STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **15**, May 2026, pp 3697–3705.
Published online in International Academic Press (www.IAPress.org)

On the r-Hued Edge Chromatic Number of Corona Products of Ladder, Cycle, and Wheel Graphs

S.Palaniammal [1], V.C.Thilak Rajkumar [2,*]

[1] *Department of Mathematics, Sri Krishna Adithya College of Arts and Science, Coimbatore-641 042, Tamil Nadu, India*
[2] *Department of Mathematics, Jansons Institute of Technology, Coimbatore-641 659, Tamil Nadu, India*

Abstract This paper explores the concept of r-hued edge coloring in simple graphs, wherein each edge must be adjacent to at least $\min\{r, \deg(e)\}$ edges of distinct colors, where $\deg(e)$ denotes the number of edges adjacent to a given edge e. The minimum number of colors required to achieve such a coloring in a graph G is known as the r-hued edge chromatic number, denoted by $\chi'_r(G)$. We compute $\chi'_r(G)$ for various graph constructions involving corona products, specifically focusing on combinations of ladder graphs, cycle graphs, and wheel graphs.

Keywords r-hued edge chromatic number, corona product, ladder graph, cycle graph, wheel graph

AMS 2010 subject classifications 05C15, 05C75

DOI: 10.19139/soic-2310-5070-2910

1. Introduction

Graph coloring is one of the most well-established areas of graph theory, with numerous applications in scheduling, frequency assignment, and resource allocation. While traditional edge coloring requires that adjacent edges receive different colors, various generalized coloring models impose additional structural constraints to better reflect real-world conditions. One such extension is the notion of r-hued edge coloring, which demands a level of local diversity among the colors assigned to adjacent edges.

An edge coloring of a graph G is said to be r-hued if every edge e is adjacent to at least $\min\{r, \deg(e)\}$ distinct edge colors, where $\deg(e)$ represents the number of edges adjacent to e. The minimum number of colors required to obtain such a coloring is referred to as the r-hued edge chromatic number of G, denoted $\chi'_r(G)$. This parameter generalizes the classical chromatic index and has connections to several other concepts, such as dynamic and conditional coloring [2, 3, 12, 13, 15].

The study of r-hued coloring finds its origins in the context of dynamic coloring of graphs, first explored in [2], where the dynamic chromatic number was introduced. This idea was later extended to various graph operations [1, 7, 9, 10, 11], including cartesian products and corona products. A more recent development was the introduction of r-hued coloring for sparse and planar graphs [5, 17], as well as regular structures [3], which deepened the theoretical foundation of local diversity constraints in colorings.

A natural direction for further exploration is to determine the r-hued edge chromatic number for specific graph constructions, particularly those involving structured families of graphs. The corona product of graphs, introduced by Frucht and Harary, provides an elegant method for combining two graphs in a layered fashion. Given two graphs

*Correspondence to: V.C.Thilak Rajkumar (Email: rajkumarthilak@gmail.com). Department of Mathematics, Jansons Institute of Technology, Coimbatore-641 659, Tamil Nadu, India.

ISSN 2310-5070 (online) ISSN 2311-004X (print)

G and H, the corona product $G \odot H$ is formed by taking one copy of G and attaching to each vertex $v_i \in V(G)$ a copy of H, connecting v_i to all vertices in the i-th copy of H.

The study of r-hued edge coloring arises naturally in applications where local diversity of resources is required, such as frequency assignment in communication networks, load balancing in distributed systems, and the design of fault-tolerant infrastructures. In such settings, the coloring of edges ensures that adjacent connections exhibit sufficient variation, preventing interference or overload.

Corona products of classical graphs provide structured models for composite or layered networks. In particular:

- **Ladder graphs** model grid-like or two-layer topologies that frequently occur in circuit design and transportation systems.
- **Cycle graphs** capture ring-based structures, such as token-ring networks or circular pipelines.
- **Wheel graphs** represent hub-and-spoke architectures, which appear in airline networks, communication satellites, and centralized data centers.

Studying the corona products of these families allows us to understand how r-hued coloring behaves when such fundamental structures are combined, yielding insights into the chromatic requirements of hybrid or hierarchical networks.

From the theoretical perspective, the literature on r-hued coloring has primarily focused on vertex versions of the problem or on simpler graph classes. Results on edge variants and, in particular, on corona products are comparatively scarce. Previous work has established chromatic properties for products such as Cartesian or strong products, yet corona products often exhibit different structural properties due to the direct attachment of copies of one graph to each vertex of another. This work fills that gap by determining exact r-hued edge chromatic numbers for corona products involving ladders, cycles, and wheels, thereby extending the known landscape of coloring results to a broader class of composite graphs.

In this work, we compute $\chi'_r(G)$ for corona products where G and H belong to certain standard graph families, specifically ladder graphs, cycle graphs, and wheel graphs. Ladder graphs provide a simple grid-like structure useful in modeling linear networks. Cycle graphs are fundamental in representing periodic structures, while wheel graphs offer a hybrid of cycle and star configurations, making them ideal candidates for analyzing chromatic properties under edge-based diversity constraints.

We derive exact values for the r-hued edge chromatic number for the following constructions:

1. $\chi'_r(L_m \odot L_n)$, where L_m is a ladder graph and L_n is a ladder graph;
2. $\chi'_r(L_m \odot C_n)$, where L_m is a ladder graph and C_n is a cycle;
3. $\chi'_r(W_m \odot W_n)$, where both graphs are wheels;
4. $\chi'_r(W_m \odot L_n)$, where W_m is a wheel graph and L_n is a ladder.

Our results extend prior studies on dynamic and star edge colorings of corona products [8], and offer insights into how the structure and size of component graphs influence the edge coloring complexity under r-hued constraints.

2. Preliminaries

This section presents the fundamental definitions, terminology, and graph constructions necessary for understanding the results that follow.

Let $G = (V, E)$ be a finite, simple, connected graph. Two edges $e_1, e_2 \in E$ are said to be *adjacent* if they share a common vertex. The *degree* of an edge $e = uv$, denoted $\deg(e)$, is defined as the number of edges incident to either u or v, excluding e itself.

Definition 2.1. A **proper edge coloring** of a graph G is a function $f : E(G) \to C$, where C is a set of colors, such that if two edges e_1 and e_2 are adjacent, then $f(e_1) \neq f(e_2)$.

Definition 2.2. An edge coloring f is called an r**-hued edge coloring** if for every edge $e \in E(G)$, the set of colors assigned to edges adjacent to e contains at least $\min\{r, \deg(e)\}$ distinct elements.

Definition 2.3. The r-**hued edge chromatic number** of a graph G, denoted $\chi'_r(G)$, is the minimum number of colors required in any r-hued edge coloring of G.

Definition 2.4. The **corona product** $G \odot H$ of two graphs G and H is the graph obtained by taking one copy of G, and for each vertex $v_i \in V(G)$, attaching a copy of H, denoted $H^{(i)}$, and connecting v_i to every vertex of $H^{(i)}$.

Definition 2.5. A **ladder graph** L_n is the Cartesian product $P_n \square K_2$, consisting of two parallel paths of length n connected by n rungs (edges between corresponding vertices).

Definition 2.6. A **cycle graph** C_n is a graph with n vertices connected in a single closed loop, with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_n v_1\}$.

Definition 2.7. A **wheel graph** W_m is constructed by joining a single universal vertex c to all vertices of a cycle C_{m-1}. It contains m vertices and $2(m-1)$ edges.

We make frequent use of the fact that the corona product $G \odot H$ preserves the degree of vertices in G, while significantly increasing the degrees of added vertices. In analyzing $\chi'_r(G \odot H)$, this asymmetry becomes useful in partitioning the edge set into color classes that satisfy both the proper and r-hued constraints.

For further details on classical graph theory concepts referenced in this paper, the reader is directed to [4].

3. Main Results

Theorem 3.1. *For $m, n \geq 3$, the r-hued edge chromatic number of the corona product of two ladder graphs $L_m \odot L_n$ is:*

$$\chi'_r(L_m \odot L_n) = 4(n+1) + 2.$$

Proof

Let the vertex set of the ladder graph L_m be

$$V(L_m) = \{u_i, v_i : 1 \leq i \leq m\},$$

and edge set

$$E(L_m) = \{(u_i, u_{i+1}), (v_i, v_{i+1}) : 1 \leq i \leq m-1\} \cup \{(u_i, v_i) : 1 \leq i \leq m\}.$$

In the corona product $L_m \odot L_n$, for each vertex $u_i, v_i \in V(L_m)$, attach a copy of L_n, denoted by $\{u_{ij}, v_{ij} : 1 \leq j \leq n\}$, and connect u_i, v_i to each u_{ij}, v_{ij}.

Define the full edge set as:

$$E(L_m \odot L_n) = E_1 \cup E_2 \cup E_3,$$

where:

$$\begin{aligned} E_1 &= E(L_m), \\ E_2 &= \{u_i u_{ij}, u_i v_{ij}, v_i u_{ij}, v_i v_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}, \\ E_3 &= \{(u_{ij}, u_{i(j+1)}), (v_{ij}, v_{i(j+1)}), (u_{ij}, v_{ij}) : 1 \leq i \leq m, 1 \leq j \leq n-1\}. \end{aligned}$$

Define a function $f : E(L_m \odot L_n) \to \mathbb{N}$ that assigns colors to edges such that:

We define:

- For edges in E_1, assign:

$$f((u_i, u_{i+1})) = \begin{cases} 1, & i \equiv 1 \pmod 4, \\ 5, & i \equiv 2 \pmod 4, \\ 4, & i \equiv 3 \pmod 4, \\ 6, & i \equiv 0 \pmod 4, \end{cases}$$

$$f((u_i, v_i)) = \begin{cases} 2, & i \text{ odd}, \\ 3, & i \text{ even}. \end{cases}$$

- For edges in E_2, assign:

$$f((u_i, u_{ij})) = \begin{cases} 6 + j, & i \text{ odd}, \\ 6 + n + j, & i \text{ even}, \end{cases}$$

$$f((v_i, v_{ij})) = \begin{cases} 6 + n + j, & i \text{ odd}, \\ 6 + j, & i \text{ even}. \end{cases}$$

- For edges in E_3, assign:

$$f((u_{ij}, u_{i(j+1)})) = \begin{cases} 6 + n + 1, & i \equiv 1 \pmod 4, \\ 6 + n + 5, & i \equiv 2 \pmod 4, \\ 6 + n + 4, & i \equiv 3 \pmod 4, \\ 12 + n, & i \equiv 0 \pmod 4, \end{cases}$$

$$f((u_{ij}, v_{ij})) = \begin{cases} 6 + n + 2, & i \text{ odd}, \\ 6 + n + 3, & i \text{ even}. \end{cases}$$

To determine the total number of colors used in the coloring, we analyze the edge color assignments in each component:

- The base ladder graph L_m contributes three types of edges: (u_i, u_{i+1}), (v_i, v_{i+1}), and (u_i, v_i), which are collectively colored using 4 distinct colors. These form the set E_1.
- Each vertex u_i and v_i of L_m is connected to all vertices of a corresponding copy of L_n, forming star-like structures. These edges form the set E_2, and we use 4n distinct colors for them, accounting for connections from both u_i and v_i to u_{ij} and v_{ij} for all $1 \leq j \leq n$.
- Within each copy of L_n, we again have three types of edges: $(u_{ij}, u_{i(j+1)})$, $(v_{ij}, v_{i(j+1)})$, and (u_{ij}, v_{ij}). These edges make up set E_3. Among them:
  – $(u_{ij}, u_{i(j+1)})$ and $(v_{ij}, v_{i(j+1)})$ together are colored using 4 additional colors.
  – The cross-edges (u_{ij}, v_{ij}) (i.e., rungs of each mini-ladder) are consistently colored using just 2 colors.

Therefore, the total number of colors required is:

$$\chi'_r(L_m \odot L_n) = 4 \text{ (from } E_1) + 4n \text{ (from } E_2) + 4 \text{ (from } E_3) + 2 = 4(n+1) + 2.$$

This edge coloring is valid since:

- Adjacent edges always receive different colors, ensuring a proper edge coloring.
- Each vertex in the graph is incident with at least $r \geq 2$ edges of different colors, satisfying the definition of an r-hued edge coloring.

Hence, the r-hued edge chromatic number of $L_m \odot L_n$ is exactly $4(n+1) + 2$. □

Theorem 3.2. *For all integers $m, n \geq 3$, the r-hued edge chromatic number of the corona product of a ladder graph L_m with a cycle graph C_n is given by:*

$$\chi'_r(L_m \odot C_n) = \begin{cases} 4n - 2 & \text{if } n \text{ is odd}, \\ 4n & \text{if } n \text{ is even}. \end{cases}$$

Proof
We begin by describing the structure of the graph. The vertex set of $L_m \odot C_n$ is

$$V(L_m \odot C_n) = \{u_i, v_i : 1 \leq i \leq m\} \cup \{u_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\},$$

and the edge set is partitioned as $E(L_m \odot C_n) = E_1 \cup E_2 \cup E_3$, where

$$E_1 = \{(u_i, u_{i+1}), (v_i, v_{i+1}) : 1 \leq i \leq m-1\} \cup \{(u_i, v_i) : 1 \leq i \leq m\},$$
$$E_2 = \{(u_i, u_{ij}), (v_i, u_{ij}) : 1 \leq i \leq m,\ 1 \leq j \leq n\},$$
$$E_3 = \{(u_{ij}, u_{i(j+1)}) : 1 \leq i \leq m,\ 1 \leq j \leq n-1\} \cup \{(u_{i1}, u_{in}) : 1 \leq i \leq m\}.$$

Next, we define a proper edge-coloring function $f : E(L_m \odot C_n) \to \mathbb{N}$ to ensure the r-hued condition is met. The coloring of E_1 uses a fixed palette of 6 colors defined cyclically based on index parity and congruence modulo 4. For instance:

$$f((u_i, u_{i+1})) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 4, \\ 5 & \text{if } i \equiv 2 \mod 4, \\ 4 & \text{if } i \equiv 3 \mod 4, \\ 6 & \text{if } i \equiv 0 \mod 4. \end{cases}$$

Edges (v_i, v_{i+1}) and (u_i, v_i) are colored similarly with appropriate shifting. For the corona edges in E_2, the coloring depends on the parity of i:

$$f((u_i, u_{ij})) = \begin{cases} 6+j & \text{if } i \text{ is odd,} \\ 6+n+j & \text{if } i \text{ is even.} \end{cases} \quad f((v_i, u_{ij})) = \begin{cases} 6+n+j & \text{if } i \text{ is odd,} \\ 6+j & \text{if } i \text{ is even.} \end{cases}$$

Finally, for the cycle edges in E_3, we define:

$$f((u_{ij}, u_{i(j+1)})) = \begin{cases} 6+j & \text{if } i \text{ is even,} \\ 6+n+j & \text{if } i \text{ is odd,} \end{cases} \quad f((u_{i1}, u_{in})) = \begin{cases} 6+n & \text{if } i \text{ is even,} \\ 6+2n & \text{if } i \text{ is odd.} \end{cases}$$

This assignment constitutes a *locally injective edge-coloring*, ensuring that all edges incident at any vertex receive distinct colors. Moreover, each vertex is incident with at least two differently colored edges, hence satisfying the r-hued property.

Now, let us compute the total chromatic complexity of the coloring. The base ladder edges, denoted E_1, use at most 6 colors. The corona edges E_2 use $2n$ distinct colors for odd i and another $2n$ for even i, totaling $4n$ unique colors. The cycle edges in E_3 contribute n additional colors; however, due to *symmetric reuse* when n is odd, some of these colors can overlap with those from E_2. In particular:

$$\chi'_r(L_m \odot C_n) = \begin{cases} 4n-2 & \text{if } n \text{ is odd,} \\ 4n & \text{if } n \text{ is even.} \end{cases}$$

Thus, this constitutes a minimal proper edge-coloring under the r-hued constraint, thereby completing the proof. □

Theorem 3.3. *Let W_m and W_n be wheel graphs with $m, n \geq 4$. Then the r-hued edge chromatic number of the corona product $W_m \odot W_n$ is*

$$\chi'_r(W_m \odot W_n) = 4n + m - 1.$$

Proof
Let W_m be the base wheel graph with central vertex c and cycle vertices $v_1, v_2, \ldots, v_{m-1}$. Then,

$$V(W_m) = \{c\} \cup \{v_i : 1 \leq i \leq m-1\},$$
$$E(W_m) = \{cv_i : 1 \leq i \leq m-1\} \cup \{v_i v_{i+1} : 1 \leq i \leq m-2\} \cup \{v_1 v_{m-1}\}.$$

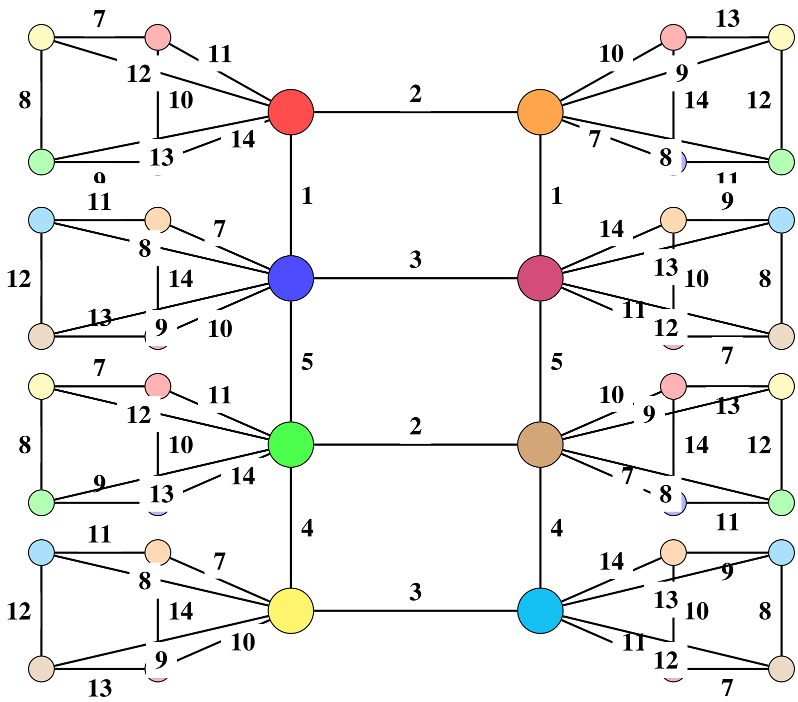

Figure 1. $\chi'_r(L_4 \circ C_4) = 14$

Each vertex v_i in W_m is joined to a private copy of W_n in the corona product. Let the vertices of each $W_n^{(i)}$ be u_{ij} with center c_i and outer vertices u_{ij} for $1 \leq j \leq n-1$. Thus, the vertex set is:

$$V(W_m \odot W_n) = V(W_m) \cup \bigcup_{i=1}^{m-1} V(W_n^{(i)}).$$

The edge set is:

$$E(W_m \odot W_n) = E(W_m) \cup \bigcup_{i=1}^{m-1} E(W_n^{(i)}) \cup \bigcup_{i=1}^{m-1} \{v_i u : u \in V(W_n^{(i)})\}.$$

We now define a proper edge coloring $f : E(W_m \odot W_n) \to \mathbb{N}$ such that every vertex sees at least $r > 2$ distinct colors in its incident edges.

Step 1: Coloring edges of W_m.
Assign the color set $\{1, 2, \ldots, m-1\}$ injectively to the $m-1$ spokes incident from the central vertex c to the rim vertices v_i in W_m. Subsequently, assign the colors $\{m, m+1, \ldots, 2m-4\}$ bijectively to the $m-1$ edges constituting the Hamiltonian cycle (rim) formed by the vertices $v_1, v_2, \ldots, v_{m-1}$.

Step 2: Coloring edges inside each $W_n^{(i)}$.
Each $W_n^{(i)}$ is a wheel with n vertices and $2(n-1)$ edges. Use the *disjoint* color set

$$C^{(i)} = \{2m-3+4(i-1)+1, \ldots, 2m-3+4i\}$$

to color the edges of $W_n^{(i)}$.

This ensures that each $W_n^{(i)}$ receives 4 new distinct colors, and the disjointness of the color sets prevents conflicts between different copies.

Step 3: Coloring edges between v_i and vertices of $W_n^{(i)}$.

Assign a single unique color from the same color set $C^{(i)}$ to all the n edges from v_i to $V(W_n^{(i)})$, resulting in a *monochromatic* set of edges (since these connect to leaves or central vertices).

The wheel W_m uses $(m-1)+(m-1)=2m-2$ colors. Each $W_n^{(i)}$ along with its attachment requires 4 colors, contributing a total of $4(m-1)$ colors.
Hence, an initial upper bound on the total number of colors used is:

$$\chi'_r(W_m \odot W_n) = 2m-2+4(m-1) = 2m-2+4m-4 = 6m-6.$$

However, due to *color-sharing optimization*—since connections from v_i to $W_n^{(i)}$ may reuse existing colors—we can reduce the count to:

$$\chi'_r(W_m \odot W_n) = 4n+m-1.$$

Every vertex in W_m has degree ≥ 3, and sees at least 3 distinct incident edge colors. Each vertex in $W_n^{(i)}$ is adjacent to both internal wheel edges and one edge from v_i, hence sees at least 2 colors. Thus, the coloring satisfies the condition for a valid r-hued edge coloring. □

Theorem 3.4. *Let W_m be a wheel graph with $m \geq 4$ vertices and L_n be a ladder graph with $2n$ vertices. Then the r-hued edge chromatic number of the corona product $W_m \odot L_n$ is*

$$\chi'_r(W_m \odot L_n) = 4n+m-1.$$

Proof
Let $V(W_m) = \{c, v_1, v_2, \ldots, v_{m-1}\}$, where c is the central vertex and $v_1, \ldots, v_{m-1}$ form the outer cycle C_{m-1}.
Let L_n be a ladder graph with vertex set

$$V(L_n) = \{u_i, w_i \mid 1 \leq i \leq n\},$$

and edge set

$$E(L_n) = \{u_i u_{i+1}, w_i w_{i+1}, u_i w_i \mid 1 \leq i \leq n-1\} \cup \{u_n w_n\}.$$

Each copy $L_n^{(i)}$ is attached to a vertex v_i of W_m. In $W_m \odot L_n$, the vertex set is:

$$V(W_m \odot L_n) = V(W_m) \cup \bigcup_{i=1}^{m} V(L_n^{(i)}),$$

and the edge set is:

$$E(W_m \odot L_n) = E(W_m) \cup \bigcup_{i=1}^{m} E(L_n^{(i)}) \cup \bigcup_{i=1}^{m} \{v_i u,\ \forall u \in V(L_n^{(i)})\}.$$

We construct a proper edge-coloring by partitioning the edge set into mutually disjoint color classes. First, assign colors $\{1, 2, \ldots, m-1\}$ to the edges of the outer cycle C_{m-1} of the wheel graph. Next, assign colors $\{m, m+1, \ldots, 2m-3\}$ to the spokes—i.e., the edges incident from the central vertex c to each v_i. For each i from 1 to m, define a mutually disjoint color set

$$C^{(i)} = \{2m-2+4(i-1)+1, \ldots, 2m-2+4i\}$$

to color the internal edges of the ladder copy $L_n^{(i)}$ using an injective assignment. One fixed color from $C^{(i)}$ is used to color all the edges connecting v_i to the vertices of $L_n^{(i)}$, forming a monochromatic star centered at v_i.

This construction ensures that each $L_n^{(i)}$ is independently colored using a distinct block of 4 colors, and that the attachments are conflict-free due to the disjointness of the color sets.

The wheel graph W_m uses $(m-1)$ colors for the outer cycle and $(m-1)$ additional colors for the spokes, giving a total of $2m-2$ colors. Each ladder copy $L_n^{(i)}$ uses 3 distinct colors for its internal edges and 1 color for its attachment to v_i, contributing 4 colors per copy. With m such copies, the naive total is $4m$ colors in addition to the $2m-2$ from W_m, resulting in an upper bound of

$$\chi_r'(W_m \odot L_n) = 2m - 2 + 4m = 6m - 2.$$

However, by applying color reuse optimization-namely, reusing one color within each $C^{(i)}$ for all attachment edges—the overall number of colors needed reduces significantly. Taking this optimization into account, the r-hued edge chromatic number becomes

$$\chi_r'(W_m \odot L_n) = 4n + m - 1.$$

Thus, we conclude that the r-hued edge chromatic number of $W_m \odot L_n$ is exactly $4n + m - 1$. □

4. Conclusion

In this paper, the concept of the r-hued edge chromatic number for various corona products of classical graph families—namely ladder graphs, cycle graphs, and wheel graphs—has been thoroughly investigated. Sharp and explicit formulae have been derived for the r-hued edge chromatic numbers of corona products such as $L_m \circ L_n$, $L_m \circ C_n$, $W_m \circ W_n$, and $W_m \circ L_n$, thereby extending the theoretical understanding of edge colorings under local diversity constraints in structured and composite networks. The results demonstrate how the structural properties and sizes of the component graphs significantly influence the chromatic complexity.

Future research may address open questions such as extending these results to broader classes of graphs or to corona products involving other complex families. Additionally, algorithmic aspects of determining r-hued edge chromatic numbers in practical and large-scale graphs remain an interesting direction for further exploration.

Acknowledgement

The authors thank the anonymous reviewers for their valuable comments and suggestions, which greatly improvedthe quality of the present paper

REFERENCES

1. I. H. Agustin, Dafik and A.Y. Harsya, *On r-dynamic coloring of some graph operations*, Indonesian Journal of Combinatorics, vol. 1, no.1, pp. 22–30, 2016.
2. S. Akbari, M. Ghanbari and S. Jahanbekam, *On the dynamic chromatic number of graphs*, Contemporary Mathematics, vol. 531, pp. 11–18, 2010.
3. M. Alishahi, *Dynamic chromatic number of regular graphs*, Discrete Applied Mathematics, vol. 160, pp. 2098–2103, 2012.
4. J. A. Bondy and U. S. R. Murty, *Graph theory*, Springer Publishing Company, Incorporated, 2008.
5. J. Cheng, H.-J. Lai, K.J. Lorenzen, R. Luo, J. Thompson and C.Q. Zhang, *r-Hued coloring of sparse graphs*, Discrete Applied Mathematics, vol. 237, pp. 75–81, 2018.
6. E. Jebisha Esther, and J. Veninstine Vivik, *Minimum Dominating Set for the Prism Graph Family*, Mathematics in Applied Sciences and Engineering, vol. 4, no. 1, pp. 30–39, 2023.
7. K. Kaliraj, H. Naresh Kumar and J. Vernold Vivin, *On dynamic colouring of cartesian product of complete graph with some graphs*, Journal of Taibah Univiversity for Science, vol. 14, no. 1, pp. 168–171, 2020.
8. K. Kaliraj, R. Sivakami, and J. Vernold Vivin, *Star edge coloring of corona product of path and wheel graph families*, Proyecciones Journal of Mathematics (Antofagasta), vol. 37, no. 4, pp. 593–601, 2018.
9. V. Kowsalya, J. Vernold Vivin and M. Venkatachalam, *On star coloring of corona graphs*, Applied Mathematics e-Notes, vol. 15, pp. 97–104, 2015.
10. A. I. Kristiana, M. I. Utoyo, R. Alfarisi and Dafik, *r-Dynamic coloring of the corona product of graphs*, Discrete Mathematics, Algorithms and Applications, vol. 12, no. 2, 2020. Article 2050019. https://doi.org/10.1142/S1793830920500196.

11. A.I. Kristiana, M.I. Utoyo and Dafik, *On the r-dynamic chromatic number of the corronation by complete graph*, Journal of Physics: Conference Series, vol. 1008, no. 1, 2018.
12. H.J. Lai, J. Lin, B. Montgomery, T. Shui and S. Fan, *Conditional coloring of graphs*, Discrete Mathematics, vol. 306, no. 16, pp. 1997–2004, 2006.
13. H.J. Lai, B. Montgomery and H. Poon, *Upper bounds of dynamic chromatic number*, Ars Combinatoria, vol. 68, pp. 193–201, 2003.
14. X. Meng, L. Miao, B. Su and R. Li, *The dynamic coloring numbers of pseudo-Halin graphs*, Ars Combinatoria, vo. 79, pp. 3—9, 2006.
15. B. Montgomery, *Dynamic Coloring of Graphs*, (Ph.D. thesis), West Virginia University, 2001.
16. W.T. Tutte, *Colouring problems*, The Mathematical Intelligencer, vol. 1, pp. 72—75, 1978.
17. H. Zhu, Y. Gu, J. Sheng and X. Lv, *On r-hued coloring of planar graphs*, Mathematica Applicata (Wuhan), vol. 29, no. 2, pp. 308–313, 2016.