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# A Stochastic Optimization Model For a Multi-Echelon Inventory System with Direct Demand that consists of Two Commodities

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**Abstract** In this paper, we present a stochastic optimization model for a multi-echelon inventory system with direct demand, handling two interrelated commodities. The system consists of a three-level continuous review inventory model, comprising a warehouse (WH), a single distribution center (DC), and a retailer (R). A $(s, S)$ inventory policy is implemented, assuming Poisson demand and exponentially distributed lead times at the retail node. The DC replenishes retailers in fixed pack sizes $Q_i(= S_i - s_i)$, while the WH provides an abundant supply. We derive the steady-state probability distribution and key performance measures, offering insights into system efficiency and operational characteristics.

**Keywords** Multi-echelon system, Two-commodity, Direct demand, Continuous review, Optimization.

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## 1. Introduction

In past, many researches have reviewed that Continuous review inventory control of a single item at a single location had been overlain. But in this paper an extend concept is introduced into multi-item and multi-location to overcome the single location of single item. The intent of the entrepreneur is to keep the inventory (stock of goods) for future sale. In order to meet the demand on time the organization must keep a track of stock goods which are waiting for sales. The purpose of maintaining an inventory theory is to determine the rules that the management can minify the costs which are associated with the inventory to meet customers demand. Inventory is studied of order to help the organization to save large amounts of cash. Inventory models answers the questions: (1) when an order be placed for a product? (2) How large should each order to be? The answer to these questions is collectively called an inventory policy.

Two different but inter related products: If a company sells two different products, main product (A) and sub product (B) i.e tire and tube, pen and refill, printer and cartridge, insulin vial and injection etc. The two products will be manufactured by a same production unit which are sold in two different demand rates. In starting, the customer will purchase the main product with the sub product, but for the next time he may purchase only the sub product. Comparatively it clearly shows the demand of sub product is higher than the main product.

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Inventory decision is an important component in supply chain management, because Inventories exist at each and every stage of the supply chain as raw material, semi-finished or finished goods. They can also be in Work-in-process between the stages or stations. Since holding of inventories can cost anywhere between 20% to 40% of their value, so their efficient management will be critical in Supply Chain operations

Stochastic inventory model has received an considerable attention in inventory literature. Inventory systems of $(s, S)$ type for single commodity has been studied quite extensively in the past. The first quantitative analysis of inventory studies was started Harris in 1915 [20]. And followed by Clark and Scarf in 1960 [4] had proposed the multi-echelon inventory system. In that they analyzed a N-echelon pipelining system without considering a lot size, in recent developments the two-echelon models have introduced by Q.M. He and E.M. Jewkes in 1998[21].Sven Axsäter in 1990 [16] has proposed an approximate model of inventory structure in SC. One of the oldest papers in the field of continuous review multi-echelon inventory system is a basic and seminal paper which was written by Sherbrooke [25] in 1968. And finally the Complete review was provided by Benita M. Beamon in 1998 [3]. The supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that the research in this area is based on the classic work of Clark and Scarf (1960)[4]. A continuous review perishable inventory system at Service Facilities was studied by Elango (2001) [18]. A continuous review $(s, S)$ policy with positive lead times in two-echelon Supply Chain in single item was considered by Krishnan. K and Elango.C. 2005[23].

The modelling of multi-item inventory system under a joint replenishment has been receiving a considerable attention for the past three decades. In continuous review inventory systems, Ballintfy [1964] and Silver [1974] have considered a coordinated reordering policy. Sivazlian [1971] considers the stationary characteristics of a multi-commodity inventory problem. Krishnamoorthy, Lakshmi and Basha 1994 [24] have dealt with a two-commodity inventory problem. Kalpakam and Arivarigan [22] analyzed that a multi-item inventory model with renewal demands under a joint replenishment policy. Anbazhagan and Arivarignan [14] have analyzed two commodity inventory systems under various ordering policies. Yadavalli et. al., [26] have analyzed a model with joint ordering policy and varying order quantities. Prabaharan and Bakthavachalam (2023) developed a perishable inventory optimization model for two commodities in a multi-echelon system. Their work emphasized minimizing wastage while maintaining product availability [27]. Sathish Kumar (2023) proposed a simulation-based approach for analyzing multi-echelon inventory systems. The study demonstrated how simulation helps evaluate inventory policies under uncertain demand conditions [29]. Zhang, Li, and Chen (2025) introduced a data-driven method for strategic inventory placement in large-scale supply networks. Their approach improved operational efficiency through real-time data analytics [30]. Rachman et al. (2025) explored reinforcement learning for multi-objective, multi-echelon supply chain optimization. Their model applied AI techniques to balance multiple goals such as cost, service level, and sustainability [28].

The paper is organized as follows. In section two, a mathematical model for the problem is presented along with some important notations used in the paper. Both transient and steady state analysis are done in section three. In section four, the operating characteristics of the system are shown. In section five, deals with the cost analysis for the operation. In section six, some numerical examples are shown. Finally, section seven concludes the paper.

## 2. The Mathematical Model

The supply chain inventory control system that is the subject of this paper's discussion is described as follows.

A serial Multi-echelon system consisting of a warehousing facility (WH), one distribution centre (DC) and single retailer (R) dealing with two different but inter related products. Assume that the finished products are supplied from WH to DC which adopts (0, M) replenishment policy then the product is supplied to R who adopts $(s_i, S_i)$ policy $(i = 1, 2)$. The demands at retailer-node follows independent Poisson distribution with rate $\lambda_A$ , $\lambda_B$ and $\lambda_C$ for main product (A), sub product (B) and both the product respectively. Demands that occur during the stock out periods are assumed to be lost sales at retailer-nodes.

Supply to the Manufacturer in packets of $Q$ items is administrated with exponential lead time having parameter $\mu(> 0)$. The Direct demand at DC is also permitted with rate $\lambda_D > 0$. The replenishment of items in terms of pockets is made from WH to DC is instantaneous. In this model the maximum inventory-level at $R$ node $S_i$ is fixed and the reorder-level $s_i(i = 1, 2)$ and the Ordering Quantity $Q_i = S_i - s_i(i = 1, 2)$ and The maximum inventory-level at $DC_i$ is $M_i(M_i = nQ_i), n > 0$.

## 3. Analysis

Let $I - i(t), (i = 1, 2, 3)$ denote the on-hand inventory-levels for main product(A), sub product (B) at Retailer and DC respectively at time $t^+$. From the assumptions on input & output processes,
$I(t) = \{ (I_i(t), : t \geq 0 \}, (i = 1, 2, 3)$ is a Markov-process with state-space

$$E = \begin{cases} (i, k, m)/i = S_1, (S_1 - 1), ..., s_1, (s_1 - 1), ..., 2, 1, 0; \\ k = S_2, (S_2 - 1), ..., s_2, (s_2 - 1) \\ m = nQ, (n - 1)Q, ..., Q \end{cases} \tag{3.0.1}$$

As $\{I(t), t \geq 0\}$ is an irreducible Markov process with state space $E$ and an ergodic process, E is finite and all of its states are recurrent non-null. Because of this, the limiting distribution is real and unaffected by the initial state.

**Theorem 3.1.** The vector process $I(t) : t \geq 0$ where $I(t) = (I_1(t), I_2(t), I_3(t))$ for $t \geq 0$ is a continuous time Markov-Chain with state-space

$$E = \{(i, k, m)/i, k = 0, 1, 2..., S; m = Q, 2Q, ..., nQ\}$$

*Proof*
The stochastic-process $\{I(t) : t \geq 0\}$ has a discrete state-space with order relation $' \leq'$ that $(j, q) \leq (k, r)$ if and only if $j \leq k$ and $q \leq r$.
To prove that $I(t) : t \geq 0$ is a Markov chain, first we do a transformation for state-space E to E' such that $(j, q) \longrightarrow j + q \in E'$,where $E' = \{Q, Q + 1, ..., Q + S, ...., nQ + S\}$.
Now we may realize that $\{I(t) : t \geq 0\}$ is a stochastic-process with discrete state-space E'.
The joint distribution of random variables $\{I(t_1), I(t_2), ..., I(t_n)\}$ and $\{I(t_1 + \tau), I(t_2 + \tau), ..., I(t_n + \tau)\}$ with $\tau > 0$ (random real number) are equal.
In specific the conditional probability $P_r\{I_n = k|I_{n-1} = j, I_{n-2} = i, .....I_0 = 1\} = P_r\{I_n = k|I_{n-1} = j\}$ because of the states' one-step transition in E.
Hence $\{I(t) : t \geq 0\}$ is a continuous time Markov-Chain. □

The following arguments can be used to determine the infinitesimal generator of this process:

$$R = (a(i, k, m : j, l, n))_{(i,k,m),(j,l,n)\in E}.$$

- In the Markov-process, the arrival of a demand for main product (A) at retailer-node causes a state shift from $(i, k, m)$ to $(i - 1, k, m)$. with the degree of change $\lambda_A$
- In the Markov-process, the arrival of a demand for subproduct (B) at the retailer-node causes a state transition with intensity of transition $\lambda_B$ from $(i, k, m)$ to $(i, k - 1, m)$.
- When a demand for both products (A and B) arrives at the retailer-node, the Markov-process changes from $(i, k, m)$ to $(i - 1, k - 1, m)$ with the degree of change $\lambda_C$.
- In the Markov-process, the arrival of a direct demand at DC causes a state transition with intensity of transition $\lambda_D$ from $(i, k, m)$ to $(i, k, m - Q)$.
- In the Markov-process, replenishing inventory at the retailer-node causes a state transition with intensity $\mu$ from $(i, k, m)$ to $(i + Q, k, m - Q)$ or $(i, k, m)$ to $(i, k + Q, m - Q)$.

Given by is the infinitesimal generator $R$.

$$R = \begin{pmatrix} A & B & 0 & 0 & 0 & \cdots & 0 \\ 0 & A & B & 0 & 0 & \cdots & 0 \\ 0 & 0 & A & B & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ B & 0 & 0 & 0 & 0 & \cdots & A \end{pmatrix}$$

It is feasible to represent the matrix $R$'s elements as

$$[A]_{m\times n} = \begin{cases} A_1 & \text{if} \quad m = n, \quad m = S_1, S_1-1, ...s_1. + 1 \\ A_2 & \text{if} \quad n = m+1, \quad m = S_1, S_1-1, ...s_1. + 1 \\ A_3 & \text{if} \quad m = n \quad m = s_1, s_1-1, ...1 \\ A_4 & \text{if} \quad n = m+1 \quad m = s_1, s_1-1, ...1 \end{cases}$$

$$B = \begin{cases} M_1 & \text{if} \quad m = n, \quad m = S, S-1, ...0 \\ M_2 & \text{if} \quad n = m+1, \quad m = s, s-1, ...0 \\ 0 & \text{otherwise} \end{cases}$$

Then, $A$ and $B$'s submatrices are provided by

$$[A_1] = \begin{cases} \lambda_B & \text{if} \quad n = m+1 \quad m = S, S-1, ...1 \\ -(\lambda_A + \lambda_B + \lambda_C + \lambda_D) & if \quad m = n_1 \quad m = S, S-1, ...s+1 \\ -(\lambda_A + \lambda_B + \lambda_C + \lambda_D + \mu) & \text{if} \quad m = n_1 \quad m = s, ...1 \\ -(\lambda_A + \lambda_D + \mu) & \text{if} \quad m = 4 \quad m = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$[A_2] = \begin{cases} \lambda_A & \text{if } m = n_1 \quad m = S, S-1, ...0 \\ \lambda_C & \text{if } n = m+1 \quad m = S, S-1, ...1 \\ 0 & \text{otherwise} \end{cases}$$

$$[A_3] = \begin{cases} \lambda_B & \text{if} \quad n = m+1 \quad m = S, ...1 \\ -(\lambda_A + \lambda_B + \lambda_C + \lambda_D) & \text{if} \quad m = n_1 \quad m = S, S-1, ...s+1 \\ -(\lambda_A + \lambda_B + \lambda_C + \lambda_D + 2\mu) & \text{if} \quad m = n_1 \quad m = s, s-1...1 \\ -(\lambda_A + \lambda_D + 2\mu) & \text{if} \quad m = 4 \quad m = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$[A_4] = \begin{cases} \lambda_B & \text{if} \quad n = m+1 \quad m = S, ...1 \\ -(\lambda_B + \lambda_D + \mu) & if \quad m = n_1 \quad m = S, S-1, ...s+1 \\ -(\lambda_B + \lambda_D + 2\mu) & \text{if} \quad m = n \quad m = s, s-1...1 \\ -(\lambda_D + 2\mu) & \text{if} \quad m = 4 \quad m = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$[B_1] = \begin{cases} \lambda_D & \text{if} \quad m = n \quad m = S, S-1, ..., 1, 0 \\ \mu & \text{if} \quad n = m+Q \quad m = s, s-1, , ...0 \\ 0 & \text{otherwise} \end{cases}$$

$$[B_2] = \begin{cases} \mu & \text{if} \quad m = n \quad m = S, S-1, , ..., 1, 0 \\ 0 & \text{otherwise} \end{cases}$$

### *3.1. Steady state analysis*

The finite and irreducible state space $E$ of the Markov chain $I(t) : t \geq 0$ can be observed through the structure of the infinitesimal matrix R. Assume that the inventory level process's limiting distribution is determined by

$$\mathrm{P}^m_{i,k} = \lim_{t\to\infty} \Pr\left\{ (I_1(t), I_2(t), I_3(t)) = (i,k,m) \right\}_{(i,k,m)\in E} \tag{3.1.1}$$

where $P^m_{i,k}$ is the steady-state probability for the system be in state $(i,k,m)$, (Cinlar[17]).
Let $P = \left(P^{nQ}, P^{(n-1)Q}, P^{(n-2)Q} ... P^{Q}\right)$ denote the steady-state probability distribution where $P^q = \left(P^q_S, P^q_{S-1} .... P^q_0\right)$ for the system under consideration.
For each $(i,k,m)$, $P^m_{i,k}$ can be obtained by solving the matrix equation $PA = 0$ together with normalizing condition $\sum_{i,k,m} P^m_{i,k} = 1$.
Assuming $P^Q_Q = a$, we Obtain the steady-state probability
$\overset{iQ}{P} =$ (-1)$^{\mathrm{k}}$ a $(BA)^k, i = 1,2,...n; k = n - i + 1$, where a = e$^1$ $\left[\sum_{i=0}^{\text{n-1}} (\text{-1})^{\mathrm{i}} \, (BA^{-1})^i\right]^{-1}$.

## 4. Performance measures

### 4.1. Mean inventory level

Let $I_A$ and $I_B$ represent the expected inventory levels for commodities A and B at the retailer node in the steady state, and $I_D$ represent the expected inventory levels for commodities 1 and 2, respectively, at the distribution center. They are described as

$$I_A = \sum_{m=Q}^{nQ} \sum_{k=0}^{S} i \sum_{i=0}^{S} i.P^m{}_{i,k},\; I_B = \sum_{m=Q}^{nQ} \sum_{i=0}^{S} k \sum_{k=0}^{S} k.P^m{}_{i,k}\,,\; I_D = \sum_{i=S}^{S} \sum_{k=0}^{S} \sum_{m=Q}^{nQ} m.P^m{}_{i,k}.$$

### 4.2. Mean reorder rate

The mean reorder-rate at retailer-node for the Commodity 1 and Commodity 2 and Distributor center are given by

$$R_A = \sum_{m=Q}^{nQ} \sum_{k=0}^{S} \lambda_A.P^m{}_{s+1,k}\, R_B = \sum_{m=Q}^{nQ} \sum_{i=0}^{S} \lambda_B.P^m{}_{i,s+1}\, R_D = \sum_{i=0}^{S} \sum_{k=0}^{S} (\mu + \lambda_D).P^Q{}_{i,k}$$

### 4.3. Mean shortage rate

Shortage occurs only at retailer-node and the shortage rate for the commodity 1 and 2 at retailer is denoted by $S_A$ and $S_B$ and $S_C$ is the shortage rate for both product, which are given by

$$S_A = \sum_{m=Q}^{nQ} \sum_{k=0}^{S} \lambda_A.P^m{}_{0,k} S_B = \sum_{m=Q}^{nQ} \sum_{i=0}^{S} \lambda_B.P^m{}_{i,0} S_C = \sum_{m=Q}^{nQ} \sum_{i=0}^{S} \lambda_C.P^m{}_{i,0} + \sum_{m=Q}^{nQ} \sum_{k=0}^{S} \lambda_C.P^m{}_{0,k}$$

## 5. Cost Analysis

By taking into account the minimization of the steady state total projected cost per time, we assess the cost structure for the suggested models in this section. The model's long-term projected cost rate is determined to be

$$TC(s,Q) = h_A\, I_A + h_B\, I_B + h_D\, I_D + k_A R_A + k_B\, R_B + k_D R_D + g_A S_A + g_B S_B + g_C S_C$$

While the convexity of the cost function $TC(s,Q)$ has not been analytically proven, our experience with a large number of numerical instances suggests that $TC(s,Q)$ for fixed $Q$ is convex in $s$. It turns out to be an increasing function of $s$ in certain instances. In order to find the ideal values $s^*$, we therefore used the numerical search technique; as a result, we were able to find the ideal $n^*$. Our computation of $TC(s,Q)$ showed a convex structure for the same for large numbers of parameters. As a result, we used a numerical search strategy to determine the best value $s^*_i$ for every $S_i (i = 1,2)$.

## 6. Numerical Example and Sensitivity Analysis

The issue of minimizing the long-term total predicted cost per unit of time under the given cost structure is covered in this section. The following numerical example might be used to show the findings achieved in the steady state scenarios.

| s | Cost | Q |
|---|---|---|
| 2 | 37.86433 | 16 |
| 3 | 35.66304 | 15 |
| 4 | 33.49332 | 14 |
| 5 | 34.84016 | 13 |
| 6 | 32.27803 | 12 |
| 7 | 29.75622* | 11 |
| 8 | 30.04207 | 10 |

Table 1. Total Expected Cost Rate Analysis (TC(s, Q)) in a Multi-Echelon System with S=18 and M=54

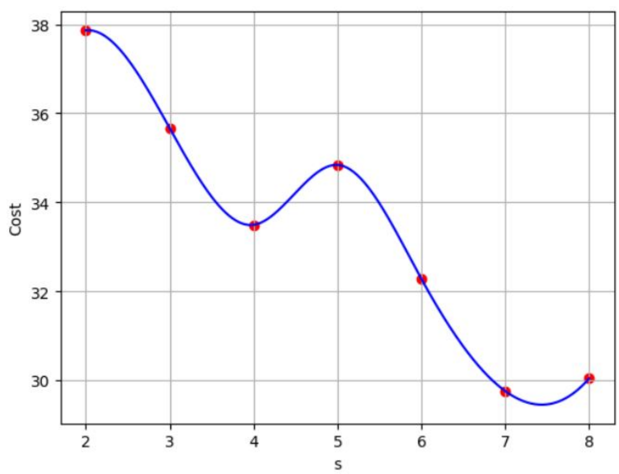


Figure 1. Total Expected Cost Rate Analysis (TC(s, Q)) in a Multi-Echelon System with S=18 and M=54

The table 1 shows the total expected cost rate as a function of the reorder level (s) and the order quantity (Q), where S=18 and M=54. The minimum cost is achieved when s=7 and Q=11, with a cost of 29.75622. This suggests that the optimum reorder level is 7 and the optimum order quantity is 11.

The figure 1 shows the table 1 data as a line graph.

| $\lambda_A \backslash \lambda_B$ | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
|---|---|---|---|---|---|---|
| 1 | 37.22789 | 37.33488 | 37.43636 | 37.53605 | 37.63497 | 37.7335 |
| 1.2 | 37.16158 | 37.30526 | 37.41231 | 37.5141 | 37.61419 | 37.71353 |
| 1.4 | 37.31188 | 37.25732 | 37.38395 | 37.48994 | 37.59184 | 37.69227 |
| 1.6 | 37.21993 | 37.38221 | 37.35493 | 37.46326 | 37.56771 | 37.66958 |
| 1.8 | 37.18446 | 37.30108 | 37.41926 | 37.44337 | 37.54261 | 37.64557 |
| 2 | 37.15509 | 37.26283 | 37.37874 | 37.47438 | 37.52621 | 37.62171 |

Table 2. Comparison of $\lambda_A$ & $\lambda_B$

It is observed that from the Table 2, the demand rate $\lambda_A$ increases then the total cost are also increased. Hence the demand rate is one of the key parameter of the system.

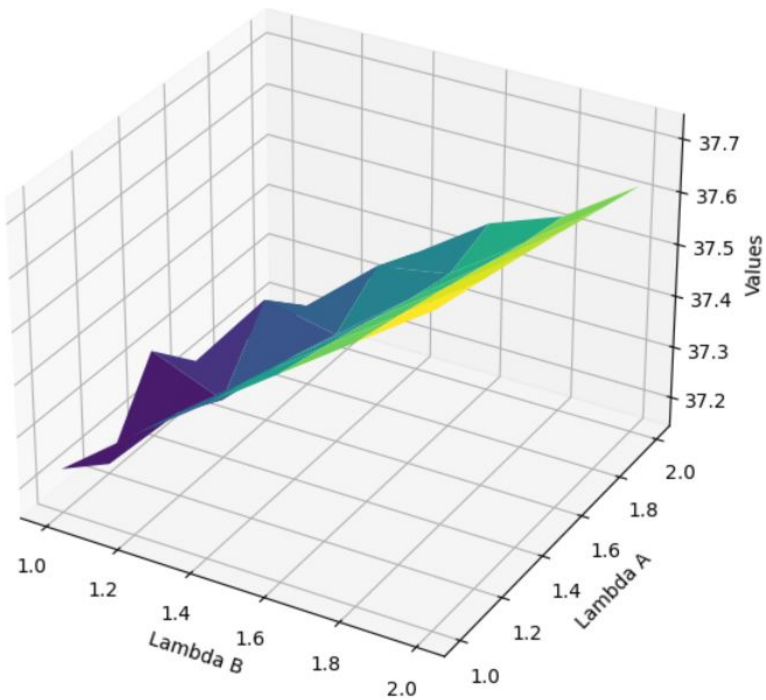


Figure 2. Comparison of $\lambda_A$ & $\lambda_B$

In our research on the "A stochastic optimization model for a multi-echelon inventory system with direct demand that consists of two commodities," we conducted a comprehensive analysis of the Total Expected Cost Rate (TC) with varying system parameters. Specifically, we examined scenarios with different values of S (the maximum inventory level) and M (the maximum demand rate).

Our findings indicate that the demand rate ($\lambda_A$ and $\lambda_B$) is a critical factor affecting the total cost. When demand rate increases, the total cost also rises. This observation underscores the importance of demand forecasting and effective inventory management strategies to mitigate increased costs.

## 7. Conclusion

This paper presents a Stochastic Optimization Model for a two-commodity Multi-Echelon Inventory System with direct demand at distribution centers. We focus on a continuous review inventory control system, addressing interdependencies between commodities to enhance efficiency and responsiveness.

Steady-state probability distribution analysis provides insights into stockouts, excess inventory, and reliability. Numerical examples illustrate the models ability to optimize inventory, minimize costs, and maximize service levels. Sensitivity analysis evaluates robustness, while graphical representations aid decision-making.

This study contributes to supply chain management by offering a tailored model that improves inventory strategies. Supported by numerical and visual analysis, our approach helps organizations enhance efficiency and resilience in dynamic environments.

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