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A Lexicographic Bi-Objective Chance-Constrained Model for Asset-Liability Management with Endogenous Reliability: Theory and Application to Banque Misr

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Abstract Banks in emerging markets face persistent commercial and regulatory challenges. Profits must be maximized while ensuring solvency against economic volatility. Traditional Asset-Liability Management (ALM) models typically address this problem by treating reliability targets as constant, static parameters. A generalized Lexicographic Bi-Objective Chance-Constrained Programming model is introduced to endogenize these regulatory reliability levels as decision variables. The framework uses a hierarchical optimization system that prioritizes Capital Adequacy Surplus over Loan Returns. Historical financial data from Banque Misr (2011–2021) supplied the empirical baseline. Dynamic calibration demonstrated superior performance compared to standard static compliance. Setting the strategic floor to 80% yields a maximized Capital Surplus of EGP 247.3 million and drives Loan Returns to a peak of EGP 28.1 million. Because the solver strictly anchors both reliability objectives at this exact boundary, the maximum possible Capital Surplus is secured alongside peak Loan Returns. The practical value of this endogenous framework becomes evident when measured against a rigid 90% static benchmark. Escaping that static rule releases EGP 2.43 million in Capital Surplus and generates an additional EGP 6.44 million in yield, preventing a 22.96% contraction in Loan Returns.

Keywords Asset-Liability Management (ALM); Chance-Constrained Programming; Endogenous Reliability; Lexicographic Optimization; Basel III; Banque Misr.

AMS 2010 subject classifications 90B50, 90C30, 90C29

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1. Introduction

Commercial banks in emerging markets operate in a uniquely challenging environment characterized by high macroeconomic volatility, fluctuating interest rates, and stringent regulatory oversight. In this context, Asset-Liability Management (ALM) serves as a crucial tool for balancing two competing goals: maximizing shareholder profit and ensuring structural solvency for depositors and regulators. The primary role of ALM extends beyond simply allocating portfolios; it seeks to immunize the bank's balance sheet against stochastic shocks while meeting the tight capital requirements set by regulatory authorities at both global (Basel Accords) and national levels.

Traditionally, Quantitative ALM has relied on Linear Programming (LP) and on Deterministic Goal Programming. However, these models often overlook the uncertainty in market returns and liability flows. To address this, Stochastic Programming, especially Chance-Constrained Programming (CCP), has become more popular. CCP allows decision-makers to meet constraints at a specified probability known as the 'reliability level' (γ).

However, a significant limitation in the current literature is that most CCP models treat the reliability level (γ) as an exogenous parameter set by the researcher, typically at 95% or 99%. This static risk assumption can be

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inefficient. It forces banks to maintain rigid capital requirements that may be too large during stable times or too small during crises, ignoring the marginal costs of this safety. As a result, few models treat regulatory reliability as a variable to be optimized alongside asset allocation.

To fill this gap, this paper introduces a generalized Lexicographic Bi-Objective Non-Linear Chance-Constrained Programming model for ALM. This approach endogenizes the reliability levels for capital adequacy (γ_1) and loan returns (γ_2). This allows the creation of an 'efficiency frontier' of risk that properly balances regulatory safety and capital surplus. This model helps bank management assess the 'cost of safety'—how much extra liquidity (surplus) must be sacrificed to reach higher confidence targets.

To evaluate the effectiveness of this framework, the model is applied to Banque Misr, one of Egypt's largest and most important financial institutions. The paper examines annual financial data from 2011 to 2021, a decade marked by considerable economic instability, including the post-2011 political transition and the 2016 currency flotation. When used on this complex real-world data, the model shows how optimizing reliability can uncover strong asset-liability structures that adhere to 'Safety First' guidelines while optimizing the allocation of available financial resources.

The rest of the paper is structured as follows: Section 2 provides background on ALM and CCP by reviewing relevant literature and identifying a gap concerning endogenous reliability levels. Section 3 outlines the general lexicographic bi-objective mathematical model. Section 4 describes the empirical application to Banque Misr, including data processing and parameter estimation. Section 5 discusses the empirical results and sensitivity analysis regarding the trade-offs between profits and reliability. Section 6 offers policy implications for bank management and regulators. Finally, the complete historical dataset and the statistical goodness-of-fit tests for the stochastic parameters are included in Appendices A and B, respectively.

2. Literature Review

Asset and Liability Management (ALM) in commercial banks has evolved from simple gap analysis to advanced mathematical optimization techniques. This paper reviews the development of quantitative ALM models, focusing on the transition from deterministic Linear Programming (LP) based methods to stochastic models, and reveals a central methodological gap in the handling of regulatory reliability levels.

2.1. Evolution of ALM in Volatile Emerging Markets

Earlier ALM approaches were primarily deterministic, using LP to maximize profit subject to fixed balance-sheet constraints. Although computationally feasible, these models are not suitable for the structural rigidities and high volatility found in emerging markets. It was noted that rigid ALM practices within the Kenyan commercial banks contribute to inefficient performance under liquidity stress [13]. In the same vein, studies of Zimbabwean and Ethiopian banks observed that the static asset allocation of Zimbabwean and Ethiopian banks is insensitive to sudden movements in inflation and interest rates [7] and [15]. Multi-objective decision-making tools have been used to resolve conflicts between objectives, such as profit maximization and risk minimization. A systematic review of banking fund distribution concluded that Goal Programming (GP) and Bi-Objective Linear Programming are more effective at reflecting the relationships among liquidity, solvency, and profitability [17]. Furthermore, this was expanded by applying bi-objective programming to portfolio optimization, showing that hierarchical (lexicographic) methods can accurately describe the "Safety First" principle of prudent financial institutions [8].

2.2. Stochastic Approaches: Dominance vs. Chance Constraints

Since banking parameters are inherently random, the literature has shifted to Stochastic Programming. Abou El-Sood [1] demonstrated that stochastic models deliver more efficient hedging strategies for banks in the MENA region than deterministic solutions.

Two major approaches have been proposed in this area:

- **Stochastic Dominance**: second-order stochastic dominance has been used to manage underfunding risk [22]. Although theoretically sound, dominance constraints are computationally costly and less transparent to regulators than explicit probability-based targets [9].
- **Chance-Constrained Programming (CCP):** Constraints can be violated with a low, prespecified probability. The CCP can be incorporated into bank capital requirements by modelling the capital adequacy ratio (CAR) as a stochastic constraint [2].

More recently, the literature has transcended traditional CCPs to Distributionally Robust settings that account for parameter uncertainty. A model providing guarantees for Basel III compliance under a worst-case scenario was proposed in [3]. Similarly, the superiority of Distributionally Robust Chance Constrained (DRCC) optimization over standard stochastic models for uncertainty management was demonstrated in [14]. Furthermore, methods have been developed to account for the risk of credit rating migration within optimal asset allocation frameworks [21]. In addition to these methodological advances, solution algorithms and distributional assumptions have been further refined. Bi-objective optimization using Particle Swarm Optimization (PSO) has been shown to outperform Genetic Algorithms (GA) in terms of risk-adjusted returns [12]. Additionally, the Multivariate Generalized Hyperbolic (mGH) model was developed to capture skewness and heavy tails, which are characteristics commonly observed in financial returns [16]. Furthermore, recent advancements have begun to integrate machine learning techniques, such as Data Envelopment Analysis (DEA), to rigorously evaluate the operational efficiency of these ALM strategies in emerging banking sectors [4]. Nevertheless, decision problems with reliability levels as a dynamic decision variable have seldom been investigated in these contexts.

2.3. The Gap: Exogenous vs. Endogenous Reliability

Despite the advances achieved with CCP, a significant drawback remains: risk tolerance is generally treated as an exogenous, static value. In the majority of published ALM work—including [18, 20, 6]-the confidence level for risk measures (such as CVaR or Reliability) is set a priori by the researcher. It has been noted that such static procedures do not account for the ”shadow price” of reliability [10]. Furthermore, while risk tolerance directly affects portfolio weights, most models do not optimize the tolerance level itself [11].

There is a scarcity of research that treats reliability targets as endogenous decision variables. A model that simultaneously optimizes asset allocation and target reliability levels would enable banks to pinpoint the ”efficient frontier” of compliance, ensuring solvency while dynamically balancing competing financial objectives. This study addresses this specific gap by proposing a Lexicographic Bi-Objective Non-Linear Chance-Constrained Programming framework where the reliability levels for competing objectives are optimized dynamically, subject to a strategic safety floor.

3. Research Methodology

This study follows a quantitative strategy and utilizes a Lexicographic Bi-Objective Non-Linear Chance-Constrained Programming framework. The methodology is designed for commercial banks operating in volatile emerging markets. It develops a stochastic optimization model in which regulatory reliability levels are treated as endogenous decision variables, enabling the model to determine the optimal trade-off between structural solvency and financial profitability.

3.1. Decision Variables and Notation

Asset Allocation Variables ($x_1 - x_7$):
x_1: Cash and balances with the Central Bank.
x_2: Due from banks (Interbank lending).
x_3: Loans and advances to customers (The primary earning asset).
x_4: Financial investments (Securities and bonds).
x_5: Investments in subsidiaries and associates.

x_6: Intangible and other assets.
x_7: Fixed assets.
Capital Structure Variables ($x_8 - x_{10}$):
x_8: Tier 1 Capital (Core equity).
x_9: Tier 2 Capital (Supplementary capital).
x_{10}: Total Risk-Weighted Assets (RWA).
Endogenous Reliability Variables (γ_k):
These are the decision variables representing the reliability levels assigned to the stochastic objectives. The model optimizes these values dynamically rather than treating them as exogenous parameters.
γ_1: The endogenous reliability level associated with the Capital Adequacy constraint.
γ_2: The endogenous reliability level associated with the Loan Return constraint.

3.2. Model Parameters and Constants

The optimization is driven by exogenous parameters derived from the bank's financial statements and historical structure. These parameters serve as the coefficients and Right-Hand Side (RHS) constants in the model constraints.
Financial Constants (RHS Values):
CD: Total Customer Deposits.
DB: Due to Banks (Liability).
LP: Other Liabilities and Provisions.
PC: Paid-in Capital.
R: Reserves.
RE: Retained Earnings.
TE: Total Equity, defined as the sum of capital components ($TE = PC + R + RE$).
Structural Coefficients:
b_i: The minimum historical liquidity or solvency ratio for an asset j (e.g.,$x_1 \geq b_1 \times CD$).
a_1: The maximum ratio of retained earnings coverage relative to total assets.

3.3. Stochastic Parameters

The model includes two principal random variables that signify the essential uncertainties in banking regulation and market performance:

1. $\tilde{c}_1$ (Capital Adequacy Ratio): A positive independent random variable representing the regulatory capital to risk-weighted assets ratio. It follows a Normal Distribution with a mean μ_{c_1} and standard deviation σ_{c_1}.
2. $\tilde{c}_2$ (Interest Rate on Loans): A positive independent random variable representing the return on lending assets. It follows a Normal Distribution with a mean μ_{c_2} and standard deviation σ_{c_2}.

3.4. Model Formulation and Objectives

The Asset-Liability Management (ALM) problem is formulated as a hierarchical optimization model. The objectives are arranged lexicographically to reflect the "Safety First" principle: Solvency (Priority 1) must be satisfied before Profitability (Priority 2).

Priority 1: Maximize Capital Adequacy Surplus (L_1) The primary objective is to maximize the surplus (L_1) between the bank's total capital and the stochastic regulatory requirement. We define L_1 such that the probability of the actual surplus exceeding this value is equal to the reliability level γ_1:

$$\Pr(x_8 + x_9 - \tilde{c}_1 x_{10} \geq L_1) = \gamma_1 \tag{1}$$

Where:
$x_8 + x_9$: Total Regulatory Capital.
$\tilde{c}_1 x_{10}$: The stochastic regulatory capital requirement (Random Ratio RWA).

L_1: The guaranteed surplus value (Objective to maximize). To transform this probabilistic objective into a deterministic equivalent, we isolate the normally distributed stochastic parameter $\tilde{c}_1 \sim N(\mu_{c_1}, \sigma_{c_1}^2)$:

$$Pr\left(\tilde{c}_1 \leq \frac{x_8 + x_9 - L_1}{x_{10}}\right) = \gamma_1$$

Standardizing the random variable $\tilde{c}_1$ yields the standard normal variable $Z \sim N(0,1)$:

$$Pr\left(Z \leq \frac{x_8 + x_9 - L_1 - \mu_{c_1} x_{10}}{\sigma_{c_1} x_{10}}\right) = \gamma_1$$

Using the inverse cumulative distribution function of the standard normal, $\Phi^{-1}(\gamma_1)$, the probabilistic statement becomes a deterministic equality:

$$\frac{x_8 + x_9 - L_1 - \mu_{c_1} x_{10}}{\sigma_{c_1} x_{10}} = \Phi^{-1}(\gamma_1)$$

Solving for the objective variable L_1, the deterministic function is:

$$\text{Max } L_1 = (x_8 + x_9) - \left[\mu_{c_1} + \Phi^{-1}(\gamma_1)\sigma_{c_1}\right] x_{10}$$

Interpretation as Value-at-Risk (VaR): The term $\left[\mu_{c_1} + \Phi^{-1}(\gamma_1)\sigma_{c_1}\right] x_{10}$ represents the Value-at-Risk (VaR_{γ_1}) of the required capital under a conservative, high-percentile risk scenario. Therefore, maximizing L_1 equates to optimizing the guaranteed Capital Surplus against a worst-case regulatory requirement at the specified reliability level γ_1.

Priority 2: Maximize Loan Returns (L_2)

Subject to maintaining the surplus achieved in Priority 1, the secondary objective is to maximize the guaranteed financial return from the loan portfolio. We seek the maximum lower bound L_2 that satisfies:

$$\Pr(\tilde{c}_2 x_3 \geq L_2) = \gamma_2 \tag{2}$$

Where $\tilde{c}_2$ is the stochastic interest rate on loans, distributed as $\tilde{c}_2 \sim N(\mu_{c_2}, \sigma_{c_2}^2)$. Standardizing this variable gives:

$$Pr\left(Z \geq \frac{L_2 - \mu_{c_2} x_3}{\sigma_{c_2} x_3}\right) = \gamma_2$$

Since $Pr(Z \geq z) = 1 - \Phi(z) = \gamma_2$, it follows that $z = \Phi^{-1}(1-\gamma_2)$. Given the symmetry of the normal distribution, $\Phi^{-1}(1-\gamma_2) = -\Phi^{-1}(\gamma_2)$. Substituting this back into the equality yields:

$$\frac{L_2 - \mu_{c_2} x_3}{\sigma_{c_2} x_3} = -\Phi^{-1}(\gamma_2)$$

Solving for L_2, the deterministic objective becomes:

$$\text{Max } L_2 = \left[\mu_{c_2} - \Phi^{-1}(\gamma_2)\sigma_{c_2}\right] x_3$$

Quantile Interpretation: This equation demonstrates that maximizing L_2 is equivalent to optimizing the γ_2-quantile of the expected loan portfolio returns, securing a deterministic lower bound for profitability at the target reliability level γ_2.

3.5. Model Constraints

The optimization is subject to a set of deterministic constraints derived from the balance sheet structure defined in Section 3.2.

1. **Structural Constraints:**
These constraints ensure the asset allocation adheres to historical liquidity ratios (b_j) relative to Customer Deposits (CD) and Total Equity (TE).

$$\begin{aligned} x_j &\geq b_j \times CD, && j = 1, \ldots, 5 \\ x_j &\geq b_j \times TE, && j = 6, \ldots, 9 \\ x_{10} &\leq b_{10} \times CD \end{aligned}$$

2. **Regulatory & Logical Constraints**:

$$\begin{aligned} x_8 - x_9 &\geq 0 && \text{(Tier 1 Capital Dominance)} \\ x_{10} - (x_8 + x_9) &\geq 0 && \text{(Risk Assets} \geq \text{Capital)} \\ a_1 \sum_{j=1}^{7} x_j &\leq RE && \text{(Retained Earnings Coverage)} \\ x_j &\geq 0, && \forall j \in \{1, \ldots, 10\} \end{aligned}$$

3. **Balance Sheet Identity:**
To ensure accounting consistency and realistic funding limitations, the model strictly enforces that total assets must equal total available funding (Liabilities + Equity):

$$\sum_{j=1}^{7} x_j = (DB + CD + LP + TE) \tag{3}$$

4. **Strategic Safety Floor (Endogenous Reliability Bounds):**

To align the model with prudent banking practices, a Strategic Safety Floor (γ_{min}) is imposed on both reliability variables:

$$\gamma_{min} \leq \gamma_k < 1, \quad \forall k \in \{1, 2\} \tag{4}$$

3.6. The Final Deterministic Model

By converting the probabilistic chance constraints into their deterministic equivalents using the inverse cumulative distribution function $\Phi^{-1}(\cdot)$, the final Lexicographic Bi-Objective Non-Linear Chance-Constrained Programming Model is summarized below.

Objectives (Lexicographic Priority):

$$\text{Priority 1:} \quad \text{Max } L_1 = (x_8 + x_9) - \left[\mu_{c_1} + \Phi^{-1}(\gamma_1)\sigma_{c_1}\right] x_{10} \tag{5}$$

$$\text{Priority 2:} \quad \text{Max } L_2 = \left[\mu_{c_2} - \Phi^{-1}(\gamma_2)\sigma_{c_2}\right] x_3 \tag{6}$$

Subject to Structural Constraints:

$$x_j \geq b_j \times CD, \qquad j = 1, \ldots, 5 \tag{7}$$

$$x_j \geq b_j \times (PC + R + RE), \qquad j = 6, \ldots, 9 \tag{8}$$

$$x_{10} \leq b_{10} \times CD \tag{9}$$

Subject to Regulatory, Logical & Strategic Constraints:

$$
\begin{aligned}
x_8 - x_9 &\geq 0 && \text{(Tier 1 Capital Dominance)} && (10)\\
x_{10} - (x_8 + x_9) &\geq 0 && \text{(Risk Assets} \geq \text{Capital)} && (11)\\
a_1 \sum_{j=1}^{7} x_j &\leq RE && \text{(Retained Earnings Coverage)} && (12)\\
\sum_{j=1}^{7} x_j &= DB + CD + LP + PC + R + RE && \text{(Balance Sheet Identity)} && (13)\\
x_j &\geq 0, && \forall j \in \{1, \ldots, 10\} && (14)\\
\gamma_{min} \leq \gamma_k &< 1, && \forall k \in \{1, 2\} && (15)
\end{aligned}
$$

The logical constraint $x_{10} - (x_8 + x_9) \geq 0$ enforces the fundamentally leveraged nature of commercial banking. It dictates that total Risk-Weighted Assets (x_{10}) must exceed or equal total equity capital ($x_8 + x_9$). Without this boundary constraint, the optimization algorithm might theoretically evaluate non-operational boundary spaces—such as an institution funded entirely by equity with zero leveraged risk assets. This ensures the solver remains strictly grounded in the operational reality of a commercial bank's balance sheet.

Where:

- b_j, a_1: Structural coefficients derived from historical asset allocation.
- CD, DB, LP, PC, R, RE: Financial constants representing the bank's funding sources.
- γ_{min}: The strategic reliability floor is defined by bank management policy.

3.7. Solution Algorithm

The resulting model constitutes a Non-Linear Programming (NLP) problem due to the inclusion of the non-linear inverse cumulative distribution function $\Phi^{-1}(\gamma)$. To address the hierarchical nature of the strategic goals, the problem is solved using a Lexicographic Optimization procedure:

1. Step 1: Maximize L_1 subject to all structural, regulatory, and safety floor constraints.
2. Step 2: Maximize L_2 subject to the same constraints, while imposing the optimal surplus value (L_1^*) found in Step 1 as a hard constraint.

The numerical solution is obtained using the Augmented Lagrange Multiplier Method, which handles non-linear constraints and endogenous variables with high precision.

4. Empirical Application to Banque Misr

To verify the theoretical framework presented in Section 3, we have empirically applied the model to Banque Misr, one of Egypt's largest and most systemic banks. In this section, data treatment and parameter estimation, as well as calibration of the structural constraints used in the optimization, are described.

4.1. Data Collection and Processing

The research uses historical financial data for the period 2011/2012 to 2020/2021. The essential data were extracted from the "Summarized Separate Financial Statements" published in annual bank reports [5].

Managing Outliers:

The observation period coincides with a period of macroeconomic instability in Egypt (the political transition in 2011, the floatation of the currency in 2016, and the COVID-19 outbreak in 2020). These events resulted in significant deviations in specific account balances. To ensure that the optimization parameters are not overly sensitive to exogenous shocks, raw data points with extreme deviations were smoothed by linearly interpolating adjacent years before calculating the model constants. The complete raw historical dataset (before smoothing) is provided in Appendix A.

4.2. Stochastic Parameter Estimation

The model includes two major random variables: the Capital Adequacy Ratio ($\tilde{c}_1$) and the Interest Rate on Loans ($\tilde{c}_2$). SPSS was used to fit probability distributions to these parameters using the historical financial data (see Appendix A). Both variables were found to be normally distributed according to the One-Sample Kolmogorov-Smirnov Test. Appendix B summarizes the statistical goodness-of-fit for each of the tests.

Table 1. Stochastic Parameters ($\tilde{c}_1, \tilde{c}_2$)

Parameter	**Description**	**Mean (μ)**	**Standard Deviation (σ)**	**Distribution**
$\tilde{c}_1$	Capital Adequacy Ratio	0.1424	0.0188	Normal
$\tilde{c}_2$	Interest Rate on Loans	0.1743	0.0632	Normal

(Source: Calculated from Banque Misr Annual Reports 2011–2021 using SPSS)

4.3. Model Calibration: Structural Constants

The deterministic bounds require specific Right-Hand Side (RHS) values and structural coefficients (b_j). These were calculated as the averages of the processed historical data over the observation period. First, the funding inputs are shown in Table 2. These refer to the average scale of funding available to the bank and serve as the constant terms (RHS) for the budget constraints. Consequently, to ensure the optimization model accurately reflects the bank's operational context, we adjusted the structural coefficients using historical asset allocation strategies from the dataset (Appendix A). Table 3 defines the minimum asset allocation ratios (b_j), which preserve the bank's historical liquidity and solvency profile.

Table 2. Funding Inputs (RHS Constants)

Symbol	**Description**	**Average Value (EGP)**
CD	Customer Deposits	521,900,733
DB	Due to Banks	41,745,272
LP	Other Liabilities & Provisions	68,864,962
PC	Paid-in Capital	13,907,769
R	Reserves	38,979,724
RE	Retained Earnings	5,948,473
TE	Total Equity ($PC + R + RE$)	58,835,966
Total Funding	(Sum of all Sources)	691,346,933

Table 3. Structural Coefficients (b_j)

Constraint Variable	Coefficient (b_j)	Structural Logic
x_1 (Cash)	0.0625	$\geq 6.25\,\%$ of Deposits (CD)
x_2 (Due from Banks)	0.1578	$\geq 15.78\,\%$ of Deposits (CD)
x_3 (Loans)	0.3618	$\geq 36.18\,\%$ of Deposits (CD)
x_4 (Financial Inv.)	0.5563	$\geq 55.63\,\%$ of Deposits (CD)
x_5 (Investments in Subs.)	0.0166	$\geq 1.66\,\%$ of Deposits (CD)
x_6 (Intangible Assets)	0.7300	$\geq 73.00\,\%$ of Total Equity (TE)
x_7 (Fixed Assets)	0.0453	$\geq 4.53\,\%$ of Total Equity (TE)
x_8 (Tier 1 Capital)	0.7923	$\geq 79.23\,\%$ of Total Equity (TE)
x_9 (Tier 2 Capital)	0.2929	$\geq 29.29\,\%$ of Total Equity (TE)
x_{10} (RWA Limit)	0.5630	$\leq 56.30\,\%$ of Deposits (CD)
a_1 (Retained Earnings)	0.0085	Ratio of RE to Total Assets

4.4. Computational Implementation

The Lexicographic bi-objective non-linear Chance-Constrained Programming was executed in the R Statistical Computing Environment (Version 4.5.1). The `Rsolnp` package was used to solve the optimization problem, utilizing the Augmented Lagrange Multiplier approach for nonlinear optimization with equality and inequality constraints.

Execution of the hierarchical lexicographic procedure relies on a two-stage algorithmic structure:

1. **Step 1:** The primary objective, Capital Surplus (L_1), is maximized under the strict enforcement of all structural, regulatory, and safety floor constraints. From this initial optimization, the maximum achievable surplus (L_1^*) is derived.
2. **Step 2:** Optimization is then redirected toward the secondary objective, Loan Returns (L_2). The original constraint set is maintained in its entirety. However, a strict inequality constraint ($L_1 \geq L_1^* - \epsilon$) is appended to safeguard the priority objective. Where, ϵ represents a negligible computational tolerance, granting the solver adequate mathematical flexibility to maximize L_2 without violating feasibility.

Non-linear, bi-objective optimization inherently introduces severe numerical challenges. Solvers tend to converge on economically degenerate boundary solutions, such as artificially driving Risk-Weighted Assets to zero. To circumvent these mathematical traps, the algorithm utilized a feasible scaled multi-start initialization procedure. The solver was executed from multiple distinct starting points across the feasible region. Concurrently, the retained earnings structural parameter (a_1) required strict mathematical calibration. While the arithmetic mean of the raw yearly ratios is 0.0091, applying this parameter to the aggregated Right-Hand Side (RHS) constants causes immediate solver infeasibility (the actual ratio of aggregate Average Retained Earnings to aggregate Average Total Assets is 0.0086). Therefore, a_1 was calibrated to 0.0085. This adjustment maintained structural consistency with the scaled baseline, preventing artificial constraint conflicts and ensuring robust global convergence during the multi-start optimization process.

The strategic safety floor (γ_{min}) was subsequently enforced as a hard lower limit governing both endogenous reliability variables:

$$\gamma_1 \geq 0.80, \quad \gamma_2 \geq 0.80 \tag{16}$$

This bounds the search space strictly to solutions that align with the bank's "Safety First" mandate, preventing the optimizer from mathematically converging on high-return but unacceptably high-risk portfolios.

5. Empirical Results and Discussion

The numerical results obtained from solving the Lexicographic Bi-Objective Non-Linear Chance-Constrained Programming model, using the parameters calibrated in Section 4, are presented in this section. The analysis focuses on the optimal asset-liability structure, the achieved reliability levels, and a sensitivity analysis regarding the strategic safety floor.

5.1. Optimal Solution: The Efficient Strategy.

The baseline model was solved with a Strategic Safety Floor of 80% ($\gamma_{min} = 0.80$). This is the "balanced" scenario, in which the bank seeks to optimize Capital Surplus and Loan Returns while avoiding excessive conservatism. The solver converged to the optimal decision variables (x_j), as detailed in Table 4. These values represent the optimal asset allocation necessary to fulfill the bank's two objectives.

Table 4. Optimal Asset Allocation and Capital Structure

Decision Variable	Description	Optimal Value (EGP)	% of Total Assets
x_1	Cash & Central Bank	32,618,799	4.7%
x_2	Due from Banks	82,355,939	11.9%
x_3	Loans & Advances	231,760,574	33.5%
x_4	Financial Investments	290,333,381	42.0%
x_5	Investments in Subs.	8,663,556	1.3%
x_6	Intangible Assets	42,950,259	6.2%
x_7	Fixed Assets	2,665,423	0.4%
Total Assets	$(\sum x_1 \dots x_7)$	691,347,931	100.0%
x_8	Tier 1 Capital	161,705,002	-
x_9	Tier 2 Capital	132,125,110	-
x_{10}	Risk-Weighted Assets	293,830,112	-

Under this optimal allocation, the bank achieved the following performance metrics, detailed in Table 5.

Table 5. Objective Function Performance

Objective	Metric	Value	Reliability Level (γ)
Priority 1	Capital Surplus (L_1)	EGP 247,339,583	80.00% (γ_1)
Priority 2	Loan Returns (L_2)	EGP 28,068,416	80.00% (γ_2)

Interpretation of Endogenous Reliability

A critical structural finding from the optimization is the behavior of the endogenous reliability targets. While the model was granted the mathematical flexibility to seek higher reliability levels, the solver anchored both objectives strictly to the 80% strategic floor ($\gamma_1 = 0.80, \gamma_2 = 0.80$).

This outcome quantifies the "cost of safety." Because the reliability parameter (γ) acts as a standard-normal quantile penalty in both deterministic objective functions, demanding higher reliability disproportionately contracts the available Capital Surplus and suppresses Loan Returns. By anchoring precisely at the 80% boundary, the solver secures the maximum possible Capital Surplus while allowing Loan Returns to reach their absolute peak. This confirms that operating at the strict limit of regulatory acceptability is the mathematically optimal strategy for simultaneously maximizing both Capital Surplus and Loan Returns.

5.2. *Sensitivity Analysis and Comparative Benchmarking: The Cost of Safety*

A comprehensive grid search evaluates the performance of the optimization model. The framework is tested across a spectrum of strategic safety floors (γ_{min}) ranging from 0.70 to 0.99. This exploration maps the efficient frontier of the dynamic optimization model. It establishes the exact mathematical trade-off between strict solvency constraints and portfolio yield.

Table 6 merges the efficient frontier mapping with a direct benchmark against static chance-constrained programming (CCP) models.

Table 6. Sensitivity Analysis and Comparative Benchmarking: The Cost of Safety

Model Constraint	**Target (γ)**	**Surplus (L_1) (EGP)**	**Surplus Loss**	**Returns (L_2) (EGP)**	**Returns Loss**
Theoretical Baseline	70.0%	249,091,913	+0.71%	32,208,610	+14.75%
Endogenous (Opt.)	**80.0%**	**247,339,583**	—	**28,068,416**	—
Exogenous (Static)	90.0%	244,909,406	-0.98%	21,624,655	-22.96%
Exogenous (Static)	95.0%	242,902,523	-1.79%	16,303,268	-41.92%
Exogenous (Static)	99.0%	239,137,945	-3.32%	6,321,231	-77.48%

Relaxing the boundary to 0.70 maximizes theoretical efficiency. Loan Returns peak at EGP 32.21 million. This specific constraint level violates baseline regulatory risk tolerances.

The endogenous formulation treats reliability as an active decision variable. By setting the strategic floor to 80% and allowing the algorithm to optimize the reliability levels, the solver identifies this exact minimum boundary as the mathematically optimal efficiency point. Crucially, when the strategic floor is artificially raised during the sensitivity analysis (e.g., to 90%, 95%, or 99%) and the solver is run again, it consistently returns these new minimum permissible bounds as the optimal values. Because the economic penalty for excess safety is severe, the dynamic solver never voluntarily exceeds the floor. Consequently, at these elevated bounds, the endogenous model's optimal outputs coincide exactly with the static CCP benchmarks presented in Table 6. This demonstrates that enforcing an arbitrary 90% safety constraint strictly shrinks Loan Returns to EGP 21.62 million.

Dynamic calibration successfully maximizes the available Capital Surplus. The model secures a maximized Capital Surplus of EGP 247.3 million simply by operating at the exact 80% boundary. If an exogenous 90% static target is imposed instead, the bank is forced to meet elevated reliability levels, thereby shrinking this available surplus by EGP 2.43 million. Escaping these rigid, static limits enables the endogenous framework to generate an additional EGP 6.44 million in Loan Returns relative to the static 90% alternative. These absolute monetary figures might seem nominal given Banque Misr's massive scale, but the relative efficiency gain is substantial preventing a 22.96% contraction in Loan Returns.

Looking at the underlying balance sheet mechanics, the data reveal a stark asymmetry. Capital Surplus (L_1) is highly inelastic when constraints tighten, dropping a mere 0.98% at the 90% boundary. Such resilience is a direct result of its massive baseline scale combined with the algorithm's strict prioritization of the primary Capital Surplus objective. Because of this rigid protection, Loan Returns (L_2) are forced to act as the primary structural shock absorber. To satisfy these mandated reliability levels, the static model destroys almost a quarter of the potential loan yield. Endogenous optimization, however, actively shields the portfolio from taking this disproportionate penalty.

5.3. *Robustness Check Under Heavy-Tailed Distributions*

Standard normal assumptions often fail to capture the heavy tails found in volatile emerging markets. We tested the framework's sensitivity by replacing the baseline normal distribution with a Student-t distribution ($df = 5$). Modeling the stochastic parameters with fat tails provides a more accurate reflection of an economy in which severe market shocks occur more frequently than parametric normality predicts. Table 7 outlines the optimization's response to this shift. Both core objectives show minimal contraction. Capital Surplus (L_1) declines 0.17% to settle at EGP 246.91 million. Loan Returns (L_2) decrease by 4.20% to reach EGP 26.89 million. The solver successfully maintains both reliability targets (γ_1 and γ_2) exactly at the 80% strategic boundary. Imposing fat-tailed market assumptions does not compel the endogenous framework to unnecessarily restrict the available Capital Surplus or trigger a disproportionate contraction in Loan Returns.

Table 7. Robustness Check: Normal vs. Student-t Distribution (Heavy Tails)

Distribution	**Capital Surplus (L_1) (EGP)**	**Change**	**Loan Returns (L_2) (EGP)**	**Change**
Normal (Baseline)	**247,339,583**	**—**	**28,068,416**	**—**
Student-t ($df = 5$)	246,909,138	-0.17%	26,890,130	-4.20%

6. Conclusion and Policy Implications

Volatile emerging markets pose a fundamental dilemma for commercial banks. Returns must be maximized. Solvency under stochastic regulatory rules must remain guaranteed. Standard Asset-Liability Management (ALM) models typically treat regulatory reliability as an exogenous, static parameter. Such assumptions inflict a dual penalty by unnecessarily restricting the available Capital Surplus while suppressing Loan Returns. A generalized Lexicographic Bi-Objective Non-Linear Chance-Constrained Programming formulation resolves this limitation. The mathematical framework embeds reliability levels as endogenous decision variables.

6.1. *Summary of Findings*

Empirical testing utilizing Banque Misr data (2011–2021) quantified the exact trade-off between dynamic and static risk optimization. Dynamic calibration consistently outperformed static compliance methods.

The endogenous solver anchored reliability for both objectives exactly at the 80% strategic floor. Optimal solvency is achieved precisely at this boundary. This precise calibration prevents available capital from being unnecessarily restricted by elevated reliability mandates. Operating at this strict 80.00% safety threshold secures the maximum possible Capital Surplus. It simultaneously allows the secondary Loan Returns objective to reach its absolute peak. Calibrating risk acceptability to the precise regulatory floor satisfies capital adequacy mandates while shifting asset allocation directly onto the true efficient frontier.

Comparative analysis quantified the exact monetary toll of regulatory conservatism. Elevating the strategic safety floor (γ_{min}) from 80% to 90% forced the endogenous solver to perfectly mimic the static penalty. Under this tightened boundary, Capital Surplus was suppressed by a marginal 0.98%, while lending efficiency absorbed a vastly disproportionate penalty. Loan Returns contracted by 22.96%, wiping out approximately EGP 6.44 million in yield. This outcome demonstrates that whether imposed statically or dynamically, excessive regulatory tightening forces a severe contraction in Loan Returns to secure purely marginal gains in Capital Surplus.

The framework demonstrated structural robustness against the heavy tails characteristic of emerging markets. When tested under fat-tailed conditions, the optimization maintained a constraint-compliant asset allocation with minimal contraction across both core objectives. The endogenous solver successfully navigates these distributional shifts without forcing a severe decline in Capital Surplus or triggering a disproportionate contraction in Loan Returns. This empirically confirms the model's overall stability when normal assumptions are relaxed.

6.2. *Managerial and Regulatory Implications*

These empirical findings carry direct implications for how banks manage their treasuries and how regulators design capital rules. For bank management, the proposed endogenous framework shifts asset-liability strategy away from passive compliance and turns it into an exercise in active risk budgeting. Because the model identifies the exact marginal cost of safety, treasurers can clearly identify the two structural penalties of being overly conservative. Treasury teams can now pinpoint the exact contraction in Capital Surplus forced by elevated regulatory mandates. They also gain the ability to calculate the precise Loan Returns sacrificed whenever higher reliability targets are enforced.

On the regulatory side, endogenous supervision offers a mathematically rigorous way to maximize the banking sector's Capital Surplus while maintaining strict compliance. It completely replaces the need for rigid static mandates. Instead of relying on exogenous limits, the regulatory boundary is determined by the institution's unique risk-return profile. When safety requirements are calibrated directly to a bank's real balance sheet capacity, the broader asset structure optimizes. The institution secures its maximum possible capital surplus and guarantees strict solvency. Crucially, it achieves this without artificially suppressing the portfolio's yield.

6.3. *Limitations and Future Research*

Although the framework demonstrates structural robustness under symmetric, fat-tailed conditions, a strict mathematical boundary remains. Periods of market distress in emerging markets frequently produce asymmetric shocks. Relying on any form of parametric symmetry means that one-sided market downturns could still mask severe contractions in capital surplus and cause unanticipated volatility in loan returns. Future research must move beyond symmetric assumptions to capture these complex dynamics. Integrating Extreme Value Theory or deploying non-parametric chance constraints provides a direct mechanism to absorb true asymmetric market shocks.

Time horizon presents another boundary. The current formulation evaluates a single, static period. Moving to a Multi-Stage Stochastic Programming (MSP) setup is the logical next step. An MSP architecture allows the framework to handle time-varying portfolio rebalancing. It captures the exact path dependence of fluctuating interest rates and dynamic liability cash flows. Beyond time dynamics, qualitative targets pose a separate challenge. Managerial aspirations rarely translate easily into hard mathematical constraints. Adding a Fuzzy Goal Programming extension provides future researchers with a concrete mechanism to capture these softer strategic goals.

A. Appendix A: Banque Misr Financial Data and Parameters

This appendix presents the historical data for Banque Misr covering the fiscal years 2011/2012 through 2020/2021. This dataset was used to estimate the stochastic parameters $(\tilde{c}_1, \tilde{c}_2)$ and calculate the structural constants used in the optimization model. All monetary values are in thousands of Egyptian Pounds (EGP).

A.1. *Financial Position Data*

Table A.1 details the liability side of the balance sheet, specifically the funding sources from banks and customers. Table A.2 presents the equity components, which are essential for calculating the bank's capital adequacy and solvency limits.

Table A.1. Raw Financial Position – Deposits and Liabilities

Fiscal Year	Due to Banks (DB)	Customer Deposits (CD)	Liabilities (LP)
2011-2012	4,436,301	162,523,605	11,132,066
2012-2013	5,334,787	188,833,818	10,876,685
2013-2014	3,460,577	240,203,665	12,263,113
2014-2015	4,109,626	290,146,318	13,649,540
2015-2016	41,775,462	341,306,939	20,534,703
2016-2017	61,728,393	532,462,756	47,713,787
2017-2018	61,560,197	669,592,542	67,759,233
2018-2019	61,392,000	745,774,837	94,524,474
2019-2020	91,247,368	927,813,051	118,769,472
2020-2021	122,206,054	1,120,349,800	119,103,527
Average	**41,745,272**	**521,900,733**	**68,864,962**

Note: The values in the ”Average” row represent the smoothed aggregate figures utilized as the structural constraints in the optimization model. Because these final constants were derived after applying linear interpolation to manage macroeconomic outliers (detailed in Section 4.1), they naturally differ from the direct arithmetic mean of the raw historical columns presented above.

Table A.2. Raw Financial Position – Equity Components

Fiscal Year	Paid-in Capital (PC)	Reserves (R)	Retained Earnings (RE)
2011-2012	11,277,692	1,761,866	708,863
2012-2013	11,400,000	3,962,586	1,160,632
2013-2014	11,400,000	6,163,305	2,515,015
2014-2015	15,000,000	6,121,451	4,181,917
2015-2016	15,000,000	6,079,597	5,506,856
2016-2017	15,000,000	41,768,244	8,176,950
2017-2018	15,000,000	45,971,440	8,389,233
2018-2019	15,000,000	54,514,967	8,601,516
2019-2020	15,000,000	63,058,936	11,883,936
2020-2021	15,000,000	70,836,820	10,800,574
Average	**13,907,769**	**38,979,724**	**5,948,473**

Note: The values in the ”Average” row represent the smoothed aggregate figures utilized as the structural constraints in the optimization model. Because these final constants were derived after applying linear interpolation to manage macroeconomic outliers (detailed in Section 4.1), they naturally differ from the direct arithmetic mean of the raw historical columns presented above.

A.2. Calculated Structural Parameters

The structural constraint parameters were derived from the historical balance sheet ratios. Table A.3 presents the asset allocation coefficients ($b_1 - b_5$) based on customer deposits.

Table A.3. Structural Constraint Parameters – Asset Allocation ($b_1 - b_5$)

Fiscal Year	b_1 **(Cash)**	b_2 **(Due From Bank)**	b_3 **(Loans)**	b_4 **(Invest)**	b_5 **(Subsidiary)**
2011-2012	0.0753	0.1127	0.2790	0.5867	0.0161
2012-2013	0.0753	0.0954	0.2613	0.6224	0.0150
2013-2014	0.0737	0.0922	0.2815	0.6626	0.0137
2014-2015	0.0743	0.0890	0.3017	0.6887	0.0134
2015-2016	0.0677	0.1539	0.3197	0.6023	0.0124
2016-2017	0.0536	0.2863	0.3376	0.4726	0.0194
2017-2018	0.0493	0.2996	0.3350	0.4053	0.0218
2018-2019	0.0449	0.2875	0.3498	0.4980	0.0220
2019-2020	0.0419	0.2755	0.3645	0.5773	0.0222
2020-2021	0.0558	0.2201	0.5105	0.4470	0.0226
Average	**0.0625**	**0.1578**	**0.3618**	**0.5563**	**0.0166**

Note: The values in the "Average" row represent the smoothed aggregate figures utilized as the structural constraints in the optimization model. Because these final constants were derived after applying linear interpolation to manage macroeconomic outliers (detailed in Section 4.1), they naturally differ from the direct arithmetic mean of the raw historical columns presented above.

Table A.4 presents the coefficients for equity and risk-weighted asset limits ($b_6 - b_{10}$), which are primarily tied to total equity (TE). Table A.5 presents the historical stochastic variables. Raw historical values for Loan Interest

Table A.4. Structural Constraint Parameters – Equity and Risk ($b_6 - b_{10}$)

Fiscal Year	b_6 **(Intangible)**	b_7 **(Fixed)**	b_8 **(Tier 1)**	b_9**(Tier 2)**	b_{10} **(RWA)**
2011-2012	1.2500	0.0407	0.9157	0.3040	0.4635
2012-2013	1.0830	0.0363	0.9241	0.2486	0.5221
2013-2014	0.6710	0.0333	0.7199	0.2298	0.5300
2014-2015	0.6591	0.0302	0.6894	0.2111	0.5379
2015-2016	0.5807	0.0366	0.7182	0.2289	0.5692
2016-2017	0.4883	0.0352	0.6534	0.3023	0.6065
2017-2018	0.4717	0.0339	0.6503	0.3667	0.6156
2018-2019	0.4240	0.0568	0.7530	0.3783	0.6247
2019-2020	0.3763	0.0556	0.9774	0.2055	0.6056
2020-2021	0.4566	0.0724	0.9220	0.2119	0.6388
Average	**0.7300**	**0.0453**	**0.7923**	**0.2929**	**0.5630**

Note: The values in the "Average" row represent the smoothed aggregate figures utilized as the structural constraints in the optimization model. Because these final constants were derived after applying linear interpolation to manage macroeconomic outliers (detailed in Section 4.1), they naturally differ from the direct arithmetic mean of the raw historical columns presented above.

Rates and Capital Adequacy Ratios were used to fit the probability distributions.

Table A.5. Historical Stochastic Variables

Fiscal Year	**Interest Rate on Loans $\tilde{c}_2$**	**Capital Adequacy Ratio $\tilde{c}_1$**	**Retained Earnings Ratio (a_1)**
2011-2012	0.1325	0.1333	0.0038
2012-2013	0.1325	0.1353	0.0053
2013-2014	0.1275	0.1314	0.0092
2014-2015	0.1325	0.1259	0.0126
2015-2016	0.1325	0.1290	0.0128
2016-2017	0.1650	0.1321	0.0104
2017-2018	0.1975	0.1484	0.0097
2018-2019	0.1800	0.1456	0.0097
2019-2020	0.3316	0.1894	0.0097
2020-2021	0.2118	0.1531	0.0074
Average	**0.1743**	**0.1424**	**0.0091**

Note: The averages shown above represent the arithmetic mean of the raw historical data. While the retained earnings ratio (a_1) possesses a raw yearly average of 0.0091, applying this specific parameter to the aggregated Right-Hand Side (RHS) constants creates a mathematical infeasibility. The actual ratio of aggregate Average Retained Earnings to aggregate Average Total Assets is strictly 0.0086. To prevent an artificial constraint conflict and guarantee a feasible solution space for the multi-start solver, the a_1 parameter was mathematically calibrated to 0.0085 in the final optimization model.

B. Appendix B: Statistical Goodness-of-Fit Tests

To validate the use of Chance-Constrained Programming (CCP), the stochastic parameters were tested for normality using the One-Sample Kolmogorov-Smirnov (K-S) Test. The results confirm that both the Interest Rate on Loans $(\tilde{c}_2)$ and the Capital Adequacy Ratio $(\tilde{c}_1)$ follow a Normal Distribution at the 5% significance level.

Table B.1. One-Sample Kolmogorov-Smirnov Test for Interest Rate on Loans ($\tilde{c}_2$)

Null Hypothesis (H_0): The distribution is Normal.	
Parameter	**Value**
N	10
Normal Parameters	Mean: 0.1743
	Std. Deviation: 0.0632
Most Extreme Differences	Absolute: 0.246
	Positive: 0.246
	Negative: -0.229
Test Statistic	0.246
Asymp. Sig. (2-tailed)	**0.088**

Interpretation: Since the p-value (0.088) is greater than the significance level ($\alpha = 0.05$), we fail to reject the null hypothesis. The data follows a Normal Distribution.

Table B.2. One-Sample Kolmogorov-Smirnov Test for Capital Adequacy Ratio ($\tilde{c}_1$)

Null Hypothesis (H_0): The distribution is Normal.	
Parameter	**Value**
N	10
Normal Parameters	Mean: 0.1424
	Std. Deviation: 0.0188
Most Extreme Differences	Absolute: 0.246
	Positive: 0.246
	Negative: -0.191
Test Statistic	0.246
Asymp. Sig. (2-tailed)	**0.087**

Interpretation: Since the p-value (0.087) is greater than the significance level ($\alpha = 0.05$), we fail to reject the null hypothesis. The data follows a Normal Distribution.

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