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Efficient Power Flow Solution in Monopolar DC Networks Using a Derivative-Free Steffensen Method

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Abstract This paper proposes and evaluates advanced solution techniques for nonlinear power flow analysis in monopolar low-voltage direct-current (DC) networks. Its key contribution lies in the systematic comparison between the classical Newton-Raphson method and a derivative-free multivariable Steffensen approach, demonstrating that the latter offers a practical alternative with superlinear convergence, reduced computational complexity, and simpler implementation. Numerical simulations on benchmark 33- and 69-bus systems show that both methods converge rapidly within less than six iterations, with Steffensen's method maintaining competitive solution times and accuracy while significantly reducing the effort needed for Jacobian evaluations. The findings confirm that the Steffensen method is highly suitable for real-time large-scale power system analysis, especially when derivative calculations are expensive or unreliable. Overall, the results validate the Steffensen approach as a robust, efficient, and scalable solution method for modern DC power systems, paving the way for improved operational reliability and the integration of renewable energy sources.

Keywords Steffensen's method, DC power flow, radial distribution networks, nonlinear analysis, numerical stability

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1. Introduction

The modern landscape of energy distribution is undergoing a transformative shift driven by the increasing integration of renewable energy sources and the rapid evolution of advanced power distribution systems. This shift has sparked the need for the development and optimization of low-voltage direct-current (DC) networks, which hold promise for revolutionizing energy efficiency and control [1, 2]. In particular, monopolar DC architectures have garnered significant attention due to their intrinsic simplicity, high efficiency, and ease of control. These attributes make monopolar DC systems particularly attractive for various applications, including microgrids, data centers, and electric vehicle charging stations [3–5]. As these applications become increasingly pervasive, the need for accurate and efficient power flow analysis has become paramount. Such analyses are critical for ensuring operational security, optimizing system performance, and enabling real-time control [6–8].

Traditional power flow models, historically applied to alternating-current (AC) systems, primarily rely on intricate nonlinear equations based on phasor representations [9, 10]. In contrast, DC power systems—especially monopolar configurations—provide a unique opportunity to leverage a linear algebraic approach [11]. This approach utilizes admittance matrices, with nonlinearities predominantly emerging from voltage-dependent power

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injections and the inverse relationship between voltage and current at the load nodes [12, 13]. Developing precise and computationally viable models for these systems is crucial for comprehensive network analysis and strategic planning [14].

The inherent nonlinearities of the monopolar DC power flow can be encapsulated by two algebraic formulations. The first hinges on inverse voltage dependence, utilizing the fundamental relationships between power, voltage, and current, which leads to characteristically reciprocal voltage terms [6]. The second formulation focuses on the multiplicative interactions between voltages and admittance parameters, resulting in equations reminiscent of quadratic forms [12]. Each formulation captures the essential physical behaviors of the network while posing distinct computational challenges, particularly concerning the convergence and efficiency of numerical solution methods.

Iterative algorithms, most notably the classical Newton-Raphson method, have been the cornerstone for solving nonlinear power flow equations, appreciated for their quadratic convergence traits [14]. Nonetheless, the computational burden involved in the computation and inversion of Jacobian matrices, especially in large-scale systems, cannot be overstated. In addressing these computational demands, derivative-free methods such as the multivariable Steffensen method have emerged as promising alternatives. These methods are characterized by simpler implementations and competitive convergence rates without the need for derivative information.

This paper undertakes a comprehensive exploration of both traditional and modern iterative algorithms applied to the power flow problem in monopolar DC networks. With a focus on computational efficiency and robustness, it rigorously analyzes their performance through extensive numerical simulations on established benchmark systems. Particular emphasis is placed on examining the influence of algorithm parameters (*e.g.*, the finite difference step-size in Steffensen's method) on the convergence behavior. Our results unequivocally demonstrate the potential of these approaches to deliver reliable, rapid, and scalable solutions suitable for contemporary DC power systems.

The remainder of this document is structured as follows. Section 2 delves into the power flow modeling of monopolar DC networks, deriving the nonlinear equations and discussing their physical significance. Section 3 introduces the solution algorithms with a detailed presentation of derivative-free Steffensen methods. Section 4 addresses implementation aspects and computational considerations. Numerical validations and the results obtained are comprehensively presented in Section 5, followed by some concluding remarks in Section 6, highlighting key findings and potential directions for future research.

2. Power flow modeling in monopolar DC networks

In monopolar DC distribution architectures, the electrical relationships governing the network can be captured through an admittance matrix that links nodal voltages to current injections via a linear model [12]. $\mathbf{Y} \in \mathbb{R}^{n \times n}$ is defined as the nodal admittance matrix, where n is the total number of buses within the system. The fundamental relation is expressed as follows [15]:

$$\mathbf{I} = \mathbf{Y}\mathbf{V}, \tag{1}$$

where $\mathbf{I} \in \mathbb{R}^n$ and $\mathbf{V} \in \mathbb{R}^n$ denote the bus current injections and voltage magnitude vectors, respectively.

For a detailed analysis, the nodes are distinguished between generation (controlled voltage sources) and load (demand) buses. Partitioning the network accordingly yields the following [16]:

$$\begin{bmatrix} \mathbf{I}_G \\ \mathbf{I}_L \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{GG} & \mathbf{Y}_{GL} \\ \mathbf{Y}_{LG} & \mathbf{Y}_{LL} \end{bmatrix} \begin{bmatrix} \mathbf{V}_G \\ \mathbf{V}_L \end{bmatrix}, \tag{2}$$

where $\mathbf{V}_G$ and $\mathbf{V}_L$ are the voltage vectors at the generation and load nodes, respectively, with dimensions corresponding to their node counts.

The core challenge involves finding the bias voltages at the load buses $\mathbf{V}_L$, such that the power flow constraints are met, given the fixed voltages at the generation nodes $\mathbf{V}_G$ and the active power demands $\mathbf{P}_L$. The currents associated with load and generation nodes can be related to their power injections via the following expression [6]:

$$\begin{bmatrix} \mathbf{I}_G \\ \mathbf{I}_L \end{bmatrix} = \begin{bmatrix} \text{diag}(\mathbf{V}_G)^{-1}\mathbf{P}_G \\ \text{diag}(\mathbf{V}_L)^{-1}\mathbf{P}_L \end{bmatrix}, \tag{3}$$

assuming constant power injections and proportional relationships.

The nonlinear power flow problem boils down to solving for $\mathbf{V}_L$ such that the network equations align with the specified power demands. By integrating the CUrrent Equations (3) into the network admittance expressions, two alternative algebraic formulations are obtained:

1. **First formulation:** DC relation with inverse voltage dependence:

$$\mathbf{F}_1(\mathbf{V}_L) = \mathbf{Y}_{LL}\mathbf{V}_L + \mathbf{Y}_{LG}\mathbf{V}_G + \text{diag}(\mathbf{V}_L)^{-1}\mathbf{P}_L = \mathbf{0}. \tag{4}$$

2. **Second formulation:** Power balance expressed through quadratic-like relations:

$$\mathbf{F}_2(\mathbf{V}_L) = \text{diag}(\mathbf{V}_L)(\mathbf{Y}_{LL}\mathbf{V}_L + \mathbf{Y}_{LG}\mathbf{V}_G) + \mathbf{P}_L = \mathbf{0}. \tag{5}$$

Both system forms, *i.e.*, $\mathbf{F}_1(\mathbf{V}_L)$ and $\mathbf{F}_2(\mathbf{V}_L)$, represent the nonlinear power flow equations for monopolar DC systems. Iterative numerical techniques, such as fixed-point iteration or Newton-Raphson methods, can be employed to solve these models depending on convergence and computational efficiency requirements. This approach provides a comprehensive framework for analyzing and solving the DC power flow problem through algebraic reformulation, avoiding direct dependence on existing formulations.

3. Multivariable Steffensen method: general theory

Solving nonlinear systems of equations is a fundamental issue in numerical analysis, particularly when the computation of derivatives is either analytically intractable or computationally expensive [17, 18]. Traditional Newton-type methods rely heavily on the explicit formation and inversion of the Jacobian matrix $J(\mathbf{x}) = \frac{\partial \mathbf{F}}{\partial \mathbf{x}}$, which can be a significant drawback in high-dimensional or complex problems.

In light of the above, the multivariable Steffensen method emerges as an effective and derivative-free iterative technique that approximates solutions to nonlinear systems by employing finite difference schemes to approximate the Jacobian matrix at each iteration [19, 20]. This approach circumvents the explicit computation of derivatives, offering a computationally attractive alternative that is particularly suited for problems where derivatives are computationally prohibitive or analytically unavailable [21].

Let $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^n$ be a nonlinear vector-valued function. The objective is to find a solution $\mathbf{x}^\star \in \mathbb{R}^n$ such that

$$\mathbf{F}(\mathbf{x}^\star) = \mathbf{0}.$$

If the Jacobian $J(\mathbf{x}) = \frac{\partial \mathbf{F}}{\partial \mathbf{x}}$ is difficult to compute analytically, the multivariable Steffensen method offers a derivative-free iterative approach that leverages finite differences for numerical approximation. Starting from an initial guess, the method approximates the Jacobian matrix via finite differences [22] and iteratively updates the solution without requiring explicit derivatives [23]. This makes it a practical and efficient alternative for solving complex nonlinear systems, especially when analytical Jacobians are challenging to obtain.

1. **Initialization.** Select an initial guess:

$$\mathbf{x}^{(0)} \in \mathbb{R}^n.$$

2. **Iterative step.** For $k = 0, 1, 2, \ldots$, perform the following:

   (a) **Evaluate the function:**

$$\mathbf{F}^{(k)} := \mathbf{F}(\mathbf{x}^{(k)}).$$

(b) **Approximate the Jacobian:** For each $i = 1, 2, \ldots, n$, use finite differences to compute:

$$\Delta \mathbf{F}_i^{(k)} := \frac{\mathbf{F}(\mathbf{x}^{(k)} + h\mathbf{e}_i) - \mathbf{F}(\mathbf{x}^{(k)})}{h},$$

where:

- $h > 0$ is a small scalar step size,
- $\mathbf{e}_i \in \mathbb{R}^n$ is the i-th canonical basis vector (*i.e.*, 1 in the i-th component and 0 elsewhere).

Assemble the approximate Jacobian as:

$$J_k \approx \begin{bmatrix} \Delta \mathbf{F}_1^{(k)} & \Delta \mathbf{F}_2^{(k)} & \ldots & \Delta \mathbf{F}_n^{(k)} \end{bmatrix} \in \mathbb{R}^{n \times n}.$$

(c) **Solve the linear system:**

$$J_k \Delta \mathbf{x}^{(k)} = -\mathbf{F}^{(k)},$$

to obtain the update step $\Delta \mathbf{x}^{(k)}$.

(d) **Update the iterate:**

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}.$$

The multivariable Steffensen method is a derivative-free iterative technique that is particularly suitable for functions obtained through simulations or other numerical procedures. It generally exhibits superlinear convergence when the underlying function is sufficiently smooth, the Jacobian approximations are accurate, and the Jacobians remain nonsingular in the vicinity of the solution [24]. This method extends the scalar Steffensen approach to vector-valued functions and is often regarded as a quasi-Newton method that utilizes forward difference approximations to estimate the Jacobian matrix [25].

This approach is highly versatile and can be effectively applied to various nonlinear formulations. For instance, in the context of DC networks, it can be used to compute voltage profiles at load nodes—arising from the inverse voltage dependence associated with constant power loads or from the multiplicative interactions between voltage and current [11]. The flexibility of the method in handling different nonlinear models makes it attractive for a wide range of applications in power systems analysis and beyond.

A key advantage of the multivariable Steffensen method lies in its ability to bypass explicit Jacobian computations. Instead, it relies on finite difference approximations based on perturbations along basis vectors, simplifying complex matrix operations such as inversion or element-wise voltage scaling. Its derivative-free nature enhances its robustness and computational efficiency, especially in scenarios where model complexity or hardware limitations restrict the feasibility of classical Newton-type methods. Consequently, this approach offers a practical and reliable alternative for solving large nonlinear systems where derivative evaluations are costly or impractical.

4. Numerical validation

In order to evaluate the computational efficiency of the proposed approach, numerical tests were carried out using MATLAB R2024b. All simulations were executed on a personal computer featuring an AMD Ryzen 7 3700 processor with a base speed of 2.3 GHz, as well as 16 GB of RAM, running a 64-bit version of Windows 10.

4.1. Test network descriptions

The corresponding performance assessments were conducted on two established test systems that are frequently employed in research studies: the 33- and 69-bus radial distribution networks. These feeders were adapted for DC operation, as described in [6]. They were selected because of their different sizes and structural complexities, which provide a broad basis for evaluating the robustness and efficiency of the proposed numerical approach across diverse system configurations [13].

Figure 1 presents the layouts of both test systems, highlighting their radial topology and layered structure. Each network is anchored by a single slack node, which functions as a steady voltage source at 12,660 V. The other nodes mainly represent load points with constant power demand, and the transmission lines are modeled as purely resistive elements [26].

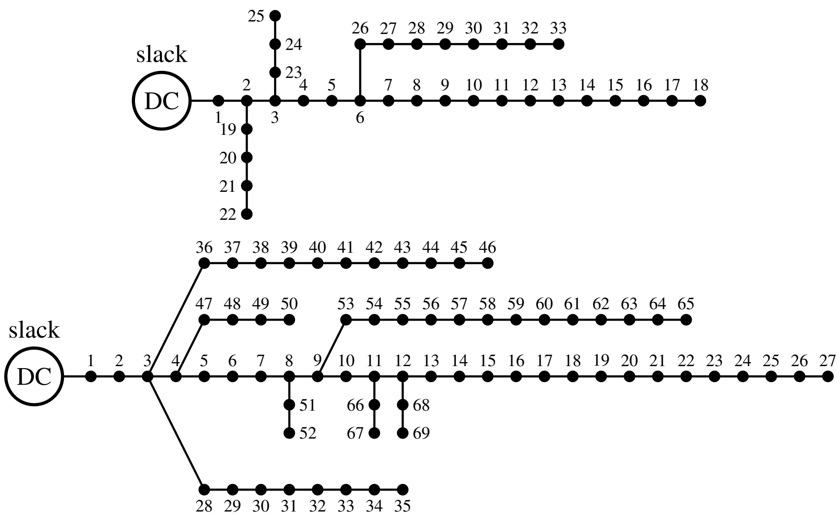

Figure 1. Topology diagrams of the 33- and 69-bus DC distribution feeders [6].

The system parameters, including the resistance of the distribution line and the constant power loads at each bus, were taken from the data provided in [6]. These parameters are detailed in Tables 1 and 2. The use of these standardized datasets ensured consistency with prior research and enabled straightforward validation through a comparison with existing results. Both test networks simulated realistic operating conditions, allowing for a comprehensive evaluation of convergence properties, computational efficiency, and voltage profile accuracy when applying the Steffensen method to solve the nonlinear DC power flow equations.

Table 1. System data for the 33-bus grid

Node k	**Node** m	R_{km} (Ω)	P_m **(kW)**	**Node** k	**Node** m	R_{km} (Ω)	P_m **(kW)**
1	2	0.0922	100	17	18	0.7320	90
2	3	0.4930	90	2	19	0.1640	90
3	4	0.3660	120	19	20	1.5042	90
4	5	0.3811	60	20	21	0.4095	90
5	6	0.8190	60	21	22	0.7089	90
6	7	0.1872	200	3	23	0.4512	90
7	8	1.7114	200	23	24	0.8980	420
8	9	1.0300	60	24	25	0.8960	420
9	10	1.0400	60	6	26	0.2030	60
10	11	0.1966	45	26	27	0.2842	60
11	12	0.3744	60	27	28	1.0590	60
12	13	1.4680	60	28	29	0.8042	120
13	14	0.5416	120	29	30	0.5075	200
14	15	0.5910	60	30	31	0.9744	150
15	16	0.7463	60	31	32	0.3105	210
16	17	1.2890	60	32	33	0.3410	60

4.2. Effect of the h-factor

The numerical performance of the Steffensen method in solving the power flow problem for monopolar DC networks is significantly influenced by h-factor selection. This parameter affects convergence and directly impacts the number of iterations required to reach a solution.

Table 2. System data for the 69-bus grid

Node k	**Node** m	R_{km} (Ω)	P_m **(kW)**	**Node** k	**Node** m	R_{km} (Ω)	P_m **(kW)**
1	2	0.0005	0	3	36	0.0044	26
2	3	0.0005	0	36	37	0.0640	26
3	4	0.0015	0	37	38	0.1053	0
4	5	0.0215	0	38	39	0.0304	24
5	6	0.3660	2.6	39	40	0.0018	24
6	7	0.3810	40.4	40	41	0.7283	102
7	8	0.0922	75	41	42	0.3100	0
8	9	0.0493	30	42	43	0.0410	6
9	10	0.8190	28	43	44	0.0092	0
10	11	0.1872	145	44	45	0.1089	39.2
11	12	0.7114	145	45	46	0.0009	39.2
12	13	1.0300	8	4	47	0.0034	0
13	14	1.0440	8	47	48	0.0851	79
14	15	1.0580	0	48	49	0.2898	384
15	16	0.1966	45	49	50	0.0822	384
16	17	0.3744	60	8	51	0.0928	40.5
17	18	0.0047	60	51	52	0.3319	3.6
18	19	0.3276	0	9	53	0.1740	4.35
19	20	0.2106	1	53	54	0.2030	26.4
20	21	0.3416	114	54	55	0.2842	24
21	22	0.0140	5	55	56	0.2813	0
22	23	0.1591	0	56	57	1.5900	0
23	24	0.3463	28	57	58	0.7837	0
24	25	0.7488	0	58	59	0.3042	100
25	26	0.3089	14	59	60	0.3861	0
26	27	0.1732	14	60	61	0.5075	1244
3	28	0.0044	26	61	62	0.0974	32
28	29	0.0640	26	62	63	0.1450	0
29	30	0.3978	0	63	64	0.7105	227
30	31	0.0702	0	64	65	1.0410	59
31	32	0.3510	0	65	66	0.2012	18
32	33	0.8390	10	66	67	0.0047	18
33	34	1.7080	14	67	68	0.7394	28
34	35	1.4740	4	68	69	0.0047	28

Figures 2a and 2b illustrate the relationship between the h-factor and the number of iterations needed for convergence in the 33- and 69-bus DC grid models. The plots display a semi-logarithmic scale, highlighting how different h values can drastically alter convergence performance.

In general, smaller h-factors tend to facilitate faster convergence, reducing the iteration count. For example, in the 33-bus system, an h-factor of 0.01 achieves convergence within approximately four iterations, whereas larger h values, such as 0.1, require more iterations (around five) or may fail to converge efficiently. Similarly, for the 69-bus system, very small h-values (*e.g.*, 10^{-6}) result in rapid convergence (around five iterations), while larger values may cause slower convergence.

The aforementioned figures further demonstrate that choosing an overly large h-factor can hinder convergence, as reflected by the higher iteration counts or the observed instability beyond certain thresholds. In this vein, selecting an appropriate h-value is crucial, and it should be tailored to the specific problem to optimize convergence speed and numerical stability.

5. Analysis of computational performance

This section provides a comprehensive evaluation of the computational efficiency of various iterative algorithms applied to the monopolar power flow problem in DC systems. The algorithms assessed include the classical Newton-Raphson method, applied to two different nonlinear formulations, and Steffensen's method, which was similarly implemented. To ensure fairness and consistency, dedicated MATLAB routines were developed for each algorithm. Each simulation was executed 1000 times, and the mean execution time was recorded in order to obtain

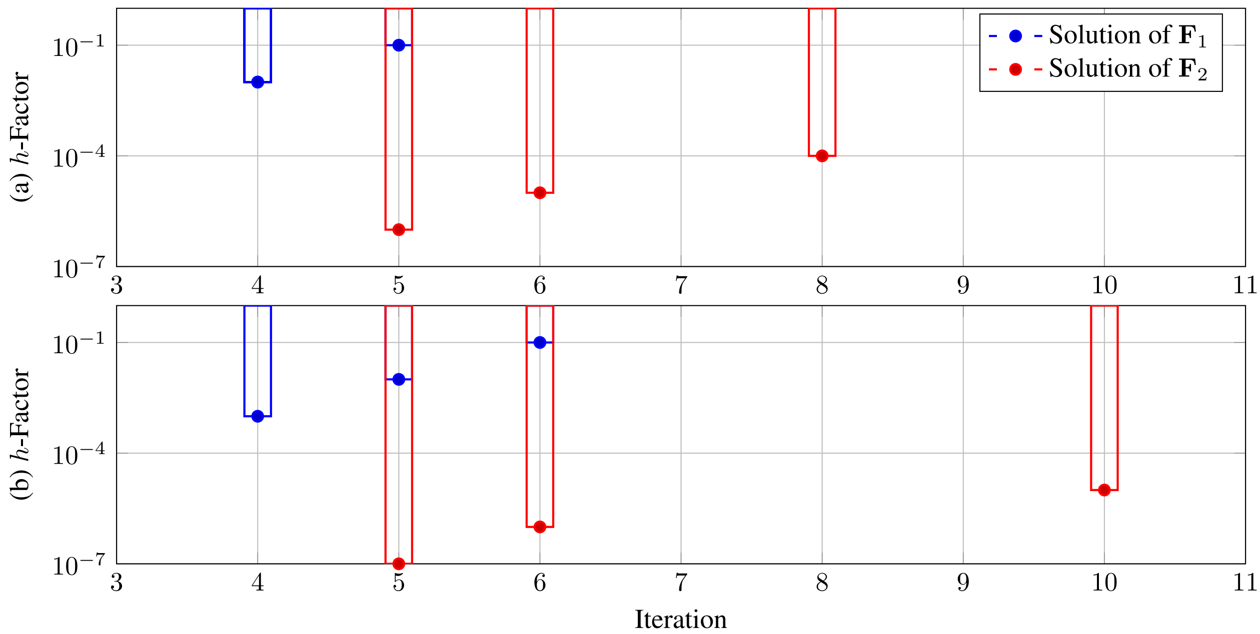

Figure 2. Number of iterations for solving the power flow problems in monopolar DC networks: (a) the 33-bus grid, and (b) the 69-bus grid

statistically robust results. All methods employed a standardized stopping criterion: a maximum of 100 iterations or a convergence threshold of $\varepsilon = 1 \times 10^{-10}$. Such standardization guaranteed the meaningful comparability of convergence behavior and computational performance.

The primary performance metrics summarized in Table 3 include the number of iterations for convergence, the average processing time (in ms), the minimum nodal voltage (in p.u.), and the system's total power losses (in kW). These results are presented for both the 33- and 69-bus grids. The data demonstrate that both approaches achieve rapid convergence, typically within fewer than six iterations, highlighting their suitability for fast power flow solutions. Although the Newton-Raphson method slightly surpasses Steffensen's method in terms of raw speed—which can be attributed to its straightforward Jacobian calculations—Steffensen's method exhibits comparable iteration counts along with marginally higher computational times, given its matrix operations and iterative correction steps.

Table 3. Numerical assessment of the monopolar power flow problem in DC systems using Newton- and Steffensen-based algorithms

Method	**Iterations**	**Avg. processing time** (ms)	**Min. voltage** (p.u.)	**Power losses** (kW)
33-bus grid				
Newton-Raphson (4)	4	0.1466	0.9339	135.26
Newton-Raphson (5)	5	0.2482		
Steffensen (4)	4	0.2499		
Steffensen (5)	5	0.2796		
The 69-bus Grid				
Newton-Raphson (4)	4	0.2823	0.9274	153.85
Newton-Raphson (5)	5	0.7173		
Steffensen (4)	4	0.8121		
Steffensen (5)	5	0.9257		

The numerical results presented in Table 3 provide valuable insights into the performance of the evaluated algorithms, revealing several key aspects:

i. **Convergence speed.** Both the Newton-Raphson and Steffensen methods exhibit rapid convergence, typically reaching the prescribed tolerance in fewer than six iterations across all test cases. This confirms their high efficiency and suitability for real-time power flow applications, where fast and reliable solutions are critical. The minimal iteration counts observed also suggest that both methods are robust against different initial conditions, maintaining high convergence rates even in complex network scenarios.
ii. **Computational efficiency.** An analysis of average processing times indicated that the Newton-Raphson approach slightly outperforms Steffensen's method in the examined test systems, primarily due to its direct Jacobian computation and linear algebraic structure, which benefits from well-optimized numerical libraries. Despite this, Steffensen's method achieved similar iteration counts but incurred a marginally higher computational overhead due to its iterative matrix operations and the finite difference approximations used to estimate the Jacobian-like matrices. These results demonstrate that, while Newton-Raphson is marginally faster in terms of raw processing speed, Steffensen's method remains a competitive alternative, especially considering its derivative-free nature and potential for simpler implementation.
iii. **Voltage and power losses.** The minimum nodal voltages obtained are well within the operational safety margins, ensuring system stability and reliability. The slight variations observed (*i.e.*, often in the form of marginally lower voltages) do not threaten system security and are indicative of convergence towards physically meaningful solutions. The total power losses confirm the algorithms' ability to accurately evaluate real power dissipation within the network, validating the physical plausibility of the solutions achieved within the specified tolerance levels.
iv. **Scalability.** Performance data for the 33- and 69-bus systems demonstrate that both methods scale effectively to larger network sizes. They maintain low iteration counts and reasonable processing times, which underscores their robustness and flexibility for application to more extensive and complex power systems. This scalability is crucial for practical deployment in large-scale power grids, where computational efficiency directly impacts operational feasibility.

This performance comparison confirms that both the Newton-Raphson and Steffensen-based algorithms are highly effective for solving the monopolar power flow problem in DC systems. Although the Newton-Raphson method offers slightly faster processing times due to its direct Jacobian evaluations, Steffensen's method provides a competitive alternative with similar convergence characteristics and a lower per-iteration computational cost, thanks to its derivative-free approach with superlinear convergence. Both methods demonstrate excellent scalability to larger network configurations, consistently maintaining low iteration counts and reasonable computational times. As a result, these techniques constitute reliable and efficient options for real-time and large-scale power system analysis, with the optimal choice depending on specific application requirements and implementation convenience.

6. Conclusions and future work

This study demonstrates the compelling advantages of employing the multivariable Steffensen method for solving nonlinear power flow problems in monopolar DC networks. Its derivative-free nature eliminates the need for explicit Jacobian calculations, significantly simplifies implementation, and reduces the computational burden, especially in large-scale or complex systems. Despite this simplification, the method consistently exhibits superlinear convergence, achieving accurate solutions within a few iterations. Its robustness and flexibility across various nonlinear formulations highlight its potential as a practical and attractive alternative to traditional methods such as the Newton-Raphson approach, particularly in scenarios where derivative evaluations are challenging, noisy, or computationally expensive.

Looking ahead, several research directions could further enhance the effectiveness and broad applicability of the Steffensen method. Incorporating adaptive step-size strategies for finite difference approximations could improve convergence speed and numerical stability, particularly in ill-conditioned or highly nonlinear problems. Developing hybrid algorithms that combine the strengths of Steffensen's approach with other iterative techniques could optimize performance across diverse problem regimes, offering a more versatile toolkit for power system analysis. Additionally, exploring the parallelization of finite difference evaluations—leveraging modern high-performance computing architectures—is crucial for scaling the method to real-time, large-scale applications. These advancements aim to position the Steffensen method as a reliable, efficient, and flexible tool for future power systems analysis and control, especially as the integration of renewable energy sources and distributed generation continues to grow.

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