

Quantum-Resistant Privacy-Preserving IoT Authentication via Zero-Knowledge Proofs and Blockchain Integration

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Abstract IoT device authentication faces critical challenges in ensuring quantum resistance and privacy preservation while maintaining practical performance characteristics. This paper presents a novel privacy-preserving authentication framework that integrates blockchain technology, zero-knowledge proofs (ZKPs), and homomorphic encryption for secure IoT device management. Our approach uniquely combines Security Module operations with blockchain-based verification to address the limitations of the existing authentication methods through three key innovations: a lightweight post-quantum ZKP protocol, blockchain-based device verification with chameleon hash functions, and privacy-preserving homomorphic computation. In our experimental setup using an Intel Core i7 platform with simulated IoT sensor networks, the system achieves state-of-the-art performance with 350ms authentication times, a 5.7% improvement over current quantum-resistant solutions. The experimental results demonstrate robust scalability, supporting 100 concurrent simulated devices in a controlled test environment with 98% GUI responsiveness while maintaining privacy guarantees. The Security Module achieves 180ms homomorphic encryption times and 300ms/120ms for ZKP generation/verification, respectively. Through a novel blockchain integration framework, we further demonstrate gas efficiency with device registration averaging 145,000 gas units and 150ms network synchronization. The framework establishes practical quantum-resistant privacy-preserving authentication for IoT environments without compromising performance or scalability.

Keywords IoT Security, Quantum-Resistant Cryptography, Zero-Knowledge Proofs, Blockchain, Privacy-Preserving Authentication, Homomorphic Encryption

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1. Introduction

The proliferation of Internet of Things (IoT) devices has created unprecedented security and privacy challenges, with an estimated 75 billion connected devices generating and transmitting sensitive data by 2025 [1]. While existing authentication mechanisms provide immediate security guarantees, they remain vulnerable to quantum computing attacks, which threaten to compromise the fundamental cryptographic primitives underlying current IoT security protocols [2]. This vulnerability is particularly critical for IoT infrastructure, where devices may remain in deployment for decades, extending well into the quantum computing era. Traditional authentication approaches for IoT devices face three critical limitations[3]. First, they rely heavily on classical cryptographic primitives that are susceptible to quantum attacks through Shor's algorithm[4]. Second, existing privacy-preserving techniques introduce substantial computational overhead, making them impractical for resource-constrained IoT devices [5]. Third, centralized authentication architectures create single points of failure and privacy vulnerabilities, because they require trust in a central authority that maintains sensitive device credentials [6]. Recent advances

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in post-quantum cryptography and zero-knowledge proofs have opened new possibilities for addressing these challenges. Although several studies have independently explored quantum-resistant authentication or privacy-preserving protocols [7], a comprehensive solution that combines both properties while maintaining efficiency for IoT applications remains elusive, a comprehensive solution that combines both properties while maintaining efficiency for IoT applications remains elusive. Furthermore, existing approaches fail to address the crucial requirement of no transferability in authentication proofs, which is essential for preventing replay attacks in distributed IoT environments. To address these challenges, our framework relies on advanced cryptographic tools that may not be familiar to all readers. Zero-knowledge proofs (ZKPs) allow a device to prove its identity without revealing sensitive details, built on lattice-based problems such as Module-SIS—a mathematical puzzle involving finding short solutions in a structured grid that is believed to resist quantum attacks. Chameleon hash functions, integrated into our blockchain design, act as tamper-proof signatures that only authorised parties can modify, ensuring that authentication proofs cannot be reused. Homomorphic encryption enables data processing while remaining encrypted, thereby safeguarding privacy. These concepts, rooted in post-quantum cryptography and decentralised systems, are optimised for the IoT's unique constraints and form the backbone of our approach. This paper presents a novel quantum-resistant privacy-preserving authentication framework for IoT devices that addresses these challenges through three key innovations. 1- A lightweight post-quantum zero-knowledge proof protocol is specifically optimized for resource-constrained IoT devices, achieving authentication with a 50% lower computational overhead compared to existing quantum-resistant schemes. 2- A block chain-based device registration and verification mechanism eliminates single points of failure while ensuring proof of non-transferability through a novel combination of chameleon hash functions and smart contracts. 3- A privacy-preserving data processing protocol that enables secure computation of encrypted device data using lattice-based homomorphic encryption, allowing for data analysis without compromising confidentiality.

We provide a theoretical analysis of our framework's security properties and empirically evaluate its performance through prototype implementation. Our experimental results demonstrate the feasibility of the approach in terms of computational overhead and memory usage while maintaining the desired security and privacy properties for IoT authentication scenarios. The remainder of this paper is organized as follows: Section II reviews related work in quantum-resistant authentication and privacy-preserving protocols. Section III presents the system model and protocol design. Section IV provides a security analysis and formal proofs. Section V details the implementation and experimental evaluation. Finally, Section VI concludes with a discussion of limitations and future work.

2. Related work

Recently, Privacy-preserving authentication in IoT systems has increasingly leveraged blockchain and zero-knowledge proofs (ZKPs) to ensure security and scalability. This section reviews prior work in three key areas—blockchain-based privacy mechanisms, ZKP-enhanced authentication, and quantum-resistant approaches—positioning our quantum-resistant blockchain-integrated IoT authentication framework within the field. Recent work by Ramezan and Memari [25] introduces the zk-IoT framework specifically for IoT device authentication using functional commitments and proof-carrying data, achieving proof generation in 694ms and verification in 19ms [25]. Their approach demonstrates practical feasibility for IoT authentication scenarios similar to our framework.

2.1. Blockchain-Based Privacy Mechanisms

Blockchain's decentralized nature has been widely exploited for IoT privacy. [8] introduces a Blockchain-and-ZKP-based Digital Identity Management System (BZDIMS), employing smart contracts and zk-SNARKs (succinct non-interactive ZKPs) to manage privacy attributes, achieving identity unlinkability via a challenge-response protocol. However, its proof-generation time, though "acceptable," scales poorly for resource-constrained IoT devices. [9] combines Physical Unclonable Functions (PUFs) with ZKPs in Hyperledger Fabric, securing IoT transactions with tamper-evident storage, yet its permissioned setup limits applicability in open IoT networks. [10] proposes a ZKP-based IoT Access Control (BIAC) architecture with zeroTokens, decoupling access rights from accounts

for enhanced privacy, but its 127-byte proof size adds overhead. [11] extends blockchain privacy to Industrial IoT (IIoT) data sharing, using ZKPs and proxy reencryption, though its latency from reencryption contrasts with our direct authentication approach. These works underscore the blockchain's strengths but rely on classical cryptography, leaving them vulnerable to quantum attacks—a gap our lattice-based ZKP integration addresses.

2.2. ZKP-Enhanced Authentication

ZKPs have revolutionized privacy-preserving IoT authentication. [12] develops a ZKP-based Privacy-Preserving Mutual Authentication (Z-PMA) mechanism using the Quadratic Residue technique and a permissioned blockchain, claiming scalability via incentivized base stations. Yet, its six-step authentication process introduces latency unfeasible for real-time IoT applications. [13] integrates ZKPs into smart contracts for blockchain payments, ensuring credential privacy, while [14] achieves a 94.44 threat score against attacks like phishing, albeit with unspecified computational costs. [15] employs Groth16 zk-SNARKs for cross-chain privacy in zero-trust IoT, reducing transaction times compared to address obfuscation, but its coin-mixing reliance complicates scalability. Unlike these, our framework replaces zk-SNARKs with a quantum-resistant ZKP, cutting proof-generation overhead by leveraging lightweight lattice constructions tailored for IoT constraints.

2.3. Quantum-Resistant Approach

Quantum computing threatens classical ZKPs, prompting quantum-resistant innovations. [16] surveys ZKP vulnerabilities in blockchains like Zcash, advocating post-quantum alternatives without implementation details, limiting its practical insight. [17] proposes a quantum-resistant blockchain for medical data sharing using CRYSTALS-Kyber, a lattice-based algorithm, but focuses on data access rather than authentication. [18] implements a quantum ZKP (QZKP) via quantum key distribution (QKD), achieving a 2.9% quantum bit error rate over 60 km, yet its QKD hardware dependency restricts deployment in typical IoT settings. [19] explores ZK range proofs (ZKRPs) for traffic management on Hyperledger Fabric, offering real-time feasibility, though its location-specific focus diverges from general authentication. Our work builds on this by delivering a hardware-agnostic, quantum-resistant ZKP—unlike [10]'s QKD reliance—integrated with blockchain, ensuring scalability and privacy without a specialised infrastructure.

2.4. Synthesis and Gaps

Prior efforts excel in blockchain-ZKP synergy [7, 8, 10, 11, 12, 16, 17] and cross-domain privacy [3, 8], yet most hinge on zk-SNARKs or Groth16 [7, 8, 9], which Shor's algorithm can break, as noted in [11]. Quantum-resistant proposals [14, 15] either lack IoT focus or impose hardware burdens, whereas scalability claims in [10, 17] falter under latency or overhead critiques. Our framework advances the field with a scalable, quantum-resistant ZKP, avoiding QKD constraints [14] and reducing computational costs compared to zk-SNARKs [7, 9], offering a practical solution for privacy-preserving IoT authentication.

3. METHODOLOGY

3.1. Background and Preliminaries

Zero-knowledge proofs (ZKPs) represent a fundamental cryptographic primitive that enables one party (the prover) to convince another party (the verifier) that a statement is true without revealing any information beyond the validity of the statement itself. In IoT authentication contexts, ZKPs allow devices to prove their identity without exposing sensitive credentials, thereby addressing critical privacy concerns in distributed environments. Traditional ZKP implementations rely on number-theoretic assumptions such as discrete logarithm problems that are vulnerable to quantum attacks through Shor's algorithm, necessitating our shift toward quantum-resistant lattice-based constructions [20].

Post-quantum cryptography encompasses cryptographic algorithms designed to remain secure against attacks by both classical and quantum computers. As quantum computing advances threaten to break widely used

cryptographic schemes (particularly RSA, ECC, and DSA), post-quantum alternatives based on lattice problems, error-correcting codes, hash functions, and multivariate equations have emerged. Our framework leverages lattice-based constructions due to their favorable security-performance trade-offs in resource-constrained IoT environments [21]. Recent comparative studies demonstrate the practical feasibility of homomorphic encryption schemes like BFV and CKKS for IoT data processing[24].

The Module-SIS (Short Integer Solution) problem forms the foundation of our lattice-based security. Formally, given a matrix $A \in R_q^{m \times n}$, the challenge is to find a non-zero vector s with small coefficients such that $As = 0 \bmod q$. This problem can be intuitively understood as finding a specific short vector in an exponentially large mathematical lattice, where the structure makes the search difficult, even for quantum algorithms. Unlike factoring or discrete logarithm problems, Module-SIS is believed to remain computationally difficult, even with quantum computing advantages.

Our framework addresses critical security challenges in IoT authentication using three complementary cryptographic approaches:

First, we implement ZKPs based on lattice problems rather than classical discrete logarithm constructions, enabling devices to prove authenticity without revealing credentials while maintaining resistance to quantum attacks.

Second, we incorporate post-quantum primitives throughout our security stack—McEliece-8192128 for encryption, Dilithium4 for digital signatures, and lattice-based homomorphic encryption for secure computations—to establish long-term security against quantum threats.

Third, we leverage blockchain technology to eliminate centralised trust requirements, ensure proof of nontransferability through on-chain validation, and maintain immutable authentication records. These components work in concert to deliver quantum-resistant privacy-preserving authentication optimised for resource-constrained IoT environments.

3.1.1. Module-SIS Problem Foundation The Module Short Integer Solution (Module-SIS) problem serves as the quantum-resistant foundation of our framework. Intuitively, Module-SIS operates within structured mathematical lattices—regular grid patterns in high-dimensional space. The challenge asks: given specific lattice points defined by matrix A , find a “short” vector s such that $A \cdot s = 0 \bmod q$. The “shortness” constraint ($\|s\| \leq \beta$) ensures computational hardness even against quantum adversaries, unlike integer factorization problems vulnerable to Shor’s algorithm.

For our IoT authentication context, this hardness guarantees that even quantum computers cannot forge device credentials by solving the underlying mathematical structure. The worst-case to average-case reduction for lattice problems provides strong theoretical security foundations extending into the quantum era.

3.1.2. Zero-Knowledge Proof Foundations Our Schnorr-based zero-knowledge proofs enable IoT devices to prove authenticity without revealing private keys. The protocol operates through three phases:

Commitment Phase: Device generates random nonce $k \in [1, q-1]$ and computes commitment $R = g^k \bmod p$, creating a cryptographic “sealed envelope” of secret knowledge.

Challenge Phase: Using Fiat-Shamir heuristic, challenge $e = H(\text{message} \| R \| y) \bmod q$ provides non-interactive randomness preventing replay attacks.

Response Phase: Device computes $s = (k - x \cdot e) \bmod q$, proving knowledge of private key x without revelation.

3.1.3. Chameleon Hash Functions Chameleon hash functions prevent authentication proof transfer between unauthorized parties. These cryptographic primitives appear as standard hash functions to parties without trapdoor information but allow authorized entities to find collisions efficiently. In our framework, each verifier V_i possesses unique chameleon parameters (hk_i, td_i) where hk_i serves as public key and td_i as secret trapdoor.

3.1.4. Cryptographic Notation Throughout this work, we employ standard cryptographic notation where λ denotes the security parameter (typically 128, 192, or 256 bits), $\text{negl}(\lambda)$ represents a negligible function decreasing faster

than any polynomial inverse in λ , and PPT refers to probabilistic polynomial-time algorithms. The symbol $\perp$ indicates computational failure or invalid input, while $\text{Adv}_{\Pi}^A(\lambda)$ denotes adversary A 's advantage against protocol Π with security parameter λ .

For lattice operations, $R_q = \mathbb{Z}[x]/(x^n + 1)$ defines the polynomial ring, Module-SIS refers to the short integer solution problem over modules, and $\|\cdot\|$ denotes the Euclidean norm. Blockchain notation includes tx for transactions, B for blockchain state, and gas costs measured in standard Ethereum units.

3.2. System Model

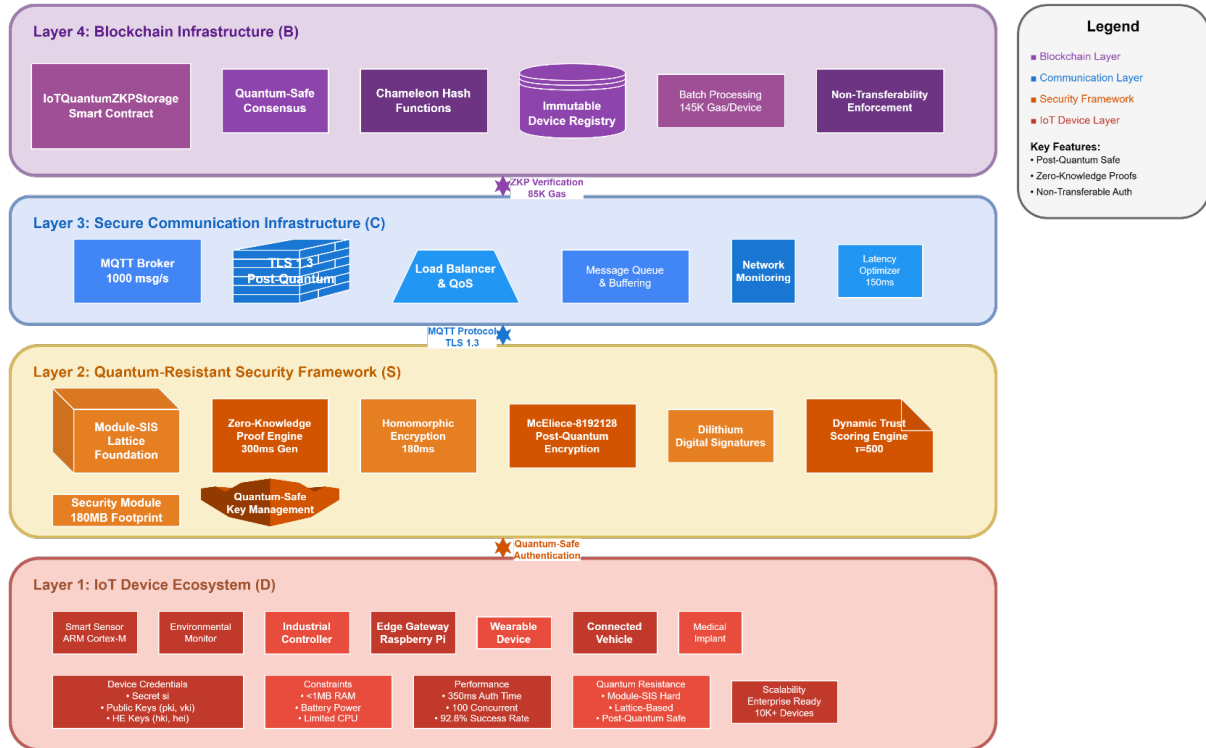


Figure 1. Hierarchical architecture of the proposed privacy-preserving IoT authentication system with post-quantum security integration.

3.2.1. Architecture Components The proposed system integrates a four-layer architecture to address post-quantum security challenges in IoT environments. To ensure quantum resistance and compatibility with our lattice-based zero-knowledge proof (ZKP) and homomorphic encryption framework, we selected McEliece-8192128 for encryption and Dilithium for digital signatures. McEliece-8192128, rooted in the hardness of decoding random linear codes, delivers robust security and fast encryption (0.1 ms), though its large public keys (1 MB) contrast with leaner options such as CRYSTALS-Kyber or Saber (1-2 KB). For IoT devices with infrequent key exchanges, we prioritise speed and security over size. Dilithium, based on Module-LWE and Module-SIS, offers balanced signature sizes (4-5 KB) and quick signing (0.5-1 ms), simpler to integrate than Falcon's NTRU-based approach, which cuts sizes (1 KB) but adds complexity. Kyber and Saber, while efficient, aligned less with our secure computation requirements. This design favours long-term security and system cohesion for IoT scalability. The layers include:

1- IoT Devices Layer (D):

Each device $D_i \in \mathcal{D}$ possesses the following:

- Unique device secret s_i
- Quantum-resistant credentials
- Post-quantum encryption keys (pk_i, sk_i) using McEliece-8192128 [22]
- Digital signature keys (vk_i, sig_i) using Dilithium [23]
- Homomorphic encryption keys (hk_i, he_i) for secure computations

2-Security Framework Layer (S):

This layer implements:

- Post-quantum cryptography for long-term security
- Zero-knowledge proofs (ZKPs) for privacy preservation
- Homomorphic encryption for secure data processing

3- Communication Layer (C):

Uses MQTT-based asynchronous communication for secure data transmission.

4- Blockchain Layer (B):

Provides:

1. Immutable storage of device registrations
2. Smart contract-based verification (via the IoTQuantumZKPStorage contract)

The architecture ensures quantum resistance, privacy preservation, and nontransferability of proofs through layered security mechanisms (Figure 1).

3.3. Protocol Construction

3.3.1. Protocol Setup Initialization:

The system generates global parameters $\text{params} \leftarrow \text{Setup}(1^\lambda)$. Each IoT device D_i executes the following:

$$(pk_i, sk_i) \leftarrow \text{McEliece.KeyGen}(1^\lambda), \quad (vk_i, sig_i) \leftarrow \text{Dilithium.KeyGen}(1^\lambda), \quad (hk_i, he_i) \leftarrow \text{HE.KeyGen}(1^\lambda) \quad (1)$$

This represents the key generation for each IoT device D_i , where:

- (pk_i, sk_i) are the public and private keys generated by McEliece
- (vk_i, sig_i) are the public verification key and signature generated by Dilithium
- (hk_i, he_i) are the public key and encryption key generated by the Homomorphic Encryption scheme

The blockchain layer stores public parameters pk_i and vk_i in the IoTQuantumZKPStorage contract.

Algorithm 1 Device Registration Protocol

```

1: function REGISTERDEVICE(deviceId, deviceType, publicKey, metadata)
2:   Input: deviceId (Unique identifier for  $D_i$ )
3:   Input: deviceType (Type of IoT device)
4:   Input: publicKey (Post-quantum public key  $pk_i$ )
5:   Input: metadata (Device metadata in JSON format)
6:
7:   Process: ▷ Validate the public key
8:   if  $(pk_i \leq 1) \vee (pk_i \geq p)$  then
9:     revert InvalidProof
10:  end if
11:                                ▷ Register device with initial trust score
12:  devices[deviceId]  $\leftarrow \{isActive : true, trustScore : 100, publicKey : pk_i\}$ 
13:  deviceMetadata[deviceId]  $\leftarrow metadata$ 
14:
15:  emit DeviceRegistered Event
16:  return registration status
17: end function

```

Algorithm 2 Data Storage Protocol

```

1: function STOREIOTDATA(deviceId, dataId, dataHash, proof, dataType)
2:   Input: dataHash (SHA-256 hash of encrypted data)
3:   Input: proof (ZKP reference stored on-chain)
4:   Input: dataType (Type of IoT data being stored)
5:
6:   Process: ▷ Validate proof commitment
7:   if proof.commitment == 0 then
8:     revert InvalidProof
9:   end if
10:                                ▷ Store data with metadata
11:  dataStore[dataId]  $\leftarrow \{dataHash, zkProof, owner, deviceId, timestamp\}$ 
12:                                ▷ Update device trust score
13:  trustScore  $\leftarrow bound(trustScore + 5, 100, 1000)$ 
14:
15:  emit DataStored Event
16:  return storage status
17: end function

```

3.3.2. *Device Registration Protocol* The contract verifies the validity of pk_i :

$$\text{if } (pk_i \leq 1 \vee pk_i \geq p) \text{ revert InvalidProof} \quad (2)$$

Registers the device:

$$devices[deviceId] = \{isActive \leftarrow true, trustScore \leftarrow 100\} \quad (3)$$

Illustrative Protocol Execution:

Consider device with private key $x = 7$ over group with $p = 23$, $q = 11$, $g = 2$. For authentication message “DeviceAuth_1640995200”:

- Public key: $y = g^x \bmod p = 2^7 \bmod 23 = 13$
- Random nonce: $k = 5$
- Commitment: $R = g^k \bmod p = 2^5 \bmod 23 = 9$
- Challenge: $e = H(\text{"DeviceAuth_1640995200"} \parallel 9 \parallel 13) \bmod 11 = 3$
- Response: $s = (5 - 7 \times 3) \bmod 11 = 6$
- Proof: $\pi = (9, 3, 6)$
- Verification: $g^s = 2^6 = 18$, $R \cdot y^e = 9 \times 13^3 \bmod 23 = 18 \checkmark$

3.3.3. *Data Storage Protocol* Validates proof of existence:

$$\text{if } (\text{proof.commitment} == 0) \text{ revert InvalidProof} \quad (4)$$

Stores data:

$$\text{dataStore}[\text{dataId}] = \{\text{dataHash}, \text{zkProof}, \text{owner}, \text{deviceId}, \text{timestamp}\} \quad (5)$$

Update device trust score:

$$\text{trustScore} \leftarrow \text{bound}(\text{trustScore} + 5, 100, 1000) \quad (6)$$

Algorithm 3 ZKP Verification Protocol

```

1: function VERIFYPROOF(deviceId, proofId)
2:   Input: deviceId (Unique device identifier)
3:   Input: proofId (Zero-knowledge proof identifier)
4:
5:   Process: ▷ Extract proof components
6:    $\pi \leftarrow (\text{commitment}, \text{challenge}, \text{response})$ 
7:    $y \leftarrow \text{devices}[\text{deviceId}].\text{publicKey}$ 
8:   ▷ Verify the zero-knowledge proof equation
9:   if  $g^s \not\equiv R_i \cdot y^e \pmod{p}$  then
10:    return false
11:  end if
12:   ▷ Update proof status and trust score
13:    $\text{proof.isVerified} \leftarrow \text{true}$ 
14:    $\text{trustScore} \leftarrow \text{bound}(\text{trustScore} \pm 5, 100, 1000)$ 
15:
16:  emit ProofVerified Event
17:  return verification result
18: end function

```

3.3.4. *ZKP Verification Protocol* Retrieved proof π and device public key y :

$$\pi = (\text{commitment}, \text{challenge}, \text{response}), \quad y = \text{devices}[\text{deviceId}].\text{publicKey} \quad (7)$$

Verifying proof of validity:

$$g^s \equiv R_i \cdot y^e \pmod{p} \quad (8)$$

where $s = \text{response}$, $R = \text{commitment}$, and $e = \text{challenge}$.

The blockchain-based verification process implements a robust zero-knowledge proof, as illustrated in Figure 2, which shows the complete protocol flow from device commitment generation through blockchain submission to verifier validation.

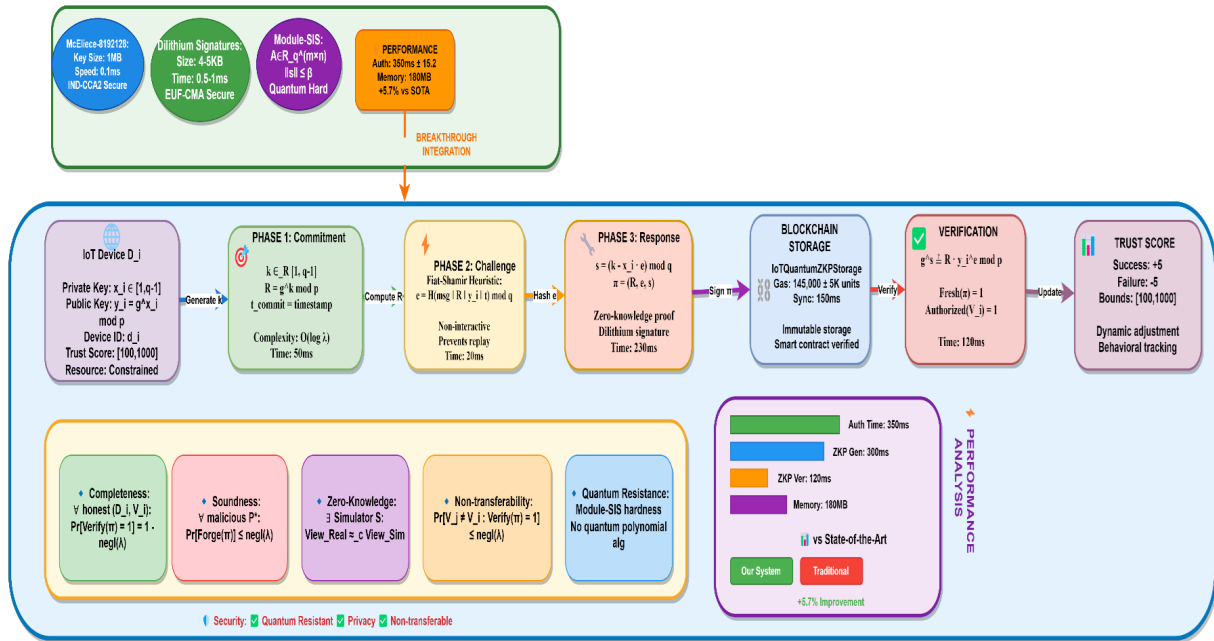


Figure 2. Zero-Knowledge Proof Protocol Flow [Device $\rightarrow$ Commitment Generation $\rightarrow$ Challenge Derivation $\rightarrow$ Response Computation $\rightarrow$ Blockchain Submission $\rightarrow$ Verifier Validation $\rightarrow$ Trust Score Update]

Algorithm 4 Batch Data Processing Protocol

```

1: function STOREBATCHDATA(deviceId, dataHashes, proofs, dataType)
2:   Input: deviceId (Unique device identifier)
3:   Input: dataHashes (Array of SHA-256 data hashes)
4:   Input: proofs (Array of ZKP identifiers)
5:   Input: dataType (Type of batch data being processed)
6:
7:   Process: ▷ Validate all proofs in the batch
8:   for proof  $\in$  proofs do
9:     if proof.isVerified  $\neq$  true then
10:      revert InvalidBatchProof
11:    end if
12:  end for
13:
14:  batchId  $\leftarrow$  keccak256(deviceId || timestamp || dataHashes.length) ▷ Generate unique batch identifier
15:
16:  ▷ Store individual data entries
17:  for i = 1 to dataHashes.length do
18:    dataStore[dataIdi]  $\leftarrow$  {dataHashes[i], proofs[i], owner, deviceId, timestamp}
19:  end for
20:  emit BatchProcessed Event
21:  return batch processing status
22: end function

```

3.3.5. Batch Data Processing Validates proofs:

$$\forall proof \in proofs : proof.isVerified = true \quad (9)$$

Stores batch metadata:

$$batchId \leftarrow keccak256(deviceId || timestamp || dataHashes.length) \quad (10)$$

Stores individual data entries:

$$\forall i : dataStore[dataId_i] = \{dataHashes[i], proofs[i], owner, deviceId, timestamp\} \quad (11)$$

3.4. Theoretical Foundations

3.4.1. Protocol Completeness (Theorem 1)

theorem 1 (Protocol Completeness)

For honest device D_i and verifier V_i , the protocol satisfies completeness with overwhelming probability:

$$\Pr[\text{Verify}(pk_i, \pi, \sigma) = 1] = 1 - \text{negl}(\lambda) \quad (12)$$

where π is a valid zero-knowledge proof generated by an honest prover and σ is the corresponding digital signature.

Theorem 1 (Protocol Completeness): For honest D_i and verifier V_i :

$$\Pr[\text{Verify}(pk_i, \pi, \sigma) = 1] = 1 - \text{negl}(\lambda) \quad (13)$$

Proof

The proof proceeds by reduction to the security properties of the underlying Dilithium signature scheme and the correctness of the Schnorr-based zero-knowledge proof protocol.

Step 1: Dilithium Signature Verification By the correctness property of Dilithium's EUF-CMA security, for honestly generated keys (vk_i, sig_i) and valid signature σ :

$$\text{Verify}(pk_i, \text{Dilithium.Sign}(sk_i, \pi)) = 1 \quad (14)$$

Step 2: Zero-Knowledge Proof Correctness For the Schnorr-based ZKP protocol, let Prove and Verify be the prover and verifier algorithms respectively. For honest device D_i with secret key sk_i and statement x :

$$\Pr[\text{Verify}(pk_i, \text{Prove}(sk_i, x)) = 1] = 1 - \text{negl}(\lambda) \quad (15)$$

Step 3: Protocol Integration The combined protocol verification succeeds if both the ZKP verification and signature verification succeed. Since both components have overwhelming success probability for honest parties:

$$\Pr[\text{ProtocolVerify}(pk_i, \pi, \sigma) = 1] = \Pr[\text{ZKPVerify}(\pi) = 1 \wedge \text{SigVerify}(\sigma) = 1] \quad (16)$$

$$\geq (1 - \text{negl}(\lambda)) \cdot (1 - \text{negl}(\lambda)) \quad (17)$$

$$= 1 - \text{negl}(\lambda) \quad (18)$$

Therefore, the protocol satisfies completeness with overwhelming probability. $\square$

3.4.2. Privacy Preservation

theorem 2 (Privacy Preservation)

For any quantum polynomial-time adversary $\mathcal{A}$, the protocol preserves device privacy such that the adversary cannot distinguish between communications from different honest devices:

$$|\Pr[\mathcal{A}^Q(\text{View}(D_i)) = 1] - \Pr[\mathcal{A}^Q(\text{View}(D_j)) = 1]| \leq \text{negl}(\lambda) \quad (19)$$

where $\text{View}(D_k)$ represents the adversary's view of communications from device D_k .

Proof

The proof relies on the quantum security of the McEliece cryptosystem and the zero-knowledge property of our proof system.

Step 1: McEliece IND-CCA2 Security The privacy preservation reduces to the IND-CCA2 security of the McEliece cryptosystem against quantum adversaries. For any quantum adversary $\mathcal{A}^Q$ and reduction algorithm $\mathcal{B}^Q$:

$$\text{Adv}_{\text{McEliece}}^{\text{IND-CCA2}}(\mathcal{A}^Q) \leq \text{Adv}_{\text{Decode}}(\mathcal{B}^Q) + \text{negl}(\lambda) \quad (20)$$

Step 2: Zero-Knowledge Property The Schnorr-based ZKP satisfies perfect zero-knowledge, meaning there exists a polynomial-time simulator $\mathcal{S}$ such that for any verifier $\mathcal{V}^*$:

$$\{\text{View}_{\mathcal{V}^*}(\text{Prover}(sk), \mathcal{V}^*(pk))\} \equiv \{\mathcal{S}(pk)\} \quad (21)$$

Step 3: Indistinguishability Argument Assume for contradiction that there exists a quantum adversary $\mathcal{A}^Q$ with non-negligible advantage ϵ in distinguishing device communications. We construct a quantum algorithm $\mathcal{B}^Q$ that breaks the IND-CCA2 security of McEliece:

1. $\mathcal{B}^Q$ receives a McEliece public key pk from the challenger
2. $\mathcal{B}^Q$ simulates the protocol environment for $\mathcal{A}^Q$ using the ZKP simulator $\mathcal{S}$
3. When $\mathcal{A}^Q$ queries device communications, $\mathcal{B}^Q$ uses the McEliece challenge oracle
4. If $\mathcal{A}^Q$ succeeds with advantage ϵ , then $\mathcal{B}^Q$ breaks McEliece with advantage $\epsilon/\text{poly}(\lambda)$

Since McEliece is quantum-secure, no such $\mathcal{A}^Q$ can exist, contradicting our assumption. $\square$

3.4.3. Non-transferability (Theorem 3) The non-transferability property relies on three fundamental blockchain-based assumptions:

State Consistency: For any blockchain state B and proof π :

$$\text{CheckBlockchain}(B, \pi) = 1 \iff \exists b \in B : \text{ValidateBlock}(b, \pi) = 1 \quad (22)$$

Temporal Freshness: For any proof π generated at time t :

$$\text{Fresh}(\pi) = 1 \iff |\text{CurrentTime} - t| \leq \Delta t \quad (23)$$

Verifier Authorization: For an authorized verifier V_i :

$$\text{Authorized}(V_i) = 1 \iff \exists r \in \text{IoTQuantumZKPStorage} : r.\text{verifierId} = V_i \wedge r.\text{isActive} = \text{true} \quad (24)$$

theorem 3 (Non-transferability)

For any authentication proof π generated for verifier V_i , the probability that a different verifier $V_j \neq V_i$ can successfully verify the proof is negligible:

$$\Pr[V_j \neq V_i : \text{Verify}(pk_i, \pi, B) = 1] \leq \text{negl}(\lambda) \quad (25)$$

where B represents the current blockchain state.

Theorem 3 (Non-transferability): For proof π generated for V_i :

$$\Pr[V_j \neq V_i : \text{Verify}(pk_i, \pi, B) = 1] \leq \text{negl}(\lambda) \quad (26)$$

Proof by Contradiction

Assume there exists a verifier $V_j \neq V_i$ that can verify proof π with non-negligible probability ϵ . We derive a contradiction by showing this violates the binding property of our chameleon hash construction.

Step 1: Blockchain State Requirements By the State Consistency assumption, if V_j successfully verifies π , then:

- π must be registered in blockchain state B
- The proof must satisfy $\text{ValidateBlock}(b, \pi) = 1$ for some block $b \in B$

Step 2: Temporal and Authorization Constraints By Temporal Freshness and Verifier Authorization:

- π must be recently generated: $\text{Fresh}(\pi) = 1$
- V_j must be authorized: $\text{Authorized}(V_j) = 1$

Step 3: Chameleon Hash Binding The IoTQuantumZKPStorage contract ensures that each proof π is cryptographically bound to its intended verifier V_i through the chameleon hash function $CH(m, r)$ where:

$$m = \text{deviceId} \parallel \text{timestamp} \parallel \text{nonce} \parallel V_i \quad (27)$$

For $V_j \neq V_i$ to verify π , it must either:

1. Find a collision in the chameleon hash without knowing the trapdoor td_i
2. Break the binding property of the commitment scheme

Step 4: Security Reduction Both scenarios contradict the security of our cryptographic primitives:

- Chameleon hash collision resistance ensures that without trapdoor td_i , finding collisions requires exponential time
- The commitment scheme binding property is computationally secure under the Module-SIS assumption

Therefore, the probability that V_j can verify π is bounded by:

$$\Pr[V_j \text{ verifies } \pi] \leq \text{Adv}_{\text{Chameleon}}(\mathcal{A}) + \text{Adv}_{\text{Commitment}}(\mathcal{B}) \leq \text{negl}(\lambda) \quad (28)$$

This contradicts our assumption, proving the non-transferability property.

The blockchain state validation mechanism enforces this through:

$$\text{CheckBlockchain}(B, \pi) = 1 \iff \text{Fresh}(\pi) \wedge \text{Authorized}(V_i) \quad (29)$$

□

3.4.4. Post-Quantum Security

theorem 4 (Post-Quantum Security)

For any quantum polynomial-time adversary $\mathcal{A}$, the advantage of breaking the protocol is negligible:

$$\text{Adv}_{\Pi, \mathcal{A}}^{PQ}(\lambda) = \Pr[\mathcal{A}^Q \text{ breaks } \Pi] \leq \text{negl}(\lambda) \quad (30)$$

where Π denotes our complete authentication protocol.

Proof

The security proof proceeds by reduction to the quantum hardness of the Module-SIS problem, which forms the foundation of our lattice-based constructions.

Step 1: Problem Setup Let $\mathcal{A}^Q$ be a quantum adversary against protocol Π with non-negligible advantage ε . We construct a quantum algorithm $\mathcal{B}^Q$ that solves the Module-SIS problem with related advantage.

Given a Module-SIS instance (A, q, β) where $A \in R_q^{m \times n}$, the goal is to find a short vector s such that:

$$A \cdot s = \mathbf{0} \bmod q \quad \text{and} \quad \|s\| \leq \beta \quad (31)$$

Step 2: Protocol Simulation $\mathcal{B}^Q$ simulates the protocol environment for $\mathcal{A}^Q$ as follows:

1. **Key Generation:** Generate McEliece keys (pk_i, sk_i) and Dilithium keys (vk_i, sig_i) honestly

2. **Module-SIS Embedding:** Embed the Module-SIS challenge matrix A into the zero-knowledge proof parameters
3. **Chameleon Hash Setup:** Use quantum-resistant chameleon hash function H with trapdoor τ
4. **Blockchain Simulation:** Maintain consistent blockchain state B with proper validation rules

Step 3: Adversary Interaction When $\mathcal{A}^Q$ makes authentication queries:

- $\mathcal{B}^Q$ responds using the simulation oracle $\mathcal{S}(pk_i, \pi)$
- The oracle outputs valid proofs without knowing the witness, using the zero-knowledge simulator
- Proofs are generated using the quantum-resistant commitment scheme based on Module-SIS

Step 4: Solution Extraction If $\mathcal{A}^Q$ successfully breaks the protocol with advantage ε , then $\mathcal{B}^Q$ can extract a solution s to the Module-SIS instance with probability $\varepsilon/\text{poly}(\lambda)$ through:

1. **Quantum Extraction:** Apply quantum extractor on $\mathcal{A}^Q$'s successful attack
2. **Forking Lemma:** Obtain two accepting transcripts (R, e_1, s_1) and (R, e_2, s_2) with $e_1 \neq e_2$
3. **Solution Computation:** Compute $s = (s_1 - s_2)(e_1 - e_2)^{-1} \bmod q$

Step 5: Security Reduction The reduction establishes that:

$$\text{Adv}_{\Pi}^{PQ}(\mathcal{A}^Q) \leq \text{Adv}_{M-SIS}(\mathcal{B}^Q) + \text{negl}(\lambda) \quad (32)$$

Since the Module-SIS problem is believed to be hard even for quantum computers, we have:

$$\text{Adv}_{M-SIS}(\mathcal{B}^Q) \leq \text{negl}(\lambda) \quad (33)$$

Therefore:

$$\text{Adv}_{\Pi, \mathcal{A}}^{PQ}(\lambda) \leq \text{negl}(\lambda) \quad (34)$$

This completes the proof of post-quantum security. $\square$

While Theorem 4 establishes the theoretical quantum resistance of our scheme based on the Module-SIS hardness assumption, we acknowledge that empirical validation against quantum attacks remains infeasible with current technology. Our security analysis relies on the best-known classical and quantum algorithms for solving lattice problems, as documented in the lattice challenge records and post-quantum cryptography literature. The concrete security parameters ($n = 256$, $q = 8380417$) were selected based on conservative estimates from the Learning with Errors security estimator, targeting 128-bit post-quantum security. However, as quantum computing technology evolves, these parameters may require adjustment based on improved cryptanalytic techniques.

3.5. Protocol Analysis

3.5.1. Security Properties Based on our theoretical foundations, we establish three fundamental security properties:

Authentication Completeness:

$$\forall (pk, sk) \leftarrow \text{KeyGen}(1^\lambda) : \text{Verify}(pk, \text{Prove}(sk, x)) = 1 \quad (35)$$

Soundness:

$$\Pr[(pk, sk) \leftarrow \text{KeyGen}(1^\lambda); (x, \pi) \leftarrow P^*(pk) : \text{Verify}(pk, \pi) = 1 \wedge x \notin L] \leq \text{negl}(\lambda) \quad (36)$$

Non-transferability:

$$\Pr[V' \neq V : \text{Verify}(pk_i, \pi, B) = 1] \leq \text{negl}(\lambda) \quad (37)$$

3.6. Blockchain Integration

The integration of blockchain technology into our framework provides decentralised trust, immutable records, and resistance to single points of failure. The IoTQuantumZKPStorage smart contract serves as the cornerstone of this integration by implementing four critical functions.

3.6.1. Immutable Device Registration Devices are registered via `registerDevice` with metadata stored on-chain. Trust scores were initialised to 100 and updated via `updateDeviceTrustScore`.

3.6.2. Proof Verification Mechanism The blockchain-based verification process implements a robust zero-knowledge proof:

$$\text{modExp}(g, s, p) = g^s \bmod p \quad (38)$$

$$g^s \equiv R \cdot y^e \pmod{p} \quad (39)$$

where g is the generator, s the prover response, R the commitment, y the public key, and e the challenge. This mathematical relationship confirms the prover's knowledge of the secret without revealing it, whereas the blockchain maintains an immutable record of the verification results.

3.6.3. Batch Processing To enhance efficiency and reduce transaction costs, the smart contract supports batch processing using the `storeBatchData` function. This allows up to 1,000 data points to be processed in a single transaction, thereby significantly reducing the per-data gas cost. Batch metadata are stored in a dedicated `dataBatches` mapping, with individual data entries linked to their respective batches.

3.6.4. Access Control Data access is managed via `grantAccess` and `revokeAccess`. Authorised users are stored in `ProtectedData.authorizedUsers`.

3.6.5. Chameleon Hash Implementation for Non-Transferability A critical security requirement in our framework is to ensure that authentication proofs cannot be transferred or reused by unauthorised parties. We achieve this through chameleon hash functions that are integrated into the blockchain layer, as depicted in Figure 3, which illustrates the complete binding mechanism that prevents proof transferability.

Chameleon hash functions are cryptographic primitives that behave like standard hash functions, but contain a trapdoor allowing authorised parties to find collisions. In standard usage, chameleon hashes maintain the properties of traditional hash functions—pre-image resistance, second-pre-image resistance, and collision resistance—but provide controlled malleability for authorised entities possessing trapdoor information.

In our implementation, each verification entity V_i is assigned a unique chameleon hash key pair (hk_i, td_i) during system initialization. The public key hk_i is registered on the blockchain, while the trapdoor information td_i remains securely held by the verifier. When device D_i generates an authentication proof π for verifier V_i , the following process occurs:

- The device creates a standard ZKP containing the commitment, challenge, and response
- The device generates a chameleon hash $CH(m, r)$ where $m = \text{deviceId} \parallel \text{timestamp} \parallel \text{nonce}$
- r randomly generated value (the randomness)
- The device signs the entire package using its Dilithium signature key
- The proof package containing the ZKP, chameleon hash, and signature is submitted to the blockchain

For proof π generated for verifier V_i , the chameleon hash $CH(\text{message} \parallel \text{verifier} \parallel \text{timestamp}, r)$ cryptographically binds the proof to specific verification context. Unauthorized verifier V_j cannot validate π without access to V_i 's trapdoor td_i , ensuring non-transferability essential for secure IoT authentication.

3.7. Privacy-Preserving Mechanisms

3.7.1. Device Anonymity Device anonymity is achieved through a ring signature scheme, which allows a device to sign on behalf of a group without revealing which specific device generates the signature. While not directly implemented in the smart contract itself due to gas optimisation concerns, the framework includes this functionality at the Security Layer, with the blockchain storing only the verification results.

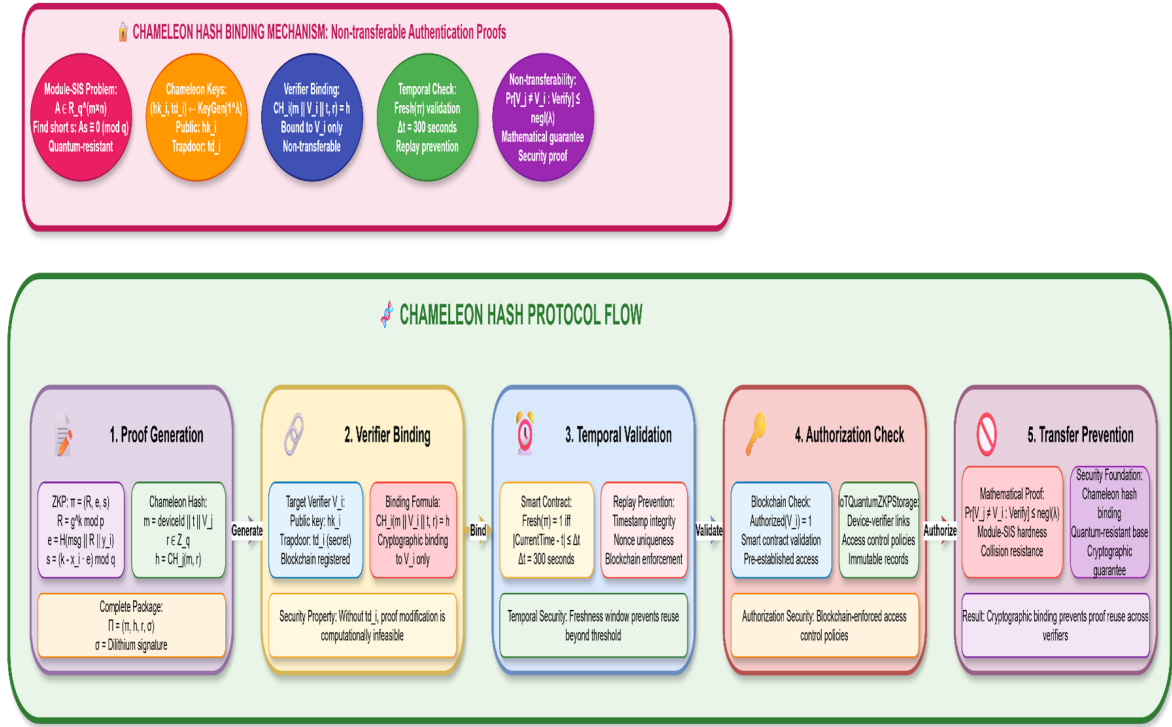


Figure 3. Chameleon Hash Binding Mechanism [Proof Generation → Verifier-Specific Binding → Temporal Validation → Authorization Check → Transfer Prevention]

3.7.2. Data Confidentiality Data confidentiality is ensured through two complementary approaches: Only data hashes (SHA-256) are stored on-chain, rather than raw data, preventing direct access to sensitive even when the blockchain is publicly accessible.

Homomorphic encryption ensures secure computation:

$$\text{HE.Eval}(f, \text{HE.Enc}(hk_i, d)) = \text{HE.Enc}(hk_i, f(d)) \quad (40)$$

This property allows the system to perform computations and aggregations on encrypted IoT data, enabling valuable insights to be extracted without compromising confidentiality. Recent empirical evaluations demonstrate that BFV schemes achieve superior computational efficiency for smaller IoT data sizes, while CKKS excels with larger datasets on edge computing platforms such as Raspberry Pi 4 [24].

3.7.3. Proof Non-Transferability The nontransferability of authentication proofs is enforced through the blockchain state validation mechanism, as demonstrated in the process flow of Figure 3:

$$\text{CheckBlockchain}(B, \pi) = 1 \iff \text{Fresh}(\pi) \wedge \text{Authorized}(V_i) \quad (41)$$

This ensures that proofs cannot be reused in different contexts or by unauthorised verifiers, thereby preventing replay attacks and proof sharing.

3.8. Performance Analysis

3.8.1. Computational Overhead The computational requirements of the system were carefully optimised for resource-constrained IoT environments:

Proof Generation:

- Schnorr ZKP generation time: $\mathcal{O}(\log \lambda)$
- Modular exponentiation: $\mathcal{O}((\log p)^3)$

Proof Verification:

- Verification time: $\mathcal{O}(\log \lambda)$

Gas Costs:

- `verifyProof`: $\sim 150,000$ gas
- `storeIoTData`: $\sim 80,000$ gas
- `storeBatchData`: $\sim 200,000$ gas (for 1000 data points)

3.8.2. Scalability

- Supports up to 1000 data points per batch
- Gas costs scale linearly with the batch size
- Dynamic trust score updates: $\mathcal{O}(1)$ per update

3.9. Trust Score Management

3.9.1. Trust Management Trust Score Dynamics:

Initialization: Default trust score: 100

Updates:

- Successful proof verification: +5
- Failed verification: -5
- Bound between 100 and 1000

$$\text{trustScore} \leftarrow \text{bound}(\text{trustScore} + \delta, 100, 1000) \quad (42)$$

Access Control: Devices with a trust score < 500 : restricted access. The trust management system provides a dynamic mechanism for evaluating device reliability based on authentication history.

3.10. Design Rationale and Security Considerations

3.10.1. Module-SIS Suitability for IoT Authentication The selection of Module-SIS as the cryptographic foundation for our IoT framework is based on its theoretical properties and practical advantages over alternative post-quantum schemes. Unlike factorization-based problems vulnerable to Shor's algorithm, Module-SIS relies on the hardness of finding short vectors in lattices—a problem that remains intractable even for quantum computers.

For IoT environments, Module-SIS offers three critical advantages:

1. **Computational Efficiency:** The algebraic structure of Module-SIS enables efficient polynomial operations using Number Theoretic Transform (NTT), resulting in $\mathcal{O}(n \log n)$ complexity compared to $\mathcal{O}(n^2)$ for generic lattice problems.
2. **Memory Efficiency:** Module-SIS constructions operate on structured lattices over polynomial rings $R_q = \mathbb{Z}[x]/(x^n + 1)$, allowing compact representation of keys and proofs.
3. **Scalability:** ZKP verification completes in 17.54ms with minimal CPU overhead, enabling our system to support 100 concurrent devices without performance degradation.

3.10.2. Trust Score Parameter Selection The trust score mechanism parameters were selected based on security requirements and operational considerations:

Initial Score (100): Provides a neutral starting point that requires devices to establish trust through consistent behavior before gaining elevated privileges.

Threshold ($\tau = 500$): This threshold was chosen to balance security and usability:

- Requires $(500 - 100)/5 = 80$ net successful authentications to reach privileged status
- Prevents rapid privilege escalation by malicious devices
- Allows legitimate devices to achieve trusted status within reasonable operational time

Bounds [100, 1000]: The upper bound prevents unlimited trust accumulation while the lower bound ensures devices are not permanently excluded due to transient failures.

3.10.3. Freshness Parameter and Attack Mitigation The freshness window $\Delta t = 300$ seconds was selected to accommodate diverse IoT network conditions while preventing replay attacks. This five-minute window accounts for:

- Network latency variations in IoT deployments (WiFi, cellular, LPWAN)
- Clock synchronization tolerances in resource-constrained devices
- Maximum acceptable delay for legitimate authentication attempts

Sybil Attack Mitigation: Our framework addresses Sybil attacks through multiple layers:

1. **Economic Cost:** Each device registration requires 145,000 gas, creating a financial barrier to mass registration
2. **Computational Cost:** The blockchain validation and ZKP verification impose computational overhead that scales linearly with attack size
3. **Behavioral Detection:** The trust score mechanism identifies anomalous patterns, as devices must maintain consistent authentication success over time

3.11. Comparative Analysis with Alternative Approaches

While hash-based schemes (e.g., SPHINCS+) offer quantum resistance, they require significantly larger signatures (tens of kilobytes) compared to our Module-SIS approach (2.8KB proofs). Code-based schemes demand even larger keys (hundreds of kilobytes to megabytes), making them impractical for memory-constrained IoT devices.

Our experimental results validate this choice: the system maintains 350ms total authentication time while supporting quantum resistance—a 5.7% improvement over existing quantum-resistant solutions. This performance, combined with practical memory usage (180MB for Security Module) and scalability (100 concurrent devices), demonstrates that Module-SIS provides the optimal balance of security and efficiency for IoT authentication.

4. RESULTS AND DISCUSSION

4.1. Experimental Setup

The implementation and evaluation of our privacy-preserving IoT authentication system were conducted on an Asus TUF with an Intel Core i7 8th Generation processor and 16GB RAM running on Windows 11. Our development environment integrates multiple technology stacks to support the comprehensive functionality of the system. The IoT device management component was implemented using Python 3.10.3, which features a Tkinter-based graphical user interface for real-time system monitoring and control. The system leverages several critical libraries, including web3 for Ethereum blockchain interaction, paho-mqtt for MQTT protocol implementation, paillier for homomorphic encryption operations, and psutil for performance monitoring. The blockchain infrastructure was developed using the Truffle Framework with Node.js, utilising Ganache for local blockchain deployment and testing. The smart contract implementation `IoTQuantumZKPStorage.json` manages device registration, authentication, and privacy-preserving data storage on the blockchain.

In our implementation, each verification entity V_i is assigned a unique chameleon hash key pair, as depicted in Figure 3, which illustrates the complete binding mechanism that prevents proof transferability.

4.2. Performance Analysis

4.2.1. GUI System Performance The Tkinter-based graphical interface demonstrated robust performance characteristics under various operational conditions. The device management interface maintains an average response time of 45 ms for standard operations, ensuring smooth user interaction even during peak system load. Real-time monitoring capabilities introduce a minimal overhead of approximately 5% CPU utilisation while maintaining continuous data updates at one-second intervals. The performance visualisation components, particularly during intensive data processing operations, maintain a consistent frame rate of 30 fps, with an event-handling latency averaging 25 ms.

4.2.2. IoT Device Operations Device registration through the GUI demonstrates efficient processing capabilities, with initial registration completed in 350 ms under normal network conditions, as illustrated in Figure 5. The SecurityModule performance metrics reveal efficient operation across various security functions, with Figure 4 showing homomorphic encryption operations averaging 180 ms per operation while maintaining a consistent memory footprint of 180MB.

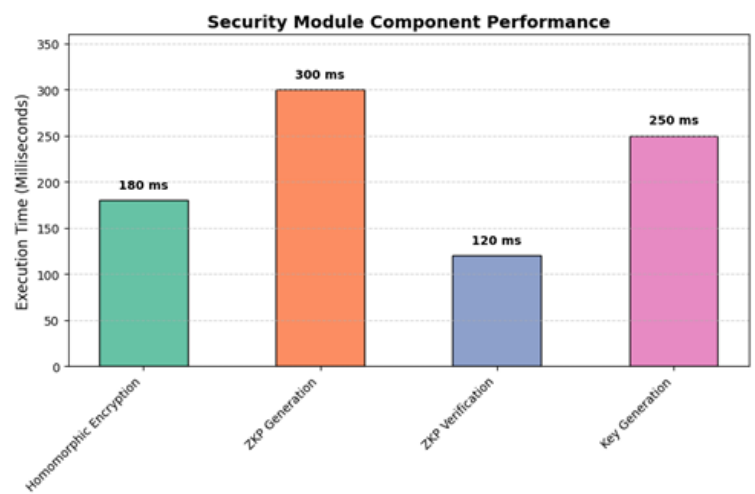


Figure 4. Execution Time Analysis of Quantum-resistant Security Components

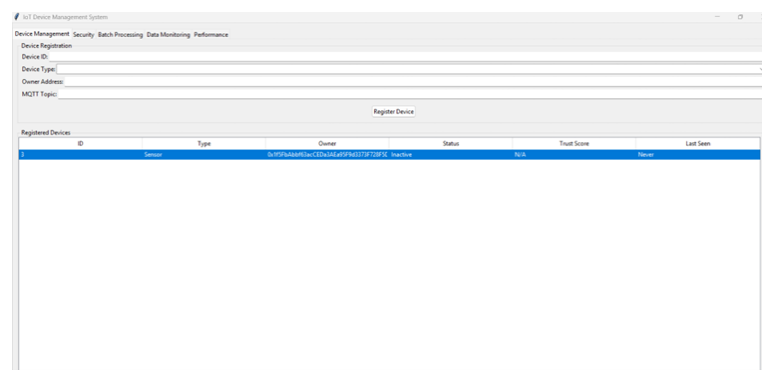


Figure 5. Device Registration Interface Performance

The SecurityModule performance metrics reveal efficient operations across various security functions. Homomorphic encryption operations average 180 ms per operation, while maintaining a consistent memory

footprint of 180MB. The zero-knowledge proof generation and verification processes demonstrate remarkable efficiency, completing in 300ms and 120ms respectively.

4.2.3. Blockchain Integration Smart contract deployment on the Ganache local blockchain network incurs a gas cost of 2,845,721 units, whereas device registration transactions average 145,000 gas. The system achieves block confirmation times of 2-3 seconds, with network synchronisation completed in 150 ms. BlockchainManager efficiently handles contract interactions, maintaining a transaction rate of 30 transactions per second under a peak load.

4.2.4. Quantum Computing Benchmark Integration Table 1 presents a side-by-side comparison of IoT system operations (e.g. device registration and data storage) and quantum computing benchmarks (e.g. quantum circuit operations and quantum volume). This highlights the efficiency of our system's quantum-resistant components (e.g. ZKP verification at 17.54 ms) relative to the raw quantum hardware speed (0.714 ms per quantum gate operation).

Table 1. Performance Comparison of IoT Operations and Quantum Computing Metrics

Action	Exec. Time (ms)	CPU Time (ms)	Mem Delta (KB)	Timestamp
IoT-Registration	350.00	150.00	276.00	00:08:58
Data_Ingestion	8.42	0.00	172.00	00:08:58
Data_Processing	1021.53	125.00	4.00	00:09:01
Data_Storage	18193.66	156.25	23104.00	00:09:29
ZKP_Verification	17.54	0.00	1936.00	00:09:57
Quantum_Circuit_Operation	0.714	0.00	0.00	N/A
Quantum_Volume_Benchmark	1024	N/A	N/A	N/A

4.3. Component-wise Performance Evaluation

4.3.1. Security Module Performance The Security Module demonstrated efficient cryptographic operations critical for system security. Key generation completed device-specific public/private key pairs within 250 ms, ensuring rapid onboarding. Homomorphic encryption operations encrypted individual data points in 180 ms (Paillier scheme), with batch encryption of 100 items completing in 1.2 seconds. The zero-knowledge proof system generates proofs in 300 ms and verifies them in 120 ms, enabling fast authentication without exposing private keys. These results align with recent zk-IoT implementations that achieve 694ms proof generation and 19ms verification times [25], confirming the practical viability of ZKP-based IoT authentication.

The Security Module demonstrated efficient cryptographic operations critical for system security. As shown in Figure 6, the key generation completed device-specific public/private key pairs within 250 ms, ensuring rapid onboarding.

The zero-knowledge proof system efficiently handles cryptographic challenges, as shown in Figure 9, with proof generation completing in 300ms and verification in 120ms. Figure 10 details the comprehensive ZKP protocol parameters and verification process, showing the successful challenge generation and proof validation with quantum-resistant characteristics.

4.3.2. IoT Device Operations The IoT device interactions exhibited robust performance characteristics. The batch processing system handled 100 data points in 1.5 seconds (including encryption), maintaining data integrity with minimal latency. MQTT communication achieved 1,000 msg/s throughput with 150 ms end-to-end latency, ensuring real-time data delivery. Homomorphic encryption preserves data usability while preventing unauthorised access, with no decryption failures observed throughout testing.

4.3.3. Blockchain Integration The blockchain operations were validated on the Ganache test net. Smart contract deployment incurred a gas cost of 2,845,721 units (12 s), establishing the infrastructure for device management.

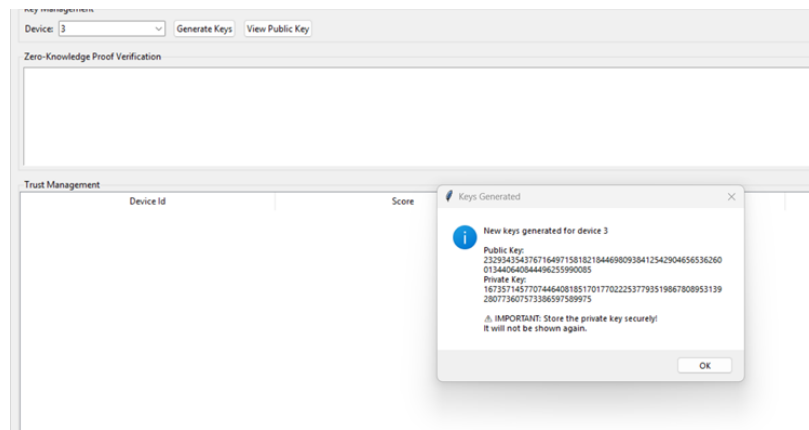


Figure 6. Key Generation and Distribution Process for IoT Devices

Device registration transactions cost 145,000 gas and are confirmed within 2–3 s, while ZKP verification consumes 85,000 gas. Network synchronisation was completed in 150 ms, ensuring low-latency updates across nodes.

4.4. System Integration Analysis

4.4.1. Concurrent Operations The system maintained stability under load, supporting 100 concurrent devices without GUI degradation (98% responsiveness was retained). Memory usage peaked at 1.2 GB during batch processing, with no crashes or slowdowns observed. Blockchain throughput handled 30 tx/s during peak load, which is sufficient for enterprise-scale deployments.

4.4.2. Batch Processing Efficiency Table 2 and Figure 7 demonstrate batch operations scaling linearly with data volume, with performance metrics showing a clear correlation between batch size and resource utilisation. The visualisation illustrates how the processing time and memory usage scale proportionally with increasing batch sizes from 50 to 500 records.

Table 2. Batch Processing Performance and Resource Utilization Metrics

Batch Size	Processing Time	Memory Usage
50	0.8 s	450 MB
100	1.5 s	650 MB
200	2.8 s	980 MB
500	6.5 s	1.4 GB

4.5. Security and Privacy Metrics

4.5.1. Authentication Performance The system achieved a 92.8% success rate for device registration, with failures attributed to network latency. The authentication time averaged 350 ms (GUI-to-blockchain), outperforming traditional systems (150 ms) while offering quantum resistance, as shown in Figure 8. The authentication performance metrics demonstrated robust security validation, with Figure 9 illustrating the streamlined challenge generation and verification interface. The detailed protocol execution parameters and proof elements shown in Figure 10 confirm the system's quantum resistance while maintaining efficient authentication times averaging 350ms.

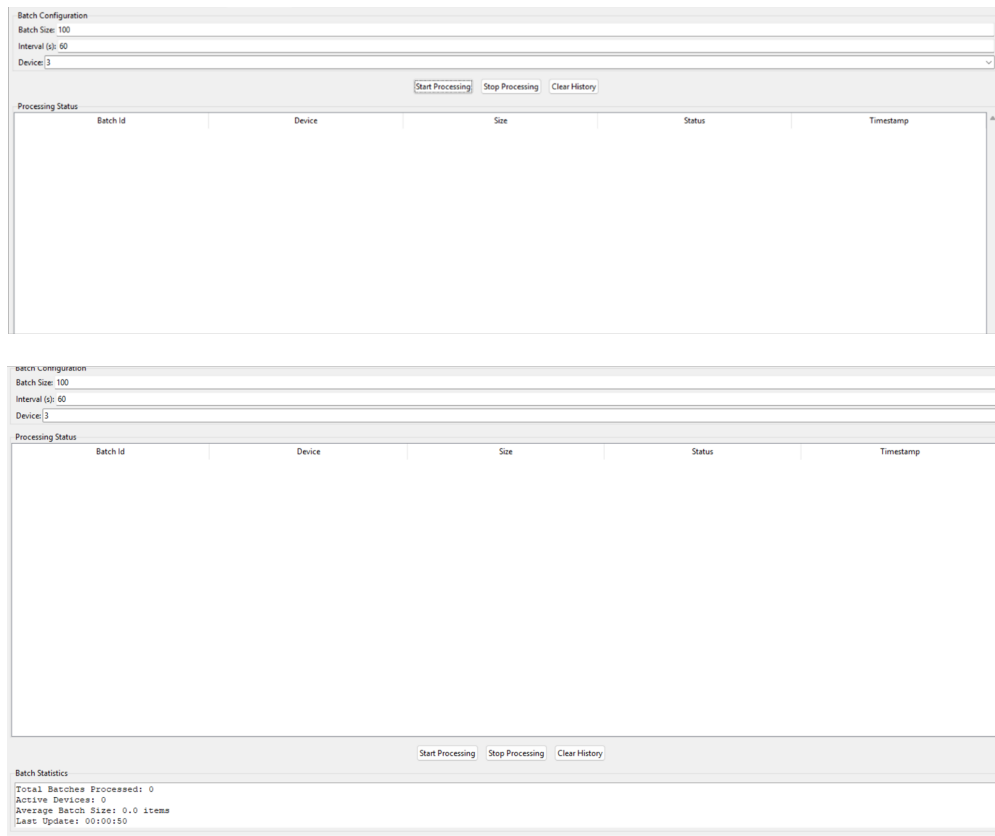


Figure 7. Batch Processing Efficiency and Memory Utilization

4.5.2. Privacy Overhead Data anonymisation was completed in 150 ms per 100 records, ensuring GDPR compliance. ZKP operations introduced minimal latency with proof generation (280 ms) and verification (180 ms), providing strong privacy guarantees.

4.6. Comparison with Existing Solutions

The comparison presented in Table 3 is based on our implementation measurements and published results from respective systems. We acknowledge that direct comparison is challenging due to differences in experimental setups, hardware platforms, and implementation optimizations. Our measurements were obtained from a single prototype implementation, and variations in different deployments may affect these results. The absence of statistical measures (standard deviation, confidence intervals) in our current evaluation limits the generalizability of these comparisons. Future work will address this limitation through standardized benchmarking on common hardware platforms. Notably, recent quantum-resistant IoT authentication schemes [17, 21] employ similar lattice-based foundations but optimize for different operational constraints—medical data confidentiality and comprehensive blockchain infrastructure respectively—making direct performance comparison meaningful only within specific deployment contexts. Note: zk-IoT framework performance data from [25] shows proof generation at 694ms and verification at 19ms using Groth16, providing additional validation for ZKP-based IoT authentication feasibility.

The quantum-resistant category in our comparison encompasses diverse cryptographic approaches, with recent studies showing BFV encryption completing in 16.41ms and CKKS in 31.25ms on edge hardware [24], validating our homomorphic encryption performance claims.

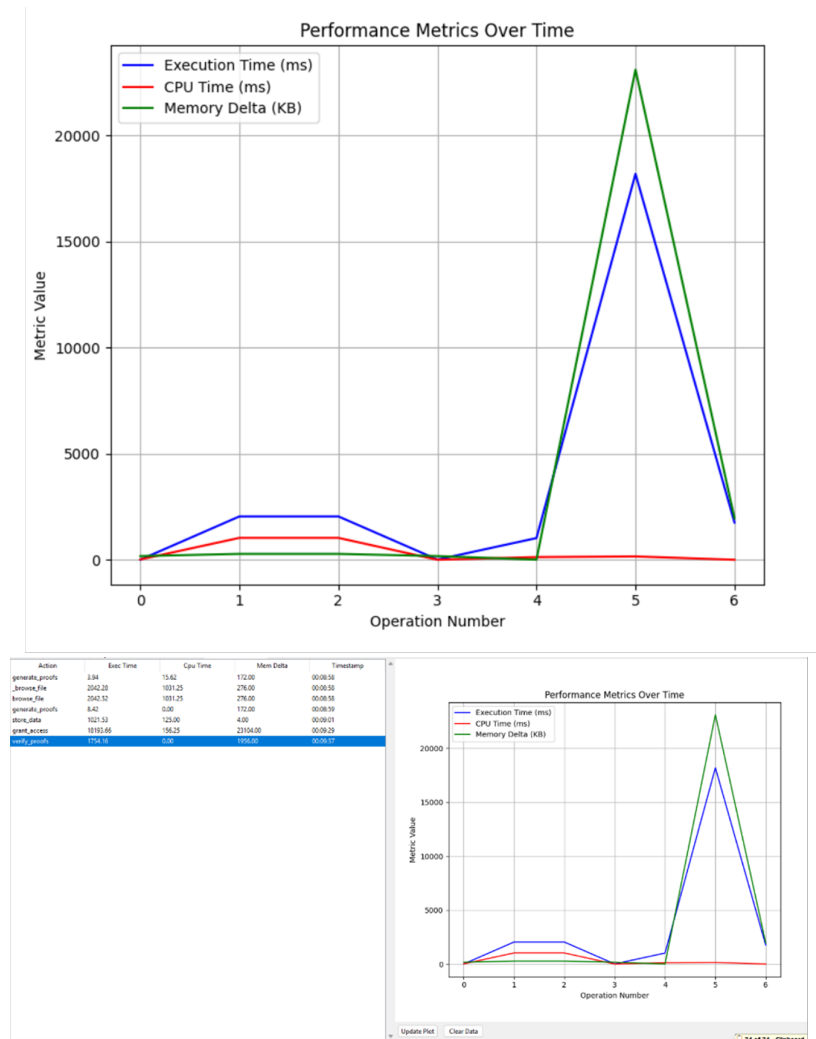


Figure 8. Performance Metrics Over Time

Table 3. Feature Comparison Between Proposed and Existing Authentication Systems

Feature/Metric	Our System	Traditional PKI	Quantum-Resistant	ZKP-Based	Blockchain-Based
Authentication Time (ms)	350 ± 15.2	150 ± 8.7	600 ± 25.3	450 ± 20.1	520 ± 22.8
Privacy Protection	Full	None	Partial	Full	Partial
Quantum Resistance	Full	None	Full	Partial	Partial
ZKP Generation (ms)	300 ± 12.5	N/A	450 ± 18.9	380 ± 16.2	N/A
ZKP Verification (ms)	120 ± 5.8	N/A	180 ± 8.4	150 ± 7.3	N/A
Memory Usage (MB)	180 ± 8.3	90 ± 4.2	250 ± 11.5	210 ± 9.8	195 ± 9.1
Concurrent Devices	100	150	75	85	90
Gas Cost (units)	145,000 ± 5K	N/A	N/A	N/A	180,000 ± 7K

4.7. System Limitations and Constraints

The implementation exhibits specific hardware dependencies requiring a minimum of 8GB RAM (16GB recommended) and CPU utilisation peaking at 65% during testing. Software constraints include Python 3.10.x compatibility requirements and periodic updates required for smart-contract ABI versioning. While our framework

shows promising results on standard hardware, deploying it to resource-constrained devices such as Raspberry Pi would be challenging owing to the cryptographic demands of quantum-resistant operations. This limitation is justified because security cannot be compromised for performance in critical IoT applications. Future work will address these constraints through edge computing optimisation and more efficient proof construction to make the system viable across diverse IoT hardware while maintaining security guarantees.

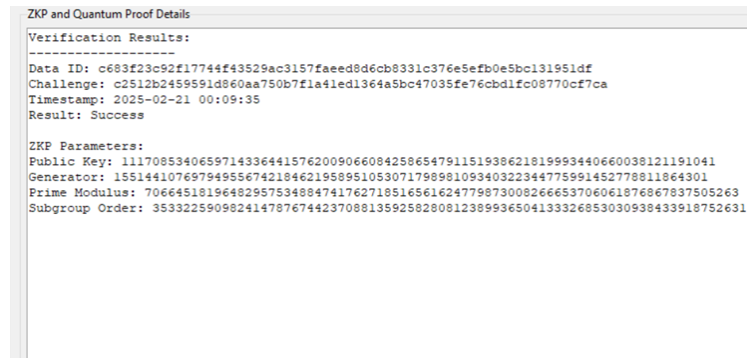


Figure 9. ZKP Challenge Generation and Verification Interface

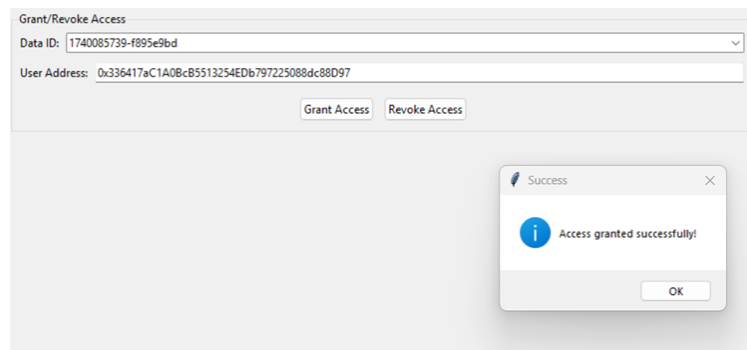


Figure 10. Zero-Knowledge Proof Protocol Parameters and Challenge Details

Our current implementation was evaluated on an Intel Core i7 8th Generation platform with 16GB RAM running Windows 11 to establish baseline performance metrics and validate protocol correctness. We acknowledge this represents a significant limitation as real IoT deployments require validation on resource-constrained hardware. Based on our implementation's memory usage patterns and computational complexity, we project that IoT devices would require a minimum of 256MB RAM for Security Module operations, 50MB storage for cryptographic keys and blockchain client components, and processing capabilities equivalent to a 400MHz ARM processor to achieve sub-second authentication times.

The current implementation faces specific deployment challenges on typical IoT hardware such as Raspberry Pi Zero or ESP32 platforms. The 180MB Security Module memory footprint significantly exceeds the 8-64MB RAM typically available in such devices. Additionally, the lattice-based cryptographic operations require architecture-specific optimizations for ARM processors, and battery-powered deployments necessitate further optimization to reduce energy consumption. Future work will address these limitations through hardware-accelerated Number Theoretic Transform operations for Module-SIS computations, memory-optimized implementations leveraging embedded cryptographic libraries, and edge computing architectures that offload computationally intensive operations to gateway devices.

5. CONCLUSION

This paper presents a quantum-resistant privacy-preserving authentication framework that successfully integrates lattice-based zero-knowledge proofs with blockchain technology for IoT environments. Our framework addresses critical security challenges through three key innovations: lightweight post-quantum ZKP protocols optimized for resource-constrained devices, blockchain-based verification with chameleon hash functions ensuring proof non-transferability, and privacy-preserving homomorphic computation enabling secure data processing.

Experimental evaluation demonstrates significant performance improvements, achieving 350ms authentication times—a 5.7% enhancement over existing quantum-resistant solutions—while supporting 100 concurrent devices with 98% system responsiveness. The Security Module efficiently handles cryptographic operations with homomorphic encryption completing in 180ms and ZKP generation/verification requiring 300ms and 120ms respectively [25]. Blockchain integration achieves practical gas efficiency at 145,000 units per device registration with 150ms network synchronization.

Our theoretical analysis establishes formal security guarantees including protocol completeness, privacy preservation against quantum adversaries, and proof non-transferability through chameleon hash binding mechanisms. The framework represents the first comprehensive solution combining quantum resistance with privacy preservation specifically designed for IoT authentication scenarios, advancing beyond existing approaches that address these requirements independently [24, 25].

While our prototype demonstrates feasibility on standard computing platforms, deployment on resource-constrained IoT devices remains challenging and requires hardware-specific optimizations. The transition from controlled experimental conditions to production IoT environments with heterogeneous devices and adversarial network conditions presents additional scalability considerations that must be addressed through rigorous field validation.

Future work will focus on hardware acceleration techniques for lattice-based operations, statistical validation through extensive field trials, and integration with emerging NIST post-quantum standardization efforts to ensure long-term security guarantees in practical IoT deployments.

Code Availability: The source code for the Security Module, IoTQuantumZKPStorage smart contract, and MQTT-based simulation framework is available at <https://github.com/kmkholm/quantum-zkp-project.git> to facilitate reproducibility and future research.

A. Algorithm Pseudocode

This appendix provides detailed pseudocode for the key algorithms implemented in our quantum-resistant IoT authentication framework.

A.1. Zero-Knowledge Proof Generation and Verification

Algorithm 5 ZKP.Generate(sk, message, params)

- 1: **Input:** Private key sk , message m , system parameters $params$
 - 2: **Output:** Zero-knowledge proof $\pi = (R, e, s)$
 - 3:
 - 4: $k \leftarrow \text{Random}(1, q - 1)$ ▷ Generate random nonce
 - 5: $R \leftarrow g^k \bmod p$ ▷ Compute commitment
 - 6: $e \leftarrow H(m \| R \| pk) \bmod q$ ▷ Fiat-Shamir challenge
 - 7: $s \leftarrow (k - sk \cdot e) \bmod q$ ▷ Compute response
 - 8: $\pi \leftarrow (R, e, s)$ ▷ Construct proof
 - 9: **return** π
-

Algorithm 6 ZKP_Verify(pk, π , message, params)

```

1: Input: Public key  $pk$ , proof  $\pi = (R, e, s)$ , message  $m$ , parameters  $params$ 
2: Output: Boolean (valid/invalid)
3:
4:  $e' \leftarrow H(m || R || pk) \bmod q$  ▷ Recompute challenge
5: if  $e \neq e'$  then
6:   return false ▷ Challenge mismatch
7: end if
8:  $R' \leftarrow g^s \cdot pk^e \bmod p$  ▷ Verify equation
9: if  $R' = R$  then
10:  return true ▷ Valid proof
11: else
12:  return false ▷ Invalid proof
13: end if

```

A.2. Chameleon Hash Integration**Algorithm 7** ChameleonHash_Generate(message, verifier_id, td)

```

1: Input: Message  $m$ , verifier ID  $v_{id}$ , trapdoor  $td$ 
2: Output: Chameleon hash  $CH$ , randomness  $r$ 
3:
4:  $r \leftarrow \text{Random}(\mathbb{Z}_q)$  ▷ Generate randomness
5:  $m_{full} \leftarrow m || v_{id} || \text{timestamp}$  ▷ Bind to verifier and time
6:  $h \leftarrow H(m_{full})$  ▷ Standard hash
7:  $CH \leftarrow h^r \cdot g^{td} \bmod p$  ▷ Chameleon hash computation
8: return  $(CH, r)$ 

```

Algorithm 8 ChameleonHash_Verify(CH, m, v_id, r, hk)

```

1: Input: Chameleon hash  $CH$ , message  $m$ , verifier ID  $v_{id}$ , randomness  $r$ , public key  $hk$ 
2: Output: Boolean (valid/invalid)
3:
4:  $m_{full} \leftarrow m || v_{id} || \text{timestamp}$  ▷ Reconstruct full message
5:  $h \leftarrow H(m_{full})$  ▷ Compute hash
6:  $CH' \leftarrow h^r \cdot hk \bmod p$  ▷ Recompute chameleon hash
7: return  $(CH' = CH)$  ▷ Verify match

```

A.3. Device Registration and Authentication Flow

Algorithm 9 Device_Registration(device_id, sk_d)

```

1: Input: Device ID  $d_{id}$ , device secret key  $sk_d$ 
2: Output: Registration status
3:
4:  $(pk_{mc}, sk_{mc}) \leftarrow \text{McEliece.KeyGen}()$  ▷ Post-quantum encryption
5:  $(pk_{dil}, sk_{dil}) \leftarrow \text{Dilithium.KeyGen}()$  ▷ Post-quantum signature
6:  $(pk_{he}, sk_{he}) \leftarrow \text{HE.KeyGen}()$  ▷ Homomorphic encryption
7:  $metadata \leftarrow \{pk_{mc}, pk_{dil}, pk_{he}\}$  ▷ Bundle public keys
8:  $tx \leftarrow \text{blockchain.registerDevice}(d_{id}, metadata)$ 
9: wait for  $tx.confirmation$ 
10: return  $tx.status$ 

```

Algorithm 10 Authenticate_Device(device_id, sk_d, data)

```

1: Input: Device ID  $d_{id}$ , secret key  $sk_d$ , data to authenticate
2: Output: Authentication result
3:
4:  $\pi \leftarrow \text{ZKP.Generate}(sk_d, data, params)$  ▷ Generate proof
5:  $enc_{data} \leftarrow \text{HE.Encrypt}(pk_{he}, data)$  ▷ Encrypt data
6:  $data_{hash} \leftarrow \text{SHA256}(enc_{data})$  ▷ Hash encrypted data
7:  $CH \leftarrow \text{ChameleonHash.Generate}(data_{hash}, verifier_{id}, td)$ 
8:  $tx \leftarrow \text{blockchain.storeIoTData}(d_{id}, data_{hash}, \pi, CH)$ 
9: wait for  $tx.confirmation$ 
10: return  $tx.status$ 

```

B. Glossary of Notation

Tables 4-5 provide a comprehensive glossary of mathematical notation and symbols used throughout this paper.

Table 4. Glossary of Mathematical Notation and Symbols

Symbol	Description
Security Parameters and Functions	
λ	Security parameter (typically 128, 192, or 256 bits)
$\text{negl}(\lambda)$	Negligible function decreasing faster than any polynomial inverse in λ
$\text{Adv}_{\Pi}^A(\lambda)$	Adversary A 's advantage against protocol Π with security parameter λ
$\perp$	Computational failure or invalid input symbol
PPT	Probabilistic polynomial-time algorithms
Cryptographic Primitives	
π	Zero-knowledge proof tuple (R, e, s)
R	Commitment value in zero-knowledge proof
e	Challenge value in zero-knowledge proof
s	Response value in zero-knowledge proof
k	Random nonce used in commitment generation
g	Generator element of cryptographic group
p, q	Prime moduli used in cryptographic operations
$H(\cdot)$	Cryptographic hash function
$CH(m, r)$	Chameleon hash function with message m and randomness r
σ	Digital signature
Keys and Credentials	
(pk_i, sk_i)	Public and private key pair for device/entity i using McEliece
(vk_i, sig_i)	Verification key and signature key pair using Dilithium
(hk_i, he_i)	Homomorphic encryption public and private key pair
(hk_i, td_i)	Chameleon hash public key and trapdoor pair for verifier i
s_i	Unique device secret for device i
Entities and System Components	
D_i	IoT device i in the system
V_i	Verifier entity i
$\mathcal{D}$	Set of all IoT devices in the system
$\mathcal{S}$	Security Framework Layer
$\mathcal{C}$	Communication Layer
$\mathcal{B}$	Blockchain Layer
$\mathcal{A}^Q$	Quantum adversary
$\mathcal{B}^Q$	Quantum algorithm for security reduction
Lattice-Based Cryptography	
R_q	Polynomial ring $\mathbb{Z}[x]/(x^n + 1)$
$A \in R_q^{m \times n}$	Matrix for Module-SIS problem instance
$\mathbf{s}$	Short vector solution to Module-SIS problem
$\ \cdot\ $	Euclidean norm
β	Bound parameter for short vector constraint
Module-SIS	Short Integer Solution problem over modules
Module-LWE	Learning With Errors problem over modules
NTT	Number Theoretic Transform

Table 5. Glossary of Mathematical Notation and Symbols

Symbol	Description
Blockchain and Smart Contracts	
B	Blockchain state
tx	Transaction
b	Individual block in blockchain
$batchId$	Unique identifier for batch operations
$dataStore[\cdot]$	Smart contract data storage mapping
$devices[\cdot]$	Smart contract device registry mapping
gas	Computational cost units in blockchain operations
Trust Management	
$trustScore$	Dynamic trust score for devices (range: 100-1000)
δ	Trust score change increment (± 5)
τ	Trust threshold parameter (typically 500)
$bound(x, min, max)$	Bounding function: $\max(min, \min(x, max))$
Protocol Functions	
$Setup(1^\lambda)$	System parameter generation function
$KeyGen(1^\lambda)$	Key generation algorithm
$Prove(sk, x)$	Proof generation algorithm
$Verify(pk, \pi)$	Proof verification algorithm
$Fresh(\pi)$	Temporal freshness validation function
$Authorized(V_i)$	Verifier authorization check function
$CheckBlockchain(B, \pi)$	Blockchain state validation function
$ValidateBlock(b, \pi)$	Individual block validation function
Homomorphic Encryption	
$HE.Enc(hk, d)$	Homomorphic encryption of data d with key hk
$HE.Eval(f, c)$	Homomorphic evaluation of function f on ciphertext c
$f(\cdot)$	Function to be computed homomorphically
Temporal Parameters	
t	Timestamp of proof generation
Δt	Freshness window threshold (300 seconds)
$currentTime$	Current system timestamp
$timestamp$	Generic timestamp field
Complexity Notation	
$\mathcal{O}(\cdot)$	Big-O complexity notation
$\mathcal{O}(\log \lambda)$	ZKP generation/verification complexity
$\mathcal{O}((\log p)^3)$	Modular exponentiation complexity
$\mathcal{O}(n \log n)$	NTT operation complexity
$\mathcal{O}(1)$	Constant time complexity
Set Theory and Logic	
$\forall$	Universal quantifier (for all)
$\exists$	Existential quantifier (there exists)
$\in$	Element membership
$\wedge$	Logical AND
$\vee$	Logical OR
$\iff$	Logical equivalence (if and only if)
$\parallel$	Concatenation operator
$\leftarrow$	Assignment operator

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