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Image reconstruction from incomplete convolution data via total variation regularization

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Abstract In this paper, we propose the use of alternating direction method for image reconstruction from highly incomplete convolution data, where an image is reconstructed as a minimizer of an energy function that sums a TV term for image regularity and a least squares term for data fitting. Our algorithm, called RecPK, takes advantage of problem structures and has an extremely low per-iteration cost. To demonstrate the efficiency of RecPK, we compare it with TwIST, a state-of-the-art algorithm for minimizing TV models. Moreover, we also demonstrate the usefulness of RecPK in image zooming.

Keywords Total Variation, Image Reconstruction, Zooming, Compressive Imaging, Fast Fourier Transform, Convolution, Alternating Direction Method

AMS 2010 subject classifications ???, ???

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1. Introduction

Total variation (TV) regularization for image restoration and reconstruction has been widely studied in the literature since its introduction by Rudin, Osher and Fatemi, mainly due to the strong ability of TV in preserving edges and object boundaries that are usually the most important features to recover. In image restoration, TV models with the ℓ_2-norm fidelity, which are suitable for data contaminated by independent and identically distributed Gaussian noise, have been studied in, e.g., [6, 8]. When observed data suffer from impulsive noise, e.g., salt-and-pepper noise, TV models with nonsmooth data fitting are desirable, e.g., TV regularization with the ℓ_1-norm fidelity was addressed in [7] with some interesting geometric properties.

1.1. Image reconstruction via TV minimization

1.2. A brief review of existing methods

1.3. Notation

1.4. Contributions

The main contribution of this paper is a very simple and fast algorithm, called RecPK, proposed by alternating direction method(ADM), takes advantage of problem structures and thus has an extremely low per-iteration cost.

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1.5. Organization

2. Basic algorithm and extensions

3. Experimental results

Figure 1. Original images.

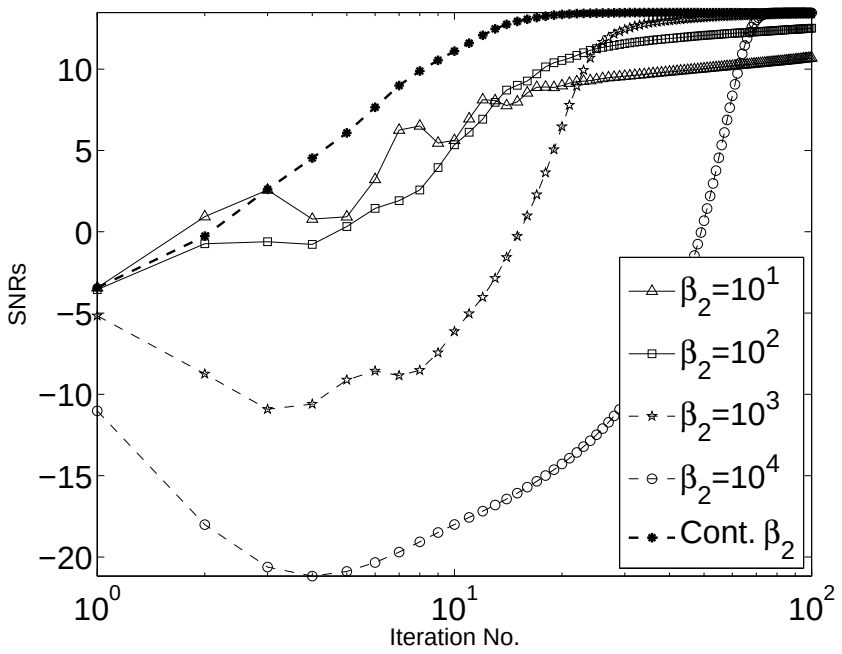

Figure 2. Test results on different values of β_2. "Cont. β_2" represents the results obtained from $\beta_2 = \min\{10 \times 1.2^k, 10^4\}$, where k counts the number of iterations.

3.1. Comparison with TwIST

In this subsection, we compare RecPK with TwIST, which...

4. Conclusion

Based on the classic augmented Lagrangian approach and a simple splitting technique, we proposed the use of ADM for solving TV image reconstruction problems with partial convolution data. Our algorithm minimizes the sum of a TV regularization term and a fidelity term measured in either ℓ_1 or ℓ_1 norm. The per-iteration cost of our algorithm contains simple shrinkages, matrix-vector multiplications and two FFTs. Extensive comparison results on single-channel images with partial convolution data indicate that RecPK is highly efficient, stable and robust and, in particular, faster than a state-of-the-art algorithm TwIST [2]. Moreover, its superior performance depends on fewer fine tuning parameters than TwIST. We hope that RecPK is useful in relevant areas of compressive sensing such as sparsity-based image reconstruction.

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