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Using Conformable Fractional Laplace Transform to Solve Fractional System

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Abstract In this study, we introduce the conformable fractional derivative, one of the most recent concepts in fractional calculus. We then employ the conformable fractional Laplace transform (CFLT) to solve a nonhomogeneous conformable fractional differential equation with variable coefficients, as well as a system of fractional differential equations, as an application.

Keywords Conformable Fractional Derivative, Conformable Fractional Laplace Transform, System of Fractional Differential Equations

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1. Introduction

Fractional calculus is a branch of applied mathematics concerned with derivatives and integrals of arbitrary order. The concept of fractional derivatives was first introduced more than 300 years ago by L'Hôpital and has since evolved through various definitions, including those of Cauchy, Riemann, Liouville, Grünwald–Letnikov, and, more recently, Khalil [10].

The importance of fractional calculus has gained significant attention due to its wide range of applications in various fields, including fluid flow, rheology, diffusion, relaxation, oscillation, anomalous diffusion, reaction–diffusion systems, turbulence, signal and image processing, diffusive transport phenomena, electrical networks, viscoelasticity, and many other physical processes [5, 9, 11, 12].

Most nonlinear fractional differential equations do not admit exact solutions; therefore, approximate and numerical methods are often the most suitable alternatives. Among these are the Adomian decomposition method, the homotopy perturbation method, the homotopy analysis method, and the differential transform method [1, 4, 7, 13].

In addition, several analytical methods have been introduced, including the Laplace transform method, Fourier transform method, vibrational iteration method, and Green's function method [3, 6, 13].

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Conformable Fractional Derivative

A completely new definition of fractional calculus, which is more influential than preceding definitions of order $\alpha \in (0,1]$. Also this definition can be generalized to involve any However, the case $\alpha \in (0,1]$ is the most important one, and the other cases become easy when it is established.

Definition 1.1. [10] Given a function $f : [0,\infty) \to \mathbb{R}$, the conformable fractional derivative of f of order α is defined by

$$f^{(\alpha)}(t) = \lim_{\varepsilon \to 0} \frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \quad t > 0,\ \alpha \in (0,1]. \tag{1.1}$$

If f is α-differentiable in some $(0,a)$, $a > 0$, and $\lim_{t\to 0^+} f^{(\alpha)}(t)$ exists, then we define

$$f^{\alpha}(0) = \lim_{t\to 0^+} f^{(\alpha)}(t). \tag{1.2}$$

We write $T_\alpha(f)$ for $f^{(\alpha)}(t)$, to denote the conformable fractional derivative of f of order α. If the conformable fractional derivative of f of order α exists at some t, then we say f is α-differentiable at t.

In this paper, we present the *conformable fractional Laplace transform*, which provides an effective tool for solving complex problems with relative ease. We now introduce some fundamental definitions and properties of the classical Laplace transform and the conformable fractional Laplace transform.

Also, we present a generalization of conformable fractional Laplace transform derivatives.

2. Basics of Conformable Fractional Laplace Transform (CFLT)

Definition 2.1. [8] Let $f : [0,\infty) \to \mathbb{R}$ be a real-valued function and $0 < \alpha \leq 1$.

The conformable fractional Laplace transform of f is defined as:

$$\begin{aligned} \mathcal{L}_\alpha\{f(x)\} &= F_\alpha(s) \\ &= \int_0^\infty e^{-\frac{sx^\alpha}{\alpha}} f(x)\, d_\alpha\, x \\ &= \int_0^\infty e^{-\frac{sx^\alpha}{\alpha}} f(x)\, x^{\alpha-1} dx, \end{aligned} \tag{2.1}$$

provided the integral exists.

In the following theorem, we present the relation between the usual and conformable fractional Laplace transform.

Theorem 2.1. *[8] Let $f : [0,\infty) \to \mathbb{R}$ be a function such that $\mathcal{L}_\alpha\{f(x) = F_\alpha(s)\}$ exists. Then*

$$\mathcal{L}_\alpha\{f(x)\} = F_\alpha(s) = \mathcal{L}\{f((\alpha x)^{1/\alpha})\}(s), \qquad 0 < \alpha \leq 1. \tag{2.2}$$

Now we display the CFLT for some well-known functions in the following Lemma.

Lemma 2.1. *Let $k, q \in \mathbb{R}$, $m \in \mathbb{N}$, and $0 < \alpha \leq 1$. Then:*

(i) $\mathcal{L}_\alpha\{k\}(s) = \frac{k}{s}, \quad s > 0.$

(ii) $\mathcal{L}_\alpha\{x^m\}(s) = \alpha^{m/\alpha} \frac{\Gamma(1+m/\alpha)}{s^{1+m/\alpha}}, \quad s > 0.$

(iii) $\mathcal{L}_\alpha\{e^{qx^\alpha/\alpha}\}(s) = \frac{1}{s-q}, \quad s > 0.$

(iv) $\mathcal{L}_\alpha\{\sin(qx^\alpha/\alpha)\}(s) = \frac{q}{s^2+q^2}, \quad s > 0.$

(v) $\mathcal{L}_\alpha\{\cos(qx^\alpha/\alpha)\}(s) = \frac{s}{s^2+q^2}, \quad s > 0.$

(vi) $\mathcal{L}_\alpha\{\sinh(qx^\alpha/\alpha)\}(s) = \frac{q}{s^2-q^2}, \quad s > |q|.$

(vii) $\mathcal{L}_\alpha\{\cosh(qx^\alpha/\alpha)\}(s) = \frac{s}{s^2-q^2}, \quad s > |q|.$

Comparison for the conformable fractional Natural transform for the derivatives in [16] and in this paper we used conformable fractional Laplace transform for a new application.

Lemma 2.2. *(Conformable fractional Natural transform is denoted by n)*
Let $k, q \in \mathbb{R}$, $m \in \mathbb{N}$ and $0 < \alpha \leq 1$. Then:

(i) $n_\alpha\{k\}(s) = \dfrac{k}{s}, \quad s > 0.$

(ii) $n_\alpha\left\{e^{\frac{qx^\alpha}{\alpha}}\right\}(s) = \dfrac{1}{s-qu}, \quad s > 0.$

(iii) $n_\alpha\left\{\sin\left(\frac{qx^\alpha}{\alpha}\right)\right\}(s) = \dfrac{qu}{s^2+q^2u^2}, \quad s > 0.$

(iv) $n_\alpha\left\{\cos\left(\frac{qx^\alpha}{\alpha}\right)\right\}(s) = \dfrac{s}{s^2+q^2u^2}, \quad s > 0.$

(v) $n_\alpha\left\{\sinh\left(\frac{qx^\alpha}{\alpha}\right)\right\}(s) = \dfrac{qu}{s^2-q^2u^2}, \quad s > |q|.$

(vi) $n_\alpha\left\{\cosh\left(\frac{qx^\alpha}{\alpha}\right)\right\}(s) = \dfrac{s}{s^2-q^2u^2}, \quad s > |q|.$

Theorem 2.2. *Generalization of (CFLT) of Derivatives*
Let $f : [0, \infty) \to \mathbb{R}$ be a continuous real-valued differentiable function and $0 < \alpha \leq 1$, then for any integer number n we have:

$$\mathcal{L}_\alpha\{f^{(n\alpha)}(x)\} = s^n F_\alpha(s) - \sum_{j=0}^{n-1} s^j f^{(n-j-1)\alpha}(0), \qquad s > 0 \tag{2.3}$$

Proof
We get the proof using mathematical induction.
Assume that the formula is true for $n = 1$:

$$\begin{aligned}
\mathcal{L}\{f^\alpha(x)\} &= \int_0^\infty e^{-sx} f^\alpha(x)\,dx \qquad \text{(Integration by parts)}\\
&= \lim_{A\to\infty} \int_0^A e^{-sx} f^\alpha(x)\,dx\\
&= \lim_{A\to\infty} \left[e^{-sx} f(x)\right]_0^A + s\int_0^A e^{-sx} f(x)\,dx\\
&= (0 - f(0)) + s\mathcal{L}\{f(x)\}\\
&= sF(s) - f(0)
\end{aligned}$$

Assume that the formula is true for n and we must prove it for $n + 1$.
So, $\mathcal{L}_\alpha\{f^{(n\alpha)}(x)\} = s^n F_\alpha(s) - \sum_{j=0}^{n-1} s^j f^{(n-j-1)\alpha}(0), \quad s > 0$ is true.

Now by using definition of CFLT and integration by parts, we get:

$$
\begin{aligned}
\mathcal{L}_\alpha\{f^{((n+1)\alpha)}(x)\} &= \int_0^\infty e^{-\frac{sx^\alpha}{\alpha}} f^{(n+1)\alpha}(x)\, d_\alpha x \\
&= \int_0^\infty e^{-\frac{sx^\alpha}{\alpha}} f^{(n+1)\alpha}(x)\, x^{\alpha-1}\, dx \\
&= \int_0^\infty e^{-\frac{sx^\alpha}{\alpha}} \frac{d}{dx} f^{n\alpha}(x)\, dx \\
&= \left[e^{-\frac{sx^\alpha}{\alpha}} f^{n\alpha}(x) \right]_0^\infty + \int_0^\infty f^{n\alpha}(x)\, s x^{\alpha-1} e^{-\frac{sx^\alpha}{\alpha}}\, dx \\
&= -f^{n\alpha}(0) + s\int_0^\infty e^{-\frac{sx^\alpha}{\alpha}} f^{n\alpha}(x)\, d_\alpha x \\
&= -f^{n\alpha}(0) + sL_\alpha\{f^{n\alpha}(x)\} \qquad \text{(Since the formula is true)} \\
&= -f^{n\alpha}(0) + s\left[s^n F_\alpha(s) - \sum_{j=0}^{n-1} s^j f^{(n-j-1)\alpha}(0) \right]
\end{aligned}
$$

□

Theorem 2.3. *Let $f, g : [0,\infty) \to \mathbb{R}$ be continuous functions, $\delta, \omega, p \in \mathbb{R}$, $0 < \alpha \leq 1$. Then:*

(i) $\mathcal{L}_\alpha\{\delta f(x) + \omega g(x)\} = \delta F_\alpha(s) + \omega G_\alpha(s), \quad s > 0.$

(ii) $\mathcal{L}_\alpha\{e^{-px^\alpha/\alpha} f(x)\}(s) = F_\alpha(s+p), \quad s > |p|.$

(iii) $\mathcal{L}_\alpha\{I^\alpha f(x)\}(s) = \frac{F_\alpha(s)}{s}, \quad s > 0.$

where F_α and G_α are the conformable fractional Laplace transform of the functions f and g respectively.

Proof

(i) Applying the definition (2.1), we have

$$
\begin{aligned}
\mathcal{L}_\alpha\{\delta f(x) + \omega g(x)\} &= \int_0^\infty x^{\alpha-1}\big[\delta f(x) + \omega g(x)\big] e^{-\frac{sx^\alpha}{\alpha}}\, dx \\
&= \delta \int_0^\infty x^{\alpha-1} f(x) e^{-\frac{sx^\alpha}{\alpha}}\, dx + \omega \int_0^\infty x^{\alpha-1} g(x) e^{-\frac{sx^\alpha}{\alpha}}\, dx \\
&= \delta F_\alpha(s) + \omega G_\alpha(s)
\end{aligned}
$$

(ii) Using Theorem (2.1), we have

$$
\begin{aligned}
\mathcal{L}_\alpha\{f(x) e^{-\frac{px^\alpha}{\alpha}}\}(s) &= \mathcal{L}\left\{ e^{-\frac{p}{\alpha}(ax)^{\alpha(1/\alpha)}} f\big((ax)^{1/\alpha}\big) \right\} \\
&= \mathcal{L}\{e^{-px} f((ax)^{1/\alpha})\} \\
&= \mathcal{L}\{f((ax)^{1/\alpha})\}_{s \to s+p} \\
&= F_\alpha(s+p)
\end{aligned}
$$

(iii) Using Theorem (2.2), we have

$$
L_\alpha\{D^\alpha I^\alpha f(x)\} = sL_\alpha[I^\alpha f(x)] - I^\alpha f(0)
$$

where $I^\alpha f(0) = 0$, then

$$
L_\alpha[I^\alpha f(x)] = \frac{F_\alpha(s)}{s}
$$

□

Theorem 2.4. *(Convolution Theorem for CFLT) If f, g have well-defined conformable fractional Laplace transform $\mathcal{L}_\alpha\{f(x)\}, \mathcal{L}_\alpha\{g(x)\}$ and $0 < \alpha \geq 1$, then*

$$\mathcal{L}_\alpha\{(f * g)(x)\}(s) = F_\alpha(s)G_\alpha(s), \qquad s > 0. \tag{2.4}$$

Theorem 2.5. *[3] Let f be continuous on $[0, \infty)$, and $0 < \alpha \leq 1$. Then $F_\alpha(s) = \mathcal{L}_\alpha\{f(x)\}$ has derivatives of all orders:*

$$\mathcal{L}_\alpha\left\{\frac{x^{n\alpha}}{\alpha^n}f(x)\right\}(s) = (-1)^n\frac{d^n}{ds^n}F_\alpha(s), \quad s > 0. \tag{2.5}$$

Proof
See [3, 8]. □

3. Applications

In this paper, we apply the CFLT to solve nonlinear fractional differential equations and systems of FDEs with constant coefficients using the idea of Cramer's rule.

Problem 1

Consider the nonlinear FDE:

$$y^\alpha(x) = 2y^2(x)\sin\left(\frac{x^\alpha}{\alpha}\right) - y(x), \tag{3.1}$$

With the initial condition $y(0) = 2$.

Solution:

$$\frac{y^\alpha(x)}{-y^2(x)} = \frac{2y^2(x)\sin\left(\frac{x^\alpha}{\alpha}\right)}{-y^2(x)} - \frac{y(x)}{-y^2(x)}$$

$$\frac{y^\alpha(x)}{-y^2(x)} = -2\sin\left(\frac{x^\alpha}{\alpha}\right) + \frac{1}{y(x)} \tag{3.2}$$

Let

$$z(x) = \frac{1}{y(x)}$$

$$z^\alpha(x) = \frac{-1}{y^2(x)}x^{1-\alpha}\frac{dy}{dx}$$

$$z^\alpha(x) = \frac{-1}{y^2(x)}y^\alpha$$

Substitution in (3.2)

$$z^\alpha(x) = -2\sin\left(\frac{x^\alpha}{\alpha}\right) + z(x)$$

Applying the CFLT to both sides of equation and using the condition we obtain,

$$sZ_\alpha(s) - z(0) = \frac{-2}{s^2+1} + Z_\alpha(s)$$

$$Z_\alpha(s)[s-1] = 2 - \frac{2}{s^2+1}$$

$$\begin{aligned} Z_\alpha(s) &= \frac{2}{s-1} - \frac{2}{(s-1)(s^2+1)} \\ &= \frac{2}{s-1} - \frac{1}{s-1} + \frac{s}{s^2+1} + \frac{1}{s^2+1} \\ &= \frac{1}{s-1} + \frac{s}{s^2+1} + \frac{1}{s^2+1} \end{aligned}$$

Applying the CFLIT to both sides of equation to obtain,

$$z(x) = e^{\frac{x^\alpha}{\alpha}} + \cos\left(\frac{x^\alpha}{\alpha}\right) + \sin\left(\frac{x^\alpha}{\alpha}\right)$$

Then, the exact solution is

$$y(x) = \frac{1}{e^{\frac{x^\alpha}{\alpha}} + \cos\left(\frac{x^\alpha}{\alpha}\right) + \sin\left(\frac{x^\alpha}{\alpha}\right)} \tag{3.3}$$

Problem 2: Consider the following equation of F.D.E

$$y^{2\alpha}(x) + 2y(x) = \cos\left(\frac{x^\alpha}{\alpha}\right),$$

With the initial conditions $y^\alpha(0) = y(0) = 0$.

The solution:

By taking C.F.L.T for both sides and using the initial conditions, we obtain

$$s^2 Y_\alpha(s) - y^\alpha(0) - sy(0) + 2Y_\alpha(s) = \frac{s}{s^2+1}$$

$$(s^2+2)Y_\alpha(s) = \frac{s}{s^2+1}$$

$$Y_\alpha(s) = \frac{s}{(s^2+1)(s^2+2)}$$

Using Partial fraction decomposition, we get

$$Y_\alpha(s) = \frac{s}{(s^2+1)} + \frac{-s}{(s^2+2)}$$

By taking the conformable fractional Laplace inverse transformation (C.F.L.I.T) for both sides for this equation:

$$Y_\alpha(s) = \frac{s}{(s^2+1)} + \frac{-s}{(s^2+2)}$$

Then the exact solution is

$$y(x) = \cos\left(\frac{x^\alpha}{\alpha}\right) - \cos\left(\sqrt{2}\frac{x^\alpha}{\alpha}\right)$$

Problem 3: Consider the following system of FDEs

$$\begin{cases} y_1^\alpha = y_1 - y_2 + y_3 \\ y_2^\alpha = 2y_1 + y_2 + -y_3 \\ y_3^\alpha = 5y_1 - y_2 + -y_3 \end{cases} \tag{3.4}$$

With the initial conditions

$$y_1(0) = y_2(0) = y_3(0) = 1.$$

Solution:

Let $\mathcal{L}_\alpha\{y_1\} = F_\alpha(s)$, $\mathcal{L}_\alpha\{y_2\} = G_\alpha(s)$, $\mathcal{L}_\alpha\{y_3\} = H_\alpha(s)$.
Applying the CFLT on all the system of fractional differential equation and using the given conditions we get,

$$sF_\alpha(s) - 1 = F_\alpha(s) - G_\alpha(s) + H_\alpha(s)$$
$$sG_\alpha(s) - 1 = 2F_\alpha(s) + G_\alpha(s) - H_\alpha(s)$$
$$sH_\alpha(s) - 1 = 5F_\alpha(s) - G_\alpha(s) + H_\alpha(s)$$

The system can be written as:

$$(s-1)F_\alpha(s) + G_\alpha(s) - H_\alpha(s) = 1$$
$$-2F_\alpha(s) + (s-1)G_\alpha(s) + H_\alpha(s) = 1$$
$$-5F_\alpha(s) + G_\alpha(s) + (s-1)H_\alpha(s) = 1$$

Now we use Cramer's rules to obtain solutions $F_\alpha(s)$, $G_\alpha(s)$ and $H_\alpha(s)$.
First, we find Determinant

$$\Delta = \begin{vmatrix} (s-1) & 1 & -1 \\ -2 & (s-1) & 1 \\ -5 & 1 & (s-1) \end{vmatrix} = s^3 - 3s^2 - s$$

Hence

$$F_\alpha(s) = \frac{1}{\Delta}\begin{vmatrix} 1 & 1 & -1 \\ 1 & (s-1) & 1 \\ 1 & 1 & (s-1) \end{vmatrix} = \frac{s^2 - 2s}{s^3 - 3s^2 - s}$$

Find $G_\alpha(s)$ using Cramer's rule again.

$$G_\alpha(s) = \frac{1}{\Delta}\begin{vmatrix} (s-1) & 1 & -1 \\ -2 & 1 & 1 \\ -5 & 1 & (s-1) \end{vmatrix} = \frac{s^2 - s - 8}{s^3 - 3s^2 - s}$$

Finally, find $H_\alpha(s)$ using Cramer's rule again.

$$H_\alpha(s) = \frac{1}{\Delta}\begin{vmatrix} (s-1) & 1 & 1 \\ -2 & (s-1) & 1 \\ -5 & 1 & 1 \end{vmatrix} = \frac{s^2 + 2s - 8}{s^3 - 3s^2 - s}$$

Using partial fraction to rewrite $F_\alpha(s)$, $G_\alpha(s)$ and $H_\alpha(s)$.

$$\begin{aligned} F_\alpha(s) &= \frac{s^2 - 2s}{s^3 - 3s^2 - s} = \frac{s-2}{s^2 - 3s - 1} \\ &= \frac{s - 2 - \frac{3}{2} + \frac{3}{2}}{s^2 - 3s - 1 + \frac{9}{4} - \frac{9}{4}} = \frac{\left(s - \frac{3}{2}\right) - \frac{1}{2}}{\left(s - \frac{3}{2}\right)^2 - \frac{13}{4}} \\ &= \frac{\left(s - \frac{3}{2}\right)}{\left(s - \frac{3}{2}\right)^2 - \frac{13}{4}} + \frac{-\frac{1}{2}}{\left(s - \frac{3}{2}\right)^2 - \frac{13}{4}} \end{aligned}$$

$$\begin{aligned} G_\alpha(s) &= \frac{s^2 - s - 8}{s^3 - 3s^2 - s} = \frac{8}{s} + \frac{-7s + 23}{s^2 - 3s - 1} \\ &= \frac{8}{s} + \frac{-7s + 23 * \frac{2}{2}}{s^2 - 3s - 1 + \frac{9}{4} - \frac{9}{4}} = \frac{8}{s} + \frac{-7s + \frac{46}{2}}{\left(s - \frac{3}{2}\right)^2 - \frac{13}{4}} \\ &= \frac{8}{s} + \frac{-7s + \frac{21}{2} + \frac{25}{2}}{\left(s - \frac{3}{2}\right)^2 - \frac{13}{4}} = \frac{8}{s} - 7\frac{\left(s - \frac{3}{2}\right)}{\left(s - \frac{3}{2}\right)^2 - \frac{13}{4}} + \frac{\frac{25}{2}}{\left(s - \frac{3}{2}\right)^2 - \frac{13}{4}} \end{aligned}$$

$$\begin{aligned} H_\alpha(s) &= \frac{s^2+2s-8}{s^3-3s^2-s} = \frac{8}{s} + \frac{-7s+26}{s^2-3s-1} \\ &= \frac{8}{s} + \frac{-7s+26*\frac{2}{2}}{s^2-3s-1+\frac{9}{4}-\frac{9}{4}} = \frac{8}{s} + \frac{-7s+\frac{21}{2}+\frac{31}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} \\ &= \frac{8}{s} - 7\frac{s-\frac{3}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} + \frac{\frac{31}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} \end{aligned}$$

Therefore, we get

$$\begin{cases} F_\alpha(s) = \frac{\left(s-\frac{3}{2}\right)}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} + \frac{-\frac{1}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} \\ G_\alpha(s) = \frac{8}{s} - 7\frac{\left(s-\frac{3}{2}\right)}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} + \frac{\frac{25}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} \\ H_\alpha(s) = \frac{8}{s} - 7\frac{s-\frac{3}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} + \frac{\frac{31}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}} \end{cases}$$

Applying the CFLIT on all systems we obtain the solution of our problem.

$$\begin{cases} \mathcal{L}_\alpha^{-1}\{F_\alpha(s)\} = \mathcal{L}_\alpha^{-1}\{\frac{\left(s-\frac{3}{2}\right)}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}\} + \mathcal{L}_\alpha^{-1}\{\frac{-\frac{1}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}\} \\ \mathcal{L}_\alpha^{-1}\{G_\alpha(s)\} = \mathcal{L}_\alpha^{-1}\{\frac{8}{s}\} - 7\mathcal{L}_\alpha^{-1}\{\frac{\left(s-\frac{3}{2}\right)}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}\} + \mathcal{L}_\alpha^{-1}\{\frac{\frac{25}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}\} \\ \mathcal{L}_\alpha^{-1}\{H_\alpha(s)\} = \mathcal{L}_\alpha^{-1}\{\frac{8}{s}\} - 7\mathcal{L}_\alpha^{-1}\{\frac{s-\frac{3}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}\} + \mathcal{L}_\alpha^{-1}\{\frac{\frac{31}{2}}{\left(s-\frac{3}{2}\right)^2-\frac{13}{4}}\} \end{cases}$$

Then, the exact solution is

$$\begin{cases} y_1(x) = e^{\frac{3}{2}\frac{x^\alpha}{\alpha}}\cosh\left(\frac{\sqrt{13}}{2}\frac{x^\alpha}{\alpha}\right) - \frac{1}{\sqrt{13}}e^{\frac{3}{2}\frac{x^\alpha}{\alpha}}\sinh\left(\frac{\sqrt{13}}{2}\frac{x^\alpha}{\alpha}\right) \\ y_2(x) = 8 - 7e^{\frac{3}{2}\frac{x^\alpha}{\alpha}}\cosh\left(\frac{\sqrt{13}}{2}\frac{x^\alpha}{\alpha}\right) + \frac{25}{\sqrt{13}}e^{\frac{3}{2}\frac{x^\alpha}{\alpha}}\sinh\left(\frac{\sqrt{13}}{2}\frac{x^\alpha}{\alpha}\right) \\ y_2(x) = 8 - 7e^{\frac{3}{2}\frac{x^\alpha}{\alpha}}\cosh\left(\frac{\sqrt{13}}{2}\frac{x^\alpha}{\alpha}\right) + \frac{31}{\sqrt{13}}e^{\frac{3}{2}\frac{x^\alpha}{\alpha}}\sinh\left(\frac{\sqrt{13}}{2}\frac{x^\alpha}{\alpha}\right) \end{cases} \tag{3.5}$$

Conclusion

In this work, we deduce the importance of the conformable fractional Laplace transform, which enables the solution of highly complicated fractional differential equations and systems in a simplified manner, as demonstrated in the presented problems. Moreover, this fractional transform has numerous applications in physics and engineering, highlighting its significance and practical value.

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