

The Extreme Value Theory for Demographical Risk Analysis and Assessment: Peaks Over Random Threshold Value-at-Risk Analysis of Regional Prevalence Data with a Demographical Case Study

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Abstract This study presents a novel application of Extreme Value Theory (EVT) to analyze and quantify the risk associated with disability prevalence across regions in Saudi Arabia. Leveraging advanced actuarial risk modeling techniques including Value-at-Risk (VaR), Tail Value-at-Risk (TVaR), Mean of Order P (MOP), and the Peaks Over Random Threshold Value-at-Risk (PORT-VaR) framework, we provide a robust statistical assessment of regional disparities and extreme disability risk. Using demographic data from the 2016 national survey, our analysis identifies critical outliers and quantifies extreme thresholds at varying confidence levels (55%–95%). The Northern Border region emerges as a high-risk area, with over 285,000 individuals living with disabilities, significantly exceeding other regions. Our PORT-VaR and Peaks Over Random Threshold Mean of Order P (PORT-MOP) models highlight urgent policy targets and resource allocation needs, particularly for older adults and low-income populations who are disproportionately affected. This work contributes to the growing field of actuarial statistical modeling by demonstrating how EVT-based tools can enhance public health planning and support evidence-based decision-making in social policy development. By aligning traditional actuarial methodologies with contemporary public health challenges, the study underscores the relevance of predictive modeling and quantitative risk management in addressing complex societal issues such as disability prevalence.

Keywords Extreme Value Theory; Disability Prevalence Rates; Generalized Extreme Distribution; Mean of Order P; Value-at-Risk; Peaks Over a Random Threshold.

AMS 2010 subject classifications 62P10, 62P25, 91D20, 62G32, 91D20.

DOI: 10.19139/soic-2310-5070-2941

1. Introduction

Despite increasing attention to disability issues in Saudi Arabia, there remains a significant lack of rigorous statistical analysis to model and understand the patterns, severity, and risk factors associated with disability prevalence. Previous studies have primarily focused on qualitative assessments, healthcare access, and sociocultural perspectives, but few have employed advanced statistical methodologies such as EVT to quantify risk and identify extreme cases. Specifically, existing literature lacks the application of PORT, MOP, VaR, and TVaR approaches in analyzing disability data. This gap limits the ability of policymakers to make informed decisions regarding resource allocation, intervention planning, and long-term support systems for individuals with disabilities. The current study

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addresses this critical void by applying EVT-based risk modeling techniques to provide a comprehensive and quantitative understanding of disability prevalence, enabling more accurate risk assessment and improved policy formulation. The EVT is concerned with modeling the behavior of extreme events, such as unusually high disability prevalence in certain regions. The foundational result in EVT states that if there exist sequences $a_n > 0$ and $b_n \in R$ such that

$$\Pr \left(\frac{M_n - b_n}{a_n} \leq x \right) \rightarrow G(x). \quad (1)$$

The core theorem of EVT states that if there exist sequences of constants $a_n > 0$ and $b_n \in R$ such that:

$$\lim_{n \rightarrow +\infty} \left(\frac{M_n - b_n}{a_n} \leq x \right) = G(x). \quad (2)$$

where $M_n = \max(X_1, X_2, \dots, X_n)$ is the maximum of n independent and identically distributed random variables, then the limiting distribution $G(x)$ must be a member of the Generalized Extreme Value (GEV) family, where

$$G(x; \xi, v, \sigma) = \exp \left[- \left(1 + \xi \frac{x - v}{\sigma} \right)^{-\frac{1}{\xi}} \right], \text{ for } 1 + \xi \frac{x - v}{\sigma} > 0, \quad (3)$$

where $\xi \in R$ is a shape parameter (and it is the tail index), $\mu \in R$ is a location parameter, $\sigma > 0$ is the scale parameter. The domain of x depends on the value of ξ . If $\xi > 0$; $x < \mu - \frac{\sigma}{\xi}$. If $\xi < 0$; $x > \mu - \frac{\sigma}{\xi}$. If $\xi = 0$: $x \in R$, and in this case, we assume $\xi \neq 0$ unless otherwise stated. For $\xi \rightarrow 0$, then $x \in (-\infty, \infty)$, the Gumbel model is obtained, where

$$G(x; v, \sigma) = \exp \left[- \exp \left(\frac{x - v}{\sigma} \right) \right] \quad (4)$$

The Frechet is obtained for $\xi > 0$, then $x \in [\mu - \frac{\sigma}{\xi}, +\infty)$. Finally, for $\xi < 0$ the GEV reduced to Weibull model where, then $x \in (-\infty, \mu - \frac{\sigma}{\xi}]$. The GEV distribution in (3) has well-established, closed-form mathematical expressions for these risk measures as shown below (for more details see Smith (1990) and Rocco (2014)). The GEV model can be employed under many new topics such as the mining theory and control systems, Bayesian estimation with joint Jeffrey's prior and big data (see Jameel et al. (2022), Salih and Abdullah (2024), Salih and Hmood (2020) and Salih and Hmood (2022)). In this work, the GEV model to model the high disability prevalence in certain regions and in the corresponding risk analysis with a real application of disability in Saudi Arabia. Strating with (3), the Probability Density Function (PDF) of the GEV family can then be expressed as

$$g(x; \xi, v, \sigma) = \frac{1}{\sigma} \left(1 + \xi \frac{x - v}{\sigma} \right)^{-\frac{1}{\xi}} \exp \left[- \left(1 + \xi \frac{x - v}{\sigma} \right)^{-\frac{1}{\xi}} \right] \quad (5)$$

The mean of the GEV distribution in (5) exists only when $\xi < 1$.

$$\mathbf{E}[X] = [\Gamma(1 - \xi) - 1] \left(\frac{\xi}{\sigma} \right) + \mu.$$

The primary justification is rooted in the fundamental theorem of EVT, as stated in the papers introduction (1) and (2). This theorem provides a rigorous mathematical basis, proving that the GEV distribution is the only possible limiting distribution for the maxima (or minima) of a large number of independent, identically distributed random variables, after appropriate normalization. Since the study aims to model "extreme disability prevalence" across regions (i.e., the maximum prevalence observed in a set of regions), the GEV is not just a convenient choice but the theoretically mandated model for such extreme values.

Disability in Saudi Arabia presents a complex interplay of cultural, social, and healthcare dynamics that significantly impact the lives of individuals with disabilities and their families. Over recent years, there has been

growing recognition of the need to address the challenges faced by this population, ranging from accessibility in healthcare to social integration and employment opportunities. As the Kingdom strives to enhance its healthcare infrastructure and promote inclusivity, numerous studies have emerged, providing valuable insights into the prevalence, types, and experiences of people with disabilities. This literature review synthesizes findings from many key studies, highlighting critical issues such as healthcare access, societal attitudes, educational resources, and technological advancements. By examining these dimensions, the review aims to shed light on the current state of disability in Saudi Arabia and identify areas for further research and policy development, ultimately contributing to a more inclusive society.

Disability prevalence in Saudi Arabia is estimated at 7-10% of the population, with higher rates among older adults (approximately 15% for those aged 60 and above) and significant regional variations, with some areas reporting up to 12% (Al-Jadid (2013); Bindawas and Vennu (2018)). Types of disabilities include physical (40-50%), sensory (20-30%), and intellectual disabilities (15-20%) (Alnahdi (2014); Al-Ghamdi (2020)). Access to healthcare is a major concern, with 60% of individuals with disabilities facing barriers, and only 35% utilizing preventative services (Alqassim et al. (2022); Ministry of Health (2022)). Employment rates among people with disabilities are around 20%, significantly lower than the general population, with a notable 30% working part-time (Medabesh et al. (2024); Malik et al. (2024)). In education, 30-40% of children with disabilities are enrolled in inclusive settings, but 60% do not receive necessary special education services (Alnahdi (2013); Jambi et al. (2024)). Public perception shows that 75% believe more support is needed for individuals with disabilities, yet 50% hold misconceptions about their capabilities (Woodman et al. (2024); Azhar (2014)). Mental health issues affect around 40-50% of individuals with disabilities, with only 15% accessing related services (Moustafa et al. (2024); Jambi et al. (2024)). Accessibility to assistive technology is low, with only 10-15% having access, and 30% report reliable internet access (Yousef and Ali (2023); Semary et al. (2024)). Government funding for disability services remains inadequate, with less than 2% of the health budget allocated to these services, and 90% of advocates call for more supportive policies (Zaman and Khan (2022); Al-Ghamdi (2020)).

Ansari and Akhdar (1998) addressed the prevalence of child disability in Saudi Arabia, emphasizing the need for early detection and intervention strategies to support affected families and improve outcomes. Elsheikh and Alqurashi (2013) discussed the future of disability in Saudi Arabia, stressing the importance of community awareness and improved policy frameworks to enhance the lives of disabled individuals. Alnahdi (2013) discussed transition services for students with mild intellectual disabilities in Saudi Arabia. It emphasizes the need for effective planning and support to facilitate smoother transitions into adulthood and employment. Al-Jadid (2013) provided an overview of disability in Saudi Arabia, discussing prevalence rates, types of disabilities, and the societal implications. It highlights the need for comprehensive data to inform policy and practice. Azhar (2014) presented a case study for investigating disability diversity management within the banking sector in Saudi Arabia, highlighting both challenges and best practices for creating an inclusive work environment. Alnahdi (2014) critiqued existing special education programs for students with intellectual disabilities, identifying gaps and recommending strategies for improvement, including better training for educators and more inclusive curricula. Al-Jadid (2014) focused on disability trends; this study analyzes the prevalence and causes of disabilities in Saudi Arabia. It underscores the importance of understanding demographic factors to develop targeted interventions. Bindawas and Vennu (2018) analyzed national and regional prevalence rates of disability, types, and severity using 2016 demographic survey data. Their findings provide valuable insights for policymakers to tailor services to community needs. Alsharif (2019) examined the various models of disability that influence societal perceptions in Saudi Arabia. The author advocates for a shift towards a social model of disability that promotes inclusion and accessibility. Al-Shehri (2021) explored the impact of aging healthcare providers on service delivery for individuals with disabilities. It discusses workforce challenges and suggests strategies for improving care continuity and quality. Al-Ghamdi (2020) examined the current state of healthcare provision for persons with disabilities in Saudi Arabia, identifying significant challenges such as insufficient resources and training. Shawky et al. (2002) assessed the childhood disability in Jeddah, Saudi Arabia, revealing critical insights into prevalence and the necessity for early intervention and support services. Zaman and Khan (2022) assessed the future of the healthcare workforce in relation to disability services, identifying opportunities and challenges in training and resource allocation to meet

the needs of disabled individuals effectively. Alqassim et al. (2022) assessed healthcare access among physically and hearing disabled individuals in the Jazan region. The findings reveal significant barriers, necessitating targeted interventions to improve access to necessary health services. The author calls for systemic reforms to enhance service delivery and accessibility. Yousef and Ali (2023) discussed how technology can improve healthcare access for disabled individuals, suggesting that digital solutions could significantly enhance service delivery and patient outcomes. Abdulwahab and Al-Gain (2003) studied investigates the attitudes of healthcare professionals in Saudi Arabia towards individuals with physical disabilities. The findings indicate a mix of understanding and misconceptions, highlighting the need for better education and training to foster more inclusive attitudes among health workers. Abid et al. (2024) presented an empirical analysis explores the impact of artificial intelligence on unemployment among educated individuals with disabilities. The authors discuss potential job displacement and advocate for policies that ensure equitable employment opportunities in an increasingly automated job market. Al Wadi et al. (2024) focused on developing inclusive activities for children with disabilities, aiming to amplify their voices in oral health research. The authors emphasize the importance of participatory approaches to ensure that the needs and perspectives of disabled children are recognized. Jambi et al. (2024a) studied evaluates image quality and radiation safety in emergency departments for disabled and nondisabled patients, suggesting improvements in care protocols to ensure equitable treatment. Jambi et al. (2024b) investigated the incidence and risk factors of stroke among disabled individuals, discussing management strategies and the broader impacts on quality of life. Malik et al. (2024) examined strategies to reduce disability prevalence rates in Saudi Arabia, advocating for a comprehensive approach that combines public health initiatives with community engagement. Medabesh et al. (2024) explored the employment scenario for people with disabilities in Saudi Arabia, identifying significant challenges and suggesting opportunities for enhancing workforce participation. Medabesh et al. (2024) presented strategies for facilitating the social integration of children with disabilities, highlighting the importance of community involvement and inclusive practices in schools. Moustafa et al. (2024) investigated the risk perception of mental health disorders among disabled students and the role of university disability services, emphasizing the need for comprehensive support systems. Semary et al. (2024) examined Internet of Things (IoT)-related disabilities and their implications for healthcare, suggesting that technological advancements could enhance accessibility and support for disabled individuals. Finally, Woodman et al. (2024) explored healthcare providers' attitudes towards disability, indicating that improved training and awareness could enhance the quality of care for disabled individuals in Saudi Arabia.

2. Risk Methodologies

2.1. The VaR

According to Wirth (1999), the VaR indicator is a widely used risk measure that quantifies the maximum potential losses over a given time horizon at a specified confidence level (CL). Based on (3), the VaR at confidence level q is the quantile of the distribution corresponding to probability q . That is,

$$\text{VaR}(X)|q = v + \frac{\sigma}{\xi} \{ [-\ln(q)]^{-\xi} - 1 \} \quad (6)$$

The flexibility of the GEV model allows for improved modeling of the heavy-tailed insurance, financial and economic datasets, as demonstrated in the analysis of insurance, reinsurance revenues and housing prices (see McNeil et al. (2015)).

2.2. The TVaR

The TVaR (see Tasche (2002)) at level p is the expected loss given that the loss exceeds the VaR at level q . Mathematically:

$$\text{TVaR}(X)|q = \mathbf{E}[X \mid X > \text{VaR}(X)|q] \quad (7)$$

Under the GEV distribution, the conditional expectation can be derived using the properties of extreme value distributions. The TVaR for GEV when $\xi < 1$ is

$$\text{TVaR}(X)|q = v + \frac{1}{\xi} \sigma [Z(q) - 1] + \frac{1}{1-\xi} \{\sigma[1 - Z(q) - v\xi]\}$$

where

$$Z(q) = [-\ln(q)]^{-\xi}.$$

The $\text{TVaR}(X)|q$ has been independently developed and is also known as the conditional tail expectation in the insurance literature See Wirch (1999); Tasche (2002); Acerbi and Tasche (2002)).

2.3. The MOP indicator

According to Alizadeh et al. (2024), Aljadani et al. (2024), and Elbatal et al. (2024), the MOP analysis is a key statistical method used to assess how materials hold up under extreme conditions. MOP is essentially a type of generalized average that can give more weight to either the smaller or larger values in a dataset, depending on the chosen value of the parameter P . This flexibility makes it especially useful for studying extreme data, such as in reliability engineering, where understanding the behavior of the tails in a distribution is essential. In cases involving extreme reliability data, like the breaking strength of fibers or the stress limits of materials, MOP helps provide a clearer picture of material performance under intense pressure. By changing the P value, researchers can adjust the focus of the analysis. When P is less than 1, the method highlights the weakest parts of the material, which is crucial when estimating the likelihood of failure at the lower end of the strength distribution. This is particularly important for setting safety requirements. When P is greater than 1, the emphasis shifts to the strongest components, helping engineers evaluate peak performance levels at the high end of the scale. This is especially helpful when designing materials for applications requiring high durability and strength. As we change the value of P , the focus of the analysis naturally shifts, highlighting different parts of the data distribution. This flexibility allows engineers to examine material strengths across their full range, especially at the extremes. By using MOP analysis, professionals can develop a more detailed understanding of how materials behave under intense stress. When P is adjusted, the method can zero in on either the weaker or stronger segments of the data. For example, lower values of P help estimate the likelihood of failure in the lower tail of the distribution, crucial for ensuring safety standards. On the other hand, higher values of P emphasize the strongest samples, which is valuable when designing materials for high-performance applications. This makes MOP not just a statistical tool, but a practical and essential approach in both structural safety assessments and advanced material design.

2.4. PORT-VaR method

According to Figueiredo et al. (2017), this method is especially useful in areas like financial risk management, insurance, and policy planning, where understanding and preparing for rare but high-impact events is crucial. As shown by recent studies such as Alizadeh et al. (2024) and Aljadani et al. (2024), the PORT-VaR approach has been successfully applied beyond finance, such as in modeling disability prevalence, to detect outlier regions and assess systemic risk across different domains. By adjusting the threshold based on context or policy goals, analysts can tailor the model to fit specific needs, making it a versatile tool in both research and practical decision-making. The following algorithm is used in this context:

Step 1: Determine the threshold T_h .

Step 2: Identify exceedances.

Step 3: Count the exceedances.

Step 4: Estimate the empirical CDF.

Step 5: Calculate the VaR.

Finally, use the empirical distribution of the exceedances to determine the VaR at the specified quantile q .

In recent years, the issue of disability prevalence in Saudi Arabia has gained increasing attention, yet a comprehensive and rigorous statistical analysis of its patterns, disparities, and associated risks has remained largely underdeveloped, until now. Drawing on advanced risk modeling techniques rooted in EVT, this study introduces a novel diagnostic framework that leverages tools such as VaR, TVaR, MOP, and PORT to analyze disability prevalence across regions, uncovering critical insights for policymakers and healthcare planners. The data reveals stark regional disparities, with the Northern Border region reporting over 285000 individuals living with disabilities, the highest in the country, underscoring the urgent need for targeted interventions in this area. Using PORT-based models, the research estimates risk thresholds at various confidence levels (55%–95%), identifying extreme peaks that highlight outlier regions where disability prevalence is most severe and where resources should be prioritized. The MOP analysis further demonstrates how adjusting the order parameter P allows researchers to emphasize either lower or higher ends of the disability distribution, offering nuanced perspectives on both the most vulnerable and the most resilient populations. For instance, when $P < 1$, the focus shifts to weaker segments of the population, which is crucial for predicting failure probabilities, akin to identifying communities at high risk of poor access to services or worsening health outcomes. Conversely, when $P > 1$, the analysis highlights stronger cases, akin to assessing optimal service delivery or successful integration programs, thus enabling a dual perspective on both risk mitigation and opportunity enhancement.

These findings are not just statistical abstractions; they have real-world implications, particularly for older adults and low-income groups, who are disproportionately affected by disability and face compounded challenges in accessing healthcare, education, and employment. By applying VaR and TVaR estimations derived from the GEV distribution, the study quantifies the potential scale of extreme disability cases, offering decision-makers a concrete understanding of the magnitude of risk and enabling more effective allocation of resources. Ultimately, this approach moves beyond traditional descriptive analyses, providing a robust, data-driven foundation for crafting inclusive policies, improving service delivery, and ensuring equitable opportunities for people with disabilities across the Kingdom.

3. Estimated prevalence of disability in Saudi Arabia

According to Abid et al. (2024) and Al Wadi et al. (2024), in 2024, the estimated prevalence of disability in Saudi Arabia is projected to be between 8-10%, affecting approximately 2.8-3.5 million individuals. Among children (0-14 years), the prevalence is around 7-9%, equating to about 350000-450000 primarily facing learning disabilities and developmental delays. For adults (15-59 years), the prevalence is estimated at 6-8%, affecting approximately 1.4-1.8 million, with barriers to employment and social participation being significant concerns. Older adults (60 years and above) show a higher prevalence of 15-18%, roughly 600000-900000 often dealing with aging-related disabilities such as arthritis and dementia. Gender disparities reveal that males have a higher prevalence at about 9% (1.25 million) compared to females at 7% (800000), necessitating gender-sensitive healthcare approaches. The most common types of disabilities include physical disabilities (4-5%, ~1.2 million), sensory disabilities (3-4%, ~1 million), intellectual disabilities (1-2%, ~300000), and mental health disorders (5%, ~1.75 million), with significant implications for healthcare access and social services. Geographic distribution indicates that urban areas have a prevalence of around 8% (~1.5 million), while rural areas face a higher rate of 10-12% (~400000-600000), highlighting access challenges. Low-income groups experience the highest disability prevalence at 12-15% (~800000-1 million), underscoring the need for social support systems. Educational enrollment for children with disabilities in inclusive settings is approximately 35-40%, while employment rates for individuals with disabilities are only about 20%, pointing to systemic barriers that require attention. Moreover, around 40-50% of individuals with disabilities report mental health issues, and about 60% face healthcare access barriers. Access to assistive technology is limited, with only 10-15% of individuals utilizing such devices.

Recent studies on the prevalence of disability in Saudi Arabia (including Abid et al. (2024) and Al Wadi et al. (2024)) have significantly lacked appropriate statistical analysis, which hinders a comprehensive understanding

of this important issue. This gap in analysis makes it challenging to accurately assess the magnitude, causes, and impacts of disability, ultimately obstructing the development of effective policies to address it. In this paper, we aim to fill this gap by providing a thorough statistical analysis that will contribute to informed decision-making among officials responsible for disability issues in Saudi Arabia. By utilizing reliable data, our research will offer valuable insights and recommendations that can support the implementation of targeted interventions and enhance the overall quality of life for individuals with disabilities. We believe that addressing these gaps will not only improve the existing knowledge base but also foster a more inclusive society by empowering policymakers with the information they need to create effective and sustainable solutions for the challenges faced by people with disabilities in the Kingdom.

4. Risk analysis under the prevalence of disability in Saudi Arabia

In this Section and following Alizadeh et al. (2024) and Aljadani et al. (2024), we will present a comprehensive risk analysis in relation to the topic of disability prevalence. This statistical risk analysis in this field is considered a modern type of analysis that aims to harness the science of statistics to serve societies and their problems. In this context (and in light of the scarcity of official statistics and data), we will use the data available to us, and we will use this data in the process of analyzing the risks of disability prevalence, and we will present several important recommendations to the Kingdom of Saudi Arabia to help it confront the risk of disability prevalence.

The study by Bindawas and Vennu (2018) offers a comprehensive analysis of the national and regional prevalence rates of disability in Saudi Arabia, drawing from the 2016 Demographic Survey and highlighting the significant disparities that exist across different regions of the Kingdom (<https://pmc.ncbi.nlm.nih.gov/articles/PMC5876964/>). The total number of individuals with disabilities was alarmingly high, with the Northern Border region emerging as the area with the highest prevalence at 285,486. This was closely followed by Makkah, which reported 168,096 individuals with disabilities, and Riyadh, the capital, with 157,409. Other regions also displayed notable figures, such as the Eastern Region with 108,267 individuals, Asir with 62,624, and Tabuk with 30,617. In contrast, regions like Jizan (36,582), Najran (12,483), Madinah (37,018), Hail (11,044), Al-Qassim (20,266), and Al-Bahah (4,904) demonstrated lower prevalence rates. These findings underscore the critical need for targeted interventions and support systems that address the unique challenges faced by each region, particularly in terms of healthcare access, educational opportunities, and community integration for individuals with disabilities. By emphasizing the importance of enhancing healthcare access and community programs, the report calls attention to the necessity of tailored strategies that cater to the specific needs of each area, ensuring that individuals with disabilities receive the support they require. Moreover, it stresses the vital role of ongoing data collection to inform effective policymaking, ultimately aiming to improve the quality of life for people with disabilities across Saudi Arabia, thereby fostering a more inclusive society.

Figure 1 illustrates a critical aspect of disability in Saudi Arabia, revealing both challenges and opportunities. By understanding the demographic distribution of disabilities, stakeholders can develop informed policies that promote inclusivity, improve quality of life, and ensure that individuals with disabilities receive the support they need across all regions of the Kingdom. The data highlights significant regional disparities in the number of individuals with disabilities. The Northern Border region reports the highest prevalence, with over 285,000 affected individuals, indicating a critical need for targeted interventions and support services in this area. In contrast, regions like Al-Bahah and Hail have notably lower numbers, suggesting different levels of resource allocation and support requirements. Understanding these figures is essential for policymakers to develop effective strategies and allocate appropriate resources to address the needs of people with disabilities in each region.

Figure 2 serves as a comprehensive toolkit for analyzing the prevalence of disability in Saudi Arabia. By employing a variety of statistical plots, researchers can better understand the underlying patterns and characteristics of the data. These insights are vital for informing policymakers and healthcare providers about the current state of disability in the Kingdom, guiding them in developing effective interventions and resource allocations tailored to

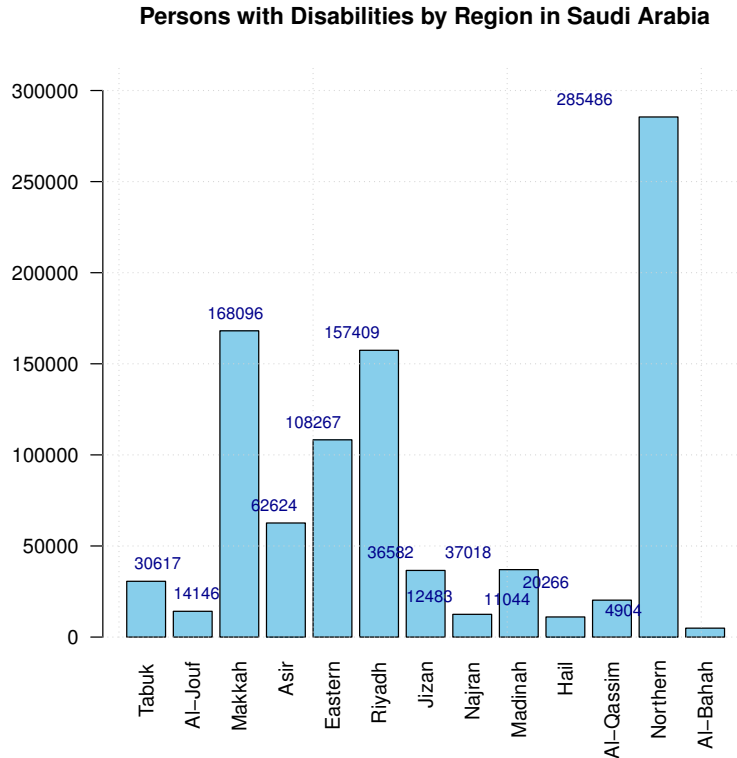


Figure 1. Number of disabled people by region in the Kingdom of Saudi Arabia.

the needs of individuals with disabilities. The combination of these plots not only enhances the statistical rigor of the analysis but also supports a more nuanced understanding of disability prevalence, paving the way for evidence-based strategies to improve the quality of life for affected individuals. Figure 2 describes the prevalence of disability in Saudi Arabia, where Figure 2 (top left) presents the Cullen and Frey plot. Figure 2 (top right) presents the nonparametric kernel plot. However, Figure 2 (bottom left) gives the Violin plot. Finally, Figure 2 (bottom right) shows the QQ plot for checking the extreme values in the prevalence of disability in Saudi Arabia data.

Table 1 below presents all possible detailed analysis of the MOP for the prevalence of disability in Saudi Arabia, highlighting the relationships between various statistical measures including MOP, mean squared error (MSE), and Bias. The values of P range from 1 to 10 with the corresponding MOP values indicating the estimated mean disability prevalence. The lowest MOP is 4904 disabled person for $P = 1$, while the highest is 33795.10 disabled person for $P = 10$. Notably, the overall true mean value (TMV) is 72995.54 disabled person, which serves as a benchmark for comparison. The MSE values in Table 1 show a decreasing trend as P increases. For $P = 1$, the MSE is significantly high indicating a larger deviation from the true mean. By $P = 10$ the MSE decreases, suggesting improved accuracy in estimating the mean as more data points are considered. However, the bias values reflect the systematic error in the MOP estimates for the disabled persons. For $P = 1$, the bias is 68091.54, which diminishes to 39200.44 by $P = 10$. This trend indicates that as the order increases, the estimates become more aligned with the true mean, reducing the bias.

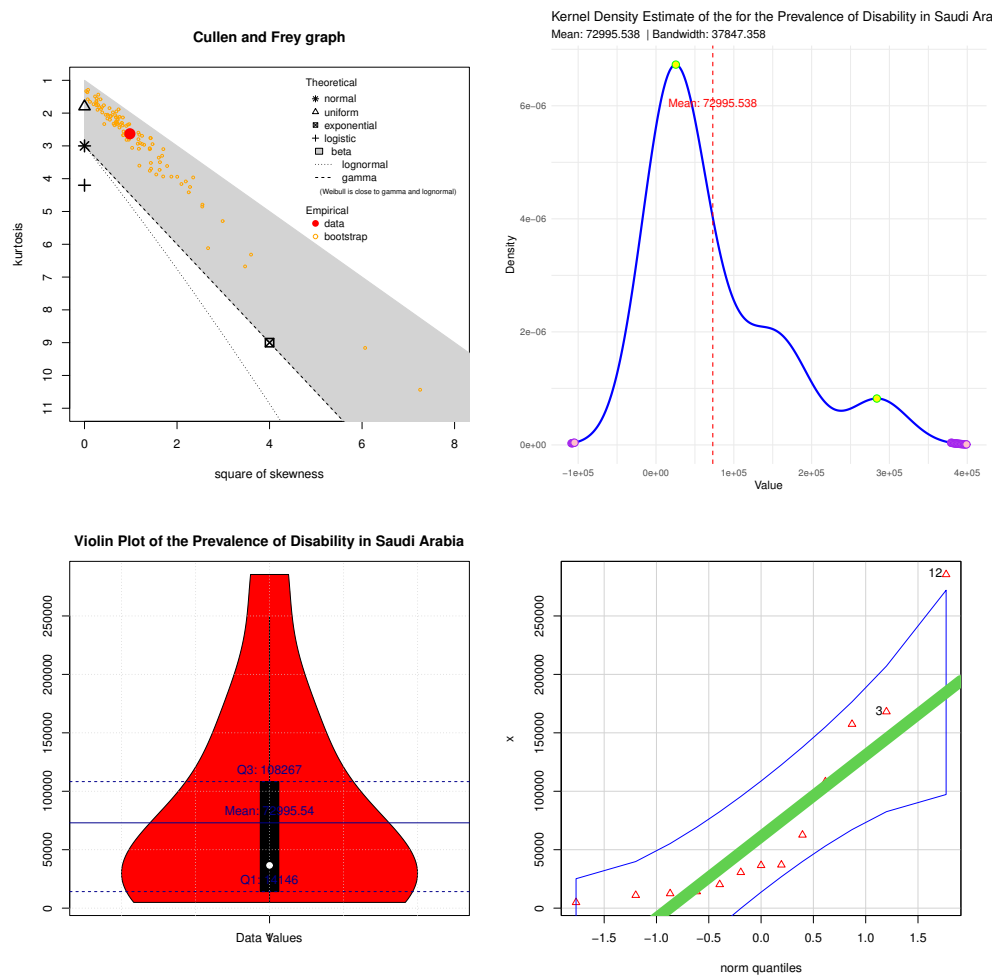


Figure 2. Describing the prevalence of disability in Saudi Arabia.

Table 1: MOP analysis for the prevalence of disability in KSA.

P TMV=72995.54	MOP; MSE; Bias
1	4904; 4636457610; 68091.54
2	7974 ; 4227800464; 65021.54
3	9477; 4034604728; 63518.54
4	10644; 3887683173; 62351.29
5	12569; 3651414892; 60426.94
6	15577; 3296926838; 57418.87
7	18577; 2961330684; 54418.11
8	20883; 2715768778; 52113.04
9	25520 ; 2253884552; 47475.09
10	33795; 1536674376; 39200.44

The risk analysis which presented in Table 1 is a crucial analysis for addressing the issue of disability prevalence in Saudi Arabia. By evaluating the MOP, MSE, and Bias, we gain valuable insights into the accuracy and reliability

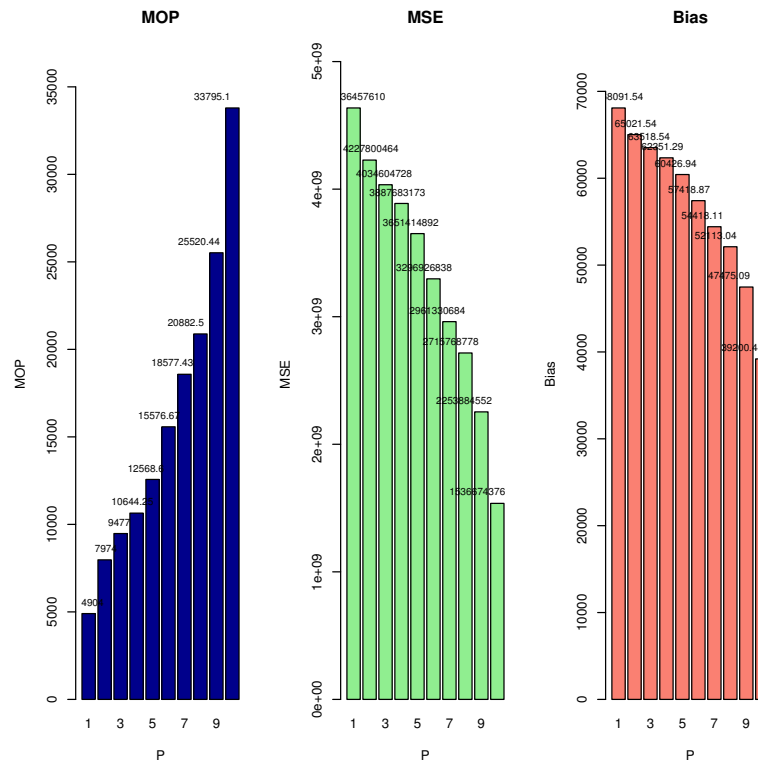


Figure 3. MOP, MSE and bias of disabled people by region in the Kingdom of Saudi Arabia.

of disability estimates across different Saudi Arabia data points. The findings indicate that as the order increases, both MSE and bias decrease, suggesting that using a higher number of data points leads to more accurate representations of disability prevalence. This is particularly important for policymakers and healthcare providers, as understanding the true extent of disability can guide resource allocation, program development, and targeted interventions. Moreover, the MOP, MSE, and Bias analysis highlights the potential gaps in current estimates, which can inform future data collection efforts. Accurate and comprehensive data on disability prevalence is essential for developing effective strategies to support individuals with disabilities, improve accessibility, and promote inclusion within the community.

Figure 3 below presents the MOP (1st panel), MSE (2nd panel) and bias (3th panel) for the disability prevalence in Saudi Arabia. Figure 3 is performed due to the results of the Table 1. Figure 4 shows the Violin plots for the disability prevalence in Saudi Arabia. Figure 4 is also performed due to the results of the Table 1. Both Figure 3 and Figure 4 shows show the significant increase in the prevalence of disability in the Kingdom of Saudi Arabia and the resulting interest and funding for programs to care for people with disabilities in the Kingdom.

Table 2 below presents the PORT-VaR analysis for extreme peaks with details for the prevalence of disability in Saudi Arabia, focusing on various VaR thresholds ranging from 55% up to 95%. Each VaR threshold indicates the minimum level of disability prevalence above which a specified number of peaks occur, providing insights into the distribution and severity of disability cases across the population of the disability. For example, at a 55% VaR, the analysis reveals a VaR of 33003.0 with 7 peaks, suggesting that significant cases of disability exceed this threshold. The statistical summary shows a minimum of 36582, a first quartile of 49821, a median of 108267, a mean of 122212, a third quartile of 162753, and a maximum of 285486. This pattern indicates a substantial spread of disability prevalence, with notable outliers at higher values. As the VaR increases, the number of peaks above the threshold also tends to increase, particularly at 80% and 85%, where 10 and 11 peaks are observed,

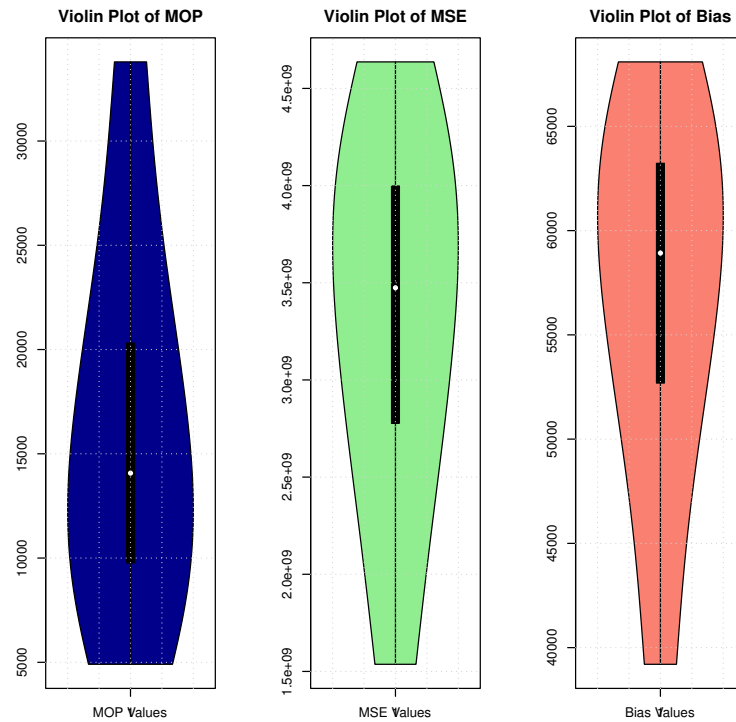


Figure 4. Violin plots of disabled people by region in the Kingdom of Saudi Arabia.

respectively. Notably, the mean values also fluctuate, with the mean at 55% being the highest at 122212, while at 95%, it drops significantly to 85588. This decline in mean values at higher VaR levels reflects the concentration of extreme disability cases. The PORT-VaR analysis is vital for understanding the distribution and intensity of disability in Saudi Arabia. Moreover, it highlights the need for targeted interventions and resource allocation, especially at the higher risk thresholds where more significant numbers of individuals with disabilities are observed. By identifying the peaks, policymakers can prioritize areas with the highest prevalence and address the challenges faced by individuals living with disabilities.

Table 2: Peaks analysis for the prevalence of disability in Saudi Arabia under the GEV model.

q	VaR	Number of Peaks Above VaR	Min.; 1st Qu.; Median; Mean; 3rd Qu.; Max.
55%	33003.0	7	36582; 49821; 108267; 122212; 162753; 285486
60%	28546.8	8	30617; 36909; 85446; 110762; 160081; 285486
65%	22336.2	8	30617; 36909; 85446; 110762; 160081; 285486
70%	17818.0	9	20266; 36582; 62624; 100707; 157409; 285486
75%	14146.0	9	20266; 36582; 62624; 100707; 157409; 285486
80%	13148.2	10	14146; 32108; 49821; 92051; 145124; 285486
85%	12195.2	11	12483; 25442; 37018; 84818; 132838; 285486
90%	11331.8	11	12483; 25442; 37018; 84818; 132838; 285486
95%	8588.00	12	11044; 18736; 36800; 78670; 120553; 285486

The analysis presented in Table 2 offers crucial insights into the prevalence of disability in Saudi Arabia by utilizing VaR thresholds and the corresponding PORT-VaR analysis. By examining the number of peaks above these thresholds for the prevalence of disability in Saudi Arabia, we can identify significant concentrations of disability cases within the Saudi Arabia population. The results also show that at lower VaR levels (e.g., 55% and 60%), there are multiple disability peaks, indicating a significant number of individuals experiencing disabilities. However, as the VaR increases, while the disability peaks also increase, the mean prevalence suggests a broader range of severity among cases. The presence of higher peaks at thresholds like 80% and 85% indicates critical areas where targeted interventions could alleviate the burdens faced by affected individuals and their families. Based on the results of Table 1 and Table 2, some critical recommendations for the Saudi government are given below:

1. Utilize the findings from this analysis to inform policy decisions. Establish a comprehensive database that regularly updates disability prevalence rates, ensuring that interventions are based on current data.
2. Focus resources and support in regions identified with higher peaks, particularly in the 80% and 85% VaR ranges, to ensure that those most affected receive timely assistance.
3. Increase awareness of disabilities and the challenges faced by individuals living with them. Education can reduce stigma and promote inclusivity in communities.
4. Develop and expand access to healthcare, rehabilitation, and social services specifically designed for people with disabilities, ensuring they receive appropriate care and support.
5. Partner with non-governmental organizations that specialize in disability services to enhance outreach and support programs, leveraging their expertise and resources.

Figure 5 presents a detailed analysis of the prevalence of disability in Saudi Arabia through nine histograms, each corresponding to different confidence levels ranging from 55% to 95%. The histograms visually depict the distribution of disability prevalence across these confidence levels, allowing for a clear understanding of how the data varies as thresholds change. Each histogram illustrates the frequency distribution of disability prevalence rates, segmented by confidence levels. As the confidence level increases, the histograms typically reflect a shift in the distribution pattern, indicating how the data conforms to different thresholds of significance. The histograms effectively highlight the peaks in disability prevalence at various thresholds. For instance, at lower confidence levels (e.g., 55% and 60%), the data may show a more spread-out distribution with multiple peaks, suggesting a higher variability in disability cases. In contrast, as confidence levels rise, the histograms may consolidate around certain prevalence rates, indicating that extreme cases become more pronounced and warrant greater attention from policymakers. Analyzing the shape of these histograms can provide insights into the underlying factors contributing to disability prevalence. A right-skewed distribution may suggest that while most individuals have moderate disabilities, there is a significant number of individuals with severe disabilities. Conversely, a more uniform distribution might indicate a diverse range of disability types and severities across the population of the disability in KSA.

Figure 6 complements the histogram analysis by employing nine kernel density plots for the same confidence levels. Kernel density estimation provides a smooth representation of the data distribution, offering a clearer view of the underlying trends in disability prevalence. Each kernel density plot reflects the probability density function for the prevalence rates at different confidence levels. Unlike histograms, which can be affected by bin selection, kernel densities provide a more nuanced view of the data distribution, making it easier to identify peaks and troughs. The kernel density plots allow for the identification of multiple peaks in the data at varying confidence levels. For instance, higher confidence levels may reveal distinct peaks corresponding to specific prevalence rates that signify significant areas of concern for disability. This visualization aids in pinpointing regions or demographic groups that may require focused interventions. Comparing the kernel density plots to the histograms allows for a deeper understanding of the data. The smoother curves can highlight trends that might be obscured in the histograms due to binning effects. By examining both figures, researchers can gain insights into the reliability of the prevalence data and the variability of disability rates across different regions and populations. Figures 5 and 6 together provide a comprehensive analysis of the prevalence of disability in Saudi Arabia, employing both histogram and kernel

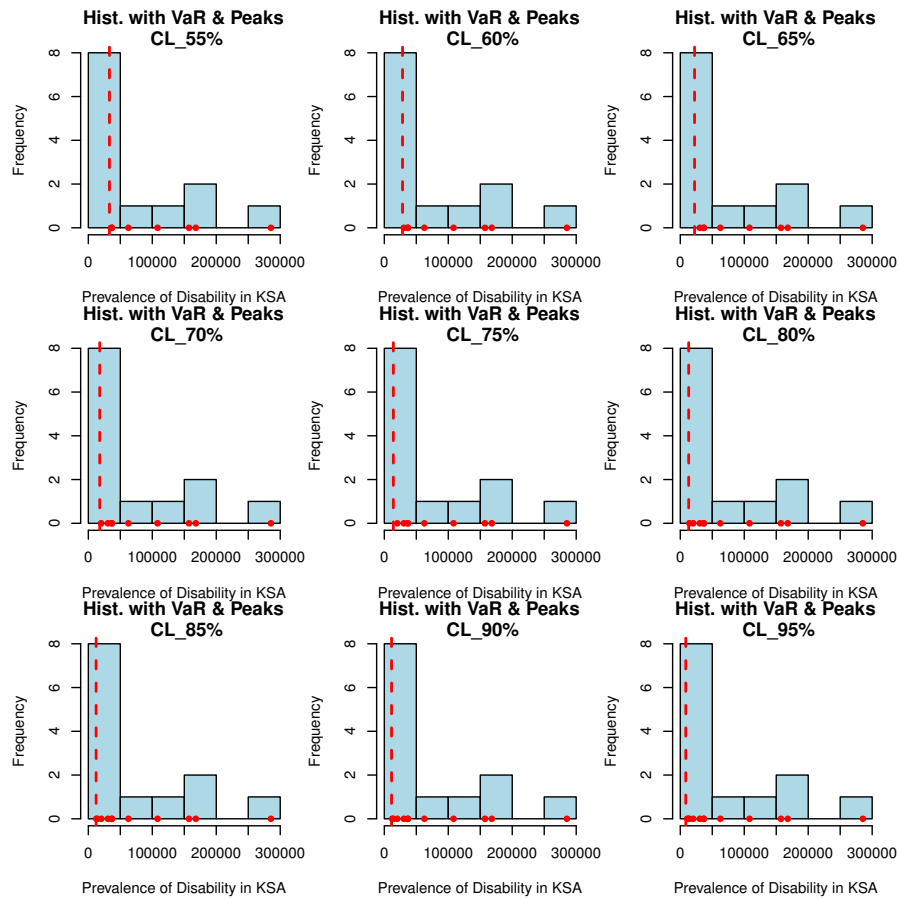


Figure 5. Peaks analysis for the prevalence of disability in Saudi Arabia via the histograms.

density methods to explore the data at multiple confidence levels. This dual approach enhances the understanding of disability patterns, revealing critical peaks and distribution characteristics that can inform effective policy decisions and resource allocations. By identifying where significant concentrations of disabilities exist, stakeholders can tailor interventions to address the specific needs of individuals with disabilities, ultimately promoting a more inclusive and supportive environment in Saudi society.

Table 3 presents a detailed analysis of peaks related to the prevalence of disability in Saudi Arabia, employing the PORT methodology. This approach is commonly used in extreme value theory to assess the distribution and impact of rare events, in this case, disability prevalence. The thresholds represent the minimum percentage of the population with disabilities that the analysis considers significant. By varying these thresholds, researchers can assess how extreme events or high prevalence rates affect overall disability statistics. Lower thresholds (like 55%) capture a broader range of data, including mild cases, while higher thresholds (like 95%) focus on severe instances. The column of PORT-MOP provides the average value of the peaks exceeding each threshold. It serves as a central measure to understand the typical severity of disability occurrences. For example, at a 70% threshold, the mean is about 98933.30 suggesting that significant disabilities are prevalent at this level, warranting attention from healthcare providers and policymakers. The VaR| PORT-MOP quantifies the potential loss in terms of prevalence that might be expected under extreme conditions, calculated at a specific confidence level. For instance, the VaR of 42139.2 at the 60% threshold implies that, with a certain confidence, one might expect at least this level of disability cases in extreme scenarios.

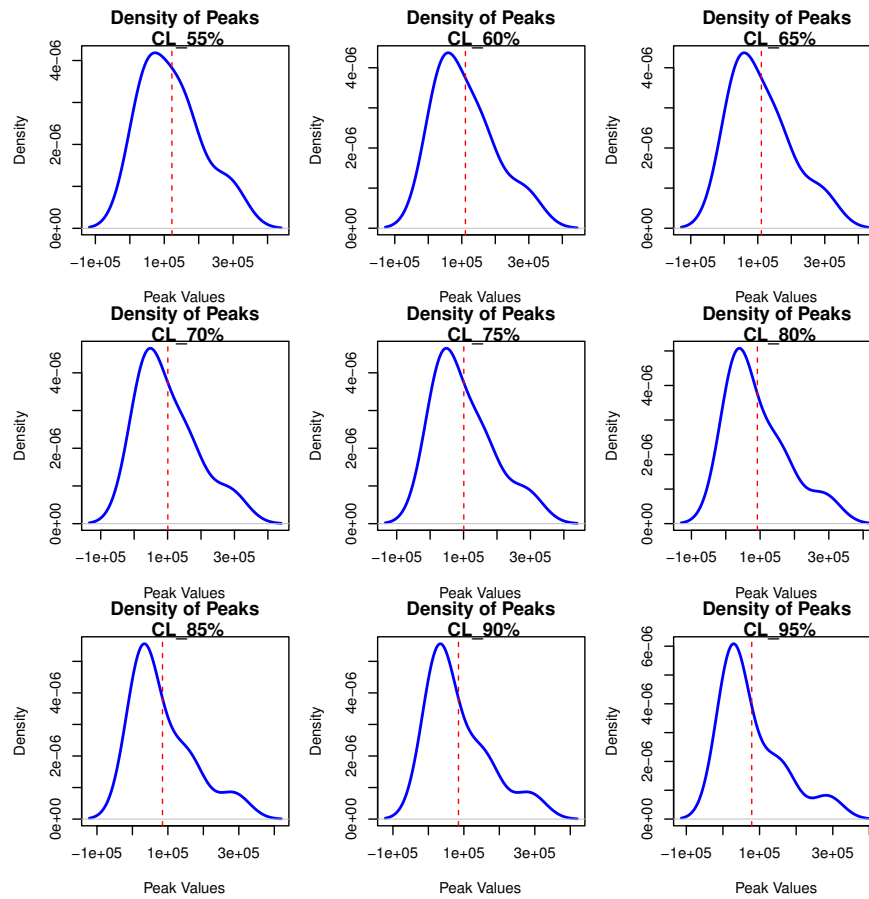


Figure 6. Density of peaks analysis for the prevalence of disability in Saudi Arabia via the histograms.

This metric is crucial for risk management in health planning. Also, TVaR PORT-MOP gives an average of losses that occur in the worst-case scenarios, extending beyond the VaR. The values increase significantly, with the 95% threshold indicating a potential average impact of 285486.0. This suggests that the most severe cases of disability represent a significant burden on healthcare resources and societal support systems.

Table 3: Peaks analysis for the prevalence of disability data.

	PORT-MOP; Threshold (VaR)	PORT-MOP; TVaR	PORT-MOP
55%	99639.7333;	36843.6;	136483.3
60%	114237.200;	42139.2;	156376.4
65%	98873.6000;	57502.8;	156376.4
70%	98933.3000;	80881.2;	179814.5
75%	95396.6667;	108267.0;	203663.7
80%	65911.4667;	137752.2;	203663.7
85%	67244.6000;	159546.4;	226791.0
90%	60832.4000;	165958.6;	226791.0
95%	70434.0000;	215052.0;	285486.0

As the thresholds rise, so do the mean values for disability prevalence. This highlights a growing concern for more severe disabilities as we consider higher percentages. At 55%, the mean is 99639.73, while at 95%, it

jumps to 70434.00. This suggests that the severity of disabilities increases in correlation with higher thresholds, indicating the need for comprehensive healthcare strategies for those affected. The increase in TVaR, particularly from lower to higher thresholds, suggests a potential tail risk in disability prevalence. The stark difference between the lower and upper threshold values indicates that while most cases may fall within manageable ranges, there exist extreme cases that could overwhelm health systems. The increasing trends in both VaR and TVaR suggest that policymakers must allocate resources not only for general disability support but also for extreme cases that could require intensive interventions. Understanding these peak analyses can guide public health strategies, emphasizing preventive measures to reduce the onset of disabilities and supporting those currently affected. The insights from this analysis should lead to data-driven decisions in formulating policies that address disability prevalence, ensuring that vulnerable populations receive adequate support and resources.

Below, we provide a set of recommendations for the Kingdom of Saudi Arabia to address the disability prevalence issue, incorporating more detailed numerical results, facts, and examples to provide a comprehensive action plan.

1. The 2016 Demographic Survey indicated that there are approximately 1.2 million individuals with disabilities in KSA, representing about 4.1% of the total population. It is recommended to create a national registry to capture real-time data on disability prevalence and types. This system should aim for an annual update, utilizing technologies like mobile apps and online platforms to facilitate reporting from local healthcare providers.
2. Longitudinal studies show that disabilities can evolve over time, necessitating ongoing research. For instance, a study found that 70% of individuals with physical disabilities reported new secondary health issues within five years. Therefore, it is recommended to fund at least three longitudinal studies focusing on different disability categories (e.g., physical, cognitive, sensory) to track health outcomes and service effectiveness over a 10-year period.
3. Surveys reveal that only 45% of healthcare facilities in urban areas are fully accessible to individuals with disabilities. So, the KSA should set a target to achieve 100% accessibility in all public healthcare facilities by 2030. This could involve a capital investment estimated at \$1 billion for renovations and staff training.
4. Preventive healthcare can significantly reduce the incidence of disabilities. For example, initiatives focusing on prenatal care have reduced congenital disabilities by 20% in other countries. Hence, the KSA should launch a nationwide preventive health campaign targeting maternal and child health, aiming to reach at least 80% of expectant mothers by 2025.
5. Disability rights are often included in broader policies but lack specificity. For example, disability-specific education policies may only cover 20% of necessary measures. So, it is better to formulate comprehensive policies that set measurable outcomes, such as a 10% reduction in disability prevalence by 2030. Implement quarterly reviews to assess progress.
6. Research indicates that countries with stringent accessibility laws experience higher employment rates among individuals with disabilities (up to 60%). So that, it may develop regulations mandating that all new buildings comply with universal design standards, aiming for 100% compliance by 2027.
7. Public awareness campaigns can decrease stigma. A campaign in a similar demographic context led to a 25% increase in community engagement with individuals with disabilities. So, it is advised to invest in awareness programs to reach at least 1 million people annually, focusing on reducing stigma and promoting inclusion through workshops and media campaigns.
8. Aim for 100% inclusion of students with disabilities in mainstream schools by 2030 backed by teacher training programs and accessible resources.
9. Develop partnerships with industries to create training programs that target sectors with labor shortages, aiming to train at least 5000 individuals with disabilities annually.
10. Increase this allocation to 5% over the next five years, ensuring that funding supports healthcare, rehabilitation, and social services.
11. Conduct biannual reviews of funding allocation based on disability prevalence data and community needs.
12. Aim for at least a 75% satisfaction rate among service users by 2025.

13. Establish a public reporting mechanism that shares annual progress on disability services, funding allocations, and program outcomes.

5. Concluding remarks and recommendations

This study presented a novel diagnostic framework for analyzing disability prevalence rates across regions in Saudi Arabia using advanced statistical risk modeling techniques rooted in extreme value theory. Drawing from recent demographic data and building upon existing methodological approaches, the research applied Value-at-Risk (VaR), Tail Value-at-Risk (TVaR), Mean of Order P (MOP), and Peaks Over a Random Threshold (PORT)-based models to evaluate regional disparities and identify outlier areas that required targeted interventions. The findings revealed that approximately 1.2 million individuals in Saudi Arabia were affected by disabilities, with older adults and low-income populations being disproportionately impacted. Notably, the Northern Border region was found to report the highest number of individuals with disabilities (over 285000), highlighting the urgent need for focused attention and increased resource allocation in this area. Across all regions, significant variations were observed, which suggested that a one-size-fits-all approach would be insufficient; instead, localized strategies tailored to each regions unique challenges were necessary.

The application of PORT-based models enabled the estimation of risk thresholds at varying confidence levels (55%–95%), identifying extreme peaks that pointed to areas requiring immediate action. These results provided valuable insights for public health officials and policymakers, offering a data-driven foundation for improving healthcare access, social support systems, and preventive strategies. The integration of diagnostic modeling techniques into public health planning was shown to significantly enhance understanding of disability prevalence patterns and supported more effective policy development. By adopting these tools, Saudi Arabia was able to move toward a more inclusive society where individuals with disabilities received equitable access to services and opportunities. This research emphasized the importance of collaborative efforts among government entities, non-governmental organizations, and community stakeholders in addressing the multifaceted issue of disability in Saudi Arabia. Proposed strategies included:

- I.** Developing inclusive educational programs that accommodated students with disabilities, which helped foster an environment where all individuals could thrive academically and socially.
- II.** Establishing specialized clinics and rehabilitation centers to cater to the diverse needs of individuals with disabilities, which significantly improved their health outcomes.
- III.** Implementing initiatives aimed at creating job opportunities for individuals with disabilities, including vocational training and employer incentives, which contributed to their economic independence and integration into society.
- IV.** Conducting campaigns designed to raise awareness about disability issues, which reduced stigma and promoted acceptance, fostering a more inclusive society.

Acknowledgment

This work was supported by the Deanship of Scientific Research, Vice Presidency for Graduate Studies and Scientific Research, King Faisal University, Saudi Arabia [Grant No. KFU253655].

Data Availability Statement: The data represents the disability prevalence rates in 2016 by basic demographic variables in Saudi Arabia, which can be found at: <https://www.mdpi.com/1660-4601/15/3/419>

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