

Estimating Concealment Behavior via Innovative and Effective Randomized Response Model

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Abstract Estimating concealment behavior via direct questioning often fails. One proposed and effective solution to tackle this challenge is the Randomized Response Technique (RRT). This study aims to present a new efficient and easily applicable randomized response model as a practical tool for estimating concealment behavior with improved reliability. Efficiency examination and privacy protection of the proposed model are analyzed. As a real-world implementation of the model, the case of COVID-19 non-disclosure among university students is investigated as an example of concealment behavior. The proposed model, with a rational choice of parameters, was tested on a sample of university students and demonstrated practical reliability in real-world settings. Health status disclosure ratio was estimated. This estimate serves as a foundation for predicting concealment behavior in different fields.

Keywords Concealment behavior, pandemics, sample surveys, randomized response technique, privacy measure

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1. Introduction

Randomized Response (RR) is a statistical technique used for collecting private and confidential data of individuals. Developed initially by Warner [33], it was meant to solicit honest answers to sensitive questions without necessarily revealing the identity of a person. The method is most appropriate in instances where people might be afraid of sharing such information due to fear of being judged by society, victimization from certain groups or they may also be aware that there are legal actions which can be taken against them if found guilty of committing some offenses. Through making the responses random, one's identity can never be disclosed hence this acts as a shield towards maintaining individuals' privacy and confidentiality.

After Warner proposed the randomized response technique, several authors have broadened the method in order to make the model more efficient and decrease the estimate's variance. Some authors recommended that parameter

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values which minimize the variance of the estimator should be used, while others suggested different estimation techniques. Certain researchers utilized auxiliary characteristics or covariates so that the estimate's precision can be improved [9, 19], whereas some adopted Bayesian approach [34]. Logit models [13] and stratified sampling [15, 12] were also employed for enhancing the sensitive attribute's estimation. Many studies aimed at improving the efficiency of the technique through design modification [11, 22, 16, 21, 31, 30, 14, 9, 10, 27, 8, 23, 29, 1, 2, 3].

This research paper aims to introduce a new efficient and easily applicable randomized response model via the utilization of the design modification approach. To verify the applicability of the suggested model in measuring concealment behavior, it is utilized to estimate the prevalence of COVID-19 non-disclosure among university students, as an example. For this purpose:

- A survey was conducted among a sample of university students using a modified version of the randomized response technique to protect privacy and increase survey response rates.
- The survey included a question about their willingness to disclose their health status during exams, to assess their experiences with COVID-19 disclosure.

2. Groundbreaking models

2.1. Warner's model

Warner [33] proposed the innovative RR model to estimate the percentage of individuals with sensitive trait (T), denoted as π . As outlined in Warner's model, the estimation of π , with appropriate notation adjustments, is expressed as follows:

$$\hat{\pi}_w = [\hat{\alpha} - q_1] [1 - 2q_1]^{-1} \quad q_1 \neq 0.5 \quad (1)$$

where $\hat{\alpha} = n'/n$ is the observed proportion of "yes" answers.

Additionally, the variance is determined by:

$$V(\hat{\pi}_w) = \frac{\pi\pi^c}{n} + \frac{q_1 [1 - q_1] [1 - 2q_1]^{-2}}{n} \quad \pi^c = (1 - \pi) \quad (2)$$

2.2. Mangat and Singh's model

Mangat & Singh [22] presented an effective two-stage RR model. In their framework, the estimation of π is defined as:

$$\hat{\pi}_{M\&S} = [\hat{\alpha} - q_1 q_2] [1 - 2q_1 q_2]^{-1} \quad q_1 q_2 \neq 0.5 \quad (3)$$

The variance for this estimate is given by:

$$V(\hat{\pi}_{M\&S}) = \frac{\pi\pi^c}{n} + q_1 q_2 \frac{[1 - q_1 q_2] [1 - 2q_1 q_2]^{-2}}{n} \quad (4)$$

Mangat & Singh [22] illustrated that their model surpasses Warner's model by appropriately selecting feasible values for p_1 and p_2 as follows:

$$p_2 > [1 - 2p_1] q_1^{-1} \quad (5)$$

2.3. Mangat's model

Mangat [21] put forth a straightforward randomized response design where the respondents are instructed to answers "yes" if they possess the sensitive trait (T). Otherwise, they are directed to use the Warner randomization device. In his framework, the estimation of π is defined as:

$$\hat{\pi}_M = [\hat{\alpha} - 1 + p_1] [p_1]^{-1} \quad (6)$$

The variance for this estimate is given by

$$V(\hat{\pi}_M) = \frac{\pi\pi^c}{n} + \frac{\pi^c q_1 [1 - q_1]^{-1}}{n} \tag{7}$$

Mangat [21] demonstrated that his model exhibits greater efficiency compared to Mangat & Singh’s model when:

$$\pi > 1 - p_1 q_1 [1 - q_1 q_2] [1 - 2q_1 q_2]^{-2} \tag{8}$$

This condition is met by selecting feasible values for p_1 and p_2 .

Furthermore, he established that his model surpasses the original Warner’s model in efficiency when:

$$\pi > 1 - p_1^2 [2p_1 - 1]^{-2} \tag{9}$$

This condition is constantly satisfied for $p_1 > 1/3$.

In the following section, we introduce a novel model that is more effective than the previously discussed randomized response models.

3. The proposed RR model

To estimate the concealment behavior of individuals having the sensitive trait (T), a random sample of “ n ” interviewees are selected. Each interviewee is provided with a set of “Yes” cards, a set of “No” cards and a two-stage random device. They are instructed to pick a ”Yes” card if they have the sensitive trait; otherwise, they are directed to utilize the two-stage random device. In the initial stage, one of two alternatives is to pick a ”No” card (with probability p_2) or proceed to the next stage (with probability q_2). In the subsequent stage, if they advance to it, they face a decision between picking a ”No” card (with probability p_1) or a ”Yes” card (with probability q_1). The process, depending on their actual status regarding the sensitive trait and the outcome of the random device, may conclude before utilizing the two-stage random device, either after using it in the initial stage, or in the second stage. The interviewee places the selected card into a container without disclosing to the interviewer which card was chosen, and at which point the process concluded. The probability of placing a ”Yes” card into the container is:

$$\alpha = \pi + \pi^c q_1 q_2 \qquad \pi^c = (1 - \pi) \tag{10}$$

where:

π : The proportion of having the sensitive trait.

p_{3-s} : The probability of choosing a ”No” card in stage s , $s = 1,2$ and $p_{3-s} + q_{3-s} = 1$.

In this case, the estimator for the population ratio of individuals having the sensitive trait ($\hat{\pi}$) is:

$$\hat{\pi} = [\hat{\alpha} - q_1 q_2] [1 - q_1 q_2]^{-1} \qquad q_1 q_2 \neq 0.5 \tag{11}$$

where $\hat{\alpha}$ is the ratio of “Yes” answer obtained from the sample.

3.1. Proposed estimator properties

Theorem 1: The suggested estimator $\hat{\pi}$ has a variance given by:

$$V(\hat{\pi}) = \frac{\pi\pi^c}{n} + \pi^c q_1 q_2 \frac{[1 - q_1 q_2]^{-1}}{n} \tag{12}$$

Proof: Based on Eq. (11),

$$V(\hat{\pi}) = V\left([\hat{\alpha} - q_1 q_2] [1 - q_1 q_2]^{-1}\right) = V(\hat{\alpha}) [1 - q_1 q_2]^{-2} \tag{13}$$

as $n\hat{\alpha} \sim Bin(n, \alpha)$, then

$$V(\hat{\alpha}) = \frac{\alpha(1-\alpha)}{n} \quad (14)$$

Substituting Eq. (14) in Eq. (13) by:

$$V(\hat{\pi}) = \frac{\alpha(1-\alpha)[1-q_1q_2]^{-2}}{n} \quad (15)$$

Eq. (10) can be use to calculate $\alpha(1-\alpha)$ as follows:

$$(1-\alpha) = \pi\pi^c [1-q_1q_2]^2 + \pi^c q_1q_2 [1-q_1q_2] \quad (16)$$

then, it is easy to get (12) by inserting (16) in (15). \square

Theorem 2: The $V(\hat{\pi})$ has an unbiased estimator given by:

$$\hat{V}(\hat{\pi}) = \frac{\hat{\alpha}(1-\hat{\alpha})[1-q_1q_2]^{-2}}{(n-1)} \quad (17)$$

Proof: Considering the expectation on both sides of Eq. (17), the proof holds.

4. Efficiency Comparison

The presented model is proposed as a proficient substitute for the pioneering randomized response models proposed by Warner, Mangat & Singh, and Mangat. Efficiency comparisons conducted by Mangat [21] concluded that his model outperforms Mangat & Singh's model [22] and Warner's model [33], in terms of efficiency. Hence, our attention will be directed towards comparing the efficiency of Mangat's model [21] with the proposed model.

Theorem 3: The proposed estimate is consistently more efficient than Mangat's estimate.

Proof: $\hat{\pi}$ will be more efficient than $\hat{\pi}_M$ iff

$$V(\hat{\pi}) < V(\hat{\pi}_M)$$

or

$$\frac{\pi\pi^c}{n} + \pi^c q_1q_2 \frac{[1-q_1q_2]^{-1}}{n} < \frac{\pi\pi^c}{n} + \pi^c \frac{q_1[1-q_1]^{-1}}{n}$$

or

$$q_1q_2 [1-q_1q_2]^{-1} < q_1 [1-q_1]^{-1}$$

The last expression simplifies to:

$$q_2 [1-q_1] < 1-q_1q_2$$

or

$$q_2 < 1$$

which always holds.

Figure 1 illustrates the efficiency difference between the proposed model and Mangat's model across all values of q_1 and q_2 . Positive values support the suggested model.

Figure 1, reveals:

- Across all values of q_1 and q_2 the proposed estimator outperforms Mangat's estimator.
- If q_2 is held fixed, the efficiency difference between the proposed estimator and Mangat's estimator increases as q_1 rises.
- If q_1 is held fixed, the efficiency difference between the proposed estimator and Mangat's estimator increases as q_2 decreases (the proposed estimator variance decreases as q_3 decreases from 0.9 to 0.1, while the Mangat's estimator variance is fixed).

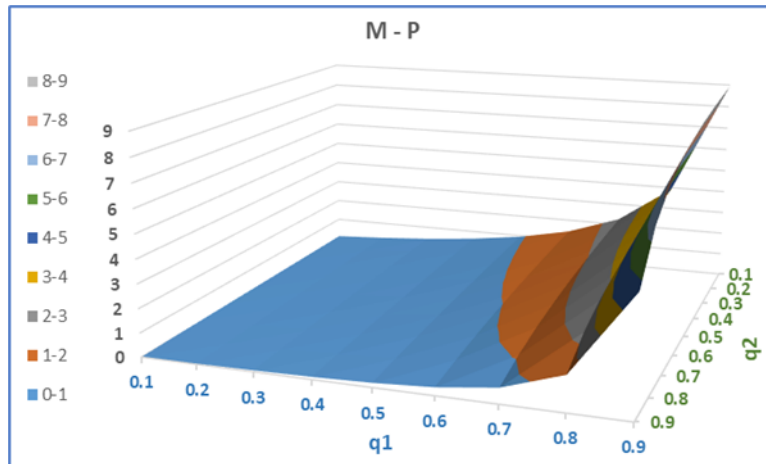


Figure 1. The efficiency difference between the proposed model and Mangat’s model.

5. Privacy protection measure

Randomized response models have a fundamental attribute of safeguarding the confidentiality of survey respondents. Several approaches have been suggested for protecting privacy in RR models [6, 17, 20, 35]. The privacy protection measure for Warner’s model is [35]:

$$M_W(R) = \frac{(1 - 2q_1)^2}{2q_1(1 - q_1)}$$

and for Mangat & Singh’s model are given as

$$M_{M\&S}(R) = \frac{(1 - 2q_1q_2)^2}{2q_1q_2(1 - q_1q_2)}$$

and for Mangat’s model

$$M_M(R) = \frac{2q_1 - 1}{2q_1}$$

and it can be derived for the proposed model as follows:

$$P(yes|T) = 1 \quad \text{and} \quad P(yes|\bar{T}) = q_1q_2$$

$$P(no|T) = 0 \quad \text{and} \quad P(no|\bar{T}) = 1 - q_1q_2$$

and

$$P(T|yes) = \frac{\pi}{\pi + (1 - \pi)P(yes|T)/P(yes|\bar{T})}$$

$$P(T|no) = \frac{\pi}{\pi + (1 - \pi)P(no|T)/P(no|\bar{T})}$$

Hence,

$$M_P(R) = \left| 1 - \frac{1}{2} \{ \tau(yes) + \tau(no) \} \right|$$

then

$$M_P(R) = \frac{2q_1q_2 - 1}{2q_1q_2} \quad (18)$$

According to Zhimin and Zaizai [35], the level of privacy protection for respondents increases as the value of the privacy protection measure $M_P(R)$ (as defined in Eq. (18)) approaches zero.

6. Real-world applications

To demonstrate the practical application of the proposed model in real-world situations, an example involving measuring individuals' concealment behavior during pandemics, like the case of COVID-19, is being explored. COVID-19, linked to the SARS-CoV-2 virus, was first identified in Wuhan city in December 2019. The World Health Organization (WHO) declared it a pandemic on March 11, 2020, leading to global lockdowns that shuttered schools and universities in over 124 nations. The repercussions of these measures had impacts on the daily routines of more than 2.2 billion students [28]. In the early stages of the pandemic, countries implemented precautionary strategies to combat the virus. However, there was a general lack of awareness among the public regarding the seriousness of COVID-19, resulting in a diminished sense of susceptibility among some individuals. Consequently, primary prevention measures did not yield the desired outcomes in practice [5].

School closures due to the pandemic were one of the most significant phenomena of the pandemic in various countries. This forced schools and universities to shift from any traditional approach towards teaching and adopt new paradigms of delivery. Schools and universities shifted immediately to online learning delivery systems to keep on with the instruction process as a way of preventing the spread of COVID-19. Also, safety protocols were implemented to continue their work while complying with health measures [7]. This has entailed measures like wearing face masks, regularly washing hands, maintaining social distance, or using hand sanitizers, and identifying individuals that interacted with COVID-19 patients. Universities have also had to include questions and requests from students and individuals regarding their health status in the event that the specified individual comes into contact with a COVID-19 patient or if they experience symptoms or have potential exposure to the virus within the university setting. From the information provided, university officials can fast track the identification of students who were affected, isolated from others, or tested positive for the virus so that effective contact tracing and isolation measures can be taken [32, 24, 25, 18, 26].

Despite the significance of self-reported health status, some concerns have been raised regarding students' compliance to disclose their particular health status during examinations. At the same time, students often get scared that disclosing information about their health status could result in negative consequences, they may, for example, be barred from taking exams or isolated by their peers. Hence, conventional approaches to gathering such information via direct questioning often fail. One suggested effective solution to address this issue is the Randomized Response Technique (RRT). In the empirical study, the proposed RR model was applied to estimate COVID-19 non-disclosure rate among university students through an experimental study in which a random sample of Saudi university undergraduates who have gone through the time of Covid-19 pandemic while attending university, was selected.

For a large population, and at a 95% confidence level and a 10% margin of error, a sample size of at least 97 is appropriate. Invitations to take part in the study were sent to 150 students, among them 143 have responded.

All interviewees were informed and made an agreement regarding the experiment's location, date, and time a few days before it. Informed consent was obtained from all participants. A brief presentation detailing the entire process and highlighting how their privacy is well-preserved by design was given at the start of the trial. Without anyone else in the room being able to see them, each respondent completes the experiment behind a partition before exiting.

An empty box, a group of “Yes” cards, a group of “No” cards, and a two-stage random device have been used for this purpose. At the start of the experiment, each respondent is instructed to select a ”Yes” card and drop it in the box if he/she has complied to disclose their health status or were willing to do so, and the experiment concludes at this point . If not, he/she is directed to use the two-stage random devices. The first stage was set to show one of two options:

- pick a ”No” card (with $p_1= 0.3$)
- proceed to the next stage (with $q_1= 0.7$)

So, if the first option appears, the interviewee places a “No” card into the container and the experiment concludes at this point. If the second option appears and the proceed to the next stage, the respondent face a decision between:

- pick a ”No” card (with $p_2= 0.3$)
- pick a ”Yes” card (with $q_2= 0.7$)

The selection of probabilities p_1 and p_2 should prioritize maintaining acceptable efficiency while minimizing the compromise on privacy. This decision should align with minimizing the measure of privacy protection as defined by Eq. (18), promoting the inclusion of sensitive questions while diminishing respondent suspicion.

Based on the sample results, where 65 “Yes” responses were obtained, and employing Eq. (11) the estimate for the ratio of students, in the selected population, who did not disclose their health status during the pandemic, ($\hat{\pi}^*$), is 0.0695 (approximately 7%) with an estimated variance ($V(\hat{\pi}^*)$) of 0.0067 (Eq. (17)).

7. Discussion

The case of COVID-19 non-disclosure among university students is considered as an application of the proposed RR model, to validate its applicability. This application indicated that the proposed RR model has proven to be an efficient practical alternative to Mangat’s model, with greater credibility. The choice of 0.7 for the values of probabilities q_1 and q_2 seemed logical because this allowed for acceptable efficiency while compromising as little as possible privacy. This selection corresponds to the lowest value of the measure of privacy protection given by Eq. (18), which enhances the likelihood of the appearance of sensitive questions, while reducing respondents’ suspicion levels, thus increasing their cooperation. Overall, this approach strikes a good balance between protecting privacy and increasing survey response rates. Figure 2 demonstrates that the proposed estimator outperforms Mangat’s estimator in efficiency when $q_2=0.7$ and all values of q_1 . The case is the same for any other value of q_2 .

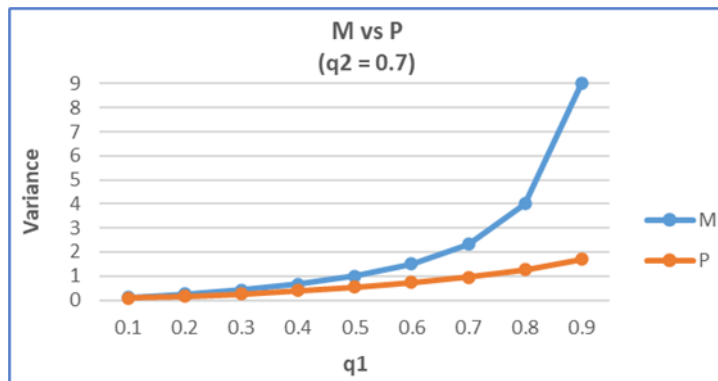


Figure 2. Efficiency of Mangat’s estimator and the suggested estimator at $q_2=0.7$.

According to the specific use of the suggested model for assessing adherence of higher education students to COVID-19 disclosure requirements, the resulting estimate (0.0695) can be considered as an initial approximation for health disclosure among the whole Saudi community during the time of the pandemic. University students' adherence to and views on COVID-19 preventive measures and their satisfaction about such measures is affected by several factors including the lack of physical distancing, lack of organization, and lack of screening measures [4].

The estimate of health status disclosure ratio (the proportion of individuals reveal their health-related information e.g., medical conditions, symptoms, or risk factors) introduced in this research can be generalized as a foundation for predicting the level of adherence to preventive measures, in general, at times of such pandemics.

Moreover, the proposed model can be utilized following the approach detailed in section 6 to assess the disclosure of other sensitive or delicate issues (such as drug use, tax compliance, alcoholism, mental health conditions, criminal activities, abortion, affiliation with political parties, dishonest behavior, theft, illegal marriages, and more).

8. Limitations and Future Research

Presenting the proposed model as a straightforward and effective tool for assessing concealment behavior solely under conditions of entirely honest disclosure presents a constraint. However, in scenarios concerning highly delicate subjects like sexual conduct, criminal activities, racial bias, unethical practices, or when survey participants lack confidence in the model, the likelihood of incomplete truthful reporting increases [2]. This sets the stage for future research endeavors - to develop a tailored iteration of the model that performs more effectively in the absence of complete honesty, thereby expanding its utility in addressing profoundly sensitive attributes.

Author Contributions

“Conceptualization; methodology; visualization; validation; formal analysis; writing—original draft preparation; writing—review and editing; Ahmad M. Aboalkhair, El-Emam El-Hosseiny, Mohammad A. Zayed, Tamer Elbayoumi, Mohamed Ibrahim and A. M. Elshehawey; funding acquisition, El-Emam El-Hosseiny. All authors have read and agreed to the published version of the manuscript.”

Ethics statement

The study was conducted according to the guidelines of the Declaration of Helsinki and supported by the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU). Informed consent was obtained from all subjects and/or their legal guardian(s).

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

The authors declare that the data supporting the findings of this study are available within the article.

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