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The Central Metric Dimension of the k-Corona Graph

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Abstract The metric dimension is the minimum cardinality of a subset of the vertex set of a graph G that uniquely represents each vertex in a graph. The central set is a set of vertices with minimum eccentricity. This central set concept can be used to determine strategic public service locations, such that accessible transportation can be reached from all regions. The central metric dimension is the minimum cardinality of a resolving set that includes the central set. This study aims to determine the central metric dimension in k-corona graph. The k-corona operation of G and H denoted by $G \odot_k H$ is a generalization of the corona operation, where a new graph is formed by connecting each vertex of a graph G to k copies of graph H. The results show that the central metric dimension of the k-corona graph depends on the central set of G, the order of G, the value of k, and the metric dimension of H.

Keywords Central Set, Metric Dimension, Central Metric Dimension, k-Corona Graph, Accessible Transportation

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1. Introduction

Graph theory is a mathematical concept that falls under the field of algebra. This theory studies elements represented as vertices and edges, where edges represent the relationship between two vertices. According to Chartrand and Lesniak, graph theory was first introduced in 1736 by Swiss mathematician Leonhard Euler, who discussed the problem of the Seven Bridges of Königsberg [1].

Over time, graph theory developed several important concepts such as distance, paths, cycles, and others, which are further detailed and are interesting topics of discussion in this field. In recent years, the study of metric dimension and its variants has been extended to a wide range of graph operations. Several works have investigated the metric dimension of corona products and related constructions. For instance, Saputro $etal.$[2] studied the metric dimension of comb product graphs, while Susilowati $etal.$ [3] [4] considered rooted product and corona-type graphs in relation to local metric dimension. More recently, Prabhu and collaborators [5, 6, 7, 8, 9, 10] examined fault-tolerant metric dimension in various interconnection networks. These studies show that corona type operations provide a fertile ground for exploring metric dimension and its generalizations. Another concept that has developed within graph

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theory is the concept of the central set. According to Sooryanarayana $etal.$, the central set is the set of all central vertices whose eccentricities are equal to the radius of the graph [11].

This central set concept can be used to determine strategic locations of public services so that accessible transportation can be reached from all regions.

Drawing from these two concepts —metric dimension and central set— a new idea emerged to combine them, known as the central metric dimension. One of the efforts to merge these concepts was conducted by Susilowati $etal.$. Their research discussed the concept of the central metric dimension, namely a resolving set that also contains the central set in specific types of graphs. One of these is the special graph resulting from the k-corona operation [12].

The k-corona operation is an extension of the corona operation that was introduced earlier. This development lies in the inclusion of a positive integer k. The k-corona operation creates a new graph by taking one copy of graph G and k copies of graph H for each vertex in G. Then, each vertex in G is connected to every vertex in its corresponding k copies of H.

The k-corona operation expands the applications and research potential of corona operations by offering more flexibility in graph composition. Therefore, this study focuses on the integration of the metric dimension and the concepts of central set in the k-corona graph. The specific graphs used in this research include path graphs, star graphs, complete graphs, and cycle graphs.

In recent developments, various extensions of the metric dimension have been introduced to capture different structural properties of graphs, such as fault-tolerant metric dimension [7, 9]. However, studies that integrate these distance-based approaches with the concept of the central set remain relatively limited. The central metric dimension provides a new perspective by requiring the resolving set to also contain the central vertices of the graph.

Furthermore, while several works have examined the metric dimension of the standard corona product $G \odot H$ [13, 14], the inclusion of a positive integer parameter k in the k-corona operation introduces new structural challenges. The interaction between the central vertices of G and the replicated structures of H under the k-corona construction has not been widely analyzed in existing literature. Hence, this study contributes by determining the central metric dimension of k-corona graphs involving fundamental families such as paths, stars, cycles, and complete graphs, thereby extending prior results on corona-type operations.

In previous studies, the metric and local metric dimensions of various graph operations—such as corona, comb, and rooted products—have been widely analyzed [13, 3, 2, 4]. However, most of these works focus on classical or local metric dimensions, without considering the structural influence of the central vertices. The introduction of the central metric dimension expands this framework by combining the concept of a resolving set with the central structure of the graph.

In this paper, we generalize these findings by determining the central metric dimension of the k-corona product $G \odot_k H$, where H can be a path, a cycle, a star, or a complete graph. This provides a unified formulation that includes several previously studied cases as special instances. Furthermore, the results establish a theoretical foundation that can be applied to identify optimal monitoring or service locations in network systems, where accessibility to central regions is a key consideration.

To support this study, the following preliminary definitions and concepts are presented.

Definition 1.1 ([15])**.** A central vertex is a vertex whose eccentricity is equal to the radius of the graph G.

Definition 1.2 ([15])**.** A central set is a set whose elements are all central vertices. The central set of a graph G is denoted by $S(G)$.

Lemma 1.3. [[3]] Let G be a connected graph. If $W \subseteq V(G)$, then for every $v_i, v_j \in W$ with $i \neq j$, it holds that $r(v_i \,|\, W) \neq r(v_j \,|\, W)$.

Definition 1.4 ([4])**.** Let G be a connected graph. An ordered set $W \subseteq V(G)$ with $W \neq \emptyset$ is called a central resolving set of G if W is a resolving set that also contains the central set. A central resolving set of the minimum cardinality is called a central basis. The cardinality of the central basis in the graph G is called the central metric dimension, denoted by $\dim_{\mathrm{cen}}(G)$.

Definition 1.5 ([4]). Let G be a connected graph with $V(G) = \{v_i \mid i = 1, 2, 3, \ldots, n\}$ and let H be a connected graph of order at least two. The k-corona of G and H, denoted by $G \odot_k H$, is the graph obtained by taking one copy of G and nk copies of H, that is,

$$H_1^1, H_1^2, H_1^3, \ldots, H_1^k, H_2^1, H_2^2, H_2^3, \ldots, H_2^k, \ldots, H_n^1, H_n^2, H_n^3, \ldots, H_n^k,$$

such that each vertex $v_i \in V(G)$ is connected to all vertices in H_i^r, for $r = 1, 2, 3, \ldots, k$.

In the generalized corona product $G \odot_k H$, each vertex v_i of the base graph G is connected to k distinct copies of the graph H, denoted by $H_i^1, H_i^2, \ldots, H_i^k$. Every vertex u_j^{ir} in the r-th copy H_i^r is adjacent only to the corresponding vertex v_i in G, and there are no edges between different copies H_i^r and H_i^s for $r \neq s$. This structure ensures that the resulting graph has $|V(G)| + k|V(G)| \cdot |V(H)|$ vertices. The vertex labeling on the resulting graph $G \odot_k H$ is defined as follows. Let G be a graph with $V(G) = \{v_i \mid i = 1, 2, 3, \ldots, n\}$ and H with $V(H) = \{u_j \mid j = 1, 2, 3, \ldots, m\}$. Based on the definition of the k-corona operation, the vertex set is

$$V(G \odot_k H) = V(G_0) \cup \bigcup_{i=1}^{n} \left(\bigcup_{r=1}^{k} V(H_i^r) \right),$$

where $V(G_0) = \{v_i^0 \in V(G \odot_k H) \mid v_i \in V(G)\}$ and

$$V(H_i^r) = \{u_j^{ir} \mid u_j \in V(H);\ r = 1, 2, 3, \ldots, k;\ i = 1, 2, 3, \ldots, n\}.$$

Here, G_0 is referred to as the central graph, while each H_i^r is referred to as a branch graph.

2. Central Metric Dimension of the *k*-Corona Graph

The initial step in this study is to determine the central sets of the specific graphs used in the research. The results are as follows.

- The central set of a cycle graph and a complete graph is the set of all vertices in the graph.
- The central vertex of a path graph P_n is the $\frac{n+1}{2}$-th vertex for odd n, or the $\frac{n}{2}$-th and $\frac{n+2}{2}$-th vertices for even n.
- The central vertex of a star graph is the central vertex of the star itself.

This section explains the central set of the graph resulting from the k-corona operation, followed by the central metric dimension of that graph.

Lemma 2.1. Suppose G and H are connected graphs, then

$$S(G \odot_k H) = \{s_i^0 \in V(G \odot_k H) \mid s_i \in S(G)\}.$$

Proof. Let G be a connected graph with $V(G) = \{v_i \mid i = 1, 2, 3, \ldots, n\}$. Let H be a connected graph with $V(H) = \{u_j \mid j = 1, 2, 3, \ldots, m\}$. The vertex set of $G \odot_k H$ is given by

$$V(G \odot_k H) = V(G_0) \cup \bigcup_{i=1}^{n} \left(\bigcup_{r=1}^{k} V(H_i^r) \right),$$

where $V(G_0) = \{v_i^0 \in V(G \odot_k H) \mid v_i \in V(G)\}$ and $V(H_i^r) = \{u_j^{ir} \mid u_j \in V(H); r = 1, \ldots, k; i = 1, \ldots, n\}$. Here, H_i^r denotes the r-th copy of H attached to vertex v_i of G. Suppose $s \in S(G)$, then $e(s) = \min\{e(v) \mid v \in V(G)\}$. Let $s_i^0 \in V(G \odot_k H)$ for $s_i \in S(G)$, then $e(s_i^0) = e(s_i) + 1$.

Next, consider a vertex u_j^{ir} that lies in the r-th copy of H attached to v_i^0. For any central vertex v_t^0 we have $d(u_j^{ir}, v_t^0) = 1 + d(v_i^0, v_t^0)$, and for any vertex u_j^{ts} in a copy of H attached to v_t^0,

$$d(u_j^{ir}, u_j^{ts}) = 1 + d(v_i^0, v_t^0) + 1 = d(v_i^0, v_t^0) + 2.$$

So, for any vertex $u_j^{ir} \in V(G \odot_k H)$, we have $e(u_j^{ir}) = e(v_i^0) + 2$.

Since $e(u_j^{ir}) \geq e(s_i^0)$ for every $u_j^{ir} \in V(G \odot_k H)$, it follows that $e(s_i^0) = \min\{e(v) \mid v \in V(G \odot_k H)\}$. Thus,

$$S(G \odot_k H) = \{s_i^0 \in V(G \odot_k H) \mid s_i \in S(G)\}.$$

In the k-corona product $G \odot_k H$, each vertex v_i^0 of G becomes the central attachment point for k copies of H. The distances between vertices in different copies of H must pass through their corresponding central vertex v_i^0, which increases every distance by exactly one compared to the original distance in G. Consequently, the eccentricity of each central vertex in $G \odot_k H$ is $e(v_i) + 1$, while the eccentricity of vertices in the attached copies of H is $e(v_i) + 2$. This explains why the central vertices of $G \odot_k H$ coincide with those of G.

Lemma 2.2. Let G be a connected graph and $U \subseteq V(G)$. If $x \in U$ or $y \in U$, then $r(x \mid U) \neq r(y \mid U)$.

Proof. Let G be a connected graph and $U \subseteq V(G)$. For any $v_i \in U$ or $v_j \in U$, there are two possible cases:

(1) $v_i, v_j \in U$: Based on Lemma 1.3, it holds that $r(v_i|U) \neq r(v_j|U)$.
(2) $v_i \in U$ and $v_j \notin U$: In the ordered pair $r(v_i|U)$ there exists an element 0, whereas in $r(v_j|U)$ there is no such element. Therefore, $r(v_i|U) \neq r(v_j|U)$.

Lemma 2.3. Let G be a connected graph. If there is no central resolving set of G with cardinality k, then any set $W \subseteq V(G)$ with $|W| < k$ is not a central resolving set.

Proof. Let G be a connected graph. Suppose there is no central resolving set of G with cardinality k, and there exists a central resolving set $T \subseteq V(G)$ with $|T| < k$ such that for every $v_i, v_j \in V(G)$, $r(v_i|T) \neq r(v_j|T)$ and T is a central set of G. Moreover, there exists a set $U \subseteq V(G) \setminus T$ such that $|T \cup U| = k$. Since T is a resolving set and a central set of G, then $T \cup U$ is also a central resolving set of G. So that, $T \cup U$ is a central resolving set of G which is a contradiction. Thus, the result follows and the proof is completed.

The following three lemmas (Lemmas 2.1–2.3) establish the fundamental structure of the central set and its relation to the metric representations in the k-corona product. In particular, Lemma 2.1 characterizes the vertices of minimum eccentricity, Lemma 2.2 ensures distinct metric representations for vertices within and outside the central set, and Lemma 2.3 guarantees the minimality of the chosen central resolving set. These results form the logical basis for the proofs of Theorems 2.4–2.10.

Theorem 2.4. Let G be a connected graph and S_n be a star graph. Then

$$\dim_{\text{cen}}(G \odot_k S_n) = |S(G)| + k|V(G)| \left(\dim(S_n) + 1\right).$$

Proof. Let $V(G) = \{v_i \mid i = 1, 2, \ldots, m\}$ and $V(S_n) = \{u_j \mid j = 1, 2, \ldots, n\}$. The vertex set of $G \odot_k S_n$ is given by

$$V(G \odot_k S_n) = V(G_m^0) \cup \bigcup_{i=1}^{m} \left(\bigcup_{r=1}^{k} V((S_n)_i^r) \right),$$

where $V(G_m^0) = \{v_i^0 \mid v_i \in V(G)\}$ and $V((S_n)_i^r) = \{u_j^{ir} \mid u_j \in V(S_n); r = 1, \ldots, k; i = 1, \ldots, m\}$, for $(S_n)_i^r$ is the r-th copy of the star graph S_n at the i-th vertex of G. Let $B = \{u_2, u_3, \ldots, u_{n-1}\}$ be a basis of S_n, and define $B_i^r = \{u_j^{ir} \mid u_j \in B\} \cup \{u_n^{ir} \mid u_n \in V(S_n)\}$. Define the set

$$W = \{s_i^0 \in V(G \odot_k S_n) \mid s_i \in S(G)\} \cup \left(\bigcup_{i=1}^{m} \bigcup_{r=1}^{k} B_i^r \cup \{u_{ir1}\} \right).$$

Based on Lemma 2.1, $S(G \odot_k S_n) \subseteq W$, and hence

$$|W| = |S(G)| + k|V(G)| \left(\dim(S_n) + 1\right).$$

To show that W is a central resolving set, note that each vertex u_j^{ir} in a copy of S_n is adjacent only to its attachment vertex v_i^0 in G. Thus, any shortest path between vertices from different copies passes through their respective attachment vertices, ensuring distinct distance representations relative to W. Within each copy, leaves u_j^{ir} have distance 1 to v_i^0 and distance 2 to each other, which guarantees that all vertices are distinguishable with respect to W. For any distinct vertices $u, v \in V(G \odot_k S_n)$ where $u \neq v$, there are three possible cases: (1) $u, v \in W$; (2) $u \in W$ and $v \in V(G \odot_k S_n) \setminus W$; (3) $u, v \in V(G \odot_k S_n) \setminus W$. For cases (1) and (2), by Lemma 2.2, it is proven that $r(u \mid W) \neq r(v \mid W)$. For case (3), there are three subcases:

1. $v_i^0 \in V(G_m^0)$ and $u_j^{yr} \in V((S_n)_y^r)$
   Since $d(v_i^0, v_x^0) = s$, where $0 \leq s \leq m-1$, and $d(u_j^{yr}, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(G_m^0)$. Since every vertex u_j^{yr} is only adjacent to its attachment vertex v_y^0, any shortest path from v_i^0 to u_j^{yr} must pass through v_y^0, it follows that $d(v_i^0, u_j^{yr}) = d(v_i^0, v_x^0) + d(u_j^{yr}, v_y^0) = s + 1$, thus $d(v_i^0, v_y^0) \leq d(v_i^0, v_x^0) \leq d(v_i^0, u_j^{yr})$. Thus, there exists at least one vertex $v_y^0 \in V(G \odot_k S_n) \setminus W$ for which $d(v_i^0, v_y^0) \neq d(u_j^{yr}, v_y^0)$, ensuring that their representation to W is distinct. Hence, for $v_i^0, u_j^{yr} \in V(G \odot_k S_n) \setminus W$, it holds that $r(v_i^0 \mid W) \neq r(u_j^{yr} \mid W)$.
2. $u_j^{xr} \in V((S_n)_x^r)$ and $u_j^{yr} \in V((S_n)_y^r)$ with $x \neq y$
   Since $d(u_j^{xr}, v_x^0) = 1$ and $d(v_x^0, v_y^0) = s$, for every $v_x^0, v_y^0 \in V(G_m^0)$, where $0 \leq s \leq m-1$. $d(v_y^0, u_j^{yr}) = 1$, we know that $d(u_j^{xr}, v_y^0) = d(u_j^{xr}, v_x^0) + d(v_x^0, v_y^0) = 1 + s$. Since any shortest path between u_j^{xr} and u_j^{yr} passes through v_y^0, we have $d(u_j^{xr}, u_j^{yr}) = d(u_j^{xr}, v_y)^0 + d(v_y^0, u_j^{yr}) = (1+s) + 1 = 2 + s$, thus $d(u_j^{xr}, v_x^0) \leq d(u_j^{xr}, v_y^0) \leq d(u_j^{xr}, u_j^{yr})$. Hence, for $u_j^{xr}, u_j^{yr} \in V(G \odot_k S_n) \setminus W$, it follows that $r(u_j^{xr} \mid W) \neq r(u_j^{yr} \mid W)$.
3. $u_j^{ir} \in V((S_n)_i^r)$, $u_j^{is} \in V((S_n)_i^s)$, $r \neq s$
   For every $u_k^{ir} \in B_i^r$, $d(u_j^{ir}, u_k^{ir}) = 1$ whereas $d(u_j^{is}, u_k^{ir}) = 2$. Similarly, for every $u_k^{is} \in B_i^s$, $d(u_j^{is}, u_k^{is}) = 1$ whereas $d(u_j^{ir}, u_k^{is}) = 2$. Since $B_i^r, B_i^s \subseteq W$, for $u_j^{ir} \in V((S_n)_i^r)$ and $u_j^{is} \in V((S_n)_i^s)$ with $r \neq s$, it follows that $r(u_j^{ir} \mid W) \neq r(u_j^{is} \mid W)$.

Next, to prove that W is a central resolving set with minimal cardinality, suppose $T \subseteq V(G \odot_k S_n)$ contains the central set and $|T| < |W|$. Let $|T| = |W| - 1$, then there exist two vertices $u_j^{ir}, u_k^{ir} \in V((S_n)_i^r)$ such that $u_j^{ir}, u_k^{ir} \notin T$. Thus, $d(u_j^{ir} \mid v) = d(u_k^{ir} \mid v)$ for all $v \in B_i^r$, meaning the distances from u_j^{ir} and u_k^{ir} to all other vertices in B_i^r are identical. This contradicts the definition of a resolving set, so T is not a central resolving set. By Lemma 2.3, W is the central resolving set with minimal cardinality. Hence,

$$\dim_{\text{cen}}(G \odot_k S_n) = |S(G)| + k|V(G)| \left(\dim(S_n) + 1\right).$$

Next, the central metric dimension is presented for the graph resulting from the operation $G \odot_k H$, where H is a complete graph, a path graph, or a cycle graph.

Theorem 2.5. Let C_q be a cycle graph and K_m be a complete graph. Then

$$\dim_{\text{cen}}(C_q \odot_k K_m) = |S(C_q)| + kq \left(\dim(K_m)\right)$$

Proof. Let $V(C_q) = \{v_i \mid i = 1, 2, \dots, q\}$ and $V(K_m) = \{u_j \mid j = 1, 2, \dots, m\}$. The vertex set of $C_q \odot_k K_m$ is given by

$$V(C_q \odot_k K_m) = V(C_q^0) \cup \bigcup_{i=1}^{q} \left(\bigcup_{r=1}^{k} V((K_m)_i^r) \right),$$

where $V(C_q^0) = \{v_i^0 \mid v_i \in V(C_q)\}$ and $V((K_m)_i^r) = \{u_j^{ir} \mid u_j \in V(K_m); r = 1, \dots, k; i = 1, \dots, q\}$, for $(K_m)_i^r$ is the r-th copy of the complete graph K_m at the i-th vertex of C_q. Let $B = \{u_1, u_2, \dots, u_{m-2}, u_{m-1}\}$ be a basis of K_m, and define $B_i^r = \{u_j^{ir} \mid u_j \in B\}$. Define the set

$$W = \{s_i^0 \in V(C_q \odot_k K_m) \mid s_i \in S(C_q)\} \cup \left(\bigcup_{i=1}^{q} \bigcup_{r=1}^{k} B_i^r \right).$$

Based on Lemma 2.1, $S(C_q \odot_k K_m) \subseteq W$, and hence

$$|W| = |S(C_q)| + kq\,(\dim(K_m)).$$

To show that W is a central resolving set, observe that each vertex in a copy of K_m is adjacent only to its attachment vertex in C_q. Hence, any shortest path between vertices belonging to different copies of K_m must pass through their corresponding attachment vertices on the cycle, ensuring distinct distance representations across copies. Within each copy, the inclusion of the copied basis B_i^r distinguishes all vertices locally. Therefore, every pair of distinct vertices in $C_q \odot_k K_m$ has a unique distance representation with respect to W. For any distinct vertices $u, v \in V(C_q \odot_k K_m)$ where $u \neq v$, there are three possible cases: (1) $u, v \in W$; (2) $u \in W$ and $v \in V(C_q \odot_k K_m) \setminus W$; (3) $u, v \in V(C_q \odot_k K_m) \setminus W$. For cases (1) and (2), by Lemma 2.2, it is proven that $r(u \mid W) \neq r(v \mid W)$. For case (3), there are three subcases:

1. $v_i^0 \in V(C_q^0)$ and $u_j^{yr} \in V((K_m)_y^r)$
   Since $d(v_i^0, v_x^0) = s$, where $0 \le s \le \left\lfloor \frac{q}{2} \right\rfloor$, and $d(u_j^{yr}, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(C_q^0)$. Since every vertex u_j^{yr} is only adjacent to its attachment vertex v_y^0, any shortest path from v_i^0 to u_j^{yr} must pass through v_y^0, it follows that $d(v_i^0, u_j^{yr}) = d(v_i^0, v_x^0) + d(u_j^{yr}, v_y^0) = s + 1$, thus $d(v_i^0, v_y^0) \le d(v_i^0, v_x^0) \le d(v_i^0, u_j^{yr})$. Thus, there exists at least one vertex $v_y^0 \in V(C_q \odot_k K_m) \setminus W$ for which $d(v_i^0, v_y^0) \neq d(u_j^{yr}, v_y^0)$, ensuring that their representation to W is distinct. Hence, for $v_i^0, u_j^{yr} \in V(C_q \odot_k K_m) \setminus W$, it holds that $r(v_i^0 \mid W) \neq r(u_j^{yr} \mid W)$.
2. $u_j^{xr} \in V((K_m)_x^r)$ and $u_j^{yr} \in V((K_m)_y^r)$ with $x \neq y$
   Since $d(u_j^{xr}, v_x^0) = 1$ and $d(v_x^0, v_y^0) = s$, for every $v_x^0, v_y^0 \in V(C_q^0)$, where $0 \le s \le \left\lfloor \frac{q}{2} \right\rfloor$. $d(v_y^0, u_j^{yr}) = 1$, we know that $d(u_j^{xr}, v_y^0) = d(u_j^{xr}, v_x^0) + d(v_x^0, v_y^0) = 1 + s$. Since any shortest path between u_j^{xr} and u_j^{yr} passes through v_y^0, we have $d(u_j^{xr}, u_j^{yr}) = d(u_j^{xr}, v_y)^0 + d(v_y^0, u_j^{yr}) = (1 + s) + 1 = 2 + s$, thus $d(u_j^{xr}, v_x^0) \le d(u_j^{xr}, v_y^0) \le d(u_j^{xr}, u_j^{yr})$. Hence, for $u_j^{xr}, u_j^{yr} \in V(C_q \odot_k K_m) \setminus W$, it follows that $r(u_j^{xr} \mid W) \neq r(u_j^{yr} \mid W)$.
3. $u_j^{ir} \in V((K_m)_i^r)$, $u_j^{is} \in V((K_m)_i^s)$, $r \neq s$
   For every $u_k^{ir} \in B_i^r$, $d(u_j^{ir}, u_k^{ir}) = 1$ whereas $d(u_j^{is}, u_k^{ir}) = 2$. Similarly, for every $u_k^{is} \in B_i^s$, $d(u_j^{is}, u_k^{is}) = 1$ whereas $d(u_j^{ir}, u_k^{is}) = 2$. Since $B_i^r, B_i^s \subseteq W$, for $u_j^{ir} \in V((K_m)_i^r)$ and $u_j^{is} \in V((K_m)_i^s)$ with $r \neq s$, it follows that $r(u_j^{ir} \mid W) \neq r(u_j^{is} \mid W)$.

Next, to prove that W is a central resolving set with minimal cardinality, suppose $T \subseteq V(C_q \odot_k K_m)$ contains the central set and $|T| < |W|$. Let $|T| = |W| - 1$, then there exist two vertices $u_j^{ir}, u_k^{ir} \in V((K_m)_i^r)$ such that $u_j^{ir}, u_k^{ir} \notin T$. Thus, $d(u_j^{ir} \mid v) = d(u_k^{ir} \mid v)$ for all $v \in B_i^r$, meaning the distances from u_j^{ir} and u_k^{ir} to all other vertices in B_i^r are identical. This contradicts the definition of a resolving set, so T is not a central resolving set. By Lemma 2.3, W is the central resolving set with minimal cardinality. Hence,

$$\dim_{\text{cen}}(C_q \odot_k K_m) = |S(C_q)| + kq\,(\dim(K_m)).$$

Theorem 2.6. Let P_n be a path graph and C_q be a cycle graph. Then

$$\dim_{\text{cen}}(P_n \odot_k C_q) = |S(P_n)| + kn(\dim(K_1 + C_q)),$$

Proof. Let $V(P_n) = \{v_i \mid i = 1, 2, \ldots, n\}$ and $V(C_q) = \{u_j \mid j = 1, 2, \ldots, q\}$. The vertex set of $P_n \odot_k C_q$ is given by

$$V(P_n \odot_k C_q) = V(P_n^0) \cup \bigcup_{i=1}^{n} \left(\bigcup_{r=1}^{k} V((C_q)_i^r) \right),$$

where $V(P_n^0) = \{v_i^0 \mid v_i \in V(P_n)\}$ and $V((C_q)_i^r) = \{u_j^{ir} \mid u_j \in V(C_q); r = 1, \ldots, k; i = 1, \ldots, n\}$, for $(C_q)_i^r$ is the r-th copy of the cycle graph C_q at the i-th vertex of P_n. Let B be a basis of the graph $K_1 + C_q$. The structure of the central set and the metric basis of $K_1 + C_q$ depends on the parity of q. Since the vertices on C_q form a symmetric structure, selecting alternating vertices ensures that every vertex of C_q has a distinct distance representation with respect to B. As an illustration, consider the following examples:

- $q = 3$. The graph $K_1 + C_3$ is isomorphic to the complete graph K_4. Since every vertex of K_4 has eccentricity 1, any resolving set must contain at least three vertices to distinguish all pairs. Thus, one possible minimal metric basis is $B = \{u_1, u_2, u_3\}$.
- $q = 4$. The graph $K_1 + C_4$ consists of a universal vertex u connected to a cycle of four vertices. The universal vertex has eccentricity 1, and each cycle vertex has eccentricity 2. A minimal resolving set that distinguishes all vertices is obtained by selecting alternating vertices on the cycle, for example $B = \{u_1, u_3\}$.
- $q \geq 5$. For larger cycles, alternating vertices along the cycle ensure that all vertices have distinct distance representations with respect to B. Hence, the general construction of B is:

$$B = \begin{cases} \{u_1, u_3, \ldots, u_{q-4}, u_{q-2}\}, & q \text{ odd}, \\ \{u_1, u_3, \ldots, u_{q-3}, u_{q-1}\}, & q \text{ even}. \end{cases}$$

These examples illustrate that the form and size of the metric basis B for $K_1 + C_q$ depend on the parity of q. Thus, the basis B is chosen as follows:

$$B = \begin{cases} \{u_1, u_3, \ldots, u_{q-4}, u_{q-2}\}, & q \text{ odd}, \\ \{u_1, u_3, \ldots, u_{q-3}, u_{q-1}\}, & q \text{ even}. \end{cases}$$

Define $B_i^r = \{u_j^{ir} \mid u_j \in B\}$. Define the set

$$W = \{s_i^0 \in V(P_n \odot_k C_q) \mid s_i \in S(P_n)\} \cup \left(\bigcup_{i=1}^{n} \bigcup_{r=1}^{k} B_i^r \right).$$

Based on Lemma 2.1, $S(P_n \odot_k C_q) \subseteq W$, and hence

$$|W| = |S(P_n)| + kn\left(\dim(K_1 + C_q)\right).$$

To show that W is a central resolving set, note that each vertex u_j^{ir} in a copy of C_q is adjacent only to its attachment vertex v_i^0 in P_n. Hence, any shortest path between vertices from different copies must pass through their respective attachment vertices, yielding distinct distance representations relative to W. Within each copy of C_q, the alternating vertices in the chosen basis B break the cycle's symmetry, ensuring that every vertex u_j^{ir} has a unique distance vector to B_i^r. Therefore, all vertices in $P_n \odot_k C_q$ are distinguishable with respect to W. For any distinct vertices $u, v \in V(P_n \odot_k C_q)$ where $u \neq v$, there are three possible cases: (1) $u, v \in W$; (2) $u \in W$ and $v \in V(P_n \odot_k C_q) \setminus W$; (3) $u, v \in V(P_n \odot_k C_q) \setminus W$. For cases (1) and (2), by Lemma 2.2, it is proven that $r(u \mid W) \neq r(v \mid W)$. For case (3), there are three subcases:

1. $v_i^0 \in V(P_n^0)$ and $u_j^{yr} \in V((C_q)_y^r)$
Since $d(v_i^0, v_x^0) = s$, where $0 \leq s \leq n-1$, and $d(u_j^{yr}, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(P_n^0)$. Since every vertex u_j^{yr} is only adjacent to its attachment vertex v_y^0, any shortest path from v_i^0 to u_j^{yr} must pass through v_y^0, it follows that $d(v_i^0, u_j^{yr}) = d(v_i^0, v_x^0) + d(u_j^{yr}, v_y^0) = s+1$, thus $d(v_i^0, v_y^0) \leq d(v_i^0, v_x^0) \leq d(v_i^0, u_j^{yr})$. Thus, there exists at least one vertex $v_y^0 \in V(P_n \odot_k C_q) \setminus W$ for which $d(v_i^0, v_y^0) \neq d(u_j^{yr}, v_y^0)$, ensuring that their representation to W is distinct. Hence, for $v_i^0, u_j^{yr} \in V(P_n \odot_k C_q) \setminus W$, it holds that $r(v_i^0 \mid W) \neq r(u_j^{yr} \mid W)$.
2. $u_j^{xr} \in V((C_q)_x^r)$ and $u_j^{yr} \in V((C_q)_y^r)$ with $x \neq y$
Since $d(u_j^{xr}, v_x^0) = 1$ and $d(v_x^0, v_y^0) = s$, for every $v_x^0, v_y^0 \in V(P_n^0)$, where $0 \leq s \leq n-1$. $d(v_y^0, u_j^{yr}) = 1$, we know that $d(u_j^{xr}, v_y^0) = d(u_j^{xr}, v_x^0) + d(v_x^0, v_y^0) = 1+s$. Since any shortest path between u_j^{xr} and u_j^{yr} passes through v_y^0, we have $d(u_j^{xr}, u_j^{yr}) = d(u_j^{xr}, v_y)^0 + d(v_y^0, u_j^{yr}) = (1+s)+1 = 2+s$, thus $d(u_j^{xr}, v_x^0) \leq d(u_j^{xr}, v_y^0) \leq d(u_j^{xr}, u_j^{yr})$. Hence, for $u_j^{xr}, u_j^{yr} \in V(P_n \odot_k C_q) \setminus W$, it follows that $r(u_j^{xr} \mid W) \neq r(u_j^{yr} \mid W)$.
3. $u_j^{ir} \in V((C_q)_i^r)$, $u_j^{is} \in V((C_q)_i^s)$, $r \neq s$
For every $u_k^{ir} \in B_i^r$, $d(u_j^{ir}, u_k^{ir}) = 1$ whereas $d(u_j^{is}, u_k^{ir}) = 2$. Similarly, for every $u_k^{is} \in B_i^s$, $d(u_j^{is}, u_k^{is}) = 1$ whereas $d(u_j^{ir}, u_k^{is}) = 2$. Since $B_i^r, B_i^s \subseteq W$, for $u_j^{ir} \in V((C_q)_i^r)$ and $u_j^{is} \in V((C_q)_i^s)$ with $r \neq s$, it follows that $r(u_j^{ir} \mid W) \neq r(u_j^{is} \mid W)$.

Next, to prove that W is a central resolving set with minimal cardinality, suppose $T \subseteq V(P_n \odot_k C_q)$ contains the central set and $|T| < |W|$. Let $|T| = |W| - 1$, then there exist two vertices $u_j^{ir}, u_k^{ir} \in V((C_q)_i^r)$ such that $u_j^{ir}, u_k^{ir} \notin T$. Thus, $d(u_j^{ir} \mid v) = d(u_k^{ir} \mid v)$ for all $v \in B_i^r$, meaning the distances from u_j^{ir} and u_k^{ir} to all other vertices in B_i^r are identical. This contradicts the definition of a resolving set, so T is not a central resolving set. By Lemma 2.3, W is the central resolving set with minimal cardinality. Hence,

$$\dim_{\text{cen}}(P_n \odot_k C_q) = |S(P_n)| + kn(\dim(K_1 + C_q))$$

The following is an example of case that satisfies Theorem 2.6. Let P_3 be a path graph of order 3 as in Figure 1. Let C_4 be cycle graph of order 4. In Figure 2, there are two copies of the graph C_4. The k-corona graph $P_3 \odot_k C_4$ presented in Figure 3 is obtained from P_3 and C_4 with k=2. Let B be a basis of the graph C_4, $B = \{u_1, u_2, u_3\}$. Define $B_i^r = \{u_j^{ir} \mid u_j \in B\}$ for $r = 1, 2; i = 1, 2, 3\}$.

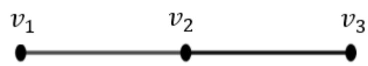

Figure 1. One copy of the graph P_3

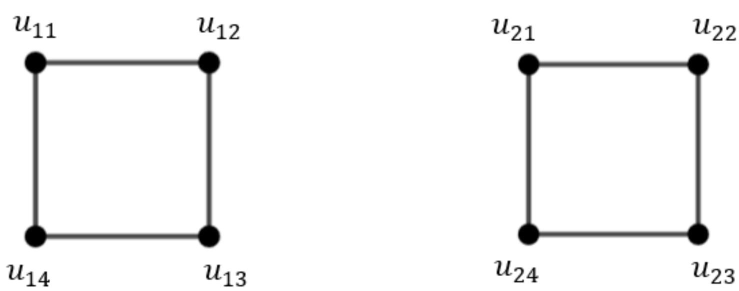

Figure 2. Two copies of the graph C_4

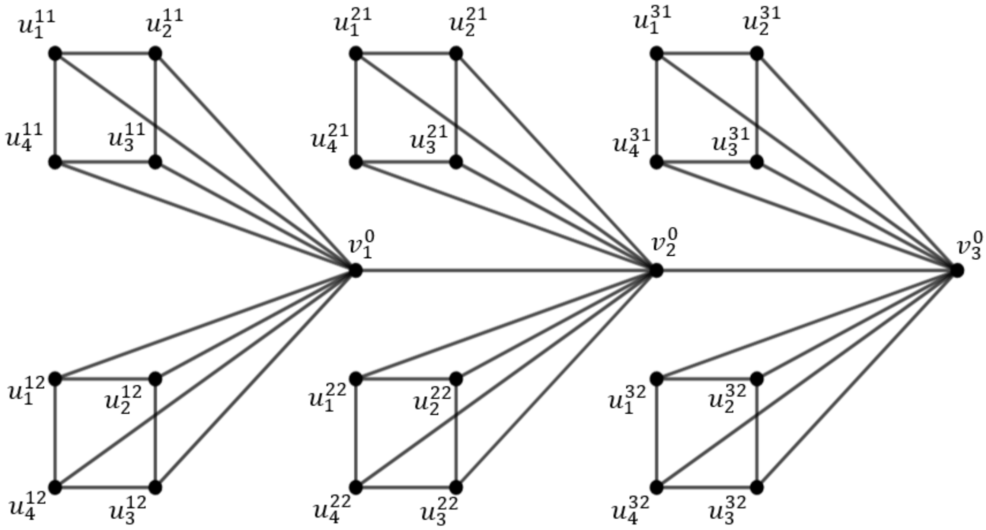

Figure 3. Structure of corona graph $P_3 \odot_k C_4$

Figure 3 shows that the central set of $P_3 \odot_k C_4$ is $S(P_3 \odot_2 C_4) = \{v_2^0\}$. We choose $W = S(P_3 \odot_2 C_4) \cup B_x^r$ or $W = \{v_2^0\} \cup \{u_j^{ir} \mid u_j \in B; r = 1, 2; i = 1, 2, 3\}$. It follows that $S(P_3 \odot_2 C_4) \subseteq W$. Therefore, it can be shown that W is a basis containing the central set of $P_3 \odot_2 C_4$, so that $|W| = |S(P_3)| + 2 \cdot 3 \cdot |B| = 19$.

Theorem 2.7. Let K_m be a complete graph and P_n be a path graph. Then

$$\dim_{\text{cen}}(K_m \odot_k P_n) = |S(K_m)| + km(\dim(K_1 + P_n)).$$

Proof. Let $V(K_m) = \{v_i \mid i = 1, 2, \ldots, m\}$ and $V(P_n) = \{u_j \mid j = 1, 2, \ldots, n\}$. The vertex set of $K_m \odot_k P_n$ is given by

$$V(K_m \odot_k P_n) = V(K_m^0) \cup \bigcup_{i=1}^{m} \left(\bigcup_{r=1}^{k} V((P_n)_i^r) \right),$$

where $V(K_m^0) = \{v_i^0 \mid v_i \in V(K_m)\}$ and $V((P_n)_i^r) = \{u_j^{ir} \mid u_j \in V(P_n); r = 1, \ldots, k; i = 1, \ldots, m\}$, for $(P_n)_i^r$ is the r-th copy of the path graph P_n at the i-th vertex of K_m. Let B be a basis of the graph $K_1 + P_n$. The structure of the central set and the metric basis of $K_1 + P_n$ depends on the parity of n. As an illustration, consider:

- $n = 3$. For P_3 with vertices u_1, u_2, u_3, a minimal metric basis for $K_1 + P_3$ is $B = \{u_1, u_3\}$: distances to u_1 and u_3 separate every pair of vertices in the join.
- $n = 4$. For P_4 with vertices u_1, u_2, u_3, u_4, a minimal metric basis for $K_1 + P_4$ is $B = \{u_1, u_3\}$. Distances to u_1 and u_3 distinguish all vertices of the joined graph.

In general one may choose

$$B = \begin{cases} \{u_1, u_3, \ldots, u_{n-2}, u_n\}, & n \text{ odd}, \\ \{u_1, u_3, \ldots, u_{n-3}, u_{n-1}\}, & n \text{ even}. \end{cases}$$

Define $B_i^r = \{u_j^{ir} \mid u_j \in B\}$. Define the set

$$W = \{s_i^0 \in V(K_m \odot_k P_n) \mid s_i \in S(K_m)\} \cup \left(\bigcup_{i=1}^{m} \bigcup_{r=1}^{k} B_i^r \right).$$

Based on Lemma 2.1, $S(K_m \odot_k P_n) \subseteq W$, and hence

$$|W| = |S(K_m)| + km\left(\dim(K_1 + P_n)\right).$$

To show that W is a central resolving set, observe that each vertex u_j^{ir} in a copy of P_n is adjacent only to its attachment vertex v_i^0 in K_m. Since K_m is complete, all attachment vertices are mutually adjacent, so any path between vertices from different copies passes through their respective attachments. This guarantees that all vertices of $K_m \odot_k P_n$ have distinct distance representations with respect to W.

For any distinct vertices $u, v \in V(K_m \odot_k P_n)$ where $u \neq v$, there are three possible cases: (1) $u, v \in W$; (2) $u \in W$ and $v \in V(K_m \odot_k P_n) \setminus W$; (3) $u, v \in V(K_m \odot_k P_n) \setminus W$. For cases (1) and (2), by Lemma 2.2, it is proven that $r(u \mid W) \neq r(v \mid W)$. For case (3), there are three subcases:

1. $v_i^0 \in V(K_m^0)$ and $u_j^{yr} \in V((P_n)_y^r)$
   Since $d(v_i^0, v_x^0) = 1$, and $d(u_j^{yr}, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(K_m^0)$. Since every vertex u_j^{yr} is only adjacent to its attachment vertex v_y^0, any shortest path from v_i^0 to u_j^{yr} must pass through v_y^0, it follows that $d(v_i^0, u_j^{yr}) = d(v_i^0, v_x^0) + d(u_j^{yr}, v_y^0) = 1 + 1 = 2$, thus $d(v_i^0, v_y^0) = d(v_i^0, v_x^0) \leq d(v_i^0, u_j^{yr})$. Thus, there exists at least one vertex $v_y^0 \in V(K_m \odot_k P_n) \setminus W$ for which $d(v_i^0, v_y^0) \neq d(u_j^{yr}, v_y^0)$, ensuring that their representation to W is distinct. Hence, for $v_i^0, u_j^{yr} \in V(K_m \odot_k P_n) \setminus W$, it holds that $r(v_i^0 \mid W) \neq r(u_j^{yr} \mid W)$.
2. $u_j^{xr} \in V((P_n)_x^r)$ and $u_j^{yr} \in V((P_n)_y^r)$ with $x \neq y$
   Since $d(u_j^{xr}, v_x^0) = 1$ and $d(v_x^0, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(K_m^0)$. $d(v_y^0, u_j^{yr}) = 1$, we know that $d(u_j^{xr}, v_y^0) = d(u_j^{xr}, v_x^0) + d(v_x^0, v_y^0) = 1 + s$. Since any shortest path between u_j^{xr} and u_j^{yr} passes through v_y^0, we have $d(u_j^{xr}, u_j^{yr}) = d(u_j^{xr}, v_y)^0 + d(v_y^0, u_j^{yr}) = (1 + s) + 1 = 2 + s$, thus $d(u_j^{xr}, v_x^0) \leq d(u_j^{xr}, v_y^0) \leq d(u_j^{xr}, u_j^{yr})$. Hence, for $u_j^{xr}, u_j^{yr} \in V(K_m \odot_k P_n) \setminus W$, it follows that $r(u_j^{xr} \mid W) \neq r(u_j^{yr} \mid W)$.
3. $u_j^{ir} \in V((P_n)_i^r)$, $u_j^{is} \in V((P_n)_i^s)$, $r \neq s$
   For every $u_k^{ir} \in B_i^r$, $d(u_j^{ir}, u_k^{ir}) = 1$ whereas $d(u_j^{is}, u_k^{ir}) = 2$. Similarly, for every $u_k^{is} \in B_i^s$, $d(u_j^{is}, u_k^{is}) = 1$ whereas $d(u_j^{ir}, u_k^{is}) = 2$. Since $B_i^r, B_i^s \subseteq W$, for $u_j^{ir} \in V((P_n)_i^r)$ and $u_j^{is} \in V((P_n)_i^s)$ with $r \neq s$, it follows that $r(u_j^{ir} \mid W) \neq r(u_j^{is} \mid W)$.

Next, to prove that W is a central resolving set with minimal cardinality, suppose $T \subseteq V(K_m \odot_k P_n)$ contains the central set and $|T| < |W|$. Let $|T| = |W| - 1$, then there exist two vertices $u_j^{ir}, u_k^{ir} \in V((P_n)_i^r)$ such that $u_j^{ir}, u_k^{ir} \notin T$. Thus, $d(u_j^{ir} \mid v) = d(u_k^{ir} \mid v)$ for all $v \in B_i^r$, meaning the distances from u_j^{ir} and u_k^{ir} to all other vertices in B_i^r are identical. This contradicts the definition of a resolving set, so T is not a central resolving set. By Lemma 2.3, W is the central resolving set with minimal cardinality. Hence,

$$\dim_{\text{cen}}(K_m \odot_k P_n) = |S(K_m)| + km(\dim(K_1 + P_n))$$

Theorem 2.8. Let S_n be a star graph and C_q be a cycle graph. Then

$$\dim_{\text{cen}}(S_n \odot_k C_q) = |S(S_n)| + kn(\dim(K_1 + C_q))$$

Proof. Let $V(S_n) = \{v_i \mid i = 1, 2, \ldots, n\}$ and $V(C_q) = \{u_j \mid j = 1, 2, \ldots, q\}$. The vertex set of $S_n \odot_k C_q$ is given by

$$V(S_n \odot_k C_q) = V(S_n^0) \cup \bigcup_{i=1}^{n} \left(\bigcup_{r=1}^{k} V((C_q)_i^r) \right),$$

where $V(S_n^0) = \{v_i^0 \mid v_i \in V(S_n)\}$ and $V((C_q)_i^r) = \{u_j^{ir} \mid u_j \in V(C_q); r = 1, \ldots, k; i = 1, \ldots, n\}$, for $(C_q)_i^r$ is the r-th copy of the cycle graph C_q at the i-th vertex of S_n. Let B be a basis of the graph $K_1 + C_q$. We choose

$$B = \begin{cases} \{u_1, u_3, \ldots, u_{q-4}, u_{q-2}\}, & q \text{ odd}, \\ \{u_1, u_3, \ldots, u_{q-3}, u_{q-1}\}, & q \text{ even}. \end{cases}$$

Define $B_i^r = \{u_j^{ir} \mid u_j \in B\}$. Define the set

$$W = \{s_i^0 \in V(S_n \odot_k C_q) \mid s_i \in S(S_n)\} \cup \left(\bigcup_{i=1}^{n} \bigcup_{r=1}^{k} B_i^r \right).$$

Based on Lemma 2.1, $S(S_n \odot_k C_q) \subseteq W$, and hence

$$|W| = |S(S_n)| + kn\left(\dim(K_1 + C_q)\right).$$

To show that W is indeed a central resolving set, we verify that any two distinct vertices in $S_n \odot_k C_q$ have different distance representations with respect to W. Each vertex u_j^{ir} in a copy of C_q is connected only to the central vertex v_i^0 of S_n. Therefore, any shortest path between two vertices belonging to different copies of C_q must pass through their respective attachment vertices v_i^0 and v_y^0. This property guarantees that vertices from different copies have distinct representations to the elements of W, because the distances to central vertices $v_i^0 \in S(S_n)$ differ depending on their positions along the path S_n. Within each copy of C_q, the cycle structure introduces symmetry, but the chosen basis B—comprising alternating vertices along the cycle—breaks this symmetry by ensuring that the distance from any vertex u_j^{ir} to B_i^r is unique. In particular, adjacent vertices on the cycle have distance 1, their second neighbors have distance 2, and so on, wrapping around the cycle modulo q. Since each u_j^{ir} has a distinct pattern of distances to B_i^r, it follows that all vertices in the same copy of C_q are distinguishable with respect to W. For any distinct vertices $u, v \in V(S_n \odot_k C_q)$ where $u \neq v$, there are three possible cases: (1) $u, v \in W$; (2) $u \in W$ and $v \in V(S_n \odot_k C_q) \setminus W$; (3) $u, v \in V(S_n \odot_k C_q) \setminus W$. For cases (1) and (2), by Lemma 2.2, it is proven that $r(u \mid W) \neq r(v \mid W)$. For case (3), there are three subcases:

1. $v_i^0 \in V(S_n^0)$ and $u_j^{yr} \in V((C_q)_y^r)$
   Since $d(v_i^0, v_x^0) = s$, where $0 \leq s \leq 2$, and $d(u_j^{yr}, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(S_n^0)$. Since every vertex u_j^{yr} is only adjacent to its attachment vertex v_y^0, any shortest path from v_i^0 to u_j^{yr} must pass through v_y^0, it follows that $d(v_i^0, u_j^{yr}) = d(v_i^0, v_x^0) + d(u_j^{yr}, v_y^0) = s + 1$, thus $d(v_i^0, v_y^0) \leq d(v_i^0, v_x^0) \leq d(v_i^0, u_j^{yr})$. Thus, there exists at least one vertex $v_y^0 \in V(S_n \odot_k C_q) \setminus W$ for which $d(v_i^0, v_y^0) \neq d(u_j^{yr}, v_y^0)$, ensuring that their representation to W is distinct. Hence, for $v_i^0, u_j^{yr} \in V(S_n \odot_k C_q) \setminus W$, it holds that $r(v_i^0 \mid W) \neq r(u_j^{yr} \mid W)$.
2. $u_j^{xr} \in V((C_q)_x^r)$ and $u_j^{yr} \in V((C_q)_y^r)$ with $x \neq y$
   Since $d(u_j^{xr}, v_x^0) = 1$ and $d(v_x^0, v_y^0) = s$, for every $v_x^0, v_y^0 \in V(S_n^0)$, where $0 \leq s \leq 2$. $d(v_y^0, u_j^{yr}) = 1$, we know that $d(u_j^{xr}, v_y^0) = d(u_j^{xr}, v_x^0) + d(v_x^0, v_y^0) = 1 + s$. Since any shortest path between u_j^{xr} and u_j^{yr} passes through v_y^0, we have $d(u_j^{xr}, u_j^{yr}) = d(u_j^{xr}, v_y)^0 + d(v_y^0, u_j^{yr}) = (1+s) + 1 = 2 + s$, thus $d(u_j^{xr}, v_x^0) \leq d(u_j^{xr}, v_y^0) \leq d(u_j^{xr}, u_j^{yr})$. Hence, for $u_j^{xr}, u_j^{yr} \in V(S_n \odot_k C_q) \setminus W$, it follows that $r(u_j^{xr} \mid W) \neq r(u_j^{yr} \mid W)$.
3. $u_j^{ir} \in V((C_q)_i^r)$, $u_j^{is} \in V((C_q)_i^s)$, $r \neq s$
   For every $u_k^{ir} \in B_i^r$, $d(u_j^{ir}, u_k^{ir}) = 1$ whereas $d(u_j^{is}, u_k^{ir}) = 2$. Similarly, for every $u_k^{is} \in B_i^s$, $d(u_j^{is}, u_k^{is}) = 1$ whereas $d(u_j^{ir}, u_k^{is}) = 2$. Since $B_i^r, B_i^s \subseteq W$, for $u_j^{ir} \in V((C_q)_i^r)$ and $u_j^{is} \in V((C_q)_i^s)$ with $r \neq s$, it follows that $r(u_j^{ir} \mid W) \neq r(u_j^{is} \mid W)$.

Next, to prove that W is a central resolving set with minimal cardinality, suppose $T \subseteq V(S_n \odot_k C_q)$ contains the central set and $|T| < |W|$. Let $|T| = |W| - 1$, then there exist two vertices $u_j^{ir}, u_k^{ir} \in V((C_q)_i^r)$ such that $u_j^{ir}, u_k^{ir} \notin T$. Thus, $d(u_j^{ir} \mid v) = d(u_k^{ir} \mid v)$ for all $v \in B_i^r$, meaning the distances from u_j^{ir} and u_k^{ir} to all other vertices in B_i^r are identical. This contradicts the definition of a resolving set, so T is not a central resolving set. By Lemma 2.3, W is the central resolving set with minimal cardinality. Hence,

$$\dim_{\text{cen}}(S_n \odot_k C_q) = |S(S_n)| + kn(\dim(K_1 + C_q))$$

Theorem 2.9. Let S_n be a star graph and K_m be a complete graph. Then

$$\dim_{\text{cen}}(S_n \odot_k K_m) = |S(S_n)| + kn\left(\dim(K_m)\right)$$

Proof. Let $V(S_n) = \{v_i \mid i = 1, 2, \ldots, n\}$ and $V(K_m) = \{u_j \mid j = 1, 2, \ldots, m\}$. The vertex set of $S_n \odot_k K_m$ is given by

$$V(S_n \odot_k K_m) = V(S_n^0) \cup \bigcup_{i=1}^{n} \left(\bigcup_{r=1}^{k} V((K_m)_i^r) \right),$$

where $V(S_n^0) = \{v_i^0 \mid v_i \in V(S_n)\}$ and $V((K_m)_i^r) = \{u_j^{ir} \mid u_j \in V(K_m); r = 1, \ldots, k; i = 1, \ldots, n\}$, for $(K_m)_i^r$ is the r-th copy of the complete graph K_m at the i-th vertex of S_n. Let $B = \{u_1, u_2, \ldots, u_{m-2}, u_{m-1}\}$ be a basis of K_m, and define $B_i^r = \{u_j^{ir} \mid u_j \in B\}$. Define the set

$$W = \{s_i^0 \in V(S_n \odot_k K_m) \mid s_i \in S(S_n)\} \cup \left(\bigcup_{i=1}^{n} \bigcup_{r=1}^{k} B_i^r \right).$$

Based on Lemma 2.1, $S(S_n \odot_k K_m) \subseteq W$, and hence

$$|W| = |S(S_n)| + kn\left(\dim(K_m)\right).$$

To show that W is a central resolving set, note that each vertex in a copy of K_m is adjacent only to the central vertex of its corresponding S_n. Consequently, any shortest path between vertices in different copies of K_m must pass through their respective central vertices in S_n, which guarantees distinct distance representations across copies. Within each copy of K_m, the inclusion of the local basis B_i^r ensures that all v For any distinct vertices $u, v \in V(S_n \odot_k K_m)$ where $u \neq v$, there are three possible cases: (1) $u, v \in W$; (2) $u \in W$ and $v \in V(S_n \odot_k K_m) \setminus W$; (3) $u, v \in V(S_n \odot_k K_m) \setminus W$. For cases (1) and (2), by Lemma 2.2, it is proven that $r(u \mid W) \neq r(v \mid W)$. For case (3), there are three subcases:

1. $v_i^0 \in V(S_n^0)$ and $u_j^{yr} \in V((K_m)_y^r)$
   Since $d(v_i^0, v_x^0) = s$, where $0 \leq s \leq 2$, and $d(u_j^{yr}, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(S_n^0)$. Since every vertex u_j^{yr} is only adjacent to its attachment vertex v_y^0, any shortest path from v_i^0 to u_j^{yr} must pass through v_y^0, it follows that $d(v_i^0, u_j^{yr}) = d(v_i^0, v_x^0) + d(u_j^{yr}, v_y^0) = s + 1$, thus $d(v_i^0, v_y^0) \leq d(v_i^0, v_x^0) \leq d(v_i^0, u_j^{yr})$. Thus, there exists at least one vertex $v_y^0 \in V(S_n \odot_k K_m) \setminus W$ for which $d(v_i^0, v_y^0) \neq d(u_j^{yr}, v_y^0)$, ensuring that their representation to W is distinct. Hence, for $v_i^0, u_j^{yr} \in V(S_n \odot_k K_m) \setminus W$, it holds that $r(v_i^0 \mid W) \neq r(u_j^{yr} \mid W)$.
2. $u_j^{xr} \in V((K_m)_x^r)$ and $u_j^{yr} \in V((K_m)_y^r)$ with $x \neq y$
   Since $d(u_j^{xr}, v_x^0) = 1$ and $d(v_x^0, v_y^0) = s$, for every $v_x^0, v_y^0 \in V(S_n^0)$, where $0 \leq s \leq 2$. $d(v_y^0, u_j^{yr}) = 1$, we know that $d(u_j^{xr}, v_y^0) = d(u_j^{xr}, v_x^0) + d(v_x^0, v_y^0) = 1 + s$. Since any shortest path between u_j^{xr} and u_j^{yr} passes through v_y^0, we have $d(u_j^{xr}, u_j^{yr}) = d(u_j^{xr}, v_y)^0 + d(v_y^0, u_j^{yr}) = (1 + s) + 1 = 2 + s$, thus $d(u_j^{xr}, v_x^0) \leq d(u_j^{xr}, v_y^0) \leq d(u_j^{xr}, u_j^{yr})$. Hence, for $u_j^{xr}, u_j^{yr} \in V(S_n \odot_k K_m) \setminus W$, it follows that $r(u_j^{xr} \mid W) \neq r(u_j^{yr} \mid W)$.
3. $u_j^{ir} \in V((K_m)_i^r)$, $u_j^{is} \in V((K_m)_i^s)$, $r \neq s$
   For every $u_k^{ir} \in B_i^r$, $d(u_j^{ir}, u_k^{ir}) = 1$ whereas $d(u_j^{is}, u_k^{ir}) = 2$. Similarly, for every $u_k^{is} \in B_i^s$, $d(u_j^{is}, u_k^{is}) = 1$ whereas $d(u_j^{ir}, u_k^{is}) = 2$. Since $B_i^r, B_i^s \subseteq W$, for $u_j^{ir} \in V((K_m)_i^r)$ and $u_j^{is} \in V((K_m)_i^s)$ with $r \neq s$, it follows that $r(u_j^{ir} \mid W) \neq r(u_j^{is} \mid W)$.

Next, to prove that W is a central resolving set with minimal cardinality, suppose $T \subseteq V(S_n \odot_k K_m)$ contains the central set and $|T| < |W|$. Let $|T| = |W| - 1$, then there exist two vertices $u_j^{ir}, u_k^{ir} \in V((K_m)_i^r)$ such that $u_j^{ir}, u_k^{ir} \notin T$. Thus, $d(u_j^{ir} \mid v) = d(u_k^{ir} \mid v)$ for all $v \in B_i^r$, meaning the distances from u_j^{ir} and u_k^{ir} to all other vertices in B_i^r are identical. This contradicts the definition of a resolving set, so T is not a central resolving set. By Lemma 2.3, W is the central resolving set with minimal cardinality. Hence,

$$\dim_{\text{cen}}(S_n \odot_k K_m) = |S(S_n)| + kn\left(\dim(K_m)\right).$$

Theorem 2.10. Let G be a connected graph and H be a graph of order at least two. If H has a basis B such that there is no vertex $v \in V(G \odot_k H)$ with $r(v|B) = (2, 2, 2, \ldots, 2)$, then

$$\dim_{\text{cen}}(G \odot_k H) = \begin{cases} |S(G)| + k|V(G)|(\dim(H)), & \text{if } H \text{ is a complete graph} \\ |S(G)| + k|V(G)|(\dim(K_1 + H)), & \text{otherwise} \end{cases}$$

Proof. Let G be a connected graph with $V(G) = \{v_i \mid i = 1, 2, 3, \ldots, n\}$. Let H be a graph of order at least two with $V(H) = \{u_j \mid j = 1, 2, 3, \ldots, m\}$. The vertex set of $G \odot_k H$ is given by

$$V(G \odot_k H) = V(G_n^0) \cup \bigcup_{i=1}^{n} \left(\bigcup_{r=1}^{k} V(H_i^r) \right),$$

where $V(G_n^0) = \{v_i^0 \mid v_i \in V(G)\}$ and $V(H_i^r) = \{u_j^{ir} \mid u_j \in V(H); r = 1, \ldots, k; i = 1, \ldots, n\}$, for H_i^r represents the r-th copy of the graph H attached to the i-th vertex of graph G. There are two possible cases in determining the central metric dimension of the graph, when H is a complete graph and otherwise.

Case 1: For H is a complete graph, let B be a basis of the graph H that there is no vertex $v \in V(G \odot_k H)$ with $r(v|B) = (2, 2, 2, \ldots, 2)$ and $B_i^r = \{u_j^{ir} \mid u_j \in B\}$. for $r = 1, 2, 3, \ldots, k$; $i = 1, 2, 3, \ldots, n$. Define the set

$$W = \{s_i^0 \in V(G \odot_k H) \mid s_i \in S(G)\} \cup B_i^r,$$

Based on Lemma 2.1, $S(G \odot_k H) \subseteq W$, and hence

$$|W| = |S(G)| + kn\,(\dim(H))\,.$$

To confirm that W is a central resolving set, observe that each vertex in a copy of H is adjacent only to its attachment vertex v_i^0 in G. Therefore, any shortest path between vertices belonging to different copies of H must pass through their respective attachment vertices. Since the vertices of G themselves have distinct distance patterns relative to $S(G)$, and every copy of H contributes its local basis B_i^r, it follows that all vertices of $G \odot_k H$ are uniquely represented with respect to W. For any distinct vertices $u, v \in V(G \odot_k H)$ where $u \neq v$, there are three possible cases: (1) $u, v \in W$; (2) $u \in W$ and $v \in V(G \odot_k H) \setminus W$; (3) $u, v \in V(G \odot_k H) \setminus W$. For cases (1) and (2), by Lemma 2.2, it is proven that $r(u \mid W) \neq r(v \mid W)$. For case (3), there are three subcases:

1. $v_i^0 \in V(G_n^0)$ and $u_j^{yr} \in V((H)_y^r)$
   Since $d(v_i^0, v_x^0) = s$, where $0 \leq s \leq n-1$, and $d(u_j^{yr}, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(G_n^0)$. Since every vertex u_j^{yr} is only adjacent to its attachment vertex v_y^0, any shortest path from v_i^0 to u_j^{yr} must pass through v_y^0, it follows that $d(v_i^0, u_j^{yr}) = d(v_i^0, v_x^0) + d(u_j^{yr}, v_y^0) = s + 1$, thus $d(v_i^0, v_y^0) \leq d(v_i^0, v_x^0) \leq d(v_i^0, u_j^{yr})$. Thus, there exists at least one vertex $v_y^0 \in V(G \odot_k H) \setminus W$ for which $d(v_i^0, v_y^0) \neq d(u_j^{yr}, v_y^0)$, ensuring that their representation to W is distinct. Hence, for $v_i^0, u_j^{yr} \in V(G \odot_k H) \setminus W$, it holds that $r(v_i^0 \mid W) \neq r(u_j^{yr} \mid W)$.
2. $u_j^{xr} \in V((H)_x^r)$ and $u_j^{yr} \in V((H)_y^r)$ with $x \neq y$
   Since $d(u_j^{xr}, v_x^0) = 1$ and $d(v_x^0, v_y^0) = s$, for every $v_x^0, v_y^0 \in V(G_n^0)$, where $0 \leq s \leq n-1$. $d(v_y^0, u_j^{yr}) = 1$, we know that $d(u_j^{xr}, v_y^0) = d(u_j^{xr}, v_x^0) + d(v_x^0, v_y^0) = 1 + s$. Since any shortest path between u_j^{xr} and u_j^{yr} passes through v_y^0, we have $d(u_j^{xr}, u_j^{yr}) = d(u_j^{xr}, v_y)^0 + d(v_y^0, u_j^{yr}) = (1+s) + 1 = 2 + s$, thus $d(u_j^{xr}, v_x^0) \leq d(u_j^{xr}, v_y^0) \leq d(u_j^{xr}, u_j^{yr})$. Hence, for $u_j^{xr}, u_j^{yr} \in V(G \odot_k H) \setminus W$, it follows that $r(u_j^{xr} \mid W) \neq r(u_j^{yr} \mid W)$.
3. $u_j^{ir} \in V((H)_i^r)$, $u_j^{is} \in V((H)_i^s)$, $r \neq s$
   For every $u_k^{ir} \in B_i^r$, $d(u_j^{ir}, u_k^{ir}) = 1$ whereas $d(u_j^{is}, u_k^{ir}) = 2$. Similarly, for every $u_k^{is} \in B_i^s$, $d(u_j^{is}, u_k^{is}) = 1$ whereas $d(u_j^{ir}, u_k^{is}) = 2$. Since $B_i^r, B_i^s \subseteq W$, for $u_j^{ir} \in V((H)_i^r)$ and $u_j^{is} \in V((H)_i^s)$ with $r \neq s$, it follows that $r(u_j^{ir} \mid W) \neq r(u_j^{is} \mid W)$.

Next, to prove that W is a central resolving set with minimal cardinality, suppose $T \subseteq V(G \odot_k H)$ contains the central set and $|T| < |W|$. Let $|T| = |W| - 1$, then there exist two vertices $u_j^{ir}, u_k^{ir} \in V((H)_i^r)$ such that

$u_j^{ir}, u_k^{ir} \notin T$. Thus, $d(u_j^{ir} \mid v) = d(u_k^{ir} \mid v)$ for all $v \in B_i^r$, meaning the distances from u_j^{ir} and u_k^{ir} to all other vertices in B_i^r are identical. This contradicts the definition of a resolving set, so T is not a central resolving set. By Lemma 2.3, W is the central resolving set with minimal cardinality. Hence,

$$\dim_{\text{cen}}(G \odot_k H) = |S(G)| + k|V(G)| \left(\dim(H)\right).$$

Case 2: For H is not a complete graph, let B be a basis of the graph H . Let $\mathcal{B}$ be the basis of the graph $K_1 + H$ and $\mathcal{B}_i^r = \{u_j^{ir} \mid u_j \in \mathcal{B}\}$. for $r = 1, 2, 3, \ldots, k$; $i = 1, 2, 3, \ldots, n$. Define the set

$$W = \{s_i^0 \in V(G \odot_k H) \mid s_i \in S(G)\} \cup \mathcal{B}_i^r,$$

Based on Lemma 2.1, $S(G \odot_k H) \subseteq W$, such that there is no vertex $v \in V(G \odot_k H)$ with $r(v \mid \mathcal{B}) = (2, 2, 2, \ldots, 2)$. Hence

$$|W| = |S(G)| + kn \left(\dim(K_1 + H)\right).$$

To verify that W is a central resolving set, note that for each copy of H, all vertices are connected only through their corresponding attachment vertex v_i^0. Thus, vertices from different copies have distinct distance patterns with respect to $S(G)$, while within the same copy, the inclusion of the local basis $\mathcal{B}_i^r$—derived from $K_1 + H$—ensures that every vertex in H_i^r is uniquely identified. Consequently, W distinguishes all vertices of $G \odot_k H$. For any distinct vertices $u, v \in V(G \odot_k H)$ where $u \neq v$, there are three possible cases: (1) $u, v \in W$; (2) $u \in W$ and $v \in V(G \odot_k H) \setminus W$; (3) $u, v \in V(G \odot_k H) \setminus W$. For cases (1) and (2), by Lemma 2.2, it is proven that $r(u \mid W) \neq r(v \mid W)$. For case (3), there are three subcases:

1. $v_i^0 \in V(G_n^0)$ and $u_j^{yr} \in V((H)_y^r)$
   Since $d(v_i^0, v_x^0) = s$, where $0 \leq s \leq n-1$, and $d(u_j^{yr}, v_y^0) = 1$, for every $v_x^0, v_y^0 \in V(G_n^0)$. Since every vertex u_j^{yr} is only adjacent to its attachment vertex v_y^0, any shortest path from v_i^0 to u_j^{yr} must pass through v_y^0, it follows that $d(v_i^0, u_j^{yr}) = d(v_i^0, v_x^0) + d(u_j^{yr}, v_y^0) = s + 1$, thus $d(v_i^0, v_y^0) \leq d(v_i^0, v_x^0) \leq d(v_i^0, u_j^{yr})$. Thus, there exists at least one vertex $v_y^0 \in V(G \odot_k H) \setminus W$ for which $d(v_i^0, v_y^0) \neq d(u_j^{yr}, v_y^0)$, ensuring that their representation to W is distinct. Hence, for $v_i^0, u_j^{yr} \in V(G \odot_k H) \setminus W$, it holds that $r(v_i^0 \mid W) \neq r(u_j^{yr} \mid W)$.
2. $u_j^{xr} \in V((H)_x^r)$ and $u_j^{yr} \in V((H)_y^r)$ with $x \neq y$
   Since $d(u_j^{xr}, v_x^0) = 1$ and $d(v_x^0, v_y^0) = s$, for every $v_x^0, v_y^0 \in V(G_n^0)$, where $0 \leq s \leq n-1$. $d(v_y^0, u_j^{yr}) = 1$, we know that $d(u_j^{xr}, v_y^0) = d(u_j^{xr}, v_x^0) + d(v_x^0, v_y^0) = 1 + s$. Since any shortest path between u_j^{xr} and u_j^{yr} passes through v_y^0, we have $d(u_j^{xr}, u_j^{yr}) = d(u_j^{xr}, v_y)^0 + d(v_y^0, u_j^{yr}) = (1+s) + 1 = 2 + s$, thus $d(u_j^{xr}, v_x^0) \leq d(u_j^{xr}, v_y^0) \leq d(u_j^{xr}, u_j^{yr})$. Hence, for $u_j^{xr}, u_j^{yr} \in V(G \odot_k H) \setminus W$, it follows that $r(u_j^{xr} \mid W) \neq r(u_j^{yr} \mid W)$.
3. $u_j^{ir} \in V((H)_i^r)$, $u_j^{is} \in V((H)_i^s)$, $r \neq s$
   For every $u_k^{ir} \in \mathcal{B}_i^r$, $d(u_j^{ir}, u_k^{ir}) = 1$ whereas $d(u_j^{is}, u_k^{ir}) = 2$. Similarly, for every $u_k^{is} \in \mathcal{B}_i^s$, $d(u_j^{is}, u_k^{is}) = 1$ whereas $d(u_j^{ir}, u_k^{is}) = 2$. Since $\mathcal{B}_i^r, \mathcal{B}_i^s \subseteq W$, for $u_j^{ir} \in V((H)_i^r)$ and $u_j^{is} \in V((H)_i^s)$ with $r \neq s$, it follows that $r(u_j^{ir} \mid W) \neq r(u_j^{is} \mid W)$.

Next, to prove that W is a central resolving set with minimal cardinality, suppose $T \subseteq V(G \odot_k H)$ contains the central set and $|T| < |W|$. Let $|T| = |W| - 1$, then there exist two vertices $u_j^{ir}, u_k^{ir} \in V((H)_i^r)$ such that $u_j^{ir}, u_k^{ir} \notin T$. Thus, $d(u_j^{ir} \mid v) = d(u_k^{ir} \mid v)$ for all $v \in \mathcal{B}_i^r$, meaning the distances from u_j^{ir} and u_k^{ir} to all other vertices in $\mathcal{B}_i^r$ are identical. This contradicts the definition of a resolving set, so T is not a central resolving set. By Lemma 2.3, W is the central resolving set with minimal cardinality. Hence,

$$\dim_{\text{cen}}(G \odot_k H) = |S(G)| + k|V(G)| \left(\dim(K_1 + H)\right).$$

Remark. The condition $r(v \mid B) \neq (2, 2, \ldots, 2)$ in Theorem 2.10 is essential to ensure that the resolving set W distinguishes all vertices in $G \odot_k H$. If a vertex $v \in V(H)$ satisfies $r(v \mid B) = (2, 2, \ldots, 2)$, then v is located at distance 2 from every vertex of the basis B. Consequently, in the corona operation, every copy of such a vertex v

will have the same distance pattern to all vertices of W. As a result, two distinct copies attached to the same central vertex will share identical distances with respect to W, and hence W fails to resolve the graph.

As an example, let S_n be a star graph of order n with one central vertex u_1 and $n-1$ leaf vertices $u_2, u_3, \ldots, u_n$. Each leaf u_i is adjacent only to the central vertex u_1. Consider any connected graph G of order m and form the corona product $G \odot_k S_n$. Let $B = \{u_2, u_3, \ldots, u_{n-1}\}$ be a basis of S_n. Define $B_i^r = \{ u_j^{ir} \mid u_j \in B \}$ for $r = 1, 2, \ldots, k$ and $i = 1, 2, \ldots, m$. We choose $W = S(G \odot_k S_n) \cup B_i^r$ as a candidate resolving set. In this structure, there exist distinct leaf copies u_j^{ir} and u_j^{is} (attached to the same central vertex v_i^0) such that $d(u_j^{ir}, v_i^0) = d(u_j^{is}, v_i^0)$ for every $v_i^0 \in W$. Consequently, the vertices u_j^{ir} and u_j^{is} cannot be distinguished by W, since $r(u_j^{ir} \mid W) = r(u_j^{is} \mid W) = (2, 2, \ldots, 2)$, and hence W fails to resolve the graph. Therefore, the restriction $r(v \mid B) \neq (2, 2, \ldots, 2)$ is crucial to prevent such a situation and to ensure that every vertex in $G \odot_k H$ has a unique distance representation with respect to W.

3. Conclusion and Suggestions

Based on this research, the central metric dimension of k-corona graphs is determined by the central set of G, the order of G, the value of k, and the metric dimension of H. The findings present a generalized case of the central metric dimension for k-corona graphs.

Each type of branch graph contributes differently to the structure of the resulting k-corona graph. For $H = S_n$, the presence of a universal vertex in each copy adds one additional element to the central resolving set. For $H = P_n$ and $H = C_q$, the parity of n or q affects both the formation of the central set and the selection of the metric basis. When the order is odd, one additional central vertex appears, while for even order, the size of the central set remains the same. For $H = K_m$, every vertex is adjacent to all others, resulting in equal eccentricities across vertices. This makes each copy of K_m highly symmetric, and the central metric dimension mainly depends on the metric dimension of K_m.

These differences produce variations in the formula of the central metric dimension. The concept of the central set can further be applied to determine strategic public service locations that are accessible from all regions through efficient transportation routes. For future research, concepts related to the metric dimension and the diameter of graphs can be extended and analyzed in other graph operations or in dynamic network models.

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REFERENCES

1. Chartrand, G. & Lesniak, L., 2016, *Graphs and Digraphs* (6th ed.), Chapman & Hall/CRC, Florida.
2. Saputro, S. W., Mardiana, N., & Purwasih, I. A., 2017, The metric dimension of comb product graphs, *Matematicki Vesnik*, **69**(4), 248–258.
3. Susilowati, L., Slamin, I., Utoyo, M. I., & Estuningsih, N., 2015, The similarity of metric dimension of rooted product graph, *Far East Journal of Mathematical Sciences*, **97**(7), 841–856.
4. Susilowati, L., Utoyo, M. I., & Slamin, I., 2016, On commutative characterization of generalized comb and corona products of graphs with respect to the local metric dimension, *Digital Repository Universitas Jember*, 647.
5. Prabhu, S., Janany, T. J., & Klavžar, S., 2025, Metric dimensions of generalized Sierpiński graphs over squares, *Applied Mathematics and Computation*, **505**, 129528.

6. Prabhu, S., Janany, T. J., & Manuel, P., 2025, An efficient way to represent the processors and their connections in omega networks, *Ain Shams Engineering Journal*, **16**, 103287.
7. Prabhu, S., Manimozhi, V., Davoodi, A., & Guirao, J. L. G., 2024, Fault-tolerant basis of generalized fat trees and perfect binary tree derived architectures, *The Journal of Supercomputing*, **80**, 15783–15798.
8. Prabhu, S., Jeba, D. S. R., Arulperumjothi, M., & Klavžar, S., 2024, Metric dimension of irregular convex triangular networks, *AKCE International Journal of Graphs and Combinatorics*, **21**, 225–231.
9. Arulperumjothi, M., Klavžar, S., & Prabhu, S., 2023, Redefining fractal cubic networks and determining their metric dimension and fault-tolerant metric dimension, *Applied Mathematics and Computation*, **452**, 128037.
10. Prabhu, S., Manimozhi, V., Arulperumjothi, M., & Klavžar, S., 2022, Twin vertices in fault-tolerant metric sets and fault-tolerant metric dimension of multistage interconnection networks, *Applied Mathematics and Computation*, **420**, 126897.
11. Sooryanarayana, B., Kunikullaya, S., & Swamy, N. N., 2019, Metric dimension of generalized wheels, *Arab Journal of Mathematical Sciences*, **25**(2), 131–144.
12. Susilowati, L., Slamin, I., Maulida, U., Estuningsih, N., Zahidah, S., & Prabhu, S., 2024, On the central resolver set of the edge koronation graphs, *Journal of Discrete Mathematical Sciences & Cryptography*, (in press).
13. Kuziak, D., Yero, I. G., & Rodríguez-Velázquez, J. A., 2011, On the metric dimension of corona product graphs, *Computers & Mathematics with Applications*, **61**(9), 2793–2798.
14. Hernando, C., Mora, M., Pelayo, I. M., Seara, C., & Wood, D. R., 2010, Extremal graph theory for metric dimension and diameter, *Electronic Journal of Combinatorics*, **17**(1), R30.
15. Chartrand, G. & Lesniak, L., 2000, *Graphs and Digraphs* (3rd ed.), Chapman & Hall/CRC, Florida.