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Properties on Micro Pre-Covering Map in Micro Topological Spaces

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Abstract The basic objective of this research work is to introduce and investigate the properties of micro semi-continuous, micro pre-continuous, micro pre-irresolute map, micro pre-covering map.

Keywords micro semi-continuous, micro pre-continuous, micro pre-irresolute map, micro pre-covering map.

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1. Introduction

Levine's introduction of semi-open sets, semi-continuity [6] in 1963 and generalized closed sets in 1970 [7], providing a foundational framework for subsequent developments. D. Andrijevic introduced pre-open sets. A. S. Mashhour et al., [9] defined pre-continuous mapping and in 1991 [1] Balachandran et.al., defined the concepts of generalized continuous functions. In 2013 Lellis Thivagar [4], introduced the concept of nano topology based on Levine's work and defined [5] nano continuity. In 2018, Sathishmohan et al., [11] introduced nano pre-neighborhoods and he [10] introduced nano semi-continuity and nano pre-continuity.

In 2019, Chandrasekar [2], introduced the concept of micro topology which is a simple extension of nano topology, with a focus on micro pre-open and micro semi-open sets. Sathishmohan et al., [12] explored many properties of micro pre-neighborhoods and defined [13] micro pre separation axioms. The aim of this paper is to introduce and investigate the properties of micro semi-continuous, micro pre-continuous functions, micro pre-irresolute and micro pre-covering map.

2. Preliminaries

Definition 2.1
[4] Let U be the universe, R be an equivalence relation on U and
$\tau_R(X) = \{U, \emptyset,\ L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq$ (U, $\tau_R(X)$) satisfies the following axioms:

1. $U\ and\ \emptyset \in \tau_R(X)$
2. The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

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That is, $\tau_R(X)$ forms a topology on U is called the nano topology on U with respect to X. $(U, \tau_R(X))$ is called the nano topological space.

Definition 2.2
[2] Let $(U, \tau_R(X))$ is a nano topological space here $\mu_R(X) = \{\{N \cup (N' \cap \mu)\} : N, N' \in \tau_R(X)\}$ and called it micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$.

Definition 2.3
[2] The micro topology $\mu_R(X)$ satisfies the following axioms.

1. $U \; and \; \emptyset \in \mu_R(X)$
2. The union of elements of any sub collection of $\mu_R(X)$ is in $\mu_R(X)$.
3. The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called micro topological spaces and the elements of $\mu_R(X)$ are called micro open sets and the complement of a micro open set is called a micro closed set.

Definition 2.4
[2] The micro closure of a set A is denoted by Mic-cl(A) and is defined as Mic-cl(A) = $\cap\{B : B \; is \; micro \; closed \; and \; A \subseteq B\}$. The micro interior of a set A is denoted by Mic-int(A) and is defined as Mic-int(A) = $\cup\{B : B \; is \; micro \; open \; and \; A \supseteq B\}$.

Definition 2.5
[12] The union of all micro pre-open sets which are contained in A is called the micro pre-interior of A and is denoted by Mic-Pint(A) or by Mic-PA_*.
As the union of micro pre-open sets is micro pre-open, Mic-PA_* is micro pre-open always. micro pre-open is denoted by Mic-PO(U) and micro pre-closed is denoted by Mic-PF(U).

Definition 2.6
[12] The intersection of micro pre-closed sets containing a set A is called the micro pre-closure of A and is denoted by Mic-Pcl(A) or by Mic-PA^*.

Definition 2.7
[2] Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and $A \subseteq U$. Then A is called micro pre-open if $A \subseteq Mic$-int(Mic-cl(A)).

Definition 2.8
[2] Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau^{'}_R(X), \mu^{'}_R(X))$ be two micro topological spaces. A function f : U $\rightarrow$ V is called micro-continuous function if f^{-1} (H) is micro-open in U for every micro-open set H in V.

Definition 2.9
[2] Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau^{'}_R(X), \mu^{'}_R(X))$ be two micro topological spaces. A function f : U $\rightarrow$ V is called micro-continuous at a point a $\in$ U if for every micro-open set H containing f(a) in V, there exist a micro-open set G containing a in U, such that f(G) $\subset$ H.

Definition 2.10
[2] Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau^{'}_R(X), \mu^{'}_R(X))$ be two micro topological spaces, then f : U $\rightarrow$ V is micro pre-continuous if f^{-1}(H) is micro pre-closed in U whenever H is micro closed in V.

Definition 2.11
[2] Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau^{'}_R(X), \mu^{'}_R(X))$ be two micro topological spaces, then f : U $\rightarrow$ V is micro semi-continuous if f^{-1}(H) is micro-semi closed in U whenever H is micro closed in V.

Definition 2.12
[13] A space U is called micro pre-T_2 (or Mic-PT_2) space, for each pair of distinct points x, y $\in$ U, there exists disjoint micro pre-open sets H and I such that x $\in$ H and y $\in$ I.

Definition 2.13
[5] A function f: $(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is a nano homeomorphism if

1. f is 1-1 and onto.
2. f is nano continuous and
3. f is nano-open.

Definition 2.14
[3] Let p: E $\to$ B be a continuous surjective map. The open set U of B is said to be evenly covered by p if the inverse image $p^{-1}(U)$ can be written as the union of disjoint open sets V_α in E such that for each α, the restriction of p to V_α is a homeomorphism of V_α onto U. The collection $\{V_\alpha\}$ will be called a partition of $p^{-1}(U)$ into slices.

Definition 2.15
[3] Let p: E $\to$ B be continuous and surjective. If every point b of B has a neighborhood U that is evenly covered by p, then p is called a covering map, and E is said to be a covering space of B.

3. Micro semi and micro pre-continuous function

In this section, we study the additional concepts of micro semi-continuous and micro pre-continuous functions and obtained some basic results.

Theorem 3.1
Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau^{'}_R(X), \mu^{'}_R(X))$ be two micro topological spaces. Then f: U $\to$ V is micro pre-continuous function if and only if f^{-1}(H) is micro pre-closed in U, whenever H is micro closed in V.
Proof: Let f: U $\to$ V is micro pre-continuous function and H be micro closed in V. Then H^C is micro-open in V. By hypothesis f^{-1}(H^C) is micro pre-open in U, i.e., $(f^{-1}$(H)$)^C$ is micro pre-open in U. Hence f^{-1}(H) is micro pre-closed in U whenever H is micro closed in V.
Conversely, suppose f^{-1}(H) is micro pre-closed in U whenever H is micro closed in V. Let I is micro closed in V then I^C is micro closed in V. By assumption f^{-1}(I^C) is micro pre-closed in U i.e., $(f^{-1}$(I)$^C)$ is micro pre-closed in X. Then f^{-1}(I) is micro pre-open in U. Hence f is micro pre-continuous.

Example 3.2
Let U = {a,b,c,d}, U/R = {{a,c},{b,d}}, X = {a,c}, $\tau_R(X)$ = {U,∅,{a,c}}, μ = {b,c} and $\mu_R(X)$ = {U,∅,{c},{a,c},{b,c},{a,b,c}}. Mic-PO(U) = {U,∅,{c},{b,c},{c,d},{a,c},{a,b,c},{b,c,d},{a,c,d}}. Let V = {1,2,3,4}, V/R = {{1,3},{2,4}}, Y = {1,3}, $\tau^{'}_R(Y)$ = {V,∅,{1,3}}, μ = {3} and $\mu^{'}_R(Y)$ = {V,∅,{3},{1,3}}. Let f:U $\to$ V be a micro pre-continuous function and defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4. Since for every micro open set H in V, there exists f^{-1}(H) in micro pre-open in U. Therefore f is micro pre-continuous function.

Theorem 3.3
Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau^{'}_R(X), \mu^{'}_R(X))$ be two micro topological spaces. Then f: U $\to$ V is micro semi-continuous function if and only if f^{-1}(H) is micro semi-closed in U, whenever H is micro closed in V.
Proof: Let f: U $\to$ V is micro semi-continuous function and H be micro closed in V. Then H^C is micro-open in V. By hypothesis f^{-1}(H^C) is micro semi-open in U, i.e., $(f^{-1}$(H)$)^C$ is micro semi-open in U. Hence f^{-1}(H) is micro semi-closed in U whenever H is micro closed in V.
Conversely, suppose f^{-1}(H) is micro semi-closed in U whenever H is micro closed in V. Let I is micro closed in V then I^C is micro closed in V. By assumption f^{-1}(I^C) is micro semi-closed in U i.e., $(f^{-1}$(I)$^C)$ is micro semi-closed in X. Then f^{-1}(I) is micro semi-open in U. Hence f is micro semi-continuous.

Example 3.4
Let U = {a,b,c,d}, U/R = {{a},{b},{c},{d}}, X = {b,c}, $\tau_R(X)$ = {U,∅,{b,c}}, μ = {a,b,d} and $\mu_R(X)$ = {U,∅,{b},{b,c},{a,b,d}}. Mic-PO(U) = {U,∅,{b},{a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}}. Let V = {1,2,3,4}, V/R = {{1},{2},{3},{4}}, Y = {2,3}, $\tau^{'}_R(Y)$ = {V,∅,{2,3}}, μ = {2,3,4} and $\mu^{'}_R(Y)$ = {V,∅,{2,3},{2,3,4}}. Let f: U $\to$ V be a micro semi-continuous function and defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4. Since for every micro open set H in V, there exists f^{-1}(H) in micro semi-open in U. Therefore f is micro semi-continuous function.

Theorem 3.5
A function f: $(U,\tau_R(X),\mu_R(X)) \rightarrow (V,\tau^{'}_R(X),\mu^{'}_R(X))$ is micro semi-continuous if and only if f(Mic-cl(H)) $\subseteq$ Mic-cl(f(H)) for every subset H of U.
Proof: Let f be micro semi-continuous function and H $\subseteq$ U. Then f(H) $\subseteq$ V. Mic-cl(f(H)) is micro closed in V. Since f is micro semi-continuous, f^{-1}(Mic-cl(f(H))) is micro semi-closed in U. Since f(H) $\subseteq$ Mic-cl(f(H)), H $\subseteq$ f^{-1}(Mic-cl(f(H))). Thus f^{-1}(Mic-cl(f(H))) is a micro semi-closed set containing H. Therefore Mic-cl(f(H)) $\subseteq$ f^{-1}(Mic-cl(f(H))). Therefore f(Mic-cl(H)) $\subseteq$ Mic-cl(f(H)).
Conversely, let f(Mic-cl(H)) $\subseteq$ Mic-cl(f(H)) for every subset H of U. If I is micro closed in V, since f^{-1}(I) $\subseteq$ U, f(Mic-cl(f^{-1}(I))) $\subseteq$ Mic-cl(f(f^{-1}(I))) $\subseteq$ Mic-cl(I). That is, Mic-cl(f^{-1}(I)) $\subseteq$ f^{-1}(Mic-cl(I)) = f^{-1}(I), since I is micro closed. Thus Mic-cl(f^{-1}(I)) $\subseteq$ f^{-1}(I). But f^{-1}(I) $\subseteq$ Mic-cl(f^{-1}(I)) = f^{-1}(I). Therefore, f^{-1}(I) is micro semi-closed in U for every micro closed set I in V. That is, f is micro semi-continuous function.

Theorem 3.6
A function f: $(U,\tau_R(X),\mu_R(X)) \rightarrow (V,\tau^{'}_R(X),\mu^{'}_R(X))$ is micro semi-continuous if and only if Mic-int(f(H)) $\subseteq$ f(Mic-int(H)) for every subset H of U.
Proof: Let f be micro semi-continuous function and H $\subseteq$ U. Then f(H) $\subseteq$ V. Mic-int(f(H)) is micro open in V. Since f is micro semi-continuous, f^{-1}(Mic-int(f(H))) is micro semi-open in U. Since Mic-int(f(H)) $\subseteq$ f(H), f^{-1}(Mic-int(f(H))) $\subseteq$ H. Thus f^{-1}(Mic-int(f(H))) is a micro semi-open set containing H. Therefore f^{-1}(Mic-int(f(H))) $\subseteq$ Mic-int(f(H)). Therefore Mic-int(f(H)) $\subseteq$ f(Mic-int(H)).
Conversely, let Mic-int(f(H)) $\subseteq$ f(Mic-int(H)) for every subset H of U. If I is micro open in V, Since f^{-1}(I) $\subseteq$ U. Since I is micro open f^{-1}(Mic-int(I)) = f^{-1}(I) $\subseteq$ Mic-int(f^{-1}(I)). Thus f^{-1}(I) $\subseteq$ Mic-int(f^{-1}(I)). But f^{-1}(I) = Mic-int(f^{-1}(I)) $\subseteq$ f^{-1}(I). Therefore, f^{-1}(I) is micro semi-open in U for every micro open set I in V. That is, f is micro semi-continuous function.

Example 3.7
Let U = {a,b,c,d}, U/R = {{a,b},{c},{d}}, X = {a}, $\tau_R(X)$ = {U,$\emptyset$,{a,b}}, μ = {a,c} and $\mu_R(X)$ = {U,$\emptyset$,{a},{a,b},{a,c},{a,b,c}}. Mic-SO(U) = {U,$\emptyset$,{a},{a,b},{a,d},{a,c},{a,b,c},{a,c,d},{a,b,d}}. Let V = {1,2,3,4}, V/R = {{1,2},{3},{4}}, Y = {1}, $\tau^{'}_R(Y)$ = {V,$\emptyset$,{1,2}}, μ = {1,2,3} and $\mu^{'}_R(Y)$ = {V,$\emptyset$,{1,2},{1,2,3}}. Let f: U $\rightarrow$ V be a micro semi-continuous function and defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4. Since for every micro open set H in V, there exists f^{-1}(H) in micro semi-open in U. Therefore f is micro semi-continuous function. Let H = {b,c,d} then f(Mic-cl(H)) = {2,3,4} and Mic-cl(f(H)) = V. Therefore f(Mic-cl(H)) $\subseteq$ Mic-cl(f(H)). Let H = {a} then Mic-int(f(H)) = $\emptyset$ and f(Mic-int(H)) = {1}. Therefore Mic-int(f(H)) $\subseteq$ f(Mic-int(H)).

Theorem 3.8
A function f : $(U,\tau_R(X),\mu_R(X)) \rightarrow (V,\tau^{'}_R(X),\mu^{'}_R(X))$ is micro pre-continuous if and only if f(Mic-cl(H)) $\subseteq$ Mic-cl(f(H)) for every subset H of U.
Proof: Let f be micro pre-continuous function and H $\subseteq$ U. Then f(H) $\subseteq$ V. Mic-cl(f(H)) is micro closed in V. Since f is micro pre-continuous, f^{-1}(Mic-cl(f(H))) is micro pre-closed in U. Since f(H) $\subseteq$ Mic-cl(f(H)), H $\subseteq$ f^{-1}(Mic-cl(f(H))). Thus f^{-1}(Mic-cl(f(H))) is a micro pre-closed set containing H. Therefore Mic-cl(f(H)) $\subseteq$ f^{-1}(Mic-cl(f(H))). Therefore f(Mic-cl(H)) $\subseteq$ Mic-cl(f(H)).
Conversely, let f(Mic-cl(H)) $\subseteq$ Mic-cl(f(H)) for every subset H of U. If I is micro closed in V, since f^{-1}(I) $\subseteq$ U, f(Mic-cl(f^{-1}(I))) $\subseteq$ Mic-cl(f(f^{-1}(I))) $\subseteq$ Mic-cl(I). That is, Mic-cl(f^{-1}(I)) $\subseteq$ f^{-1}(Mic-cl(I)) = f^{-1}(I), since I is micro closed. Thus Mic-cl(f^{-1}(I)) $\subseteq$ f^{-1}(I). But f^{-1}(I) $\subseteq$ Mic-cl(f^{-1}(I)) = f^{-1}(I). Therefore, f^{-1}(I) is micro pre-closed in U for every micro closed set I in V. That is, f is micro pre-continuous function.

Theorem 3.9
A function f : $(U,\tau_R(X),\mu_R(X)) \rightarrow (V,\tau^{'}_R(X),\mu^{'}_R(X))$ is micro pre-continuous if and only if Mic-int(f(H)) $\subseteq$ f(Mic-int(H)) for every subset H of U.
Proof: Let f be micro pre-continuous function and H $\subseteq$ U. Then f(H) $\subseteq$ V. Mic-int(f(H)) is micro open in V. Since f is micro pre-continuous, f^{-1}(Mic-int(f(H))) is micro pre-open in U. Since Mic-int(f(H)) $\subseteq$ f(H), f^{-1}(Mic-int(f(H))) $\subseteq$ H. Thus f^{-1}(Mic-int(f(H))) is a micro pre-open set containing H. Therefore f^{-1}(Mic-int(f(H))) $\subseteq$ Mic-int(f(H)). Therefore Mic-int(f(H)) $\subseteq$ f(Mic-int(H)).
Conversely, let Mic-int(f(H)) $\subseteq$ f(Mic-int(H)) for every subset H of U. If I is micro open in V, since f^{-1}(I) $\subseteq$ U.

Since I is micro open f^{-1}(Mic-int(I)) = f^{-1}(I) $\subseteq$ Mic-int(f^{-1}(I)). Thus f^{-1}(I) $\subseteq$ Mic-int(f^{-1}(I)). But f^{-1}(I) = Mic-int(f^{-1}(I)) $\subseteq$ f^{-1}(I). Therefore, f^{-1}(I) is micro pre-open in U for every micro open set I in V. That is, f is micro pre-continuous function.

Example 3.10
Let U = {a,b,c,d}, U/R = {{a,b},{c,d}}, X = {b,c}, $\tau_R(X)$ = {U,∅,{b,c}}, μ = {a} and $\mu_R(X)$ = {U,∅,{a},{b,c},{a,b,c}}. Mic-PO(U) = {U,∅,{a},{b},{c},{a,b},{b,c},{a,c},{a,b,d},{a,b,c},{a,c,d}}. Let V = {1,2,3,4}, V/R = {{1,2},{3,4}}, Y = {2,3}, $\tau^{'}_R(Y)$ = {V,∅,{2,3}}, μ = {1,2,3} and $\mu^{'}_R(Y)$ = {V,∅,{2,3},{1,2,3}}. Mic-PO(V) = {V,∅,{2},{3},{1,2},{2,3},{3,4},{1,3},{2,4},{1,2,3},{2,3,4},{1,3,4},{1,2,4}}. Let f :U $\rightarrow$ V be a micro pre-continuous function and defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4. The micro pre-open sets in U are {a},{b},{c},{a,b},{b,c},{a,c},{a,b,d},{a,b,c},{a,c,d} and micro open set in V are {2,3},{1,2,3}. Therefore for every micro open set H in V, f^{-1}(H) is micro pre-open in U. Then f is micro pre-continuous function. Let H = {a} then f(Mic-cl(H)) = {2,3,4} and Mic-cl(f(H)) = V. Therefore f(Mic-cl(H)) $\subseteq$ Mic-cl(f(H)). Let H = {a,b} then Mic-int(f(H)) = ∅ and f(Mic-int(H)) = {1}. Therefore Mic-int(f(H)) $\subseteq$ f(Mic-int(H)).

Definition 3.11
Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau^{'}_R(Y), \mu^{'}_R(Y))$ be two micro topological spaces. A function f : U $\rightarrow$ V is called micro semi-continuous at a point a $\in$ U if for every micro-open set H containing f(a) in V, there exist a micro semi-open set I containing a in U, such that f(I) $\subset$ H.

Definition 3.12
Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau^{'}_R(Y), \mu^{'}_R(Y))$ be two micro topological spaces. A function f : U $\rightarrow$ V is called micro pre-continuous at a point a $\in$ U if for every micro-open set H containing f(a) in V, there exist a micro pre-open set I containing a in U, such that f(I) $\subset$ H.

Theorem 3.13
A function f : $(U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau^{'}_R(Y), \mu^{'}_R(Y))$ is micro semi-continuous if and only if f is micro semi-continuous at each point of U.
Proof: Let f : $U \rightarrow V$ be micro semi-continuous. Let a $\in$ U, and H be a micro open set in V containing f(a). Since f is micro semi-continuous, f^{-1}(V) is micro semi-open in U containing a. Let I = f^{-1}(H), then f(I) $\subseteq$ H, and f(a) $\in$ I. Hence f is micro semi-continuous at a.
Conversely, suppose f is micro semi-continuous at each point of U. Let H be micro open set in V. If f^{-1}(H) = ∅ then it is micro open. So let f^{-1}(H) $\neq$ ∅. Take any point as a f^{-1}(H), then f(a) $\in$ H. Since f is micro semi-continuous at each point there exist a micro semi-open set I_a containing a such that f(I_a) $\subseteq$ H. Let I = f^{-1}(H). If x $\in$ f^{-1}(H) then x $\in$ I_x $\subseteq$ I. Hence f^{-1}(H) $\subseteq$ I. On the other hand, suppose y $\in$ I then y $\in$ I_x for some x and y $\in$ f^{-1}(H). Hence U = f^{-1}(H). Since I_x is micro semi-open, by Definition 3.11 I is micro semi-open and hence I = f^{-1}(H) is micro semi-open for every micro open set H in V. Hence f is micro semi-continuous.

Theorem 3.14
A function f : $(U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau^{'}_R(Y), \mu^{'}_R(Y))$ is micro pre-continuous if and only if f is micro pre-continuous at each point of U.
Proof: Let f : $U \rightarrow V$ be micro pre-continuous. Let a $\in$ U, and H be a micro open set in V containing f(a). Since f is micro pre-continuous, f^{-1}(V) is micro pre-open in U containing a. Let I = f^{-1}(H), then f(I) $\subseteq$ H, and f(a) $\in$ I. Hence f is micro pre-continuous at a.
Conversely, suppose f is micro pre-continuous at each point of U. Let H be micro open set in V. If f^{-1}(H) = ∅ then it is micro open. So let f^{-1}(H) $\neq$ ∅. Take any point as a f^{-1}(H), then f(a) $\in$ H. Since f is micro pre-continuous at each point there exist a micro pre-open set I_a containing a such that f(I_a) $\subseteq$ H. Let I = f^{-1}(H). If x $\in$ f^{-1}(H) then x $\in$ I_x $\subseteq$ I. Hence f^{-1}(H) $\subseteq$ I. On the other hand, suppose y $\in$ I then y $\in$ I_x for some x and y $\in$ f^{-1}(H). Hence U = f^{-1}(H). Since I_x is micro pre-open, by Definition 3.12 I is micro pre-open and hence I = f^{-1}(H) is micro pre-open for every micro open set H in V. Hence f is micro pre-continuous.

Theorem 3.15
Let $(U, \tau_R(X), \mu_R(X))$, $(V, \tau^{'}_R(Y), \mu^{'}_R(Y))$ and $(W, \tau^{''}_R(Z), \mu^{''}_R(Z))$ be three micro topological spaces. If f : $U \rightarrow V$ is a micro semi-continuous function and g : $V \rightarrow W$ be a micro continuous function then g $\circ$ f : $U \rightarrow W$

is micro semi-continuous function.

Proof: Let H be micro open set in W. Since by g is micro continuous function, then g^{-1}(H) is micro open set in V. Now, $(g \circ f)^{-1}$(H) = $(f^{-1} \circ g^{-1})$H = $f^{-1} \circ (g^{-1}$(H)). Take g^{-1}(H) = I which is micro open in V, then f^{-1}(I) is micro semi-open in U, since f is micro semi-continuous function. Hence g ∘ f : $U \to W$ is micro semi-continuous function.

Theorem 3.16
Let $(U, \tau_R(X), \mu_R(X))$, $(V, \tau_R^{'}(Y), \mu_R^{'}(Y))$ and $(W, \tau_R^{''}(Z), \mu_R^{''}(Z))$ be three micro topological spaces. If f : $U \to V$ is a micro pre-continuous function and g : $V \to W$ be a micro continuous function then g ∘ f : $U \to W$ is micro pre-continuous function.

Proof: Let H be micro open set in W. Since by g is micro continuous function, then g^{-1}(H) is micro open set in V. Now, $(g \circ f)^{-1}$(H) = $(f^{-1} \circ g^{-1})$H = $f^{-1} \circ (g^{-1}$(H)). Take g^{-1}(H) = I which is micro open in V, then f^{-1}(I) is micro pre-open in U, since f is micro pre-continuous function. Hence g ∘ f : $U \to W$ is micro pre-continuous function.

4. Micro pre-irresolute function

In this section, we defined and studied the concept of micro pre-irresolute functions and obtained some common results.

Definition 4.1
Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R^{'}(Y), \mu_R^{'}(Y))$ be two micro topological spaces, then f : U → V is micro pre-irresolute if f^{-1}(H) is micro pre-closed in U whenever H is micro pre-closed in V.

Remark 4.2
Every micro pre-irresolute function is micro pre-continuous function but the converse need not be true.

Example 4.3
Let U = {a,b,c,d}, U/R = {{a,c},{b,d}}, X = {a,c}, $\tau_R(X)$ = {U,∅,{a,c}}, μ = {a} and $\mu_R(X)$ = {U,∅,{a},{a,c}}. Mic-PO(U) = {U,∅,{a},{a,b},{a,d},{a,c},{a,b,c},{a,c,d},{a,b,d}}. Let V = {1,2,3,4}, V/R = {{1,2},{3},{4}}}, Y = {1}, $\tau_R^{'}(Y)$ = {V,∅,{1,2}}, μ = {1,2,4} and $\mu_R^{'}(Y)$ = {V,∅,{1,2},{1,2,4}}. Mic-PO(V) = {V,∅,{1},{2},{1,2},{2,3},{1,4},{1,3},{2,4},{1,2,3},{2,3,4},{1,3,4},{1,2,4}}. Let f :U → V be a micro pre-continuous function which is defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4. It is not a micro pre-irresolute function.

Theorem 4.4
Let $(U, \tau_R(X), \mu_R(X))$, $(V, \tau_R^{'}(Y), \mu_R^{'}(Y))$ and $(W, \tau_R^{''}(Z), \mu_R^{''}(Z))$ be three micro topological spaces. If f : $U \to V$ is a micro pre-irresolute function and g : $V \to W$ be a micro pre-continuous function then g ∘ f : $U \to W$ is micro pre-continuous function.

Proof: Let H be micro open set in W. Since by g is micro pre-continuous function, then g^{-1}(H) is micro pre-open in V. Now, $(g \circ f)^{-1}$(H) = $(f^{-1} \circ g^{-1})$H = $f^{-1} \circ (g^{-1}$(H)). Take g^{-1}(H) = I which is micro pre-open in V, then f^{-1}(I) is micro pre-open in U, since f is micro pre-irresolute function. Hence g ∘ f : $U \to W$ is micro pre-continuous function.

Example 4.5
Let U = {a,b,c,d}, U/R = {{a,b},{c},{d}}, X = {a}, $\tau_R(X)$ = {U,∅,{a,b}}, μ = {c,d} and $\mu_R(X)$ = {U,∅,{a,b},{c,d}}. Mic-PO(U) = P(U). Let V = {1,2,3,4}, V/R = {{1,2},{3},{4}}}, Y = {1}, $\tau_R^{'}(Y)$ = {V,∅,{1,2}}, $\mu^{'}$ = {2,4} and $\mu_R^{'}(Y)$ = {V,∅,{2},{1,2},{2,4},{1,2,4}}. Mic-PO(V) = {V,∅,{2},{1,2},{2,3},{2,4},{1,2,3},{2,3,4},{1,2,4}}. Let W = {p,q,r,s}, W/R = {{p,q},{r},{s}}}, Z = {p}, $\tau_R^{''}(Z)$ = {W,∅,{p,q}}, $\mu^{''}$ = {q,r,s} and $\mu_R^{''}(Z)$ = {W,∅,{q},{p,q},{q,r,s}}. Let f :U → V be a micro pre-irresolute function which is defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4 and g :V → W be a micro pre-continuous function which is defined as g(1) = p, g(2) = q, g(3) = r, g(4) = s. Therefore g ∘ f : $U \to W$ is micro pre-continuous function.

Theorem 4.6
Let $(U, \tau_R(X), \mu_R(X))$, $(V, \tau_R^{'}(Y), \mu_R^{'}(Y))$ and $(W, \tau_R^{''}(Z), \mu_R^{''}(Z))$ be three micro topological spaces. If f : $U \to V$ is a micro pre-irresolute function and g : $V \to W$ be a micro pre-irresolute function then g ∘ f : $U \to W$ is also micro pre-irresolute function.
Proof: Let H be micro pre-open set in W. Since by g is micro pre-irresolute function, then g^{-1}(H) is micro pre-open in V. Now, (g ∘ f)$^{-1}$(H) = (f^{-1} ∘ g^{-1})H = f^{-1} ∘ (g^{-1}(H)). Take g^{-1}(H) = I which is micro pre-open in V, then f^{-1}(I) is micro pre-open in U, since f is micro pre-irresolute function. Hence g ∘ f : $U \to W$ is micro pre-irresolute function.

Example 4.7
Let U = {a,b,c,d}, U/R = {{a,b,c},{d}}, X = {a,c}, $\tau_R(X)$ = {U,∅,{a,b,c}}, μ = {d} and $\mu_R(X)$ = {U,∅,{d},{a,b,c}}. Mic-PO(U) = P(U). Let V = {1,2,3,4}, V/R = {{1,2},{3},{4}}}, Y = {1}, $\tau_R^{'}(Y)$ = {V,∅,{1,2}}, $\mu^{'}$ = {1,2,3} and $\mu_R^{'}(Y)$ = {V,∅,{1,2},{1,2,3}}. Mic-PO(V) = {V,∅,{1},{2},{1,2},{2,3},{1,4},{1,3},{2,4},{1,2,3},{2,3,4},{1,3,4},{1,2,4}}. Let W = {p,q,r,s}, W/R = {{p,q},{r},{s}}}, Z = {p}, $\tau_R^{''}(Z)$ = {W,∅,{p,q}}, $\mu^{''}$ = {p,r,s} and $\mu_R^{''}(Z)$ = {W,∅,{p},{p,q},{p,r,s}}. Mic-PO(W) = {W,∅,{P},{p,q},{p,s},{p,r},{p,q,r},{p,r,s},{p,q,s}}. Let f :U → V be a micro pre-irresolute function which is defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4 and g :V → W be a micro pre-irresolute function which is defined as g(1) = p, g(2) = q, g(3) = r, g(4) = s. Therefore g ∘ f : $U \to W$ is micro pre-irresolute function.

5. Micro pre-covering map

In this section, we defined the concept of micro pre-covering map from the algebraic topology concept covering map and obtained some exclusive results.

Definition 5.1
A function f : U → V is said to be a micro pre-homeomorphism if

1. f is bijective.
2. f is micro pre-irresolute functions.
3. f^{-1} is micro pre-irresolute functions.

Example 5.2
Let U = {a,b,c,d}, U/R = {{a,b},{c},{d}}, X = {a}, $\tau_R(X)$ = {U,∅,{a,b}}, μ = {b} and $\mu_R(X)$ = {U,∅,{b},{a,b}}. Mic-PO(U) = {U,∅,{b},{a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}}. Let V = {1,2,3,4}, V/R = {{1,2},{3},{4}}, Y = {1}, $\tau_R^{'}(Y)$ = {V,∅,{1,2}}, $\mu^{'}$ = {1,3} and $\mu_R^{'}(Y)$ = {V,∅,{1},{1,2},{1,3},{1,2,3}}. Mic-PO(V) = {V,∅,{1},{1,2},{1,4},{1,3},{1,2,3},{1,3,4},{1,2,4}}. Let f be a micro pre-irresolute function which is defined as f(a) = 2, f(b) = 1, f(c) = 3, f(d) = 4. Clearly, f is a bijective function and a micro pre-irresolute function. Also, f^{-1} is also a micro pre-irresolute function. Therefore f is a micro pre-homeomorphism function.

Definition 5.3
Let f : U → V be a micro pre-continuous surjective map. The micro open set A of V is said to be micro pre-evenly covered by f if the inverse image $f^{-1}(A)$ can be written as the union of disjoint micro pre-open sets K_α in U such that for each α, the restriction of f to K_α is a homeomorphism of K_α onto A. The collection $\{K_\alpha\}$ is said to be a partition of $f^{-1}(A)$ into slices.

Definition 5.4
Let f : U → V be micro pre-continuous and surjective. If every point b of V has a neighborhood U that is micro pre-evenly covered by f, then f is called a micro pre-covering map, and U is said to be a micro pre-covering space of V.

Example 5.5
Let U = {a,b,c,d}, U/R = {{a,b},{c},{d}}, X = {a}, $\tau_R(X)$ = {U,∅,{a,b}}, μ = {a,b,d} and

$\mu_R(X)$ = {U,∅,{a,b},{a,b,d}}. Mic-PO(U) = {U,∅,{a},{b},{a,b},{b,c},{a,d},{a,c},{b,d},{a,b,c}, {b,c,d},{a,c,d},{a,b,d}}. Let V = {1,2,3,4}, V/R = {{1},{2},{3},{4}}, Y = {2,3}, $\tau^{'}_R(Y)$ = {V,∅,{2,3}}, $\mu^{'}$ = {1,2,3} and $\mu^{'}_R(Y)$ = {V,∅,{2,3},{1,2,3}}. Let f be micro pre-continuous and surjective map and defined as f(a) = 3, f(b) = 2, f(c) = 1, f(d) = 4. Since f is micro pre-continuous, for every micro open set H in V there exist inverse image in micro pre-open set in U. The inverse image of {2,3} and {1,2,3} is {a,b} and {a,b,c} which can be written as union of disjoint micro pre-open sets {a},{b} and {a},{b,c} respectively which is referred as K_α and the restriction of f to K_α is homeomorphism of K_α onto H. The collection {{a},{b}}, {{a},{b,c}} will be partition of f^{-1}{2,3} and f^{-1}{1,2,3} respectively is said to be slices. Hence f is said to be micro pre-covering map.

Theorem 5.6
Let f : U → V be a micro pre-covering map. If B is a subspace of V and A is the subspace of U, A = $f^{-1}(B)$, then the restriction map f_0: A → B is a micro pre-covering map.
Proof: Let b ∈ B and H be an micro open set in V containing b that is micro pre-evenly covered by f. Let {k_α} be the partitions of $f^{-1}(H)$ into slices. Then H ∩ B is a neighborhood of b in B, and the sets k_α ∩ A are disjoint micro pre-open sets in A whose union is $f^{-1}(H \cap B)$, and each is mapped homeomorphically onto H ∩ B by f_0. Hence f_0 is a micro pre-covering map.

Example 5.7
Let U = {a,b,c,d}, U/R = {{a,b},{c},{d}}, X = {a}, $\tau_R(X)$ = {U,∅,{a,b}}, μ = {c,d} and $\mu_R(X)$ = {U,∅,{a,b},{c,d}}. Mic-PO(U) = P(U). Let V = {1,2,3,4}, V/R = {{1},{2,3,4}}, Y = {2}, $\tau^{'}_R(Y)$ = {V,∅,{2,3,4}}, $\mu^{'}$ = {3,4} and $\mu^{'}_R(Y)$ = {V,∅,{3,4},{2,3,4}}. Let f be a micro pre-continuous function and micro pre-covering map which is defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4. Since f is micro pre-covering map let A be subspace which contains {c,d} in U and B be a subspace which contains {3,4} in V. Then the restriction map f_0: A → B is also a micro pre-covering map.

Theorem 5.8
If f : U → V and $f^{'}: U^{'} \to V^{'}$ are micro pre-covering maps, then f × $f^{'}$: U × $U^{'}$ → V × $V^{'}$ is a micro pre-covering map.
Proof: Let a ∈ V, $a^{'} \in V^{'}$ and H, $H^{'}$ be the neighborhoods of a and $a^{'}$ respectively, which are micro pre-evenly covered by f and $f^{'}$ respectively. Let {K_α} and {$K^{'}_\alpha$} be the partitions of $f^{-1}(H)$ and $(f^{'})^{-1}(H^{'})$ respectively into slices. Then the inverse image of f × $f^{'}$ of micro pre-open set H × $H^{'}$ is the union of all the sets {K_α} × {$K^{'}_\alpha$}. These are disjoint micro pre-open sets of U × $U^{'}$, and each is mapped homeomorphically onto H × $H^{'}$ by f × $f^{'}$ is a micro pre-covering map.

Theorem 5.9
If f : U → V be micro pre-irresolute function, micro pre-covering map and g : V → W be a micro pre-covering map then g ∘ f is a micro pre-covering map.
Proof: Let f be a micro pre-irresolute function and micro pre-coveing map. Also, g be a micro pre-covering map. Let Q be a micro open set in Z. Since g is a micro pre-covering map, the micro pre-open set $g^{-1}(Q)$ in V can be written as the union of disjoint micro pre-open sets {K_α} in which each {K_α} is homeomorphically mapped onto g. Since f is a micro pre-irresolute function and micro pre-covering map, $f^{-1}(g^{-1}(Q)) = (g \circ f)^{-1}Q$ is micro pre-open in U which can be written as the union of disjoint micro pre-open sets {K_α} in which each {K_α} is homeomorphically mapped onto f. This implies that g ∘ f is a micro pre-covering map.

Example 5.10
Let U = {a,b,c,d}, U/R = {{a},{b,c,d}}, X = {b}, $\tau_R(X)$ = {U,∅,{b,c,d}}, μ = {a} and $\mu_R(X)$ = {U,∅,{a},{b,c,d}}. Mic-PO(U) = P(U). Let V = {1,2,3,4}, V/R = {{1,2,3},{4}}}, Y = {1,3}, $\tau^{'}_R(Y)$ = {V,∅,{1,2,3}}, $\mu^{'}$ = {4} and $\mu^{'}_R(Y)$ = {V,∅,{4},{1,2,3}}. Mic-PO(V) = P(V). Let W = {p,q,r,s}, W/R = {{p},{q,r,s}}, Z = {q}, $\tau^{''}_R(Z)$ = {W,∅,{q,r,s}}, $\mu^{''}$ = {p,q,s} and $\mu^{''}_R(Z)$ = {W,∅,{q,s},{q,r,s},{p,q,s}}. Let f be micro pre-irresolute function and micro pre-coveing map which is defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4. Let g be a micro pre-covering map which is defined as g(1) = p, g(2) = q, g(3) = r, g(4) = s. Hence for every Q in Z, there exist a inverse image

$f^{-1}(g^{-1}(Q))$ in U which can be written as a union of disjoint micro pre-open sets $\{K_\alpha\}$ is homeomorphically mapped onto f for each α. Therefore g $\circ$ f is a micro pre-covering map.

Theorem 5.11
If U is a Mic-PT$_2$ space then f: U $\rightarrow$ V is a micro pre-covering map.
Proof: Let U be a Mic-PT2 space. By definition, [13] for each pair of distinct points there exist disjoint micro pre-open sets containing those points. Let f: U $\rightarrow$ V be a micro pre-continuous and surjective map. Suppose that T is a micro open set in V then $f^{-1}(T)$ is micro pre-open set in U. Since U is the Mic-PT$_2$ space, $f^{-1}(T)$ can be written as union of disjoint micro pre-open sets which will be homeomorphically mapped onto U. This implies that T is micro pre-evenly covered. Therefore, f: U $\rightarrow$ V is a micro pre-covering map.

Remark 5.12
The converse part of the above theorem need not be true which is shown in the following example.

Example 5.13
Let U = {a,b,c,d}, U/R = {{a,c},{b,d}}, X = {a,c}, $\tau_R(X)$ = {U,$\emptyset$,{a,c}}, μ = {b} and $\mu_R(X)$ = {U,$\emptyset$,{b},{a,c},{a,b,c}}. Mic-PO(U) = {U,$\emptyset$,{a},{b},{c},{a,b},{b,c},{a,c},{a,b,c},{b,c,d} ,{a,b,d}}. Let V = {1,2,3,4}, V/R = {{1,3},{2,4}}, Y = {1,3}, $\tau^{'}_R(Y)$ = {V,$\emptyset$,{1,3}}, $\mu^{'}$ = {3} and $\mu^{'}_R(Y)$ = {V,$\emptyset$,{3},{1,3}}. Let f be micro pre-continuous and surjective map and defined as f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4. Since f is micro pre-continuous, for every micro open set H in V there exist inverse image in micro pre-open set in U. The inverse image of {3} and {1,3} is {c} and {a,c} which can be written as union of disjoint micro pre-open sets {c} and {a},{c} respectively which is referred as K$_\alpha$ and the restriction of f to K$_\alpha$ is homeomorphism of K$_\alpha$ onto H. The collection {{c}}, {{a},{c}} will be partition of $f^{-1}\{3\}$ and $f^{-1}\{1,3\}$ respectively is said to be slices. Hence f is said to be micro pre-covering map. But U is not a Mic-PT$_2$ space since the distinct points like c and d does not have disjoint neighborhoods.

6. Conclusion

In this paper, we defined micro semi-continuous, micro pre-continuous, micro pre-irresolute, and micro pre-covering map. Continuous functions are used in some real life applications. Analog signals are modeled as continuous functions. Understanding their topological properties helps filtering, transmission, and reconstruction. In Biology topological continuity helps in studying gradual shape transformations in organisms or organs.

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