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A Parametric Exponential Entropy Measure for Neutrosophic Sets and It's Application in Decision Making

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Abstract In this paper, we propose a novel exponential entropy measure for Single-Valued Neutrosophic Sets. Neutrosophic sets as an extension of fuzzy and intuitionistic fuzzy sets, designed to handle uncertain, indeterminate, and inconsistent information in a more refined manner. The proposed entropy measure captures the degree of uncertainty inherent in single valued neutrosophic sets by simultaneously considering the truth-membership, indeterminacy-membership, and falsity-membership degrees. We establish the essential mathematical properties of the proposed entropy measure, including validation. Furthermore, illustrative examples and potential applications in decision-making is presented to validate the practical utility of the proposed measure.

Keywords Neutrosophic sets, Exponential Entropy, Generalized Exponential fuzzy Entropy, Entropy, Fuzzy Sets, Fuzzy Entropy , Shannon Entropy, Renyi entropy, decision making.

AMS 2010 subject classifications 03E72, 62C86, 94A17

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1. Introduction

Uncertainty is a fundamental aspect of many real world phenomena and it plays a very imprortant role in mathematical modelling of many real life problems. Classical set theory along with several extensions of fuzzy sets , has provided effective means to represent and handle several types of uncertainty. Originally, entropy is conceptualized within the framework of thermodynamics, as a measure of uncertainty. In information theory, it has been used as a measure of uncertainty associated with probability distribution. Initially Shannon [18], defined this concept entropy, later on a number of generalizations of entropy measure have been developed and studied for real life applications. Renyi [13] , Havrda and Charvat[33], Kapur [**?**] defined one parametric generalization of Shannon's entropy. Sharma and Mittal [17] proposed two parametric measure of entropy. Simultaneously, the notion of fuzzy sets was coined by Zadeh[36] , which deals with the models having non-statistical or vague phenomena. Fuzzy set theory becomes a dynamic area of research in different fields such as image processing, medical science and engineering. An entropy measure of a fuzzy sets with axioms of valid measure is given by De Luca and Termini[19]. Bhandari and Pal [16] introduced parametric fuzzy entropy corresponding to Renyi entropy. Verma and Sharma [43], Mishra and Hooda [14] introduced a parametric exponential fuzzy entropy. In classical fuzzy set theory uncertainty is associated with the degree of membership only, but in case of real life applications, there is a need to incorporate both the membership and non-membership degree of an element for

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dealing with the lack of knowledge or insufficient data. By considering this requirement Atanassov [34], defined an extension of fuzzy sets known as intuitionistic fuzzy sets, measures not only the fuzziness due to partial membership but also captures the uncertainty arising from hesitation and incomplete information. Burillo and Bustince [15], proposed a measure of intuitionistic fuzzy sets and interval valued fuzzy sets. Schmidt and Kacprzyk [21] extend the axioms of fuzzy entropy to intuitionistic fuzzy sets. Verma and Sharma [42],[43] , Joshi and Kumar [20], Vlachos and Sergidddis [25], Thao and Duang [6], have generalized the entropy in intuitionistic fuzzy sets. In real life decision making problem, intuitionistic fuzzy sets deal with only incomplete and vague information, but it cannot deal with indeterminate or inconsistent information. Due to this limitation, Smarandache [22], proposed an idea of neutrosophic set, effectively deal with imprecise, indeterminate and inconsistent information. In Neutrosophic set, each element is associated with truth membership function (T), indeterminacy membership function (I), falsity membership function (F), each of which can independently take values in real standard interval $[0, 1]$ or non-standard interval $]^{-}0, 1^{+}[$. For real applications of neutrosophic set, some researchers have defined these three membership components in standard interval $[0, 1]$ which is a subclass of neutrosophic set. Wang and Smarandache [24], defined single valued neutrosophic sets and interval valued neutrosophic sets. As a generalization of intuitionistic fuzzy entropy and similarity measure, Majumdar and Samanta [37], defined some similarity measures and an entropy measure of single valued neutrosophic sets. Ye and Du [30] defined distance, similarity measures of interval valued neutrosophic sets and its applications in decision making problem. In recent years, Aydogdu[2][3][4] , Peng J. [28], Guleria A. [12] , Ye J [30] [45], Liu P. [38] , Sahin R. [4], Chal J. et.al.[26] have proposed similarity measure of neutrosophic sets and its applications in decision making, medical diagnosis, taxonomy and clustering analysis. Over the past decade, numerous researchers have proposed entropy formulations tailored to the neutrosophic environment. These approaches often extend existing entropy functions from fuzzy and intuitionistic fuzzy frameworks, adapting them to accommodate the tri-component nature of neutrosophic sets. Despite this progress, significant challenges persist particularly in ensuring that the defined entropy measures account equitably for all three components (truth, indeterminacy, and falsity), while simultaneously satisfying key mathematical properties such as normalization, monotonicity (reflecting increased uncertainty) and symmetry.

Decision making means choosing the best alternative from a finite set of alternatives according to multiple criteria. It is an important branch which relates to the human activities. In multi-attribute decision-making (MADM) problems, the mathematical representation of preference information and the efficient processing of uncertain data remain challenging, particularly in complex decision environments. Within the framework of single-valued neutrosophic sets (SVNSs), several classical MADM techniques, including TODIM, TOPSIS, and VIKOR, have been extended and applied by various researchers, such as Liu [8] and Wang et.al. [7].

To facilitate neutrosophic MADM, a variety of mathematical tools have been proposed, including distance measures, similarity measures, score and accuracy functions, correlation coefficients, entropy measures, and aggregation operators [9]. These constructs collectively establish a formal decision-making framework capable of handling uncertainty, indeterminacy, and inconsistency.

However, the determination of criterion weights, which significantly affects the ranking of alternatives, remains an open and nontrivial issue. In the absence of reliable a priori information, subjective weighting approaches may lead to biased or unstable results. Consequently, entropy-based weighting methods have attracted increasing attention, as they derive objective criterion weights from the intrinsic information dispersion of neutrosophic data, thereby enhancing the robustness and discrimination capability of SVNS-based MADM models.

In this paper we propose a novel exponential entropy function for neutrosophic sets of order α that better captures the uncertainty inherent in the triple-valued framework. This paper is organized as follows: In section 2, some basic definitions , operations on single valued neutrosophic sets are discussed. In section 3, proposed a new exponential entropy of order α and validated through theoretical proofs and numerical examples. In section 4, discussed a comparative study with some existing measures with numerical example. In section 5, an applications in area of decision-making using TOPSIS method with numerical illustration. Lastly the conclusion and future work of this study is given in section 6.

2. Prelimanaries:

Let us begin with the introduction of the concept of neutrosophic set, which is a generalization of fuzzy sets and intutionistic fuzzy sets. Single-valued neutrosophic sets and some set theoretic operations as they are important for the rest of paper are given in this section.

Definition 2.1: A neutrosophic set A in universal set X is characterized by three functions namely truth-membership function $T_A(x) : X \to]^-0, 1^+[$, false-membership function $F_A(x) : X \to]^-0, 1^+[$ and indeterminacy-membership function $I_A(x) : X \to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x), F_A(x), I_A(x)$ such that $^-0 \leq SupT_A(x) + SupF_A(x) + SupI_A(x) \leq 3^+$

Definition 2.2: Let X be the universal set and A be the single valued neutrosophic set defined as $S = \{(x, T_A(x), I_A(x), F_A(x)), x \in X\}$, where $T_A : X \to [0, 1]$, $F_A : X \to [0, 1]$ and $I_A : X \to [0, 1]$ and the sum of truth-membership, falsity-membership and indeterminacy-membership lies in the interval $[0, 3]$.

Definition 2.3: Let $A = \{x, T_A(x), I_A(x), F_A(x)\}$ and $B = \{x, T_B(x), I_B(x), F_B(x)\}$ be any two single valued neutrosophic sets defined on universal set X then, the set theoretic operations on these two sets are defined as:

a) **Subset** : $A \subseteq B$ if and only if $T_A(x) \leq T_B(x)$, $F_A(x) \geq F_B(x)$ and $I_A(x) \geq I_B(x)$, for all $x \in X$.

b) **Union of two sets**: Union of two single valued neutrosophic sets is defined as
$A \cup B = \{(max\{T_A(x), T_B(x)\}, min\{I_A(x), I_B(x)\}, min\{F_A(x), F_B(x)\})\}$

c) **Intersection of two sets**: Intersection of two single valued neutrosophic sets is defined as $A \cap B = \{(min\{T_A(x), T_B(x)\}, max\{I_A(x), I_B(x)\}, max\{F_A(x), F_B(x)\})\}$

d) **Complement of a set**: The complement of set A is denoted by A^c , defined as $A^c = \{(x, T^c_A(x), F^c_A(x), I^c_A(x))\}$ where $T^c_A(x) = F_A(x), F^c_A(x) = T_A(x), I^c_A(x) = 1 - I_A(x)$, for all $x \in X$.

Definition 2.4: For any two single valued neutrosophic sets A and B , the addition and multiplication operation is defined as
$A \bigoplus B = \{(x, T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x))\}$
$A \bigotimes B = \{(x, T_A(x)T_B(x), I_A(x) + I_B(x) - I_A(x)I_B(x), F_A(x) + F_B(x) - F_A(x)F_B(x))\}$

Definition 2.5: For any two single valued neutrosophic sets A and B , the subtraction and division [29] operation is defined as
$A - B = \{(x, \frac{T_A(x) - T_B(x)}{1 - T_B(x)}, \frac{I_A(x)}{I_B(x)}, \frac{F_A(x)}{F_B(x)}), \forall x \in X\}$
$A \backslash B = \{(x, \frac{T_A(x)}{T_B(x)}, \frac{I_A(x) - I_B(x)}{1 - I_B(x)}, \frac{F_A(x) - F_B(x)}{1 - F_B}), \forall x \in X\}$.

3. New exponential entropy measure of order α in neutrosophic set:

In this section, we introduce a new parametric measure of exponential entropy in single valued neutrosophic set based on Verma and Sharma [43] generalized exponetial fuzzy entropy.

Definition 3.1: The entropy of order α for single valued neutrosophic set A is defined as

$$\begin{aligned} E_\alpha(A) = \frac{1}{3n(e^{(1-0.5^\alpha)} - 1)} \sum_{i=1}^{n} & [T_A(x_i)e^{(1-T_A^\alpha(x_i))} + (1 - T_A(x_i))e^{(1-(1-T_A(x_i))^\alpha)} - 1] + \\ & [F_A(x_i)e^{(1-F_A^\alpha(x_i))} + (1 - F_A(x_i))e^{(1-(1-F_A(x_i))^\alpha)} - 1] + \\ & [I_A(x_i)e^{(1-I_A^\alpha(x_i))} + (1 - I_A(x_i))e^{(1-(1-I_A(x_i))^\alpha)} - 1], \alpha > 0. \end{aligned} \quad (1)$$

Majumdar and Samanta [37] proposed an entropy measure for single-valued neutrosophic sets and established a collection of axioms to ensure the validity of entropy measures in the neutrosophic environment. The axioms are presented below:

Definition 3.2: Let $N(X)$ be all single valued nutrosophic set defined on X and $A \in N(X)$. An entropy on single valued neutrosophic set is a function $E : N(X) \to [0, 1]$ which satisfies the following axioms:

a) $E_N(A) = 0$ if and only if A is a crisp set, for all $x \in X$

b) $E_N(A) = 1$ if and only if $T_A(x) = 0.5, F_A(x) = 0.5, I_A(x) = 0.5$, for all $x \in X$.

c) If $A \subseteq B$ then $E_N(A) \geq E_N(B)$.

d) $E_N(A) = E_N(A^c)$, for all $A \in N(X)$.
Theorem 3.1: The generalized parametric entropy measure on single valued neutrosophic entropy

$$E_\alpha(A) = \frac{1}{3n(e^{(1-0.5^\alpha)}-1)} \sum_{i=1}^{n} [T_A(x_i)e^{(1-T_A^\alpha(x_i))} + (1-T_A(x_i))e^{(1-(1-T_A(x_i))^\alpha)} - 1] + [F_A(x_i)e^{(1-F_A^\alpha(x_i))} + (1-F_A(x_i))e^{(1-(1-F_A(x_i))^\alpha)} - 1] + [I_A(x_i)e^{(1-I_A^\alpha(x_i))} + (1-I_A(x_i))e^{(1-(1-I_A(x_i))^\alpha)} - 1], \alpha > 0.$$

is a valid measure.
Proof: In order to prove that $E_\alpha(A)$ is a valid entropy measure for single valued neutrosophic set, we need to prove all axioms defined in the definition 3.2.
a) A is a crisp set means, $T_A(x_i) = 1, F_A(x_i) = 0, I_A(x_i) = 0$ or $T_A(x_i) = 0, F_A(x_i) = 1, I_A(x_i) = 0$, for all $x_i \in X$. Substituting $T_A(x_i) = 1, F_A(x_i) = 0, I_A(x_i) = 0$ in definition 3.1, we get $E_\alpha(A) = 0$. Similarlly, on substitution of $T_A(x_i) = 0, F_A(x_i) = 1, I_A(x_i) = 0$, gives $E_\alpha(A) = 0$.
Conversely, if $E_\alpha(A) = 0$, Substituting in above equation we get , $T_A(x_i) = 1, F_A(x_i) = 0$,
$I_A(x_i) = 0$ or $T_A(x_i) = 0, F_A(x_i) = 1, I_A(x_i) = 0$, for all $x_i \in X$.
This means that A as a crisp set. (Refer Table 1)
b) Let $T_A(x_i) = 0.5, F_A(x_i) = 0.5, I_A(x_i) = 0.5$ then,

$$E_\alpha(A) = \frac{1}{3n(e^{(1-0.5^\alpha)}-1)} \sum_{i=1}^{n} [(0.5)e^{(1-0.5^\alpha)} + (1-0.5)e^{(1-(1-0.5))^\alpha)} - 1] + [(0.5)e^{(1-0.5^\alpha)} + (1-0.5)e^{(1-(1-0.5))^\alpha)} - 1] + [(0.5)e^{(1-0.5^\alpha)} + (1-0.5)e^{(1-(1-0.5))^\alpha)} - 1]$$

$\implies E_\alpha(A) = 1$.
Now Let $E_\alpha(A) = 1$

$$\implies \sum_{i=1}^{n} [T_A(x_i)e^{(1-T_A^\alpha(x_i))} + (1-T_A(x_i))e^{(1-(1-T_A(x_i))^\alpha)} - 1] + [F_A(x_i)e^{(1-F_A^\alpha(x_i))} + (1-F_A(x_i)) e^{(1-(1-F_A(x_i))^\alpha)} - 1] + [I_A(x_i)e^{(1-I_A^\alpha(x_i))} + (1-I_A(x_i))e^{(1-(1-I_A(x_i))^\alpha)} - 1] = 3n(e^{(1-0.5^\alpha)} - 1)$$

$$\implies [T_A(x_i)e^{(1-T_A^\alpha(x_i))} + (1-T_A(x_i))e^{(1-(1-T_A(x_i))^\alpha)} + F_A(x_i)e^{(1-F_A^\alpha(x_i))} + (1-F_A(x_i))e^{(1-(1-F_A(x_i))^\alpha)} + I_A(x_i)e^{(1-I_A^\alpha(x_i))} + (1-I_A(x_i))e^{(1-(1-I_A(x_i))^\alpha)} - 3 = 3e^{(1-0.5^\alpha)} - 3$$

$$\implies [T_A(x_i)e^{(1-T_A^\alpha(x_i))} + (1-T_A(x_i))e^{(1-(1-T_A(x_i))^\alpha)} + F_A(x_i)e^{(1-F_A^\alpha(x_i))} + (1-F_A(x_i))e^{(1-(1-F_A(x_i))^\alpha)} + I_A(x_i)e^{(1-I_A^\alpha(x_i))} + (1-I_A(x_i))e^{(1-(1-I_A(x_i))^\alpha)} = 3e^{(1-0.5^\alpha)}.$$

It is possible if and only if $T_A(x_i) = 0.5, F_A(x_i) = 0.5, I_A(x_i) = 0.5$.
Now let us consider the following function

$$f(z(x_i)) = \frac{1}{3(e^{(1-0.5^\alpha)}-1)} [z(x_i)e^{(1-z^\alpha(x_i))} + (1-z(x_i))e^{(1-(1-z(x_i))^\alpha)} - 1]$$

Now differentiate $f(z(x_i))$ with respect to $z(x_i)$, we get,

$$\frac{\partial f}{\partial z} = \frac{1}{3(e^{(1-0.5^\alpha)}-1)} [(1-\alpha z^\alpha)e^{(1-z^\alpha)} - (1-\alpha(1-z)^\alpha)e^{1-(1-z)^\alpha}]$$

equating to zero we get,

$$(1-\alpha(1-z)^\alpha)e^{1-(1-z)^\alpha} = (1-\alpha z^\alpha)e^{1-z^\alpha}$$

$Components$	$E_\alpha A(x)$			
$T_A(x) = F_A(x) = I_A(x)$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 1.2$	$\alpha = 2$
0.0	0	0	0	0
0.1	0.4602	0.4276	0.3540	0.3200
0.2	0.7167	0.6956	0.6353	0.5989
0.3	0.8790	0.8692	0.8376	0.8156
0.4	0.9704	0.9679	0.9594	0.9529
0.5	1.0000	1.0000	1.0000	1.0000
0.6	0.9704	0.9679	0.9594	0.9529
0.7	0.8790	0.8692	0.8376	0.8156
0.8	0.7167	0.6956	0.6353	0.5989
0.9	0.4602	0.4276	0.3540	0.3200
1.0	0	0	0	0

Table 1. Values of proposed exponetial entropy measure for Single valued neutrosophic sets

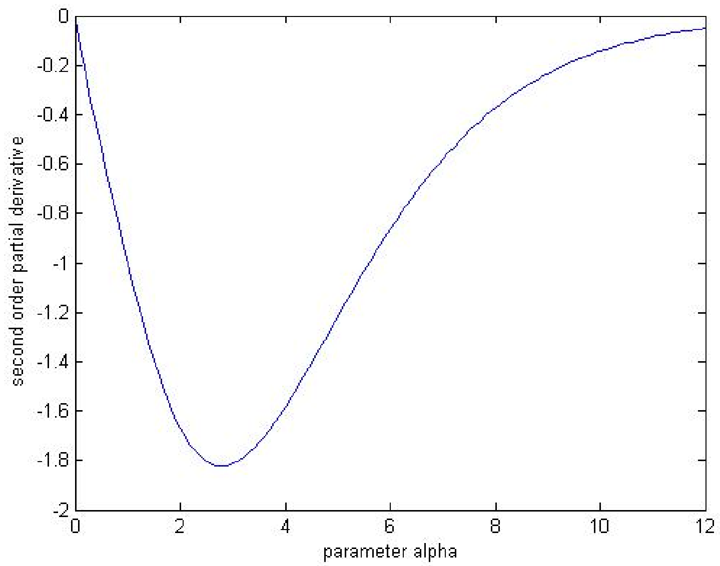

Figure 1. $\frac{\partial^2 f}{\partial z^2} < 0$ at $z = \frac{1}{2}$

This equation gets satisfied at $z = 0.5$ therefore, the critical point is at $z = 0.5$, $\forall x \in X$
Further differentiating the equation partially with respect to z ,

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{3(e^{(1-0.5^\alpha)} - 1)}[e^{1-z^\alpha}(-\alpha^2 z^{\alpha-1} - \alpha z^{\alpha-1}(1 - \alpha z^\alpha) - e^{1-(1-z)^\alpha}(\alpha^2(1-z)^{\alpha-1} + \alpha(1-z)^{\alpha-1}(1 - \alpha(1-z)^\alpha)]$$

At $z = 0.5$, the value of $\frac{\partial^2 f}{\partial z^2} < 0$, $\implies$ The function is concave function. Hence the function $f(z)$ has a global maximum at $z = 0.5$ and its value is 1. Therefore, rewriting the single valued neutrosophic entropy function as

$$Q(S) = \frac{1}{3n(e^{(1-0.5^\alpha)} - 1)} \sum_{i=1}^{n} [f(T_A(x_i)) + f(I_A(x_i)) + f(F_A(x_i))]$$

$\implies Q(S) = 1$ for $S = \{< x_i, 0.5, 0.5, 0.5 > /x_i \in X\}$ for single valued neutrosophic set. (Refer Fig :1 and 2)
c) Consider

$$\frac{\partial f}{\partial z} = [(1-\alpha z^{\alpha})e^{(1-z^{\alpha})} - (1+\alpha(1-z)^{\alpha})e^{1-(1-z)^{\alpha}}]$$

To check : For any $z \in [0,1]$, the function $f(z)$ is increasing when $z < 0.5$, while $f(z)$ is decreasing when $z > 0.5$.
Suppose that $z < 0.5 \implies 1-z > z \implies (1-z)^{\alpha} > z^{\alpha} \implies$

$$(1-\alpha(1-z)^{\alpha}) < (1-\alpha z^{\alpha}) \implies (1-\alpha z^{\alpha})e^{(1-z^{\alpha})} - (1-\alpha(1-z)^{\alpha})e^{1-(1-z)^{\alpha}} > 0$$

This means $\frac{\partial f}{\partial z} > 0$, when $z < 0.5$. Hence the function $f(z)$ is increasing. Similarlly, when $z > 0.5$, the function is decreasing. Therefore the function $f(z)$ increases as z approaches to 0.5 and decreases as z moves away from 0.5. Since the set A is defined by all membership values being 0.5, a single valued neutrosophic set is closed to A than P. Therefore the term contributed by S to the sum will be greater than or equal to terms contributed by $P \implies E_{\alpha}(P) \leq E_{\alpha}(S)$.

d) Now consider

$$E_{\alpha}(A^c) = \frac{1}{3n(e^{(1-0.5^{\alpha})}-1)} \sum_{i=1}^{n} [T_{A^c}(x_i)e^{(1-T^{\alpha}_{A^c}(x_i))} + (1-T_{A^c}(x_i))e^{(1-(1-T_{A^c}(x_i))^{\alpha})} - 1] + [F_{A^c}(x_i)e^{(1-F^{\alpha}_{A^c}(x_i))} + (1-F_{A^c}(x_i))e^{(1-(1-F_{A^c}(x_i))^{\alpha})} - 1] + [I_{A^c}(x_i)e^{(1-I^{\alpha}_{A^c}(x_i))} + (1-I_{A^c}(x_i))e^{(1-(1-I_{A^c}(x_i))^{\alpha})} - 1]$$

$= E_{\alpha}(A)$. (Using definition 2.3 of complement of set A)
All the axioms are true, hence proposed exponential entropy measure is a valid measure in single valued neutrosophic sets. Table 1 illustrates the behavior of the proposed entropy measure, showing that it attains a maximum value one and reduces to zero when applied to a crisp sets and figures 1 to 4 present the 2D and 3D graphical representations of the proposed entropy measure.
Special Case: As $\alpha \to 1$, proposed measure is reduced to exponential entropy for simplified neutrosophic sets [32]

$$E(A) = \frac{1}{3n(e^{0.5}-1)} \sum_{i=1}^{n} [T_A(x_i)e^{(1-T_A(x_i))} + (1-T_A(x_i))e^{(T_A(x_i))} - 1] + [F_A(x_i)e^{(1-F_A(x_i))} + (1-F_A(x_i))e^{(F_A(x_i))} - 1] + [I_A(x_i)e^{(1-I_A(x_i))} + (1-I_A(x_i))e^{(I_A(x_i))} - 1] \quad (2)$$

The proposed measure can be viewed as a natural extension of generalized exponential fuzzy entropy to the neutrosophic domain. When the indeterminacy and falsity components vanish or are functionally dependent on the truth-membership (as in classical fuzzy or intuitionistic fuzzy sets), the proposed entropy measure reduces to the well-known exponential fuzzy entropy form [39].

Example 3.1: Let $A = (0.6, 0.9, 0.6)$ and $B = (0.8, 0.2, 0.5)$ be the single valued neutrosophic sets and $\alpha = 1.5$ then by using definition 3.1, $E_{\alpha}(A) = 0.7494$, $E_{\alpha}(B) = 0.74444$. Using definition 2.4 and 2.5, the values of addition, subtraction, multiplication and division of two neutrosophic sets is : $A \bigoplus B$ is $(0.92, 0.18, 0.30)$, $A \bigotimes B$ is $(0.48, 0.92, 0.8)$ and $A \backslash B$ is $(0.75, 0.8750, 0.2)$. The corresponding entropy values are 0.5549 , 0.6290 and 0.5866 respectively.
Example 3.2: Let $A = (0.8, 0.2, 0.5)$ and $B = (0.6, 0.9, 0.6)$ be two single valued neutrosophic sets, the entropy of $A - B$ is 0.7334.

Theorem 3.2: For any two neutrosophic sets A and B of universe of discourse X , the following properties holds.

(i) $E_{\alpha}(A \cup B) + E_{\alpha}(A \cap B) = E_{\alpha}(A) + E_{\alpha}(B)$

(ii)$E_{\alpha}(A \cup B) = E_{\alpha}((A' \cap B')')$

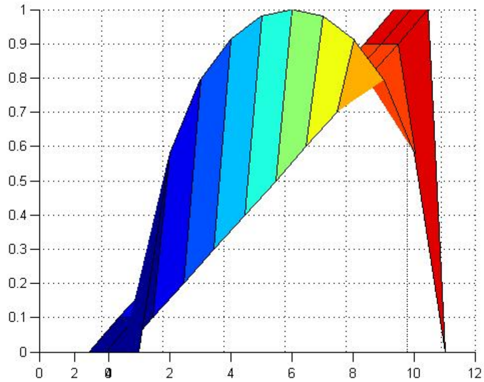

Figure 2. 3D representation of $E_\alpha(A)$

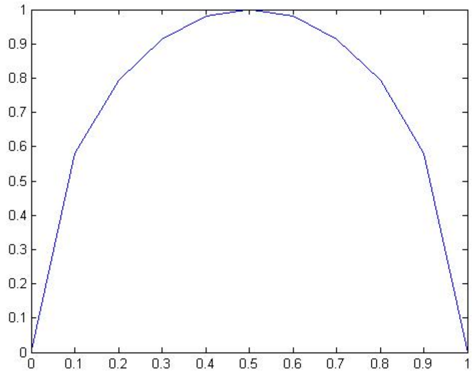

Figure 3. 2D representation of $E_\alpha(A)$

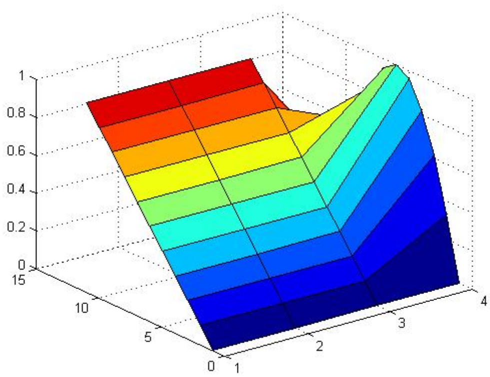

Figure 4. 3D representation of $\alpha = 1.5$

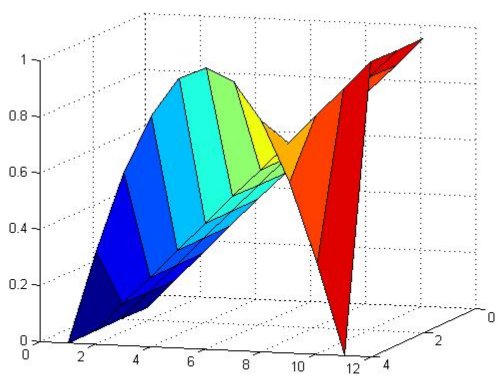

Figure 5. 3D representation of $\alpha = 1.7$

(iii) $E_\alpha(A \cap B) = E_\alpha((A' \cup B')')$

Proof: Let X be the universal set and X_1 and X_2 be the partition of set X such that $X_1 = \{A \subseteq B, x_1 \in X\}$ and $X_2 = \{B \subseteq A, x_2 \in X\}$.
$E_\alpha(A \cup B) =$

$$\frac{1}{3n(e^{(1-0.5^\alpha)}-1)} \sum_{X_1 \in X} [T_{A\cup B}(x_i) e^{(1-T^\alpha_{A\cup B}(x_i))} + (1-T_{A\cup B}(x_i)) e^{(1-(1-T_{A\cup B}(x_i))^\alpha)} - 1] + [F_{A\cup B}(x_i) e^{(1-F^\alpha_{A\cup B}(x_i))}$$
$$+(1-F_{A\cup B}(x_i)) e^{(1-(1-F_{A\cup B}(x_i))^\alpha)} - 1] + [I_{A\cup B}(x_i) e^{(1-I^\alpha_{A\cup B}(x_i))} + (1-I_{A\cup B}(x_i)) e^{(1-(1-I_{A\cup B}(x_i))^\alpha)} - 1] +$$
$$\frac{1}{3n(e^{(1-0.5^\alpha)}-1)} \sum_{X_2 \in X} [T_{A\cup B}(x_i) e^{(1-T^\alpha_{A\cup B}(x_i))} + (1-T_{A\cup B}(x_i)) e^{(1-(1-T_{A\cup B}(x_i))^\alpha)} - 1] + [F_{A\cup B}(x_i) e^{(1-F^\alpha_{A\cup B}(x_i))}$$
$$+(1-F_{A\cup B}(x_i)) e^{(1-(1-F_{A\cup B}(x_i))^\alpha)} - 1] + [I_{A\cup B}(x_i) e^{(1-I^\alpha_{A\cup B}(x_i))} + (1-I_{A\cup B}(x_i)) e^{(1-(1-I_{A\cup B}(x_i))^\alpha)} - 1]$$
$$\implies \frac{1}{3n(e^{(1-0.5^\alpha)}-1)} \sum_{X_1 \in X} [T_B(x_i) e^{(1-T^\alpha_B(x_i))} + (1-T_B(x_i)) e^{(1-(1-T_B(x_i))^\alpha)} - 1] + [F_B(x_i) e^{(1-F^\alpha_B(x_i))}$$
$$+(1-F_B(x_i)) e^{(1-(1-F_B(x_i))^\alpha)} - 1] + [I_B(x_i) e^{(1-I^\alpha_B(x_i))} + (1-I_B(x_i)) e^{(1-(1-I_B(x_i))^\alpha)} - 1] +$$

$$\frac{1}{3n(e^{(1-0.5^{\alpha})}-1)}\sum_{X_2\in X}[T_A(x_i)e^{(1-T_A^{\alpha}(x_i))}+(1-T_A(x_i))e^{(1-(1-T_A(x_i))^{\alpha})}-1]+[F_A(x_i)e^{(1-F_A^{\alpha}(x_i))} \\ +(1-F_A(x_i))e^{(1-(1-F_A(x_i))^{\alpha})}-1]+[I_A(x_i)e^{(1-I_A^{\alpha}(x_i))}+(1-I_A(x_i))e^{(1-(1-I_A(x_i))^{\alpha})}-1]. \tag{3}$$

$E_\alpha(A\cap B)$=

$$\frac{1}{3n(e^{(1-0.5^{\alpha})}-1)}\sum_{X_1\in X}[T_{A\cap B}(x_i)e^{(1-T_{A\cap B}^{\alpha}(x_i))}+(1-T_{A\cup B}(x_i))e^{(1-(1-T_{A\cup B}(x_i))^{\alpha})}-1]+[F_{A\cup B}(x_i)e^{(1-F_{A\cup B}^{\alpha}(x_i))} \\ +(1-F_{A\cap B}(x_i))e^{(1-(1-F_{A\cap B}(x_i))^{\alpha})}-1]+[I_{A\cap B}(x_i)e^{(1-I_{A\cap B}^{\alpha}(x_i))}+(1-I_{A\cap B}(x_i))e^{(1-(1-I_{A\cap B}(x_i))^{\alpha})}-1]+$$

$$\frac{1}{3n(e^{(1-0.5^{\alpha})}-1}\sum_{X_2\in X}[T_{A\cap B}(x_i)e^{(1-T_{A\cap B}^{\alpha}(x_i))}+(1-T_{A\cap B}(x_i))e^{(1-(1-T_{A\cap B}(x_i))^{\alpha})}-1]+[F_{A\cap B}(x_i)e^{(1-F_{A\cap B}^{\alpha}(x_i))} \\ +(1-F_{A\cap B}(x_i))e^{(1-(1-F_{A\cap B}(x_i))^{\alpha})}-1]+[I_{A\cap B}(x_i)e^{(1-I_{A\cap B}^{\alpha}(x_i))}+(1-I_{A\cap B}(x_i))e^{(1-(1-I_{A\cap B}(x_i))^{\alpha})}-1]$$

$$\implies \frac{1}{3n(e^{(1-0.5^{\alpha})}-1)}\sum_{X_1\in X}[T_A(x_i)e^{(1-T_A^{\alpha}(x_i))}+(1-T_A(x_i))e^{(1-(1-T_A(x_i))^{\alpha})}-1]+[F_A(x_i)e^{(1-F_A^{\alpha}(x_i))} \\ +(1-F_A(x_i))e^{(1-(1-F_A(x_i))^{\alpha})}-1]+[I_A(x_i)e^{(1-I_A^{\alpha}(x_i))}+(1-I_A(x_i))e^{(1-(1-I_A(x_i))^{\alpha})}-1]+$$

$$\frac{1}{3n(e^{(1-0.5^{\alpha})}-1)}\sum_{X_2\in X}[T_B(x_i)e^{(1-T_B^{\alpha}(x_i))}+(1-T_B(x_i))e^{(1-(1-T_B(x_i))^{\alpha})}-1]+[F_B(x_i)e^{(1-F_B^{\alpha}(x_i))} \\ +(1-F_B(x_i))e^{(1-(1-F_B(x_i))^{\alpha})}-1]+[I_B(x_i)e^{(1-I_B^{\alpha}(x_i))}+(1-I_B(x_i))e^{(1-(1-I_B(x_i))^{\alpha})}-1] \tag{4}$$

From equation (3) and (4) , we get the result

$$E_\alpha(A\cup B)+E_\alpha(A\cap B)=E_\alpha(A)+E_\alpha(B)$$

(ii) Let $D=(A^{'}\cap B^{'})^{'}$ then the value of

$$E_\alpha((A^{'}\cap B^{'})^{'})=\frac{1}{3n(e^{(1-0.5^{\alpha})}-1)}\sum_{x\in X}[T_D(x_i)e^{(1-T_D^{\alpha}(x_i))}+(1-T_D(x_i))e^{(1-(1-T_D(x_i))^{\alpha})}-1]+[F_D(x_i) \\ e^{(1-F_D^{\alpha}(x_i))}+(1-F_D(x_i))e^{(1-(1-F_D(x_i))^{\alpha})}-1]+[I_D(x_i)e^{(1-I_D^{\alpha}(x_i))}+(1-I_D(x_i))e^{(1-(1-I_D(x_i))^{\alpha})}-1]$$

$T_D(x)=1-T_{(A'\cap B')}(x)=min\{1-T_A(x),1-T_B(x)\}=max\{T_A(x),T_B(x)\}$
Similarlly,
$F_D(x)=min\{F_A(x),F_B(x)\}$ and $I_D(x)=min\{I_A(x),I_B(x)\}$
Hence $E_\alpha(A\cup B)=E_\alpha((A^{'}\cap B^{'})^{'})$
In the similar way we can prove $E_\alpha(A\cap B)=E_\alpha((A^{'}\cup B^{'})^{'})$.

4. Comparative Study of Proposed Measure :

In this section , the performance of proposed measure will be validated based on the following examples. To illustrate the effectiveness of the proposed measure for neutrosophic sets, some existing measures are taken for comparison. These are listed as below:

Majumdar and Samantha [37]

$$E_{MS}(A)=1-\frac{1}{n}\sum_{i=1}^{n}(T_A(x_i)+F_A(x_i))|i_A(x_i)-i_A^c(x_i)| \tag{5}$$

Thao and Smarandache [44]

$$E_{TS}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n} \frac{|T_A(x_i) - 0.5| + |F_A(x_i) - 0.5| + |I_A(x_i) - 0.5| + |I_A^c(x_i - 0.5|}{2} \tag{6}$$

Aydogdu [2]

$$E_{AY}(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{2 - |T_A(x_i) - F_A(x_i)| - |I_A(x_i) - I_A^c(x_i)|}{2 + |T_A(x_i) - F_A(x_i)| + |I_A(x_i) - I_A^c(x_i)|} \tag{7}$$

Harish Garg and Nancy [10]

$$E_{HG}(A) = \frac{1}{n(1-\alpha)}\sum_{i=1}^{n} log_3[(T_A^{\alpha}(x_i) + F_A^{\alpha}(x_i) + I_A^{\alpha}(x_i))(T_A(x_i) + F_A(x_i) + I_A(x_i))^{1-\alpha} + 3^{1-\alpha}(1 - T_A(x_i) - F_A(x_i) - I_A(x_i))], \alpha > 0, \alpha \neq 1 \tag{8}$$

Jun Ye [32]

$$E_{JY}(A) = \frac{1}{3n(e^{0.5}-1)}\sum_{i=1}^{n}[T_A(x_i)e^{(1-T_A(x_i))} + (1 - T_A(x_i))e^{(T_A(x_i))} - 1] + [F_A(x_i)e^{(1-F_A(x_i))} + (1 - F_A(x_i))e^{(F_A(x_i))} - 1] + [I_A(x_i)e^{(1-I_A(x_i))} + (1 - I_A(x_i))e^{(I_A(x_i))} - 1] \tag{9}$$

Example 4.1: Let $A = \{< x, T_A(x_i), I_A(x_i), F_A(x_i) >, \forall x_i \in X\}$ be a neutrosophic set defined on $X = \{x_1, x_2, ..., x_n\}$ then for a positive real number λ , A^{λ} is defined as

$$A^{\lambda} = \{< x, T_A^{\lambda}(x), 1 - (1 - F_A(x))^{\lambda}, 1 - (1 - I_A(x))^{\lambda} >, \forall x \in X\} \tag{10}$$

Consider a neutrosophic fuzzy set A of $X = \{1, 2, 3, 4, 5\}$. The set A is defined as
$A = \{< 1, 0.1, 0.5, 0.6 >, < 2, 0.3, 0.5, 0.4 >, < 3, 0.6, 0.5, 0.1 >, < 4, 0.8, 0.5, 0.1 >, < 5, 1, 0.5, 0 >\}$.
By taking into consideration the characterization of linguistic variables, A as "LARGE" on X.
then
A^2 treated as "Very LARGE"
A^3 treated as "Quiet Very LARGE"
A^4 may be treted as " Very Very LARGE".
Using equation (10) the operational result is given in the table below: The effect of linguistic modifiers on the

A^{λ}	x_1	x_2	x_3	x_4	x_5
A	$(0.1, 0.5, 0.6)$	$(0.3, 0.5, 0.4)$	$(0.6, 0.5, 0.1)$	$(0.8, 0.5, 0.1)$	$(1, 0.5, 0)$
A^2	$(0.01, 0.75, 0.84)$	$(0.09, 0.75, 0.64)$	$(0.36, 0.75, 0.19)$	$(0.64, 0.75, 0.19)$	$(1.0, 0.75, 0)$
A^3	$(0.001, 0.87, 0.93)$	$(0.027, 0.87, 0.78)$	$(0.21, 0.87, 0.27)$	$(0.51, 0.87, 0.27)$	$(1.0, 0.87, 0)$
A^4	$(0.0001, 0.93, 0.97)$	$(0.0081, 0.93, 0.87)$	$(0.12, 0.93, 0.34)$	$(0.40, 0.93, 0.34)$	$(1, 0.93, 0)$

Table 2. Operational results on A^{λ}

fuzzy set LARGE was analyzed using power operations (Table : 2). The entropy values satisfy $E_{\alpha}(A) > E_{\alpha}(A^2) > E_{\alpha}(A^3) > E_{\alpha}(A^4)$ indicating a reduction in uncertainty with increasing linguistic intensity (refer table no 3). This confirms that the entropy measure consistently captures uncertainty variations induced by linguistic transformations in fuzzy and neutrosophic environments.

5. Application in Multi- Criteria Decision Making of Proposed Measure :

In this section, the proposed exponential measure for single-valued neutrosophic sets is applied to multiple attribute decision-making (MADM) problems. The ranking of alternatives is carried out using two approaches:

	$Entropy\ for\ A^{\lambda}$			
$Existing\ Measures$	A	A^2	A^3	A^4
$E_{MS}(A)$	1.0	0.6038	0.3973	0.2860
$E_{TS}(A)$	0.5500	0.3792	0.2911	0.2250
$E_{AY}(A)$	0.1084	0.0558	0.0359	0.0257
$E_{HG}(A)$	0.7643	0.6581	0.5643	0.5021
$E_{JY}(A)$	0.6997	0.5837	0.4386	0.3374
$E_{\alpha}(A)$	0.7096	0.6004	0.4582	0.3540

Table 3. Values of different entropy measure for Single valued neutrosophic sets

direct priority ranking based on entropy and the TOPSIS method, where the attribute weights are determined using the proposed entropy measure. The applicability and effectiveness of the proposed approach are illustrated through two numerical examples.

Example 5.1 : Let us consider the problem of investment of sum of money into the best project.The potential alternatives are car company P_1 , food company P_2 ,Tourism company P_3 and Air conditiner company P_4 . According to this set , attributes are C_1 , C_2 and C_3 which denote the risk, growth and environmental impact respectively. Now the problem is to decide the best investment company from these four alternatives. These four alternatives are represented by single valued neutrosophic set as below:

$P_1 = \{(C_1, 0.4, 0.5, 0.3), (C_2, 0.4, 0.1, 0.2), (C_3, 0.7, 0.2, 0.4)\}$

$P_2 = \{C_1, 0.6, 0.1, 0.3), (C_2, 0.7, 0.2, 0.3), (C_3, 0.3, 0.5, 0.9)\}$

$P_3 = \{(C_1, 0.8, 0.1, 0.2), (C_2, 0.7, 0.6, 0.2), (C_3, 0.6, 0.4, 0.9\}$

$P_4 = \{(C_1, 0.7, 0.6, 0.3), (C_2, 0.6, 0.3, 0.1), (C_3, 0.4, 0.6, 0.3)\}$

The proposed measure of exponential entropy for single valued neutrosophic set is applied to this decision making problem and ranking is given, as shown in Table 4. All priority ranking order of the four alternatives based on

$Entropy$	P_1	P_2	P_3	P_4	$Ranking\ Order$
$E_{TS}(A)$	0.6500	0.5833	0.5333	0.6667	$P_3 > P_2 > P_1 > P_4$
$E_{AY}(A)$	0.1555	0.1239	0.1235	0.1750	$P_3 > P_2 > P_1 > P_4$
$E_{JY}(A)$	0.8056	0.7489	0.7140	0.8448	$P_3 > P_2 > P_1 > P_4$
$E_{\alpha}(A)$	0.8195	0.7656	0.7329	0.8563	$P_3 > P_2 > P_1 > P_4$

Table 4. Results and ranking order of proposed measure with existing entropy measure in neutrosophic set

proposed measure and various entropy measures of single valued neutrosophic sets are evaluated. In all measures , the priority ranking is $P_3 > P_2 > P_1 > P_4$. Thus, the decision indicates , effectiveness of the proposed measure with exiasting measure in single valued neutrosophic set. In this example, the company P_3 is the best choce since the alternative with the lowest entropy value is considered as the best choice among all the alternatives. Therefore the best investment to be made in Tourism company.

In multicriteria decision making (MCDM), one of the most popular method to find the best alternative among all the alternatives, which is near to positive ideal solution and farthest to negative ideal solution (Hwang and Yoon 1981). This model has gained the attaintion of researchers for its effectiveness. In this part , the weighted determination of criteria is evaluated using proposed entropy measure. Initially , consider m alternatives $A = \{A_1, A_2, ..., A_m\}$ and $C = \{C_1, C_2, ..., C_n\}$ attributes. The process of TOPSIS is outlined as follows:

Step 1: Determine the decision matrix : Execute the single valued neutrosophic decision matrix of alternatives under criteria in the form of $M = [x_{ij}]_{mn}$, where $x_{ij} = (T_{ij}, I_{ij}, F_{ij})$. The decision makers determine the alternatives with respect to criteria using linguistic variables. The single valued neutrosophic set is employed to transfer the rating of scale of linguistic variables. The neutrosophic decision matrix is formed from the evaluation of each expert for each alternative subject to each criteria and the integrated value is implemented for the final assessment

(Wang et. al.(2010))
Step 2: Determine the weight w_j for each criteria C_j : The importance of criteria is usually decided by experts, but this can be subjective. To make the process more objective, the performance values of alternatives under each criterion are used. Let C_j represent the set of values related to criterion j. Using Eq. () , the entropy value e_j for each criterion is calculated. A smaller entropy value means the criterion provides more useful information.The weight w_j of each criterion is then calculated using the formula

$$w_j = \frac{(1-e_j)}{\sum_{i=1}^{n}(1-e_j)} \tag{11}$$

Step 3: Determine the positive ideal solution and negative ideal solution: The PIS and NIS are

$$A^+ = \{max_{i=1,..,m}(T_{ij}), min_{i=1,...,m}(I_{ij}), min_{i=1,...,m}(F_{ij})\}$$
$$A^- = \{min_{i=1,..,m}(T_{ij}), max_{i=1,...,m}(I_{ij}), max_{i=1,...,m}(F_{ij})\}$$

Step 4: Determine the relative closeness coefficient: Use weighted euclidean distance formula to find the distance from alternatives to PIS and NIS , S^+ and S^- respectively. The relative closeness coefficient of A_i is defined as

$$CL_i = \frac{S_i^+}{(S_i^+ + S_i^-)} \tag{12}$$

Step 5: Ranking of Alternatives: The best alternative is nearest to A^+ and farthest to A^- , thus the ranking of alternative is given $A_i > A_k$ if $CL_i > CL_k$, $\forall\, i, k = 1, 2, ..., m$.
Let us illustrate it by using numerical example. The lingustic variable scale on four alternatives and eight criterias is taken from Thao et.al. [5] , Tian et. al [1].
Example 5.2: Consider a problem with four alternatives $\{A_1, A_2, A_3, A_4\}$ with eight benefit criterians , including identify probability C_1 , the growth of market C_2 , size of market C_3, likely customer satisfaction C_4 , sales volume C_5 , likelihood of sustainable differential advantage C_6 , development opportunities C_7 and the differentiation of product C_8 as shown below: The decision makes determine alternatives with respect to criteria using lingustivc

$Linguistic\ scale\ for\ alternative\ performance$	$Fuzzy\ scale$
$Very\ low(VI)$	$(0.1, 0.85, 0.9)$
$Low(L)$	$(0.2, 0.75, 0.8)$
$Medium(F)$	$(0.5, 0.5, 0.45)$
$Good(H)$	$(0.8, 0.2, 0.15)$
$Very\ Good(Vh)$	$(0.9, 0.1, 0.05)$

Table 5. Linguistic variable rating scale

variables and then its converted to single valued neutrosophic set using rating scale. Evaluation is shown in the Table 6. Using proposed entropy measure weights for each criterion is calculated (use equation 11).
The weight vector is $w_i = \{0.1811, 0.1883, 0.1430, 0.0683, 0.0683, 0.0839, 0.1255, 0.1398\}$.
For each criteria , the positive ideal solution is
$A^+ = \{(0.85, 0.15, 0.1), (0.9, 0.1, 0.05), (0.85, 0.15, 0.1), (0.85, 0.125, 0.075), (0.9, 0.1, 0.05),$
$(0.85, 0.125, 0.075), (0.85, 0.125, 0.075), (0.85, 0.15, 0.1)\}$
the negative ideal solution is
$A^- = \{(0.725, 0.275, 0.225), (0.575, 0.425, 0.375), (0.65, 0.35, 0.3), (0.5, 0.5, 0.45),$
$(0.575, 0.425, 0.375), (0.725, 0.275, 0.225), (0.65, 0.35, 0.3), (0.275, 0.6875, 0.7125)\}$
Using weighted Euclidean distance formula
$S_1^+ = 0.2135$, $S_1^- = 0.4614$,$S_2^+ = 0.4173$, $S_2^- = 0.2437$, $S_3^+ = 0.0316$, $S_3^- = 0.5379$,
$S_4^+ = 0.4560$, $S_4^- = 0.2613$.
Using equation number 12 , evaluated the relative closeness coefficient

$Criteria$	A_i	DM	NS	A_i	DM	NS
C_1	A_1	Vh, V, H, H	$(0.85, 0.15, 0.1)$	A_3	H, H, H, Vh	$(0.825, 0.175, 0.125)$
C_2		H, H, Vh, Vh	$(0.85, 0.15, 0.1)$		Vh, Vh, Vh, Vh	$(0.9, 0.1, 0.05)$
C_3		H, Vh, H, F	$(0.75, 0.25, 0.2)$		Vh, Vh, H, H	$(0.85, 0.15, 0.1)$
C_4		F, H, F, F	$(0.575, 0.425, 0.375)$		H, Vh, Vh, Vh	$(0.875, 0.125, 0.075)$
C_5		H, F, F, F	$(0.575, 0.425, 0.375)$		Vh, Vh, Vh, Vh	$(0.9, 0.1, 0.05$
C_6		Vh, H, Vh, H	$(0.875, 0.125, 0.075)$		H, H, Vh, H	$(0.825, 0.175, 0.125)$
C_7		H, H, Vh, H	$(0.875, 0.175, 0.125)$		H, H, Vh, H	$(0.85, 0.15, 0.1)$
C_8		H, H, Vh, Vh	$(0.85, 0.15, 0.1)$		H, Vh, Vh, H	$(0.85, 0.15, 0.1)$
C_1	A_2	H, F, H, H	$(0.725, 0.275, 0.225)$	A_4	Vh, Vh, H, H	$(0.85, 0.15, 0.1)$
C_2		H, F, F, F	$(0.575, 0.425, 0.375)$		Vh, Vh, H, H	$(0.85, 0.15, 0.1)$
C_3		F, H, H, F	$(0.65, 0.35, 0.3)$		H, H, H, Vh	$(0.825, 0.175, 0.125)$
C_4		F, F, F, F	$(0.5, 0.5, 0.45)$		F, F, F, F	$(0.5, 0.5, 0.45)$
C_5		H, F, F, F	$(0.575, 0.425, 0.375)$		H, F, F, F	$(0.575, 0.425, 0.375)$
C_6		H, H, H, F	$(0.725, 0.275, 0.225)$		Vh, Vh, H, H	$(0.85, 0.15, 0.1)$
C_7		H, H, F, F	$(0.65, 0.35, 0.3)$		F, F, H, H	$(0.65, 0.35, 0.3)$
C_8		H, F, F, H	$(0.65, 0.35, 0.3)$		F, L, L, L	$(0.275, 0.688, 0.713)$

Table 6. The Neutrosophic Decision Matrix

$CL_1 = 0.3163$, $CL_2 = 0.6313$, $CL_3 = 0.0555$, $CL_4 = 0.6357$.
The higher relative closeness value suggests a more favorable evaluation while a lower value suggests a less favorable one. Hence alternative A_4 is selected as a most suitable for a company to invest their money.

6. Conclusion:

In this paper, we propose an exponential entropy measure for single-valued neutrosophic sets, parameterized by α and examine its validity as an entropy function. The behavior of the proposed measure is analyzed under various set-theoretic operations within the single-valued neutrosophic framework. Several key properties are formally stated and proven. To illustrate the effectiveness of the measure, numerical examples and graphical analyses are presented. Additionally, we explore its application in a multi-criteria decision-making process using TOPSIS method. This study opens avenues for further generalization of fuzzy entropy and divergence measures in the context of parametric single-valued neutrosophic sets. Future work may include comparative studies and potential applications in pattern recognition, image processing and related fields.

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