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Economic Order Quantity Model with Green Technology Approach Considering Time Dependent Demand in Fuzzy Environment

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Abstract In this study, we consider the problem of determining the effect of carbon tax and green technologies over infinite planning horizon. We have developed two economic order quantity models considering constant demand and exponential time dependent demand. Carbon emissions may also occur from storing undelivered or unsold products due to some factors. Investment in green technology is also considered. In this article, we fuzzify the inventory parameters such as demand, ordering cost, holding cost and the amount of carbon emitted when storing the products. To incorporate uncertainty, pentagonal fuzzy numbers are utilized to fuzzify the parameters of the inventory system. The graded mean integration method is used for defuzzification. The optimal values of the order quantity, cycle time and optimal inventory cost in both crisp and fuzzy sense is determined for both the models. Numerical illustrations are given to demonstrate the solution procedure and the sensitivity of various parameters are analysed. When the demand is constant, our results implies that 4.94% savings can be obtained by investing in green technology and reducing carbon tax. A significant percentage (4.39%) of reduction in the total cost can also be achieved by green investment, when the nature of the demand is exponentially time dependant.

Keywords Inventory,Time Dependent Demand, Infinite Planning Horizon, Pentagonal Fuzzy number, Green Technology

AMS 2010 subject classifications 90B05, 94D05

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1. Introduction

Constant demand refers to steady, predictable demand, which simplifies inventory planning and minimizes costs. It is suitable for stable products with regular consumption patterns, but may not account for market fluctuations. Exponential time-sensitive demand involves rapid growth or decline, often driven by factors such as promotions or seasonality. It requires flexible inventory management to capture sales opportunities during spikes and prevent overstocking during declines. This is critical for perishable, seasonal, or trending products but involves more complex forecasting. Effective inventory strategies often combine approaches for both types of demand, leveraging technology and safety stock to balance cost efficiency and responsiveness. Sarkar [12] established an EOQ model considering finite replenishment rates, where both demand and deterioration rates change over time. Shukla et al. [15] developed an inventory model for deteriorating products with exponentially growing demand. This model allows for partial backorders during stockouts. Tripathi and Mishra [17] developed a model for situations where demand increases linearly with time, and holding costs change over time. Tripathi et al. [18] investigated an EOQ model with demand increasing exponentially over time under variable deterioration. Tripathi and Mishra [19] developed an inventory model with constant and exponential time-dependent demand.

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Carbon emissions from inventory storage have a significant impact on sustainable inventory management. Various studies have explored strategies to minimize these emissions while maintaining profitability and efficiency in inventory systems. Managing carbon emissions in inventory systems requires a comprehensive approach that integrates cost considerations with environmental impacts. By adopting integrated models and investing in green technologies, businesses can achieve more sustainable inventory practices while complying with regulatory requirements. Hincent and Bironneau [4] explained an EOQ model that is carbon-constrained, which enables a company to maximize its profit while considering and managing its carbon emissions. Huang et al. [6] examined the effects of carbon regulations and green technology on a two-echelon supply chain, accounting for carbon emissions generated during manufacturing, shipping, and storage processes. Priyan [9] investigated the effect of green investment to reduce carbon emission in an imperfect production system. Toptal et al. [16] analyzed an inventory model that integrates investment in carbon emission reduction. Wang and Song [21] focused on pricing tactics for a dual-channel supply chain, emphasizing the role of green investment in the context of uncertain demand.

To deal with uncertainties and imprecision in various inventory parameters such as costs, demand, etc., the concept of fuzziness provides a robust framework in inventory management. This approach enhances decision-making, optimizes costs, and improves profitability by allowing for more flexible and adaptable inventory models. In the modeling of uncertain and imprecise parameters, fuzzy numbers are often employed to represent vagueness in a structured manner. The most commonly used membership functions are triangular, trapezoidal, and pentagonal fuzzy numbers. Triangular fuzzy numbers (TFN) are simple and computationally efficient, but they provide only a single peak with linear rise and fall. This oversimplification may fail to capture the realistic uncertainty patterns of many practical problems. Trapezoidal fuzzy numbers (TrFN) extend the triangular form by introducing a flat plateau, thereby allowing a range of equally possible values. While this improves flexibility, the plateau assumes uniform plausibility across the interval, which is not always accurate. Pentagonal fuzzy numbers (PFN) provide a more refined representation of uncertainty by introducing five defining points. This structure enables the model to represent both a specific most-likely value (the central point) and a tolerance range around it.

Hemalatha and Annadurai [1] developed an inventory model with advance payment under a fuzzy environment. Hemalatha and Annadurai [2] framed a fuzzy mathematical model of an integrated production- distribution inventory model for deteriorating items. Hemalatha and Annadurai [3] designed an inventory model in which the carrying cost, ordering cost, and replenishing processing cost are assumed to be pentagonal fuzzy numbers. A fuzzy production model for deteriorating items was developed by Indrajitsingha et al. [7], considering demand to be constant. Kaur [8] discussed about pentagonal fuzzy numbers and their arithmetic operations. Priyan et al. [10] examined an EOQ inventory system with advance payment considering fuzzy parameters. Roy and Maiti [11] established the fuzzy EOQ model with demand-dependent unit cost and limited storage capacity. Sarkar and Mahapatra [13] developed a fuzzy inventory model with variable lead time and fuzzy demand with periodic reviews. Shaikh et al.[14] developed an inventory model for perishable goods with variable demand in a fuzzy environment. The model incorporates permissible delay in payments, allowing for partial backlogging of shortages, and operates under the Shortage Follows Policy. Valliathal and Uthayakumar [20] designed replenishment policies for non-instantaneous deteriorating items under a fuzzy environment. Wang[22] expalined the concept of Fuzzy points and local properties of fuzzy topology.

To the best of our knowledge, no prior research has focused on developing the EOQ model under fuzzy environment considering carbon emission and green investment with time-dependent demand over an infinite planning horizon. Hence, in this study, we develop two inventory models that consider carbon emission where the inventory parameters such as demand, inventory costs, the amount of carbon emissions in storing a unit product are treated to be fuzzy. In Model I, inventory models considering carbon emission and carbon tax is developed. In model II, inventory models considering carbon emission with green investment to reduce carbon tax are considered. In both models, demand is considered to be constant in the first case and exponential time-dependent demand in the second case. The fuzzy nature of ordering cost, holding cost, demand and the amount of carbon emissions in storing a unit product is addressed. We set pentagonal fuzzy numbers to fuzzify the parameters, and defuzzification is performed using the graded mean integration method. The influence of fuzziness on total inventory cost, as well as its impact on decision variables such as cycle time and order quantity, is also analyzed. The model determines the optimal total cost, aiming to fill a significant gap in the existing literature.

The subsequent sections of this article are designed as follows: In section 2, preliminaries are given. The various assumptions made and the notation used in the article are given in section 3. In section 4, the mathematical model formulation with carbon tax is given. In section 5, the mathematical model formulation with carbon tax and green investment is given. In section 6, a numerical example and insights and discussion are provided. The managerial implications obtained from our research are given in section 7. Finally, section 8 provides some concluding remarks.

2. Preliminaries

Fuzzy Point

A fuzzy point p in X is a fuzzy set with membership function

$$\mu_p(x) = \begin{cases} y, & \text{for } x = x_0 \\ 0, & \text{otherwise} \end{cases}$$

where $0 < y < 1$. p is said to have support x_0 and value y.

Fuzzy Set

A Fuzzy set $\tilde{A}$ in $\mathbb{R}$ is defined to be a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X, \mu_{\tilde{A}}(x) \in [0,1]\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set.

Fuzzy number

A fuzzy number is a fuzzy subset of the real line which is both normal and convex. For a fuzzy number $\tilde{A}$, its membership function can be denoted by

$$\mu_{\tilde{A}}(x) = \begin{cases} l(x), & x < m \\ 1, & m \leq x \leq n \\ u(x), & x > n \end{cases}$$

where $l(x)$ is upper semi-continuous, strictly increasing for $x < m$ and there exists $m_1 < m$ such that $l(x) = 0$ for $x \leq m_1$. $u(x)$ is continuous, strictly decreasing function for $x > n$ and there exists $n_1 \geq n$ such that $u(x) = 0$ for $x > n_1$. $l(x)$ and $u(x)$ are called the left and right reference functions, respectively.

Pentagonal Fuzzy Number

Pentagonal Fuzzy Numbers combine the advantages of triangular (clear peak) and trapezoidal (flat core) fuzzy numbers while offering:

1. Greater flexibility in capturing expert opinions and realistic uncertainty.
2. Smooth transition of membership values, avoiding abrupt changes in plausibility.
3. Improved precision in decision-making, since they distinguish between close alternatives more effectively.
4. Balanced complexity and expressiveness, requiring only one additional parameter compared to trapezoidal fuzzy numbers but significantly enhancing representation power.

A fuzzy number $\tilde{A} = (a, b, c, d, e)$ is called pentagonal fuzzy number if its membership function is represented as

$$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \frac{x-a}{b-a}, & a \le x \le b \\ L_2(x) = \frac{x-b}{c-b}, & b \le x \le c \\ 1, & x = c \\ R_1(x) = \frac{d-x}{d-c}, & c \le x \le d \\ R_2(x) = \frac{e-x}{e-d}, & d \le x \le e \\ 0, & x < a, e \le x \end{cases}$$

Following Hemalatha and Annadurai [3], the α-cut of a pentagonal fuzzy number $\tilde{A} = (a, b, c, d, e),\ 0 \le \alpha \le 1$ is $A(\alpha) = \left[A_L(\alpha), A_R(\alpha)\right]$ where $A_L(\alpha) = a + (b-a)\alpha$ and $A_R(\alpha) = d - (d-c)\alpha$ are the left and right end point of A_α.

In this context we have, $A_{L_1}(\alpha) = a + (b-a)\alpha = L_1^{-1}(\alpha), \quad A_{L_2}(\alpha) = b + (c-b)\alpha = L_2^{-1}(\alpha)$ and

$A_{R_1}(\alpha) = d - (d-c)\alpha = R_1^{-1}(\alpha), \quad A_{R_2}(\alpha) = e - (e-d)\alpha = R_2^{-1}(\alpha)$

So, $L^{-1}(\alpha) = \frac{L_1^{-1}(\alpha)+L_2^{-1}(\alpha)}{2} = \frac{a+b+\alpha(c-a)}{2}$ and $R^{-1}(\alpha) = \frac{R_1^{-1}(\alpha)+R_2^{-1}(\alpha)}{2} = \frac{d+e-\alpha(e-c)}{2}$

For example, consider the fuzzy demand represented by a pentagonal fuzzy number $(1200, 2700, 4500, 5200, 6740)$.
This set of five values represents different levels of possibility for the demand:

- 1200 (a): The lowest possible demand. Demand values below this are considered impossible.
- 2700 (b): The point where demand starts to be considered reasonably possible.
- 4500 (c): The most likely or expected demand value (the “core” or peak).
- 5200 (d): A higher but still plausible demand level.
- 6740 (e): The maximum possible demand. Values beyond this are impossible.

Hence, in the proposed inventory optimization model, pentagonal fuzzy numbers are adopted to represent uncertain cost parameters more accurately and to enhance the reliability of the derived policies.

Graded Mean Integration Method

If $\tilde{A} = (a, b, c, d, e)$ is a pentagonal fuzzy number then the graded mean integration representation of $\tilde{A}$ is defined as

$$\begin{aligned} G(\tilde{A}) &= \frac{1}{\int_0^{W_A} \alpha\, d\alpha} \int_0^{W_A} \frac{\alpha(L^{-1}(\alpha) + R^{-1}(\alpha))}{2}\, d\alpha, \text{with } 0 \le \alpha \le W_A, \quad 0 < W_A \le 1 \\ &= \frac{1}{\int_0^1 \alpha\, d\alpha} \int_0^1 \frac{\alpha(L^{-1}(\alpha) + R^{-1}(\alpha))}{2}\, d\alpha \\ &= \int_0^1 \alpha(L^{-1}(\alpha) + R^{-1}(\alpha))\, d\alpha \\ &= \int_0^1 \alpha \left[\left(\frac{a+b+\alpha(c-a)}{2}\right) + \left(\frac{d+e-\alpha(e-c)}{2}\right)\right] d\alpha \\ &= \frac{1}{12}[a + 3b + 4c + 3d + e] \end{aligned} \tag{1}$$

3. Assumptions and Notations

Assumptions

1. An infinite planning horizon is considered.
2. Shortages are not permitted.
3. Holding cost is time-dependent.
4. The demand rate is considered to be time-dependent.
5. Exponentially time dependent demand is considered to be $D(t) = \alpha e^{\beta t}, \quad \alpha > 0, \beta > 0.$
6. In the fuzzy model, the demand rate, ordering cost, holding cost, and the amount of carbon emissions are represented using pentagonal fuzzy numbers.
7. The method of graded mean integration is utilized for defuzzification.
8. Emissions produced from holding inventory should be taken into account.
9. Green technology investment is taken into account.
10. Two models are examined. In the first model, only carbon tax is considered. In the second model, both carbon tax and green technology were considered.
11. $R(G) = \phi G - \omega G^2$, where ϕ denotes the carbon reduction efficiency factor, and ω denotes the offsetting carbon reduction factor.

Notations

$I(t)$	Inventory level at instant t
α	Demand rate
$h(t)$	$h + \gamma t$ = Holding cost per unit time, where $h > 0$, $\gamma > 0$
A	Ordering cost per order
T_1	Cycle time for Model I with constant demand
T_2	Cycle time for Model I with exponentially time-dependent demand
T_3	Cycle time for Model II with constant demand
T_4	Cycle time for Model II with exponentially time-dependent demand
TC_1	Total cost per cycle time for Model I with constant demand
TC_2	Total cost per cycle time for Model I with exponentially time-dependent demand
TC_3	Total cost per cycle time for Model II with constant demand
TC_4	Total cost per cycle time for Model II with exponentially time-dependent demand
Q_1	Order quantity for Model I with constant demand
Q_2	Order quantity for Model I with exponentially time-dependent demand
Q_3	Order quantity for Model II with constant demand
Q_4	Order quantity for Model II with exponentially time-dependent demand
C_E	Carbon reduction effective cost
E_h	The amount of carbon emissions in storing a unit product
c_1	Carbon tax of unit carbon emission
G	Green investment amount

4. Model I (Inventory models considering carbon tax)

On each ton of carbon emissions, the government imposes a tariff, which is usually converted into a tax on the use of oil consumption, natural gas, and electricity. A carbon tax on holding inventory is a fee imposed on businesses for the greenhouse gas emissions generated during the storage of goods. These emissions arise from energy used for activities like lighting, cooling, and internal transport within warehouses. This tax incentivizes businesses to reduce emissions by optimizing inventory levels, improving energy efficiency, and adopting sustainable practices, ultimately aligning inventory management with environmental sustainability. Hence we consider carbon tax in this model.

Case (i)

Model with constant demand

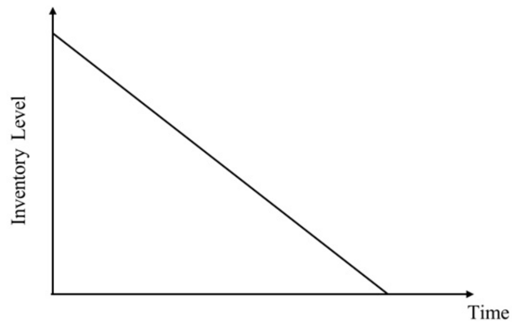

Figure 1. A graphical representation of the inventory model

Constant demand in inventory refers to a steady and predictable need for a product over time. It simplifies inventory management, reduces stockout risks, and lowers carrying costs. While beneficial for planning, real-world demand often requires adjustments to handle occasional variations. The item's demand rate is assumed to be constant in this model. During the time interval $[0, T]$, the consumer's demand causes the inventory of products to decline. This situation can be represented by the following differential equation (Depicted in Figure 1)

$$\frac{dq(t)}{dt} = -\alpha, \quad 0 < t < T. \tag{2}$$

With the boundary conditions, $q(0) = Q$ and $q(T) = 0$. The solution of (2) is,

$$q(t) = \alpha(T - t). \tag{3}$$

The total inventory cost consists of the following components:
The ordering cost $= OC = \frac{A}{T}$.
The holding cost during the time interval $[0, T]$ is given by $= HC = \frac{1}{T}\int_0^T (h + \gamma t)q(t)\, dt = \frac{\alpha T}{6}(3h + \gamma T)$.
Carbon emissions may also occur from storing undelivered or unsold products due to product characteristics or other factors. The carbon tax for storing the inventory can be obtained as follows.
The carbon tax $= C_T = \frac{E_h c_1}{T}\int_0^T q(t)\, dt = \frac{c_1 E_h \alpha T}{2}$
Total inventory cost is obtained by summing up ordering cost (OC), holding cost (HC) and carbon tax (C_T). Therefore,

$$\text{Total cost} = TC_1 = OC + HC + C_T = \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 E_h \alpha T}{2}. \tag{4}$$

The optimal value of T is obtained by solving $\frac{d(TC_1)}{dT} = 0$.

Differentiating (4) with respect to 'T' and equating to zero. We have,

$$\frac{d(TC_1)}{dT} = \frac{\alpha}{6}(3h + 2\gamma T) + \frac{c_1 E_h \alpha}{2} - \frac{A}{T^2} = 0. \tag{5}$$

Further, $\frac{d^2(TC_1)}{dT^2} = \frac{2A}{T^3} + \frac{\alpha\gamma}{6} > 0$.

Therefore, TC_1 is convex with respect to T.

Fuzzification of the inventory parameters

In real-world inventory management, parameters like demand, ordering costs, holding costs, and carbon emissions often exhibit significant uncertainty. For instance, the actual amount of carbon emissions can vary unpredictably due to factors such as the condition of the company's carbon filtering equipment, the quality of raw materials used, and other unforeseen circumstances. Traditional inventory models often rely on precise and deterministic data, which may not accurately represent the inherent uncertainties and vagueness present in real-world situations. To address these challenges, fuzzy numbers provide a valuable framework for decision-making. By incorporating fuzzy sets, inventory models can better account for the imprecise and ambiguous nature of these parameters, leading to more robust and realistic solutions.

Here, three subcases are considered. In the first case, the demand is considered to be fuzzy in nature, whereas the cost parameters and the amount of carbon emission from storing the product are crisp. In the second case, the cost parameters are treated as fuzzy. In the third case, the amount of carbon emission from storing the product is fuzzy. The graded mean integration method is further utilized for the defuzzification of the parameters.

(i) Fuzzification of demand parameter α

We express the demand parameter α by pentagonal fuzzy number, which is given as follows

$\tilde{\alpha} = (\alpha - \Delta_1, \alpha - \Delta_2, \alpha, \alpha + \Delta_3, \alpha + \Delta_4) = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$ where $\alpha > \Delta_1 > \Delta_2$ and $\Delta_3 < \Delta_4$.

Inventory total cost in the fuzzy sense is given by

$\tilde{TC_1} = \frac{A}{T} + \frac{\tilde{\alpha}T}{6}(3h + \gamma T) + \frac{c_1 E_h \tilde{\alpha} T}{2}.$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the total cost is given by

$$
\begin{aligned}
G_1(\tilde{TC_1}) &= \frac{1}{12}\left[TC_{11} + 3TC_{12} + 4TC_{13} + 3TC_{14} + TC_{15}\right] \quad \text{where} \\
TC_{11} &= \frac{A}{T} + \frac{\alpha_1 T}{6}(3h + \gamma T) + \frac{c_1 E_h \alpha_1 T}{2}, \\
TC_{12} &= \frac{A}{T} + \frac{\alpha_2 T}{6}(3h + \gamma T) + \frac{c_1 E_h \alpha_2 T}{2}, \\
TC_{13} &= \frac{A}{T} + \frac{\alpha_3 T}{6}(3h + \gamma T) + \frac{c_1 E_h \alpha_3 T}{2}, \\
TC_{14} &= \frac{A}{T} + \frac{\alpha_4 T}{6}(3h + \gamma T) + \frac{c_1 E_h \alpha_4 T}{2}, \\
TC_{15} &= \frac{A}{T} + \frac{\alpha_5 T}{6}(3h + \gamma T) + \frac{c_1 E_h \alpha_5 T}{2}.
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{A}{T} + [\frac{(\alpha_1 + 3\alpha_2 + 4\alpha_3 + 3\alpha_4 + \alpha_5)}{12}]\frac{T(3h + \gamma T)}{6} \\
&\quad + [\frac{(\alpha_1 + 3\alpha_2 + 4\alpha_3 + 3\alpha_4 + \alpha_5)}{12}]\frac{Tc_1 E_h}{2}. \\
&= \frac{A}{T} + \frac{L_1 T}{6}(3h + \gamma T) + \frac{c_1 E_h M T}{2}.
\end{aligned}
$$

Here $L_1 = \left[\frac{\alpha_1+3\alpha_2+4\alpha_3+3\alpha_4+\alpha_5}{12}\right]$.

(ii) Fuzzification of inventory costs

We express the parameters A, h by pentagonal fuzzy numbers, which is given as follows

$\tilde{A} = (A-\Delta_5, A-\Delta_6, A, A+\Delta_7, A+\Delta_8) = (A_1, A_2, A_3, A_4, A_5)$ where $A > \Delta_5 > \Delta_6$ and $\Delta_7 < \Delta_8$

$\tilde{h} = (h-\Delta_9, h-\Delta_{10}, h, h+\Delta_{11}, h+\Delta_{12}) = (h_1, h_2, h_3, h_4, h_5)$ where $h > \Delta_9 > \Delta_{10}$ and $\Delta_{11} < \Delta_{12}$.

Inventory total cost in the fuzzy sense is given by

$\tilde{TC_1} = \frac{A}{T} + \frac{\tilde{\alpha}T}{6}(3h+\gamma T) + \frac{c_1 E_h \tilde{\alpha} T}{2}$.

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the defuzzified total cost is given by

$$
\begin{aligned}
G_2(\tilde{TC_1}) &= \frac{1}{12}\left[TC_{11} + 3TC_{12} + 4TC_{13} + 3TC_{14} + TC_{15}\right] \quad \text{where} \\
TC_{11} &= \frac{A_1}{T} + \frac{\alpha T}{6}(3h_1+\gamma T) + \frac{c_1 E_h \alpha T}{2}, \\
TC_{12} &= \frac{A_2}{T} + \frac{\alpha T}{6}(3h_2+\gamma T) + \frac{c_1 E_h \alpha T}{2}, \\
TC_{13} &= \frac{A_3}{T} + \frac{\alpha T}{6}(3h_3+\gamma T) + \frac{c_1 E_h \alpha T}{2}, \\
TC_{14} &= \frac{A_4}{T} + \frac{\alpha T}{6}(3h_4+\gamma T) + \frac{c_1 E_h \alpha T}{2}, \\
TC_{15} &= \frac{A_5}{T} + \frac{\alpha T}{6}(3h_5+\gamma T) + \frac{c_1 E_h \alpha T}{2}. \\
&= \frac{1}{T}\left[\frac{(A_1+3A_2+4A_3+3A_4+A_5)}{12}\right] + \frac{\alpha T}{6}\left[3\left(\frac{(h_1+3h_2+4h_3+3h_4+h_5)}{12}\right)+\gamma T\right] \\
&\quad + \frac{c_1 \alpha E_h T}{2}. \\
&= \frac{M_1}{T} + \frac{\alpha T}{6}\left(3N_1+\gamma T\right) + \frac{c_1 E_h \alpha T}{2}.
\end{aligned}
$$

Here $M_1 = \left[\frac{A_1+3A_2+4A_3+3A_4+A_5}{12}\right]$ and $N_1 = \left[\frac{h_1+3h_2+4h_3+3h_4+h_5}{12}\right]$.

(iii) Fuzzification of carbon emission parameter E_h

We express the parameter E_h by pentagonal fuzzy number, which is given below

$\tilde{E_h} = (E_h-\Delta_{13}, E_h-\Delta_{14}, E_h, E_h+\Delta_{15}, E_h+\Delta_{16})$
$= (E_{h1}, E_{h2}, E_{h3}, E_{h4}, E_{h5})$ where $E_h > \Delta_{13} > \Delta_{14}$ and $\Delta_{15} < \Delta_{16}$.

Inventory total cost in the fuzzy sense is given by

$\tilde{TC_1} = \frac{A}{T} + \frac{\alpha T}{6}(3h+\gamma T) + \frac{c_1 \tilde{E_h} \alpha T}{2}$.

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the defuzzified total cost is given by

$$
\begin{aligned}
G_3(\tilde{TC_1}) &= \frac{1}{12}\left[TC_{11} + 3TC_{12} + 4TC_{13} + 3TC_{14} + TC_{15}\right] \quad \text{where} \\
TC_{11} &= \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 E_{h1} \alpha T}{2}, \\
TC_{12} &= \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 E_{h2} \alpha T}{2}, \\
TC_{13} &= \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 E_{h3} \alpha T}{2}, \\
TC_{14} &= \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 E_{h4} \alpha T}{2}, \\
TC_{15} &= \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 E_{h5} \alpha T}{2}. \\
&= \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 O_1 \alpha T}{2}.
\end{aligned}
$$

Here $O_1 = \left[\frac{E_{h1}+3E_{h2}+4E_{h3}+3E_{h4}+E_{h5}}{12}\right]$.

Case (ii)

Model with exponential time dependent demand

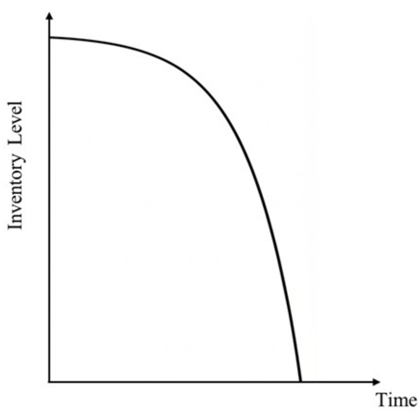

Figure 2. A graphical representation of the inventory model

Exponential time-dependent demand is an effective approach for evaluating and controlling commodities with rapidly shifting demand patterns. It is especially effective in sectors that rely on time-sensitive processes. When a new flagship smartphone is released, initial demand is often low. However, as reviews, marketing campaigns, and word-of-mouth spread, demand can increase exponentially. For example, companies like Apple and Samsung can use exponential models to forecast initial demand, plan production accordingly, and avoid stockouts or overstocking. In this model, it's assumed that demand is time sensitive and it increases exponentially. Thus, the differential equation governing the situation is given by (Depicted in Figure 2)

$$\frac{dq(t)}{dt} = -\alpha e^{\beta t}, \quad 0 < t < T \tag{6}$$

With the boundary conditions, $q(0) = Q$ and $q(T) = 0$. The solution of (6) is,

$$q(t) = \frac{\alpha}{\beta}\left(e^{\beta T} - e^{\beta t}\right). \tag{7}$$

The total inventory cost consists of the following components:

The ordering cost = $OC = \frac{A}{T}$

The holding cost from carrying the inventory during the time interval $[0, T]$ is given by
$HC = \frac{1}{T}\int_0^T (h + \gamma t)q(t)\,dt = \frac{\alpha T}{2}\left(h(1 + \beta T) + \frac{\beta\gamma T^2}{2}\right)$

Carbon emissions may also occur from storing undelivered or unsold products due to product characteristics or other factors. The carbon tax for storing the inventory can be obtained as follows

The carbon tax=$C_T = \frac{E_h c_1}{T}\int_0^T q(t)\,dt = E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right)$

Total inventory cost is obtained by summing up ordering cost (OC), holding cost (HC) and carbon tax (C_T).

Therefore, Total cost $= TC_2 = OC + HC + C_T$

$$= \frac{A}{T} + \frac{\alpha T}{2}\left(h(1 + \beta T) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right) \tag{8}$$

The optimal value of T is obtained by solving $\frac{d(TC_2)}{dT} = 0$.

Differentiating (8) with respect to 'T' and equating to zero. We have

$$\frac{d(TC_2)}{dT} = \frac{\alpha h}{2} + \alpha h\beta T + \frac{3\alpha\beta\gamma T^2}{4} + \frac{E_h c_1 \alpha}{2} + E_h c_1 \alpha\beta T - \frac{A}{T^2} = 0. \tag{9}$$

Further, $\frac{d^2(TC_1)}{dT^2} = \frac{2A}{T^3} + \alpha h\beta + \frac{3\alpha\beta\gamma T}{2} + c_1 E_h \beta^2 > 0.$

Therefore, TC_2 is convex with respect to T.

Fuzzification of the inventory parameters

Real-world inventory management faces uncertainties in factors like demand and costs, including carbon emissions. Traditional models struggle with these uncertainties. Fuzzy numbers offer a solution by allowing for imprecise and ambiguous information. Here, three subcases are established. In the first case, demand is uncertain, while inventory costs and the amount of carbon emissions are crisp. Inventory costs are fuzzy in the second case. In the third case, the amount of carbon emissions is uncertain, while demand and costs are crisp. Here, we use the pentagonal fuzzy number for fuzzification.

(i)Fuzzification of demand parameter α

We express the parameter α by pentagonal fuzzy numbers, which is given as follows

$\tilde{\alpha} = (\alpha - \Delta_{17}, \alpha - \Delta_{18}, \alpha, \alpha + \Delta_{19}, \alpha + \Delta_{20})$
$= (\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})$ where $\alpha > \Delta_{17} > \Delta_{18}$ and $\Delta_{19} < \Delta_{20}$.

Inventory total cost in the fuzzy sense is given by

$T\tilde{C}_2 = \frac{A}{T} + \frac{\tilde{\alpha}T}{2}\left(h(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right).$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the total cost is given by

$$G_4(T\tilde{C}_2) = \frac{A}{T} + \frac{L_2 T}{2}\left(h(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right).$$

Here, $L_2 = \frac{\alpha_6 + 3\alpha_7 + 4\alpha_8 + 3\alpha_9 + \alpha_{10}}{12}$.

(ii) Fuzzification of inventory costs

We express the parameters A, h by pentagonal fuzzy numbers, which is given as follows

$\tilde{A} = (A - \Delta_{21}, A - \Delta_{22}, A, A + \Delta_{23}, A + \Delta_{24}) = (A_6, A_7, A_8, A_9, A_{10})$ where $A > \Delta_{21} > \Delta_{22}$ and $\Delta_{23} < \Delta_{24}$

$\tilde{h} = (h - \Delta_{25}, h - \Delta_{26}, h, h + \Delta_{27}, h + \Delta_{28}) = (h_6, h_7, h_8, h_9, h_{10})$ where $h > \Delta_{25} > \Delta_{26}$ and $\Delta_{27} < \Delta_{28}$.

Inventory total cost in the fuzzy sense is given by

$T\tilde{C}_2 = \frac{\tilde{A}}{T} + \frac{\alpha T}{2}\left(\tilde{h}(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right).$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the defuzzified total cost is given by

$$G_5(T\tilde{C}_2) = \frac{M_2}{T} + \frac{\alpha T}{2}\left(N_2(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right).$$

Here, $M_2 = \frac{A_6 + 3A_7 + 4A_8 + 3A_9 + A_{10}}{12}$ and $N_2 = \frac{h_6 + 3h_7 + 4h_8 + 3h_9 + h_{10}}{12}$.

(iii) Fuzzification of carbon emission parameter E_h

We express the parameter E_h by pentagonal fuzzy number, which is given below

$\tilde{E_h} = (E_h - \Delta_{29}, E_h - \Delta_{30}, E_h, E_h + \Delta_{31}, E_h + \Delta_{32})$
$= (E_{h6}, E_{h7}, E_{h8}, E_{h9}, E_{h10})$ where $E_h > \Delta_{29} > \Delta_{30}$ and $\Delta_{31} < \Delta_{32}$.

Inventory total cost in the fuzzy sense is given by

$T\tilde{C}_2 = \frac{A}{T} + \frac{\alpha T}{2}\left(h(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + \tilde{E}_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right).$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the defuzzifed total Cost is given by

$$G_6\left(T\tilde{C}_2\right) = \frac{A}{T} + \frac{\alpha T}{2}\left(h(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + O_2 c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right)).$$

Here, $O_2 = \frac{E_{h6} + 3E_{h7} + 4E_{h8} + 3E_{h9} + E_{h10}}{12}$.

5. Model II (Inventory models considering carbon tax and green technology)

When carbon emissions exceed regulatory thresholds, investment in green technology is considered a necessary measure to mitigate the environmental impact. A green technology investment of amount G is assumed to result in a reduction of carbon emissions by ϕG, where ϕ denotes the carbon reduction efficiency factor. Nevertheless, the implementation of such technologies may lead to additional energy consumption, which in turn contributes to further emissions, expressed as ωG^2. The parameter ω accounts for the offsetting reduction factor that contributes to additional emissions. The net carbon emission reduction from such technology is described by the function $R(G) = \phi G - \omega G^2 6$ (Huang and Rust [5]).

Here, ϕ represents the carbon reduction efficiency parameter, which reflects the rate at which investment in carbon reduction translates into actual emission savings, whereas ω denotes the carbon reduction cost intensity parameter, capturing the marginal cost of achieving additional reductions.

To estimate these parameters in practice, managers can use historical operational and environmental data. By recording previous levels of green investment (G) and the corresponding emission reductions $(R(G))$, the parameters ϕ and ω can be obtained through the relationship $R(G) = \phi G - \omega G^2$. Alternatively, these values can be benchmarked using industry data or obtained through expert consultation, where ϕ is guided by the efficiency of adopted technologies (e.g., renewable energy systems, energy-efficient logistics) and ω is inferred from the observed diminishing returns or additional energy costs at higher investment levels. Such estimation enables managers to calibrate the model realistically and make data-driven decisions regarding optimal levels of green investment.Hence both carbon tax and green technology were considered in model II.

Case(i)

Model with constant demand

Here the demand is considered to be constant. During the time interval $[0, T]$, consumer's demand makes the inventory of products to decline. Then the differential equation is given by

$$\frac{dq(t)}{dt} = -\alpha, \quad 0 < t < T \tag{10}$$

with the boundary conditions, $q(0) = Q$ and $q(T) = 0$. The solution of (10) is

$$q(t) = \alpha(T - t). \tag{11}$$

Similar to that of model I, we obtain ordering cost, holding cost and carbon tax.

We invest the green cost G, this investment yields a reduction in carbon emissions, represented by the value φG. However, the utilization of these green technologies also necessitates energy consumption, consequently leading to additional carbon emissions, represented by ωG^2. Here, φ and ω can be obtained from the past data of carbon emission reduction and the amount in green technology.

Carbon reduction effective cost = $C_E = R(G)c_1$

Green investment = G

Total inventory cost is obtained by summing up ordering cost (OC), holding cost (HC), carbon tax (C_T), green investment amount (G) and then subtracting the carbon reduction effective cost. Therefore,

$$\text{Total cost} = TC_3 = OC + HC + C_T + G - C_E = \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 E_h \alpha T}{2} + G - (\phi G - \omega G^2)c_1 \tag{12}$$

The optimal values of T and G is obtained by solving $\frac{d(TC_3)}{dT} = 0$ and $\frac{d(TC_3)}{dG} = 0$.

Differentiating (12) with respect to 'T' and 'G' and equating to zero. We have

$$\frac{d(TC_3)}{dT} = \frac{\alpha}{6}(3h + 2\gamma T) + \frac{c_1 E_h \alpha}{2} - \frac{A}{T^2} = 0. \tag{13}$$

and,

$$\frac{d(TC_3)}{dG} = 1 - \phi c_1 + 2\omega G c_1 = 0. \tag{14}$$

Further, we estimate the hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2 TC_3}{\partial T^2} & \frac{\partial^2 TC_3}{\partial T\,\partial G} \\ \frac{\partial^2 TC_3}{\partial G\,\partial T} & \frac{\partial^2 TC_3}{\partial G^2} \end{bmatrix}$$

We have,

$$\frac{\partial^2 (TC_3)}{\partial T^2} = \frac{\alpha\gamma}{3} + \frac{2}{T^3}, \quad \frac{\partial^2 (TC_3)}{\partial G^2} = 2\omega c_1, \quad \frac{\partial^2 (TC_3)}{\partial G\,\partial T} = \frac{\partial^2 (TC_3)}{\partial T\,\partial G} = 0$$

$$\therefore H = \begin{bmatrix} \frac{\alpha\gamma}{3} + \frac{2}{T^3} & 0 \\ 0 & 2\omega c_1 \end{bmatrix}, |H| = \left(\frac{\alpha\gamma}{3} + \frac{2}{T^3}\right)(2\omega c_1) > 0$$

Therefore, $TC_3(T, G)$ is convex.

(i)Fuzzification of demand parameter α

We express the parameter α by pentagonal fuzzy number, which is given as follows

$\tilde{\alpha} = (\alpha - \Delta_{33}, \alpha - \Delta_{34}, \alpha, \alpha + \Delta_{35}, \alpha + \Delta_{36})$
$= (\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}, \alpha_{15})$ where $\alpha > \Delta_{33} > \Delta_{34}$ and $\Delta_{35} < \Delta_{36}$.

Inventory total cost in the fuzzy sense is given by

$\tilde{\text{TC}}_3 = \frac{A}{T} + \frac{\tilde{\alpha} T}{6}(3h + \gamma T) + \frac{c_1 E_h \tilde{\alpha} T}{2} + G - (\phi G - \omega G^2)c_1.$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the defuzzified total cost is given by

$$G_7(\tilde{\text{TC}}_3) = \frac{A}{T} + \frac{L_3 T}{6}(3h + \gamma T) + \frac{c_1 E_h L_3 T}{2} + G - (\phi G - \omega G^2)c_1.$$

Here $L_3 = \frac{(\alpha_{11} + 3\alpha_{12} + 4\alpha_{13} + 3\alpha_{14} + \alpha_{15})}{12}$.

(ii)Fuzzification of inventory costs

We express the parameters A, h by pentagonal fuzzy numbers, which is given as follows

$\tilde{A} = (A - \Delta_{37}, A - \Delta_{38}, A, A + \Delta_{39}, A + \Delta_{40})$
$= (A_{11}, A_{12}, A_{13}, A_{14}, A_{15})$ where $A > \Delta_{37} > \Delta_{38}$ and $\Delta_{39} < \Delta_{40}$

$\tilde{h} = (h - \Delta_{41}, h - \Delta_{42}, h, h + \Delta_{43}, h + \Delta_{44})$
$= (h_{11}, h_{12}, h_{13}, h_{14}, h_{15})$ where $h > \Delta_{41} > \Delta_{42}$ and $\Delta_{43} < \Delta_{44}$.

Inventory total cost in the fuzzy sense is given by

$\tilde{TC_3} = \frac{\tilde{A}}{T} + \frac{\alpha T}{6}(3\tilde{h} + \gamma T) + \frac{c_1 E_h \alpha T}{2} + G - \left(\phi G - \omega G^2\right) c_1.$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the total Cost is given by

$$G_8(\tilde{TC_3}) = \frac{M_3}{T} + \frac{\alpha T}{6}(3N_3 + \gamma T) + \frac{c_1 E_h \alpha T}{2} + G - (\phi G - \omega G^2)c_1.$$

Here, $M_3 = \frac{A_{11}+3A_{12}+4A_{13}+3A_{14}+A_{15}}{12}$ and $N_3 = \frac{h_{11}+3h_{12}+4h_{13}+3h_{14}+h_{15}}{12}$.

(iii)Fuzzification of carbon emission parameter E_h

We express the parameter E_h by pentagonal fuzzy number, which is given below

$\tilde{E_h} = (E_h - \Delta_{45}, E_h - \Delta_{46}, E_h, E_h + \Delta_{47}, E_h + \Delta_{48})$
$= (E_{h11}, E_{h12}, E_{h13}, E_{h14}, E_{h15})$ where $E_h > \Delta_{45} > \Delta_{46}$ and $\Delta_{47} < \Delta_{48}$.

Inventory total cost in the fuzzy sense is given by

$\tilde{TC_3} = \frac{C_O}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 \tilde{E_h} \alpha T}{2} + G - (\phi G - \omega G^2)c_1.$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the defuzzified total cost is given by

$$G_9(\tilde{TC_3}) = \frac{A}{T} + \frac{\alpha T}{6}(3h + \gamma T) + \frac{c_1 O_3 \alpha T}{2} + G - (\phi G - \omega G^2)c_1.$$

Here, $O_3 = \frac{E_{h11}+3E_{h12}+4E_{h13}+3E_{h14}+E_{h15}}{12}$.

case(ii)

Model with exponential time dependent demand

Here the demand is considered to be exponential time dependent demand. In this model, it's assumed that demand is time sensitive and it increases exponentially. Thus, the differential equation is given by

$$\frac{dq(t)}{dt} = -\alpha e^{\beta t}, \quad 0 < t < T \tag{15}$$

with the boundary conditions, $q(0) = Q$ and $q(T) = 0$. The solution of (15) is

$$q(t) = \frac{\alpha}{\beta}\left(e^{\beta T} - e^{\beta t}\right). \tag{16}$$

Ordering cost, Holding cost and Carbon tax are obtained, which are similar to those of model I.

Carbon reduction effective cost = $C_E = R(G)c_1$

Carbon emissions are a major contributor to climate change, and businesses have a responsibility to reduce their environmental impact. By investing in green technologies and practices, businesses can reduce their carbon footprint and contribute to a more sustainable future.

Green investment = G

Total cost of the inventory model is calculated by summing up the cost of ordering cost (OC), holding cost (HC), carbon tax (C_T), green investment amount (G) and then subtracting the carbon reduction effective cost.

Total cost = $TC_4 = OC + HC + C_T + G - C_E$

$$= \frac{A}{T} + \frac{\alpha T}{2}\left(h\left(1+\beta T\right) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha \left(\frac{\beta T^2}{2} + \frac{T}{2}\right) + G - \left(\phi G - \omega G^2\right) c_1 \tag{17}$$

The optimal values of T and G is obtained by solving $\frac{d(TC_4)}{dT} = 0$ and $\frac{d(TC_4)}{dG} = 0.$

Differentiating (17) with respect to 'T'and 'G' and equating to zero. We have

$$\frac{d(TC_4)}{dT} = \frac{\alpha h}{2} + \alpha h \beta T + \frac{3\alpha\beta\gamma T^2}{4} + E_h c_1 \alpha \beta T + \frac{E_h c_1 \alpha}{2} - \frac{A}{T^2} = 0. \tag{18}$$

and,

$$\frac{d(TC_4)}{dG} = 1 - \phi c_1 + 2\omega G c_1 = 0. \tag{19}$$

Further, we estimate the hessian matrix

$$H = \begin{bmatrix} \dfrac{\partial^2 TC_4}{\partial T^2} & \dfrac{\partial^2 TC_4}{\partial T\,\partial G} \\ \dfrac{\partial^2 TC_4}{\partial G\,\partial T} & \dfrac{\partial^2 TC_4}{\partial G^2} \end{bmatrix}$$

We have,

$$\frac{\partial^2 (TC_4)}{\partial T^2} = \alpha\beta h + \frac{3\alpha\beta\gamma T}{2} + E_h c_1 \alpha\beta + \frac{2}{T^3}, \quad \frac{\partial^2 (TC_4)}{\partial G^2} = 2\omega c_1, \quad \frac{\partial^2 (TC_4)}{\partial G\,\partial T} = \frac{\partial^2 (TC_4)}{\partial T\,\partial G} = 0$$

$$\therefore H = \begin{bmatrix} \alpha\beta h + \frac{3\alpha\beta\gamma T}{2} + E_h c_1 \alpha\beta + \frac{2}{T^3} & 0 \\ 0 & 2\omega c_1 \end{bmatrix},$$

$$|H| = \left(\alpha\beta h + \frac{3\alpha\beta\gamma T}{2} + E_h c_1 \alpha\beta + \frac{2}{T^3}\right)(2\omega c_1) > 0$$

Therefore, $TC_4(T, G)$ is convex.

(i)Fuzzification of demand parameter α

We express the parameter α by pentagonal fuzzy number, which is given as follows

$\tilde{\alpha} = (\alpha - \Delta_{49}, \alpha - \Delta_{50}, \alpha, \alpha + \Delta_{51}, \alpha + \Delta_{52})$
$= (\alpha_{16}, \alpha_{17}, \alpha_{18}, \alpha_{19}, \alpha_{20})$ where $\alpha > \Delta_{49} > \Delta_{50}$ and $\Delta_{51} < \Delta_{52}$.

Inventory total cost in the fuzzy sense is given by

$$\tilde{TC}_4 = \frac{A}{T} + \frac{\alpha T}{2}\left(h\left(1+\beta T\right) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right) + G - \left(\phi G - \omega G^2\right) c_1$$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the defuzzified total cost is given by

$$G_{10}\left(\tilde{TC}_4\right) = \frac{A}{T} + \frac{L_4 T}{2}\left(h(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right) + G - \left(\phi G - \omega G^2\right) c_1.$$

Here, $L_4 = \frac{(\alpha_{16}+3\alpha_{17}+4\alpha_{18}+3\alpha_{19}+\alpha_{20})}{12}$.

(ii)Fuzzification of inventory costs

We express the parameters A, h by pentagonal fuzzy numbers, which is given as follows

$\tilde{A} = (A-\Delta_{53}, A-\Delta_{54}, A, A+\Delta_{55}, A+\Delta_{56})$
$= (A_{16}, A_{17}, A_{18}, A_{19}, A_{20})$ where $A > \Delta_{53} > \Delta_{54}$ and $\Delta_{55} < \Delta_{56}$

$\tilde{h} = (h-\Delta_{57}, h-\Delta_{58}, h, h+\Delta_{59}, h+\Delta_{60})$
$= (h_{16}, h_{17}, h_{18}, h_{19}, h_{20})$ where $h > \Delta_{57} > \Delta_{58}$ and $\Delta_{59} < \Delta_{60}$.

Inventory total cost in the fuzzy sense is given by

$$\tilde{TC}_4 = \frac{\tilde{A}}{T} + \frac{\alpha T}{2}\left(\tilde{h}(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right) + G - \left(\phi G - \omega G^2\right) c_1$$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the defuzzified total cost is given by

$$G_{11}\left(\tilde{TC}_4\right) = \frac{M_4}{T} + \frac{\alpha T}{2}\left[N_4(1+\beta T) + \frac{\beta\gamma T^2}{2}\right] + E_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right) + G - (\phi G - \omega G^2) c_1.$$

Here, $M_4 = \frac{A_{16}+3A_{17}+4A_{18}+3A_{19}+A_{20}}{12}$ and $N_4 = \frac{h_{16}+3h_{17}+4h_{18}+3h_{19}+h_{20}}{12}$.

(iii)Fuzzification of carbon emission parameter E_h

We express the parameter E_h by pentagonal fuzzy number, which is given below

$\tilde{E_h} = (E_h-\Delta_{61}, E_h-\Delta_{62}, E_h, E_h+\Delta_{63}, E_h+\Delta_{64})$
$= (E_{h16}, E_{h17}, E_{h18}, E_{h19}, E_{h20})$ where $E_h > \Delta_{61} > \Delta_{62}$ and $\Delta_{63} < \Delta_{64}$

Inventory total cost in the fuzzy sense is given by

$$\tilde{TC}_4 = \frac{A}{T} + \frac{\alpha T}{2}\left(h(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + \tilde{E}_h c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right) + G - (\phi G - \omega G^2) c_1.$$

We defuzzify the fuzzy total cost by graded mean integration method. Using (1), the total cost is given by

$$G_{12}(\tilde{TC}_4) = \frac{A}{T} + \frac{\alpha T}{2}\left(h(1+\beta T) + \frac{\beta\gamma T^2}{2}\right) + O_4 c_1 \alpha\left(\frac{\beta T^2}{2} + \frac{T}{2}\right) + G - (\phi G - \omega G^2) c_1.$$

Here $O_4 = \frac{E_{h16}+3E_{h17}+4E_{h18}+3E_{h19}+E_{h20}}{12}$.

6. Numerical Examples and Sensitive Analysis

6.1. Numerical Examples

Example 1

Let us consider the following parameters in appropriate units $\alpha = 4500$ units, $A = 150$ units, $h = 20$, $c_1 = 0.5$, $E_h = 3$, and $\gamma = 0.5$ (Model I with constant demand). Solving the equation (5), we get the optimal cycle time $T_1^* = 0.0556605$ yrs. Optimal order quantity and total cost are obtained from the equations (3) & (4), and we get $Q_1^* = 250.4723$ units and $TC_1^* = \$5388.6$.

A comprehensive numerical analysis has been conducted to investigate the impact of varying levels of fuzziness in the input parameters on the decision variables. Inventory parameters such as α, h, A, and E_h are fuzzified using pentagonal fuzzy numbers. Subsequently, for each of these parameters, the inherent variations in the respective values are considered, and defuzzification is performed via the graded mean integration method. The defuzzified value and the corresponding percentage difference under the fuzzy case from the crisp values are obtained and given in Tables 1 & 2. Further, optimal values of T_1^*, TC_1^*, and Q_1^* are also obtained for varying levels of fuzziness in the demand parameter α and are given in Table 5.

Example 2

In the following example, we consider the same data as in Example 1, together with $\beta = 0.2$ (Model I with Exponential time-dependent demand). Solving the equation (9), we get the optimal cycle time $T_2^* = 0.0550806$ yrs. Optimal order quantity and total cost are obtained from the equations (7) & (8), and we get $Q_2^* = 249.2279$ units and $TC_2^* = \$5417.2$.

Some pentagonal fuzzy numbers are set for the input parameters α, h, A, and E_h, and the corresponding variations are given in Tables 3 & 4. Their defuzzified values are also given in these Tables. An analysis made based on the fuzzy nature of the inventory parameters such as α, h, A, and E_h are given in Tables 8–10.

Example 3

In the following example, we consider the same data as in Example 1, together with $\phi = 10$ and $\omega = 0.03$ (Model II with constant demand). Solving the equation (13), we get the optimal cycle time $T_3^* = 0.0556605$ yrs. Optimal order quantity, total cost, and green investment amount are obtained from the equations (11), (12), and (14), respectively. We get $Q_3^* = 250.4723$ units, $TC_3^* = \$5122$, and $G = \$133.3333$.

Inventory parameters undergo fuzzification via pentagonal fuzzy numbers, followed by defuzzification using the graded mean integration method. The resulting defuzzified values and their percentage differences from crisp values are presented in Tables 1 & 2. When the fuzziness of the cost parameters is varied, the optimal values are obtained and are given in Table 6. Table 7 provides the optimal values for varying levels of fuzziness in the parameter E_h. Further, optimal values of T_3^*, TC_3^*, and Q_3^* are also obtained for varying levels of fuzziness in the demand parameter α and are given in Table 5. From Table 11, an analysis carried out by varying the parameter E_h under pentagonal fuzzy numbers is presented. Figure 5 illustrates the impact of carbon emissions on total cost.

Example 4

In the following example, we consider the same data as in Example 3, together with $\beta = 0.2$ (Model II with exponential time-dependent demand). Solving the equation (18), we get the optimal cycle time $T_4^* = 0.0550806$ yrs. Optimal order quantity, total cost, and green investment amount are obtained from the equations (16), (17), and (19), respectively. We get $Q_4^* = 249.2279$ units, $TC_4^* = \$4883.8$, and $G = \$133.3333$.

Inventory parameters are fuzzified using pentagonal fuzzy numbers. The resulting defuzzified values, along with the corresponding fuzzy percentages expressed with respect to the crisp values, are presented in Tables 3 & 4, respectively. From Tables 8 & 9, optimal values for various levels of fuzziness in the cost components and demand are obtained by fuzzifying the parameters using pentagonal fuzzy numbers. An analysis carried out by varying the parameter E_h is given in Table 12. Using pentagonal fuzzy numbers, the effect of carbon emissions on the total cost is shown in Figure 6.

A comparative study between Model I and Model II has been done to analyze the effect of green investment on the total cost (Tables 5–10). From the Tables, we infer that the total cost in both models increases significantly with respect to various levels of fuzziness in the parameters α, h, A, and E_h. Further, we also find that when comparing the cost in Model I and Model II, the cost in Model II is comparatively lower. This is because of the investment in green technology. When an investment is made in green technology, the total cost is significantly reduced. The optimal values of cycle time, order quantity and total inventory cost are found by varying E_h and the fuzziness of the cost parameters A and h, and are given in Tables 11 & 12.

Table 1. Fuzzy pentagonal values for the input parameters α and E_h with constant demand (Model I)

$\tilde{\alpha}$	% variation $\tilde{\alpha}$	$G_1(\tilde{\alpha})$	$\tilde{E_h}$	% variation $\tilde{E_h}$	$G_3(\tilde{E_h})$
(1200,2700,4500,5200,6740)	-8%	4136.6	(1,2,3,3.64,4.64)	-4%	2.88
(1264,2900,4500,5600,7100)	-4%	4322	(1,2,3,3.82,4.82)	-2%	2.94
(1500,3000,4500,6000,7500)	0%	4500	(1,2,3,4,5)	0%	3
(1800,3200,4500,6200,8291)	4%	4690.9	(1.09,2.09,3,4.09,5.09)	2%	3.06
(2000,3400,4500,6300,9410)	8%	4875.8	(1.2,2.2,3,4.16,5.16)	4%	3.12

Table 2. Fuzzy pentagonal values for the input parameters A ,h with constant demand (Model I)

$\tilde{A}$	% variation $\tilde{A}$	$G_2(\tilde{A})$	$\tilde{h}$	% variation $\tilde{h}$	$G_2(\tilde{h})$
(24,38,150,170,200)	-20%	120.6667	(5,10,20,23,27)	-12%	17.6
(26,40,150,208,251)	-10%	135.8833	(7,13,20,24,28)	-6%	18.8
(50,87,150,217,238)	0%	150	(8,14,20,26,32)	0%	20
(60,100,150,230,327)	10%	164.7500	(12,16,20,27,33)	6%	21.2
(70,140,150,240,357)	20%	180.5833	(13,18,20,29,35)	12%	22.4

Table 3. Fuzzy pentagonal values for the input parameters α and E_h with exponentially time dependent demand (Model I)

$\tilde{\alpha}$	% variation $\tilde{\alpha}$	$G_4(\tilde{\alpha})$	$\tilde{E_h}$	% variation $\tilde{E_h}$	$G_6(\tilde{E_h})$
(1700,3100,4500,5200,6198)	-6%	4233.1	(0.8,1.27,3,3.15,3.46)	-18%	2.46
(1900,3300,4500,5300,6600)	-3%	4358.3	(1,1.6,3,3.5,4.5)	-9%	2.73
(2000,3600,4500,5500,6700)	0%	4500	(1.2,2.4,3,3.6,4.8)	0%	3
(2400,3800,4500,5700,6816)	3%	4643	(1.78,2.78,3,3.78,5.78)	9%	3.27
(2490,4000,4500,5900,6892)	6%	4756.8	(2.25,2.94,3,4.53,5.82)	18%	3.54

Table 4. Fuzzy pentagonal values for the input parameters A,h with exponentially time dependent demand (Model I)

$\tilde{A}$	% variation $\tilde{A}$	$G_5(\tilde{A})$	$\tilde{h}$	% variation $\tilde{h}$	$G_5(\tilde{h})$
(60,100,150,160,187)	-10%	135.5833	(2,5,20,21,22)	-24%	15.2
(70,110,150,170,206)	-5%	143	(5,12,20,22,25)	-12%	17.6
(80,120,150,180,220)	0%	150	(10,15,20,25,30)	0%	20
(90,130,150,190,237)	5%	157.2500	(12,18,20,29,36)	12%	22.4
(100,140,150,200,264)	10%	165.3333	(14,19,20,34,44)	24%	24.8

Table 5. Comparison between model I and II with fuzzy demand (constant demand)

% variation $\tilde{\alpha}$	T_1^*	TC_1^*	Q_1^*	T_3^*	TC_3^*	Q_3^*
-8%	0.0580529	5166.5	240.1416	0.0580529	4899.9	240.1416
-4%	0.0567947	5281	245.4667	0.0567947	5014.4	245.4667
0%	0.0556605	5388.6	250.4723	0.0556605	5122	250.4723
4%	0.0545167	5501.7	255.7324	0.0545167	5235.1	255.7324
8%	0.0534735	5609.1	260.7261	0.0534735	5342.4	260.7261

Table 6. Comparison between model I and II with fuzzy costs (constant demand)

% variation $\tilde{A}$	% variation $\tilde{h}$	T_1^*	TC_1^*	Q_1^*	T_3^*	TC_3^*	Q_3^*
-20%	-12%	0.0529645	4555.5	238.3402	0.0529645	4288.8	238.3402
-10%	-6%	0.0545191	4983.7	245.3359	0.0545191	4717	245.3359
0%	0%	0.0556605	5388.6	250.4723	0.0556605	5122	250.4723
10%	6%	0.0567711	5802.8	255.4699	0.0567711	5536.1	255.4699
20%	12%	0.0579259	6233.7	260.6666	0.0579259	5967	260.6666

Table 7. Comparison between model I and II with fuzzy carbon emission (constant demand)

% variation $\tilde{E}_h$	T_1^*	TC_1^*	Q_1^*	T_3^*	TC_3^*	Q_3^*
-4%	0.0557382	5381.1	250.8219	0.0557382	5114.5	250.8219
-2%	0.0556994	5384.9	250.6473	0.0556994	5118.2	250.6473
0%	0.0556605	5388.6	250.4723	0.0556605	5122	250.4723
2%	0.0556219	5392.4	250.2986	0.0556219	5125.7	250.2986
4%	0.0555832	5396.2	250.1244	0.0555832	5129.5	250.1244

Table 8. Comparison between model I and II with fuzzy demand (exponential time dependent demand)

% variation $\tilde{\alpha}$	T_2^*	TC_2^*	Q_2^*	T_4^*	TC_4^*	Q_4^*
-6%	0.0567716	5255	241.6842	0.0567716	4721.6	241.6842
-3%	0.0559592	5331.7	245.2518	0.0559592	4798.5	243.6508
0%	0.0550806	5417.2	249.2279	0.0550806	4883.8	249.2279
3%	0.0542341	5502.1	253.1746	0.0542341	4968.8	253.1746
6%	0.0535882	5568.8	256.2744	0.0535882	5035.4	256.2744

Table 9. Comparison between model I and II with fuzzy costs (exponential time dependent demand)

% variation $\tilde{A}$	% variation $\tilde{h}$	T_2^*	TC_2^*	Q_2^*	T_4^*	TC_4^*	Q_4^*
-10%	-24%	0.0593677	4541	268.7407	0.0593677	4007.7	268.7407
-5%	-12%	0.0570369	4986.3	258.1300	0.0570369	4453	258.1300
0%	0%	0.0550806	5417.2	249.2279	0.0550806	4883.8	249.2279
5%	12%	0.0535065	5847	242.0676	0.0535065	5313.7	242.0676
10%	24%	0.0523134	6288.5	236.6418	0.0523134	5755.2	236.6418

Table 10. Comparison between model I and II with fuzzy carbon emission (exponential time dependent demand)

% variation $\tilde{E}_h$	T_2^*	TC_2^*	Q_2^*	T_4^*	TC_4^*	Q_4^*
-18%	0.0554259	5383.2	250.7990	0.0554259	4849.9	250.7990
-9%	0.0552524	5400	250.0096	0.0552524	4866.9	250.0096
0%	0.0550806	5417.2	249.2279	0.0550806	4883.8	249.2279
9%	0.0549103	5434.1	248.4532	0.0549103	4900.7	248.4532
18%	0.0547416	5450.9	247.6857	0.0547416	4917.6	247.6857

Table 11. Effect of carbon emission on the optimal values with fuzzy costs (Model II with constant demand)

E_h	% Variation $\tilde{A}$	% Variation $\tilde{h}$	T^*	Q^*	TC^*
2.4	-20%	-12%	0.0533848	240.2316	4252.9
	-10%	-6%	0.0549260	247.1670	4680.1
	0	0	0.0560525	252.2362	5084.3
	10%	6%	0.0571495	257.1728	5497.7
	20%	12%	0.0582925	262.3162	5927.8
2.6	-20%	-12%	0.0532436	239.5962	4264.9
	-10%	-6%	0.0547893	246.5519	4692.4
	0	0	0.0559209	251.6441	5096.9
	10%	6%	0.0570226	256.6017	5510.5
	20%	12%	0.0581695	261.7627	5940.9
2.8	-20%	-12%	0.0531035	238.9657	4276.9
	-10%	-6%	0.0546538	245.9421	4704.7
	0	0	0.0557903	251.0564	5109.4
	10%	6%	0.0568964	256.0338	5523.3
	20%	12%	0.0580474	261.2133	5954
3	-20%	-12%	0.0529645	238.3402	4288.8
	-10%	-6%	0.0545191	245.3359	4717
	0	0	0.0556605	250.4723	5122
	10%	6%	0.0567711	255.4699	5536.1
	20%	12%	0.0579259	260.6666	5967

Table 12. Effect of carbon emission on the optimal values with fuzzy costs (Model II with exponential time dependent demand)

E_h	% Variation $\tilde{A}$	% Variation $\tilde{h}$	T^*	Q^*	TC^*
2.4	-10%	-24%	0.0599019	271.1733	3967
	-5%	-12%	0.0574851	260.1700	4413.9
	0%	0%	0.0554647	250.9755	4846.1
	5%	12%	0.0538415	243.5913	5277.1
	10%	24%	0.0526109	237.9946	5719.4
2.6	-10%	-24%	0.0597223	270.3554	3980.6
	-5%	-12%	0.0573345	259.4845	4426.9
	0%	0%	0.0553357	250.3886	4858.7
	5%	12%	0.0537289	243.0791	5289.3
	10%	24%	0.0525111	237.5408	5731.3
2.8	-10%	-24%	0.0595442	269.5444	3994.2
	-5%	-12%	0.0571851	258.8045	4440
	0%	0%	0.0552077	249.8062	4871.3
	5%	12%	0.0536172	242.5711	5301.5
	10%	24%	0.0524118	237.0892	5743.3
3	-10%	-24%	0.0593677	268.7407	4007.7
	-5%	-12%	0.0570369	258.1300	4453
	0%	0%	0.0550806	249.2279	4883.8
	5%	12%	0.0535060	242.0676	5313.7
	10%	24%	0.0523131	236.6418	5755.3

Table 13. Impact of carbon tax on the optimal values (Model II with constant demand)

c_1	G	T_3^*	TC_3^*	Q_3^*
0.45	129.6296	0.0558555	5143	251.3498
0.475	131.5789	0.0557578	5132.5	250.9101
0.5	133.3333	0.0556605	5122	250.4723
0.525	134.9206	0.0555638	5111.3	250.0375
0.55	136.3636	0.0554676	5100	249.6042

Table 14. Impact of carbon tax on the optimal values (Model II with exponential time dependent demand)

c_1	G	T_4^*	TC_4^*	Q_4^*
0.46	130.4327	0.0552332	4932.6	249.9222
0.48	131.9444	0.0551567	4908.3	249.5742
0.5	133.3333	0.0550806	4883.8	249.2279
0.52	134.6158	0.0550046	4859.3	248.8822
0.54	135.8042	0.0549291	4834.7	248.5387

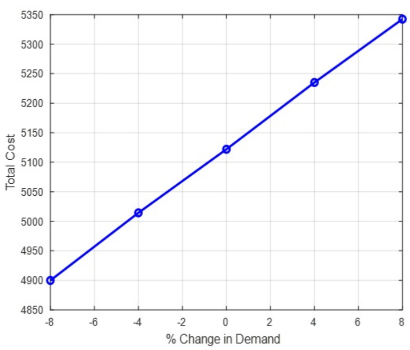

Figure 3a: Demand vs Total Cost (Model II with constant demand)

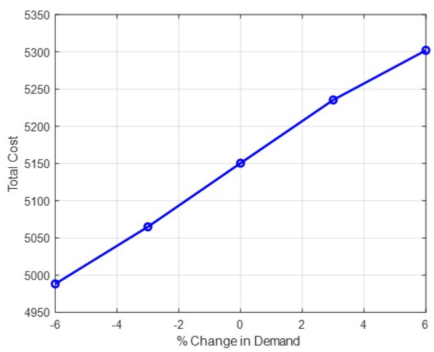

Figure 3b: Demand vs Total Cost (Model II with exponential time dependent demand)

Figure 3. The effect of demand on Total Cost

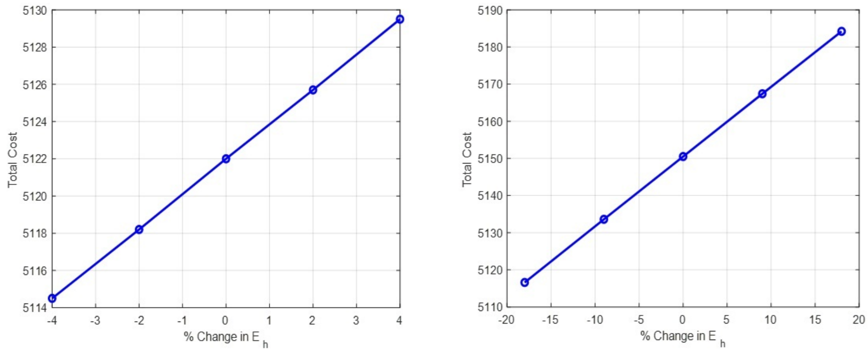

Figure 4a: E_h vs Total Cost (Model II with constant demand)

Figure 4b: E_h vs Total Cost (Model II with exponentially time dependent demand)

Figure 4. The effect of carbon emission on Total Cost

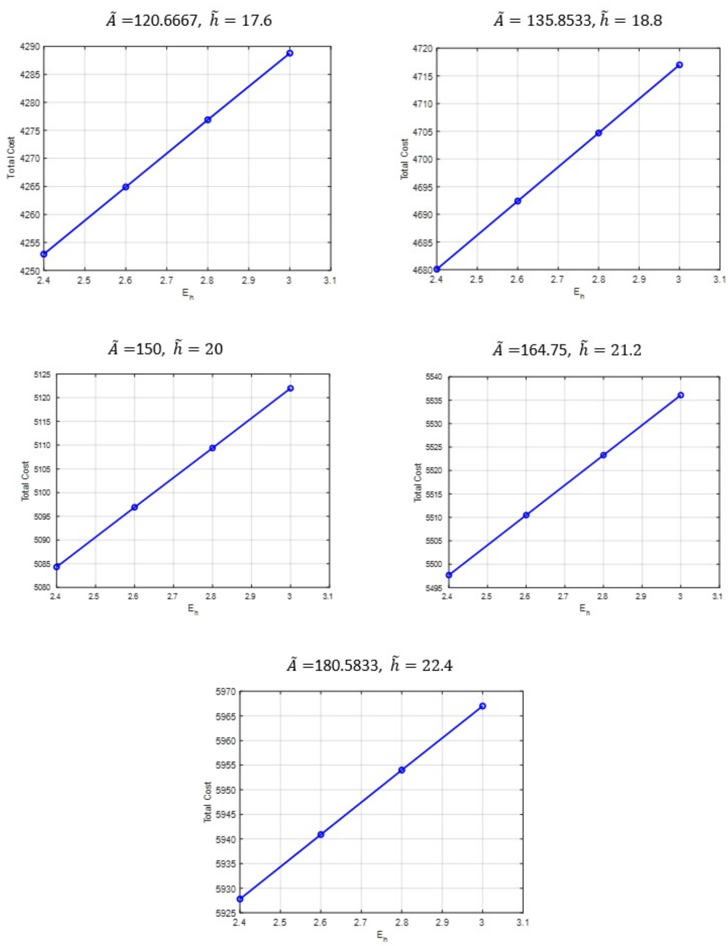

Figure 5. E_h vs total cost (Model II with constant demand)

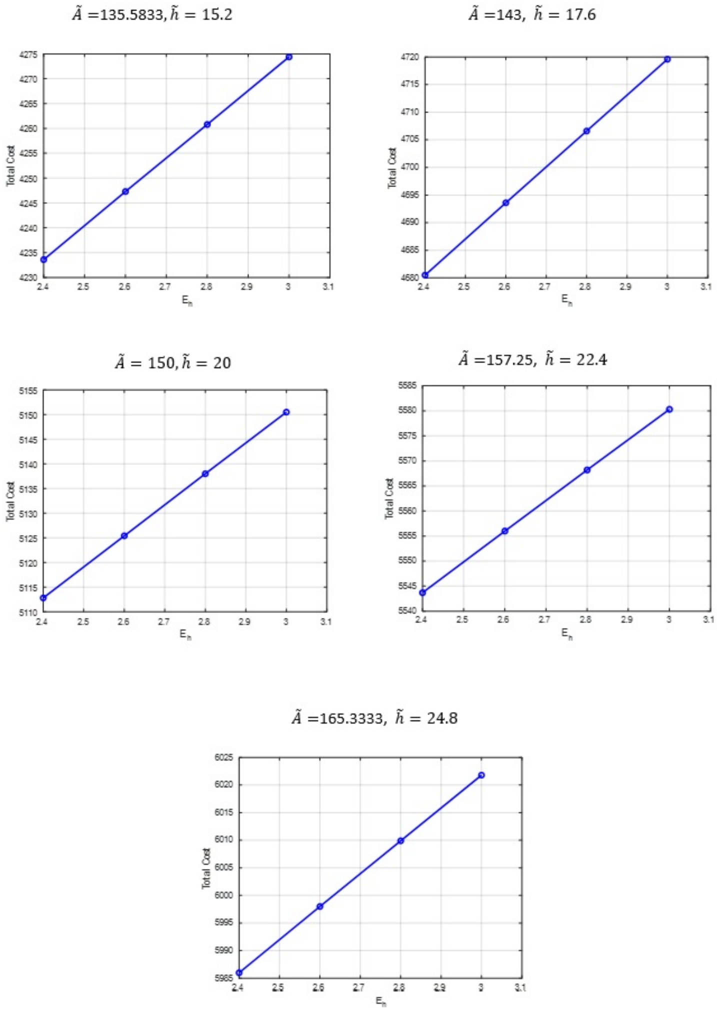

Figure 6. E_h vs total cost (Model II with exponential time dependent demand)

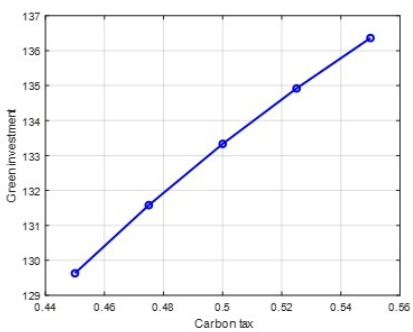

Figure 7a: Carbon tax vs Green investment
(Model II with constant demand)

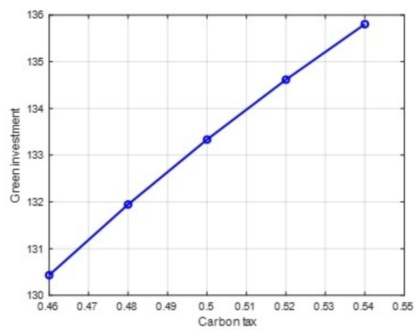

Figure 7b: Carbon tax vs Green investment
(Model II with exponential time demand)

Figure 7. Carbon tax vs Green investment

6.2. Insights and Discussion

The analysis reveals that fuzziness in demand has a more substantial impact on the total cost than on the cycle time, as evidenced in Table 5, Table 8 and Figure 3. Hence the effect of uncertainty in demand on the cycle time remains comparatively moderate. Tables 6 and 9 shows that ambiguity in ordering and holding costs exerts a stronger influence on total cost than on cycle time. The results suggest that fluctuations in these cost parameters considerably affect the total cost, while their influence on cycle time remains comparatively less significant in both models.

Uncertainty in the carbon emission parameter E_h also produces a notable effect on the total cost of the inventory system, as seen in Table 7, Table 10 and Figure 4. However, the fuzzy nature of E_h has a greater influence on the total cost than on the cycle time, while its effect on the order quantity is negligible. This finding highlights that emission-related uncertainty predominantly affects the economic performance of the system rather than its operational decisions.

Further sensitivity assessment of E_h(Tables 11 and 12) reveals a positive correlation between carbon emissions and total cost. As emission levels rise, the total cost of the system increases proportionally, emphasizing the financial importance of emission control measures. This relationship reinforces the necessity of adopting green technologies and emission-reducing strategies to achieve cost efficiency under uncertain conditions.

Tables 5–10 reveals that green investments effectively reduce total inventory costs. Furthermore, as observed in Table 13, Table 14 and illustrated in Figure 7, as the carbon tax increases, the green investment also increases than on other decision variables such as order quantity and cycle time. This suggests that the system prioritizes adjustments in green investment in response to rising carbon taxes, while the other parameters exhibit lower sensitivity. Overall, these findings underscore the strategic importance of green investments as a key mechanism for managing both environmental and economic objectives in uncertain, carbon-regulated environments.

7. Managerial implications

Key Managerial Implications from model I

The analysis presented in Table 5 demonstrates that a 4% uncertainty in the demand parameter results in a 2.09% variation in the total cost in model I. This finding emphasizes the critical importance for retailers to consider the inherent uncertainty and imprecise nature of demand while formulating their inventory policies. By integrating this uncertainty into the design and implementation of inventory models, retailers can achieve more robust, resilient, and cost-efficient decision-making frameworks that better reflect real-world market conditions.

Furthermore, a significant positive correlation is observed between total cost and carbon emissions, suggesting that an increase in emission levels contributes directly to higher operational costs. As shown in Table 10, a 9% uncertainty in the carbon emission parameter (E_h) leads to a 0.31% fluctuation in total cost. This result underscores the importance of effective carbon management strategies, as reducing emissions can substantially mitigate overall costs. Hence, adopting sustainable practices and investing in emission-reducing technologies are essential for achieving both environmental and economic efficiency in inventory management.

Key Managerial Implications from model II

Table 7 indicates that even a 2% uncertainty in the carbon emission parameter (E_h) can cause a 0.069% change in both the optimal order quantity and cycle time. This results indicates that order quantity and cycle time are less sensitive to the ambiguity in the parameter (E_h). Such a strategy not only reduces exposure to emission-related cost fluctuations but also enhances inventory flexibility and responsiveness to environmental regulations.

Moreover, the results in Tables 13 and 14 reveal that an increase in the carbon tax (c_1) leads to a higher optimal green investment (G), while simultaneously reducing the optimal order quantity. This indicates that under stringent carbon taxation policies, the optimal strategy involves frequent, smaller replenishments coupled with greater investment in green technologies. The imposition of a progressive carbon tax thus acts as a strong incentive for industries to intensify their investments in cleaner and more energy-efficient technologies. Consequently, managers must carefully evaluate and select the most suitable green technologies for adoption to ensure compliance with regulatory requirements while maintaining operational profitability.

Our results demonstrates that the total costs are significantly lower with the adoption of green technology, regardless whether the demand is steady or growing exponentially with respect to time. This reduction demonstrates that the integration of environmentally sustainable practices, such as green technology, effectively minimizes the financial burden caused by carbon taxes and conventional inventory operations. Overall, these findings reveals that the critical role of green technology in enhancing both environmental and economic performance. The incorporation of such technologies not only reduces carbon emissions but also contributes to long-term cost savings by optimizing resource utilization and reducing operational inefficiencies. From a managerial perspective, this evidence suggests that investing in green technology should be considered a strategic priority rather than a discretionary expense. Managers and decision-makers are encouraged to integrate green initiatives into their inventory and supply chain planning, as these investments lead to sustainable competitive advantages achieving lower total costs, improving corporate image, and aligning business operations with environmental regulations and sustainability goals.

8. Conclusion

In this study, we develop an inventory model for time-dependent demand integrating with carbon tax and green investment. Models I and model II are formulated in a fuzzy environment where the parameters such as demand, ordering cost, holding cost and the amount of carbon emission are fuzzified using pentagonal fuzzy numbers. In both the models, the total inventory cost is very much sensitive to the fuzziness of the various factors such as demand, ordering cost and the amount of carbon emission. Consequently, inventory managers are advised to consider the inherent flexibility of these parameters when formulating optimal inventory policies. Climate change poses a significant threat to global markets and economic growth. In this context, businesses must prioritize both sustainability and financial stability. Investing in green technologies can contribute to ecological conservation and promote global sustainability. Our research indicates that companies can combat climate change by implementing green technologies that minimize CO_2 emissions during storage. Hence, it helps to reduce the carbon tax and total inventory cost. This work can be extended to address the problems considering deteriorating items and partial backlogging of shortages.

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