STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **15**, March 2026, pp 1784–1790.
Published online in International Academic Press (www.IAPress.org)

Local Total Distance Irregularity Labeling of Graph

Eric Dwi Putra [1,3], Slamin [2], Arika Indah Kristiana [1,*], Abi Suwito [1], Ridho Alfarisi[4]

[1]*Department of Mathematics Education, University of Jember, Indonesia*
[2]*Department of Informatics, University of Jember, Indonesia*
[3]*Department of Mathematic Education, University of PGRI Argopuro Jember, Indonesia*
[4]*Department of Mathematics, Universitas Jember, Indonesia*

Abstract We introduce the notion of total distance irregular labeling, called the local total distance irregular labeling. All edges and vertices are labeled with positive integers 1 to k such that the weight calculated at the vertices induces a vertex coloring if two adjacent vertices has different weight. The weight of a vertex $u \in V(G)$ is defined as the sum of the labels of all vertices adjacent and edges incident to u (distance 1 from u). The minimum cardinality of the largest label over all such irregular assignment is called the local total distance irregularity strength, denoted by $tdis_l(G)$. In this paper, we established the lower bound of the local total distance irregularity strength of graphs G and determined exact values of some classes of graphs namely path, cycle, star, bipartite complete, fan and sun graph.

Keywords Local total, coloring, local irregularity

AMS 2010 subject classifications 05C78

DOI: 10.19139/soic-2310-5070-3025

1. Introduction

In this paper, we assume that all graphs are finite, simple, and connected. For detailed definitions and notations, the reader is referred to [4, 5, 6]. Slamin [2] introduced the distance irregular labeling of graph, denoted by $dis(G)$, the minimum cardinality of the largest label k over all such irregular assignment. The term irregularity strength of H, denoted by $s(H)$, stand for the least of maximum label among all of the possible irregular assignments of H. Furthermore, Kristiana, et al. introduced local distance irregularity, the minimum cardinality of the largest label over all such assignment. Bača et al. [3] established Irregular Total Labeling, which was motivated by the concepts of irregularity strength and total labeling. For graph $H(T,S)$, a labeling $\sigma : T \cup S \to \{1,2,\ldots,k\}$ is a Vertex Irregular Total Labeling (VITL) if for every $x, y \in T, x \neq y$ then $w_\sigma(x) \neq w_\sigma(y)$. For every $x \in T, w_\sigma(x)$ is defined as $w_\sigma(x) = \sigma(x) + \Sigma_{ux\in S}\sigma(ux)$. The labeling σ is an Edge Irregular Total Labeling (EITL) if for every $xy, uv \in S, xy \neq uv$ then $w_\sigma(xy) \neq w_\sigma(uv)$. For every $xy \in E, w_\sigma(xy)$ is defined as $w_\sigma(xy) = \sigma(xy) + \sigma(x) + \sigma(y)$. Furthermore, Kristiana, et.al [1] introduce local distance irregularity labeling, and found local distance irregularity strength of path graph, cycle graph and planary tree graph.

Lemma 1
[2] Let u and w be any two adjacent vertices in a connected graph G. If $N(u) - \{w\} = N(w) - \{u\}$, then the labels of u and w must be distinct, that is $\lambda(u) \neq \lambda(w)$.

*Correspondence to: Arika Indah Kristiana (Email: arika.fkip@unej.ac.id). Department of Mathematics Education, Universitas Jember, Indoensia.

ISSN 2310-5070 (online) ISSN 2311-004X (print)
Copyright © 2026 International Academic Press

2. Results and Discussion

In this paper, we integrate the concepts of distance irregular labeling and distance vertex irregular total k-labeling. As a result, we introduce a new notion, referred to as the local total distance irregularity labeling.

Definition 1
Suppose $\sigma : V(G) \to \{1, 2, \ldots, k\}$ and $\lambda : E(G) \to \{1, 2, \ldots, k\}$ such that the weight calculated at the vertices induces a vertex coloring if $w(u) \neq w(v)$ for any edge uv. The weight of a vertex $u \in V(G)$ is defined as the sum of the labels of all vertices adjacent and all edges incident to u (distance 1 from u), that is $w(u) = \Sigma_{y \in N(u)} \sigma(y) + \Sigma_{e \in N(u)} \lambda(e)$.

Definition 2
The local total distance irregularity strength, denoted by $tdis_l(G)$, is defined as the minimum among the maximum labels assigned in all such irregular total distance labels.

Lemma 2
Let G be a connected graph on $n \geq 3$ vertices with the chromatic number χ, the minimum degree δ and the maximum degree Δ and there is no vertex having identical neighbors, then we have the lower bound of the local total distance irregular labeling of G is

$$tdis_l(G) \geq \lceil \frac{\chi + \delta - 1}{\Delta} \rceil$$

In this paper, we introduce a new concept in graph labeling, referred to as the local total distance irregularity labeling. We establish a lower bound for this labeling and determine its exact values for several graph classes, including path graphs, cycle graphs, sun graphs, fan graphs, star graphs, and complete bipartite graphs, as presented in the following theorems.

Theorem 1
Let P_n be a path graph with order $n \geq 3$, the local total distance irregularity labeling of P_n is $tdis_l(P_n) = 2$

Proof
The vertex set $V(P_n) = \{x_1, x_2, \ldots, x_n\}$ and the edge set $E(P_n) = \{x_i x_{i+1}; 1 \leq i \leq n-1\}$. We have $\Delta(P_n) = 2$, $\delta(P_n) = 1$ and $\chi(P_n) = 2$. The cardinality of the vertex set and the edge set, respectively, are $|V(P_n)| = n$ and $|E(P_n)| = n - 1$. This proof divided into two cases are as follows. For $n \geq 3$, Based on Lemma 2 that the lower bound of local total distance irregularity labeling of path graph P_n is $tdis_l(P_n) \geq \lceil \frac{\chi+\delta-1}{\Delta} \rceil = \lceil \frac{2+1-1}{2} \rceil = 1$. However, the sharpest lower bound cannot be achieved. If all vertices and all edges are assigned the label 1, there will exist at least two adjacent vertices with the same weights, namely $w(x_2) = \sigma(x_1) + \sigma(x_2) + \lambda(x_1 x_2) + \lambda(x_2 x_3) = 1 + 1 + 1 + 1 = 4$ and $w(x_3) = \sigma(x_2) + \sigma(x_4) + \lambda(x_2 x_3) + \lambda(x_3 x_4) = 1 + 1 + 1 + 1 = 4$, so that $w(x_2) = w(x_3)$ for the edge $x_2 x_3 \in E(P_n)$. Hence, the lower bound of the local total distance irregularity labeling of P_n is $tdis_l(P_n) \geq 2$. Furthermore, we prove that the upper bound of the local total distance irregularity labeling of P_n is $tdis_l(P_n) \leq 2$.

$$\sigma(x_i) = \begin{cases} 1, & for\ i \equiv\ 1, 2, 3\ mod\ 4, 1 \leq i \leq n \\ 2, & for\ i \equiv\ 0\ mod\ 4, 1 \leq i \leq n \end{cases}$$
$$\lambda(x_i x_{i+1}) = 1, 1 \leq i \leq n-1$$

This labeling provides vertex-weight as follows.

$$w(x_i) = \begin{cases} 2, & for\ \imath\ = 1, n \\ 4, & for\ i\ even, 2 \leq i \leq n-1 \\ 5, & for\ i\ odd, 2 \leq i \leq n-1 \end{cases}$$

It is easy to see that w is vertex coloring. Hence, we obtain the upper bound of the local total distance irregularity labeling of P_n is $tdis_l(P_n) \leq 2$. It concludes that $tdis_l(P_n) = 2$. □

Theorem 2
Let C_n be a cycle graph with order $n \geq 3$, then the local total distance irregularity labeling of C_n is

$$tdis_l(C_n) = \begin{cases} 2, & \text{if } n \text{ is even} \\ 3, & \text{if } n \text{ is odd} \end{cases}$$

Proof
The vertex set $V(C_n) = \{x_1, x_2, \dots, x_n\}$ and the the edge set $E(C_n) = \{x_i x_{i+1}; 1 \leq i \leq n-1\}$. We have $\Delta(C_n) = 2$, $\delta(C_n) = 1$ and $\chi(C_n) = 2$. The cardinality of the vertex set and the edge set, respectively, are $|V(C_n)| = n$ and $|E(C_n)| = n-1$. This proof divided into two cases are as follows.
Case 1: For n is even
For $n \geq 3$, Based on Lemma 2 that the lower bound of local total distance irregularity labeling of cycle graph C_n is $tdis_l(C_n) \geq \lceil \frac{\chi+\delta-1}{\Delta} \rceil = \lceil \frac{2+2-1}{2} \rceil = 2$. However, the sharpest lower bound cannot be achieved. If all vertices and all edges are assigned the label 1, there will exist at least two adjacent vertices with the same weights, namely $w(x_2) = \sigma(x_1) + \sigma(x_3) + \lambda(x_1x_2) + \lambda(x_2x_3) = 1+1+1+1+1 = 4$ and $w(x_3) = \sigma(x_2) + \sigma(x_4) + \lambda(x_2x_3) + \lambda(x_3x_4) = 1+1+1+1 = 4$, so that $w(x_2) = w(x_3)$ for the edge $x_2x_3 \in E(C_n)$. Hence, the lower bound of the local total distance irregularity labeling of C_n is $tdis_l(C_n) \geq 2$. Furthermore, we prove that the upper bound of the local total distance irregularity labeling of C_n is $tdis_l(C_n) \leq 2$.

$$\sigma(x_i) = \begin{cases} 1, & for\ i\ odd, 1 \leq i \leq n \\ 2, & for\ i\ even, 1 \leq i \leq n \end{cases} \tag{1}$$

$$\lambda(x_i x_{i+1}) = \begin{cases} 1, & for\ i\ odd, 1 \leq i \leq n-1 \\ 2, & for\ i\ even, 1 \leq i \leq n-1 \end{cases} \tag{2}$$

$$\lambda(x_1 x_n) = 2$$

This labeling provides vertex-weight as follows.

$$w(x_i) = \begin{cases} 5, & for\ i\ even, 1 \leq i \leq n \\ 7, & for\ i\ odd, 1 \leq i \leq n \end{cases} \tag{3}$$

It is easy to see that w is vertex coloring. Hence, we obtain the upper bound of the local total distance irregularity labeling of C_n is $tdis_l(C_n) \leq 2$. It concludes that $tdis_l(C_n) = 2$.

Case 2: For n is odd
For $n \geq 3$, Based on Lemma 2 that the lower bound of local total distance irregularity labeling of cycle graph C_n is $tdis_l(C_n) \geq \lceil \frac{\chi+\delta-1}{\Delta} \rceil = \lceil \frac{2+1-1}{2} \rceil = 1$. However, the sharpest lower bound cannot be achieved. If all vertices and all edges are assigned the label 1 or 2, there will exist at least two adjacent vertices with the same weights, namely $w(x_n) = \sigma(x_1) + \sigma(x_{n-1}) + \lambda(x_1x_n) + \lambda(x_nx_{n-1}) = 1+2+1+1 = 5$ and $w(x_{n-1}) = \sigma(x_n) + \sigma(x_{n-2}) + \lambda(x_nx_{n-1}) + \lambda(x_{n-1}x_{n-2}) = 1+1+1+2 = 5$, so that $w(x_n) = w(x_{n-1})$ for the edge $x_nx_{n-1} \in E(C_n)$. Hence, the lower bound of the local total distance irregularity labeling of C_n is $tdis_l(C_n) \geq 3$. Furthermore, we prove that the upper bound of the local total distance irregularity labeling of C_n is $tdis_l(C_n) \leq 3$.

$$\sigma(x_i) = \begin{cases} 1, & for\ i\ odd, 1 \leq i \leq n-1 \\ 2, & for\ i\ even, 1 \leq i \leq n-1 \\ 3, & for\ i\ = n \end{cases} \tag{4}$$

$$\lambda(x_i x_{i+1}) = \begin{cases} 1, & for\ i\ \equiv 2, 3\ mod\ 4 \\ 2, & for\ i\ \equiv 0, 1\ mod\ 4 \end{cases} \tag{5}$$

$$\lambda(x_1x_n) = 1$$

This labeling provides vertex-weight as follows.

$$w(x_i) = \begin{cases} 5, & for\ i\ \equiv 0,2\ mod\ 4, 1 \leq i \leq n-2, i = n \\ 6, & for\ i\ \equiv 3\ mod\ 4, 1 \leq i \leq n-2 \\ 7, & for\ i\ = n-1 \\ 8, & for\ i\ \equiv 1\ mod\ 4, 1 \leq i \leq n-2 \end{cases} \tag{6}$$

It is easy to see that w is vertex coloring. Hence, we obtain the upper bound of the local total distance irregularity labeling of C_n is $tdis_l(C_n) \leq 3$. It concludes that $tdis_l(C_n) = 3$. □

Theorem 3
Let M_n be a sun graph with order $n \geq 3$, then the local total distance irregularity labeling of M_n is

$$tdis_l(M_n) = \begin{cases} 2, & \text{if } n \text{ is even} \\ 3, & \text{if } n \text{ is odd} \end{cases}$$

Proof
The vertex set $V(M_n) = \{x_1, x_2, \dots, x_n\} \cup \{y_1, y_2, \dots, y_n\}$ and the edge set $E(M_n) = \{x_ix_{i+1}; 1 \leq i \leq n-1\} \cup \{x_1x_n\} \cup \{x_iy_i\}$. We have $\Delta(M_n) = 3, \delta(M_n) = 1$ and $\chi(M_n) = 2$. The cardinality of the vertex set and the edge set, respectively, are $|V(M_n)| = 2n$ and $|E(M_n)| = 2n$. This proof divided into two cases are as follows.
Case 1: For n is even
For $n \geq 3$, Based on Lemma 2 that the lower bound of local total distance irregularity labeling of sun graph M_n is $tdis_l(M_n) \geq \lceil \frac{\chi+\delta-1}{\Delta} \rceil = \lceil \frac{3+1-1}{2} \rceil = 2$. However, the sharpest lower bound cannot be achieved. If all vertices and all edges are assigned the label 1, there will exist at least two adjacent vertices with the same weights, namely $w(x_3) = \sigma(x_2) + \sigma(x_4) + \sigma(y_3) + \lambda(x_2x_3) + \lambda(x_3x_4) + \lambda(y_3x_3) = 1+1+1+1+1+1 = 6$ and $w(x_4) = \sigma(x_3) + \sigma(x_5) + \sigma(y_4) + \lambda(x_3x_4) + \lambda(x_4x_5) + \lambda(y_4x_4) = 1+1+1+1+1+1 = 6$, so that $w(x_3) = w(x_4)$ for the edge $x_3x_4 \in E(M_n)$. Hence, the lower bound of the local total distance irregularity labeling of M_n is $tdis_l(M_n) \geq 2$. Furthermore, we prove that the upper bound of the local total distance irregularity labeling of M_n is $tdis_l(M_n) \leq 2$.

$$\sigma(x_i) = \begin{cases} 1, & for\ i\ odd, 1 \leq i \leq n \\ 2, & for\ i\ even, 1 \leq i \leq n \end{cases} \tag{7}$$

$$\sigma(y_i) = 1,\ 1 \leq i \leq n$$

$$\lambda(x_ix_{i+1}) = \begin{cases} 1, & for\ i\ odd, 1 \leq i \leq n-1 \\ 2, & for\ i\ even, 1 \leq i \leq n-1 \end{cases} \tag{8}$$

$$\lambda(x_1x_n) = 2$$
$$\lambda(x_iy_i) = 1,\ 1 \leq i \leq n$$

This labeling provides vertex-weight as follows.

$$w(x_i) = \begin{cases} 7, & for\ i\ even, 1 \leq i \leq n \\ 9, & for\ i\ odd, 1 \leq i \leq n \end{cases} \tag{9}$$

$$w(y_i) = \begin{cases} 2, & for\ i\ odd, 1 \leq i \leq n \\ 3, & for\ i\ even, 1 \leq i \leq n \end{cases} \tag{10}$$

It is easy to see that w is vertex coloring. Hence, we obtain the upper bound of the local total distance irregularity labeling of M_n is $tdis_l(M_n) \leq 2$. It concludes that $tdis_l(M_n) = 2$ for n even.

Case 2: For n is odd
For $n \geq 3$, Based on Lemma 2 that the lower bound of local total distance irregularity labeling of sun graph M_n is $tdis_l(M_n) \geq \lceil \frac{\chi+\delta-1}{\Delta} \rceil = \lceil \frac{3+1-1}{3} \rceil = 2$. However, the sharpest lower bound cannot be achieved. If all vertices and all edges are assigned the label 1 or 2, there will exist at least two adjacent vertices with the same weights, namely $w(x_n) = \sigma(x_1) + \sigma(x_{n-1}) + \sigma(y_n) + \lambda(x_1x_n) + \lambda(x_nx_{n-1}) + \lambda(x_ny_n) = 1+1+2+1+1+2 = 8$ and $w(x_{n-1}) = \sigma(x_n) + \sigma(x_{n-2}) + \sigma(y_{n-1} + \lambda(x_nx_{n-1}) + \lambda(x_{n-1}x_{n-2}) + \lambda(x_{n-1}y_{n-1}) = 2+1+1+2+1+1 = 8$, so that $w(x_n) = w(x_{n-1})$ for the edge $x_nx_{n-1} \in E(M_n)$. Hence, the lower bound of the local total distance irregularity labeling of M_n is $tdis_l(M_n) \geq 3$. Furthermore, we prove that the upper bound of the local total distance irregularity labeling of M_n is $tdis_l(M_n) \leq 3$.

$$\sigma(x_i) = \begin{cases} 1, & for\ i\ odd, 1 \leq i \leq n-1 \\ 2, & for\ i\ even, 1 \leq i \leq n-1 \\ 3, & for\ i\ = n \end{cases} \tag{11}$$

$$\sigma(y_i) = 1,\ 1 \leq i \leq n$$

$$\lambda(x_ix_{i+1}) = \begin{cases} 1, & for\ i\ \equiv 2, 3\ mod\ 4 \\ 2, & for\ i\ \equiv 0, 1\ mod\ 4 \end{cases} \tag{12}$$

$$\lambda(x_1x_n) = 1$$
$$\lambda(x_iy_i) = 1,\ 1 \leq i \leq n$$

This labeling provides vertex-weight as follows.

$$w(x_i) = \begin{cases} 5, & for\ i\ \equiv 0, 2\ mod\ 4, 1 \leq i \leq n-2, i = n \\ 6, & for\ i\ \equiv 3\ mod\ 4, 1 \leq i \leq n-2 \\ 7, & for\ i\ = n-1 \\ 8, & for\ i\ \equiv 1\ mod\ 4, 1 \leq i \leq n-2 \end{cases} \tag{13}$$

$$w(y_i) = \begin{cases} 2, & for\ i\ odd, 1 \leq i \leq n-1 \\ 3, & for\ i\ even, 1 \leq i \leq n-1 \\ 4, & for\ i\ = n \end{cases} \tag{14}$$

It is easy to see that w is vertex coloring. Hence, we obtain the upper bound of the local total distance irregularity labeling of M_n is $tdis_l(M_n) \leq 3$. It concludes that $tdis_l(M_n) = 3$. □

Theorem 4
Let F_n be a fan graph with order $n \geq 3$, then the local total distance irregularity labeling of F_n is $tdis_l(F_n) = 2$

Proof
The vertex set $V(F_n) = \{x_1, x_2, \ldots, x_n\} \cup \{x\}$ and the edge set $E(F_n) = \{x_ix\} \cup \{x_ix_{i+1}; 1 \leq i \leq n-1\}$. We have $\Delta(F_n) = n, \delta(F_n) = 2$ and $\chi(F_n) = 3$.
For $n \geq 3$, Based on Lemma 2 that the lower bound of local total distance irregularity labeling of fan graph F_n is $tdis_l(F_n) \geq \lceil \frac{\chi+\delta-1}{\Delta} \rceil = \lceil \frac{3+2-1}{3} \rceil = 2$. However, the sharpest lower bound cannot be achieved. If all vertices and all edges are assigned the label 1, there will exist at least two adjacent vertices with the same weights, namely $w(x_3) = \sigma(x_2) + \sigma(x_4) + \sigma(x) + \lambda(x_2x_3) + \lambda(x_3x_4) + \lambda(xx_3) = 1+1+1+1+1+1 = 6$ and $w(x_4) = \sigma(x_3) + \sigma(x_5) + \sigma(x) + \lambda(x_3x_4) + \lambda(x_4x_5) + \lambda(xx_4) = 1+1+1+1+1+1 = 6$, so that $w(x_3) = w(x_4)$ for the edge $x_3x_4 \in E(F_n)$. Hence, the lower bound of the local total distance irregularity labeling

of F_n is $tdis_l(F_n) \geq 2$. Furthermore, we prove that the upper bound of the local total distance irregularity labeling of F_n is $tdis_l(F_n) \leq 2$.

$$\sigma(x_i) = \begin{cases} 1, & for\ i\ odd, 1 \leq i \leq n \\ 2, & for\ i\ even, 1 \leq i \leq n \end{cases} \tag{15}$$

$$\sigma(x) = 1$$

$$\lambda(x_i x_{i+1}) = 1, 1 \leq i \leq n-1$$

$$\lambda(x_i x) = 1, 1 \leq i \leq n$$

This labeling provides vertex-weight as follows.

$$w(x_i) = \begin{cases} 4, & fori = n \\ 5, & for\ i = 1 \\ 6, & for\ i\ even, 1 \leq i \leq n-2 \\ 8, & for\ i\ odd, 1 \leq i \leq n-1 \end{cases} \tag{16}$$

$$w(x) = \tfrac{5n-5}{2}$$

It is easy to see that w is vertex coloring. Hence, we obtain the upper bound of the local total distance irregularity labeling of F_n is $tdis_l(F_n) \leq 2$. It concludes that $tdis_l(F_n) = 2$.

□

Theorem 5
Let S_n be a star graph with order $n \geq 2$, then the local total distance irregularity labeling of S_n is $tdis_l(S_n) = 1$

Proof
The vertex set $V(S_n) = \{x_1, x_2, \ldots, x_n\} \cup \{x\}$ and the edge set $E(S_n) = \{x_i x; 1 \leq i \leq n-1\}$. We have $\Delta(S_n) = n, \delta(S_n) = n+1$ and $\chi(S_n) = 2$. For $n \geq 3$, Based on Lemma 2 that the lower bound of local total distance irregularity labeling of star graph S_n is $tdis_l(S_n) \geq \lceil \frac{\chi+\delta-1}{\Delta} \rceil = \lceil \frac{n+1-1}{2} \rceil = n$.
However, we attain the sharpest lower bound. Furthermore, we prove the upper bound of the local total distance irregularity labeling of star graph is $tdis_l(S_n) \leq 1$. We define the label vertices using formula $\sigma(x) = \sigma(x_i) = 1$ for $1 \leq i \leq n$. This labeling provides vertex-weight such as $w(x) = n$ and $w(x_i) = 1$. Each leaf adjacent to central vertex and $w(x) \neq w(x_i)$. Therefore, it is easy to see that w is vertex coloring. Hence, we obtain the upper bound of the local total distance irregularity labeling of S_n is $tdis_l(S_n) \leq 1$. It concludes that $tdis_l(S_n) = 1$. □

Theorem 6
Let $K_{m,n}$ be a bipartite complete graph with order $m, n \geq 2$, then the local total distance irregularity labeling of $K_{m,n}$ is $tdis_l(K_{m,n}) = 2$

Proof
The vertex set $V(K_{m,n}) = \{x_1, x_2, \ldots, x_n\} \cup \{y_1, y_2, \ldots, y_m\}$ and the edge set $E(K_{m,n}) = \{x_i y_j\}, 1 \leq i \leq n,\ 1 \leq j \leq m\}$. We have $\Delta(K_{m,n}) = n, \delta(K_{m,n}) = m$ and $\chi(K_{m,n}) = n$.
For $m, n \geq 2$, Based on Lemma 2 that the lower bound of local total distance irregularity labeling of bipartite complete graph $K_{m,n}$ is $tdis_l(K_{m,n}) \geq \lceil \frac{\chi+\delta-1}{\Delta} \rceil = \lceil \frac{m+n-1}{n} \rceil = n$. However, we cannot attain the sharpest lower bound.
Hence, the lower bound of the local total distance irregularity labeling of $K_{m,n}$ is $tdis_l(K_{m,n}) \geq 2$. Furthermore, we prove that the upper bound of the local total distance irregularity labeling of $K_{m,n}$ is $tdis_l(K_{m,n}) \leq 2$.

$$\sigma(x_i) = 2, 1 \leq i \leq m$$

$$\sigma(y_i) = 1, 1 \leq i \leq n$$

$$\lambda(x_i y_i) = 1, 1 \leq i \leq m, 1 \leq i \leq n$$

$$w(x_i) = 2n$$

$$w(y_i) = 3m$$

It is easy to see that w is vertex coloring. Hence, we obtain the upper bound of the local total distance irregularity labeling of $K_{m,n}$ is $tdis_l(K_{m,n}) \leq 2$. It concludes that $tdis_l(K_{m,n}) = 2$.

□

3. Concluding Remarks

In this paper, we integrate the concepts of distance irregular labeling and distance vertex irregular total k-labeling. As a result, we introduce a new notion, referred to as the local total distance irregularity labeling. and obtained the local total distance irregularity labeling of some graphs.

Acknowledgments

We gratefully acknowledge the support from University of Jember of year 2025 and University of PGRI Argopuro Jember, Indonesia.

REFERENCES

1. Dafik A.I Kristiana, R. Alfarisi. Local distance irregular labeling of graphs. *TWMS Journal of Applied and Engineering Mathematics*, 13(1):1–10, 2023.
2. Milan Bača, Mirka Miller, and John Ryan. On irregular total labellings. *Discrete Mathematics*, 307(11–12):1378–1388, 2007.
3. Gary Chartrand and Linda Lesniak. *Graphs and Digraphs*. Chapman and Hall, London, 3rd edition, 2000.
4. Jonathan L. Gross, Jay Yellen, and Ping Zhang. *Handbook of Graph Theory*. CRC Press, Taylor and Francis Group, 2nd edition, 2014.
5. Nora Hartsfield and Gerhard Ringel. *Pearls in Graph Theory*. Academic Press, United Kingdom, 1994.
6. Slamin. On distance irregular labelings of graphs. *Far East Journal of Mathematical Sciences*, 102(5):919–932, 2017.