

An Optimal Adaptive Variable Sample Size Scheme for the Multivariate Coefficient of Variation

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Abstract Development of an efficient process monitoring system has always received great attention. Previous studies revealed that the coefficient of variation (CV) is important in ensuring process quality, especially for monitoring a process where its process mean and variance are highly correlated. The fact that almost all industrial process monitoring involves a minimum of two or more related quality characteristics being monitored simultaneously, this paper incorporates the salient feature of the adaptive sample size VSS scheme into the standard multivariate CV (MCV) chart, called the VSS MCV chart. A Markov chain model is developed for the derivation of the charts performance measures, i.e the average run length (ARL), the standard deviation of the run length (SDRL), the average sample size (ASS), the average number of observations to signal (ANOS) and the expected average run length (EARL). The numerical comparison shows that the proposed chart prevails over the existing standard MCV chart for detecting small and moderate upward and downward MCV shifts.

Keywords Control charting technique, efficient process monitoring, Markov-chain, multivariate coefficient of variation, variable sample size.

AMS 2010 subject classifications 90B25, 62P30

DOI:10.19139/soic-2310-5070-996

1. Introduction

Control charting techniques are common-used techniques in the process signal detection for the purpose of improving the quality of manufacturing and service processes. Most of the control charts are used to monitor the process mean and variance, subject to the process mean is constant while the process standard deviation is independent of the mean. In the past few years, increasing attention has been paid to the application of the control charting techniques to various industries, such as healthcare, chemical, biological, agriculture, service, etc. However, the mean and standard deviation of some processes are not independent of each other, this may cause erroneous conclusions. To circumvent this drawback, the coefficient of variation (CV) control chart is preferred to be used. The CV is very common and it is widely applied in various disciplines. For example, Babu and Sudha [7] applied the CV for reducing the speckle noise in ultrasound images. According to Karthik and Manjunath [23], the CV can be used to correct the unduly non-uniform distance between grid lines in noisy microarray images. Amelio [13] and Bakowshi et al. [1] presented the use of CV on sensory evaluation of virgin olive oil and injection pressure in a compression ignition engine, respectively. Papatsouma et al. [6] investigated whether the CV is a good estimator for the population CV using a discrete uniform distribution. Additionally, the CV can be implemented to

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evaluate the global solar radiation in terms of time scale separation [20]. Wachter et al. [15] examined the sincerity of effort detection for grip strength based on the CV while Doring and Reckling [26] adjusted the CV for detecting the global trends of cereal yield stability.

In the last decade, numerous intermediate and advanced CV charts were introduced to increase the sensitivity of the standard CV chart of Kang et al. [3], in detecting small and moderate CV shifts, such as those by [16], [19], [10], [24], [11], to name a few. In the industrial setting, multivariate process monitoring is considered as an important procedure. A standard (Std) multivariate CV (MCV) chart was proposed to monitor the multivariate processes [30]. The variable sample size and sampling interval (VSSI) MCV has then surpassed the Std MCV chart [9]. Subsequently, Chew et al. [34], [33] implemented the variable parameter and run rules MCV charts. The findings revealed that the statistical performances of the proposed charts are significantly better than the Std MCV chart of Yeong et al. [30]. More recently, Khaw et al. [12] considered the synthetic MCV chart whereas Giner-Bosch et al. [27] recommended the exponentially weighted moving average MCV chart.

Psarakis [22] reviewed the development of the adaptive control charts. The variable sample size (VSS) scheme is one of the best adaptive features, especially when compared to the non-adaptive counterparts [4]. Prabhu et al. [25] and Costa [2] were the pioneers to develop a VSS \bar{x} chart where the proposed chart is proved to be outperformed the Shewhart \bar{x} chart. A VSS Hotellings T^2 chart was then proposed [5]. Castagliola et al. [18] and Hu et al. [32] investigated the VSS \bar{x} chart with estimated parameters and measurement errors, respectively, while Teoh et al. [31] evaluated the performance of the VSS \bar{x} chart in terms of the median run length and expected median run length. Kazemzadeh et al. [21] incorporated the VSS scheme into a simple linear profile monitoring. Chukhrova and Johannssen [14] suggested a VSS hypergeometric p-chart whereas Lim et al. [24] presented a VSS CV chart based on the median run length. The VSS scheme was also incorporated into the CV charts by Castagliola et al. [16] and Yeong et al. [29], respectively, where the charts performance in the latter was better than the former.

In the existing literature, the adoption of the VSS scheme into a Std MCV chart was briefly explained by Khaw et al. [9], where the main concept was adopted from the VSSI scheme, by letting the short and long sampling intervals to be set equal to the fixed sampling interval. This method cannot fully reflect the essentials of the VSS scheme. The goal of this paper is to further discuss on the VSS MCV chart, which includes (1) evaluate additional important performance measures for the VSS scheme such as the average sample size (ASS) and the average number of observations to signal (ANOS), (2) investigate the detection of small and moderate downward process MCV as this is vital for process improvement, and (3) consider the selected first sample as small sample size or large sample size. Note that these discussions were not been considered in the existing VSSI MCV chart of Khaw et al. [9]. In view of the importance of monitoring the MCV, coupled with the salient feature of the VSS scheme, this paper proposes a new VSS chart for monitoring the MCV. The VSS scheme is considered as a practical and flexible approach in increasing the effectiveness of the Std MCV chart, for detecting small and moderate MCV shifts. The VSS MCV chart provides the flexibility to quality practitioners in varying the sample size. As it will be shown in Section 4, the VSS MCV chart generally outperforms the existing Std MCV chart, in terms of the average run length (ARL), the standard deviation of the run length (SDRL), the expected average run length (EARL), the ASS and the ANOS criteria.

The remainder of the sections are organized as follows: Section 2 discusses the design of the two one-sided VSS MCV charts. The derivations of the formulae and algorithms to compute the ARL, SDRL, ASS, EARL and ANOS values, by adopting the Markov-chain method are also illustrated. The next section provides an optimization procedure of the proposed chart. Statistical comparisons among the VSS MCV and Std MCV charts are enumerated in Section 4. Lastly, the research findings and suggestions for future research are shown in the last section.

2. The Two One-sided VSS MCV Charts

Voinov and Nikulin [28] derived the population MCV statistic as $\gamma = (\mu^T \Sigma^{-1} \mu)^{-\frac{1}{2}}$, where μ and σ refer to the mean vector and covariance matrix, respectively. γ can be estimated by the sample MCV, $\hat{\gamma}$ when μ and σ are unspecified. Thus, $\hat{\gamma} = (\bar{x}^T S^{-1} \bar{x})^{-\frac{1}{2}}$ by replacing μ and σ to \bar{x} and S . Here, \bar{x} is the sample mean vector whereas S is the sample covariance matrix. The computations of \bar{x} and S are shown as follows:

$$\bar{\mathbf{x}} = \frac{1}{n} \sum_{t=1}^n \mathbf{X}_t \quad (1)$$

and

$$\mathbf{S} = \frac{1}{n-1} \sum_{t=1}^n (\mathbf{X}_t - \bar{\mathbf{x}})(\mathbf{X}_t - \bar{\mathbf{x}})^T, \quad (2)$$

respectively, where $\bar{\mathbf{x}}$ and \mathbf{S} are independent of one another.

The Std MCV chart of Yeong et al. [30] uses a fixed sample size, n_0 . The Std MCV chart consists of two regions, i.e. the safe region and the action region. Contrary to the Std MCV chart, the VSS MCV chart comprises three regions, which represent the safe, warning and action regions. The VSS scheme varies the sample size to improve the performance of the Std MCV chart in the detection of small and moderate MCV shifts. The sample size adopted for the proposed chart is varied between the small sample size, n_1 and the large sample size, n_2 , where $n_1 < ASS_0 < n_2$. Note that $ASS_0 (= n_0)$ is the in-control ASS, where ASS_0 of the VSS MCV chart is equal to the fixed sample size of the Std MCV chart, for ensuring a fair comparison. This paper suggests two one-sided VSS MCV charts, that are the upward and downward VSS MCV charts. The VSS MCV chart works as follows:

1. When the t -th sample MCV, $\hat{\gamma}$ falls in the safe region ($\hat{\gamma}$ falls below the warning limit for the upward chart, while $\hat{\gamma}$ falls above the warning limit for the downward chart), the process shows no indication of trouble. Hence, n_1 is taken to compute the $(t+1)$ -th sample MCV, $\hat{\gamma}_{t+1}$;
2. When $\hat{\gamma}$ falls in the warning region ($\hat{\gamma}$ falls above the warning limit and below the control limit for the upward chart, while $\hat{\gamma}$ falls below the warning limit and above the control limit for the downward chart), the process still shows no indication of trouble but there is a higher tendency for it to become out-of-control. Hence, n_2 is taken to compute $\hat{\gamma}_{t+1}$; and
3. When $\hat{\gamma}$ falls in action region ($\hat{\gamma}$ falls above the control limit for the upward chart, while $\hat{\gamma}$ falls below the control limit for the downward chart), the process shows an indication of trouble at the t -th sample due to the presence of assignable causes. In this case, an immediate investigation should be taken.

The upper control and warning limits of the upward VSS MCV chart can be obtained as:

$$UCL = F_{\hat{\gamma}}^{-1}(1 - \alpha | n_a, p, \delta_0) \quad (3)$$

and

$$LCL = F_{\hat{\gamma}}^{-1}(\alpha | n_a, p, \delta_0) \quad (4)$$

respectively, whereas the lower control and warning limits of the downward VSS MCV chart can be obtained as:

$$UWL = F_{\hat{\gamma}}^{-1}(1 - \alpha' | n_a, p, \delta_0) \quad (5)$$

and

$$LWL = F_{\hat{\gamma}}^{-1}(\alpha' | n_a, p, \delta_0) \quad (6)$$

respectively, where $\delta_0 = n/\gamma_0^2$ and the α' value is determined based on desired in-control ARL (ARL_0) value when the process MCV is in-control and $\alpha' > \alpha$. Here, $a = 1$ when $n(i) = n_1$ and $a = 2$ when $n(i) = n_2$. The $n(i)$ denotes the selected sample size. Equations (3) C (6) follows an inverse cumulative distribution function (cdf) of $\hat{\gamma}$, i.e. $F_{\hat{\gamma}}^{-1}(\alpha | n, p, \delta) = \sqrt{[(n(n-p))/((n-1)p)] \times [1/F_F^{-1}(1 - \alpha | p, n-p, \delta)]}$. Here, $F_F^{-1}(\cdot)$ refers to an inverse cdf of a noncentral F distribution while p denotes the number of quality characteristics, where this distribution is only valid when $p \leq n$ due to the positive degree of freedom and δ is defined as $n/(\tau\gamma_0)^2$, where the shift size $\tau = 1$ when the process shows no indication of trouble, while $\gamma_1 = \tau\gamma_0$ is an out-of-control MCV when $\tau \neq 1$. When $\tau \neq 1$, the values of $\tau > 1$ and $0 < \tau < 1$ refer to upward and downward MCV shifts, respectively. A three-state

Table 1. Optimal parameter combinations (n_1, n_2, α') of the VSS MCV chart for minimizing ARL_1 , and $EARL_1$, when $p = 2$, $ASS_0 \in (5, 10)$, $\gamma_0 \in (0.1, 0.3, 0.5)$, $\tau \in (0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.5)$ and $(\tau_{min}, \tau_{max}) \in ([0.5, 1), (1, 2])$

τ	Minimize ARL_1					
	$n(1) = n_1$			$n(1) = n_2$		
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
$ASS_0=5$						
0.5	3,16,0.1561	3,16,0.1561	3,16,0.1561	3,31,0.0739	3,31,0.0739	3,31,0.0739
0.6	3,22,0.1077	3,22,0.1077	3,24,0.0977	3,31,0.0739	3,31,0.0739	3,31,0.0739
0.7	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739
0.8	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739
0.9	3,10,0.2876	3,27,0.0858	3,31,0.0739	3,10,0.2876	3,27,0.0858	3,31,0.0739
1.1	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396
1.2	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739
1.3	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739
1.4	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739
1.5	4,27,0.0461	3,30,0.0766	3,31,0.0739	3,31,0.0739	3,31,0.0739	3,31,0.0739
$ASS_0=10$						
0.5	8,28,0.1024	8,27,0.1077	7,29,0.1387	3,31,0.252	3,31,0.252	3,31,0.252
0.6	7,31,0.1274	7,31,0.1274	6,31,0.1623	3,31,0.252	3,31,0.252	3,31,0.252
0.7	5,31,0.1945	5,31,0.1945	4,31,0.2243	3,31,0.252	3,31,0.252	3,31,0.252
0.8	3,31,0.252	3,31,0.252	3,31,0.252	3,31,0.252	3,31,0.252	3,31,0.252
0.9	3,19,0.439	3,28,0.2819	3,31,0.252	3,19,0.439	3,28,0.2819	3,31,0.252
1.1	5,31,0.1945	5,31,0.1945	5,31,0.1945	5,31,0.1945	5,31,0.1945	5,31,0.1945
1.2	4,31,0.2243	4,31,0.2243	4,31,0.2243	4,31,0.2243	4,31,0.2243	4,31,0.2243
1.3	5,31,0.1945	5,31,0.1945	4,31,0.2243	4,31,0.2243	4,31,0.2243	4,31,0.2243
1.4	7,31,0.1274	6,31,0.1623	5,31,0.1945	4,31,0.2243	4,31,0.2243	3,31,0.252
1.5	8,31,0.0894	7,31,0.1274	6,31,0.1623	4,31,0.2243	4,31,0.2243	4,31,0.2243
(τ_{min}, τ_{max})	Minimize $EARL_1$					
	$n(1) = n_1$			$n(1) = n_2$		
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
$ASS_0=5$						
[0.5, 1.0)	3,31,0.0739	3,31,0.0739	3,31,0.0739	4,6,0.5014	4,6,0.5014	4,6,0.5014
(1.0, 2.0]	4,30,0.0411	4,30,0.0411	3,30,0.0766	3,6,0.6676	3,6,0.6676	3,6,0.6676
$ASS_0=10$						
[0.5, 1.0)	3,31,0.252	3,31,0.252	3,31,0.252	9,11,0.5014	9,11,0.5014	9,11,0.5014
(1.0, 2.0]	6,30,0.1689	6,30,0.1689	6,30,0.1689	3,11,0.8753	3,11,0.8753	3,11,0.8753

Markov-chain model is adopted from Yeong et al. [29] to derive the formulae for the ARL and EARL of the VSS MCV chart, where 1^{st} , 2^{nd} and 3^{rd} states correspond to safe, warning and action regions, respectively. Here, the first two are transient and the last one is the absorbing state. Subsequently, the resulting transition probability matrix \mathbf{A} can be obtained as

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & 1 - (A_{11} + A_{12}) \\ A_{21} & A_{22} & 1 - (A_{21} + A_{22}) \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

where

$$A_{11} = Pr(\hat{\gamma} \leq UWL|n_1, p, \delta_1) = F_{\hat{\gamma}}(UWL|n_1, p, \delta_1), \tag{8}$$

Table 2. Optimal parameter combinations (n_1, n_2, α') of the VSS MCV chart for minimizing ARL_1 and $EARL_1$, when $p = 3$, $ASS_0 \in (5, 10)$, $\gamma_0 \in (0.1, 0.3, 0.5)$, $\tau \in (0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.5)$ and $(\tau_{min}, \tau_{max}) \in ([0.5, 1), (1, 2])$

τ	Minimize ARL_1					
	$n(1) = n_1$			$n(1) = n_2$		
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
	$ASS_0=5$					
0.5	4,16,0.0858	4,15,0.0934	4,16,0.0858	4,31,0.0396	4,31,0.0396	4,31,0.0396
0.6	4,24,0.0526	4,22,0.0581	4,25,0.0502	4,31,0.0396	4,31,0.0396	4,31,0.0396
0.7	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396
0.8	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396
0.9	4,6,0.5014	4,27,0.0461	4,31,0.0396	4,6,0.5014	4,28,0.0443	4,31,0.0396
1.1	4,31,0.0396	4,31,0.0396	4,6,0.5014	4,31,0.0396	4,31,0.0396	4,31,0.0396
1.2	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396
1.3	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396
1.4	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396
1.5	4,25,0.0502	4,30,0.0411	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396
	$ASS_0=10$					
0.5	8,27,0.1077	8,26,0.1135	7,27,0.1523	4,31,0.2243	4,31,0.2243	4,31,0.2243
0.6	7,31,0.1274	7,31,0.1274	7,31,0.1274	4,31,0.2243	4,31,0.2243	4,31,0.2243
0.7	5,31,0.1945	5,31,0.1945	5,31,0.1945	4,31,0.2243	4,31,0.2243	4,31,0.2243
0.8	4,31,0.2243	4,31,0.2243	4,31,0.2243	4,31,0.2243	4,31,0.2243	4,31,0.2243
0.9	4,19,0.4016	4,28,0.252	4,31,0.2243	4,19,0.4016	4,28,0.252	4,31,0.2243
1.1	6,31,0.1623	6,31,0.1623	6,31,0.1623	5,31,0.1945	6,31,0.1623	6,31,0.1623
1.2	5,31,0.1945	5,31,0.1945	5,31,0.1945	4,31,0.2243	5,31,0.1945	5,31,0.1945
1.3	6,31,0.1623	5,31,0.1945	5,31,0.1945	4,31,0.2243	4,31,0.2243	4,31,0.2243
1.4	7,31,0.1274	6,31,0.1623	6,31,0.1623	4,31,0.2243	4,31,0.2243	4,31,0.2243
1.5	8,31,0.0894	7,31,0.1274	6,31,0.1623	5,31,0.1945	4,31,0.2243	4,31,0.2243
(τ_{min}, τ_{max})	Minimize $EARL_1$					
	$n(1) = n_1$			$n(1) = n_2$		
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
	$ASS_0=5$					
[0.5, 1.0)	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,6,0.5014	4,6,0.5014	4,6,0.5014
(1.0, 2.0]	4,30,0.0411	4,30,0.0411	4,30,0.0411	4,6,0.5014	4,6,0.5014	4,6,0.5014
	$ASS_0=10$					
[0.5, 1.0)	5,31,0.1945	5,31,0.1945	5,31,0.1945	9,11,0.5014	9,11,0.5014	9,11,0.5014
(1.0, 2.0]	7,30,0.1328	6,30,0.1689	6,30,0.1689	4,11,0.8575	4,11,0.8575	4,11,0.8575

$$A_{12} = Pr(UWL < \hat{\gamma} \leq UCL|n_1, p, \delta_1) = F_{\hat{\gamma}}(UCL|n_1, p, \delta_1) - F_{\hat{\gamma}}(UWL|n_1, p, \delta_1), \tag{9}$$

$$A_{21} = Pr(\hat{\gamma} \leq UWL|n_2, p, \delta_1) = F_{\hat{\gamma}}(UWL|n_2, p, \delta_1), \tag{10}$$

$$A_{22} = Pr(UWL < \hat{\gamma} \leq UCL|n_2, p, \delta_1) = F_{\hat{\gamma}}(UCL|n_2, p, \delta_1) - F_{\hat{\gamma}}(UWL|n_2, p, \delta_1), \tag{11}$$

for the upward VSS MCV chart, while

$$A_{11} = Pr(\hat{\gamma} \geq LWL|n_1, p, \delta_1) = 1 - F_{\hat{\gamma}}(LWL|n_1, p, \delta_1), \tag{12}$$

$$A_{12} = Pr(LCL < \hat{\gamma} \leq LWL|n_1, p, \delta_1) = F_{\hat{\gamma}}(LWL|n_1, p, \delta_1) - F_{\hat{\gamma}}(LCL|n_1, p, \delta_1), \tag{13}$$

Table 3. Optimal parameter combinations (n_1, n_2, α') of the VSS MCV chart for minimizing ASS_1 , when $p \in (2, 3)$, $ASS_0 \in (5, 10)$, $\gamma_0 \in (0.1, 0.3, 0.5)$ and $\tau \in (0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.5)$

τ	Minimize ASS_1					
	$p = 2$					
	$n(1) = n_1$			$n(1) = n_2$		
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
	$ASS_0=5$					
0.5	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676
0.6	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
0.7	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
0.8	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
0.9	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
1.1	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
1.2	3,6,0.6676	3,6,0.6676	4,6,0.5014	3,6,0.6676	3,6,0.6676	4,6,0.5014
1.3	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676
1.4	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676
1.5	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676	3,6,0.6676
	$ASS_0=10$					
0.5	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753
0.6	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753
0.7	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753
0.8	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014
0.9	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014
1.1	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753
1.2	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753
1.3	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753
1.4	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753
1.5	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753	3,11,0.8753
τ	$p = 3$					
	$n(1) = n_1$			$n(1) = n_2$		
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
		$ASS_0=5$				
0.5	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396	4,31,0.0396
0.6	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
0.7	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
0.8	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
0.9	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
1.1	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
1.2	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
1.3	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
1.4	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
1.5	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014	4,6,0.5014
	$ASS_0=10$					
0.5	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575
0.6	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575
0.7	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575
0.8	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014
0.9	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014	9,11,0.5014
1.1	4,11,0.8575	4,11,0.8575	9,11,0.5014	4,11,0.8575	4,11,0.8575	9,11,0.5014
1.2	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575
1.3	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575
1.4	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575
1.5	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575	4,11,0.8575

$$A_{21} = Pr(\hat{\gamma} \geq LWL|n_2, p, \delta_1) = 1 - F_{\hat{\gamma}}(LWL|n_2, p, \delta_1), \tag{14}$$

Table 4. ARL_1 , $SDRL_1$ and $EARL_1$ of the Std MCV chart, when $p \in (2, 3)$, $ASS_0 \in (5, 10)$, $n(1) = n_1$, $\gamma_0 \in (0.1, 0.3, 0.5)$, $\tau \in (0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.5)$ and $(\tau_{min}, \tau_{max}) \in ([0.5, 1], (1, 2])$

τ	$ARL_1, SDRL_1$					
	$p = 2$			$p = 3$		
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
	$ASS_0=5$					
0.5	56.43,55.93	52.15,51.65	56.37,55.87	102.21,101.71	97.27,96.77	103.33,102.83
0.6	94.03,93.53	87.72,87.22	93.57,93.07	145.17,144.67	139.23,138.73	146.12,145.62
0.7	143.67,143.17	135.3,134.8	142.82,142.31	193.46,192.96	186.39,185.89	194.52,194.02
0.8	206.98,206.48	198.12,197.62	204.62,204.12	248.48,247.98	242.14,241.64	247.78,247.28
0.9	281.24,280.74	277.14,276.64	282.74,282.24	305.76,305.26	304.71,304.21	309.54,309.04
1.1	118.63,118.13	125.71,125.21	142.99,142.49	133.52,133.02	140.67,140.17	158.14,157.64
1.2	50.45,49.95	55.77,55.27	69.68,69.18	61.45,60.95	67.36,66.86	82.78,82.28
1.3	26.16,25.65	29.85,29.34	39.97,39.46	33.59,33.09	37.98,37.47	50.02,49.52
1.4	15.64,15.14	18.28,17.77	25.78,25.28	20.8,20.3	24.1,23.6	33.54,33.04
1.5	10.39,9.88	12.37,11.86	18.14,17.63	14.13,13.62	16.7,16.19	24.3,23.79
	$ASS_0=10$					
0.5	6.59,6.07	6.14,5.62	7.21,6.69	9.22,8.71	8.46,7.95	9.88,9.37
0.6	17.5,16.99	15.94,15.44	18.55,18.05	23.33,22.83	21.05,20.55	24.27,23.76
0.7	43.56,43.05	39.28,38.77	44.45,43.94	54.02,53.52	48.6,48.1	54.48,53.97
0.8	99.02,98.52	89.55,89.05	97.66,97.16	113.49,112.99	103.03,102.53	111.62,111.12
0.9	200.53,200.03	190.22,189.72	197.42,196.92	213.16,212.66	203.7,203.2	210.64,210.14
1.1	81.09,80.59	88.92,88.42	105.06,104.56	86.27,85.77	94.3,93.8	110.93,110.43
1.2	27.3,26.79	31.77,31.27	41.83,41.32	30.1,29.59	34.9,34.4	45.74,45.23
1.3	12.33,11.82	14.91,14.4	21.01,20.5	13.86,13.35	16.7,16.2	23.49,22.98
1.4	6.85,6.33	8.46,7.95	12.42,11.91	7.76,7.25	9.58,9.07	14.09,13.58
1.5	4.41,3.88	5.51,4.98	8.27,7.75	5.01,4.48	6.26,5.74	9.45,8.94
(τ_{min}, τ_{max})	$EARL_1$					
	$p = 2$			$p = 3$		
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
	$ASS_0=5$					
[0.5, 1.0]	180.24	185.92	186.14	220.18	224.01	226.33
(1.0, 2.0]	39.08	41.85	49.29	44.59	47.81	56.62
	$ASS_0=10$					
[0.5, 1.0]	100.21	105.57	106.20	109.10	114.82	115.53
(1.0, 2.0]	27.60	29.90	34.97	29.02	31.46	36.94

$$A_{22} = Pr(LCL < \hat{\gamma} \leq LWL|n_2, p, \delta_1) = F_{\hat{\gamma}}(LWL|n_2, p, \delta_1) - F_{\hat{\gamma}}(LCL|n_2, p, \delta_1), \tag{15}$$

for the downward VSS MCV chart. Equations (8) to (15) follow the cdf of $\hat{\gamma}$, i.e. $F_{\hat{\gamma}}(x|n, p, \delta) = 1 - F_F[(n(n-p))/((n-1)px^2)|p, n-p, \delta]$, where $F_F(\cdot)$ denotes the cdf of a non-central F distribution, and $\delta_1 = n_a/\gamma_1^2$, where $a = 1$ (corresponds to small sample size) and 2 (corresponds to large sample size).

Subsequently, the out-of-control ARL (ARL_1) and the out-of-control SDRL ($SDRL_1$) of the VSS MCV chart are obtained as

$$ARL_1 = s^T(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1} \tag{16}$$

and

$$SDRL_1 = \sqrt{2s^T(\mathbf{I} - \mathbf{Q})^{-2}\mathbf{Q}\mathbf{1} - ARL^2 + ARL}, \tag{17}$$

respectively, where

$\mathbf{s} = 2 \times 1$ vector of starting probabilities such that $\mathbf{s} = (1, 0)^T$ and $\mathbf{s} = (0, 1)^T$ with first sample size n_1 and n_2 , respectively.

$\mathbf{I} = 2 \times 2$ identity matrix

$\mathbf{Q} = 2 \times 2$ transition probability matrix associated with the transient states

$\mathbf{1} = (1, 1)^T$

The out-of-control ASS (ASS_1) denotes the ratio between the expected total number of observations taken in an infinite production horizon, which can be computed as [29]

$$ASS_1 = (n_1, n_2, n(1))\theta, \quad (18)$$

where θ can be obtained from $\theta = \mathbf{B}^{-1}(\mathbf{s}, 0)^T$. Consequently, the matrix \mathbf{B} shows

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & 1 \\ A_{12} & A_{22} - 1 & 0 \\ 1 - A_{11} - A_{12} & 1 - A_{21} - A_{22} & -1 \end{pmatrix}, \quad (19)$$

for $n(1) = n_1$, when the first sample size is considered as n_1 , while

$$\mathbf{B} = \begin{pmatrix} A_{11} - 1 & A_{21} & 0 \\ 1 & 1 & 1 \\ 1 - A_{11} - A_{12} & 1 - A_{21} - A_{22} & -1 \end{pmatrix}, \quad (20)$$

for $n(1) = n_2$, when the first sample size is considered as n_2 . It is worth noting that the performance evaluation of the VSS-type chart does not solely depend on ARL since the actual number of observations taken in a sample could not be ignored. In this case, it is important to use the out-of-control ANOS ($ANOS_1$) as one of the performance measures, where the equation shows as follows:

$$ANOS_1 = ARL_1 \times ASS_1. \quad (21)$$

Practitioners need to determine τ for the computation of the ARL. However, this could be a challenge in real industry applications since τ is not easily specified. The EARL can be applied to evaluate the charts performance. Here, the in-control EARL ($EARL_0$) value of the VSS MCV charts is set equal to the ARL_0 value, whereas the $EARL_1$ formula can be written as

$$EARL_1 = \int_{\tau_{min}}^{\tau_{max}} ARL_1 \times f_{\tau}(\tau) d\tau \quad (22)$$

where $f_{\tau}(\tau)$ denotes the probability density function (pdf) of τ . Additionally, τ_{min} is the lower bound while τ_{max} is the upper bound for τ . It is often hard to fit the actual shape of $f_{\tau}(\tau)$ due to a lack of information on $f_{\tau}(\tau)$. To circumvent this, uniform distribution can be used. According to Siddall [8], a uniform distribution is one of the best options if practitioners are unable to specify the random variable distribution, and can only specify its upper and lower bounds. Castagliola et al. [17] suggested the interval (τ_{min}, τ_{max}) for the downward and upward $EWMACV^2$ charts as $= [0.5, 1)$ and $(1, 2]$, respectively.

3. Optimization Algorithm

This section presents an optimization procedure to obtain the optimal parameters of the VSS MCV chart to minimize the ARL_1 and ASS_1 . It is assumed that both ARL_0 and $EARL_0$ are 370. Three optimization criteria are considered to compute the optimal parameter combinations (n_1, n_2, α') for minimizing the following objective function:

- $\text{Min}_{(n_1, n_2, \alpha')} ARL_1(\tau)$, subject to ARL_0 and ASS_0
- $\text{Min}_{(n_1, n_2, \alpha')} ASS_1(\tau)$, subject to ARL_0 and ASS_0
- $\text{Min}_{(n_1, n_2, \alpha')} EARL_1(\tau)$, subject to $EARL_0$ and ASS_0

The optimization procedure of the VSS MCV chart is employed, i.e.

1. Determine ARL_0 (or $EARL_0$), ASS_0 , p and τ (for minimizing $ARL_1(\tau)$ and $ASS_1(\tau)$), or (τ_{min}, τ_{max}) (for minimizing $EARL_1(\tau_{min}, \tau_{max})$).
2. Start with $n_1 = p + 1$, followed by $n_2 = ASS_0 + 1$, where $n_1 > p$.
3. Compute α' from Equation (23) subject to ARL_0 (or $EARL_0$)=370 and $ASS_0 = n_0$, which is adopted from Khaw et al. [9].

$$\alpha' = 1 - \frac{F_{\gamma}(UCL|ASS_0, p, \delta_0)(ASS_0 - n_2)}{n_1 - n_2} \quad (23)$$

4. Compute $ARL_1(\tau)$, $SDRL_1(\tau)$, $ASS_1(\tau)$, $ANOS_1(\tau)$ and $EARL_1(\tau_{min}, \tau_{max})$ from Equations (16), (17), (18), (21) and (22), respectively, from the parameter combination (n_1, n_2, α') .
5. Increase n_1 by 1 while maintaining the value n_2 .
6. Repeat Steps 3 C 5 until $n_1 = ASS_0 - 1$.
7. Reset $n_1 = p + 1$ and increase n_2 by 1.
8. Repeat Steps 3 C 7 until $n_2 = 31$. According to Castagliola et al. [16], extremely large sample sizes are unlikely to be used in the industrial context. Here, the maximum sample size of 31 set in this paper only serves as guide, since the actual number of sample size taken depends on the specific process.
9. Select the optimal parameter combination (n_1, n_2, α') which minimizes the $ARL_1(\tau)$ (for objective function (i)), the $ASS_1(\tau)$ (for objective function (ii)) and the $EARL_1(\tau)$ (for objective function (iii)).

4. Results and Discussion

The performance of the Std MCV and the proposed VSS MCV charts will be compared in this section. The Std MCV chart has a fixed sample size, n_0 . For ensuring a fair comparison with the Std MCV chart in terms of the ARL and EARL criteria, the ASS_0 of the VSS MCV chart is set as n_0 . In this paper, $ASS_0 (= n_0) \in (5, 10)$ are considered. According to Castagliola et al. [16], an extremely large n_2 value is not practical in the industry. Thus, the maximum n_2 value is set as 31 to minimize the ARL_1 , ASS_1 and $EARL_1$ values, for detecting both the increase and decrease of MCV shifts. In addition, $\tau \in (0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.5)$, $(\tau_{min}, \tau_{max}) \in ([0.5, 1], (1, 2])$, $p \in (2, 3)$, $ASS_0 \in (5, 10)$ and $\gamma_0 \in (0.1, 0.3, 0.5)$ are considered, with $n(1) = n_1$. Subsequently, ARL_0 is assumed to be 370. Due to space constraint, the ARL_1 , ASS_1 , $SDRL_1$, $ANOS_1$ and $EARL_1$ values of the VSS MCV charts with $n(1) = n_2$ are not presented here. However, these values can be obtained from the first author, upon request. Tables 1 C 3 present the optimal parameter combinations (n_1, n_2, α') of the VSS MCV chart for minimizing the ARL_1 , ASS_1 and $EARL_1$ values, for detecting both the downward and upward MCV shifts, when $\tau \in (0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.5)$, $(\tau_{min}, \tau_{max}) \in ([0.5, 1], (1, 2])$, $p \in (2, 3)$, $ASS_0 \in (5, 10)$ and $\gamma_0 \in (0.1, 0.3, 0.5)$. For example, from Table 1, when $p = 2$, $ASS_0 = 10$, $\gamma_0 = 0.3$, $n(1) = n_1$ and $\tau = 1.2$, the optimal parameter combination (n_1, n_2, α') of the VSS MCV chart to minimize the ARL_1 value is (4,31,0.2243).

Tables 4 C 6 show the ARL_1 , ASS_1 , $SDRL_1$, $ANOS_1$ and $EARL_1$ values of the VSS MCV chart, together with the ARL_1 , $SDRL_1$ and $EARL_1$ values of the Std MCV chart [30], for detecting both the upward and downward MCV shifts. The results show that the VSS MCV chart outperforms the Std MCV chart, for detecting both the small and moderate upward and downward MCV shifts, in terms of the ARL_1 , $SDRL_1$ and $EARL_1$ criteria. For example, from Table 6, when $p = 3$, $ASS_0 = 5$, $\gamma_0 = 0.1$ and $\tau = 1.4$, the VSS MCV chart shows the $ARL_1 = 10.69$ (for minimizing ARL_1) and $ARL_1 = 18.95$ (for minimizing ASS_1), while the Std MCV chart gives an ARL_1 of 20.8. For another example from Table 5, when $p = 2$, $ASS_0 = 10$, $\gamma_0 = 0.5$ and $(\tau_{min}, \tau_{max}) = (1, 2]$, the VSS MCV and Std MCV charts give $EARL_1=30.39$ and 34.97, respectively. These results show that the VSS MCV chart has smaller ARL_1 , $SDRL_1$ and $EARL_1$ values than the Std MCV chart in detecting both the upward

Table 6. Optimal ARL_1 , ASS_1 , $SDRL_1$, $ANOS_1$ and $EARL_1$ of the VSS MCV chart for minimizing ARL_1 , ASS_1 and $EARL_1$, when $p=3$, $ASS_0 \in (5, 10)$, $n(1) = n_1$, $\gamma_0 \in (0.1, 0.3, 0.5)$, $\tau \in (0.5, 0.6, 0.7, 0.8, 0.9, 1.1, 1.2, 1.3, 1.4, 1.5)$ and $(\tau_{min}, \tau_{max}) \in \{(0.5, 1), (1, 2)\}$

τ	$ARL_1, ASS_1, SDR L_1, ANOS_1$										
	VSS MCV chart (Minimize ARL_1)					VSS MCV chart (Minimize ASS_1)					
	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$ASS_0=5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$ASS_0=5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
0.5	8.33,6.93,5.89,57.69	8.31,6.95,5.82,57.78	9.17,6.95,6.7,63.78	13.63,5.8,12.14,79.06	13.79,5.76,12.32,79.48	14.22,5.77,12.72,82.09	15.51,6.99,13.02,108.43	15.67,6.84,13.26,107.21	18.19,6.67,15.92,121.39	100.76,5.75,99.6,579.15	96.28,5.73,95.1,551.33
0.6	41.08,6.91,38.75,283.97	39.57,6.46,37.5,255.79	49.91,6.29,48.3,14.12	159.04,5.54,158.08,881.19	154.32,5.52,153.36,851.76	163.87,5.48,162.93,898.5	168.66,5.78,167.55,974.18	148.93,5.66,147.76,842.8	167.94,5.55,166.87,932.85	225.14,5.31,224.37,1195.62	233.36,5.29,232.61,1233.36
0.7	299.22,5.14,298.61,1538.33	293.86,5.19,293.23,1525.34	300.13,5.17,299.51,1551.54	299.22,5.14,298.61,1538.33	298.75,5.13,298.09,1533.59	303.27,5.12,302.66,1553.42	131.77,5.61,131.09,738.89	140.39,5.57,139.74,781.6	133.22,5.1,132.56,680.01	140.59,5.1,139.95,716.95	158.26,5.09,157.63,805.64
0.8	46.01,6.42,44.96,295.43	53.6,6.31,52.63,338.41	70.6,6.13,69.72,432.58	59.42,5.19,58.62,308.15	65.58,5.18,64.81,339.59	81.14,5.17,80.41,419.13	19.7,09.17,66.134.78	23.12,7.21,85.161.91	33.47,6.78,32.3,1226.86	35.88,5.23,35.02,187.59	47.77,5.22,46.95,249.28
0.9	10.69,7.3,9.28,78.06	12.76,7.34,11.3,793.63	18.34,7.25,17.2,134.44	18.95,5.25,17.99,99.44	22.23,5.25,21.3,116.65	31.27,5.25,30.38,164.17	7.51,7.12,6.11,53.43	8.76,7.35,7.37,64.38	12.22,7.46,10.8,91.18	15.16,5.24,14.19,79.5	22.21,5.27,21.27,116.95
1.0	2.38,13.88,0.8,33.01	2.39,13.54,0.82,32.29	2.54,13.31,0.92,33.81	7.75,9.4,6.27,72.88	7.16,9.28,5.68,66.5	8.31,9.49,6.83,78.9	3.63,15.36,1.79,56.47	3.57,14.96,1.77,53.35	3.96,15.44,2.08,61.1	19.01,10.29,17.57,195.68	19.86,10.32,18.43,205.03
1.1	8.45,19.28,5.88,162.82	7.61,17.78,5.16,135.22	9.13,18.14,6.7,165.59	46.08,10.69,44.72,492.38	40.61,10.64,39.23,432.1	46.36,10.67,45.494.73	41.45,18.38,39.18,761.93	32.94,17.15,30.73,564.83	40.77,16.26,38.72,662.77	95.1,10.57,94.38,1004.98	104.72,10.53,104.04,1102.29
1.2	200.44,12.04,199.45,2412.46	170.2,12.33,169.07,2097.86	182.17,12.09,181.1,2203.16	213.93,10.3,213.34,2202.58	199.05,10.28,198.47,2045.92	208.39,10.25,207.81,2136.3	74.92,12.58,73.98,942.76	84.17,12.39,83.28,1042.57	102.55,12.08,101.72,1238.48	93.52,10.2,92.57,954.22	109.81,10.19,109.24,119.36
1.3	17.43,15.23,15.88,265.46	21.49,15.02,20.02,322.81	30.74,14.55,29.38,447.27	29.12,10.13,27.92,295.07	33.85,10.17,32.344.34	44.49,10.22,43.38,454.76	6.95,15.37,5.35,106.81	8.52,15.48,6.82,131.91	13.51,9.76,12.21,131.84	16.21,9.89,14.94,160.27	22.66,10.07,21.43,228.11
1.4	4.18,14.46,2.74,60.44	4.96,14.78,3.39,73.33	6.93,15.61,5.33,108.12	7.85,9.21,6.51,72.27	9.52,9.44,8.21,89.91	13.69,9.79,12.41,134.03	3.09,13.71,1.85,42.42	3.58,14.06,2.19,50.35	4.79,14.83,3.21,71.09	6.48,8.94,5.14,57.92	9.38,9.44,8.06,88.53
1.5	3.09,13.71,1.85,42.42	3.58,14.06,2.19,50.35	4.79,14.83,3.21,71.09	5.36,8.61,3.99,46.12	6.48,8.94,5.14,57.92	9.38,9.44,8.06,88.53	$EARL_1$				
(τ_{min}, τ_{max})	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$ASS_0=5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$	$ASS_0=5$	$\gamma_0=0.1$	$\gamma_0=0.3$	$\gamma_0=0.5$
[0.5, 1.0]	75.33	78.41	80.16	78.41	78.41	80.16	78.41	78.41	78.41	80.16	80.16
(1.0, 2.0]	39.14	41.89	48.56	41.89	41.89	48.56	41.89	41.89	41.89	48.56	48.56
[0.5, 1.0]	63.94	67.74	72.93	67.74	67.74	72.93	67.74	67.74	67.74	72.93	72.93
(1.0, 2.0]	25.66	27.67	31.94	27.67	27.67	31.94	27.67	27.67	27.67	31.94	31.94

and downward MCV shifts. The $ANOS_1$ value is presented in Tables 5 C 6. The results show a decreasing trend from small to large MCV shifts. For example, from Table 5, when when $p = 2$, $ASS_0 = 5$, $\gamma_0 = 0.1$ and τ increases from 0.5 to 0.9, the $ANOS_1$ values of the VSS MCV chart increases from 37.76 to 1460.70 (for minimizing ARL_1), and increases from 193.12 to 1423.94 (for minimizing ASS_1). Note that the τ which is closer to one indicates a smaller MCV shift. Thus, $\tau = 0.5$ is a larger MCV shift than $\tau = 0.9$.

5. Conclusion

Two one-sided VSS MCV charts are developed in this study to monitor the MCV, in terms of the ARL_1 , ASS_1 , $SDRL_1$, $ANOS_1$ and $EARL_1$ criteria. The two one-sided charts can circumvent the biased statistical performances of their two-sided counterpart. From the results, it reveals that the VSS MCV chart performs better than the Std MCV charts, for detecting both small and moderate upward and downward MCV shifts. The two one-sided VSS MCV charts have the flexibility in allowing the sample size to be varied based on the current process quality. This flexibility enhances the effectiveness of the process monitoring system and simultaneously cost saving in the production. In future research, the two one-sided VSS MCV charts can be further investigated when measurement errors exist.

6. Acknowledgment

This work was funded by the Ministry of Higher Education Malaysia, Fundamental Research Grant Scheme [Grant Number: 203.PMGT.6711755].

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