



# Modeling and Dynamical Analysis of Corruption with the Influence of Anti-Corruption Education

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**Abstract** This study presents a dynamical model describing the spread of corruption by incorporating the influence of anti-corruption education. The proposed model consists of six compartments, namely the susceptible, exposed, corrupt, imprisoned, reformed, and honest groups. The model assumes that corruption propagates analogously to an infectious disease. We analyze the local stability of the corruption-free and corruption-endemic fixed points and show that their stability depend on the basic reproduction number. To examine the effect of anti-corruption education on the reduction of corrupt individuals, numerical simulations are performed for both fixed states. The results demonstrate that continuous anti-corruption education can effectively reduce the number of corrupt individuals in the population.

**Keywords** Dynamical modeling, Corruption, Stability, Basic reproduction number

**AMS 2010 subject classifications** 34D23, 34A34, 34D05

**DOI:** 10.19139/soic-2310-5070-3328

## 1. Introduction

Corruption constitutes a form of deviant social behavior that prevails across nearly all nations. According to the literature [1, 2], corruption is defined as an unlawful act committed to obtain personal or collective benefits through the misuse of authority or power by public (government) or private officials. Despite the establishment of various regulations and anti-corruption measures by many countries, incidents of corruption continue to persist. Even in several regions of the world, corruption cases have spread widely and reached epidemic levels within society. Various scientific studies related to this corrupt behavior have been carried out by various researchers, one of which is the use of dynamic models.

The application of dynamic modeling to corruption phenomena has emerged as a growing and influential research direction within mathematical modeling. Previous studies [3, 4, 5, 6, 7, 8, 9, 10, 11] demonstrate that corrupt behavior can propagate analogously to infectious diseases when non-corrupt individuals interact with corrupt agents. In these frameworks, the population is typically stratified into several compartments, including susceptible individuals who have not engaged in corruption, exposed individuals who are suspected but not yet penalized, active corrupt individuals, those currently serving a sentence, and former offenders who have completed their punishment. While these models provide valuable insights—particularly through analyses of local stability under parameter variation—most of the existing literature does not incorporate explicit control strategies aimed at mitigating or preventing corruption.

In the literature of social sciences, it is stated that it is important for rulers to raise public awareness about and increase knowledge of the threats posed by corruption, with a special emphasis on the role of targeted

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public education programs as an attempt to prevent the corruption [12, 13, 14]. According to the literature [15], anti-corruption education is considered essential in helping to decrease the number of individuals who may engage in corruption in the future. Therefore, in this paper, we propose a dynamical model approach consisting of six compartments that takes into account the influence of anti-corruption education in controlling corruption perpetrators, thereby reducing the number of offenders. Those compartments are the compartment of the susceptible individuals, the compartment of the exposed individuals, the compartment of the corrupt individuals, the compartment of the imprisoned individuals, the compartment of individuals who are already free from punishment, and the compartment of the honest individuals. We establish the local stability criteria of the fixed points for the proposed model as a means of understanding its long-term behavior. Finally, the numerical simulations are provided to demonstrate how anti-corruption education contributes to the reduction of the corrupt subpopulation. As far as the authors know, there is no mathematical model in this form and its stability analysis. As a result, the findings of this study represent both a novel and a fresh advancement in the field of epidemic dynamics.

The paper is organized as follows: Section 2 presents the main result consist of model formulation, stability analysis and numerical simulation. Finally, Section 3 concludes the study.

## 2. Main Results

### 2.1. Model Formulation

The total population in the age possible to corruption at time  $t$ , denoted as  $\mathcal{N}(t)$ , is divided into six compartments, as follows:

- i. Susceptible class, denoted as  $\mathcal{S}(t)$ . This compartment contains individuals who have not been involved so far in any type of corruption. People enter into this compartment naturally by immigration at rate  $\beta\mathcal{N}$ . In addition, it contains individuals who are already reformed and thus become susceptible. However, some of them can leave this compartment and move to exposed and honest compartments.
- ii. The exposed class, denoted as  $\mathcal{E}(t)$ . This compartment contains individuals who are already exposed to corruption. Although these people are already corrupted, they cannot influence or convert any susceptible individual into being corrupt. People will enter from a susceptible compartment only. However, some of them can leave this compartment and move to the corrupt compartment.
- iii. Corrupt class, denoted as  $\mathcal{C}(t)$ . These individuals possess the ability to influence or corrupt any vulnerable person. Put another way, these people have the power to persuade and enable vulnerable people to engage in corrupt practices. This compartment contains individuals who are generated only from the exposed compartment. Some of them, meanwhile, have the option to leave this compartment and transfer to one that is imprisoned and reformed.
- iv. Imprisoned class, denoted as  $\mathcal{I}(t)$ . These are the individuals that were previously involved in corrupt practices and were apprehended. Consequently, these individuals are sent to imprisoned for a predetermined amount of time. People from the corrupt compartment will enter this. In a similar vein, individuals can transfer to the reform section once their incarceration is over.
- v. Reformed class, denoted as  $\mathcal{R}(t)$ . These are people who have completed their prison sentences. Additionally, this compartment also includes individuals who committed corruption but did not serve prison sentences because they evaded the judicial process. Some of them, meanwhile, have the option to leave this compartment and transfer to one that is honest and susceptible.
- vi. Honest class, denoted as  $\mathcal{H}(t)$ . The progress of their nation is neither negatively impacted by these individuals, nor do they engage in any corrupt practices. Both reformed and vulnerable people are allowed to enter this compartment.

A compartment diagram for dynamics corruption model is given in the Figure 1, where the involving parameters are given in Table 1.

As illustrated in the compartment diagram in Figure 1, the population transitions among the compartments that characterize the dynamics of corruption behavior spread are formulated through the following nonlinear differential

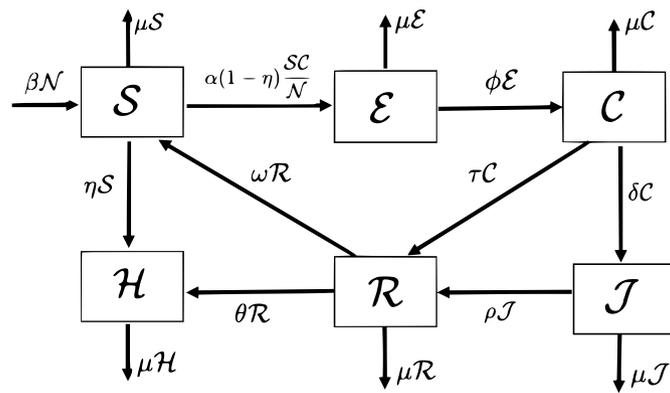


Figure 1. Compartment diagram of dynamic corruption model

| Parameter | Description   |
|-----------|---|
| $\beta$   | Immigration rate  |
| $\alpha$  | Effective rate of interaction between corrupt and susceptible people            |
| $\eta$    | The proportion of susceptible subpopulation receiving anti-corruption education |
| $\mu$     | Rate of natural mortality   |
| $\phi$    | The rate at which people who have been exposed turn corrupt                     |
| $\delta$  | The transition rate of corrupt individuals who receive imprisoned punishment    |
| $\tau$    | The rate at which corrupt individuals become reformed                           |
| $\rho$    | The rate at which imprisoned individuals become reformed                        |
| $\theta$  | The rate at which reformed individuals become honest                            |
| $\omega$  | The rate at which reformed individuals become susceptible                       |

Table 1. Parameters involved in the formation of the dynamic corruption model

equation system.

$$\begin{aligned}
 \dot{S} &= \beta N - \alpha(1-\eta)\frac{SC}{N} - (\eta + \mu)S + \omega R \\
 \dot{E} &= \alpha(1-\eta)\frac{SC}{N} - (\phi + \mu)E \\
 \dot{C} &= \phi E - (\delta + \tau + \mu)C \\
 \dot{J} &= \delta C - (\rho + \mu)J \\
 \dot{R} &= \rho J + \tau C - (\omega + \mu + \theta)R \\
 \dot{H} &= \theta R + \eta S - \mu H
 \end{aligned} \tag{1}$$

where

$$N(t) = S(t) + E(t) + C(t) + J(t) + R(t) + H(t), \tag{2}$$

with all of the initial conditions  $S(0) = S_0, E(0) = E_0, C(0) = C_0, J(0) = J_0, R(0) = R_0, H(0) = H_0$  are nonnegative.

To ensure that the model is well-posed, we must establish that the solution to model (1) remains nonnegative and bounded. These properties confirm that the model is both epidemiologically and mathematically sound.

We begin by demonstrating that the solution of model (1) cannot take negative values. Consider the dynamics of the susceptible subpopulation (1), characterized by

$$\dot{\mathcal{S}} = - \left( \alpha(1 - \eta) \frac{\mathcal{C}}{\mathcal{N}} + (\eta + \mu) \right) \mathcal{S} + \beta\mathcal{N} + \omega\mathcal{R} \geq -y\mathcal{S}, \tag{3}$$

which follows from the fact that  $\beta\mathcal{N} + \omega\mathcal{R} \geq 0$ , where  $y = \alpha(1 - \eta) \frac{\mathcal{C}}{\mathcal{N}} + (\eta + \mu)$ . Multiplying (3) by integrating factor

$$x = \exp \left( \int_0^t y(z) dz \right),$$

we obtain

$$x\dot{\mathcal{S}} + xy\mathcal{S} \geq 0,$$

which is equivalent to

$$\frac{d}{dt} (x\mathcal{S}) \geq 0. \tag{4}$$

Inequality (4) show that  $x\mathcal{S}$  is increase, and implies that

$$x\mathcal{S} = \exp \left( \int_0^t y(z) dz \right) \mathcal{S}(t) \geq x(0)\mathcal{S}(0) = \mathcal{S}(0),$$

and hence

$$\mathcal{S} \geq \mathcal{S}(0) \exp \left( - \int_0^t y(z) dz \right) \geq 0,$$

due to nonnegativity of  $\exp \left( - \int_0^t y(z) dz \right)$  for each  $t \geq 0$ .

Furthermore, the dynamics of the corruption subpopulation (1), characterized by

$$\dot{\mathcal{C}} \geq -(\delta + \tau + \mu)\mathcal{C},$$

which follows from the fact that  $\phi\mathcal{E} \geq 0$ , and this is equivalent to

$$\dot{\mathcal{C}} + (\delta + \tau + \mu)\mathcal{C} \geq 0. \tag{5}$$

The solution of (5) is given by

$$\mathcal{C}(t) \geq \mathcal{C}(0)e^{-(\delta+\tau+\mu)t} \geq 0, \tag{6}$$

since  $e^{-(\delta+\tau+\mu)t} \geq 0$  for all  $t \geq 0$ . Following a similar logical procedure, it can be demonstrated that the remaining state variables, namely  $\mathcal{E}(t)$ ,  $\mathcal{J}(t)$ , and  $\mathcal{R}(t)$  are nonnegative for all  $t \geq 0$

To proceed, we verify the boundedness of the solution to model (1). By differentiating equation (2) with respect to  $t$ , incorporating the relations provided in equation (1), and carrying out algebraic simplifications, we arrive at the rate of change of the total population:

$$\dot{\mathcal{N}} = (\beta - \mu)\mathcal{N}. \tag{7}$$

The resulting solution to equation (7) takes the form:

$$\mathcal{N}(t) = \mathcal{N}_0 e^{(\beta-\mu)t}. \tag{8}$$

It follows from equation (8) that  $\mathcal{N}(t) \leq \mathcal{N}_0$  for all  $t > 0$  under the condition  $\beta \leq \mu$ . This establishes the boundedness of the solution of model (1).

## 2.2. Stability Analysis

Following the dynamical system theory [16], the stability of model (1) is examined by analyzing the asymptotic behavior of its solutions near the fixed points, which are determined by setting

$$\dot{S} = \dot{E} = \dot{C} = \dot{J} = \dot{R} = \dot{H} = 0. \quad (9)$$

There are two fixed points that need to be considered when using an epidemiological model, namely the corruption-free fixed point, denoted by

$$\mathbb{E}^0 = (S^0, E^0, C^0, J^0, R^0, H^0),$$

and the corruption-endemic fixed point, denoted by

$$\mathbb{E}^* = (S^*, E^*, C^*, J^*, R^*, H^*).$$

The corruption-free fixed point signifies a regime devoid of corruption within the population, captured mathematically through the condition  $C = 0$ . Under this assumption and using (9), the corruption-free fixed point is given by

$$\mathbb{E}^0 = \left( \frac{\beta N}{\eta + \mu}, 0, 0, 0, 0, \frac{\eta \beta N}{\mu(\eta + \mu)} \right).$$

By invoking Hartman's Theorem [16], one concludes that  $\mathbb{E}^0$  is asymptotically stable whenever all eigenvalues of the Jacobian evaluated at  $\mathbb{E}^0$  (denoted by  $\mathfrak{J}_{\mathbb{E}^0}$ ) have real parts less than zero. A straightforward computation yields

$$\mathfrak{J}_{\mathbb{E}^0} = \begin{bmatrix} -m_1 & 0 & -\frac{\alpha(1-\eta)\beta}{\eta+\mu} & 0 & \omega & 0 \\ 0 & -m_2 & \frac{\alpha(1-\eta)\beta}{\eta+\mu} & 0 & 0 & 0 \\ 0 & \phi & -m_3 & 0 & 0 & 0 \\ 0 & 0 & \delta & -m_4 & 0 & 0 \\ 0 & 0 & \tau & \rho & -m_5 & 0 \\ \eta & 0 & 0 & 0 & \theta & -\mu \end{bmatrix}, \quad (10)$$

where  $m_1 = \eta + \mu$ ,  $m_2 = \phi + \mu$ ,  $m_3 = \delta + \tau + \mu$ ,  $m_4 = \rho + \mu$ ,  $m_5 = \omega + \theta + \mu$ . The Jacobian matrix  $\mathfrak{J}_{\mathbb{E}^0}$  admits the following characteristic polynomial

$$p_{\mathbb{E}^0}(\lambda) = (-\mu - \lambda)(-m_1 - \lambda)(-m_5 - \lambda)(-m_4 - \lambda) \left( \lambda^2 + (m_2 + m_3)\lambda + m_2 m_3 - \frac{\phi \alpha (1 - \eta) \beta}{\eta + \mu} \right). \quad (11)$$

The eigenvalues of  $\mathfrak{J}_{\mathbb{E}^0}$  are given by the roots of  $p_{\mathbb{E}^0}(\lambda)$ . Four of them are explicitly  $\lambda_1 = -\mu$ ,  $\lambda_2 = -m_1$ , and  $\lambda_3 = -m_5$ ,  $\lambda_4 = -m_4$ ; the last two eigenvalues, labeled  $\lambda_5, \lambda_6$ , are the roots of the following polynomial

$$p_1(\lambda) = \lambda^2 + (m_2 + m_3)\lambda + m_2 m_3 - \frac{\phi \alpha (1 - \eta) \beta}{\eta + \mu}. \quad (12)$$

It is obvious that  $\lambda_i < 0$ , for  $i = 1, 2, 3, 4$ . Using Routh-Hurwitz criterion [17], the real part of  $\lambda_5, \lambda_6$  is negative if  $m_2 + m_3 > 0$  and  $m_2 m_3 - \frac{\phi \alpha (1 - \eta) \beta}{\eta + \mu} > 0$ . It can be readily shown that  $m_2 m_3 - \frac{\phi \alpha (1 - \eta) \beta}{\eta + \mu} > 0$  if and only if  $\mathfrak{R}_0 < 1$ , where

$$\mathfrak{R}_0 = \frac{\phi \alpha (1 - \eta) \beta}{(\eta + \mu)(\phi + \mu)(\delta + \tau + \mu)}. \quad (13)$$

By applying the next-generation matrix approach [18, 19, 20], it is evident that the quantity  $\mathfrak{R}_0$  defined in equation (13) represents the basic reproduction number of model (1). This observation gives rise to the following result.

*Theorem 2.1*

The corruption-free fixed point  $\mathbb{E}^0$  is asymptotically stable if and only if  $\mathfrak{R}_0 < 1$ .

We now turn to the analysis of the corruption-endemic fixed point. This fixed point corresponds to a regime in which corrupt behavior persists within the population and is mathematically characterized by  $\mathcal{C} > 0$ . Under this assumption and using (9), the corruption-endemic fixed point is given by

$$\mathcal{S}^* = \frac{(\phi + \mu)(\delta + \tau + \mu)\mathcal{N}}{\phi\alpha(1 - \eta)}, \tag{14}$$

$$\mathcal{E}^* = \frac{\delta + \tau + \mu}{\phi}\mathcal{C}^*, \tag{15}$$

$$\mathcal{J}^* = \frac{\delta}{\rho + \mu}\mathcal{C}^*, \tag{16}$$

$$\mathcal{R}^* = \frac{\left(\frac{\rho\delta}{\rho + \mu} + \tau\right)}{\omega + \theta + \mu}\mathcal{C}^*, \tag{17}$$

$$\mathcal{H}^* = \frac{\theta\mathcal{R}^* + \eta\mathcal{S}^*}{\mu}. \tag{18}$$

In line with epidemiological theory,  $\mathcal{C}^* > 0$  holds if and only if  $\mathfrak{R}_0 > 1$ . In order to prove this assertion, we observe the following. It should be noted that the basic reproduction number (13) can be given by

$$\mathfrak{R}_0 = \frac{\alpha(1 - \eta)\phi}{(\phi + \mu)(\delta + \tau + \mu)} \frac{\mathcal{S}_0}{\mathcal{N}},$$

due to

$$\mathcal{S}_0 = \frac{\beta\mathcal{N}}{\eta + \mu}. \tag{19}$$

From equation (14), we can rewrite  $\mathcal{S}^*$  as

$$\mathcal{S}^* = \frac{\mathcal{S}_0}{\mathfrak{R}_0} \tag{20}$$

Under steady state condition  $\dot{\mathcal{S}} = 0$ , and neglect the contribution of  $\mathcal{R}^*$  temporarily (since it depends linearly on  $\mathcal{C}^*$  and thus will eventually appear as a positive constant), we have

$$\beta\mathcal{N} = \alpha(1 - \eta) \frac{\mathcal{S}^*\mathcal{C}^*}{\mathcal{N}} + (\eta + \mu)\mathcal{S}^*. \tag{21}$$

Substituting (19) and (20) into (21), we have

$$\begin{aligned} (\eta + \mu)\mathcal{S}_0 &= \alpha(1 - \eta) \frac{\mathcal{C}^*}{\mathcal{N}} \frac{\mathcal{S}_0}{\mathfrak{R}_0} + (\eta + \mu) \frac{\mathcal{S}_0}{\mathfrak{R}_0} \\ (\eta + \mu)\mathcal{S}_0 \left(1 - \frac{1}{\mathfrak{R}_0}\right) &= \alpha(1 - \eta) \frac{\mathcal{C}^*}{\mathcal{N}} \frac{\mathcal{S}_0}{\mathfrak{R}_0} \\ (\eta + \mu)\mathcal{S}_0 \left(\frac{\mathfrak{R}_0 - 1}{\mathfrak{R}_0}\right) &= \alpha(1 - \eta) \frac{\mathcal{C}^*}{\mathcal{N}} \frac{\mathcal{S}_0}{\mathfrak{R}_0} \\ (\eta + \mu)(\mathfrak{R}_0 - 1) &= \alpha(1 - \eta) \frac{\mathcal{C}^*}{\mathcal{N}} \\ \mathcal{C}^* &= \frac{(\eta + \mu)\mathcal{N}}{\alpha(1 - \eta)} (\mathfrak{R}_0 - 1). \end{aligned} \tag{22}$$

From (22), we have  $C^* > 0$  if and only if  $\mathfrak{R}_0 > 1$ .

Hartman's Theorem implies that the corruption-endemic fixed point  $\mathbb{E}^*$  exhibits asymptotic stability provided that the real parts of all eigenvalues of the Jacobian matrix at  $\mathbb{E}^*$  (denoted by  $\mathfrak{J}_{\mathbb{E}^*}$ ) are negative. After a routine computation, we obtain the Jacobian matrix  $\mathfrak{J}_{\mathbb{E}^*}$  as follows:

$$\mathfrak{J}_{\mathbb{E}^*} = \begin{bmatrix} -m_6 & 0 & -\frac{m_2 m_3}{\phi} & 0 & \omega & 0 \\ \alpha(1-\eta)\frac{C^*}{\mathcal{N}} & -m_2 & \frac{m_2 m_3}{\phi} & 0 & 0 & 0 \\ 0 & \phi & -m_3 & 0 & 0 & 0 \\ 0 & 0 & \delta & -m_4 & 0 & 0 \\ 0 & 0 & \tau & \rho & -m_5 & 0 \\ \eta & 0 & 0 & 0 & \theta & -\mu \end{bmatrix}, \quad (23)$$

where

$$m_6 = \alpha(1-\eta)\frac{C^*}{\mathcal{N}} + \eta + \mu. \quad (24)$$

The characteristic polynomial of  $\mathfrak{J}_{\mathbb{E}^*}$  is given by

$$p_{\mathbb{E}^*}(x) = (-\mu - x)(-m_4 - x)(-m_5 - x)(x^3 + a_1 x^2 + a_2 x + a_3), \quad (25)$$

where

$$\begin{aligned} a_1 &= m_1 + m_2 + m_3, \\ a_2 &= m_1 m_2 + m_2 m_3 + m_3 m_4, \\ a_3 &= m_1 m_2 m_3 (\mathfrak{R}_0 - 1) \end{aligned} \quad (26)$$

The eigenvalues of the Jacobian matrix  $\mathfrak{J}_{\mathbb{E}^*}$  are given by the roots of the associated characteristic polynomial  $p_{\mathbb{E}^*}(x)$ . Specifically, three eigenvalues can be explicitly identified as  $x_1 = -\mu$ ,  $x_2 = -(\rho + \mu)$ ,  $x_3 = -(\omega + \theta + \mu)$ , all of which are strictly negative. The remaining three eigenvalues, denoted by  $x_i$ ,  $i = 4, 5, 6$ , are obtained as the roots of the cubic polynomial

$$p_2(x) = x^3 + a_1 x^2 + a_2 x + a_3. \quad (27)$$

By applying the Routh–Hurwitz criterion, the real parts of the remaining eigenvalues are negative if

- (c1).  $a_1 > 0$ ,
- (c2).  $a_1 a_2 - a_3 > 0$ ,
- (c3).  $a_3(a_1 a_2 - a_3) > 0$ .

From equation (26), it is straightforward to verify that conditions (c1) and (c2) are satisfied. Furthermore, it can be demonstrated that condition (c2) ensures  $a_3 > 0$ , which in turn implies that the basic reproduction number  $\mathcal{R}_0$  exceeds unity. Therefore, the subsequent result is rigorously established.

### Theorem 2.2

The corruption-endemic fixed point  $\mathbb{E}^*$  is asymptotically stable if and only if  $\mathfrak{R}_0 > 1$ .

### 2.3. Numerical Simulation

In this section, numerical simulations are carried out in MATLAB using the classical fourth-order Runge–Kutta method to examine the time evolution of each subpopulation for different values of the parameter  $\eta$ . All parameter values and initial conditions are specified a priori and are based on assumed values. The initial conditions are given by  $\mathcal{S}(0) = 500$ ,  $\mathcal{E}(0) = 200$ ,  $\mathcal{C}(0) = 200$ ,  $\mathcal{J}(0) = 100$ ,  $\mathcal{R}(0) = 0$ , and  $\mathcal{H}(0) = 0$ . The parameter values associated with model (1) are listed in Table 2. By straightforward computations based on the parameter values reported in Table 2, the corruption-free fixed points are obtained as follows:

| Parameter | $\beta$ | $\alpha$ | $\omega$ | $\delta$ | $\tau$ | $\rho$ | $\mu$ | $\phi$ | $\theta$ |
|-----------|---------|----------|----------|----------|--------|--------|-------|--------|----------|
| Value     | 0.06    | 0.7      | 0.03     | 0.1      | 0.25   | 0.8    | 0.06  | 0.8    | 0.1      |

Table 2. The parameter values of the model

- (i).  $\mathbb{E}^0 = (375, 0, 0, 0, 0, 625)$  with  $\mathfrak{R}_0 = 0.5360$  for  $\eta = 0.1$ ,
- (ii).  $\mathbb{E}^0 = (230.769, 0, 0, 0, 0, 769.231)$  with  $\mathfrak{R}_0 = 0.2932$  for  $\eta = 0.2$ , and
- (iii).  $\mathbb{E}^0 = (166.667, 0, 0, 0, 0, 833.333)$  with  $\mathfrak{R}_0 = 0.1853$  for  $\eta = 0.3$ .

Figure 2 illustrates the temporal dynamics of the susceptible, exposed, corrupt, imprisoned, reformed, and honest subpopulations for different values of the anti-corruption education parameter  $\eta$  when  $\mathfrak{R}_0 < 1$ . The numerical results depicted in Figure 2 demonstrate that increasing the value of the anti-corruption education parameter  $\eta$  significantly alters the long-term behavior of the system. Higher levels of  $\eta$  accelerate the decline of the exposed and corrupt subpopulations, while simultaneously increasing the proportion of honest individuals. This behavior is consistent with the condition  $\mathfrak{R}_0 < 1$ , under which corruption fails to sustain itself and eventually dies out. From a policy perspective, these findings suggest that sustained investment in anti-corruption education plays a crucial role in suppressing corruption dynamics by reducing both the likelihood of transition from susceptibility to corruption and the persistence of corrupt behavior.

Furthermore, by varying the parameter  $\eta$ , the corresponding corruption-endemic fixed points are obtained as follows:

- (i).  $\mathbb{E}^* = (629.651, 29.4597, 57.4819, 6.6839, 103.773, 172.951)$  with  $\mathfrak{R}_0 = 1.5882$ , for  $\eta = 0$ ,
- (ii).  $\mathbb{E}^* = (635.986, 20.5222, 40.0441, 4.6564, 72.2988, 226.493)$  with  $\mathfrak{R}_0 = 1.3477$  for  $\eta = 0.01$ , and
- (iii).  $\mathbb{E}^* = (645.548, 2.4836, 4.8700, 0.5671, 8.8642, 337.68)$  with  $\mathfrak{R}_0 = 1.0270$  for  $\eta = 0.03$ .

Figure 3 depicts the temporal evolution of the susceptible, exposed, corrupt, imprisoned, recovered, and honest subpopulations for several values of the anti-corruption education parameter  $\eta$  under the condition  $\mathfrak{R}_0 > 1$ .

The results presented in Figure 3 show that, under the condition  $\mathfrak{R}_0 > 1$ , corruption persists in the long term despite variations in the anti-corruption education parameter  $\eta$ . Increasing  $\eta$  reduces the size of the exposed and corrupt subpopulations and enhances the proportion of honest individuals, indicating that anti-corruption education effectively mitigates the intensity of corruption dynamics. However, since the basic reproduction number remains above unity, these measures alone are insufficient to eradicate corruption, and the system ultimately converges to a corruption-endemic fixed point. This highlights the necessity of complementing educational interventions with enforcement mechanisms and institutional reforms to reduce  $\mathfrak{R}_0$  below the critical threshold and achieve sustainable corruption control.

### 3. Conclusion

In this study, a compartmental dynamical model is proposed to describe the spread of corruption while accounting for the influence of anti-corruption education. Corruption is modeled as a social contagion process, through which interactions among the susceptible, exposed, corrupt, imprisoned, reformed, and honest subpopulations are characterized. Analytical results demonstrate that the local stability of both the corruption-free and corruption-endemic fixed point is determined by the basic reproduction number  $\mathfrak{R}_0$ , with corruption dying out when  $\mathfrak{R}_0 < 1$  and persisting when  $\mathfrak{R}_0 > 1$ . Numerical simulations indicate that increasing the level of anti-corruption education effectively reduces corruption prevalence and promotes honest behavior; however, education alone is insufficient to eradicate corruption when  $\mathfrak{R}_0$  remains above unity. These findings underscore the necessity of combining sustained anti-corruption education with enforcement mechanisms and institutional reforms to achieve long-term corruption control.

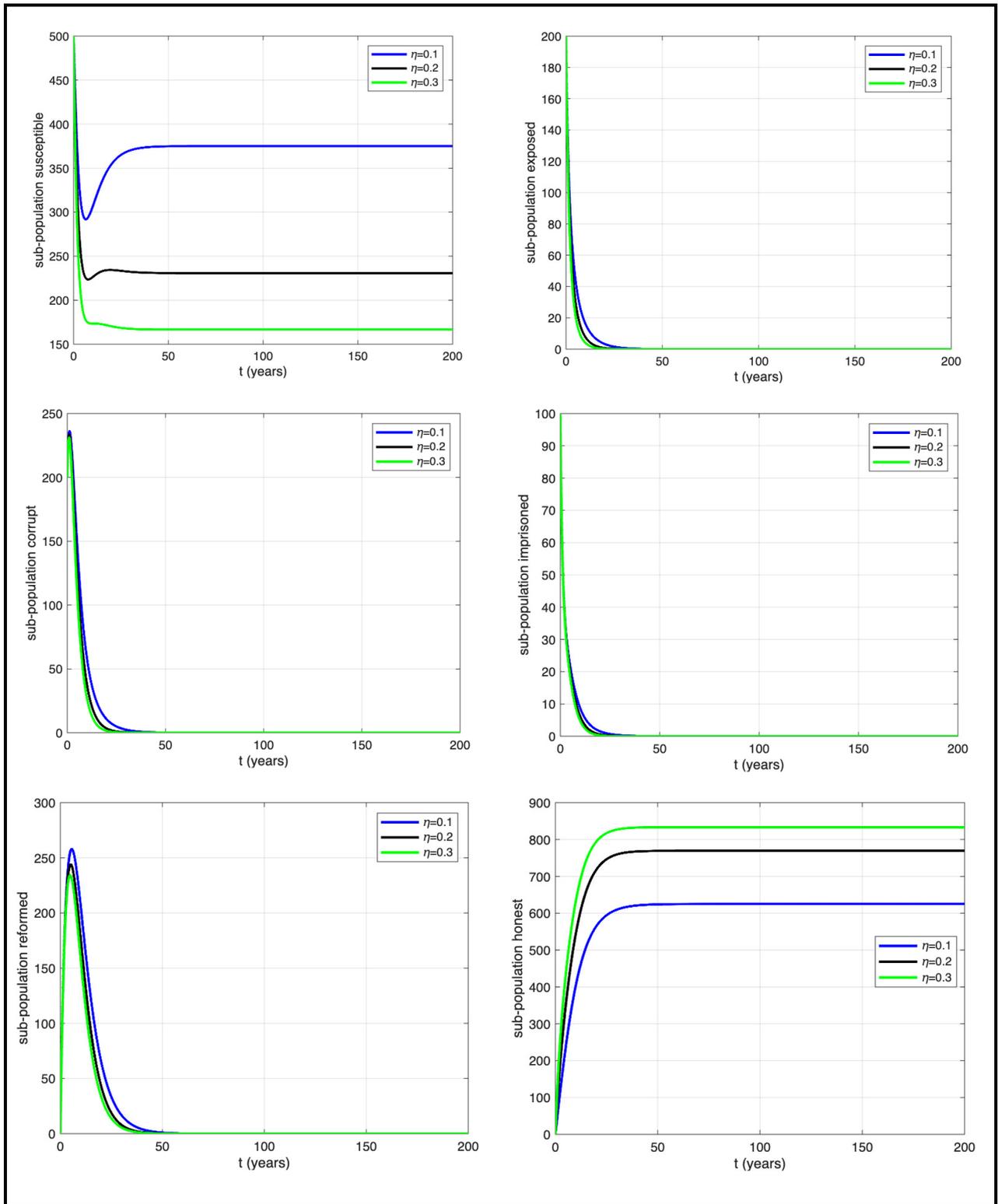


Figure 2. Curves of each subpopulation for  $\mathfrak{R}_0 < 1$  of several values  $\eta$

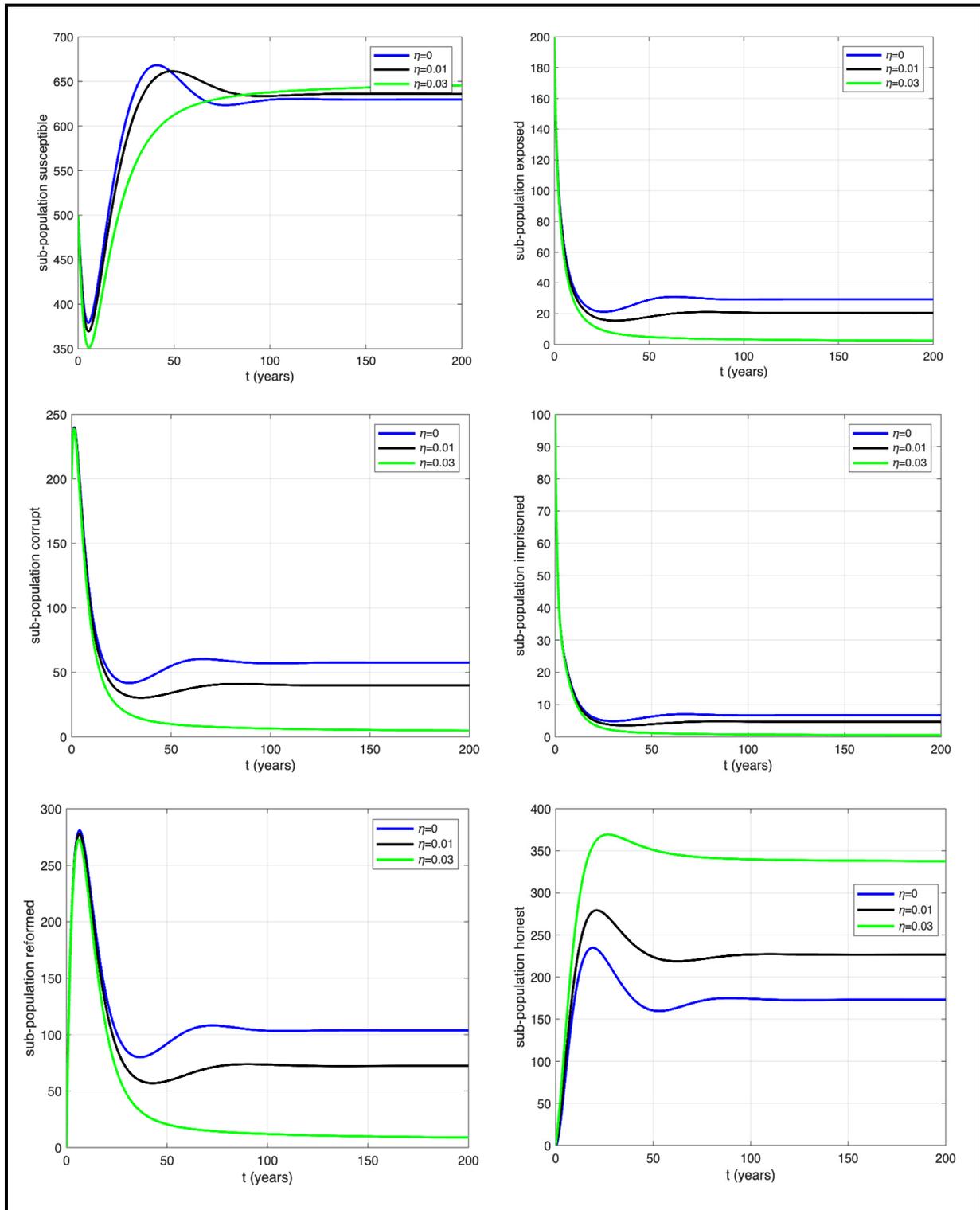


Figure 3. Curves of each subpopulation for  $\mathfrak{R}_0 > 1$  of several values  $\eta$

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