2.5	2.6
	2.7	2.7	2.9	3.1	3.1	3.2	3.4	3.4	3.5	3.9	4.0
	4.2	4.5	4.7	5.3	5.6	5.6	6.2	6.3	6.6	6.8	7.3
	7.5	7.7	7.7	8.0	8.0	8.5	8.5	8.7	9.5	10.7	10.9
	11.0	12.1	12.3	12.8	12.9	13.2	13.7	14.5	16.0	16.5	28.0
Bank B	1.3	1.5	1.8	1.9	1.9	2.1	2.6	2.7	2.9	3.1	3.2
	3.3	3.5	3.6	4.0	4.1	4.2	4.2	4.3	4.3	4.4	4.4
	4.6	4.7	4.7	4.8	4.9	4.9	5.0	5.3	5.5	5.7	5.7
	6.1	6.2	6.2	6.2	6.3	6.7	6.9	7.1	7.1	7.1	7.1
	7.4	7.6	7.7	8.0	8.2	8.6	8.6	8.6	8.8	8.8	8.9
	8.9	9.5	9.6	9.7	9.8	10.7	10.9	11.0	11.0	11.1	11.2
	11.2	11.5	11.9	12.4	12.5	12.9	13.0	13.1	13.3	13.6	13.7
	13.9	14.1	15.4	15.4	17.3	17.3	18.1	18.2	18.4	18.9	19.0
	19.9	20.6	21.3	21.4	21.9	23.0	27.0	31.6	33.1	38.5	

Table 2. Test statistic and p-value associated with each test for data 1 and data 2

Data	K-S(p-value)	A-D(p-value)	chi-squared(p-value)
Bank A	0.1405 (0.3071)	0.693 (0.2250)	1.911 (0.3461)
Bank B	0.2185 (0.4133)	1.652 (0.1233)	2.245 (0.3165)

Three different artificial Adaptive-IIHP censored data are created for both sets using the same censoring schemes as those in Section 4. The associated stopping time for each scheme and the generated censored samples are given in Tables 3 and 4.

The OVL estimates are calculated using $m_1 = 11$ and $m_2 = 11$. Table 5 presents the corresponding MLEs, $|Bias|$, asymptotic variance, and 95% confidence intervals for the OVLs, using Taylor approximation and bootstrap methods. The results illustrate the estimates' proximity to the values obtained from the complete data (see Table 5, "Complete" case). This demonstrates that the methodology produces sensible estimates even when applied to censored samples, with the estimates based on Scheme I showing the closest resemblance to the complete case results. The $|Bias|$, asymptotic variances, and confidence interval lengths are notably minimal. The asymptotic variances approach zero, especially in the cases of ρ and λ estimates within schemes II and III. Notably, estimates based on Scheme I demonstrate the closest resemblance to the complete case.

Table 3. Artificial Adaptive-IIHP censored samples for Bank A

scheme	T	censored data for Bank A									
I	T_1	1.1	1.2	1.8	1.9	2.0	2.2	2.3	2.3	2.3	2.5
		2.6	4.2	4.5	4.7	5.3	5.6	5.6	6.2	6.3	6.6
		6.8	7.3	11.0	12.1	12.3	12.8	12.9	13.2	13.7	14.5
		16.0	16.5	28.0							
II	T_2	1.1	1.8	2.0	2.2	2.3	2.5	2.6	2.7	2.9	3.1
		3.2	3.4	3.5	4.0	4.2	4.5	5.3	5.6	6.3	6.6
		7.5	8.0	8.0	8.7	9.5	10.7	10.9	11.0	12.1	12.3
		13.2	13.7	14.5							
III	T_3	2.2	2.3	2.3	2.5	2.6	2.7	3.1	3.1	3.2	3.4
		3.9	4.0	4.2	4.5	4.7	5.3	6.6	7.3	7.5	8.0
		8.0	8.5	8.5	8.7	9.5	10.7	11.0	12.8	12.9	13.2
		13.7	14.5	28.0							

Table 4. Artificial Adaptive-IIHP censored samples for Bank B

scheme	T	censored data for Bank B										
I	T_1	1.3	1.5	1.8	1.9	1.9	2.1	2.6	2.7	2.9	3.1	3.2
		4.6	4.7	4.7	4.8	4.9	4.9	5.0	5.3	5.5	5.7	5.7
		6.1	7.7	8.0	8.2	8.6	8.6	8.6	8.8	8.8	8.9	8.9
		9.5	9.6	12.5	12.9	13.0	13.1	13.3	13.6	13.7	13.9	14.1
		15.4	15.4	17.3	27.0	31.6	33.1	38.5				
II	T_2	1.3	1.5	1.8	1.9	2.9	3.1	3.2	3.3	3.5	3.6	4.0
		4.1	4.2	4.2	4.3	4.3	4.4	4.4	6.2	6.2	6.2	6.3
		6.7	6.9	7.1	7.1	7.1	7.1	7.4	7.6	9.7	9.8	10.7
		10.9	11.2	11.2	11.5	11.9	12.4	17.3	18.1	18.2	18.4	18.9
		19.0	19.9	20.6	21.3	21.4	21.9	23.0				
III	T_3	1.8	2.1	2.9	3.3	4.0	4.1	4.2	4.3	4.6	4.7	4.9
		4.9	5.5	5.7	6.2	6.2	6.2	6.3	6.7	6.9	7.1	7.1
		7.1	7.1	7.4	7.6	8.2	8.6	8.8	8.8	9.5	9.6	10.7
		10.9	11.1	11.2	11.9	12.4	13.0	13.1	13.7	13.9	17.3	18.4
		19.9	20.6	21.9	23.0	27.0	33.1	38.5				

Table 5. Results based on the real data of bank waiting times.

Scheme	**OVL**	MLE (**—Bias—**)	**Asymp. Var.**	**Asymptotic 95% CI**		**Bootstrap 95% CI**	
				Lower	Upper	Lower	Upper
Complete	ρ	0.9941 (0.0043)	0.000084	0.9804	1	0.9839	0.9991
	λ	0.9882 (0.0084)	0.000332	0.9609	1	0.9680	0.9982
	Δ	0.9199 (0.0050)	0.00385	0.8032	1	0.8677	0.9691
1	ρ	0.9925 (0.0077)	0.00019	0.9732	1	0.9104	0.9999
	λ	0.9850 (0.0152)	0.00075	0.9466	1	0.8288	0.9999
	Δ	0.9097 (0.0088)	0.00684	0.7564	1	0.6863	0.9942
2	ρ	0.9826 (0.0082)	0.0004	0.9504	1	0.9150	0.9999
	λ	0.9656 (0.0157)	0.0016	0.9018	1	0.8371	0.9998
	Δ	0.8627 (0.0084)	0.0067	0.7106	1	0.6945	0.9905
3	ρ	0.9872 (0.0080)	0.0003	0.9603	1	0.9241	0.9998
	λ	0.9746 (0.0155)	0.0012	0.9212	1	0.8539	0.9997
	Δ	0.8823 (0.0086)	0.0068	0.7296	1	0.7115	0.9866

5. Simulation Study

This simulation study aims to rigorously compare the performance of maximum likelihood estimators for the measures of overlap. These estimators are derived from diverse sets of Adaptive-IIHP censoring samples, as described by [30], generated from two independent Gompertz distributions.

5.1. Simulation Setup and Parameters

To ensure the reproducibility of our simulation study, we explicitly define the parameters used in it. The known scale parameter for the Gompertz distribution was set to $\beta = 1$ for both populations. This choice simplifies the transformation $X = \frac{1}{\beta}\left(e^{\beta U} - 1\right)$ to $X = e^U - 1$, leading to exponentially distributed data with mean θ, but does not affect the generality of our findings regarding the OVL measures, which are functions of the ratio $R = \theta_1/\theta_2$.

The threshold times T_1, T_2, T_3 were determined relative to the order statistics of the generated samples to ensure they are feasible within each simulation run. Specifically:

- T_1 was set to the $\lfloor m/4 \rfloor$-th order statistic, i.e., $T_1 = X_{\lfloor m/4 \rfloor}$.
- T_2 was set to the $\lfloor 4m/5 \rfloor$-th order statistic, i.e., $T_2 = X_{\lfloor 4m/5 \rfloor}$.
- T_3 was set to a time beyond the final failure, specifically $T_3 = X_m + 2$.

This approach creates three distinct censoring scenarios: an early stopping time (T_1), a late stopping time (T_2), and a very late stopping time that often results in the experiment concluding under Scenario 1 (before T is reached).

The algorithm proceeds as follows:

1. Generate two independent progressive type-II censored samples, denoted as $U_1, U_2, ..., U_{m_1}$ and $V_1, V_2, ..., V_{m_2}$ from Gompertz (θ_1, β) and Gompertz (θ_2, β), respectively. Use censoring schemes $\mathbf{K} = (K_1, K_2, \ldots, K_{m_1})$ and $\mathbf{L} = (L_1, L_2, \ldots, L_{m_2})$ as proposed by [10].
2. Determine the values of J_1 and J_2, such that $U_{J_1} < T_1 < U_{J_1+1}$ and $V_{J_2} < T_2 < V_{J_2+1}$. Then, remove $U_{J_1+2}, \ldots, U_{m_1}$ and $V_{J_2+2}, \ldots, V_{m_2}$.
3. Generate the first $m_1 - j_1 - 1$ order statistics from the truncated distribution $\frac{f(u)}{1-F(u_{J_1+1})}$ as $U_{J_1+2}, \ldots, U_{m_1}$, and adjust the censoring scheme to $\mathbf{K} = (K_1, \ldots, K_{J_1}, 0, \ldots, 0, K^* = n_1 - m_1 - \sum_{i=1}^{J_1} K_i)$. Similarly, generate the first $m_2 - j_2 - 1$ order statistics from the truncated distribution $\frac{f(v)}{1-F(v_{J_2+1})}$ as $V_{J_2+2}, \ldots, V_{m_2}$, and update the censoring scheme to $\mathbf{L} = (L_1, \ldots, L_{J_2}, 0, \ldots, 0, L^* = n_2 - m_2 - \sum_{i=1}^{J_2} L_i)$. Use the transformation $X = \frac{1}{\beta}\left(e^{\beta U} - 1\right)$ and $Y = \frac{1}{\beta}\left(e^{\beta V} - 1\right)$ as in Section 2.
4. Calculate $\hat{\theta}_1$ and $\hat{\theta}_2$, and subsequently obtain the estimates of the measures of overlap $\hat{\rho}$, $\hat{\lambda}$, and $\hat{\Delta}$.

In this study, we executed a total of 10000 simulations, each corresponding to one of four distinct values of R. Specifically:

1. When $R = 0.005$, the resulting parameter values are as follows: $\rho = 0.14, \lambda = 0.02$, and $\Delta = 0.03$.
2. For $R = 0.05$, we observed $\rho = 0.42, \lambda = 0.17$, and $\Delta = 0.18$.
3. When $R = 0.2$, the associated parameter values are $\rho = 0.70, \lambda = 0.50$, and $\Delta = 0.42$.
4. Lastly, $R = 0.8$ yielded parameter values of $\rho = 0.98, \lambda = 0.96$, and $\Delta = 0.85$.

These simulations are conducted based on four distinct sets of population parameters: $(\theta_1, \theta_2) = (0.005, 1), (0.1, 2), (0.1, 0.5)$, and $(4, 5)$. This comprehensive range of parameter combinations allowed us to explore varying degrees of similarity between the two Gompertz distributions. Additionally, three primary stopping times are considered: $T_1 = X_{\lfloor \frac{m}{4} \rfloor}$, $T_2 = X_{\lfloor \frac{4m}{5} \rfloor}$, and $T_3 = X_m + 2$.

We then computed the associated approximate 95% confidence intervals, bias ($|Bias|$), mean squared error (MSE), length of the confidence intervals (L), and coverage probability (Cov) using Taylor and bootstrap approximation techniques. The bootstrap approximation is based on $B = 1000$ resamples. For illustrative purposes we generated the censoring samples using $n = n_1 = n_2 = 20, 30, m = m_1 = m_2 = 5, 10, 20,$ and set $\mathbf{K} = \mathbf{L}$, employing three censoring schemes:

- Scheme-I: $(n-m, 0^{*(m-1)})$, known as scheme-I, where $n-m$ units are removed just after the first failure.
- Scheme-II: $\left(0^{*(m-1)}, n-m\right)$, known as scheme-II, where $n-m$ units are removed after the last failure.
- Scheme-III: $\left(\frac{n-m}{2}, 0^{*(m-2)}, \frac{n-m}{2}\right)$, known as scheme-III, where $\frac{n-m}{2}$ units are removed after the first and the last failures. For brevity, we use the notation 0^{*p} to denote p successive zeros. Thus, the scheme $(9,0,0,0,0,0)$ is denoted by $(9,0^{*5})$.

5.2. Analysis and Comparison of Simulation Results

The simulation results, detailed in Tables 6-13, provide a comprehensive comparison of the estimators' performance under various conditions. The key findings are synthesized and interpreted below.

Overall Performance: The estimators $\hat{\rho}$, $\hat{\lambda}$, and $\hat{\Delta}$ exhibit excellent properties when the two Gompertz distributions are substantially different (i.e., when the true OVL is low). For instance, when $R = 0.005$ ($\rho = 0.14, \lambda = 0.02, \Delta = 0.03$), as shown in Tables 6 and 10, the absolute Bias (|Bias|) and mean squared error (MSE) are nearly zero across all censoring schemes and sample sizes. The confidence intervals are short, and the coverage probability (Cov) is close to the nominal 95% level.

Overall Performance: The estimators $\hat{\rho}$, $\hat{\lambda}$, and $\hat{\Delta}$ demonstrate superior performance when the two Gompertz distributions are substantially different (i.e., $\rho = 0.14, \lambda = 0.02, \Delta = 0.03$). Tables 6 and 13 reveal nearly negligible absolute bias (|Bias|) and MSE values across all censoring schemes and sample sizes. Additionally, the confidence intervals remain narrow while maintaining coverage probabilities close to the nominal 95% level.

Impact of Increasing Similarity: As the OVL values approach 1, the performance of $\hat{\rho}$ and $\hat{\lambda}$ follows a predictable pattern. Their |Bias| and MSE increase, while the length (L) of the confidence intervals decreases. This trend is evident when comparing results for $R = 0.2$ (Table 8) to those for $R = 0.8$ (Table 9). In contrast, the $\hat{\Delta}$ estimator demonstrates unique stability. As $\hat{\Delta}$ approaches 1, its |Bias| actually decreases. However, its MSE and L increase, and it maintains high coverage probability even at the extremes (see Table 9, where Cov for $\hat{\Delta}$ is above 0.97 in many cases). This underscores the robustness of $\hat{\Delta}$ for highly similar distributions.

Comparison of Censoring Schemes: The differences between Scheme-I, Scheme-II, and Scheme-III are most pronounced when the m/n ratio is small. For example, with $(n, m) = (20, 6)$ and $R = 0.05$ (Table 7), Scheme-I generally provides lower Bias and MSE compared to Schemes II and III, particularly for the threshold time T_2. However, when the m/n ratio is large (e.g., $(30, 20)$), the performance differences between the schemes diminish significantly, as the effective sample size is larger regardless of the removal pattern.

Bootstrap vs. Taylor Approximation: A comparison of Tables 6-9 (Taylor approximation) with Tables 10-13 (Bootstrap) reveals that both methods perform similarly when the distributions are dissimilar. However, in cases of high similarity (e.g., $R = 0.8$), the bootstrap method tends to produce confidence intervals with coverage probabilities that are more stable and often closer to the nominal level than the Taylor approximation method, which can exhibit under-coverage for $\hat{\rho}$ and $\hat{\lambda}$ (e.g., Cov around 0.75–0.78 in Table 9).

In summary, the most favorable results, characterized by minimal Bias, low MSE, and short confidence intervals with good coverage, are achieved when there is substantial disagreement between the two distributions. The $\hat{\Delta}$ estimator is particularly recommended for applications where high similarity between distributions is expected. Furthermore, a larger m/n ratio consistently improves estimator performance, reducing the impact of the specific censoring scheme chosen.

6. Concluding Remarks

In this study, we conducted an in-depth analysis of overlap estimators within the Adaptive Type-II Progressive Hybrid Censoring framework, focusing on samples generated from two Gompertz distributions exhibiting varying levels of similarity. Our results indicate that these estimators possess favorable properties, including low bias, reduced mean squared error (MSE), and narrow confidence intervals, particularly when the underlying distributions differ substantially. However, as the similarity between the distributions increases, the performance of the estimators tends to decline. Among the estimators, $\hat{\Delta}$ demonstrated exceptional stability in cases where the two

distributions were either nearly identical or significantly distinct, consistently maintaining coverage probabilities close to the nominal level. This underscores the robustness of $\hat{\Delta}$ in extreme scenarios. Furthermore, our study highlights the critical influence of the m/n ratio on estimator performance, with larger values yielding improvements in bias, MSE, and coverage probability.

It is crucial to acknowledge a key limitation of the present study: the assumption that the scale parameter β is known and identical for both populations. This assumption was adopted as a necessary simplification to facilitate the initial development of inference procedures for the OVL measures under the complex Adaptive-IIPH censoring scheme. However, in practical applications, this assumption is often unrealistic. Treating β as known ignores a significant source of uncertainty, which likely results in the confidence intervals presented here being narrower than they would be if β were estimated from the data.

Therefore, the most critical direction for future work is to extend this methodology to the more realistic scenario where β is unknown. This extension would involve a more complex likelihood function and require advanced techniques, such as profile likelihood or Bayesian inference, to properly account for the additional uncertainty in the estimation process. Alternatively, a sensitivity analysis could be conducted to evaluate the robustness of the OVL estimates to misspecification of the β parameter. Addressing this limitation is essential to enhance the practical applicability of the proposed method.

Future research may build on this work by extending the analysis to alternative distributional models beyond the Gompertz model. Additionally, examining the behavior of overlap estimators in the presence of outliers or under different censoring mechanisms would further enhance their applicability in diverse statistical settings.

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Table 6. Taylor approximation: absolute value of bias ($|Bias|$), length L, mean squared error (MSE) & coverage probability (Cov), when $R = 0.005, \rho = 0.14, \lambda = 0.019$ and $\Delta = 0.030$

(n,m)	Scheme		$T_1 = X_{[\frac{m}{4}]}$				$T_2 = X_{[\frac{4\times m}{5}]}$				$T_3 = X_{[m]+2}$			
		Estimate	Bias	MSE	L	Cov	Bias	MSE	L	Cov	Bias	MSE	L	Cov
(20,6)	I	ρ	0.0078085	0.0022101	0.1749119	0.971	0.0079028	0.0022689	0.1768213	0.971	0.0078367	0.0022588	0.1751505	0.951
		λ	0.0001149	0.0002226	0.0503133	0.931	0.0001223	0.0002368	0.0516725	0.927	0.0001311	0.0002523	0.0514849	0.905
		Δ	0.0005958	0.0003356	0.0651015	0.942	0.0006130	0.0003523	0.0664152	0.936	0.0006123	0.0003606	0.0658346	0.921
	II	ρ	0.0079897	0.0023738	0.1779255	0.947	0.0083940	0.0026961	0.1851026	0.888	0.0078382	0.0022598	0.1751844	0.955
		λ	0.0001599	0.0003048	0.0542113	0.900	0.0002428	0.0004544	0.0618579	0.842	0.0001309	0.0002520	0.0515079	0.904
		Δ	0.0006488	0.0004060	0.0681106	0.914	0.0007515	0.0005349	0.0743893	0.854	0.0006125	0.0003607	0.0658590	0.924
	III	ρ	0.0079593	0.0023491	0.1773779	0.950	0.0082169	0.0025488	0.1820922	0.902	0.0078376	0.0022596	0.1751751	0.954
		λ	0.0001545	0.0002947	0.0536291	0.906	0.0001991	0.0003760	0.0583360	0.861	0.0001307	0.0002516	0.0515006	0.906
		Δ	0.0006410	0.0003966	0.0676276	0.927	0.0007034	0.0004710	0.0716195	0.875	0.0006124	0.0003605	0.0658537	0.922
(20,12)	I	ρ	0.0033578	0.0009300	0.1165151	0.960	0.0033840	0.0009444	0.1173534	0.961	0.0034008	0.0009557	0.1178517	0.951
		λ	0.0000431	0.0000842	0.0332730	0.937	0.0000449	0.0000876	0.0337951	0.944	0.0000471	0.0000917	0.0342128	0.947
		Δ	0.0002537	0.0001362	0.0434507	0.946	0.0002579	0.0001400	0.0439799	0.941	0.0002615	0.0001440	0.0443528	0.954
	II	ρ	0.0034244	0.00097770	0.1184877	0.926	0.0034562	0.0010085	0.1192615	0.878	0.0041238	0.0011436	0.1285224	0.963
		λ	0.0000511	0.0000994	0.0350271	0.901	0.0000602	0.0001160	0.0361904	0.865	0.0000542	0.0001057	0.0366528	0.932
		Δ	0.0002684	0.0001523	0.0450112	0.908	0.0002787	0.0001665	0.0458684	0.874	0.0003113	0.0001684	0.0478188	0.940
	III	ρ	0.0034018	0.0009603	0.1178386	0.947	0.0034153	0.0009726	0.1181740	0.925	0.0034031	0.0009567	0.1179274	0.952
		λ	0.0000479	0.0000933	0.0343847	0.915	0.0000516	0.0001001	0.0348485	0.908	0.0000472	0.0000918	0.0342491	0.947
		Δ	0.0002629	0.000146	0.0444612	0.926	0.0002671	0.0001518	0.0448040	0.914	0.0002617	0.0001442	0.0443945	0.952
(30,10)	I	ρ	0.0041062	0.0011336	0.1280183	0.959	0.0041200	0.0011454	0.1283417	0.952	0.0041208	0.001142	0.1284365	0.964
		λ	0.0000530	0.0001033	0.0363261	0.938	0.0000562	0.0001093	0.0367251	0.940	0.0000540	0.0001053	0.0365995	0.930
		Δ	0.0003083	0.0001657	0.0474913	0.944	0.0003122	0.0001708	0.0478015	0.944	0.0003108	0.000168	0.0477648	0.942
	II	ρ	0.0041585	0.0011867	0.1291837	0.911	0.0042790	0.0013020	0.1318252	0.806	0.0041238	0.0011436	0.1285224	0.963
		λ	0.0000659	0.0001274	0.0381024	0.883	0.0001004	0.0001902	0.0420079	0.801	0.0000542	0.0001057	0.0366528	0.932
		Δ	0.0003256	0.0001883	0.0488368	0.896	0.0003642	0.0002411	0.0517413	0.812	0.0003113	0.0001684	0.0478188	0.940
	III	ρ	0.0041476	0.0011758	0.1289448	0.925	0.0042512	0.0012673	0.1313689	0.841	0.0041266	0.0011453	0.1285991	0.961
		λ	0.0000630	0.0001221	0.0377365	0.897	0.0000871	0.0001663	0.0408114	0.828	0.0000545	0.0001062	0.0367094	0.933
		Δ	0.0003220	0.0001835	0.0485623	0.907	0.0003519	0.0002225	0.0509605	0.832	0.0003118	0.0001690	0.0478721	0.941
(30,20)	I	ρ	0.0019306	0.0005331	0.0891429	0.963	0.0019036	0.0005178	0.0879972	0.962	0.0019164	0.0005264	0.0885173	0.947
		λ	0.0000237	0.0000465	0.0254958	0.939	0.0000220	0.0000432	0.0247526	0.948	0.0000233	0.0000456	0.0251727	0.930
		Δ	0.0001459	0.0000771	0.0333695	0.947	0.0001414	0.0000730	0.0326213	0.954	0.000144	0.0000757	0.0330088	0.936
	II	ρ	0.0019364	0.0005422	0.0892683	0.905	0.0019274	0.0005412	0.0888184	0.850	0.0019170	0.0005269	0.0885384	0.944
		λ	0.0000263	0.0000513	0.0259579	0.898	0.0000268	0.0000523	0.0259128	0.848	0.0000234	0.0000458	0.0251982	0.930
		Δ	0.0001491	0.0000816	0.0336702	0.901	0.0001489	0.0000824	0.0335263	0.853	0.0001442	0.0000759	0.0330293	0.931
	III	ρ	0.0019323	0.0005374	0.0891467	0.925	0.0019139	0.0005284	0.0883461	0.920	0.0019174	0.0005271	0.0885532	0.942
		λ	0.0000251	0.00004900	0.0257131	0.921	0.0000242	0.0000473	0.0252768	0.897	0.0000234	0.0000458	0.0252105	0.929
		Δ	0.0001474	0.0000794	0.0334955	0.928	0.0001448	0.0000773	0.0330256	0.903	0.0001443	0.0000760	0.0330405	0.933

Table 7. Taylor approximation: absolute value of bias ($|Bias|$), length L, mean squared error (MSE) & coverage probability (Cov), when $R = 0.05, \rho = 0.42, \lambda = 0.17$ and $\Delta = 0.18$

(n,m)	Scheme		$T_1 = X_{[\frac{m}{4}]}$				$T_2 = X_{\left[\frac{4\times m}{5}\right]}$				$T_3 = X_{[m]+2}$			
		Estimate	Bias	MSE	L	Cov	Bias	MSE	L	Cov	Bias	MSE	L	Cov
(20,6)	I	ρ	0.0271432	0.0162017	0.4773089	0.969	0.0275186	0.0164389	0.4800898	0.962	0.0272555	0.0161282	0.4739996	0.937
		λ	0.0082224	0.0126661	0.4018969	0.929	0.0086909	0.0132607	0.409272	0.932	0.0088643	0.0132047	0.4027312	0.914
		Δ	0.0057672	0.0080358	0.3317536	0.952	0.0059062	0.0082998	0.3361419	0.948	0.0058525	0.0082399	0.3317387	0.914
	II	ρ	0.0278018	0.0163118	0.4751452	0.931	0.0292605	0.0168082	0.4762262	0.842	0.0272639	0.0161346	0.4740551	0.938
		λ	0.0099552	0.0141453	0.4110664	0.891	0.0130307	0.0169014	0.4341055	0.818	0.0088747	0.013224	0.4029332	0.914
		Δ	0.0060881	0.0086977	0.3373685	0.920	0.0067479	0.0100066	0.3520131	0.854	0.0058555	0.0082462	0.3318260	0.931
	III	ρ	0.0276843	0.0162676	0.4750558	0.933	0.0286572	0.0166965	0.4770592	0.872	0.0272635	0.0161354	0.4740463	0.936
		λ	0.0097115	0.0139002	0.4090867	0.902	0.0115533	0.0157595	0.4260858	0.857	0.0088739	0.0132282	0.4029562	0.915
		Δ	0.0060361	0.0085920	0.3362340	0.931	0.0064618	0.0094406	0.3462917	0.877	0.0058552	0.0082462	0.3318125	0.930
(20,12)	I	ρ	0.0116889	0.0069290	0.3197230	0.957	0.0117894	0.0069927	0.3212041	0.956	0.0118532	0.0070200	0.3216806	0.943
		λ	0.0033185	0.0053161	0.2717015	0.943	0.0034214	0.0054439	0.2747356	0.940	0.0035249	0.0055477	0.2764523	0.942
		Δ	0.0033185	0.0034438	0.2231695	0.952	0.0025259	0.0035069	0.2251114	0.950	0.0025512	0.0035552	0.2261863	0.948
	II	ρ	0.0119578	0.0070561	0.3213324	0.907	0.0120750	0.0070473	0.3201387	0.864	0.0118599	0.0070254	0.3218321	0.942
		λ	0.0037634	0.0058286	0.2795356	0.900	0.0041233	0.0061152	0.2817254	0.852	0.0035278	0.0070254	0.2766694	0.939
		Δ	0.0026001	0.0036615	0.2275200	0.915	0.0026599	0.0037874	0.2287992	0.861	0.0025532	0.0035583	0.2263348	0.944
	III	ρ	0.0118663	0.0070199	0.3210459	0.935	0.0119143	0.0070131	0.320648	0.916	0.0118617	0.0070265	0.3218542	0.943
		λ	0.0035909	0.0056399	0.2769721	0.916	0.0037366	0.0057469	0.2777373	0.902	0.0035300	0.0055554	0.2767239	0.941
		Δ	0.0025613	0.0035827	0.2261315	0.930	0.0025846	0.0036305	0.2266673	0.918	0.0025538	0.0035596	0.2263688	0.942
(30,10)	I	ρ	0.0142779	0.0084519	0.3516678	0.951	0.0143282	0.0084564	0.3514929	0.946	0.0143348	0.0084887	0.3523688	0.959
		λ	0.0040366	0.0064319	0.2962694	0.939	0.0041667	0.0065368	0.2972069	0.941	0.0040976	0.0065095	0.2978460	0.934
		Δ	0.0030239	0.0041698	0.2442824	0.947	0.0030475	0.0042177	0.2448807	0.949	0.0030444	0.0042073	0.2452597	0.948
	II	ρ	0.0144924	0.0084800	0.3499235	0.896	0.0149266	0.0084759	0.3462146	0.781	0.0143461	0.0084960	0.3525212	0.959
		λ	0.0046419	0.0070202	0.3010025	0.883	0.0059375	0.0080481	0.3086760	0.771	0.0041090	0.0065231	0.2981502	0.934
		Δ	0.0031329	0.0044066	0.2464480	0.893	0.0033574	0.0048756	0.2510218	0.794	0.0030484	0.0042144	0.2454582	0.947
	III	ρ	0.0144501	0.0084768	0.3502916	0.909	0.0148382	0.0085345	0.3487925	0.816	0.0143567	0.0085016	0.3526236	0.956
		λ	0.0045187	0.0069103	0.3001373	0.894	0.0055075	0.0077625	0.3080507	0.803	0.0041234	0.0065389	0.2984235	0.935
		Δ	0.0031111	0.0043598	0.2460144	0.909	0.0032969	0.0047424	0.2506196	0.819	0.0030524	0.0042218	0.2456298	0.946
(30,20)	I	ρ	0.0067265	0.0039962	0.2448918	0.955	0.0066201	0.0039275	0.2429445	0.963	0.0066722	0.0039546	0.2435249	0.946
		λ	0.0018708	0.0030461	0.2097421	0.942	0.0017590	0.0028958	0.2052955	0.952	0.0018350	0.0029890	0.2073784	0.930
		Δ	0.0014400	0.0019894	0.1716749	0.952	0.0014018	0.0019190	0.1689887	0.954	0.0014222	0.0019588	0.1701598	0.942
	II	ρ	0.0067539	0.0039836	0.2437065	0.897	0.0067247	0.0039495	0.2420644	0.837	0.0066746	0.0039552	0.2435164	0.938
		λ	0.0019971	0.0031665	0.2104179	0.895	0.0020259	0.0031900	0.2090956	0.847	0.0018407	0.0029951	0.2074645	0.927
		Δ	0.0014576	0.0020316	0.1717445	0.896	0.0014524	0.0020296	0.1705934	0.848	0.0014233	0.0019611	0.1702014	0.937
	III	ρ	0.0067359	0.0039852	0.2441088	0.922	0.0066665	0.0039366	0.2424619	0.910	0.0066761	0.0039559	0.2435317	0.938
		λ	0.0019357	0.0031063	0.2098730	0.924	0.0018815	0.0030337	0.2070011	0.897	0.0018430	0.0029978	0.2075231	0.929
		Δ	0.0014477	0.0020092	0.171557	0.928	0.0014247	0.0019697	0.1696612	0.910	0.0014239	0.0019623	0.1702344	0.937

Table 8. Taylor approximation: absolute value of bias ($|Bias|$), length L, mean squared error (MSE) & coverage probability (Cov), when $R = 0.2, \rho = 0.7, \lambda = 0.5$ and $\Delta = 0.42$

(n,m)	Scheme		$T_1 = X_{[\frac{m}{4}]}$				$T_2 = X_{\left[\frac{4\times m}{5}\right]}$				$T_3 = X_{[m]+2}$			
		Estimate	Bias	MSE	L	Cov	Bias	MSE	L	Cov	Bias	MSE	L	Cov
(20,6)	I	ρ	0.05625460	0.0281264	0.6126518	0.903	0.0557921	0.0279431	0.6106681	0.905	0.0559284	0.0277509	0.6071673	0.894
		λ	0.0575088	0.0537097	0.8535895	0.899	0.056688	0.0529309	0.8459899	0.894	0.0574538	0.0528118	0.8426389	0.884
		Δ	0.0176547	0.0302777	0.6626746	0.961	0.0174599	0.0299434	0.6580928	0.952	0.0175964	0.0302156	0.6604309	0.947
	II	ρ	0.0557064	0.0268148	0.5923238	0.852	0.0538973	0.0248834	0.5626493	0.763	0.0559201	0.0277480	0.6071495	0.894
		λ	0.0588714	0.0513039	0.8202386	0.834	0.0585295	0.0472085	0.7665091	0.735	0.0574381	0.0527998	0.8425271	0.884
		Δ	0.0176569	0.0306192	0.6609926	0.921	0.0166425	0.0305830	0.6516884	0.849	0.0175924	0.0302082	0.6603335	0.947
	III	ρ	0.0558320	0.0269591	0.5945166	0.858	0.0545661	0.0256274	0.5744046	0.798	0.0559050	0.0277328	0.6069438	0.895
		λ	0.0588837	0.0516343	0.8243695	0.839	0.0585119	0.0486076	0.7858314	0.771	0.0574315	0.0527647	0.8420961	0.884
		Δ	0.0177059	0.0306268	0.6616996	0.926	0.0171406	0.0304478	0.6536668	0.875	0.017574	0.0302032	0.660211	0.947
(20,12)	I	ρ	0.0245060	0.0116603	0.4109311	0.929	0.0246464	0.0116073	0.4096799	0.920	0.0246004	0.0115860	0.4092171	0.921
		λ	0.0250462	0.0236956	0.5894077	0.926	0.0255019	0.0237796	0.589966	0.920	0.0254305	0.0236847	0.5885137	0.917
		Δ	0.0076707	0.0131185	0.4428679	0.956	0.0077515	0.0132614	0.4450848	0.952	0.0077348	0.0132328	0.4444544	0.948
	II	ρ	0.0243908	0.0112985	0.4031750	0.874	0.0244312	0.0108853	0.3934397	0.804	0.0246007	0.0115786	0.4090475	0.918
		λ	0.0254828	0.0229066	0.5760317	0.861	0.0264199	0.0221931	0.5626014	0.792	0.0254459	0.0236706	0.5882654	0.915
		Δ	0.0077289	0.0131720	0.4416812	0.909	0.0078591	0.0134279	0.4438710	0.854	0.0077376	0.0132366	0.4444810	0.949
	III	ρ	0.0244383	0.0114477	0.4064483	0.897	0.0245632	0.0112544	0.4019322	0.863	0.0246011	0.0115743	0.4089504	0.917
		λ	0.0253018	0.0232319	0.5816347	0.887	0.0260001	0.0230153	0.5770663	0.855	0.0254559	0.0236631	0.5881299	0.915
		Δ	0.0077022	0.0131463	0.442106	0.930	0.0078165	0.0133497	0.4446501	0.908	0.0077392	0.0132390	0.4444979	0.948
(30,10)	I	ρ	0.0301362	0.0143943	0.4530857	0.919	0.0301874	0.014358	0.4522663	0.916	0.0302199	0.0143381	0.4517276	0.912
		λ	0.0309343	0.0288916	0.6461867	0.912	0.031137	0.0288954	0.6457965	0.910	0.0312608	0.0288975	0.6455462	0.907
		Δ	0.0094673	0.0161706	0.4900649	0.953	0.0095007	0.0162344	0.4908796	0.949	0.0095151	0.0162800	0.4915098	0.947
	II	ρ	0.0299532	0.0136703	0.4383383	0.834	0.0292229	0.0127344	0.4179451	0.732	0.0302211	0.0143349	0.4516607	0.9116
		λ	0.0319459	0.0274364	0.6222027	0.821	0.0320912	0.0251123	0.5839731	0.709	0.0312701	0.0288946	0.6454683	0.9067
		Δ	0.0095902	0.0163688	0.4892762	0.882	0.0093565	0.0163694	0.4840696	0.785	0.0095166	0.0162823	0.4915269	0.9472
	III	ρ	0.0299872	0.0137968	0.4409804	0.850	0.0295406	0.0130943	0.4259188	0.768	0.0302203	0.0143277	0.4515170	0.912
		λ	0.0317714	0.0276851	0.6264288	0.837	0.0321122	0.0260237	0.5989469	0.754	0.0312827	0.0288826	0.6452509	0.906
		Δ	0.0095710	0.0163303	0.4893699	0.895	0.0094781	0.0163720	0.4861507	0.824	0.0095185	0.0162848	0.4915263	0.946
(30,20)	I	ρ	0.0141938	0.0066069	0.3133374	0.938	0.0141488	0.0065994	0.3131467	0.935	0.0141637	0.0065852	0.3127598	0.931
		λ	0.014666	0.0139679	0.4575892	0.938	0.0145704	0.0138924	0.4562539	0.930	0.0146346	0.0138839	0.4560091	0.925
		Δ	0.0044456	0.0076289	0.3396541	0.957	0.0044277	0.0075940	0.3387812	0.950	0.0044382	0.0076116	0.3390946	0.945
	II	ρ	0.0141373	0.0064204	0.3082446	0.863	0.0140679	0.0062692	0.3039185	0.801	0.0141623	0.0065813	0.3126556	0.929
		λ	0.0148938	0.0135253	0.4485924	0.855	0.0150231	0.0131398	0.4405622	0.793	0.0146389	0.0138742	0.4558187	0.923
		Δ	0.0044732	0.0076520	0.3388453	0.891	0.0044842	0.0076584	0.3378817	0.831	0.0044386	0.0076119	0.3390736	0.944
	III	ρ	0.0141629	0.0065038	0.3105594	0.896	0.0141133	0.0064347	0.3086631	0.868	0.0141622	0.0065798	0.3126168	0.929
		λ	0.0147926	0.0137238	0.4526672	0.892	0.0148066	0.0135222	0.4486665	0.859	0.0146414	0.0138711	0.4557565	0.923
		Δ	0.0044600	0.0076406	0.3391822	0.921	0.0044578	0.0076257	0.3383269	0.892	0.0044390	0.0076123	0.3390736	0.944

Table 9. Taylor approximation: absolute value of bias ($|Bias|$), length L, mean squared error (MSE) & coverage probability (Cov), when $R = 0.8, \rho = 0.98, \lambda = 0.96$ and $\Delta = 0.85$

(n,m)	Scheme		$T_1 = X_{[\frac{m}{4}]}$				$T_2 = X_{[\frac{4\times m}{5}]}$				$T_3 = X_{[m]+2}$			
		Estimate	Bias	MSE	L	Cov	Bias	MSE	L	Cov	Bias	MSE	L	Cov
(20,6)	I	ρ	0.0629316	0.013083	0.3136123	0.751	0.0628740	0.0134689	0.3215902	0.762	0.0620400	0.0135154	0.3232825	0.758
		λ	0.1102586	0.0414021	0.5723596	0.749	0.1093838	0.0421911	0.5843029	0.759	0.1076603	0.0422024	0.5862373	0.756
		Δ	0.0150567	0.0570747	0.9227707	0.979	0.0146734	0.0567018	0.9195845	0.976	0.0137353	0.0565873	0.9184364	0.973
	II	ρ	0.0590975	0.0141734	0.3397860	0.774	0.0537394	0.0158385	0.3770439	0.790	0.0620410	0.0135230	0.3233951	0.758
		λ	0.100483	0.0430643	0.6075083	0.765	0.0868790	0.0450793	0.6505005	0.762	0.1076447	0.0422167	0.5863889	0.755
		Δ	0.0107958	0.0556012	0.9091969	0.956	0.0071651	0.0529574	0.8838795	0.898	0.0137556	0.0565805	0.9183799	0.973
	III	ρ	0.0594711	0.0140324	0.3368168	0.774	0.0560893	0.0153106	0.3648882	0.788	0.0620064	0.013542	0.3237355	0.759
		λ	0.1014769	0.0428360	0.6037576	0.766	0.0923564	0.0446436	0.6383941	0.767	0.1075356	0.042248	0.5868227	0.756
		Δ	0.0111250	0.0557894	0.9109468	0.960	0.0083894	0.0539651	0.8938710	0.924	0.0137865	0.0565591	0.9181924	0.973
(20,12)	I	ρ	0.0275208	0.0032187	0.1628338	0.774	0.0272212	0.0031885	0.1615351	0.771	0.0272009	0.0032354	0.1632825	0.773
		λ	0.0507949	0.0111295	0.3086751	0.773	0.0502370	0.0110056	0.3060242	0.769	0.0501144	0.0111321	0.3089275	0.772
		Δ	0.0082111	0.0246413	0.6116880	0.978	0.0076198	0.0246492	0.6117445	0.977	0.0074989	0.0246109	0.6112407	0.975
	II	ρ	0.0265605	0.0036448	0.1776207	0.789	0.0252343	0.003995	0.1905921	0.798	0.0271785	0.0032424	0.1635548	0.772
		λ	0.0480718	0.0121881	0.3319599	0.784	0.0448373	0.0129953	0.3518632	0.788	0.0500579	0.0111493	0.3093672	0.771
		Δ	0.0064388	0.0242585	0.6065633	0.953	0.0048296	0.023906	0.601707	0.924	0.0074038	0.0246045	0.6111543	0.975
	III	ρ	0.0269620	0.0034738	0.1717513	0.783	0.0262530	0.0035898	0.1767399	0.793	0.0271649	0.0032464	0.1636988	0.773
		λ	0.0491913	0.0117718	0.3228442	0.780	0.0475789	0.0120399	0.3308045	0.789	0.0500232	0.0111591	0.3095925	0.771
		Δ	0.0071758	0.024416	0.6086845	0.965	0.0060913	0.0242971	0.6070468	0.954	0.0074158	0.0246007	0.6111032	0.975
(30,10)	I	ρ	0.0336360	0.0045955	0.1932728	0.773	0.0334671	0.0045954	0.1930931	0.772	0.0333632	0.0045893	0.1931137	0.773
		λ	0.0613047	0.0155428	0.3631435	0.772	0.0609635	0.0155067	0.3624600	0.770	0.0607800	0.0154833	0.3624353	0.772
		Δ	0.0092964	0.0302925	0.6771687	0.977	0.0090529	0.0302793	0.6769795	0.976	0.0089251	0.0302767	0.6769169	0.974
	II	ρ	0.0315784	0.0053300	0.2160390	0.787	0.0290956	0.0063966	0.2484373	0.810	0.0333520	0.0045913	0.1931229	0.772
		λ	0.0558305	0.0171604	0.3970567	0.780	0.0491309	0.0193890	0.4433215	0.787	0.0607525	0.0154848	0.3623991	0.771
		Δ	0.0064249	0.0295431	0.6679282	0.939	0.0040293	0.0283724	0.6531454	0.868	0.0089111	0.0302739	0.6768811	0.974
	III	ρ	0.0319478	0.0052097	0.2123686	0.787	0.0300684	0.0059971	0.2362642	0.808	0.0333304	0.0045981	0.1932623	0.772
		λ	0.0567872	0.0169152	0.3918320	0.781	0.0516861	0.0185810	0.4263932	0.790	0.0606956	0.0154975	0.3625554	0.771
		Δ	0.0068682	0.0296759	0.6695963	0.946	0.0046989	0.0288378	0.6591284	0.896	0.0088116	0.0302666	0.6767912	0.974
(30,20)	I	ρ	0.0157053	0.0012227	0.1023877	0.773	0.0157279	0.0012593	0.1043478	0.776	0.0156734	0.0012672	0.1044796	0.774
		λ	0.0297394	0.0044292	0.1976157	0.773	0.0297213	0.0045462	0.2011134	0.776	0.0295968	0.0045672	0.2012317	0.774
		Δ	0.0052661	0.0141598	0.4649246	0.980	0.0053669	0.0141353	0.4645130	0.978	0.0051457	0.0141286	0.4643954	0.975
	II	ρ	0.0152284	0.0014716	0.1147454	0.789	0.0148511	0.0016746	0.1247492	0.800	0.0156638	0.0012727	0.1047359	0.775
		λ	0.0283352	0.0051574	0.2186248	0.784	0.0272243	0.0057232	0.2352612	0.791	0.0295677	0.0045838	0.2016696	0.775
		Δ	0.0040723	0.0139703	0.4616445	0.946	0.0034117	0.0138066	0.4587685	0.909	0.0050910	0.0141246	0.4643273	0.976
	III	ρ	0.0154432	0.0013625	0.1093403	0.783	0.0152934	0.0014704	0.1148985	0.791	0.0156599	0.0012745	0.1048255	0.776
		λ	0.0289600	0.0048435	0.2095048	0.782	0.0284677	0.0051597	0.2190266	0.788	0.0295566	0.0045893	0.2018241	0.776
		Δ	0.0044994	0.0140551	0.4631190	0.962	0.0041435	0.0139732	0.4617028	0.946	0.0050641	0.0141233	0.4643049	0.974

Table 10. Bootstrap results: absolute value of bias ($|Bias|$), length L, mean squared error (MSE) & coverage probability (Cov), when $R = 0.005, \rho = 0.14, \lambda = 0.019$ and $\Delta = 0.030$

(n,m)	Scheme		$T_1 = X_{[\frac{m}{4}]}$				$T_2 = X_{[\frac{4\times m}{5}]}$				$T_3 = X_{[m]+2}$			
		Estimate	Bias	MSE	L	Cov	Bias	MSE	L	Cov	Bias	MSE	L	Cov
(20,6)	I	ρ	0.0057719	0.0021693	0.1718689	0.937	0.0060681	0.0023285	0.1767447	0.938	0.0063525	0.0025073	0.1797093	0.950
		λ	0.0039358	0.0003375	0.0633661	0.937	0.004181	0.0003834	0.0661163	0.938	0.0043293	0.0005461	0.0673430	0.950
		Δ	0.0037974	0.0004283	0.073176	0.937	0.0040114	0.0004724	0.0758094	0.938	0.0041714	0.0005809	0.0770663	0.950
	II	ρ	0.0082319	0.0031671	0.2095153	0.945	0.0131046	0.0060223	0.2746659	0.947	0.0063667	0.0025124	0.1799279	0.949
		λ	0.0059441	0.0006063	0.0843087	0.945	0.0105502	0.0017741	0.1310053	0.947	0.0043435	0.0005450	0.0674629	0.949
		Δ	0.0055539	0.0006954	0.0931233	0.945	0.0093248	0.0016828	0.1334303	0.947	0.0041827	0.0005814	0.0771835	0.949
	III	ρ	0.0079313	0.0030251	0.2053284	0.944	0.0111045	0.0048143	0.2485872	0.939	0.0064118	0.0025226	0.1808699	0.948
		λ	0.0056887	0.0005595	0.0818932	0.944	0.0085835	0.0012233	0.1112053	0.939	0.0043854	0.0005398	0.0679887	0.948
		Δ	0.0053357	0.0006527	0.0908633	0.944	0.007732	0.0012374	0.1166626	0.939	0.0042177	0.0005804	0.0776895	0.948
(20,12)	I	ρ	0.0027247	0.0008590	0.1149772	0.957	0.0029247	0.0009988	0.1166813	0.945	0.00301170	0.0010065	0.1200596	0.942
		λ	0.0017041	0.0000969	0.0369412	0.957	0.0018071	0.0001251	0.0378388	0.945	0.0018786	0.0001246	0.0392068	0.942
		Δ	0.0017315	0.0001432	0.0457371	0.957	0.0018366	0.0001757	0.0466124	0.945	0.0019038	0.0001766	0.0481555	0.942
	II	ρ	0.0039793	0.0013400	0.1397905	0.948	0.0054716	0.0020278	0.1650620	0.948	0.0030345	0.0010107	0.1205079	0.941
		λ	0.0025686	0.0001772	0.0474050	0.948	0.0036613	0.0003309	0.0595455	0.948	0.0018938	0.0001248	0.0393854	0.941
		Δ	0.0025604	0.0002427	0.0570842	0.948	0.0035662	0.0004092	0.0694439	0.948	0.0019192	0.0001771	0.0483513	0.941
	III	ρ	0.0034443	0.0011387	0.1296862	0.947	0.0041116	0.0015030	0.1412575	0.940	0.0030488	0.0010151	0.120799	0.943
		λ	0.0021937	0.0001415	0.0430086	0.947	0.0026542	0.0002145	0.0483154	0.940	0.0019035	0.0001255	0.0395058	0.943
		Δ	0.0022046	0.0001997	0.0523862	0.947	0.0026382	0.0002833	0.0578976	0.940	0.0019287	0.0001779	0.0484832	0.943
(30,10)	I	ρ	0.0034625	0.0011171	0.1290878	0.952	0.0035330	0.0011183	0.1302099	0.960	0.0036491	0.0011874	0.1328062	0.955
		λ	0.0021874	0.0001373	0.0427940	0.952	0.0022231	0.0001455	0.0431090	0.960	0.0023068	0.0001494	0.0443288	0.955
		Δ	0.0022027	0.0001949	0.0521607	0.952	0.0022414	0.0002013	0.0525420	0.960	0.0023200	0.0002098	0.0538257	0.955
	II	ρ	0.0057561	0.0020389	0.1698244	0.946	0.0091697	0.0035282	0.2225386	0.956	0.0036538	0.0011857	0.1328795	0.955
		λ	0.0038776	0.0003315	0.0619040	0.946	0.0067314	0.0007644	0.0920474	0.956	0.0023099	0.0001491	0.0443387	0.955
		Δ	0.0037657	0.0004109	0.0718598	0.946	0.0062372	0.0008297	0.1001930	0.956	0.0023234	0.0002094	0.0538427	0.955
	III	ρ	0.0053110	0.0018380	0.1623589	0.948	0.0076540	0.0028450	0.2005434	0.954	0.0036705	0.0011861	0.1331724	0.955
		λ	0.0035336	0.0002845	0.0580703	0.948	0.0054279	0.0005503	0.0785065	0.954	0.0023216	0.0001491	0.0444427	0.955
		Δ	0.0034563	0.0003610	0.0680675	0.948	0.0051285	0.0006295	0.0878331	0.954	0.0023349	0.0002094	0.0539614	0.955
(30,20)	I	ρ	0.0017891	0.0005424	0.0877089	0.945	0.0017255	0.0004811	0.0887829	0.955	0.0017623	0.0005162	0.0898892	0.949
		λ	0.0010244	0.0000549	0.0267887	0.945	0.0010205	0.0000478	0.0272002	0.955	0.0010433	0.0000508	0.0275441	0.949
		Δ	0.0010708	0.0000854	0.0340056	0.945	0.0010590	0.0000751	0.0345083	0.955	0.0010829	0.0000801	0.0349269	0.949
	II	ρ	0.0027930	0.0008605	0.1127807	0.951	0.0035817	0.0012324	0.1318379	0.945	0.0017852	0.0005201	0.0904284	0.952
		λ	0.0016880	0.0000970	0.0360656	0.951	0.0022949	0.0001614	0.0442643	0.945	0.0010574	0.0000515	0.0277290	0.952
		Δ	0.0017317	0.0001432	0.0447414	0.951	0.0022980	0.0002225	0.0536219	0.945	0.0010977	0.0000809	0.0351495	0.952
	III	ρ	0.0023316	0.0007117	0.1019545	0.948	0.0025873	0.0008291	0.1109394	0.951	0.0017928	0.0005225	0.0906322	0.951
		λ	0.0013793	0.0000762	0.0319410	0.948	0.0016046	0.0000945	0.0355510	0.951	0.0010623	0.0000518	0.0278004	0.951
		Δ	0.0014268	0.0001153	0.0400356	0.948	0.0016357	0.0001391	0.0440948	0.951	0.0011027	0.0000813	0.0352344	0.951

Table 11. Bootstrap results: absolute value of bias ($|Bias|$), length L, mean squared error (MSE) & coverage probability (Cov), when $R = 0.05, \rho = 0.42, \lambda = 0.17$ and $\Delta = 0.18$

(n,m)	Scheme		$T_1 = X_{[\frac{m}{4}]}$				$T_2 = X_{\left[\frac{4\times m}{5}\right]}$				$T_3 = X_{[m]+2}$			
		Estimate	Bias	MSE	L	Cov	Bias	MSE	L	Cov	Bias	MSE	L	Cov
(20,6)	I	ρ	0.0094978	0.0129907	0.4235773	0.937	0.0097378	0.0136084	0.4323874	0.938	0.0104861	0.0132736	0.4396057	0.950
		λ	0.0194719	0.0138432	0.4140023	0.937	0.0202544	0.0146998	0.4247872	0.938	0.0213622	0.0147013	0.4287571	0.950
		Δ	0.0129532	0.0083320	0.3305778	0.937	0.0135303	0.0088960	0.3395280	0.938	0.0142336	0.0090488	0.3437481	0.950
	II	ρ	0.0126486	0.0171257	0.4917633	0.945	0.0191223	0.0247619	0.5792681	0.947	0.0104968	0.0132940	0.4400079	0.949
		λ	0.0259875	0.0194883	0.4928500	0.945	0.0378904	0.0321148	0.6001988	0.947	0.0214023	0.0147710	0.4293260	0.949
		Δ	0.0178690	0.0119328	0.3985085	0.945	0.0270900	0.0203964	0.5010937	0.947	0.0142657	0.0090909	0.3441934	0.949
	III	ρ	0.0122549	0.016639	0.4843928	0.944	0.0165280	0.0218074	0.5471864	0.939	0.0105486	0.0133754	0.4417017	0.948
		λ	0.0252023	0.0187734	0.4843507	0.944	0.0333555	0.0275673	0.5609553	0.939	0.0215318	0.0149129	0.4315642	0.948
		Δ	0.0172891	0.0114536	0.3910387	0.944	0.0230782	0.0172850	0.4621257	0.939	0.0143736	0.0091768	0.3459848	0.948
(20,12)	I	ρ	0.0050470	0.0058923	0.3024659	0.957	0.00545150	0.0065995	0.3056943	0.945	0.0056654	0.0066636	0.3136868	0.942
		λ	0.0101427	0.0053418	0.2787546	0.957	0.01063530	0.0062976	0.2824768	0.945	0.0110002	0.0063983	0.2914379	0.942
		Δ	0.0063694	0.0032874	0.2224686	0.957	0.00677380	0.0038402	0.2256195	0.945	0.0069784	0.0038800	0.2323326	0.942
	II	ρ	0.0069741	0.0086450	0.3589377	0.948	0.0091878	0.0119409	0.4112578	0.948	0.0057017	0.0066987	0.3147307	0.941
		λ	0.0142612	0.008501	0.3387726	0.948	0.0187339	0.0128143	0.3966464	0.948	0.0110855	0.0064175	0.2924645	0.941
		Δ	0.0091621	0.0051477	0.2698208	0.948	0.0123369	0.0077661	0.3171392	0.948	0.0070336	0.0038944	0.2331630	0.941
	III	ρ	0.0061811	0.0075306	0.3364148	0.947	0.0072120	0.0093945	0.3612012	0.940	0.0057249	0.0067243	0.3154023	0.943
		λ	0.0125412	0.0071878	0.3144940	0.947	0.0145194	0.0095797	0.3412721	0.940	0.0111350	0.0064462	0.2931757	0.943
		Δ	0.0079828	0.0043711	0.2505796	0.947	0.0094188	0.0057851	0.2723580	0.940	0.0070667	0.0039113	0.2337210	0.943
(30,10)	I	ρ	0.0062764	0.0074177	0.3350111	0.952	0.0064567	0.0072630	0.3380036	0.960	0.0066154	0.0078107	0.3435340	0.955
		λ	0.0125689	0.0070565	0.3136631	0.952	0.0127941	0.0071344	0.3153762	0.960	0.0131511	0.0075272	0.3222788	0.955
		Δ	0.0080038	0.0042902	0.2496368	0.952	0.0081689	0.0043136	0.2515303	0.960	0.0084151	0.0045634	0.2566152	0.955
	II	ρ	0.0095358	0.0120137	0.4209876	0.946	0.0142513	0.0177257	0.5140453	0.956	0.0066236	0.0078020	0.3437599	0.955
		λ	0.0196853	0.0128972	0.4086553	0.946	0.0288333	0.0212418	0.5156304	0.956	0.0131709	0.0075149	0.3223657	0.955
		Δ	0.0129611	0.0078073	0.3263921	0.946	0.0197366	0.0131431	0.4201960	0.956	0.0084293	0.0045565	0.2567261	0.955
	III	ρ	0.0089342	0.0110818	0.4063776	0.948	0.0121448	0.0152767	0.4782279	0.954	0.0066512	0.0078042	0.3444821	0.955
		λ	0.0183643	0.0116627	0.3919223	0.948	0.0249387	0.0176339	0.4730264	0.954	0.0132333	0.0075152	0.3229916	0.955
		Δ	0.0120308	0.0070491	0.3125371	0.948	0.0168537	0.0107832	0.3819409	0.954	0.0084704	0.0045572	0.2572593	0.955
(30,20)	I	ρ	0.0036759	0.0038644	0.2354817	0.945	0.0035030	0.0034464	0.2382119	0.955	0.0035674	0.0037094	0.2411036	0.949
		λ	0.0065141	0.0032788	0.2108528	0.945	0.0064497	0.0029083	0.2142501	0.955	0.0065897	0.0030976	0.2166533	0.949
		Δ	0.0041307	0.0020634	0.1696722	0.945	0.0040288	0.0018320	0.1720391	0.955	0.0041215	0.0019586	0.1740503	0.949
	II	ρ	0.0053210	0.0059082	0.2972158	0.951	0.0063791	0.0079782	0.3402727	0.945	0.0036095	0.0037318	0.2424851	0.952
		λ	0.0101865	0.0053265	0.2729093	0.951	0.0129548	0.0078616	0.3205403	0.945	0.0066767	0.0031270	0.2179933	0.952
		Δ	0.0064683	0.0032858	0.2180943	0.951	0.0082746	0.0047494	0.2550044	0.945	0.0041767	0.0019747	0.1751001	0.952
	III	ρ	0.0045583	0.0049761	0.2709806	0.948	0.0048714	0.0056654	0.2922562	0.951	0.0036235	0.0037476	0.243001	0.951
		λ	0.0085267	0.0043581	0.2461344	0.948	0.0096118	0.0051763	0.2692787	0.951	0.0067057	0.0031430	0.2184961	0.951
		Δ	0.0054068	0.0027121	0.1972199	0.948	0.0060411	0.0031786	0.2148612	0.951	0.0041951	0.0019842	0.1754943	0.951

Table 12. Bootstrap results: absolute value of bias ($|Bias|$), length L, mean squared error (MSE) & coverage probability (Cov), when $R = 0.2, \rho = 0.7, \lambda = 0.5$ and $\Delta = 0.42$

(n,m)	Scheme		$T_1 = X_{[\frac{m}{4}]}$				$T_2 = X_{[\frac{4\times m}{5}]}$				$T_3 = X_{[m]+2}$			
			Bias	MSE	L	Cov	Bias	MSE	L	Cov	Bias	MSE	L	Cov
(20,6)	I	ρ	0.0117477	0.015306	0.4520037	0.938	0.0124744	0.0159014	0.4582487	0.938	0.0123077	0.0151538	0.4711041	0.953
		λ	0.0205008	0.0305288	0.6375286	0.938	0.0214902	0.0309625	0.6459255	0.938	0.0216617	0.0292849	0.6600009	0.953
		Δ	0.0164230	0.0221759	0.5580971	0.938	0.0170966	0.0225292	0.5678643	0.938	0.017472	0.0210157	0.5780715	0.953
	II	ρ	0.0173613	0.0189994	0.5056227	0.957	0.0283296	0.0226172	0.5720952	0.978	0.0123457	0.0151013	0.4713209	0.952
		λ	0.0304653	0.0351571	0.6961822	0.957	0.0492816	0.0381734	0.7637851	0.978	0.0216993	0.0292323	0.6604776	0.952
		Δ	0.0238102	0.0257647	0.6240167	0.957	0.0393319	0.0282494	0.6979926	0.978	0.0174671	0.0210003	0.5786043	0.952
	III	ρ	0.0167805	0.0186198	0.4996590	0.951	0.0236159	0.0205869	0.5471290	0.974	0.0124580	0.0151167	0.4721985	0.951
		λ	0.0294620	0.0347652	0.6898956	0.951	0.0416742	0.0364771	0.7410927	0.974	0.0218503	0.0292777	0.6617918	0.951
		Δ	0.0229306	0.0254761	0.6168704	0.951	0.0330614	0.0271488	0.6735641	0.974	0.0175051	0.0210673	0.5800995	0.951
(20,12)	I	ρ	0.0051924	0.0081654	0.3522493	0.957	0.0053191	0.0086741	0.3547387	0.945	0.0057734	0.0086649	0.3613832	0.942
		λ	0.0090004	0.0174741	0.5136336	0.957	0.0095769	0.0186152	0.5160287	0.945	0.0101476	0.0187232	0.5263887	0.942
		Δ	0.0075834	0.0113431	0.4190711	0.957	0.0083219	0.0124018	0.4220805	0.945	0.0087366	0.0126256	0.4329965	0.942
	II	ρ	0.0077081	0.0111095	0.4034235	0.948	0.0110582	0.0140089	0.4477545	0.948	0.005830	0.0087364	0.3624388	0.941
		λ	0.0135373	0.0230542	0.5792616	0.948	0.0195532	0.0280009	0.6321406	0.948	0.010222	0.0188477	0.5275483	0.941
		Δ	0.0109682	0.0159236	0.4893388	0.948	0.0153288	0.0200989	0.5478277	0.948	0.008782	0.0126957	0.4342903	0.941
	III	ρ	0.0066184	0.0099493	0.3835531	0.947	0.0080926	0.0115844	0.4054907	0.940	0.0058606	0.0087640	0.3630609	0.943
		λ	0.011589	0.0209350	0.5541380	0.947	0.0142447	0.0240177	0.5805475	0.940	0.0102687	0.0189043	0.5283669	0.943
		Δ	0.0094657	0.0141743	0.462099	0.947	0.0116148	0.0168391	0.4907971	0.940	0.0088164	0.0127407	0.4351819	0.943
(30,10)	I	ρ	0.0063220	0.0098149	0.3813168	0.952	0.0065258	0.0092576	0.3864415	0.960	0.0067700	0.0102060	0.3896721	0.955
		λ	0.0111349	0.020749	0.5524642	0.952	0.0117229	0.0198060	0.5582560	0.960	0.0121018	0.0214703	0.5625002	0.955
		Δ	0.0095183	0.0140212	0.4607232	0.952	0.0099746	0.0135339	0.4643317	0.960	0.0101267	0.0146796	0.4707832	0.955
	II	ρ	0.0111383	0.0141172	0.4539873	0.946	0.0193504	0.0183464	0.5289440	0.975	0.0067787	0.0102024	0.3901201	0.955
		λ	0.0194449	0.0280094	0.6410094	0.946	0.0337041	0.0333715	0.7223986	0.975	0.0121252	0.0214542	0.5629440	0.955
		Δ	0.0157761	0.0201779	0.5586898	0.946	0.0271128	0.0245698	0.6482635	0.975	0.0101429	0.0146647	0.4710832	0.955
	III	ρ	0.0101599	0.0133057	0.4425067	0.948	0.0159464	0.0165252	0.5018451	0.962	0.0068184	0.0102117	0.3909943	0.955
		λ	0.0177450	0.0266800	0.6278937	0.948	0.0278609	0.0312857	0.6940647	0.962	0.0121964	0.0214546	0.5639524	0.955
		Δ	0.0145597	0.0190826	0.5437723	0.948	0.0225508	0.0229354	0.6162352	0.962	0.0101831	0.0146654	0.4719769	0.955
(30,20)	I	ρ	0.0033043	0.0056780	0.2852449	0.945	0.0033912	0.0050746	0.2872892	0.955	0.0035233	0.0055372	0.2909235	0.949
		λ	0.0060500	0.0123430	0.4193755	0.945	0.0055839	0.0111278	0.4246590	0.955	0.0059573	0.0119694	0.4288250	0.949
		Δ	0.0054071	0.0075049	0.3276875	0.945	0.0051606	0.0067049	0.3318073	0.955	0.0052665	0.0072067	0.3357459	0.949
	II	ρ	0.0050870	0.0082616	0.3477700	0.951	0.0067238	0.0101398	0.3842908	0.945	0.0035242	0.0055508	0.2924221	0.952
		λ	0.0090466	0.0175064	0.5065918	0.951	0.0117173	0.0215014	0.5568929	0.945	0.0059851	0.0120183	0.4310227	0.952
		Δ	0.0079416	0.0113462	0.4116783	0.951	0.0100717	0.0148207	0.4668933	0.945	0.0053283	0.0072512	0.3376696	0.952
	III	ρ	0.0042563	0.0071328	0.3219558	0.948	0.0049234	0.0077694	0.3408670	0.951	0.0035335	0.0055702	0.2929728	0.951
		λ	0.0076027	0.0152661	0.4711108	0.948	0.0081987	0.0167407	0.4991944	0.951	0.0060020	0.0120601	0.4318050	0.951
		Δ	0.0067645	0.0096235	0.3764679	0.948	0.0072995	0.0108885	0.4052772	0.951	0.0053489	0.0072820	0.3383823	0.951

Table 13. Bootstrap results: absolute value of bias ($|Bias|$), length L, mean squared error (MSE) & coverage probability (Cov), when $R = 0.8, \rho = 0.98, \lambda = 0.96$ and $\Delta = 0.85$

(n,m)	Scheme		$T_1 = X_{[\frac{m}{4}]}$				$T_2 = X_{\left[\frac{4\times m}{5}\right]}$				$T_3 = X_{[m]+2}$			
			Bias	MSE	L	Cov	Bias	MSE	L	Cov	Bias	MSE	L	Cov
(20,6)	I	ρ	0.0308060	0.0072922	0.2657358	0.958	0.0320181	0.0083925	0.2712829	0.953	0.0334323	0.0084632	0.2797605	0.961
		λ	0.0530997	0.0225076	0.4527855	0.958	0.0554572	0.0251130	0.4595161	0.953	0.0577409	0.0253653	0.4722637	0.961
		Δ	0.0592748	0.0314768	0.5230580	0.958	0.0632343	0.0337075	0.5272644	0.953	0.0645894	0.0341479	0.5372285	0.961
	II	ρ	0.0404262	0.0123259	0.3363409	0.954	0.0553779	0.0223033	0.4382527	0.953	0.0334900	0.0084394	0.2797889	0.960
		λ	0.0680466	0.0359072	0.5489102	0.954	0.0901309	0.0598864	0.6716731	0.953	0.0578427	0.0253067	0.4723510	0.960
		Δ	0.0715550	0.0456126	0.5919078	0.954	0.0870255	0.0690497	0.6798790	0.953	0.0646732	0.0340842	0.5374120	0.960
	III	ρ	0.0393436	0.0116744	0.3284257	0.953	0.0495402	0.0176619	0.3964957	0.957	0.0336891	0.0084849	0.2806474	0.958
		λ	0.0663976	0.0342370	0.5386186	0.953	0.0820931	0.0489835	0.6238621	0.957	0.0581917	0.0254326	0.4735348	0.958
		Δ	0.0700636	0.0439369	0.5844043	0.953	0.0815505	0.0585026	0.6455887	0.957	0.0650003	0.0342020	0.5382088	0.958
(20,12)	I	ρ	0.0173379	0.0020832	0.1513348	0.973	0.0175658	0.0024003	0.1556503	0.971	0.0182105	0.0023980	0.1595477	0.971
		λ	0.0316969	0.0071062	0.2761606	0.973	0.0319776	0.0080826	0.2829716	0.971	0.0331436	0.0080897	0.2895726	0.971
		Δ	0.0425090	0.0126440	0.3902280	0.973	0.0427863	0.0139027	0.3950117	0.971	0.0439004	0.0139004	0.4009329	0.971
	II	ρ	0.0235105	0.0039179	0.2017465	0.971	0.0297500	0.0065997	0.2532006	0.965	0.0183343	0.0024280	0.1610704	0.970
		λ	0.0418336	0.0128028	0.3572402	0.971	0.0517037	0.0205691	0.4345758	0.965	0.0333583	0.0081828	0.2921088	0.970
		Δ	0.0509934	0.0201239	0.4538805	0.971	0.0595242	0.0292889	0.5102579	0.965	0.0438742	0.0140371	0.4030005	0.970
	III	ρ	0.0209405	0.0031072	0.1814503	0.970	0.0236037	0.0043345	0.2070162	0.967	0.0184068	0.0024423	0.1617829	0.969
		λ	0.0376597	0.0103269	0.3252193	0.970	0.0418290	0.0140166	0.3650784	0.967	0.0334830	0.0082276	0.2933013	0.969
		Δ	0.0472376	0.0169756	0.4291940	0.970	0.0513068	0.0215928	0.4585986	0.967	0.0439778	0.0141028	0.4040131	0.969
(30,10)	I	ρ	0.0206886	0.0030118	0.1777876	0.971	0.0211355	0.0028474	0.1815704	0.981	0.0215274	0.0032947	0.1864027	0.975
		λ	0.0373451	0.0099889	0.3192320	0.971	0.0381079	0.0095286	0.3259365	0.981	0.0386286	0.0108660	0.3330923	0.975
		Δ	0.0479382	0.0164325	0.4243346	0.971	0.0471415	0.0160237	0.4306349	0.981	0.0485550	0.0176450	0.4353935	0.975
	II	ρ	0.0313163	0.0069072	0.2591363	0.965	0.0448860	0.0134974	0.3585237	0.965	0.0215706	0.003295	0.1868645	0.974
		λ	0.0545513	0.0213105	0.4430167	0.965	0.0750240	0.0388637	0.5778535	0.965	0.0386985	0.010871	0.3338478	0.974
		Δ	0.0622430	0.0298801	0.5166619	0.965	0.0782035	0.0487759	0.6136568	0.965	0.0485744	0.017670	0.4359764	0.974
	III	ρ	0.0294368	0.0060963	0.2441454	0.965	0.0394689	0.0103934	0.3189418	0.971	0.0216841	0.0033105	0.1876902	0.978
		λ	0.0516392	0.0190040	0.4211409	0.965	0.0669488	0.0309312	0.5265924	0.971	0.0388869	0.0109200	0.3351916	0.978
		Δ	0.0598637	0.0272054	0.5009776	0.965	0.0725808	0.0406911	0.5773566	0.971	0.0486067	0.0177410	0.4371220	0.978
(30,20)	I	ρ	0.0108596	0.0010398	0.1034992	0.973	0.0112099	0.0008851	0.1001572	0.978	0.0113659	0.0010517	0.1043244	0.976
		λ	0.0202895	0.0036895	0.1940241	0.973	0.0210902	0.0031436	0.1883089	0.978	0.0212949	0.0036870	0.1955236	0.976
		Δ	0.0311329	0.0076617	0.3178154	0.973	0.0343604	0.0065596	0.3145213	0.978	0.0332594	0.0073536	0.3206024	0.976
	II	ρ	0.0166170	0.0021866	0.1484602	0.974	0.0213490	0.0032993	0.1817661	0.969	0.0114955	0.0010519	0.1049895	0.977
		λ	0.0303940	0.0073928	0.2710037	0.974	0.0385094	0.0108234	0.3253150	0.969	0.0215325	0.0036904	0.1967327	0.977
		Δ	0.0413493	0.0128827	0.3854701	0.974	0.0484320	0.0173575	0.4287055	0.969	0.0334982	0.0073816	0.3218615	0.977
	III	ρ	0.0140299	0.0016392	0.1283727	0.973	0.0162253	0.0019463	0.1415079	0.971	0.0115421	0.0010583	0.1052925	0.977
		λ	0.0259133	0.0056444	0.2371304	0.973	0.0298870	0.0066069	0.2593496	0.971	0.0216161	0.0037126	0.1972620	0.977
		Δ	0.0371079	0.0104482	0.3568513	0.973	0.0419273	0.0116938	0.3758565	0.971	0.0336998	0.0074208	0.3223643	0.977